Topological Defects in the Topological Insulator

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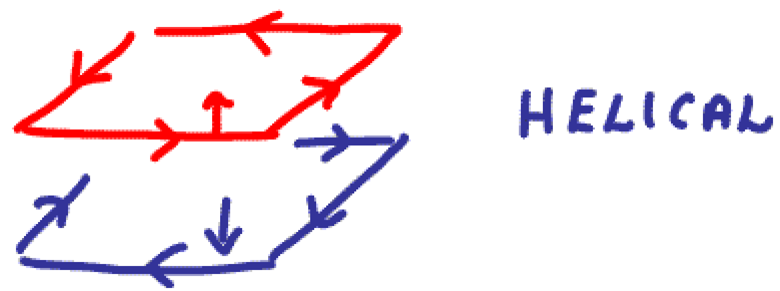
Exotic Band Topology

- Quantum Hall States

- ‘Topological’ band Insulators (quantum spin hall)



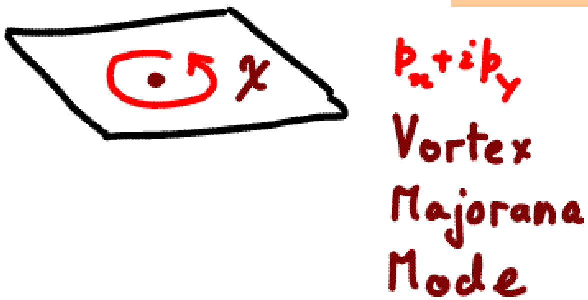
- Superconducting analogs
  - p+ip Superconductors
  - B-phase of He_3

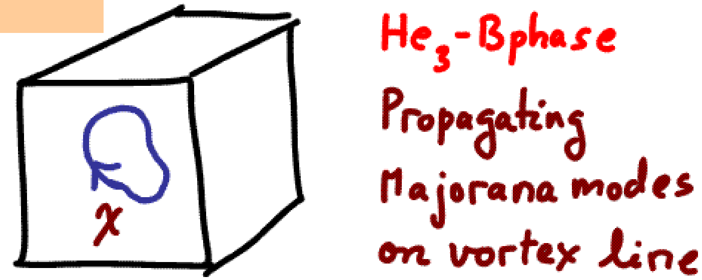
Unconventional Surface/Edge states.

“The edge is the window into the bulk” – X.G. Wen

Broken Symmetry + Exotic Band Topology

- Superconducting order parameter: $|\Psi_0| e^{i\varphi}$
- Vortex defects: $\oint \nabla \varphi \cdot dr = 2\pi m$



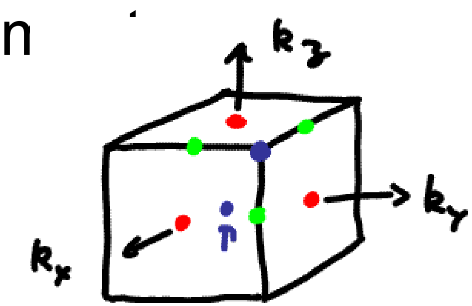


- Crystalline Solid – also broken symmetry phase.
  - Analog in topological insulators? YES
    - Dislocations host counter-propagating 1D modes. Like the edge of 2D QSH Insulator - Helical Metal.
    - Protected against disorder scattering – ideal quantum wire inside a bulk solid.

Topological Band Insulators

Fu, Kane & Mele PRL 07
Moore & Balents PRB 07
Roy, cond-mat 06

- 3-D Topological Insulator
  - Classified by 4 Z_2 Invariants $(\nu_0 = 0,1; \nu_1, \nu_2, \nu_3)$
  - $\nu_0 = 0,1$ weak and strong Top. Ins.
  - (ν_1, ν_2, ν_3) with respect to Reciprocal vectors $(\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3)$ defines a time reversal invariant mon



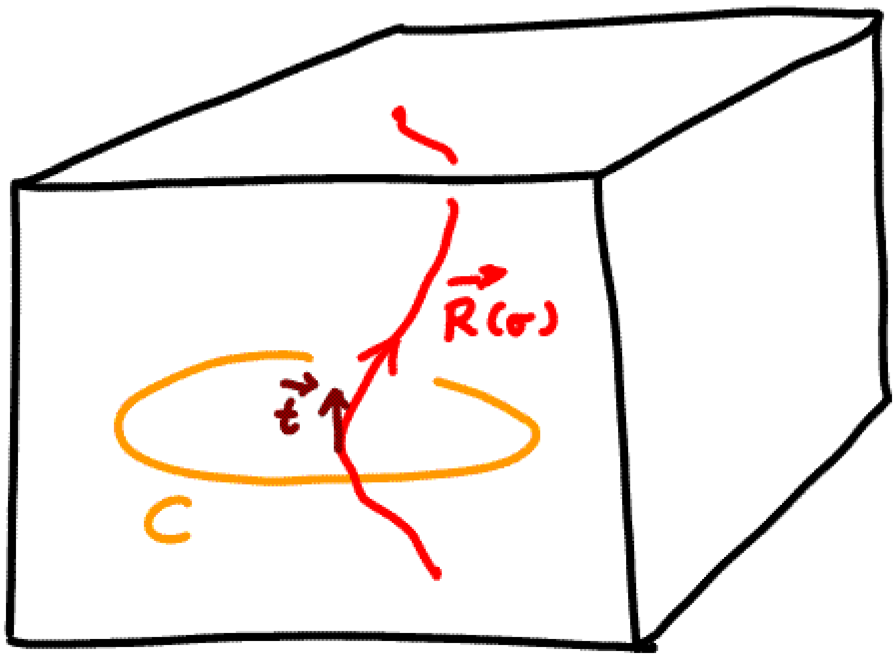
$$\mathbf{M}_\nu = (\nu_1 \mathbf{G}_1 + \nu_2 \mathbf{G}_2 + \nu_3 \mathbf{G}_3)/2$$

  - STRONG Topological Insulator
    - Odd number of Dirac modes on Surface Brillouin Zone.
    - $\mathbf{M}_\nu$ Fixes location of Dirac nodes in 2D surface BZ
  - WEAK Topological Insulator
    - Connected to decoupled layers of 2D QSHE stacked along $\mathbf{M}_\nu$

Line Defects in a Crystal

- Dislocations:

Defined by location **R**(σ) and 'strength' **B** .



$\vec{r}_n = \vec{r}_n^{(0)} + \vec{u}(r_n)$

- $\vec{r}_n$: atom locations
- $\vec{r}_n^{(0)}$: perfect crystal
- $\vec{u}(r_n)$: order parameter NOT single valued.

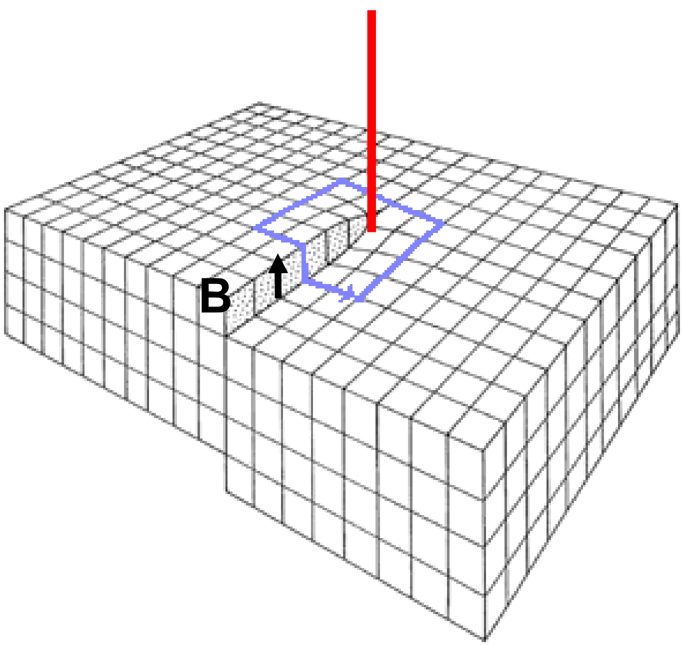
$$\oint_C d\vec{l} \cdot \vec{\nabla} \vec{u} = \vec{B} \quad \{\text{a lattice vector}\}$$

B – Burgers Vector, must stay constant along the length and is quantized to lattice vectors. (Like vorticity)

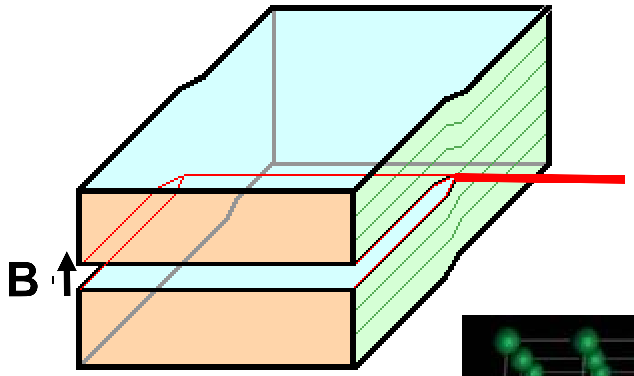
Visualizing Dislocations

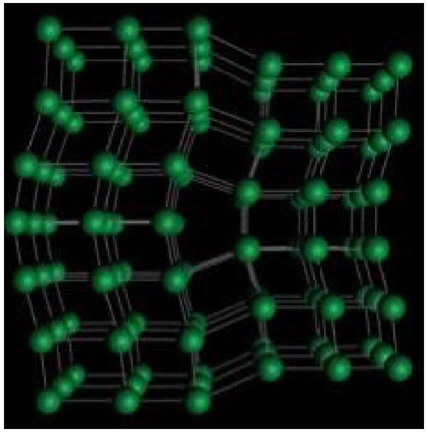
- Volterra Process:
  - Cut with an imaginary plane, that ends on the dislocation line **R**(σ)
  - Move all atoms on one side of the plane by the Burgers vector **B**
  - Add/remove atoms if required.

SCREW DISLOCATION: **t // B**



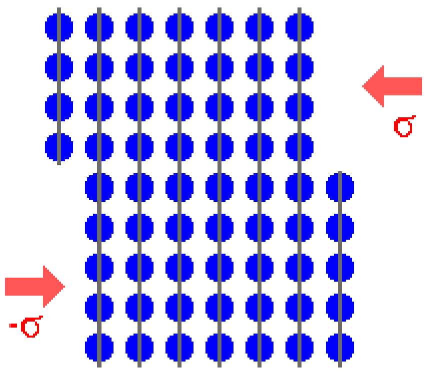
EDGE DISLOCATION: **t ⊥ B**

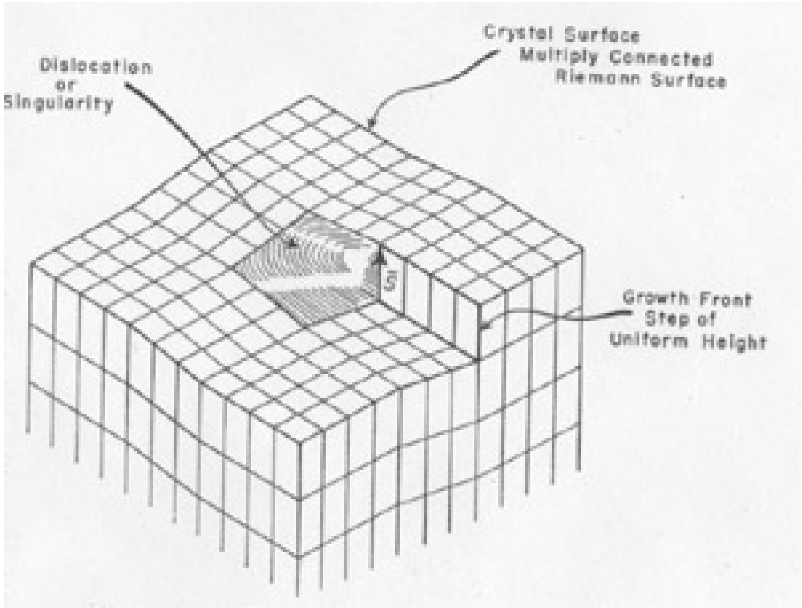




Dislocations in Solids

- Always present $\mathbf{n_d} \approx 10^{10}\,\mathbf{to}\,10^{12}\,\mathbf{m^{-2}}$
- Control mechanical properties eg. Plastic Flow
- Crystal Growth – aided by screw dislocations.

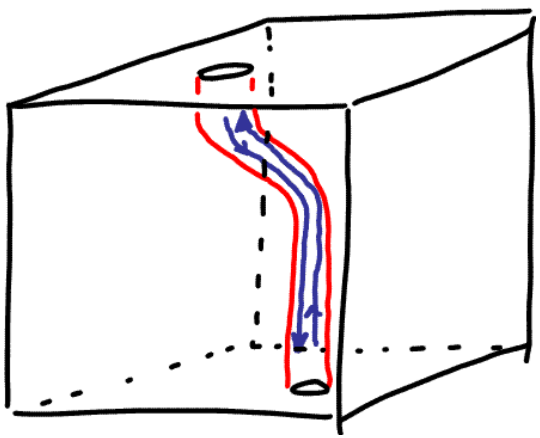


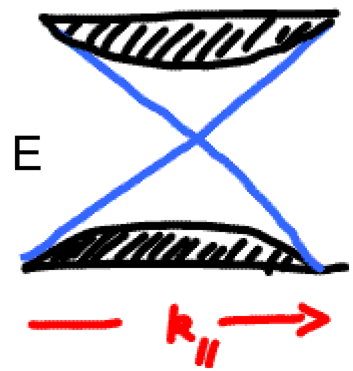


Dislocation in a Topological Insulator

- 1D Helical Metal occurs in a dislocation {**R**(σ), **B**} embedded in a topological insulator $\{\nu_0 = 0,1; \mathbf{M}_\nu\}$ iff:

$$\mathbf{B}\cdot\mathbf{M}_\nu = \pi \left(\bmod 2\pi\right)$$



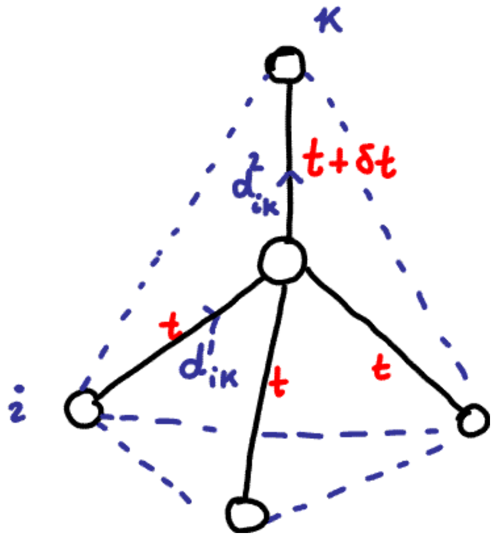


1D Modes are topologically protected:

- Cannot be gapped if Time Reversal Symmetry + bulk gap are present.
- Not localized by disorder
- Half of a regular quantum wire.

- Not all Top. Ins. have dislocation Helical modes. $\nu_0 = 1; \mathbf{M}_\nu = 0$
- Modes occur for both Weak and Strong Top.Ins.

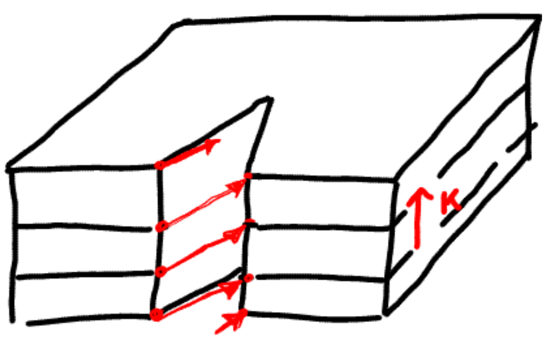
Illustration – Diamond Lattice Top. Ins.



$$H = t\sum_{\langle ij\rangle} c^{\dagger}_{i\sigma}c_{j\sigma} + i\frac{\lambda_{SO}}{8a^2}\sum_{\langle\langle ik\rangle\rangle} c^{\dagger}_{i\sigma}(\vec{d}^{1}_{ik}\times\vec{d}^{2}_{ik})\cdot\vec{\sigma}_{\sigma\sigma'}c_{k\sigma'}$$

$$\nu_0 = 1; \mathbf{M}_\nu = \frac{\pi}{2}(1,1,1)$$

- Introduce a screw dislocation: **B**=(1,1,0).
  - Easily introduced in tight binding. Momentum dependent phase factor for cut bonds.

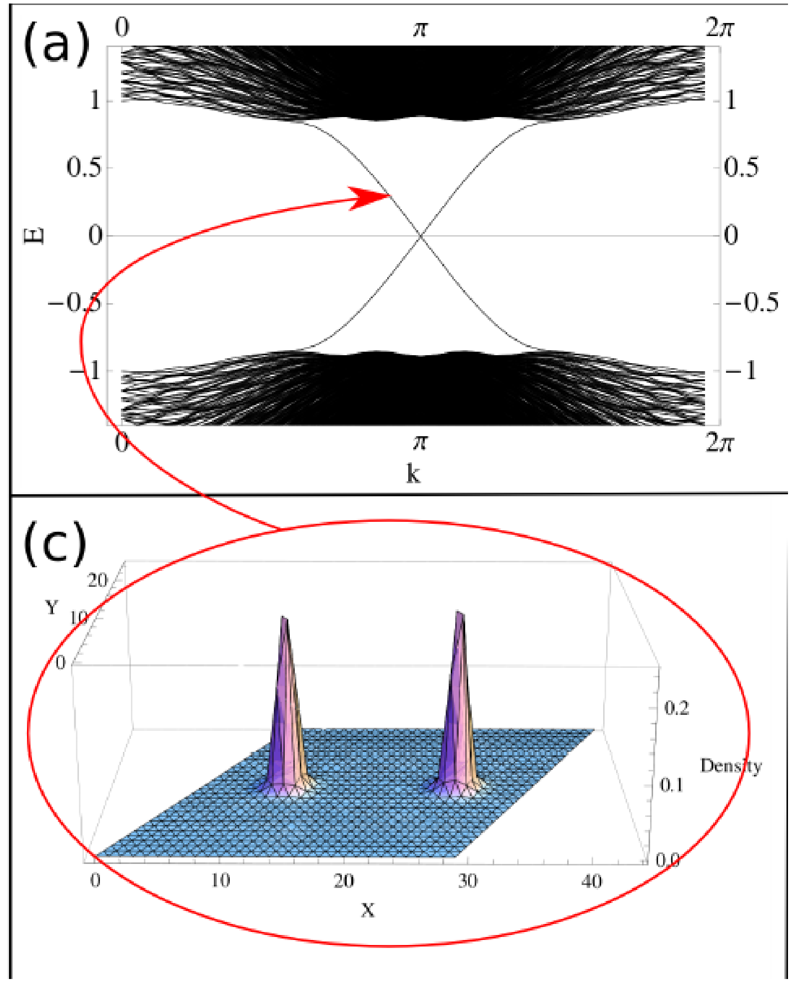


$$t \to t\, e^{\pm i\vec{K}\cdot\vec{B}}$$

Results: Screw Dislocation in Diamond Lattice Top. Ins.

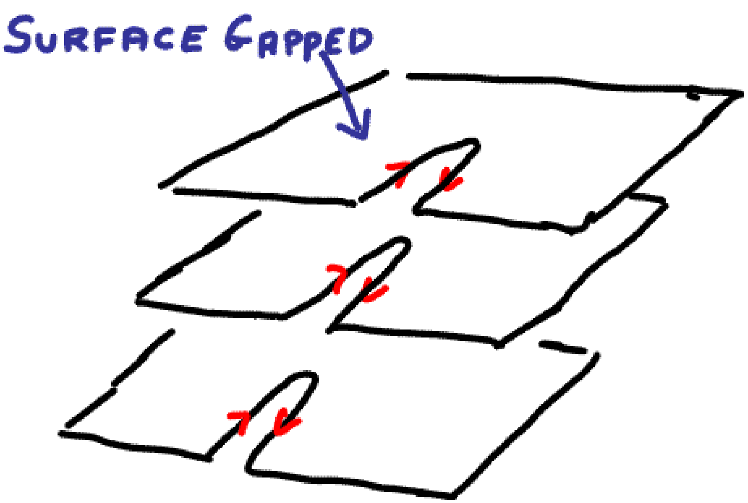
- Insert a pair of screw dislocations (36x36x18 periodic BC). Momentum along the dislocations is a good quantum number.

- Two propagating modes per dislocation. `Helical metal’.

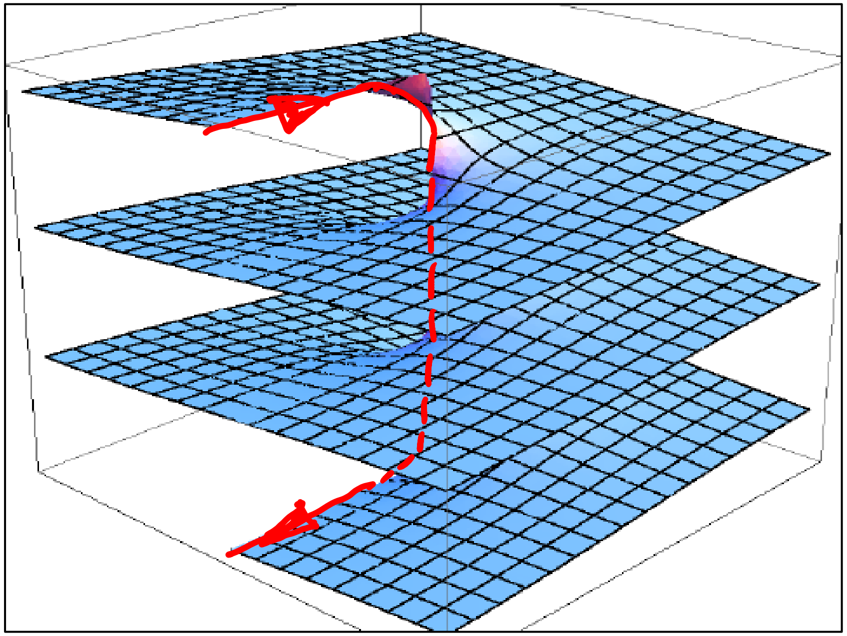


Proof for Weak Top. Ins.

- Weak Top.Ins. Adiabatically connected to a stack of decoupled 2D Top.Ins.,
  - stacking along $\mathbf{M}_\nu$
  - Different proof for Strong TI

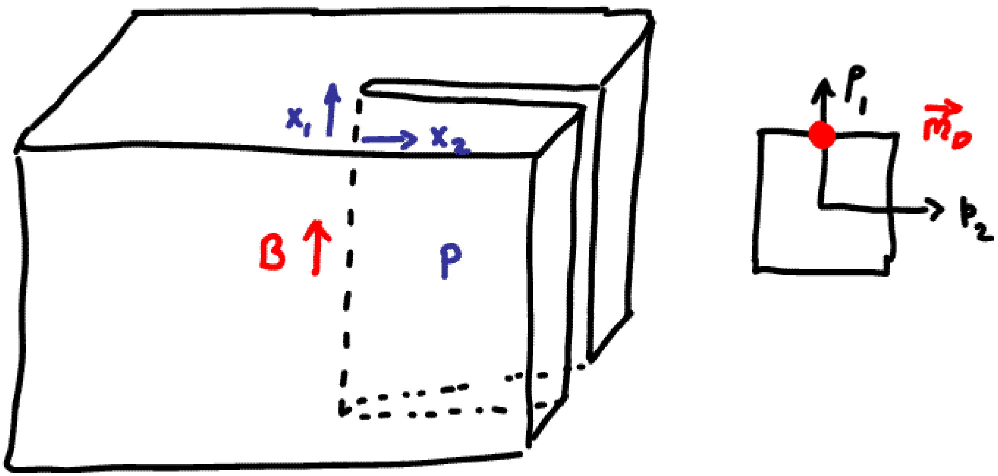


Cut Surface – only one of the helical mode pairs is shown.



Glued Surface – Dislocation must carry helical modes

Proof For General Top. Ins. 1



- Screw dislocation – **if** surface Dirac node is at momentum $\mathbf{m}_{\mathbf{Dirac}} \cdot \mathbf{B} = \pi$ then (-1) phase acquired on crossing the dislocation.
- In the weak surface connection limit => Dirac equation that changes mass term sign.

$$H = \left(p_1\sigma_1 + p_2\sigma_2\right)\mu_z + m(x_2)\mu_x$$

$$m(x_2 > 0) = -m$$
$$m(x_2 < 0) = +m$$

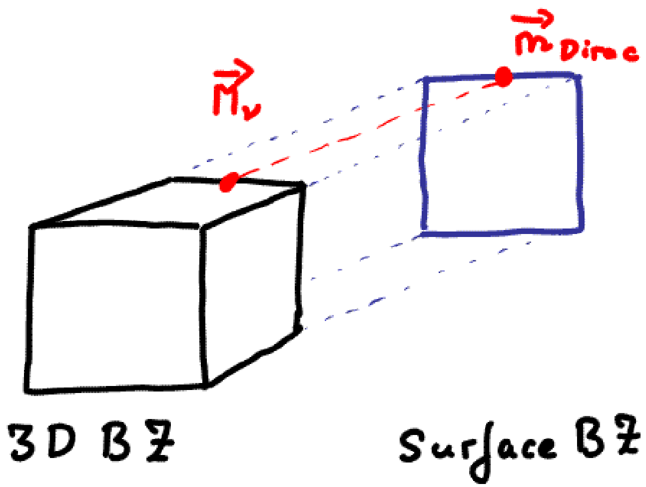
Proof For General Top. Ins. 2

$$H = (p_1\sigma_1 + p_2\sigma_2)\mu_z + m(x_2)\mu_x$$

$$m(x_2 > 0) = -m$$
$$m(x_2 < 0) = +m$$

- Pair of zero modes at p_1=0. $\psi(x_2) = \psi_0 e^{-\int_0^{x_2} |m(x')| dx'}$
- Propagating 1D helical modes for general p_1.

- Location of Surface Dirac Node – controlled by $\mathbf{M}_\nu$

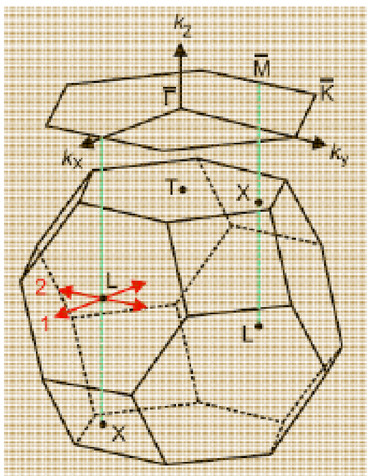


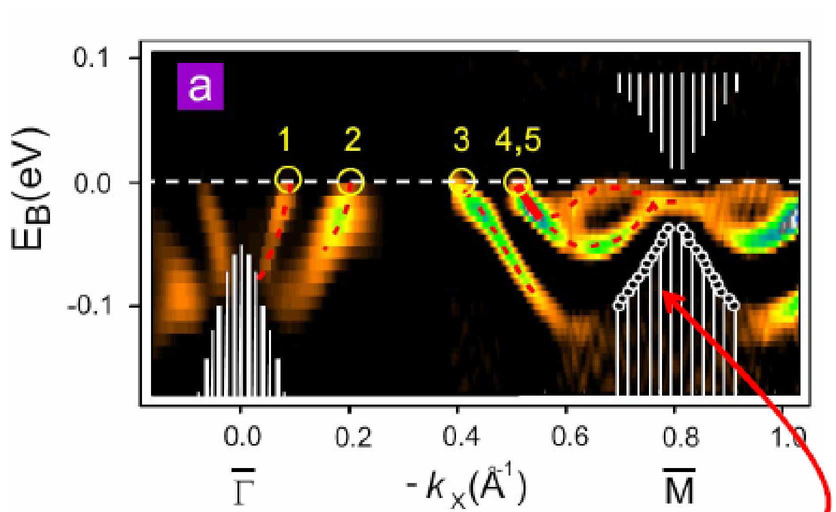
$$\mathbf{m}_{\mathbf{Dirac}} \cdot \mathbf{B} = \pi$$
$$\Downarrow$$
$$\mathbf{M}_\nu \cdot \mathbf{B} = \pi \ (\text{mod}\, 2\pi)$$

3D Topological Band Insulators

- Experimental Candidate $Bi_{0.9}Sb_{0.1}$

D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava and M. Z. Hasan, Nature (08) in press





- Bulk Dirac points at L project to M in surface Brillouin Zone

$$\nu_0 = \mathbf{1}; \mathbf{M}_\nu = (1,1,1)$$

Experimental Signatures

- Resistivity: dislocation contribution could dominate over surface conduction.

$$\rho = \frac{h}{2e^2}\frac{1}{l\,n_d} \approx 10^{-2}\,\Omega\mathbf{m}$$

$$n_d \approx 10^{12}\,m^{-2}$$

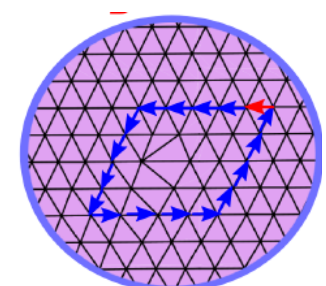
$$l \approx 1\mu m$$

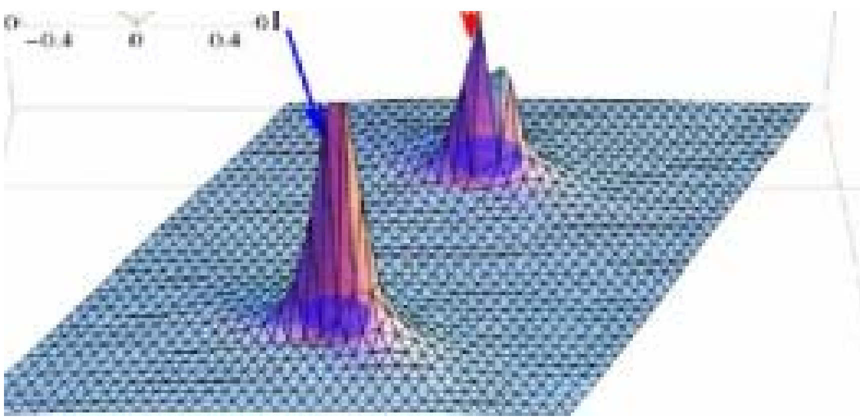
- Scanning Tunneling Microscopy: Can determine atomic defect structure and Local Density of States (LDOS). 1D modes – finite DOS. Dirac point – vanishing density of states.

Experimental Signatures - STM

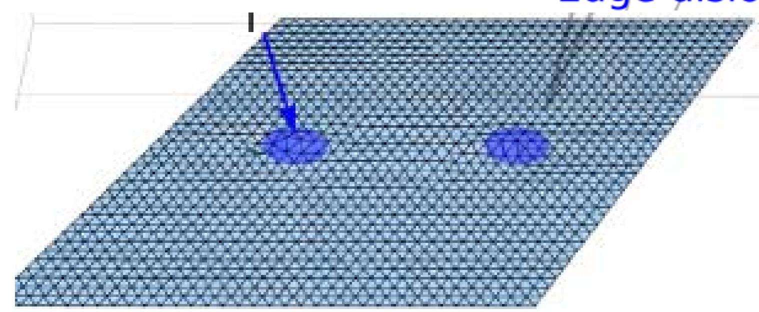
- Diamond lattice strong Top.Ins. for demonstration. Edge dislocations.

Edge dislocation on (-1,-1,1) Surface.
B=(1,1,0) OR **B**=(-1,0,1).
•Integrated LDOS in [-0.1,0.1].





$$\mathbf{B}\cdot\mathbf{M}_{\nu}=\pi$$



$$\mathbf{B}\cdot\mathbf{M}_{\nu}=0$$

Effect of Disorder

- Very Strong Disorder – dislocations proliferate; no meaning to $\mathbf{M}_\nu$
- Moderate disorder – dilute dislocation density. Can still characterize using gapless modes in dislocations and define $\mathbf{M}_\nu$. Weak insulators can be defined even *with* disorder.

  but Surface states localized.

- However, if unit cell is doubled with wave-vector $\mathbf{M}_\nu$ effective $\mathbf{M}_\nu = 0$ Now, elementary dislocations are forbidden. (different from disorder)

Conclusions

- 3D topological Band Insulator has protected helical mode in those dislocations that satisfy

$$\mathbf{B}\cdot\mathbf{M}_{\nu}=\pi\left(\bmod 2\pi\right)$$

- Indicates weak topological insulator stable to disorder if dislocations do not proliferate.

Future Directions

- Applications – quantum computing?
- Interaction effects – Luttinger liquid physics?
- Derivation from ‘field theory’?