The Thermodynamics of Causal Modelling

Felix Binder

NANYANG TECHNOLOGICAL UNIVERSITY

29 June 2018
KITP

Collaborators

Andrew Garner

IQOQI (Vienna)

Jayne Thompson

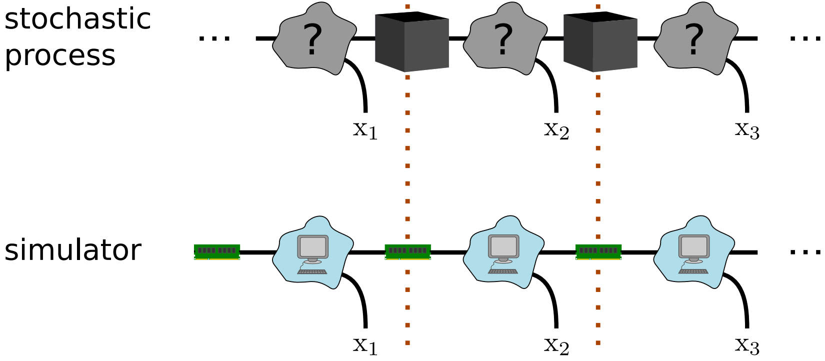
CQT (Singapore)

Mile Gu

NTU & CQT (Singapore)

e-mail: **quantum@felix-binder.net**

group website: **quantumcomplexity.org**



In this talk: classical processes, quantum modelling

Think of:

stock price, weather, neural spike trains, scan through spin chain...

Example: perturbed coin

- probability to flip: p
- probability to show the same side again: $1 - p$

Stochastic Processes (disrete-time & discrete-valued)

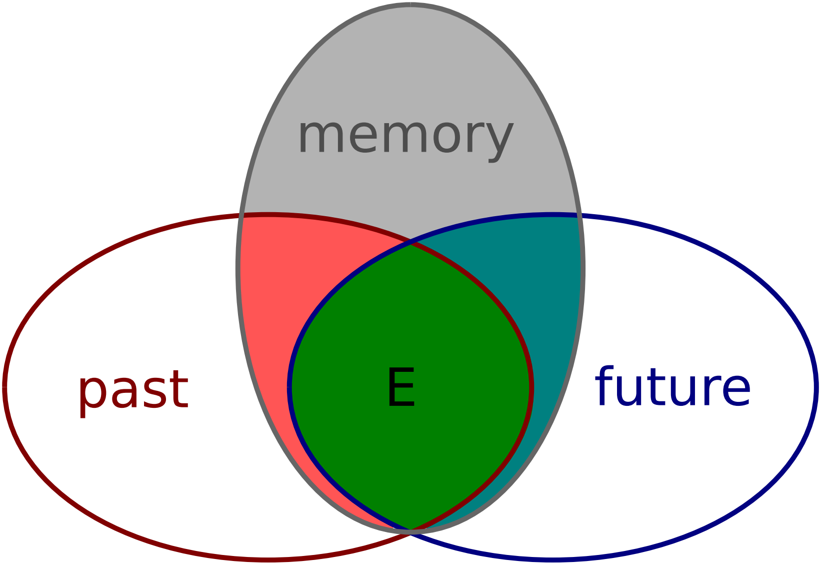
Ingredients

- Alphabet $\mathcal{A} = \{r_1, r_2, \ldots, r_\alpha\}$
- Bi-infinite sequence of random variables X_t:
  $\overleftrightarrow{X} := \ldots X_{t-2} X_{t-1} X_t X_{t+1} \ldots$
- Conditional probabilities:
  $p(X_{t:\infty} = x_{t:\infty} | X_{-\infty:t} = x_{-\infty:t})$
- Stationarity:
  $$p(X_{t:\infty} = \overrightarrow{x} | X_{-\infty:t} = \overleftarrow{x}) \\ = p(X_{t+L:\infty} = \overrightarrow{x} | X_{-\infty:t+L} = \overleftarrow{x}) \\ = p(\overrightarrow{x} | \overleftarrow{x})$$

perturbed coin

- $\mathcal{A} = \{\mathrm{H}, \mathrm{T}\}$
- …HHHTTHTTTHH…
- $p(H_t \ldots | \ldots T_{t-1}) = p$
- $p(H \ldots | \ldots T) = p$

crypticity, statistical complexity, and oracular information



Applying Ockham's razor to Ockham's pool

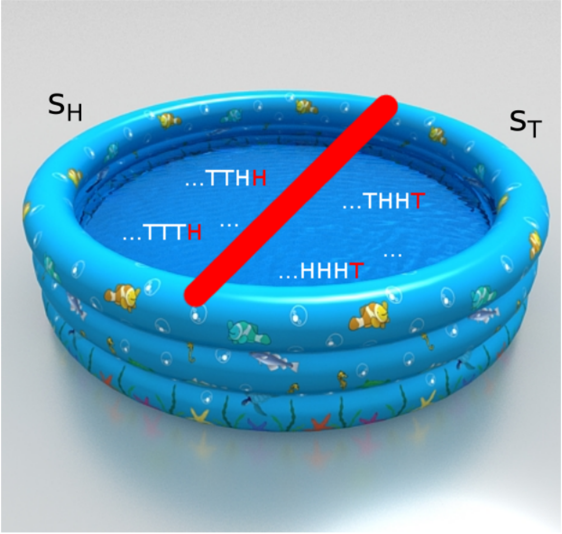
Causal states

Group all histories according to equivalence relation:

$$\overleftarrow{x} \sim_\epsilon \overleftarrow{x}'$$

iff

$$p(\overrightarrow{X}|\overleftarrow{x}) = p(\overrightarrow{X}|\overleftarrow{x}')$$



ϵ-machines

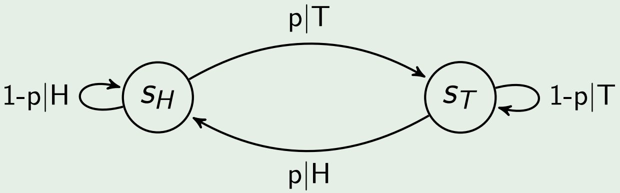
- Alphabet $\mathcal{A}$
- Set of causal states

$$\Sigma = \{s_1, s_2, ..., s_N\}$$

- Transition matrix

$$T^x_{j|i} = p(x, s_i|s_j)$$

Perturbed coin



[Crutchfield & Young, PRL **63** (1989)]

[Shalizi & Crutchfield, J. Stat. Phys. **104** (2001)]

Complexity: amount of memory required for simulation

Statistical Complexity

$$C_\mu := H[\pi] = -\sum \pi_i \log \pi_i$$

$\pi = \{\pi_i\}$: stationary distribution
ϵ-machines: minimal and unique

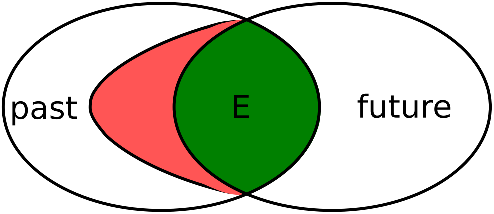
Perturbed coin

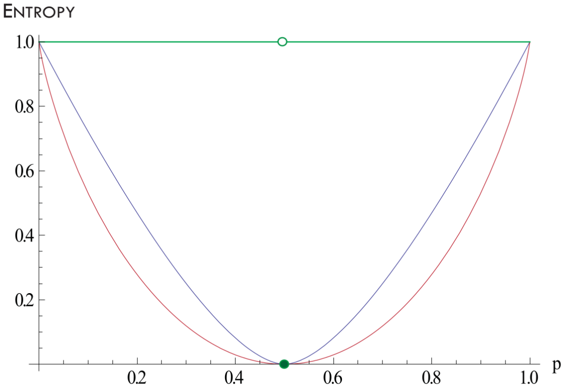
$$C_\mu = 1$$

because: $\pi_H = \pi_T = \frac{1}{2}$
(unless $p = \frac{1}{2}$! Then: $C_\mu = 0$)

[Crutchfield & Young, PRL **63** (1989)]

[Shalizi & Crutchfield, J. Stat. Phys. **104** (2001)]





[Gu et al., Nat. Comm. **3** (2012)]

Replacing bits with qubits: reduced complexity

Quantum causal states:

$$s_i \to |\sigma_i\rangle$$

Stationary state:

$$\rho = \sum_i \pi_i |\sigma_i\rangle\langle\sigma_i|$$

Quantum statistical complexity:

$$C_q := S(\rho) = -\mathrm{tr}[\rho \log \rho]$$

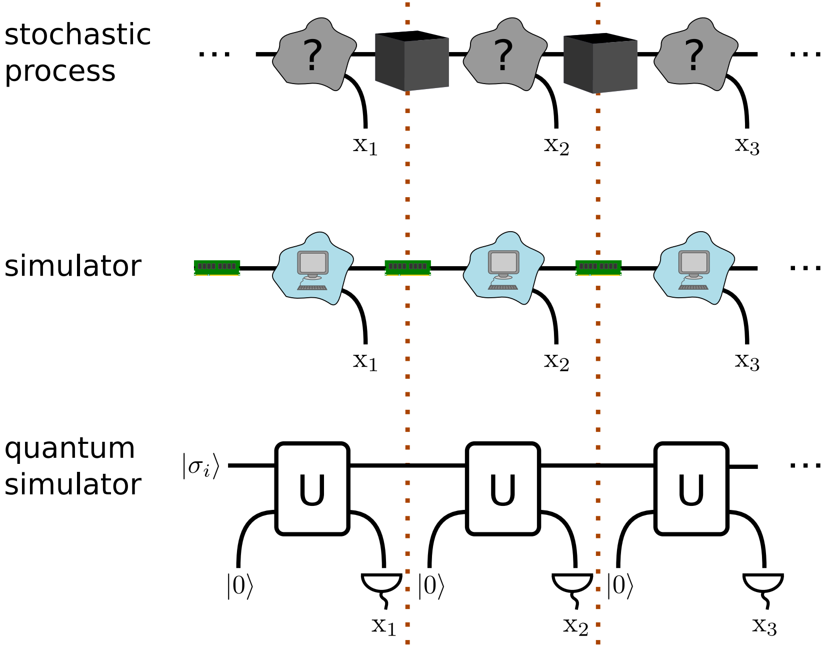
Example: perturbed coin

$$|S_H\rangle := \sqrt{p}\,|T\rangle + \sqrt{1-p}\,|H\rangle$$
$$|S_T\rangle := \sqrt{p}\,|H\rangle + \sqrt{1-p}\,|T\rangle$$

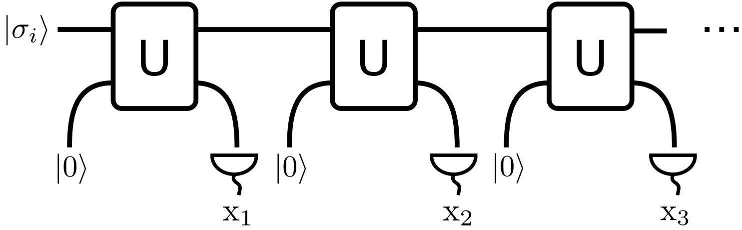
[Gu et al., Nat. Comm. **3** (2012)]

[Mahoney et al., Sci. Rep. **6** (2016)]

[Riechers et al., PRA **93** (2016)]



stochastic process
?
x1
x2
x3
simulator
quantum simulator
|σ_i⟩
U
|0⟩



We want:

$$U\,|\sigma_i\rangle\,|0\rangle = \sum_{j,x} \sqrt{p(x, s_j|s_i)}\,|\sigma_j\rangle\,|x\rangle \equiv |1_i\rangle$$

Such U exists iff

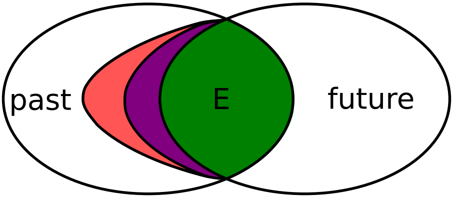
$$\langle 1_i|\, 1_j\rangle = \langle \sigma_i|\, \sigma_j\rangle$$

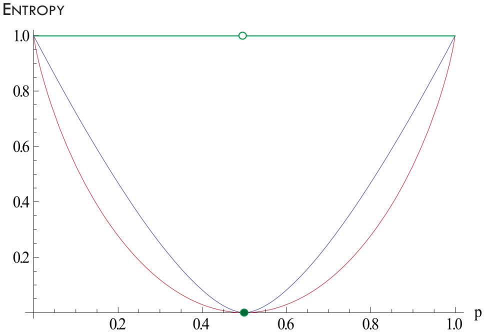
Construction:

Solve for $\langle \sigma_i|\, \sigma_j\rangle$ and proceed with Gram-Schmidt.

[FB, Thompson, Gu, PRL **120**, 240502 (2018)]

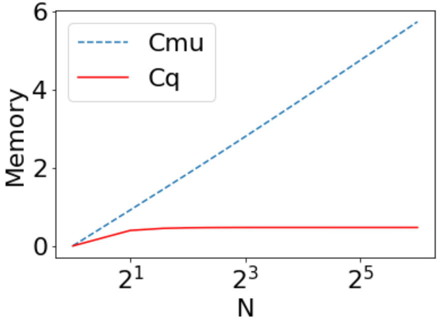
Quantum encoding saves memory but C_q still exceeds E

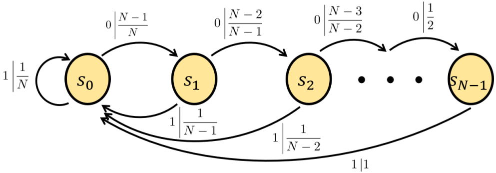




[Gu et al., Nat. Comm. (2012)]

Quantum advantage $C_\mu - C_q$ can be unbounded

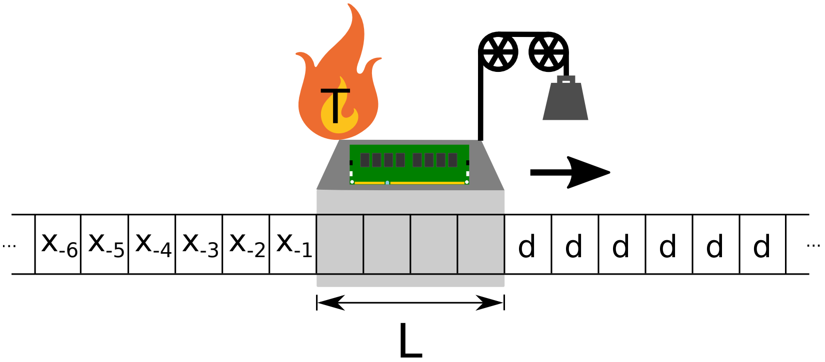




[Yang, FB, Narasimhachar, Gu, 1803.08220]

see also: [Garner et al., NJP **19**, 103009 (2017)]

Information ratchet → prescient pattern generator



$$W_{min}^{(L)} = \Delta E = W_{er} + T\chi_q + W_{mod}^{(L)}$$

- $W_{er} = TL[S(d) - h_\mu]$ $\quad (h_\mu = \lim_{L\to\infty} H[Y_{0:L}]/L)$
- $\chi_q = C_q - E$
- $W_{mod}^{(L)} = 0 \; \forall \; L \geq R$

(classical info ratchets: [Garner et al., PRE **95**, 042140 (2017)], [Boyd et al., 1708.03030])

Summary

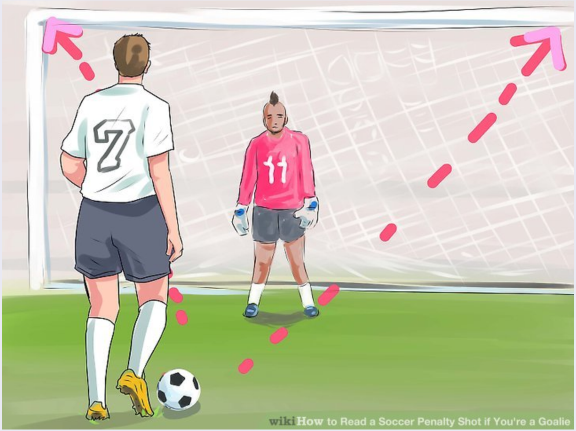
- In stochastic process simulation, there is an unavoidable work cost $\propto \chi_q$ due to causality.
- Quantum encoding reduces this cost, compared to classical simulation.
- The difference between classical and quantum encoding can become unbounded for some families of processes.

Thank you for your attention.

e-mail: **quantum@felix-binder.net**
group website: **quantumcomplexity.org**

Information Thermodynamics

Will the particle be found on the left or on the right?



Maxwell's Demon