Heat pumping from Majorana zero modes

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In collaboration with:

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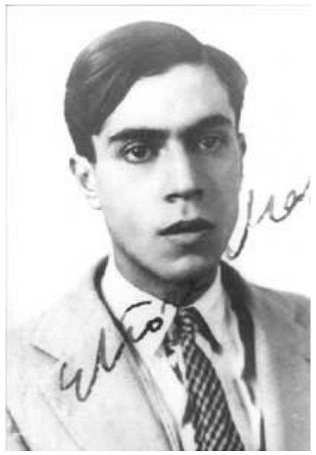
25.06.2018, KITP, Quantum thermodynamics

Outline



- Exchange statistics beyond bosons and fermions
- Quantum topologically protected operations
- Majorana zero modes
- Heat pumping from a Majorana exchange cycle

Exchange statistics and anyons

Exchange statistics in low dimensions is constrained by topology

3d: $P^2 = 1$

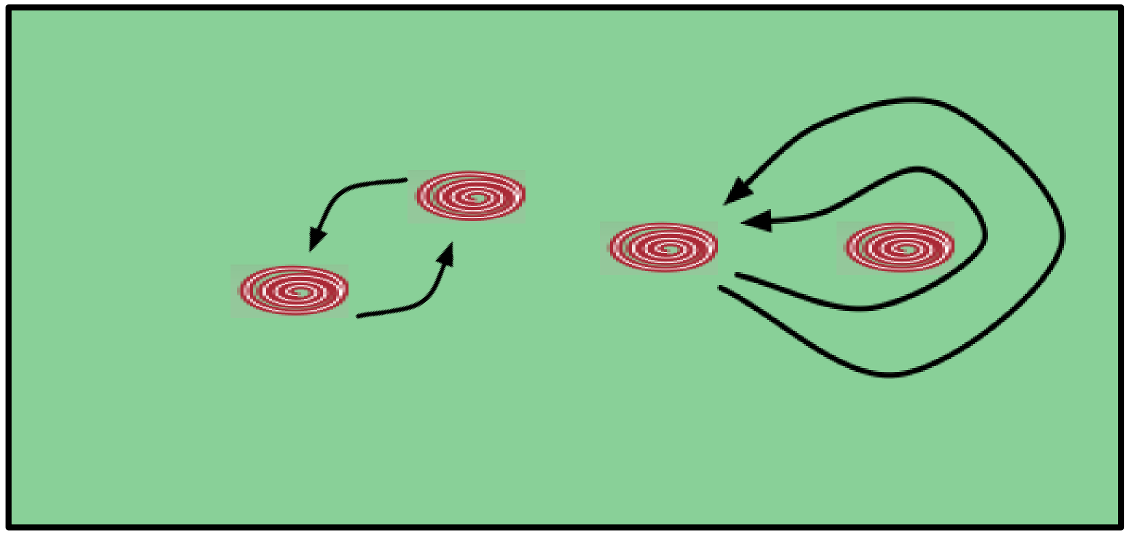
$$\psi(\mathbf{r}_1, \mathbf{r}_2) \to \pm\psi(\mathbf{r}_1, \mathbf{r}_2)$$

permutations

2d: $P^2 \neq 1$

$$\psi(\mathbf{r}_1, \mathbf{r}_2) \to e^{i\theta}\psi(\mathbf{r}_1, \mathbf{r}_2)$$

braiding



Unitary evolution of the ground state by exchange

$$|\psi\rangle \to \hat{U}|\psi\rangle$$

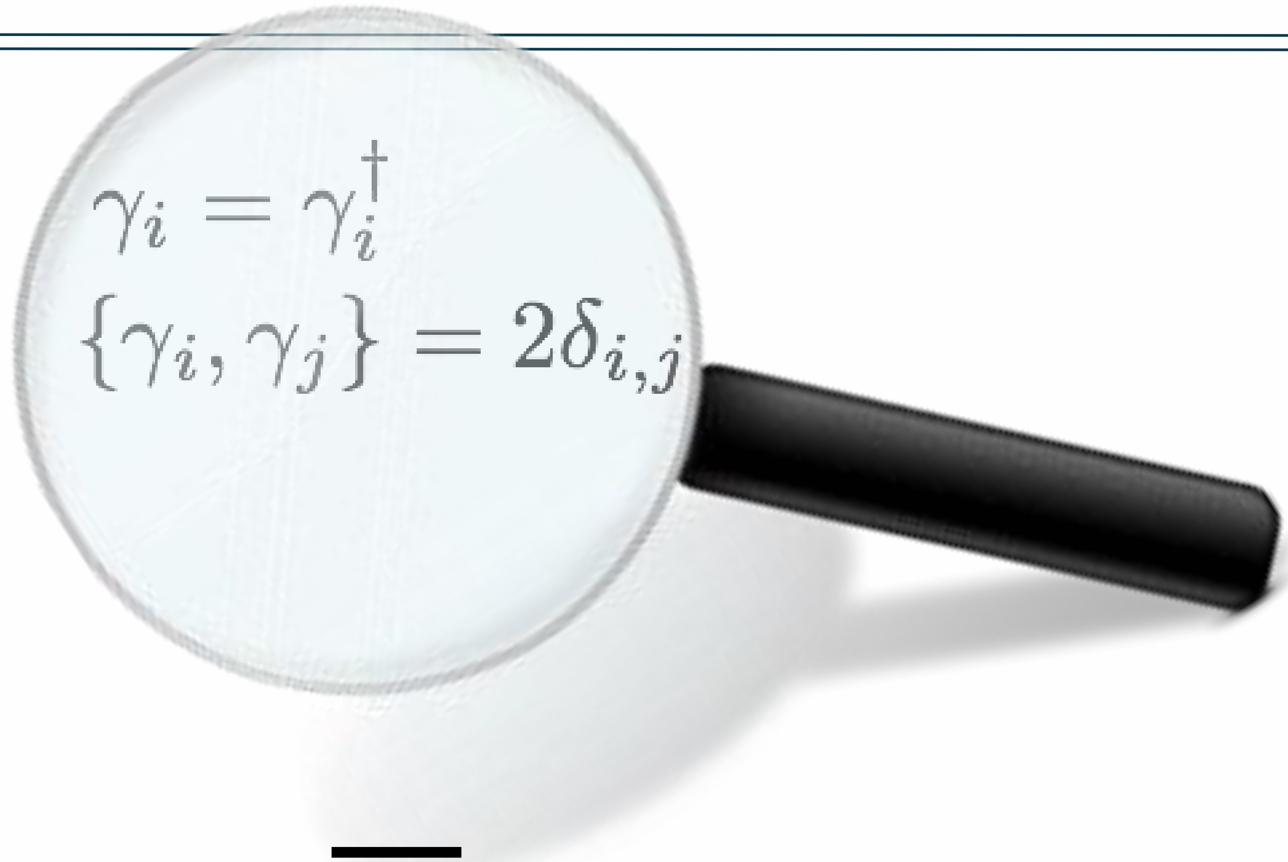
$$|\psi\rangle \to \hat{U}\hat{U}|\psi\rangle \neq |\psi\rangle \to \hat{U}\hat{U}|\psi\rangle$$

$\hat{U}$ Is fixed only by the exchange statistics

[Wilczek (1982), Moore & Read (1991)] Read &Green (2000), Kitaev (2001)...]

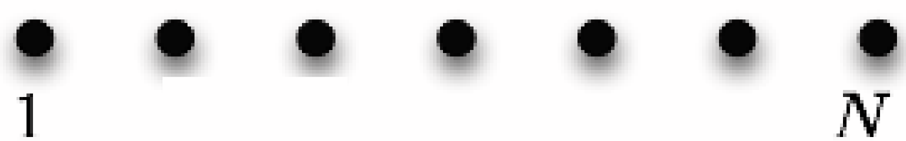
Majorana zero modes

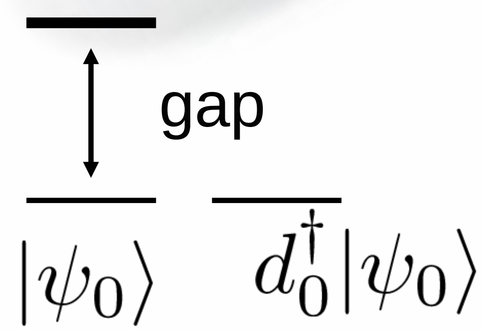
Zero energy excitations which
are their own antiparticles

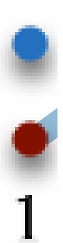


$$\gamma_j = c_j + c_j^\dagger$$

$$\Gamma_j = -i(c_j + c_j^\dagger)$$



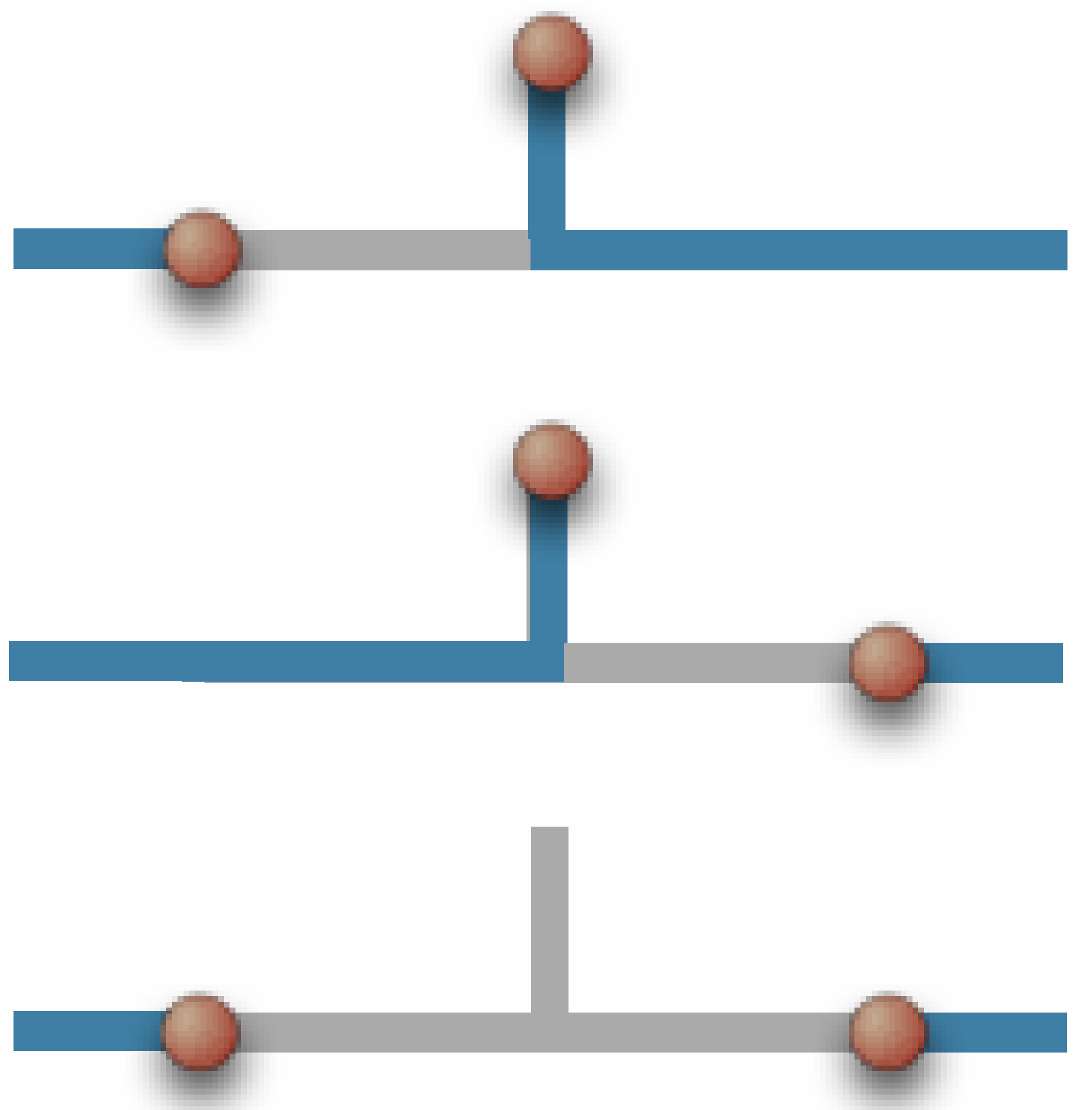


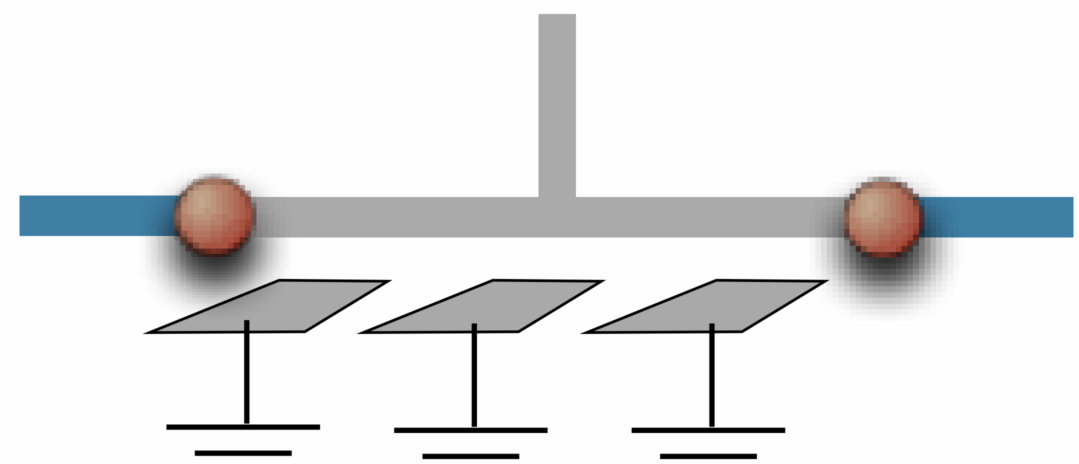


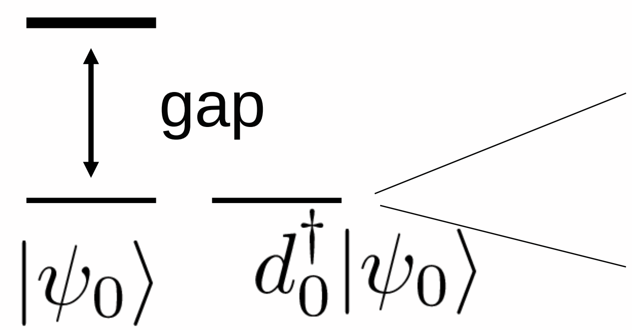
$$d_0 = (\gamma_1 + i\Gamma_N)/2$$

$$H = \Delta \sum_j (c_{j+1}^\dagger c_j + c_{j+1} c_j) + h.c.$$

Protected exchange







$$\gamma_1 \to \Gamma_N \qquad \Gamma_N \to -\gamma_1$$

$$|\psi_0\rangle + d_0^\dagger|\psi_0\rangle \to |\psi_0\rangle + e^{i\frac{\pi}{2}} d_0^\dagger|\psi_0\rangle$$

Topologically protected exchange

$$H_0 = i\Delta(L)\,\gamma_1\Gamma_N$$

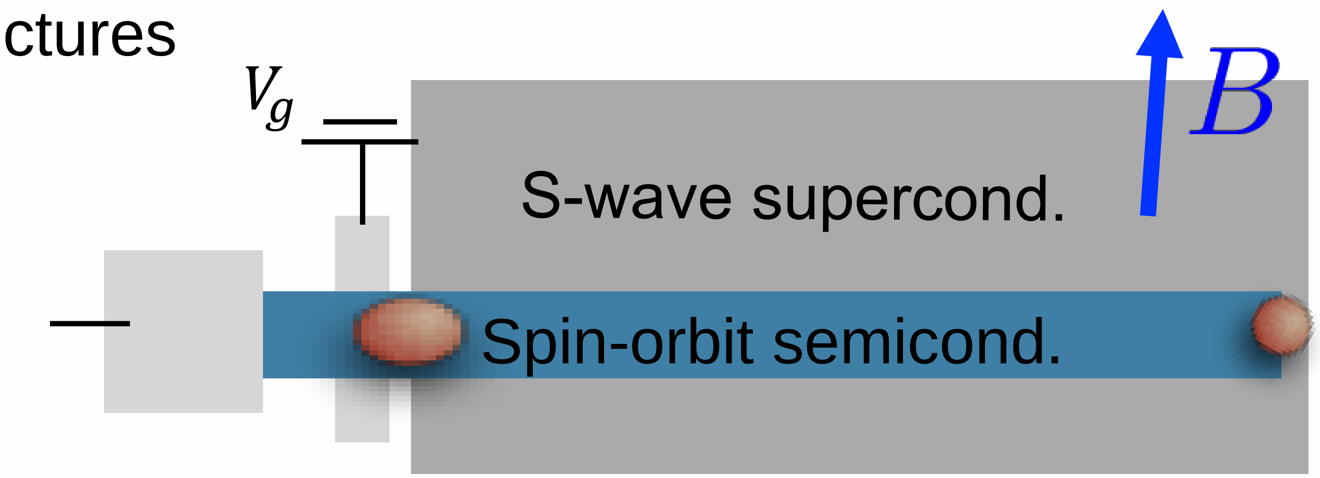
$$\Delta(L) \sim e^{-L/\xi}$$

Exponentially protected degeneracy

Majorana zero modes in nanostructures

Engineering topological phases in nanostructures

[L. Fu and C.L. Kane (2008)]

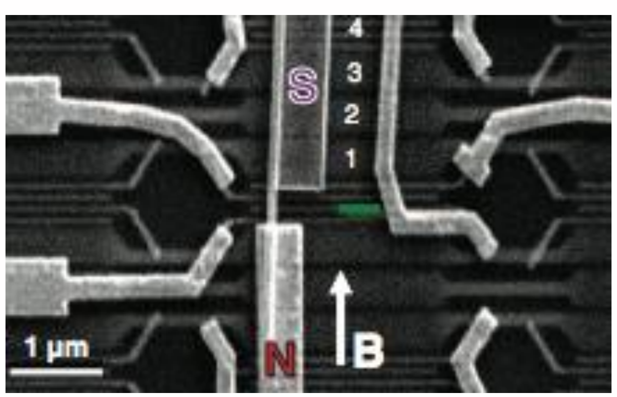


Semiconductor wires → experiments

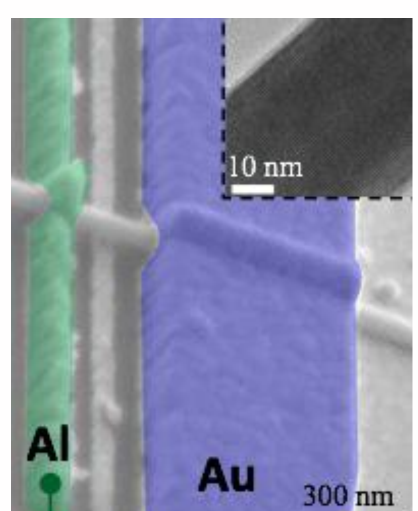
[Y. Oreg et al. (2010), J. D. Sau et al. (2010)]

Experiments in nanostructures

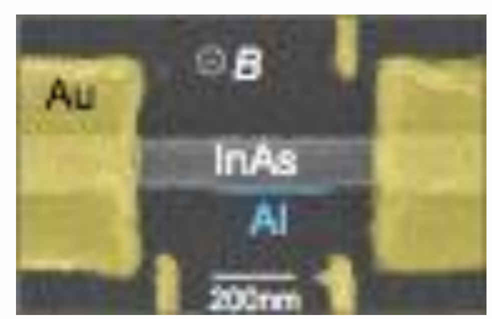
InSb wires
Delft (2012)



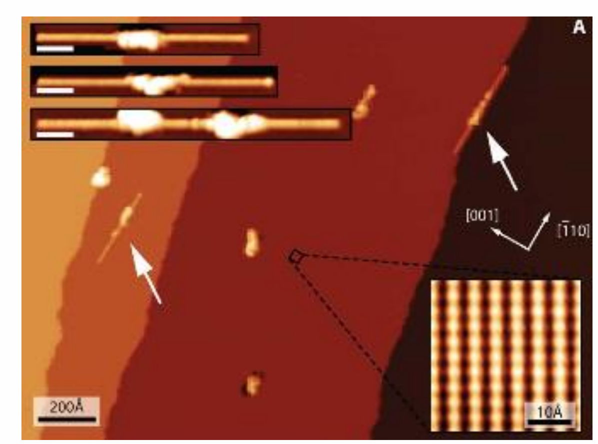
InAs wires
Weizmann (2012)



InAs wires
Copenhagen (2014)



Fe atomic chains
Princeton (2014)



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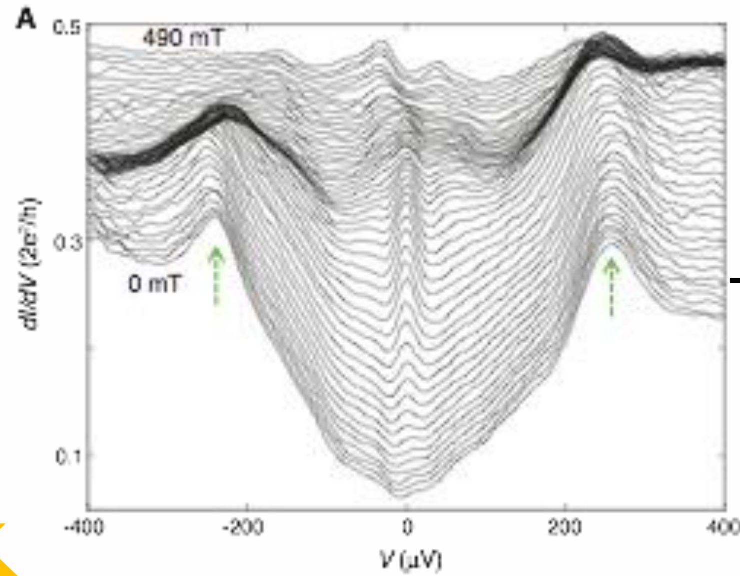
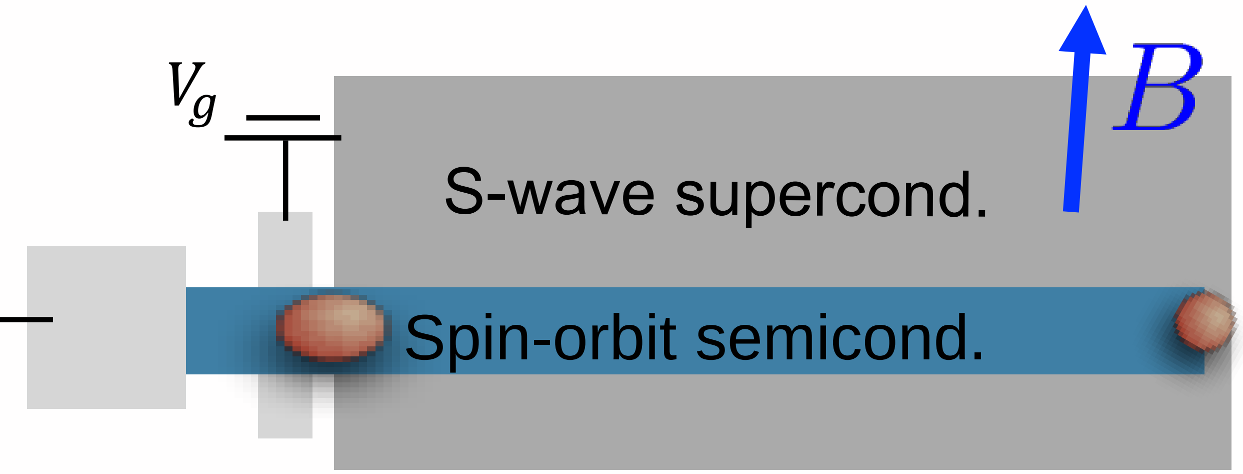
Transport & signatures

$$G = 2ne^2/h$$

Experimental signatures in transport



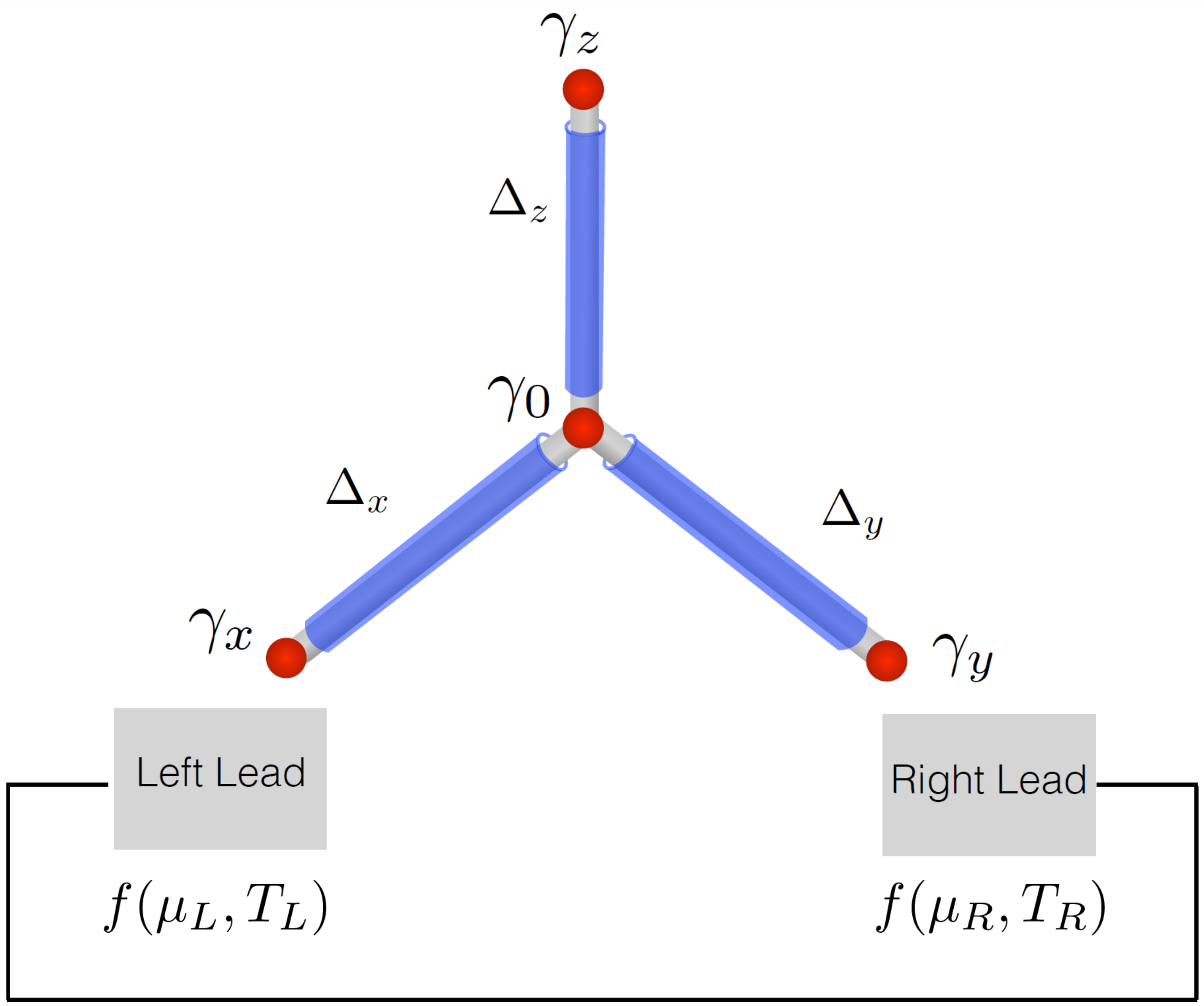


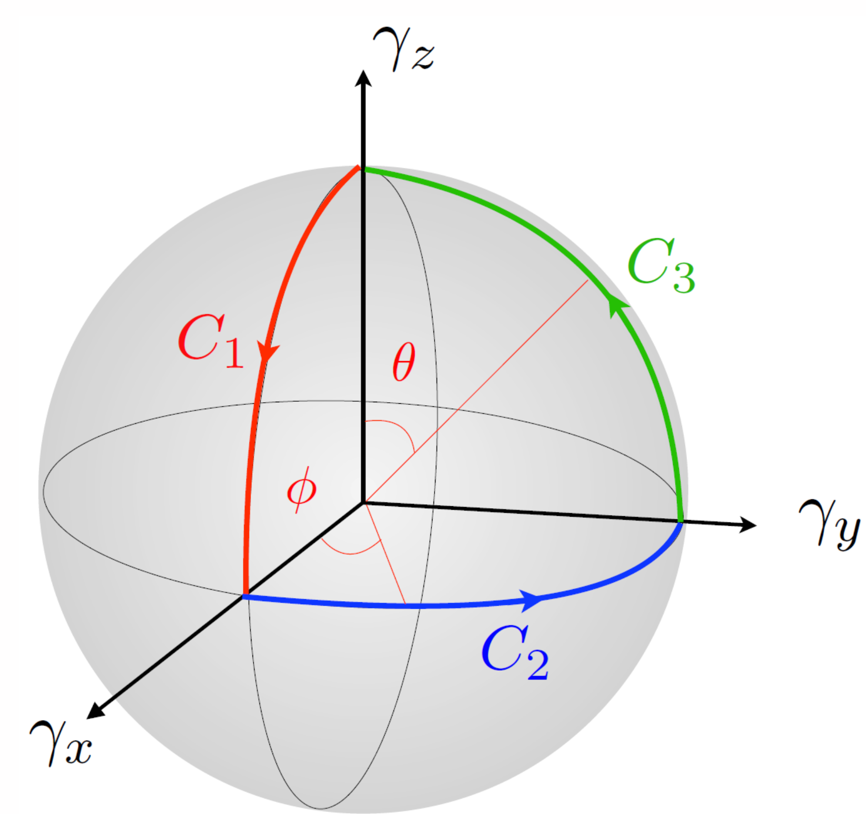
Theoretical tool for topological classification

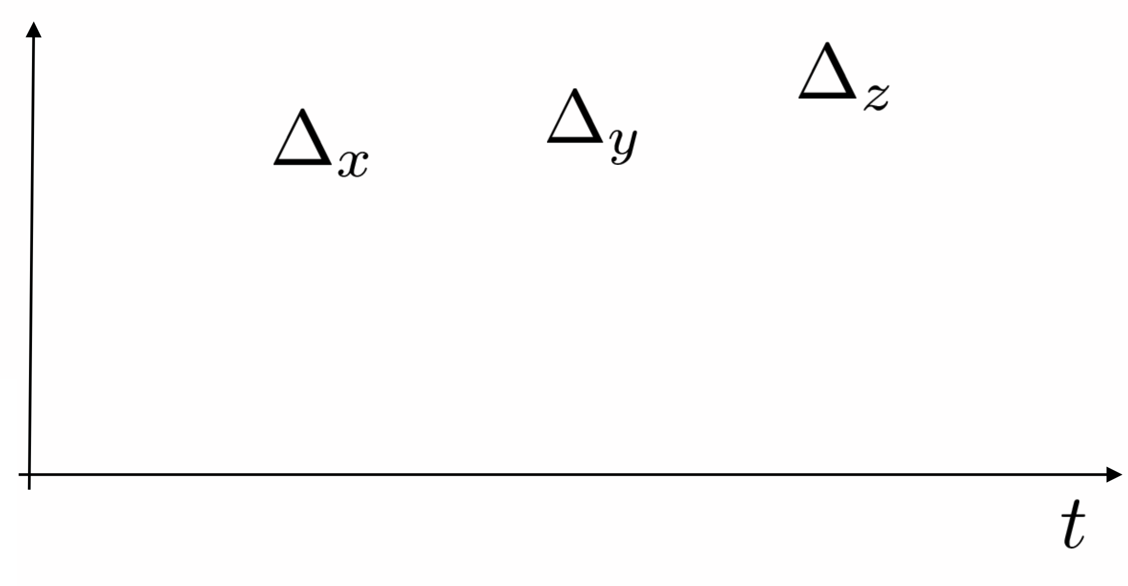
$$\mathrm{Tr}\, r_{eh}(\epsilon = 0) = \#\mathrm{Majoranas}$$

Charge measurements give only partial indication

Setup



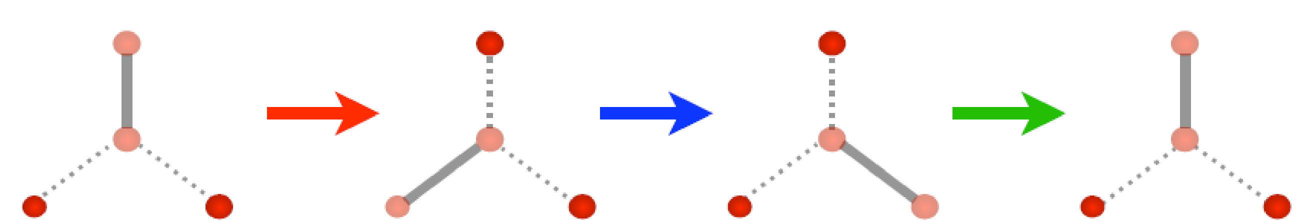




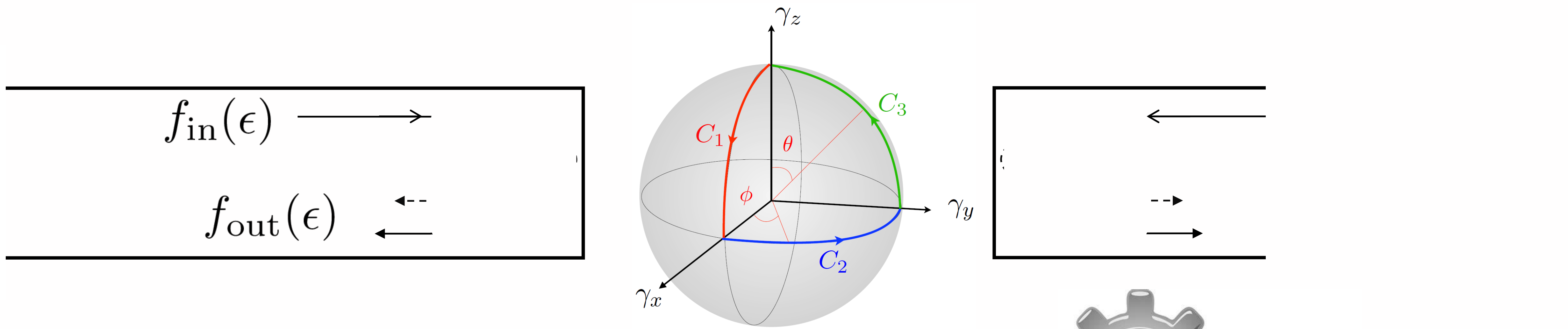
$$H_T = t_L \left(c_{Lk} - c^\dagger_{Lk} \right) \gamma_x + t_R \left(c_{Rk} - c^\dagger_{Rk} \right) \gamma_y$$

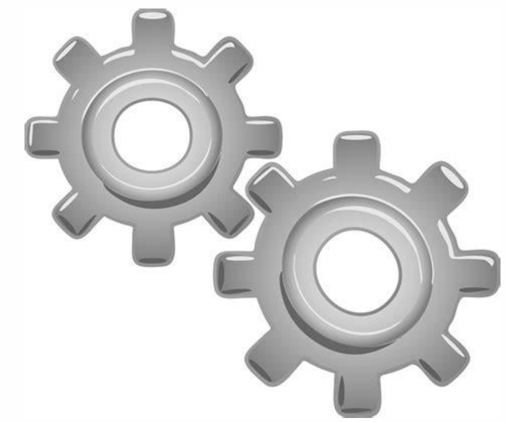
$$H_\alpha = \sum_k \xi_k c^\dagger_{\alpha k} c_{\alpha k},$$

$$H_Y = i\gamma_0 \vec{\Delta}(t) \cdot \vec{\gamma}$$



Pumped heat and charge





$I_{e,\alpha}$ $\quad$ e

$$\sout{I_{\epsilon,\alpha}} = 1/h \int d\epsilon\, (\sout{\epsilon - \mu})[f_{\text{in},\alpha}(\epsilon) - f_{\text{out},\alpha}(\epsilon)]$$

$$\mathcal{Q}_{\epsilon,\alpha} = \int_0^{\tau} dt\, I_{\epsilon,\alpha}(t)$$

$$\mathcal{Q}_{\alpha} \equiv \mathcal{Q}_{\epsilon,\alpha} + \frac{\mu}{e}\mathcal{Q}_{e,\alpha}$$

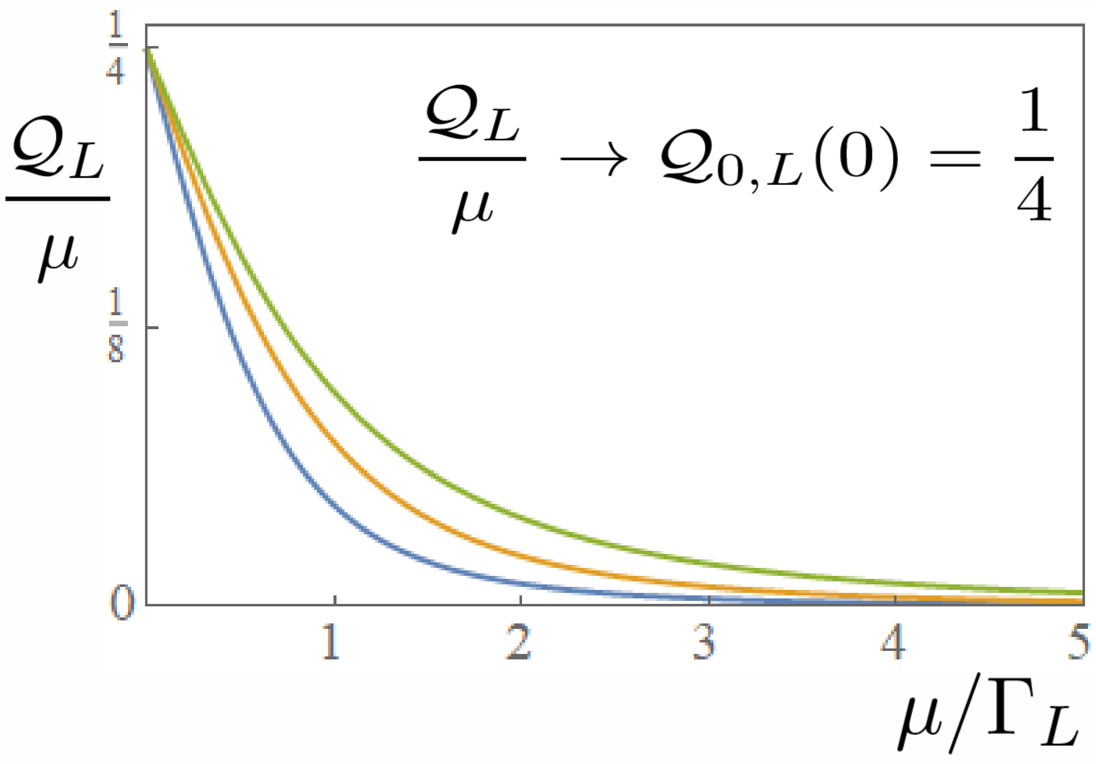
$$\mathcal{Q}_{\alpha} = \int_0^{\infty} d\epsilon\, \epsilon\, \mathcal{Q}_{o,\alpha}(\epsilon) \left[\frac{\partial f(\epsilon-\mu)}{\partial \epsilon} + \frac{\partial f(\epsilon+\mu)}{\partial \epsilon}\right]$$

System $\qquad$ Leads

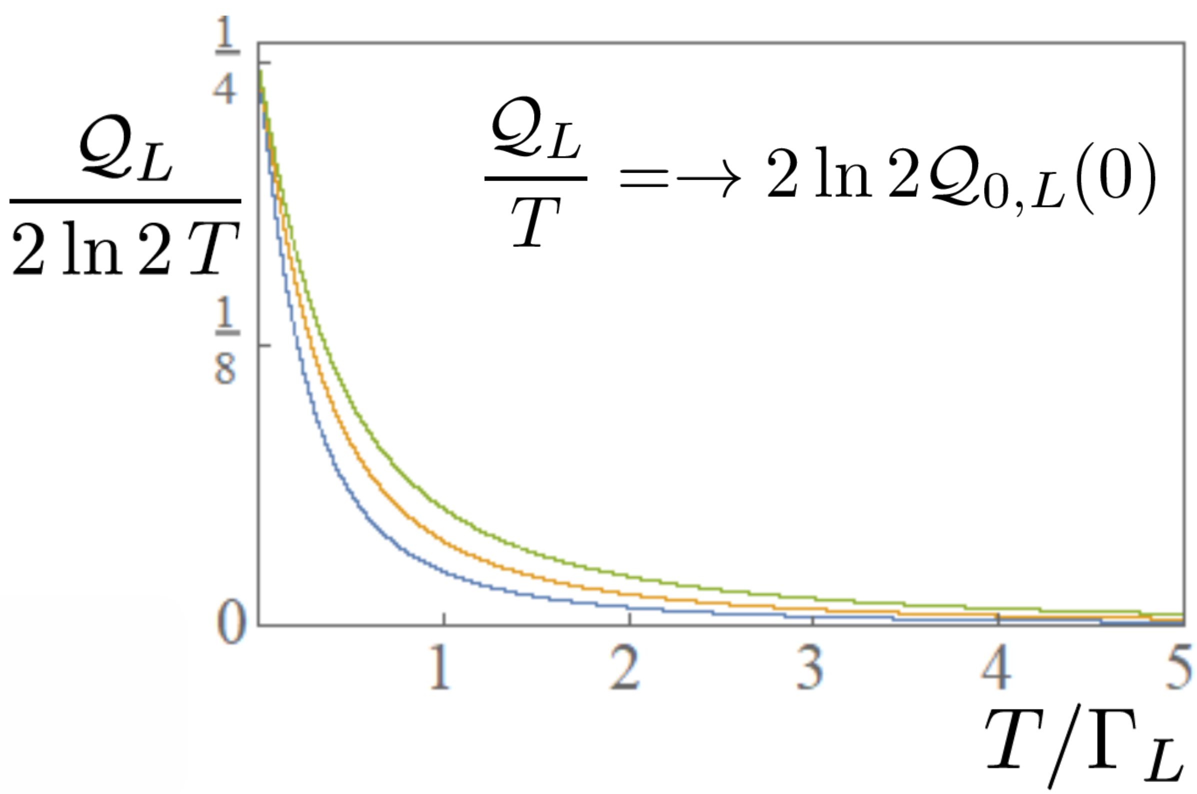
low temperature $\qquad \mathcal{Q}_{0,\alpha}(0) \qquad \mu \text{ or } T$

Results

$T \ll \mu \ll \min\{\Gamma_L, \Gamma_R\}$



$\mu \ll T \ll \min\{\Gamma_L, \Gamma_R\}$

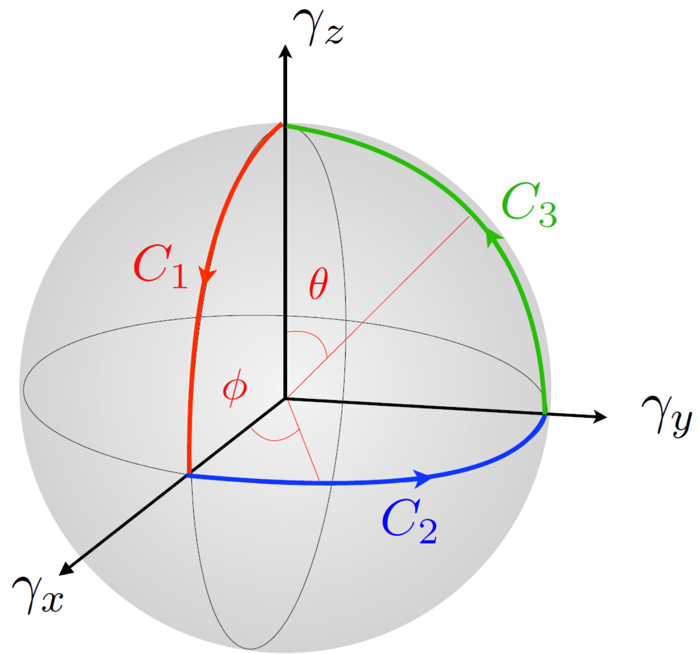


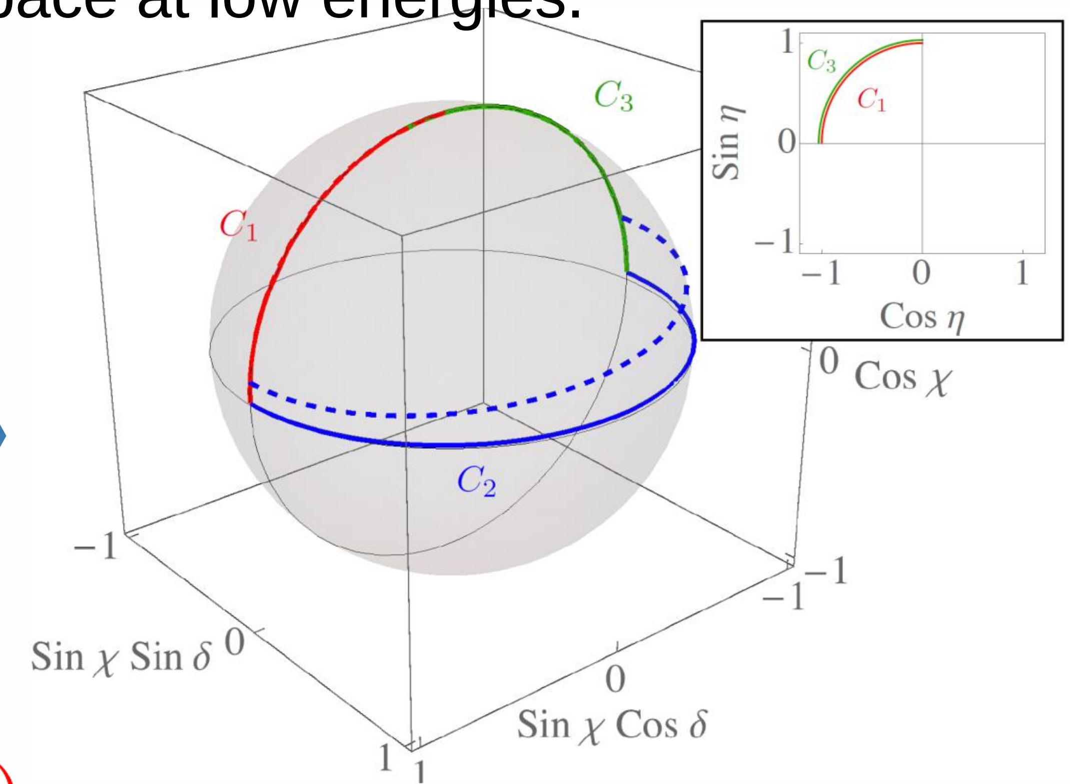
Heat pumped at low energy is **fixed by braiding**, independent on driving details

No charge pumped by driving

Protection …

The protected evolution of the state in the degenerate subspace is mapped to a protected cycle in the scattering matrix space at low energies.





$$\mathcal{Q}_{0,\alpha}(0)$$

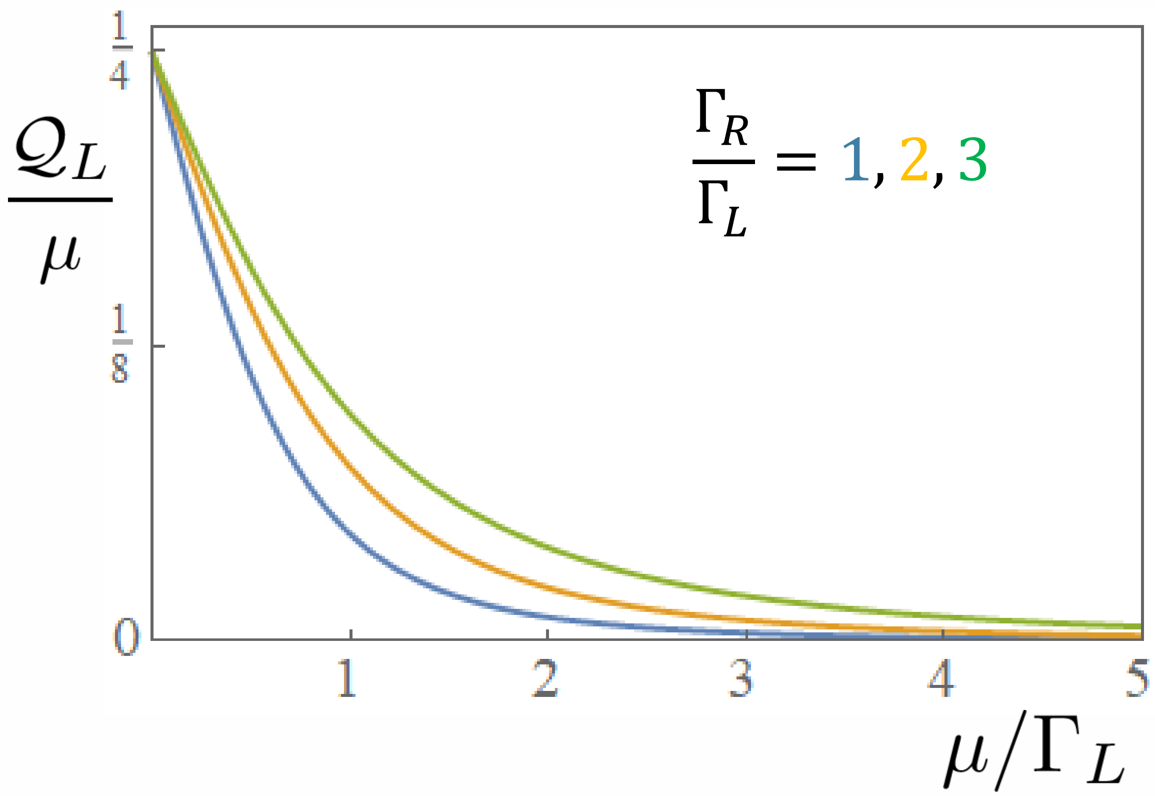
$$|\psi_0\rangle + d_0^\dagger|\psi_0\rangle \rightarrow |\psi_0\rangle + e^{i\frac{\pi}{2}} d_0^\dagger|\psi_0\rangle$$

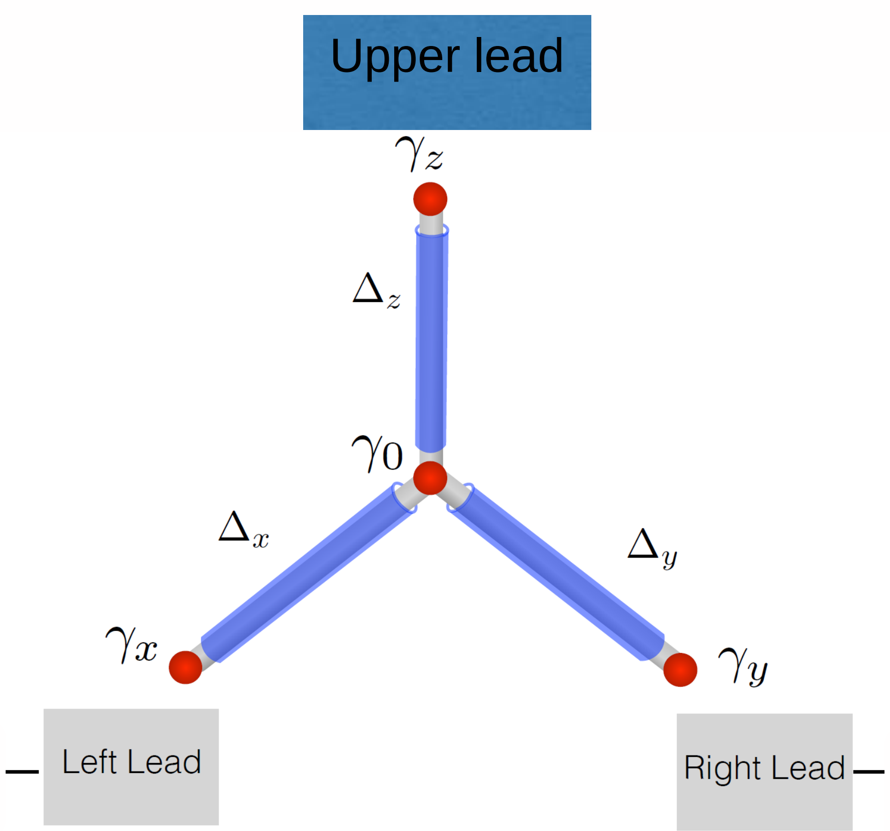
$$\mathcal{Q}_{0,\alpha}(\epsilon) = \oint_C \left(\frac{dn(\alpha)}{d\theta} d\theta + \frac{dn(\alpha)}{d\phi} d\phi \right)$$

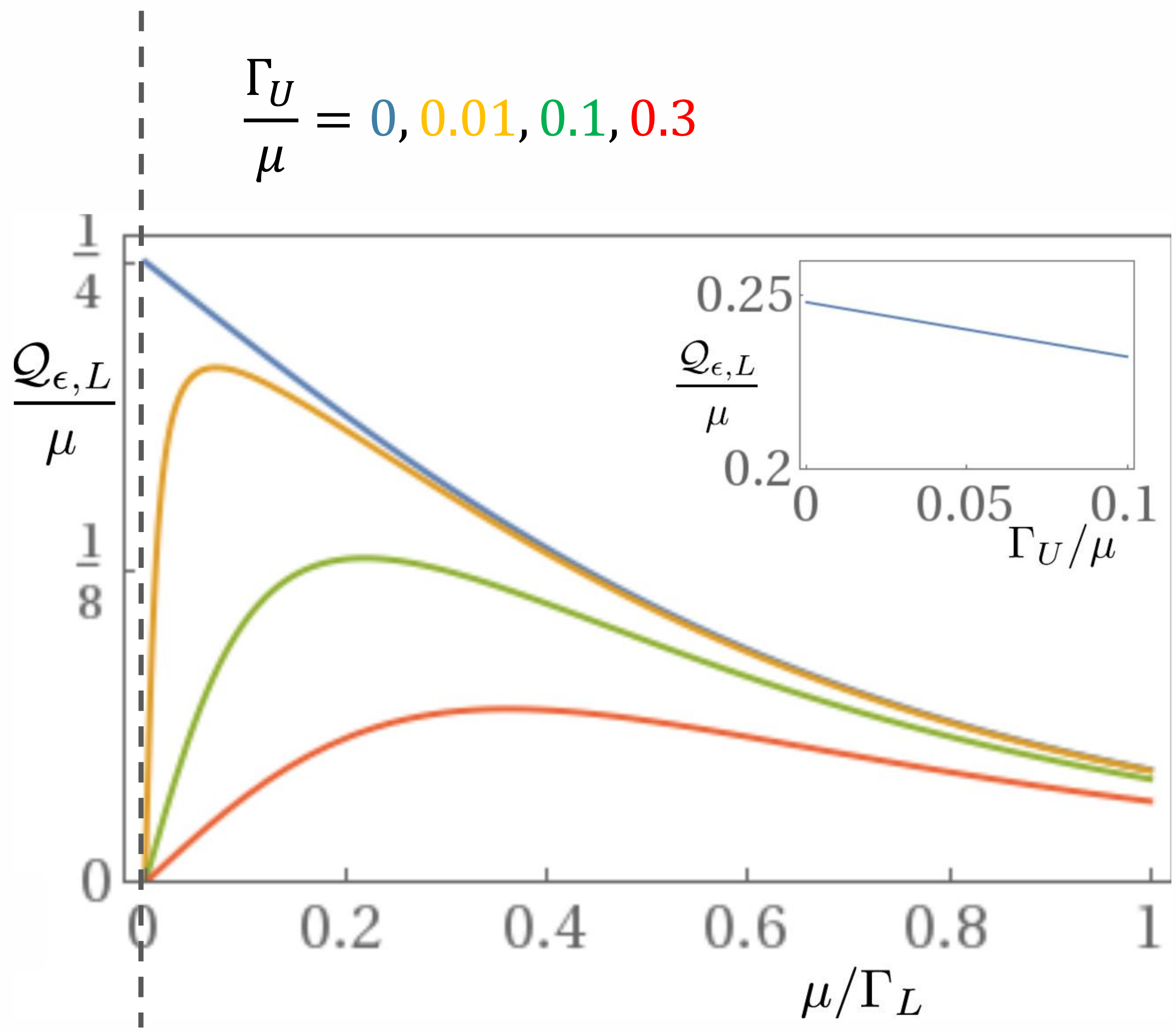
$$\frac{dn(\alpha)}{dX} = \frac{1}{2\pi} \sum_{\beta,\nu} \mathrm{Im} \frac{\partial S^{e,\nu}_{\alpha,\beta}}{\partial X} S^{e,\nu\,*}_{\alpha,\beta}$$

$$\tilde{S}(\epsilon) = e^{-i\eta(\epsilon)} e^{-i\chi(\epsilon)(\cos\delta(\epsilon)\sigma_z + \sin\delta(\epsilon)\sigma_x)}$$

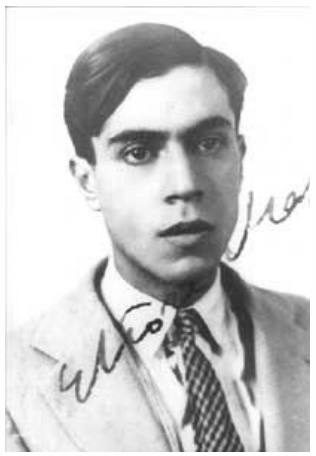
…and non-protection







Outlook



- Heat (vs. charge) senses Majorana zero modes
- Heat pumped at low energy is fixed by the braiding
- topological protected operations and thermodynamics…