Partial Thermalizations Allow for Optimal Thermodynamic Processes

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Szilard Engine
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Quantum Thermodynamics Conference, KITP, UC Santa Barbara
Szilard Engine
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contradicts second law of thermodynamics?!
Resolution of the paradox
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\[ W = k_B T \log 2 \]
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- „Erasure“: Reset of information, i.e., an unknown bit is reset to a known value, e.g., „0“ → after erasure, we have full information about the bit
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    \( \Rightarrow \) since the measurement would act on box and demon together, it could not be described in this view
  
  * Box and demon viewed together from the outside: whole cycle can be described \( \Rightarrow \) here the demon’s bit is unknown
Szilard Engine
(view from outside)
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\[ \rho_{BD}^{(i)} = \frac{1}{2} (|0\rangle\langle 0|_B + |1\rangle\langle 1|_B) \otimes |i\rangle\langle i|_D \]
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\[
\rho'_{BD} = \frac{1}{2} (|0\rangle\langle 0|_{B} + |1\rangle\langle 1|_{B}) \otimes \frac{1}{2} (|0\rangle\langle 0|_{D} + |1\rangle\langle 1|_{D})
\]

work extraction
\[
\langle W \rangle = k_B T \log 2
\]

measurement
Szilard Engine
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work extraction \[ \langle W \rangle = k_B T \log 2 \]

measurement
reset
\[ \langle W \rangle = -k_B T \log 2 \]
Motivation
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• **Goal:** Test the robustness of a work extraction protocol for an error model as general as possible

• **Main result:** Optimal isothermal processes are possible for any $\alpha < 1$
Framework 1: 
Collisional Model
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• 3 systems:
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  - System S of one information qubit
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→ Using the information of the system qubit, we apply $N$ thermal operations to convert heat from the coupled thermal bath B into work stored in system W:

In the $k^{th}$ interaction step the energy-conserving unitary $U_{SBW}^{(k)}$ acts on S, W and the $k^{th}$ bath qubit
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• All errors of this form yield the same noise model, describing the reduced state of the system as

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\rho^{(k)}_S = \alpha_k \rho^{(k-1)}_S + (1 - \alpha_k) \tau^{(k)}_B
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  ➔ arbitrary thermal operations restricted on the relevant degenerate subspace
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\[ \rho_S^{(k)} = \alpha_k \rho_S^{(k-1)} + (1 - \alpha_k) \tau_B^{(k)} \]

➤ Partial thermalization, where the degree of thermalization is quantified by \(\alpha\):
  - For \(\alpha = 0\) : standard case of full thermalization
  - For \(\alpha = 1\) : no interaction between S, B, W
Results
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\[ \langle W \rangle = \Delta F - \gamma - \varepsilon \]

\* \( \gamma = \mathcal{O} \left( \frac{1}{N} \right) \) : Error due to finite number of steps

\* \( \varepsilon = \mathcal{O} \left( \frac{1}{N} \frac{\alpha}{1 - \alpha} \right) \) : Error due to noise quantified by \( \alpha \)
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\[ \Rightarrow \text{Optimal isothermal processes can be constructed for any } \alpha < 1 \text{ with sufficiently many steps} \]
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\[ h_W = F \]

Optimal isothermal processes can be constructed for any \( \alpha < 1 \) with sufficiently many steps

- We can trade the number of steps \( N \) for precision
- Proof can be extended to qudits
Results

- Determined an almost tight upper bound for a specific example

\[
\text{error as a function of } N \text{ for } \alpha = 1/2 \text{ and of } \alpha \text{ for } N = 1000, \text{ respectively, with } k_B T \log 2 = 1, p_k = k/2N.
\]
Results

• Characterized the work fluctuations which decrease for large $N$

Histograms showing the fluctuations for $N = 100$, $N = 200$, $N = 500$ and $N = 1000$
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- As experimentally the interaction time is finite, the system will only get partially thermalized and its state is again of the form

$$\rho^{(k)} = \alpha_k \rho^{(k-1)} + (1 - \alpha_k) \tau^{(k)}$$

where $\alpha$ depends on the interaction time and strength
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→ Again, the extracted work is given by

$$\langle W \rangle = \Delta F - \mathcal{O} \left( \frac{1}{N} \right)$$
Extension: Qudits
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- Evolution described by $N$ Gibbs preserving maps $G_k$:
  \[ G_k(\tau^{(k)}) = \tau^{(k)} \]
  \[ \| G_k(\rho) - \tau^{(k)} \|_1 \leq \alpha_k \| \rho - \tau^{(k)} \|_1 \quad (\alpha_k < 1) \]
  with
  \[ \| \tau^{(k)} - \tau^{(k-1)} \|_1 = \mathcal{O} \left( \frac{1}{N} \right) \]
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$$\| \tau^{(k)} - \tau^{(k-1)} \|_1 = \mathcal{O}\left(\frac{1}{N}\right)$$

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→ Simplifies experimental implementation of optimal processes e.g. for small engines
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• Big freedom in distribution of bath qubits / Hamiltonians

  ➔ Simplifies experimental implementation of optimal processes e.g. for small engines

  ➔ Optimal processes are much more common than previously expected in small quantum systems