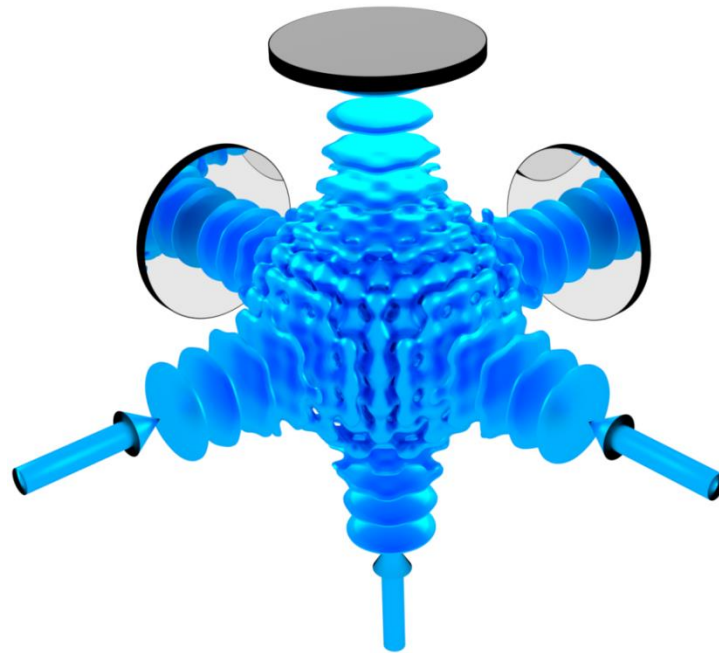


# Quantum thermodynamics (and its breakdown) in optical lattices



Ulrich Schneider

Ludwig-Maximilian Universität München  
Max-Planck Institut für Quantenoptik  
University of Cambridge

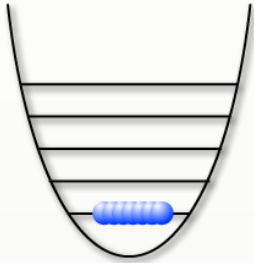
**EPSRC**  
Engineering and Physical Sciences  
Research Council



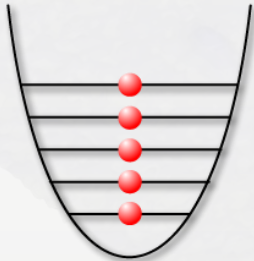
@CaMBQD

[www.manybody.phy.cam.ac.uk](http://www.manybody.phy.cam.ac.uk)

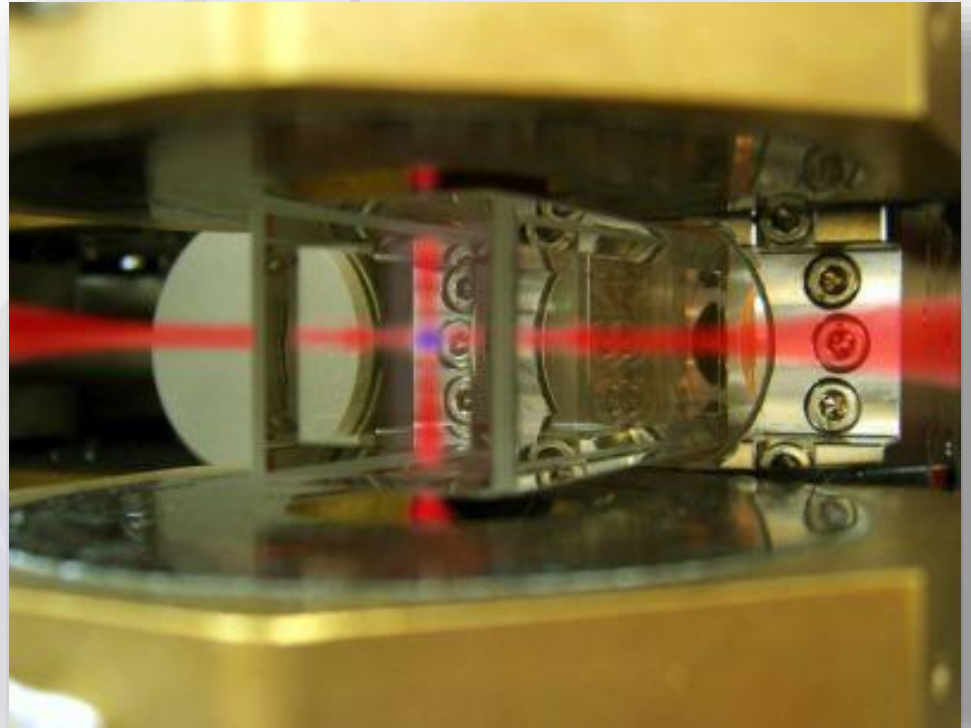
# Ultracold atoms: Isolated many-body systems



Bose-Einstein condensate of Potassium  $^{39}\text{K}$  atoms



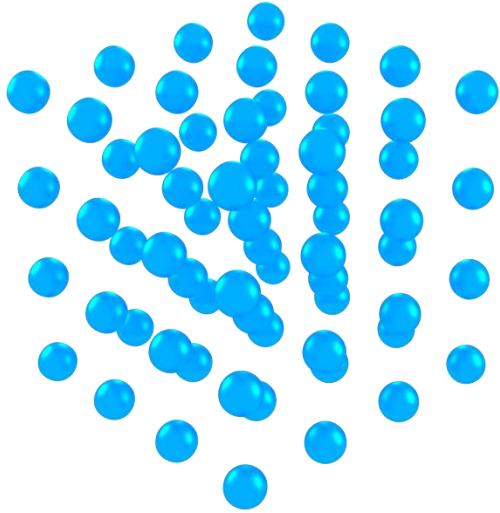
Degenerate Fermi gas  $^{40}\text{K}$



Held by classical magnetic and laser fields

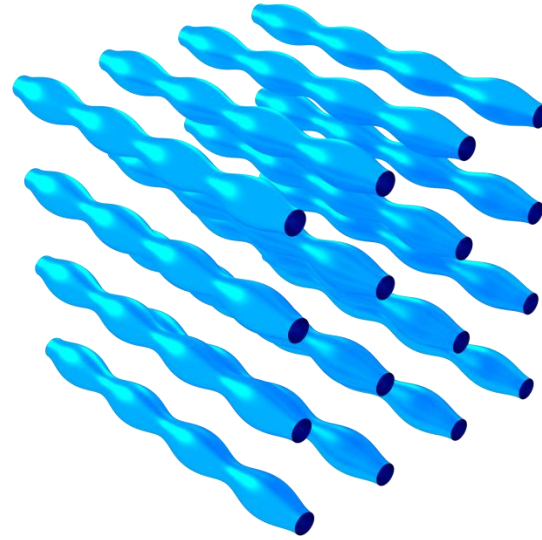
**No thermal environment**

# Controlling dimensionality



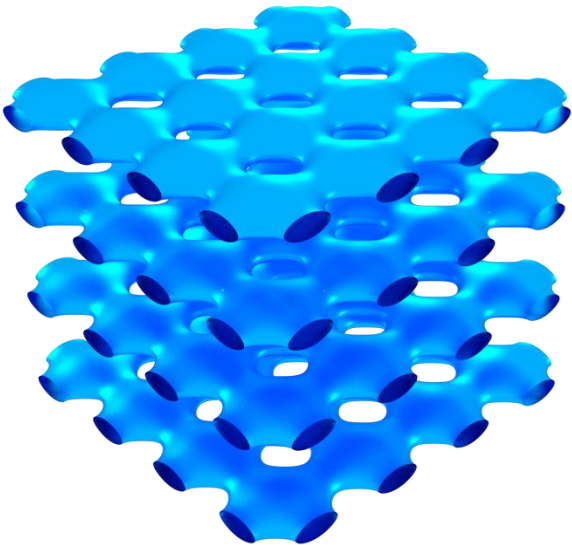
**0D**

- 3D deep lattice
- isolated wells
- no hopping

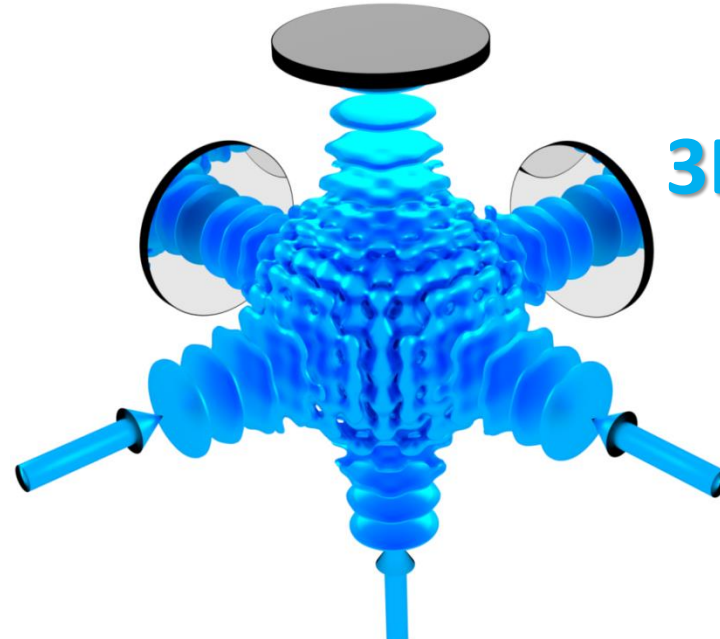


**1D**

- 2D deep lattice
- +1D weaker lattice
- 1D hopping



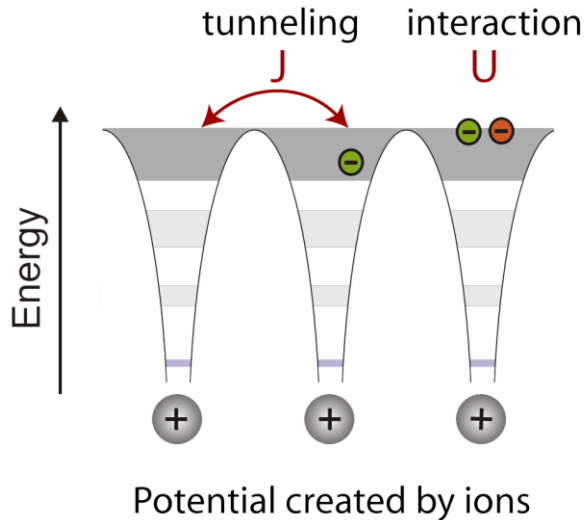
**2D**



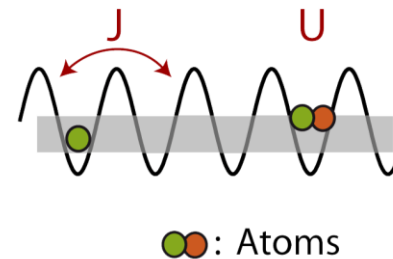
**3D**

# Simulating condensed-matter

- ▶ Realizes important model Hamiltonians from solid-state physics:
  - e.g. Hubbard models



$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

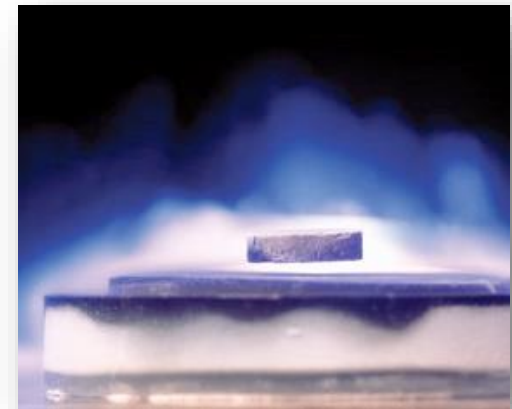


Potential created by standing light wave

## Understand and Design Quantum Materials

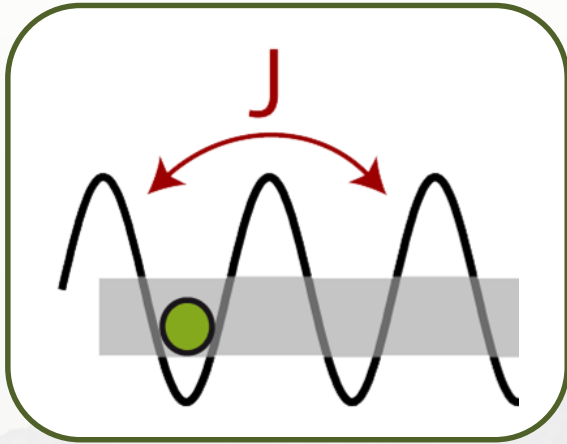
- ▶ High temperature superconductivity
- ▶ Quantum Magnetism

Emergent many-body phenomena



YBCO

# Non-Equilibrium physics



$$J \sim 1 - 1000 \text{ Hz}$$

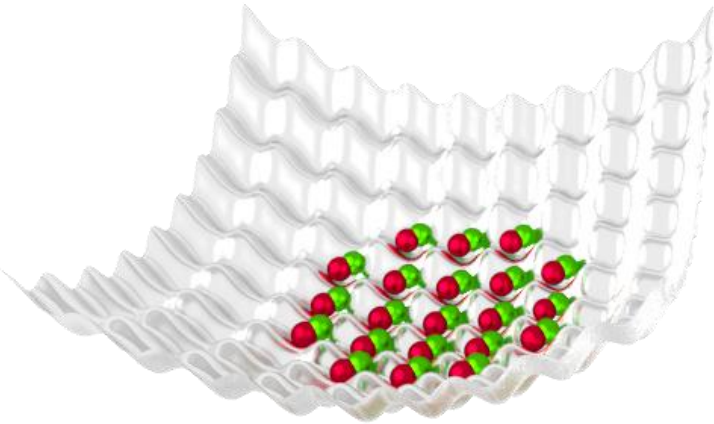
$$\tau = \frac{\hbar}{J} \sim 0.1 \text{ ms} - 1 \text{ s}$$

- ▶ Can control and observe real-time dynamics

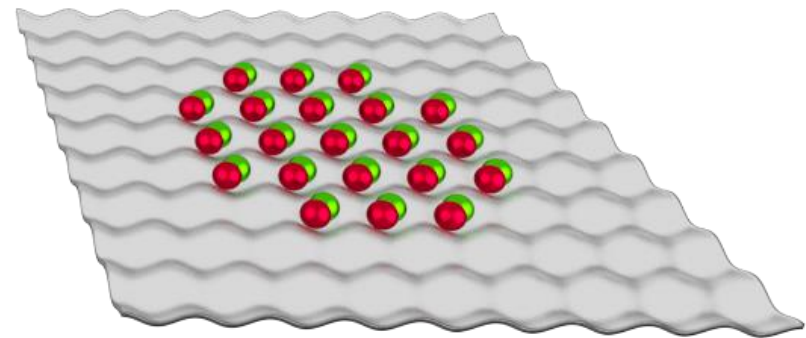
# Fermionic Expansion

fermionic  $^{40}\text{K}$

2D

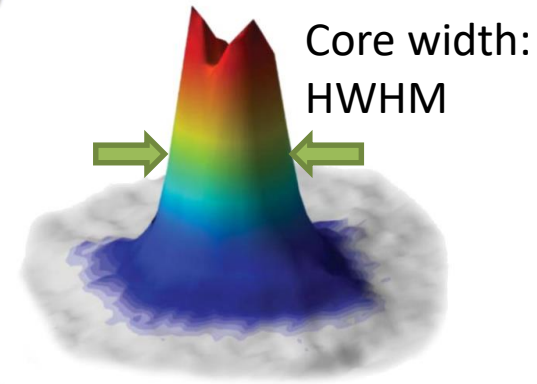
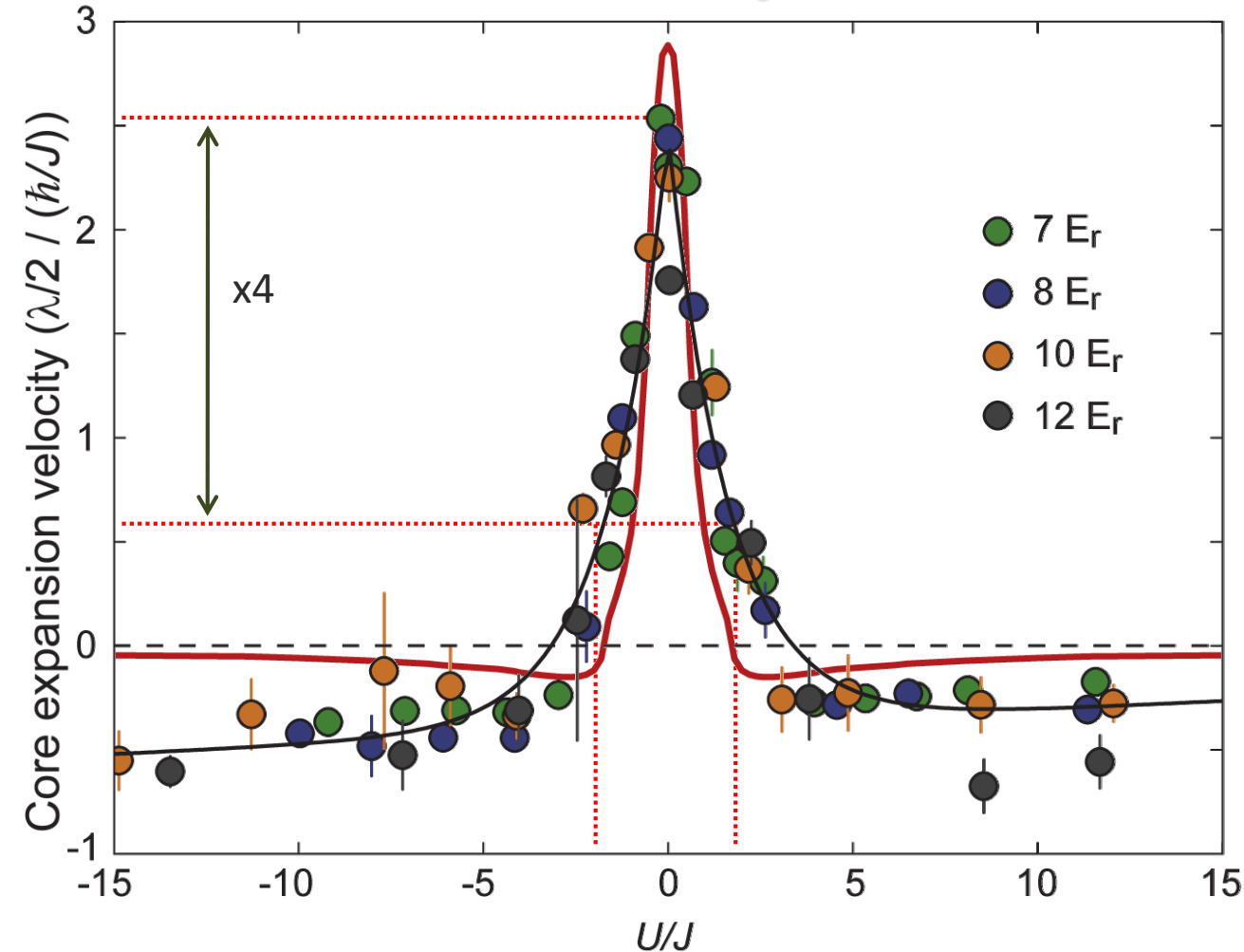


band insulator: two spin states



Dynamics **within** lattice

# Fermionic Expansion velocity in 2D



No qm calculations possible  
Too complex??

red line:

Boltzmann equation  
in relaxation time  
approximation  
(A. Rosch *et al.*)

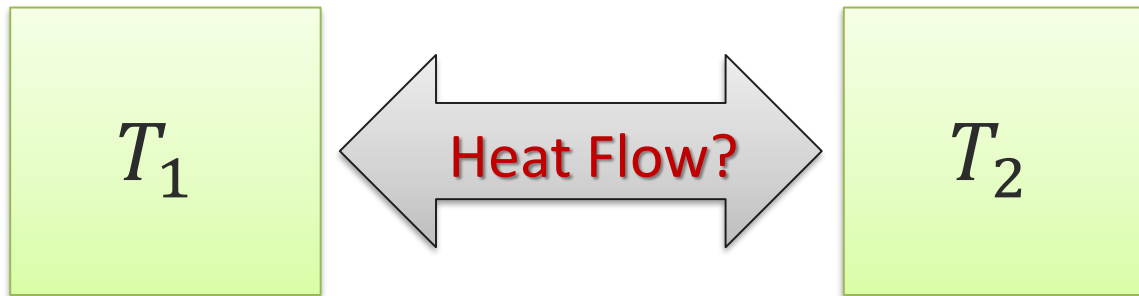
- ▶ **fast local thermalization** due to frequent scattering
- ▶ slower global dynamics driven by gradients in temperature & chemical potential



**Negative absolute temperatures**



# Temperature



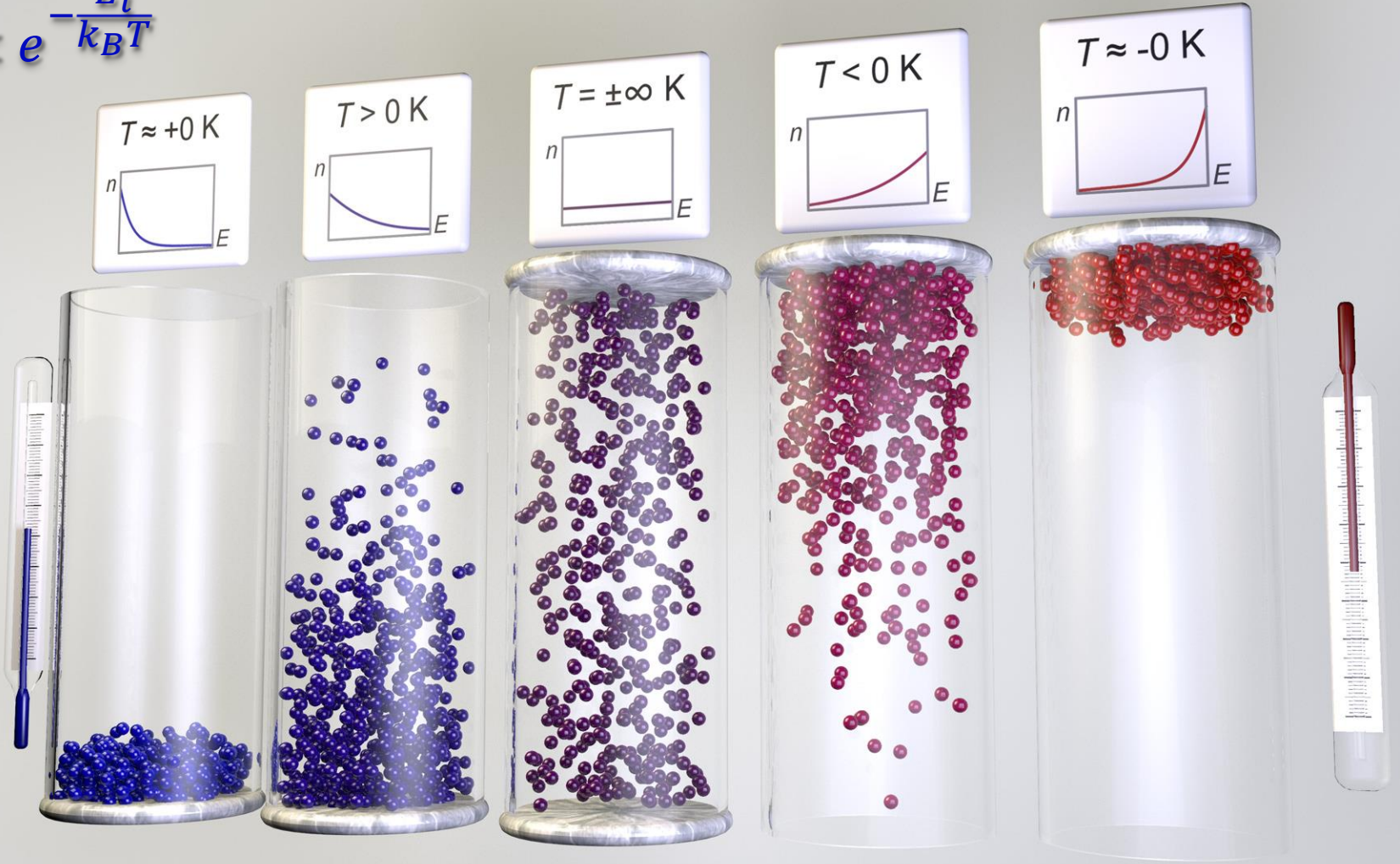
- ▶ Heat always flows from the hotter to the colder system, until both systems have the same temperature

**Temperature defines an ordering relation between systems!**

# Thermal states: Canonical distribution

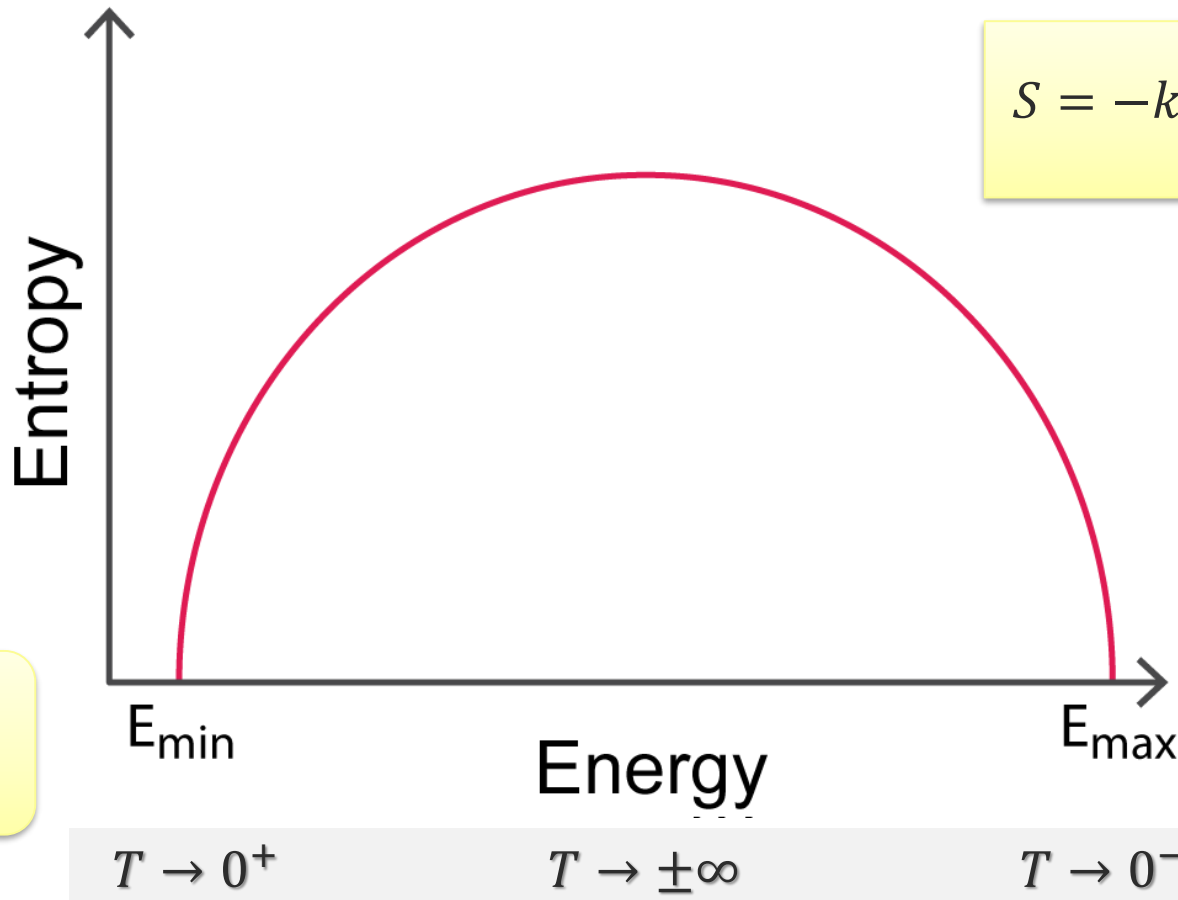
$$p_i \propto e^{-\frac{E_i}{k_B T}}$$

Total energy per particle ↑



Negative Temperatures are *hotter* than all positive temperatures

# Energy-Entropy relation



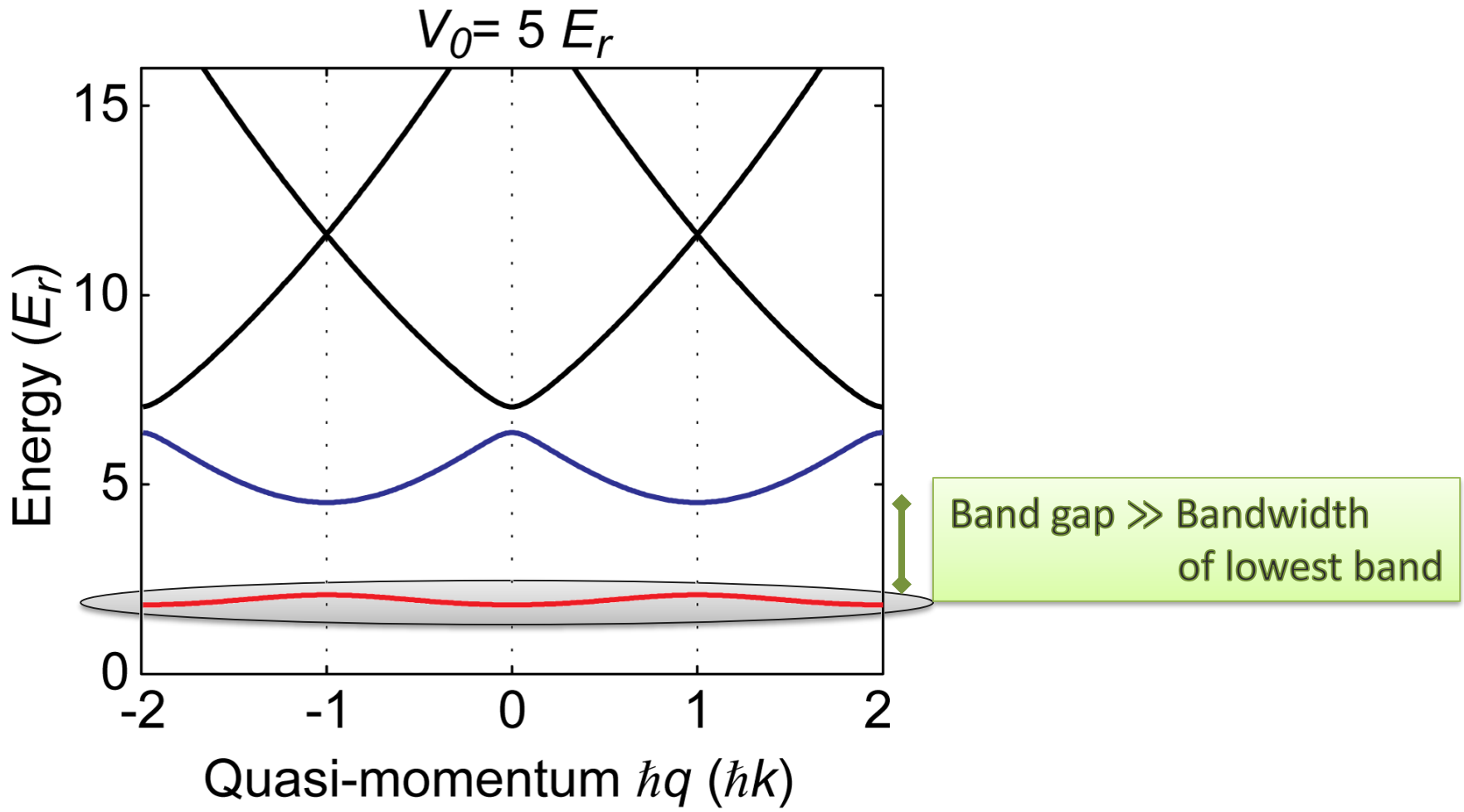
$$S = -k_B \sum_i p_i \log p_i$$

(canonical)

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

Requirement: Hamiltonian locally bounded from above:  $\frac{E}{N} \leq \epsilon_{\max}$

# Optical lattice band structure (1D)



→ kinetic energy is bounded from above and below

# How to get to negative Temperatures?

- ▶ Heat, Heat, Heat, ?

*Impossible*: Above  $T = \infty$  entropy decrease again  
→ Cannot dissipate work in heat anymore

- ▶ Quasi-static state change ?

*Impossible*: No (classical) adiabatic path can change sign of  $T$  (Landsberg 1959)

- ▶ „Flip“ the energy axis:

$$" \hat{H} \Rightarrow -\hat{H} "$$



Mott insulator:

$$U \Rightarrow -U$$

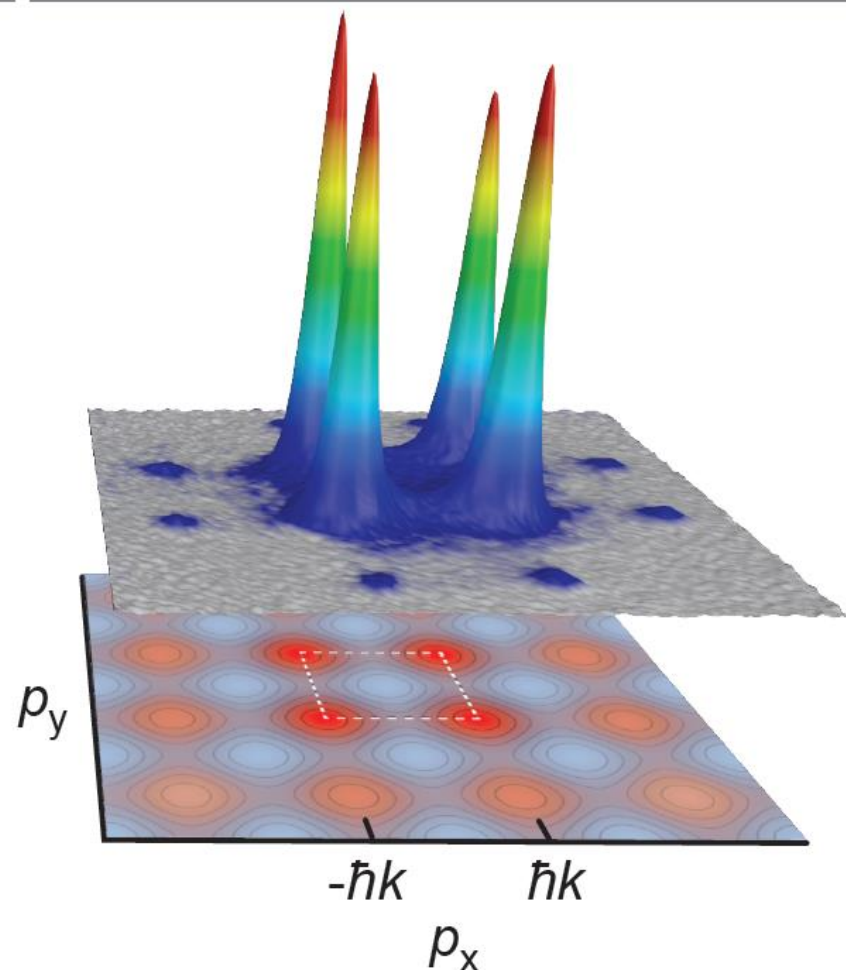
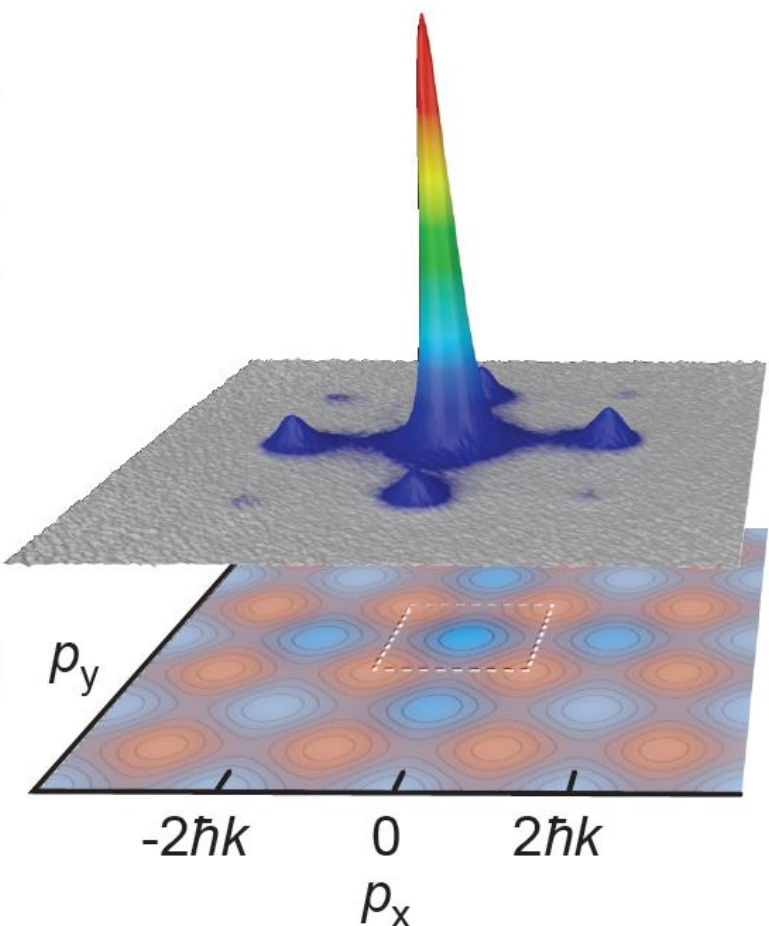
→ Feshbach resonance

# Bose gas at pos. and negative Temperature

$T, U, V > 0$

$T, U, V < 0$

Occup. (a. u.)  
Energy



# Are negative temperatures stable?

*In isolated systems:* Yes!

Due to energy conservation they cannot relax to positive temperatures.

(Same argument as stability of isolated large positive temperatures.)

*In contact with an environment:*

*Yes, if environment also at negative  $T$ .*

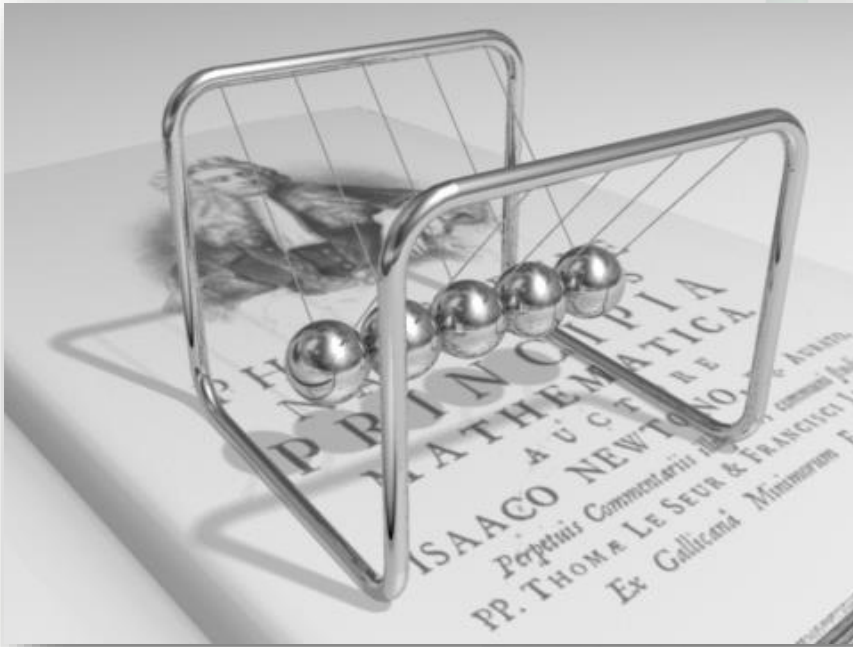
$T > 0$   $\leftrightarrow$   $T < 0$

“equivalent” to

matter  $\leftrightarrow$  antimatter

both stable on their own,  
but do not mix!

# Dynamics in different dimensions



1D: **NO** Thermalization

(Proximity to) Integrability



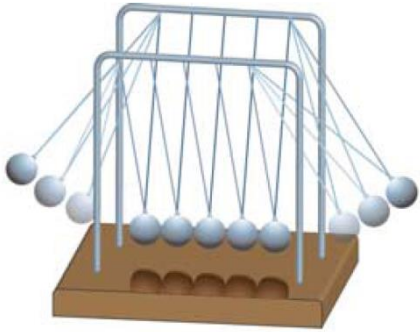
2D: Thermalization

$$p_i \propto \exp\left(-\frac{E_i}{k_B T}\right)$$

Independent of all other  
initial conditions



# What can be different in 1D?



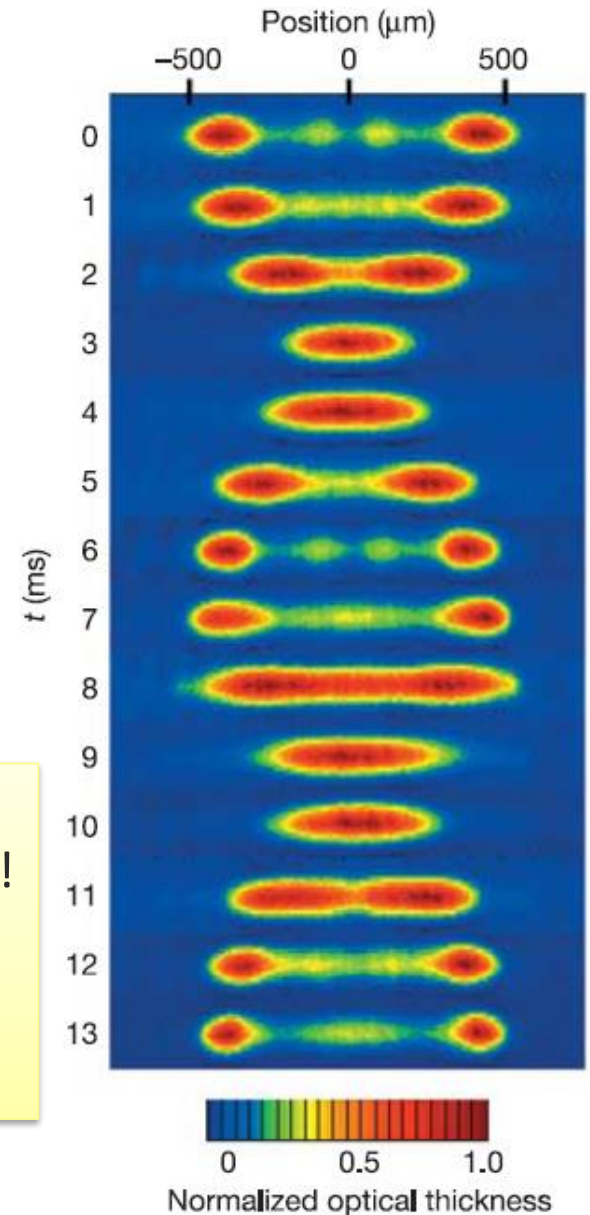
Classically: Two-body collisions can only exchange momentum, but *not redistribute* it!

$$n(k, t) = \text{const. w.r.t. } t$$

Repulsive 1D Bosons with point-like interaction without a lattice are *integrable* in homogeneous case!

→ Lieb-Liniger model

Thermalization constrained by conserved quantities.

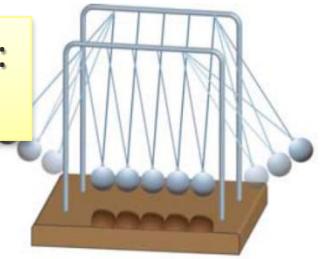
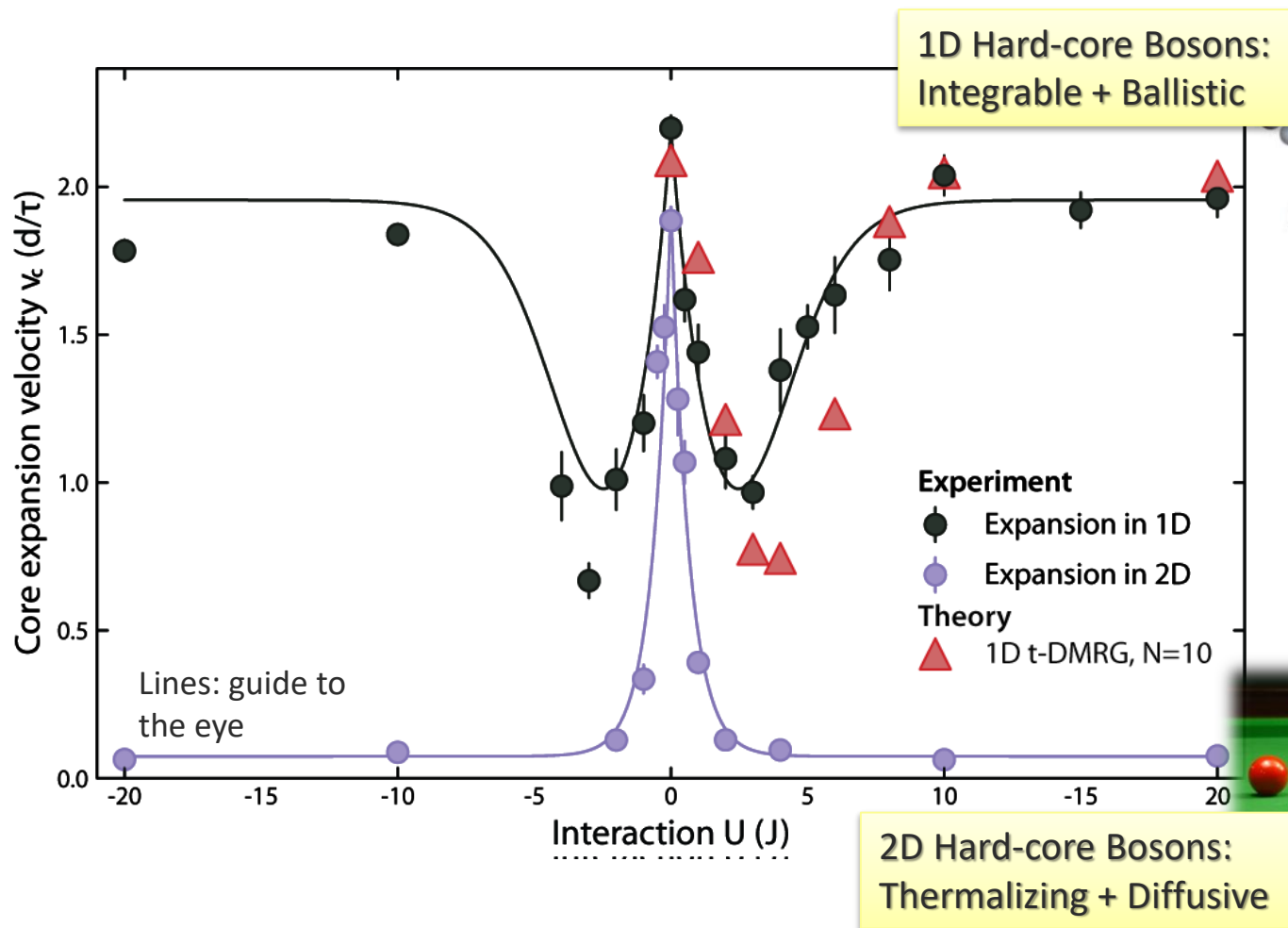


T. Kinoshita *et al.*, Nature (2006)

# 1D Bosons on a lattice

- ▶ 1D Bose–Hubbard model is (in general) *not integrable!*  
classically chaotic for intermediate  $U$  and intermediate energy  
M. Hiller et al. PRA **79**, 023621 (2009)
- ▶ Integrable limits:
  - Non-interacting
  - Hard-core Bosons:  $U \gg J$ ,  $n \in \{0, 1\}$   
i.e. no higher occupancies  
equivalent to **non-interacting spinless Fermions**  
(Jordan–Wigner transformation)

# Bosonic Expansion velocities



In general, no exact calculations available for  $D > 1$

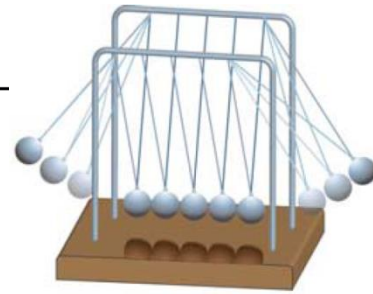
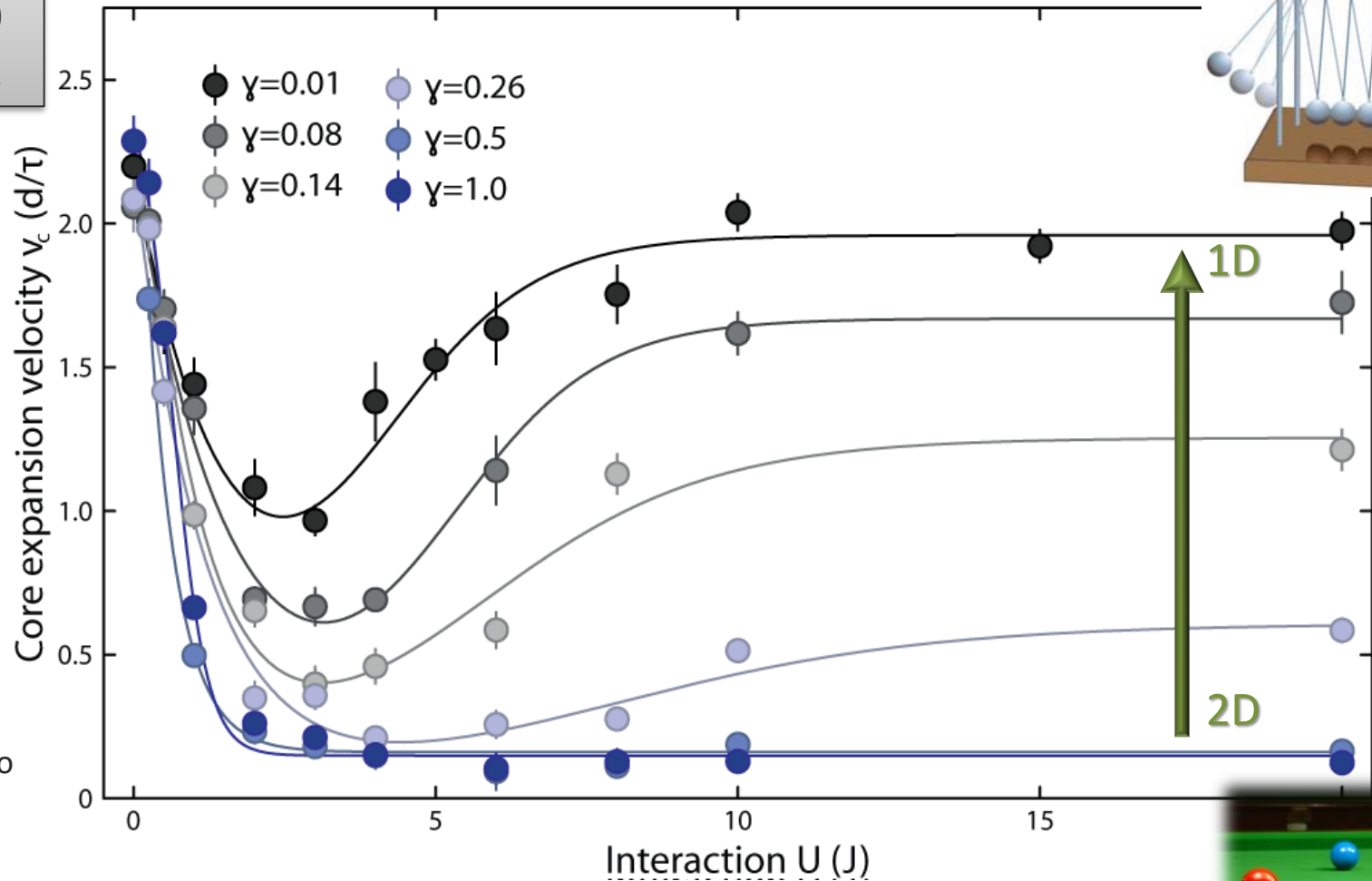
→ Quantum Simulations

$$\gamma = \frac{J_y}{J_x}$$

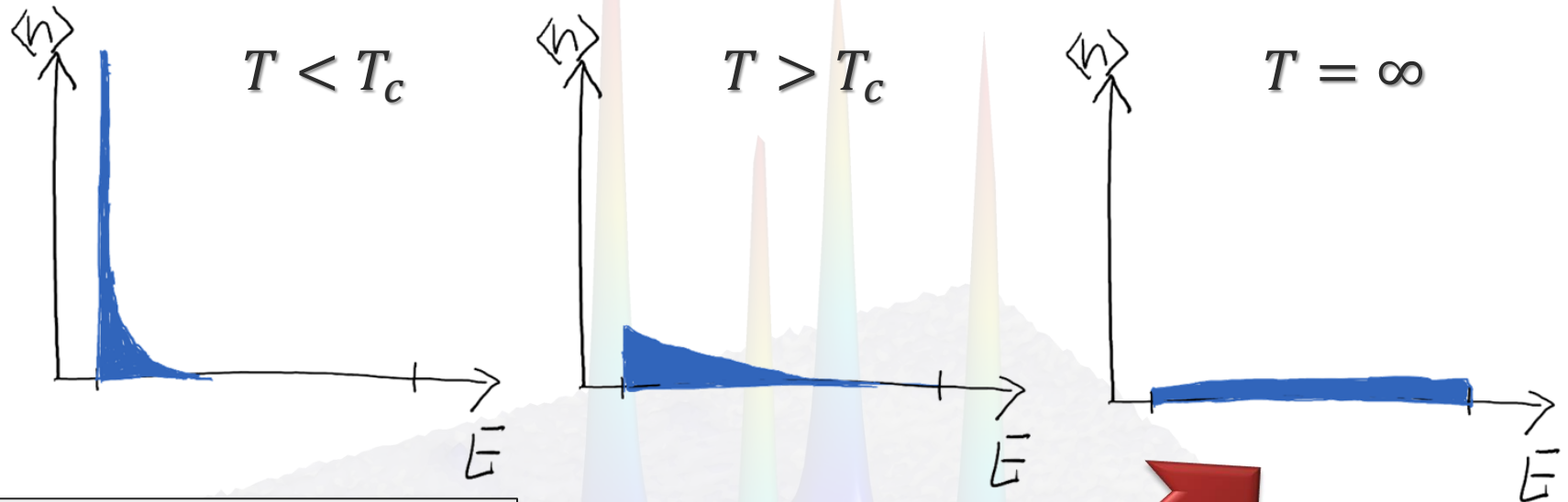
1D:  $\gamma = 0$

2D:  $\gamma = 1$

# 1D-2D Crossover



# Thermal states of nearly free Bosons



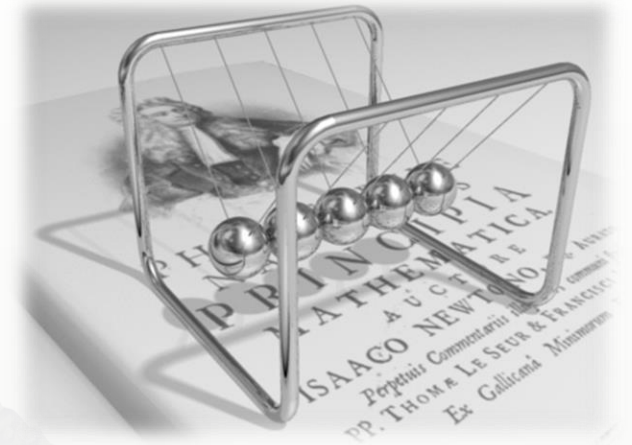
Can Bosons also bunch in the middle of the spectrum?

Not in thermal equilibrium!

Thermalisation

# 1D Hard-Core Bosons on a lattice

- Hard-core Bosons:  
Jordan-Wigner Transformation



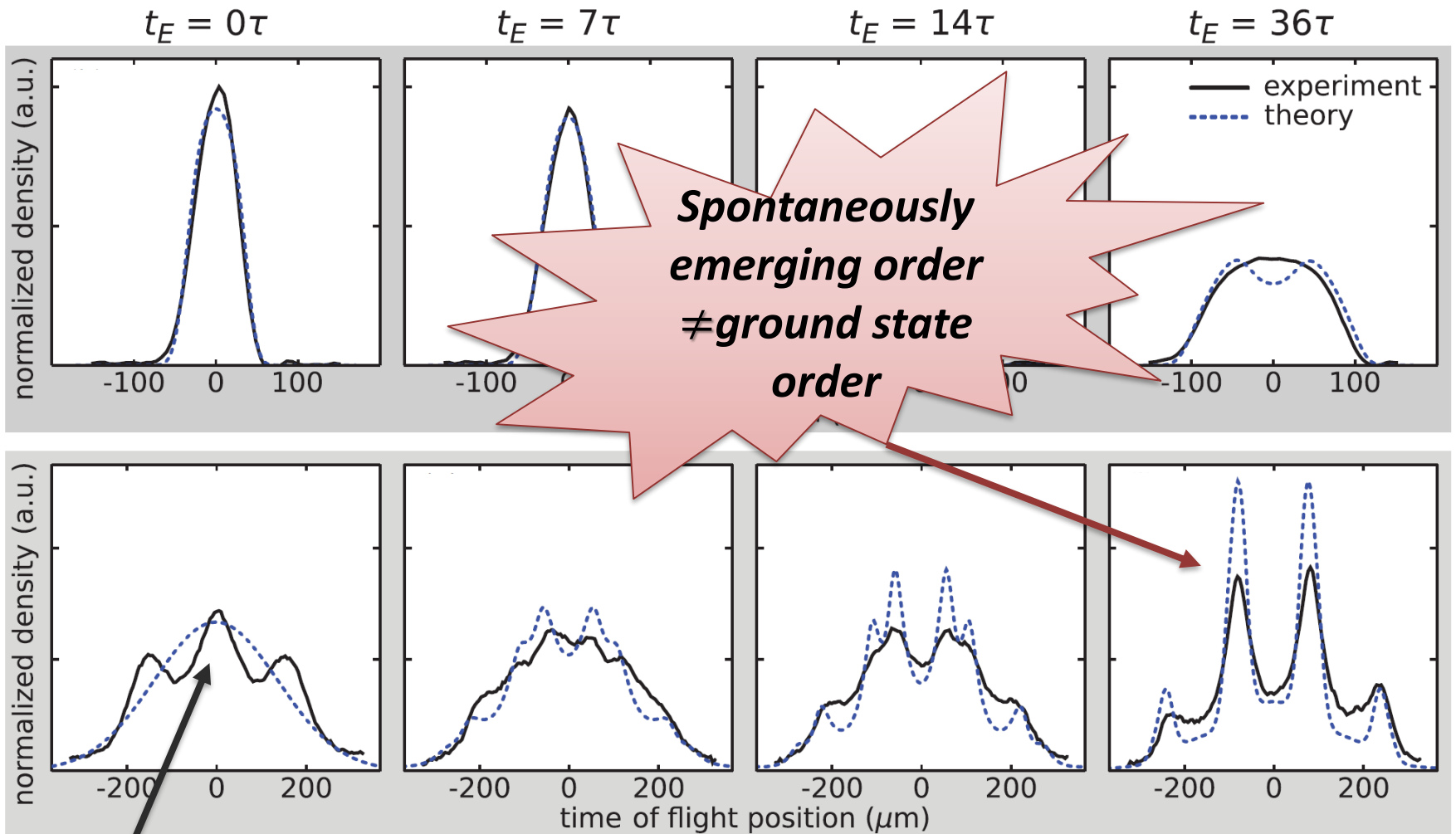
$$n_r^B(t) = n_r^F(t)$$

$$n_k^B(t) \neq n_k^F(t)$$

Experiments:

Paredes, Bloch, Weiss, Nägerle,...

# Emergence of correlations

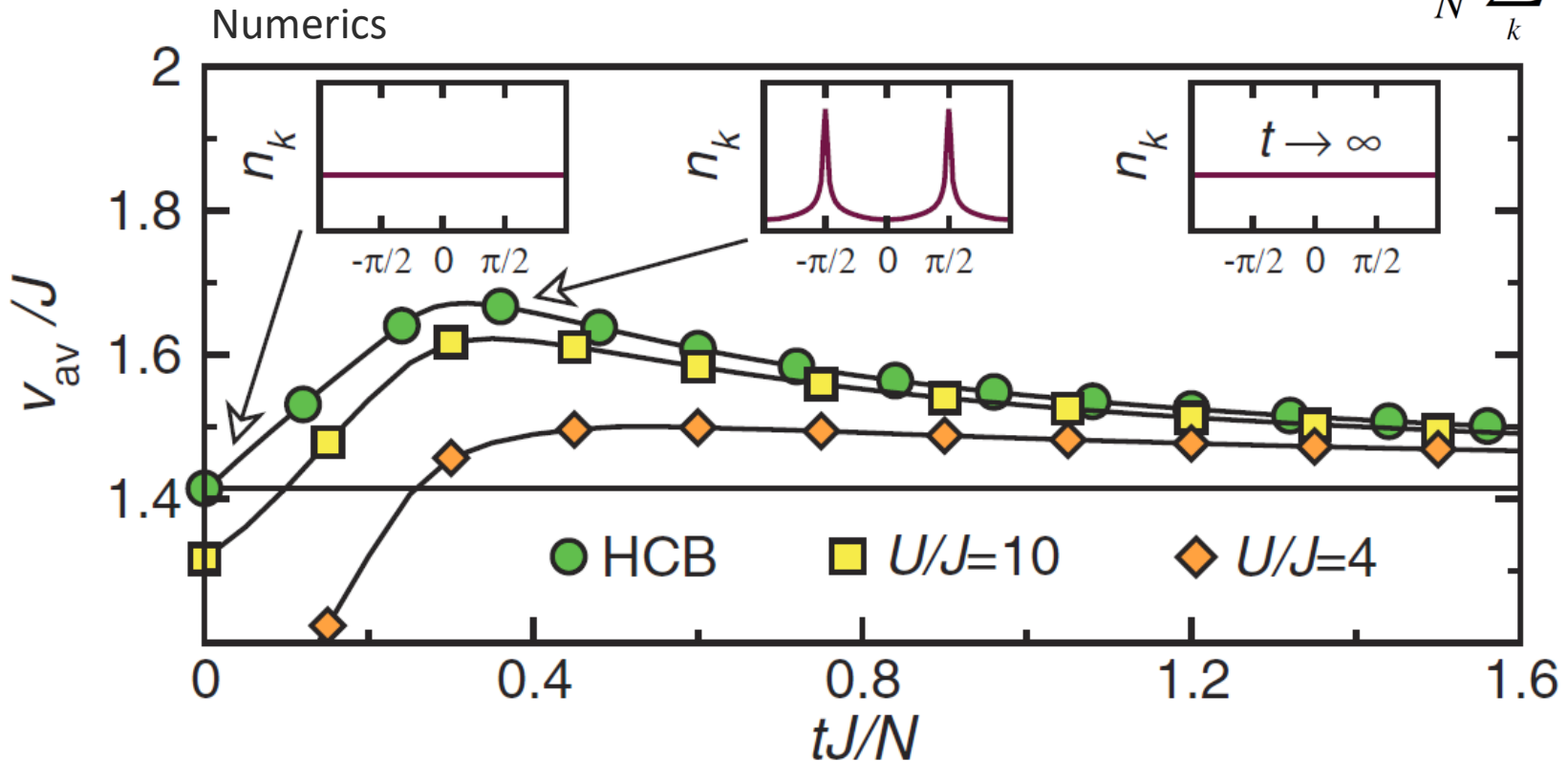


Residual  $q = 0$   
coherence  
in initial state

- ▶ Lattice pulsed to broaden Wannier envelope
- ▶ Finite time-of-flight

# Long time behaviour of expanding 1D HCB

$$v_{\text{av}}^2(t) = \frac{1}{N} \sum_k n_k(t) v_k^2$$



- ▶ Quasicondensation is transient effect
- ▶ Long times: Fermionization
- ▶ Timescales depends on chain length
- ▶ Experiment done in parallel on different chains





**Robust alternatives  
to thermalization ?**

# Localization

▶ Anderson (1958):

A single particle in a disordered potential can become localized by disorder  
→ **Anderson localization**



**1D:** arbitrarily small disorder localizes Eigenstates *at all energies*  
→ *quantum-mechanical interference effect*

▶ **Interactions: Many-body localization**

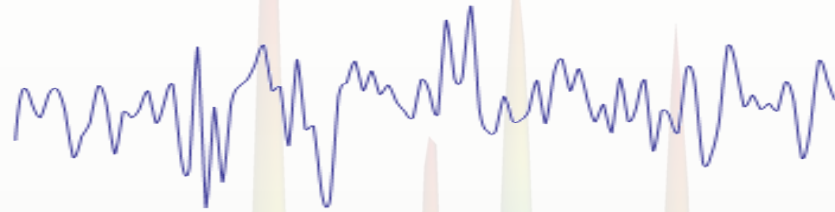
**Theory: Yes!** D. M. Basko, I. L. Aleiner, B. L. Altshuler  
+ *essentially everyone* (since 2005)

**Experiments:**

Cold Atoms (Aspect, Modugno, DeMarco, Schneble, ...)

Ions (Monroe), NV Centers (Lukin), Disordered superconductors (Sharhar)

# Many-body localization



Stability of (disorder induced) Anderson localization  
**in the presence of interactions (and finite energy density)**

***So what?***

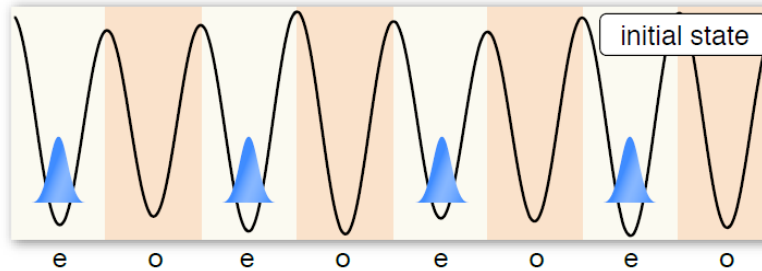
**Non-ergodic behaviour!**

*No thermalization, no standard statistical mechanics*

**→ *Potential for novel long-time dynamics***

# Ergodicity breaking in Many-body localization

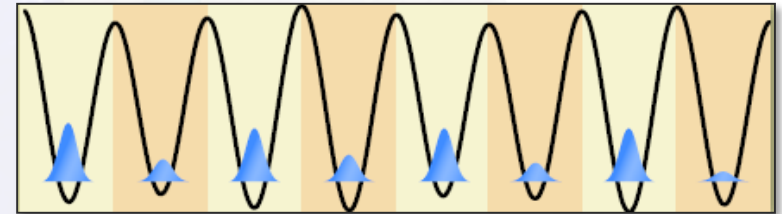
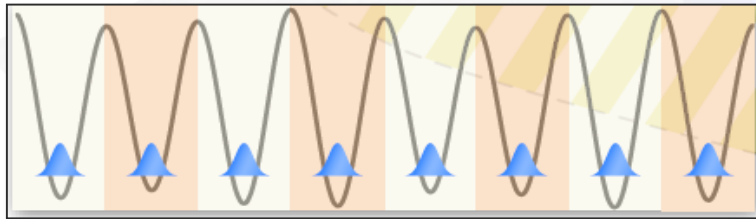
Initial state



► fermionic  $^{40}\text{K}$



► free evolution



Ergodic time evolution  
destroys initial CDW

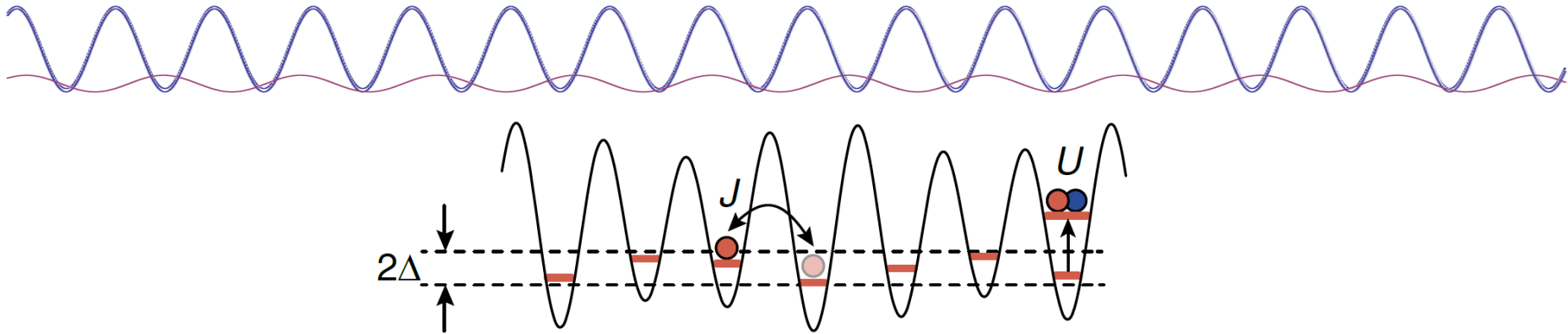
*Imbalance*

$$I = \frac{N_e - N_o}{N_e + N_o}$$

***Persistent CDW signals  
non-ergodic behavior  
→ localization***

# Aubry-André model

- ▶ Superimpose two *in-commensurable* lattices ( $\lambda_s \approx 532 \text{ nm}$ ,  $\lambda_d \approx 738 \text{ nm}$ )  
→ projected version of 2D Harper hamiltonian



$$H = -J \sum_{i,\sigma} \left( \hat{c}_{i,\sigma}^+ \hat{c}_{i+1,\sigma} + h.c \right) + \Delta \sum_{i,\sigma} \sin(2\pi\beta i + \phi) \hat{c}_{i,\sigma}^+ \hat{c}_{i,\sigma}$$

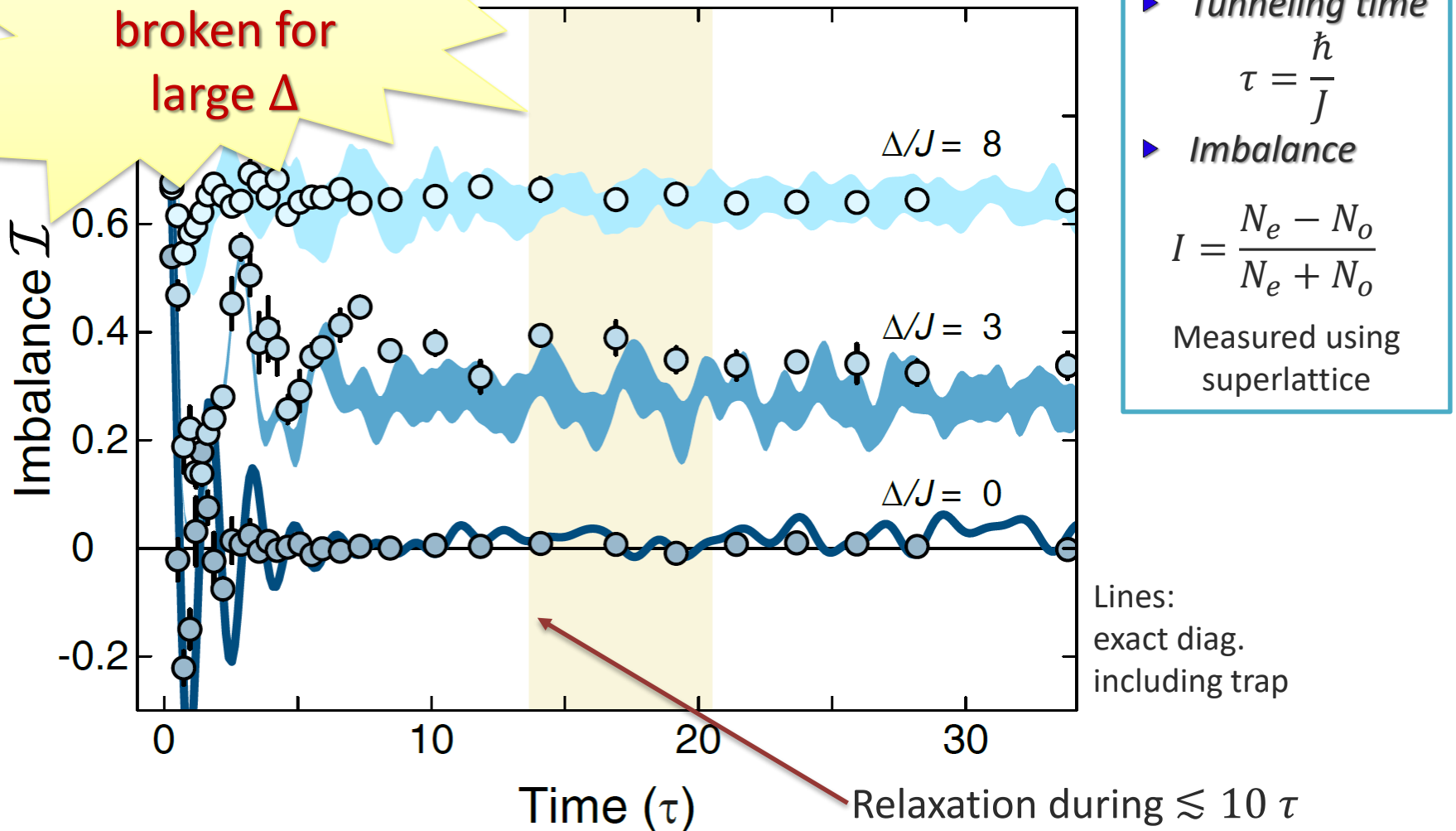
↑  
irrational

- Real random: Localization for  $\Delta > 0$
- Quasi-periodic: Localization for  $\Delta > 2J$
- Critical behaviour controlled by  $\beta$ !

# Localization in Aubry-André model

- ▶ Interacting (spin) fermions

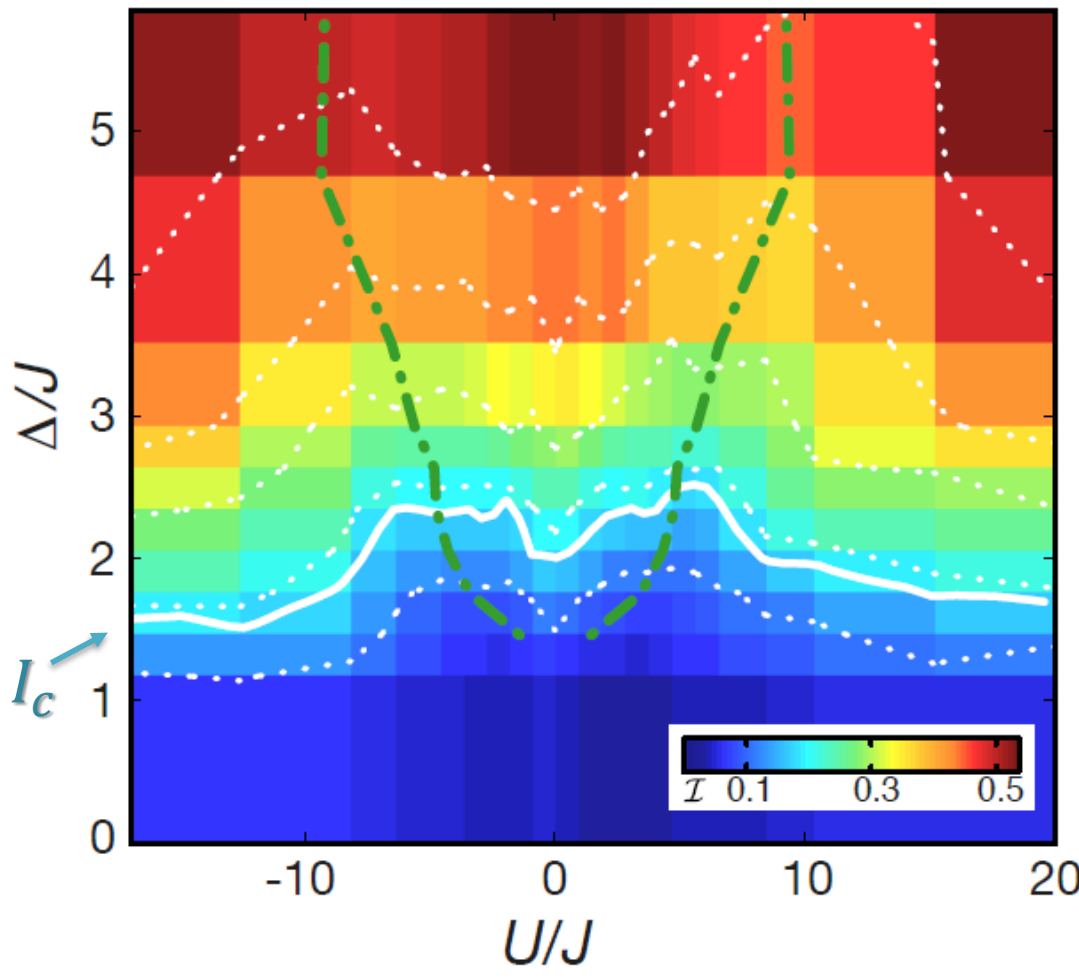
Ergodicity  
broken for  
large  $\Delta$



# Many-body localization

- ▶ Remaining CDW after  $t \approx 15 - 20 \tau$

Two-component  
Fermi gas



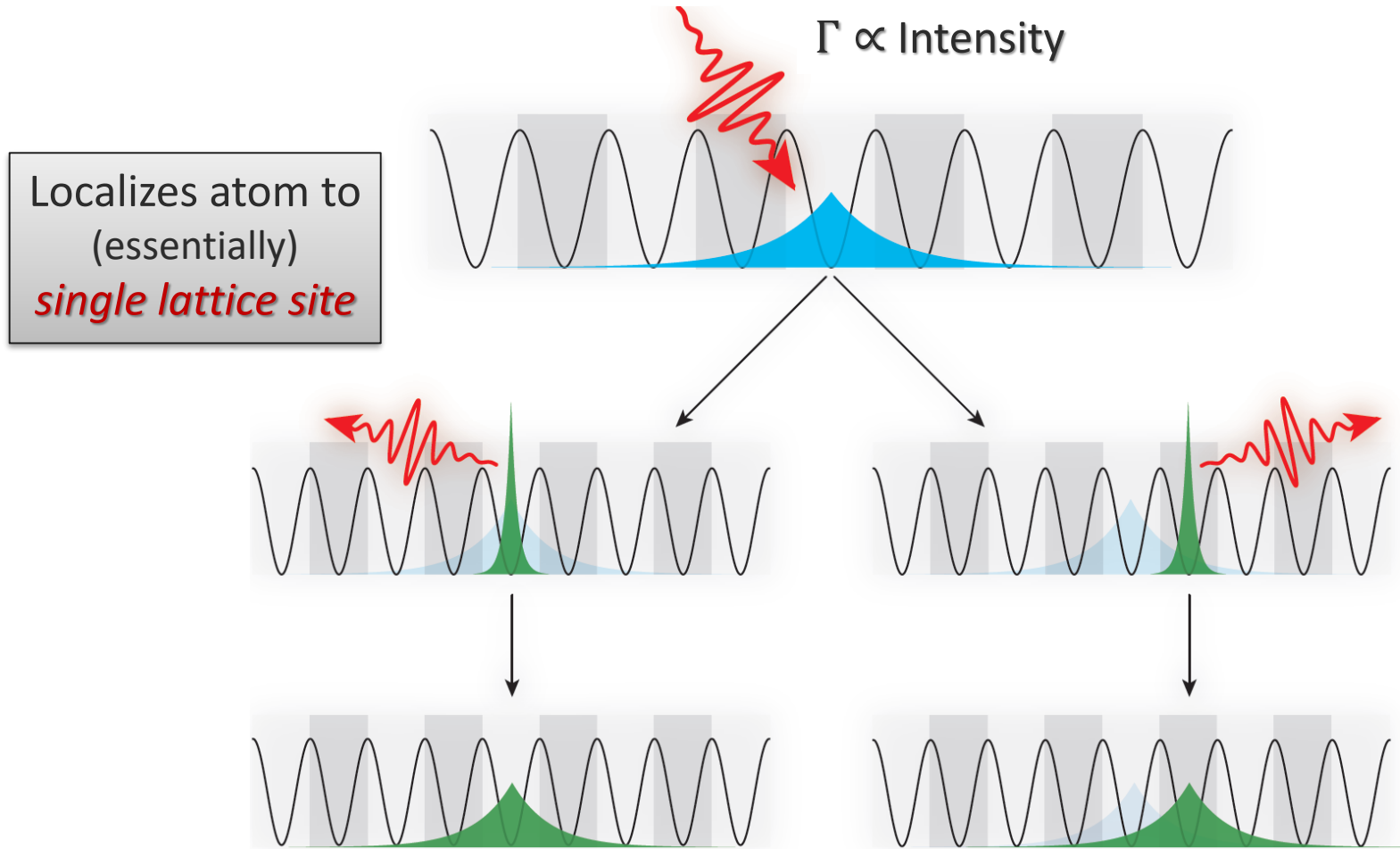
Ergodicity broken also for  
interacting atoms

→ direct observation of  
Many-body localization

Deep in localized phase  
→ Particles only probe their  
direct surrounding,  
→ no differences between  
quasi-periodic and  
disordered

# Photon Scattering

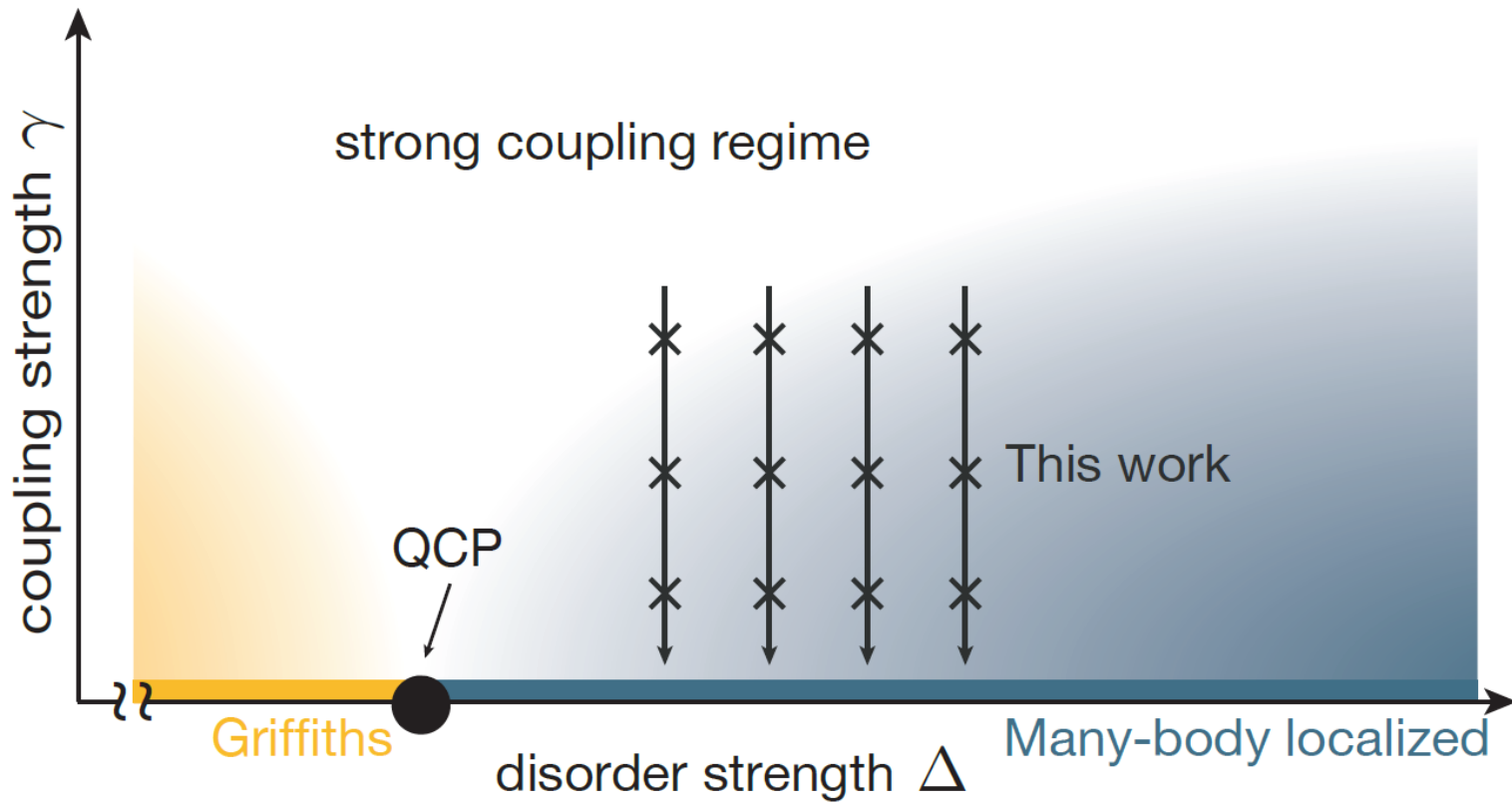
Coupling an MBL system to a  $T = \infty$  bath.



→ Photon induced hopping: Randomizing positions for  $\Gamma t \rightarrow \infty$

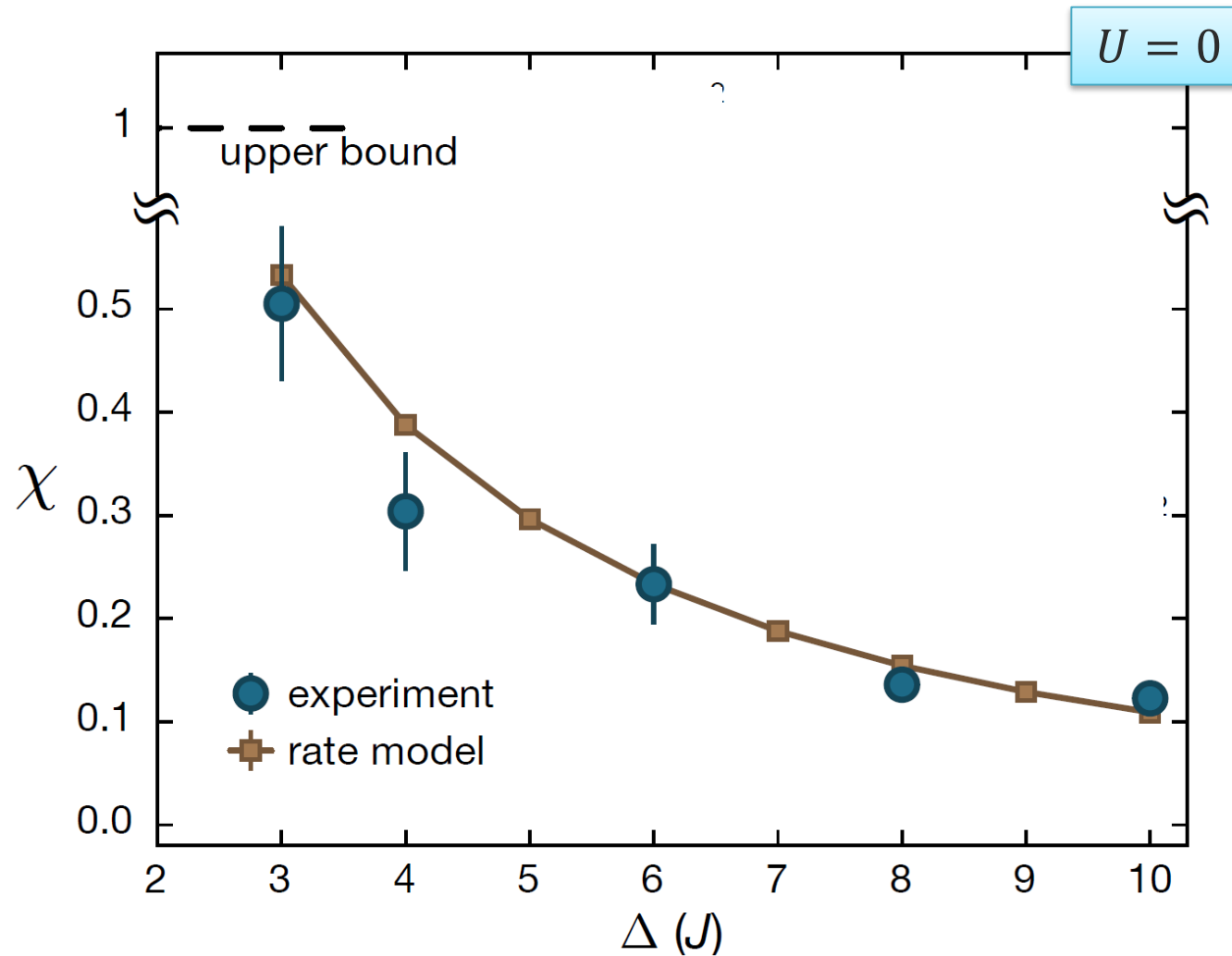


# Closed System Phase transition?



→ Susceptibility  $\chi$  expected to diverge at phase transition

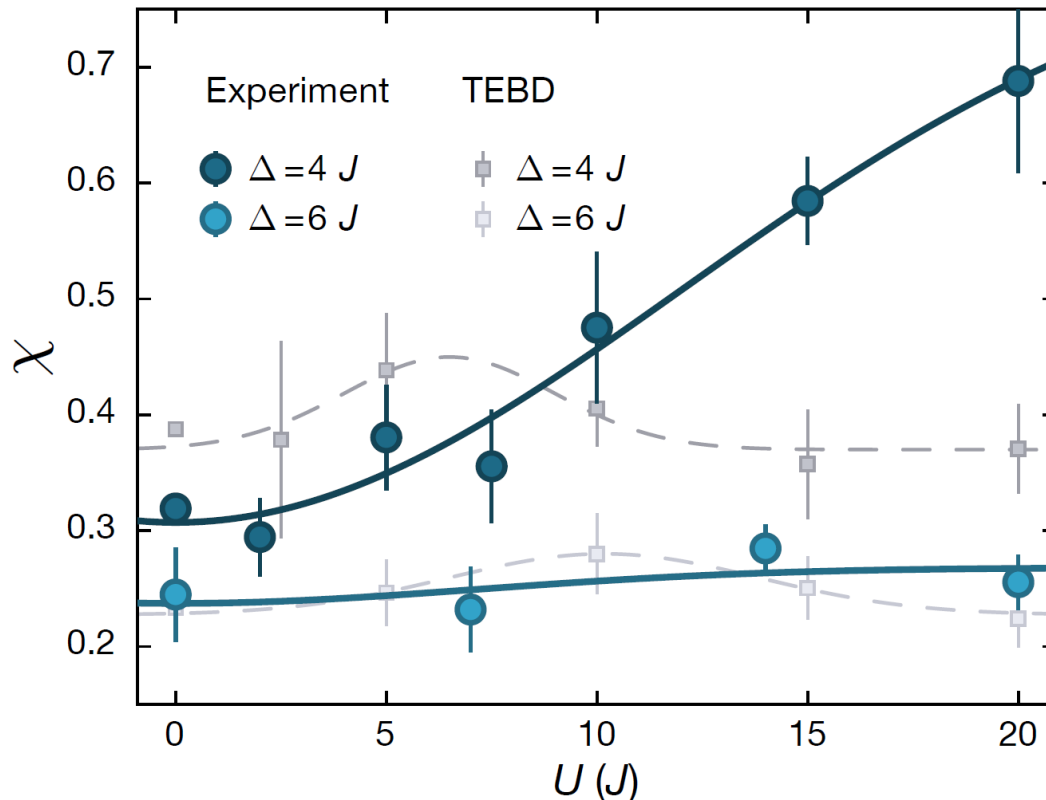
# Imbalance dynamics



► Strong dependence on localization length

# Disorder & Interactions

No re-entrant  
behaviour in  
Experiment!

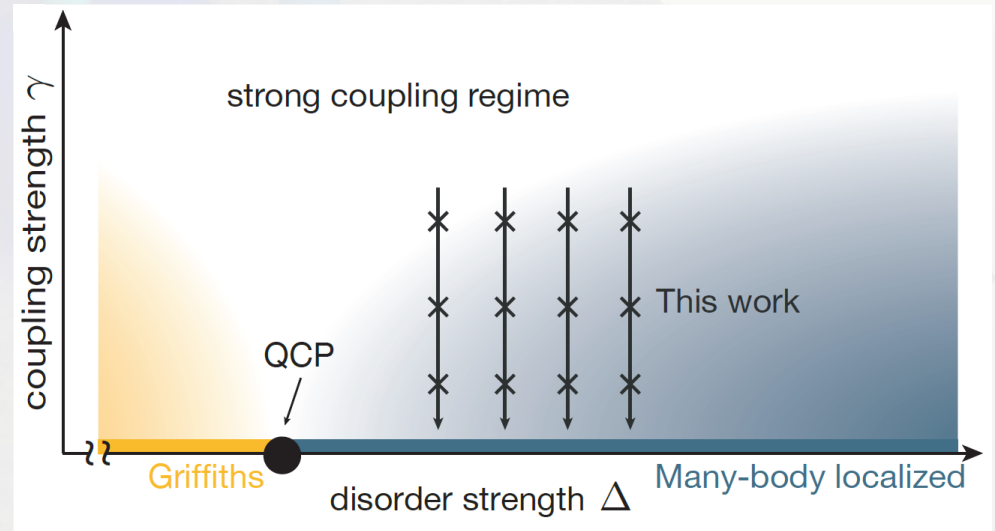
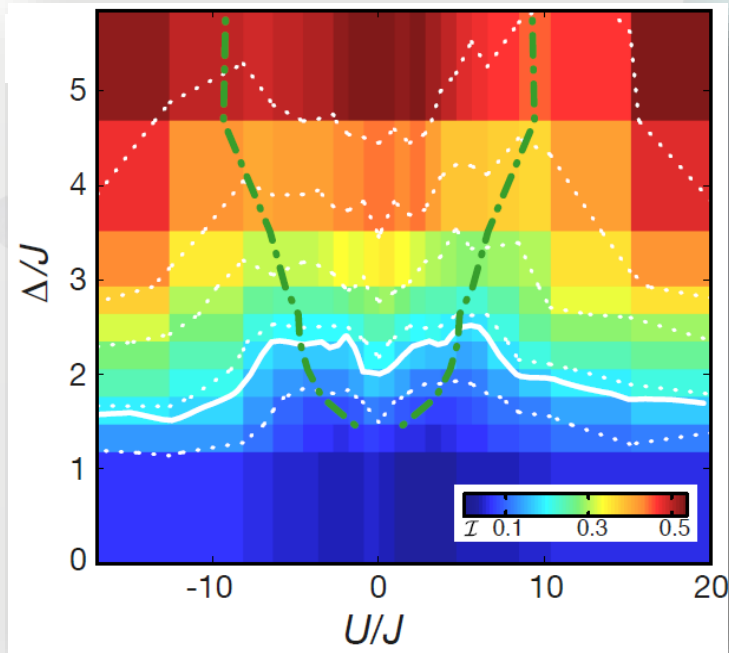
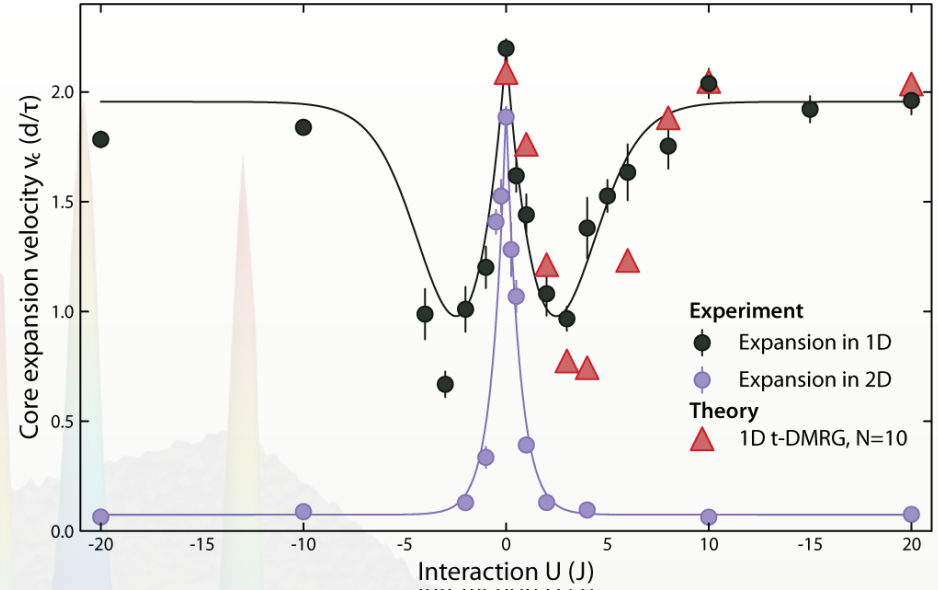
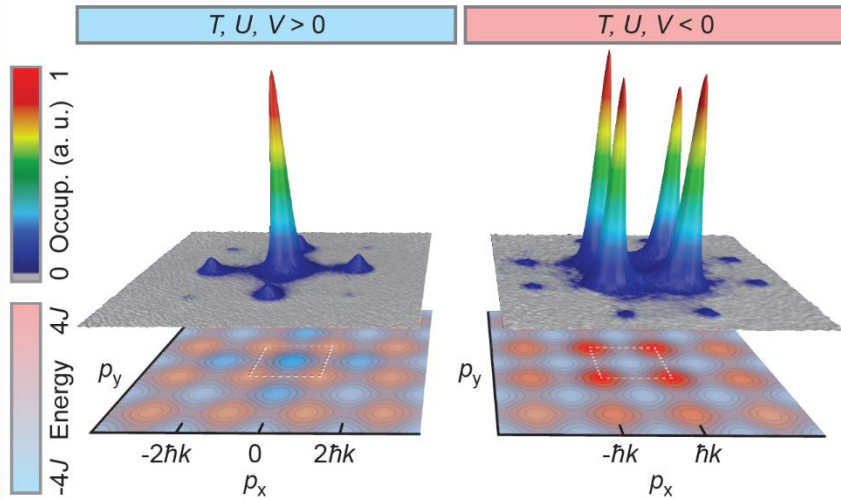


Increasing susceptibility  
with interaction  
at small disorder!

→ *consistent with MBL*

**Challenge:**

*Losses and population of non-localized band become relevant*



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www.manybody.phy.cam.ac.uk