soon on the arXiv!
(also 1709.00506)

Asymptotically reversible simulation of quantum processes

Philippe Faist[1], Mario Berta[2], Fernando Brandão[1]

[1]*Institute for Quantum Information and Matter, Caltech*
[2]*Imperial College London, U.K.*

FNSNF Swiss National Science Foundation

Caltech

IQIM

KITP, Santa Barbara, June 2018

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Thermodynamics of quantum processes

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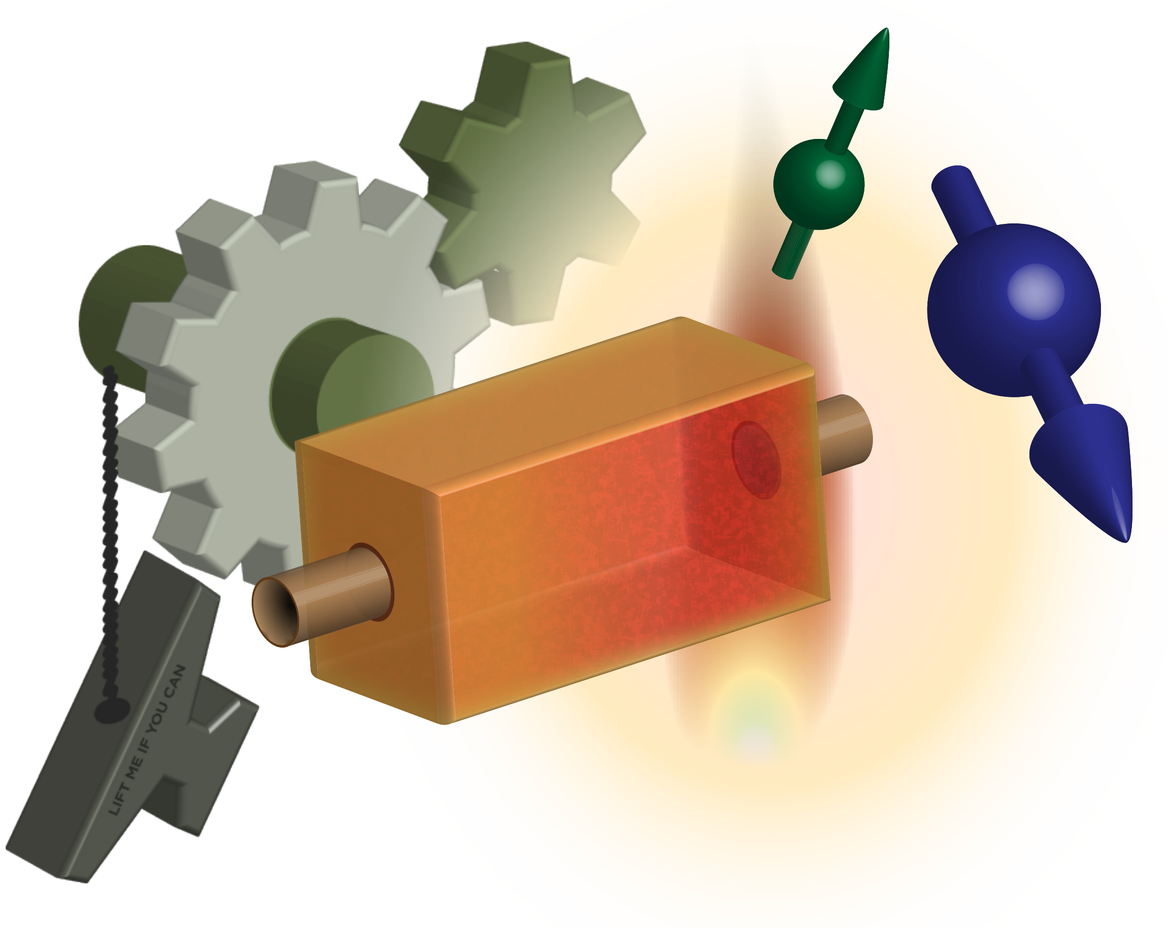
Quantum thermodynamics

- Laws of thermodynamics for quantum systems, in full generality?



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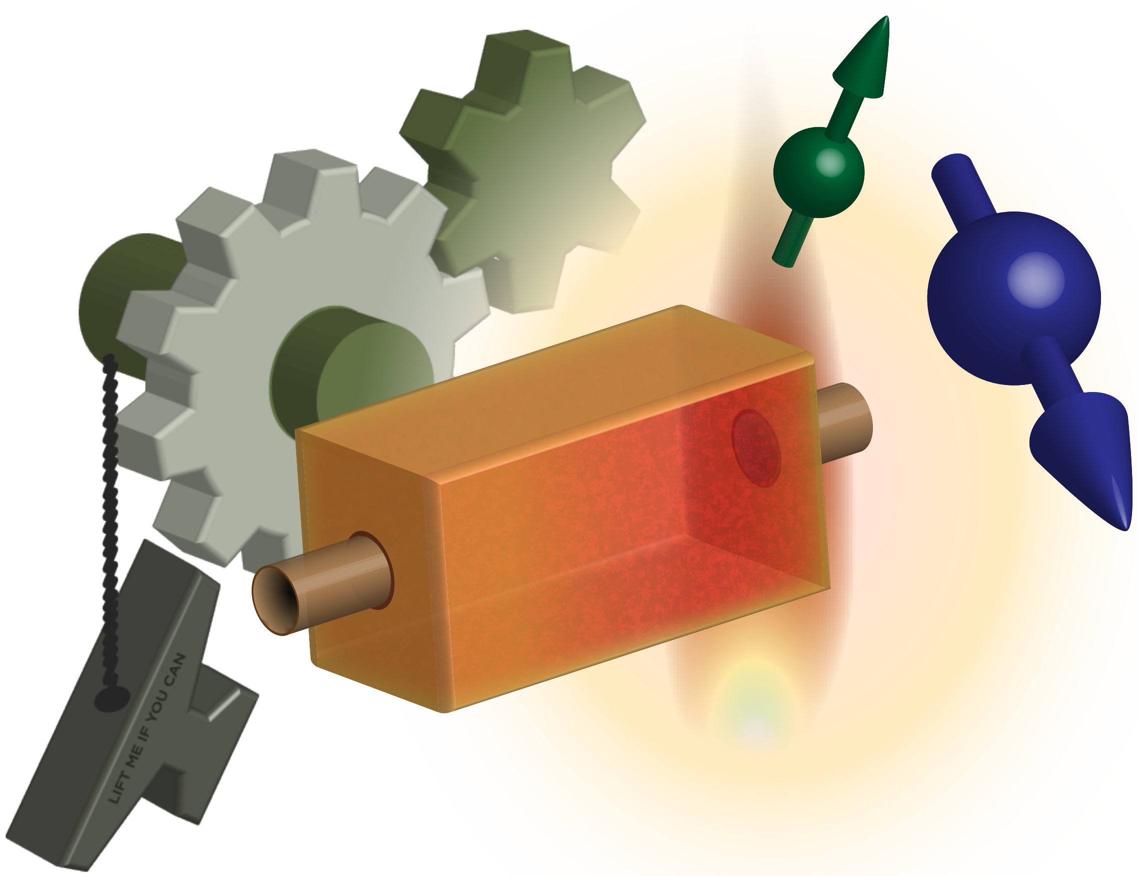

  - small quantum systems
  - information-bearing systems (memory registers)
  - observer-dependent, side information

Quantum thermodynamics

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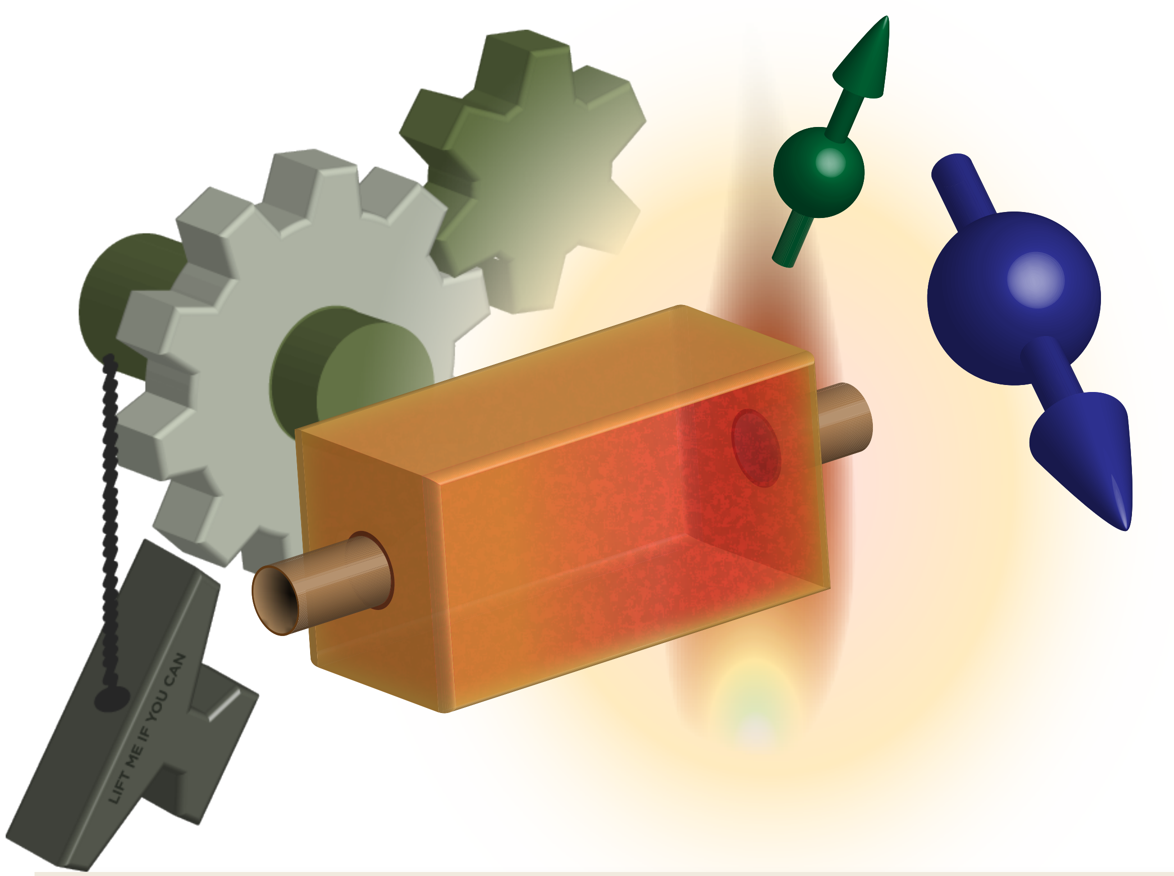

  - small quantum systems
  - information-bearing systems (memory registers)
  - observer-dependent,
    formation

generality expected because the laws of thermodynamics are so universal!

An approach for thermodynamics

- Framework with desired level of generality?

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- Often, dynamics naturally conserve a particular state
  - → energy-preserving interactions with bath preserves thermal state
  - → energy-conserving unitary evolution preserves microcanonical state
  - → steady state of Master equation

An approach for thermodynamics

- Framework with desired level of generality?

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  → energy-conservi
  microcanonical s

  → steady state of N

Natural basis for a formulation of a general "second law"

Γ-sub-preserving maps

Γ

To each system of interest S is associated an operator $\Gamma_S \geqslant 0$

Choose $\Gamma_S = e^{-\beta H_S}$

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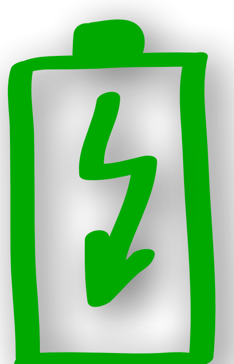
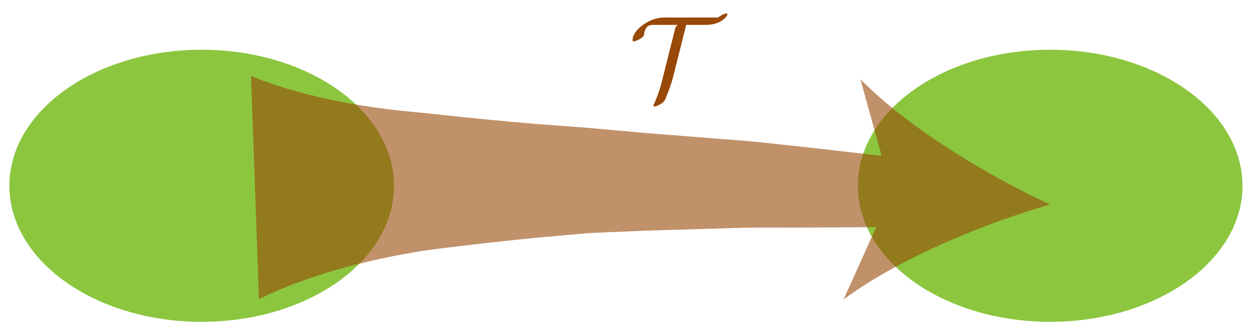


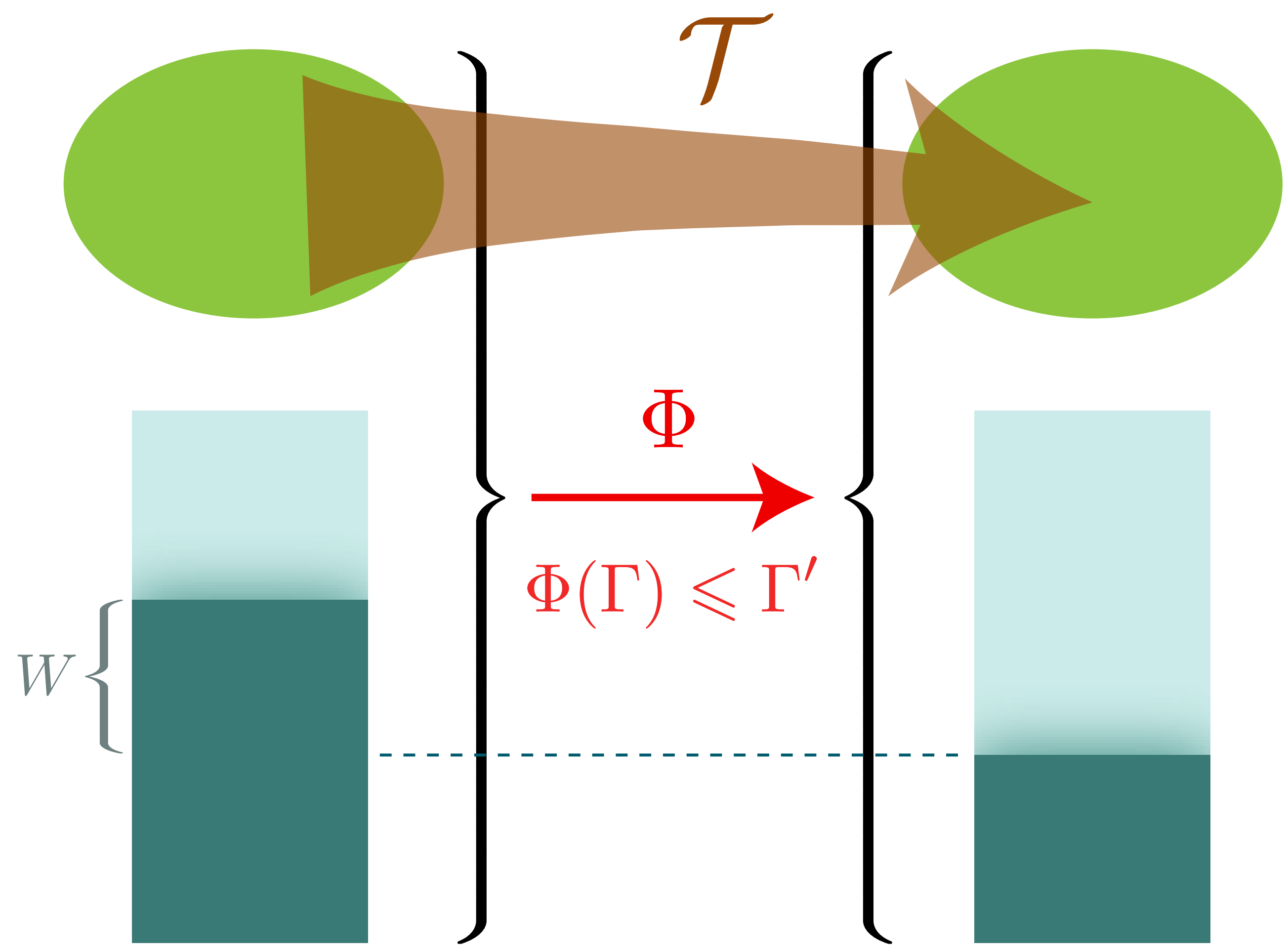
battery = work storage system

Large family of battery models are equivalent

Work cost of an exact process



Work cost of an exact process



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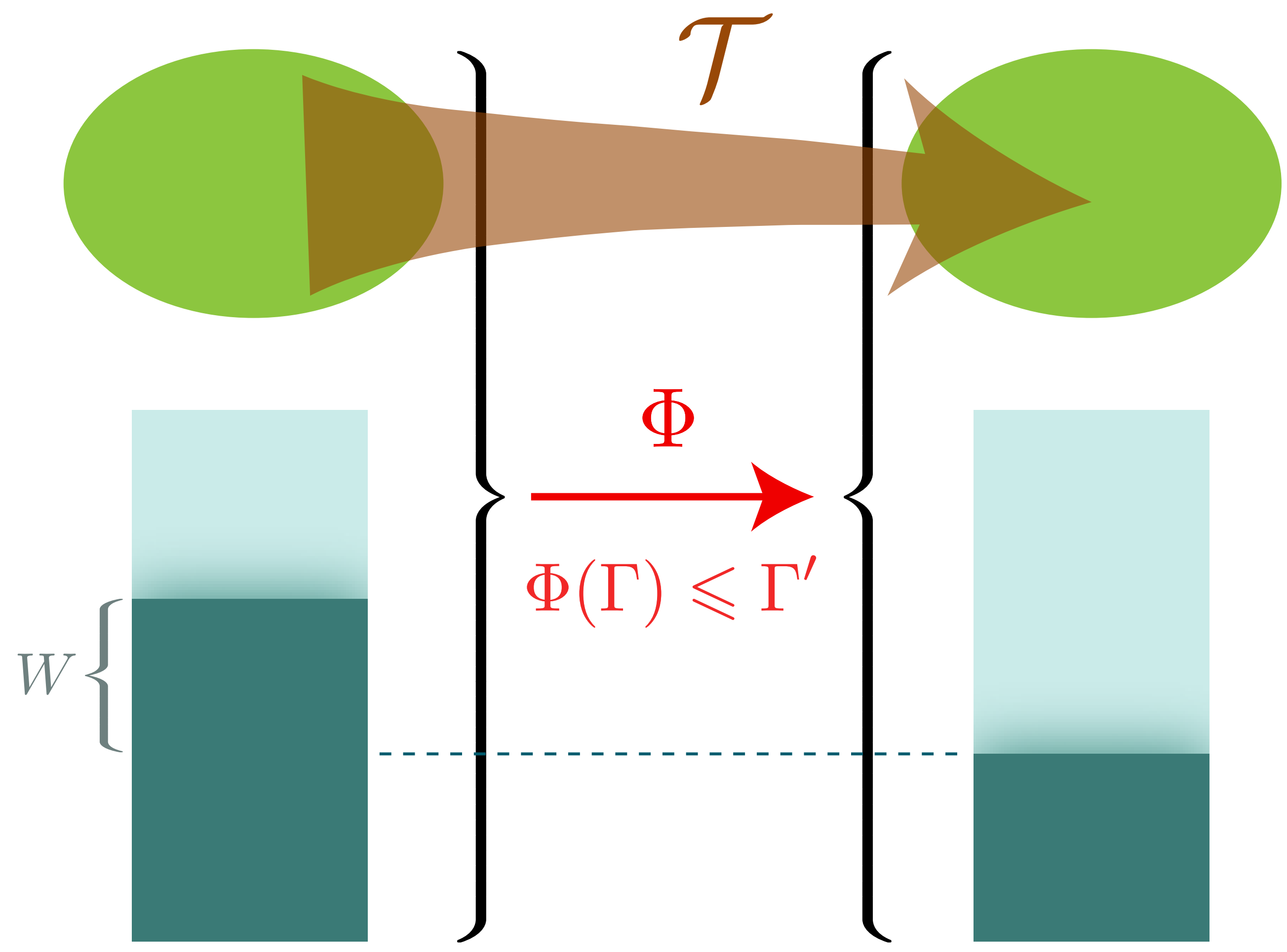


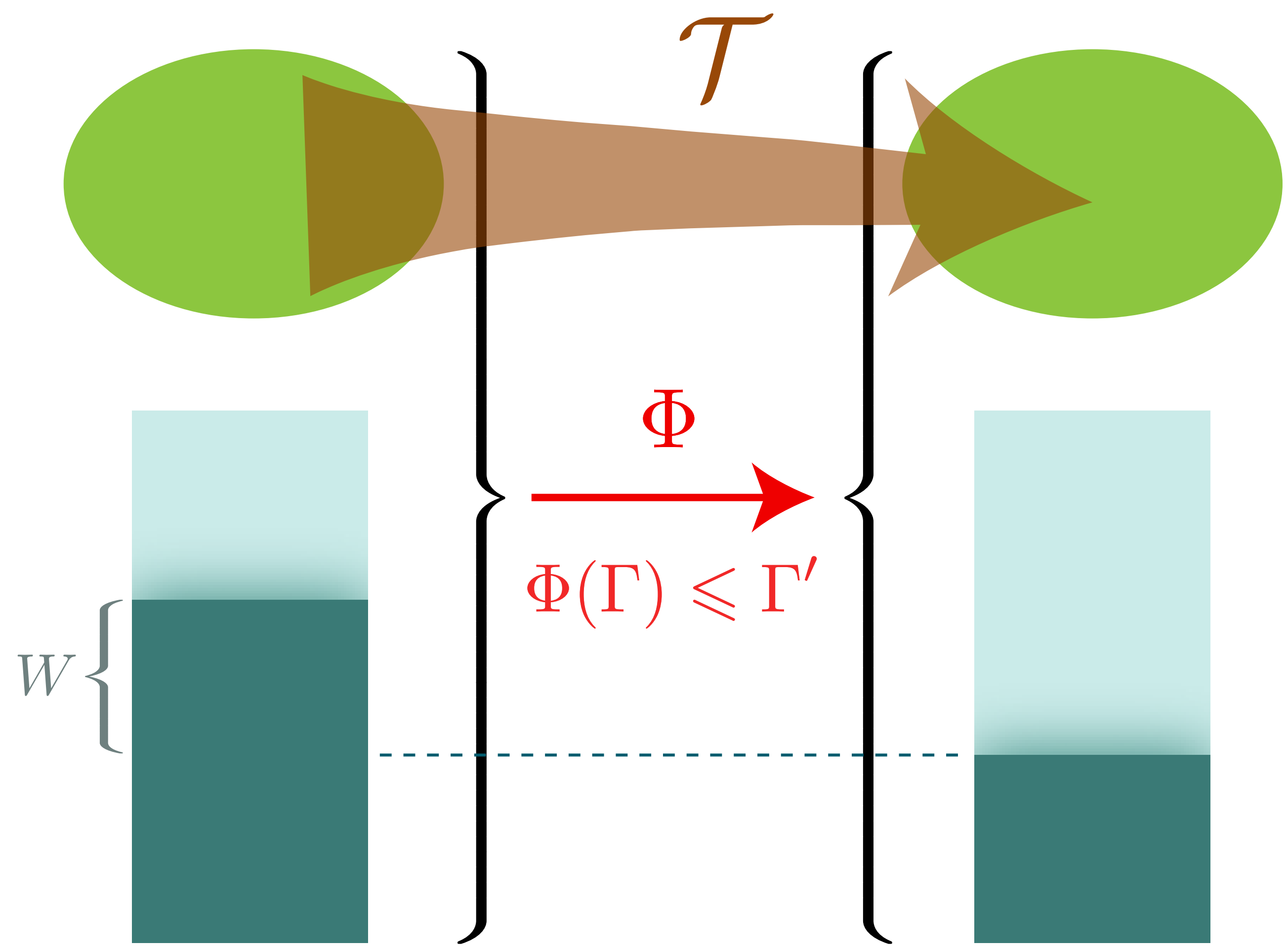
$\exists\ \Phi$:

$$\Phi(\Gamma_{\mathrm{tot}}) \leqslant \Gamma'_{\mathrm{tot}}$$

$$\Leftrightarrow$$

$$\mathcal{T}(\Gamma) \leqslant e^{\beta W}\, \Gamma'$$

Work cost of an exact process



$$\exists\ \Phi :$$
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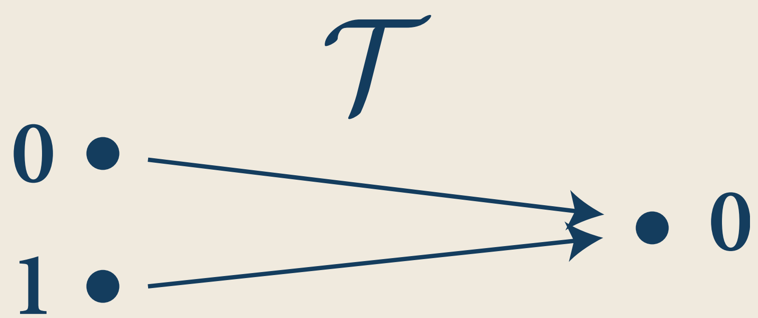
$$\Longleftrightarrow$$
$$\mathcal{T}(\Gamma) \leqslant e^{\beta W}\,\Gamma'$$

simple mathematical characterization how much work a specific process requires

Framework: Example

$$\mathcal{T}(\Gamma) \leqslant e^{\beta W}\, \Gamma'$$



$$\mathcal{T}(\mathbb{1}) = 2|0\rangle\langle 0| \leqslant 2\,\mathbb{1}$$

$$\Rightarrow \quad W \geqslant kT\ln(2)$$

States and processes

Predominant thinking in thermodynamics:

$$(\text{state } A) \qquad\qquad (\text{state } B)$$

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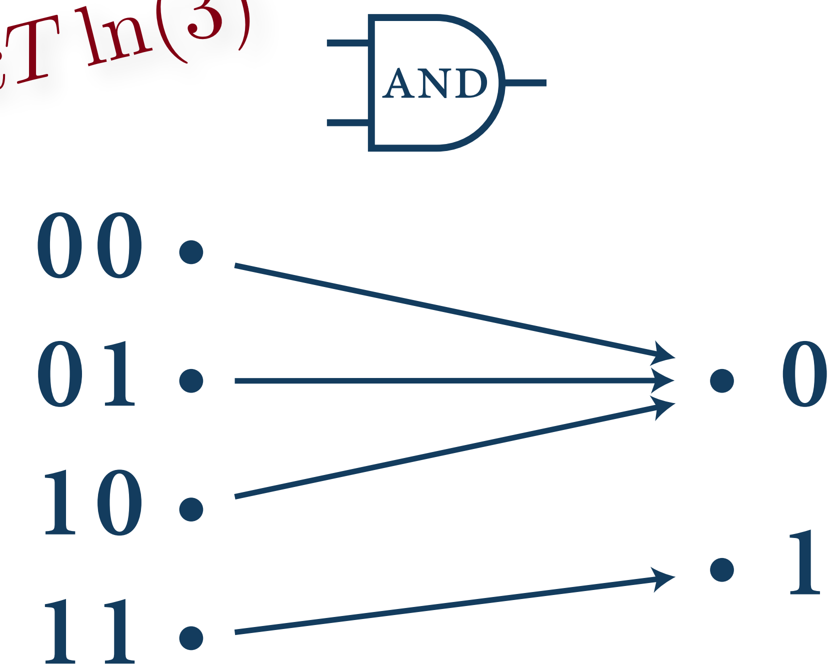
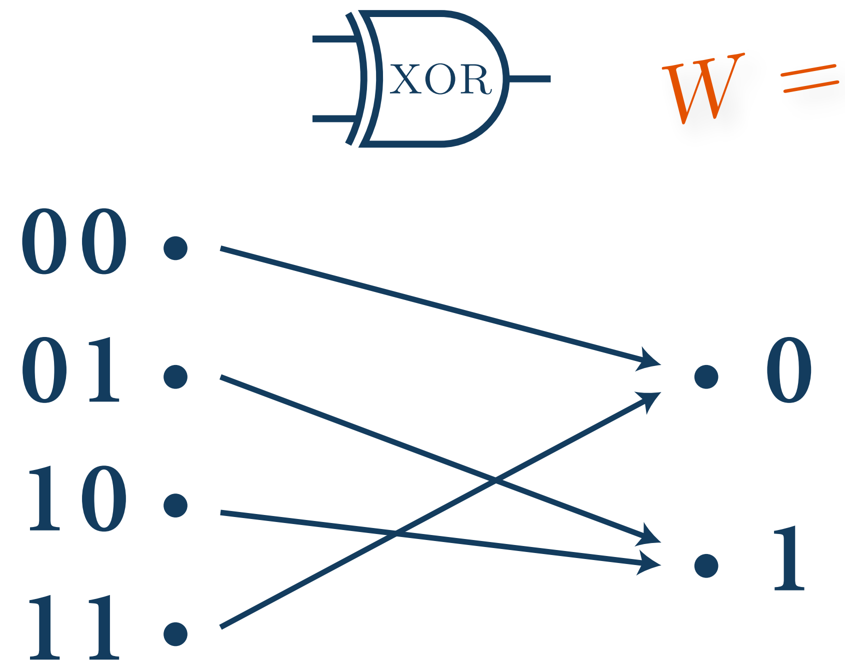
For information-bearing devices, the actual process is important

Relevance of processes

$W = kT \ln(3)$





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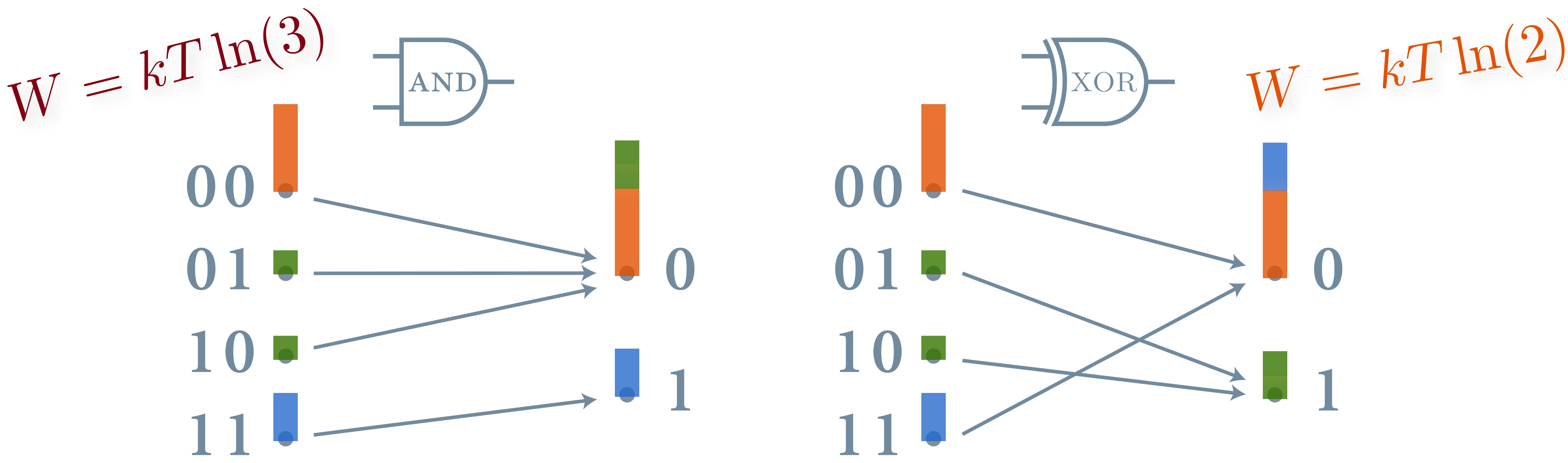

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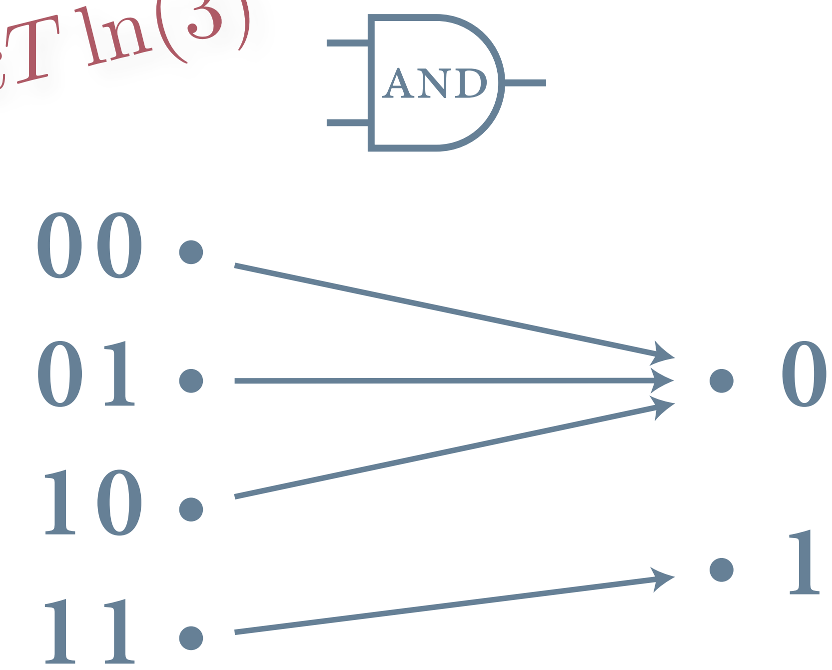

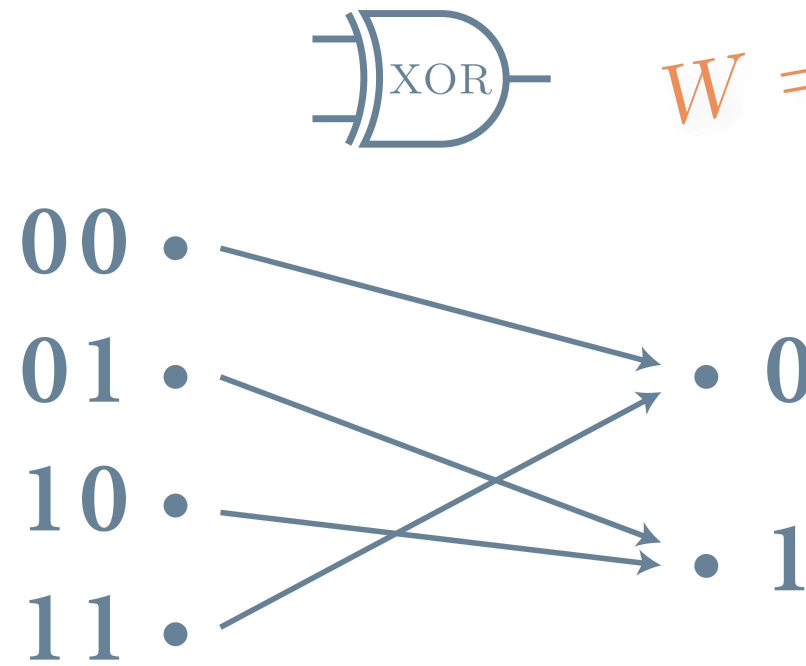

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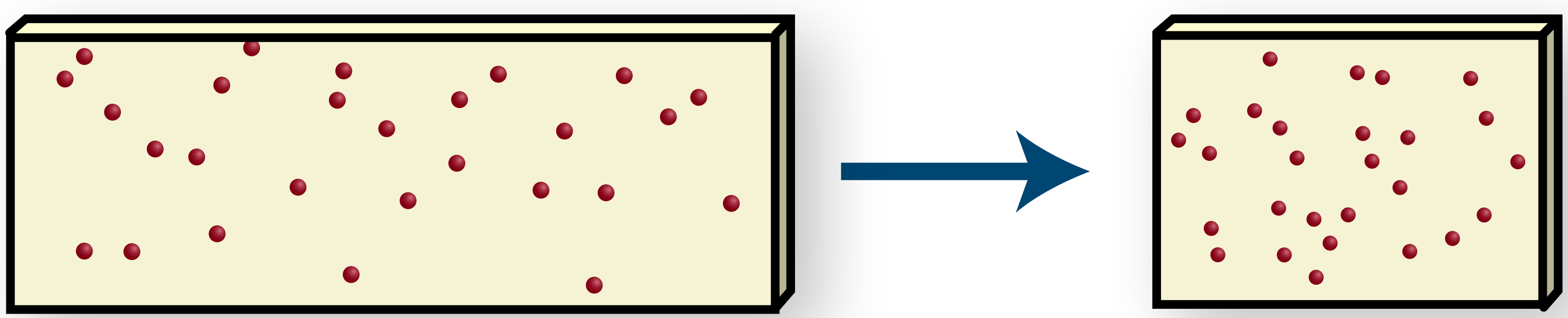

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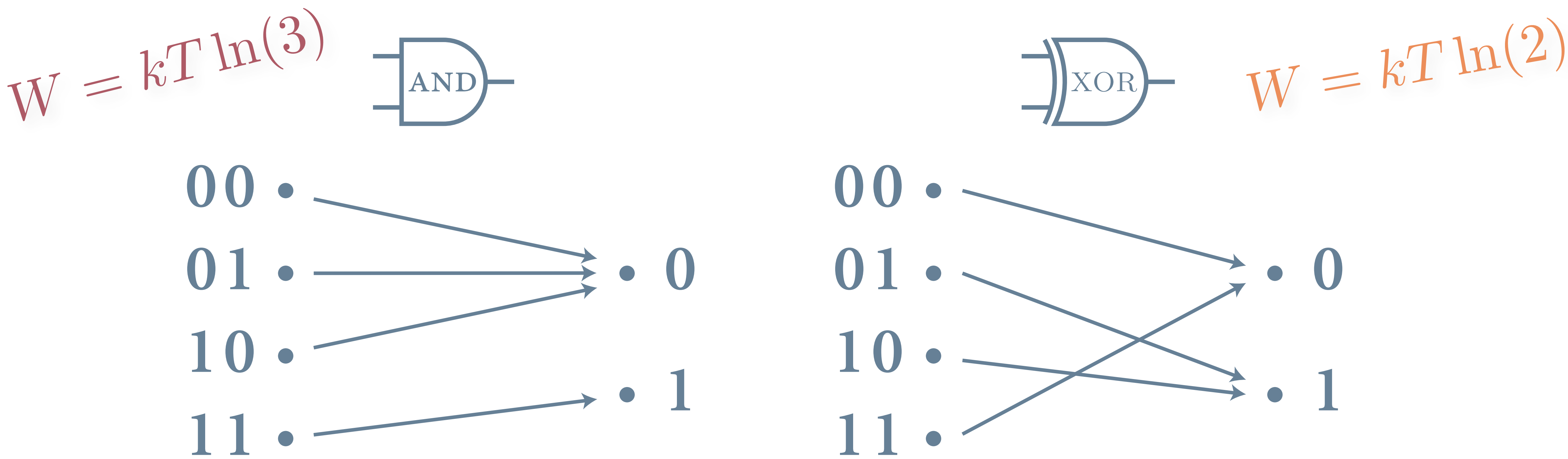

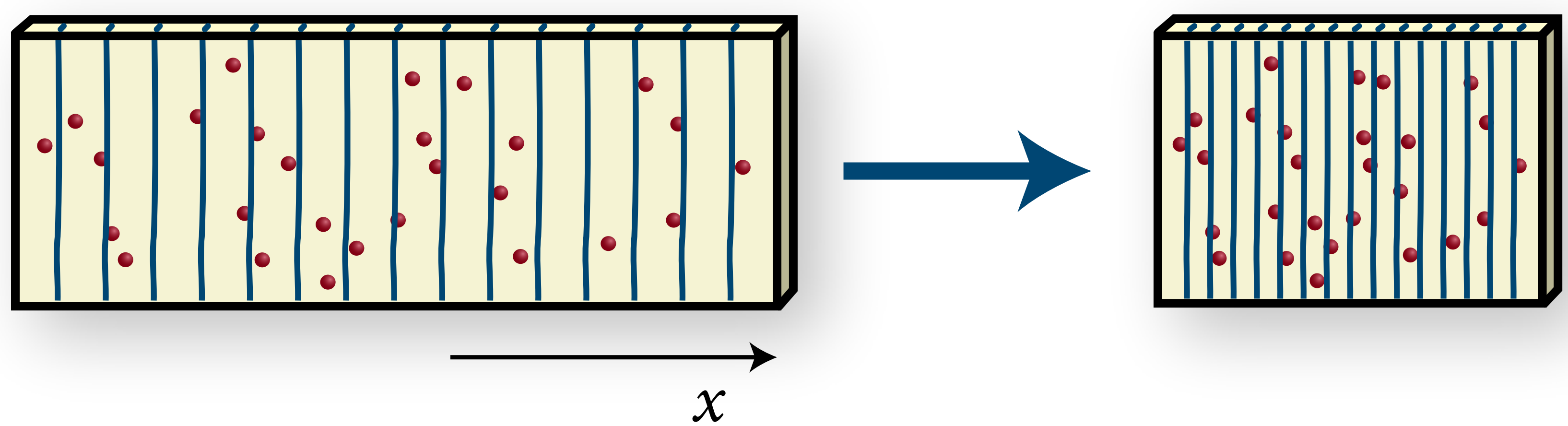

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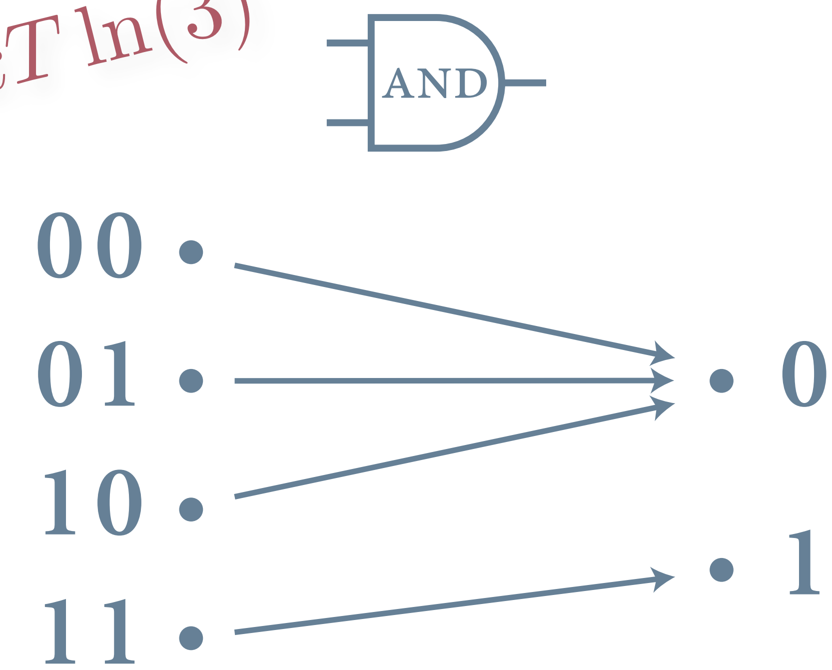

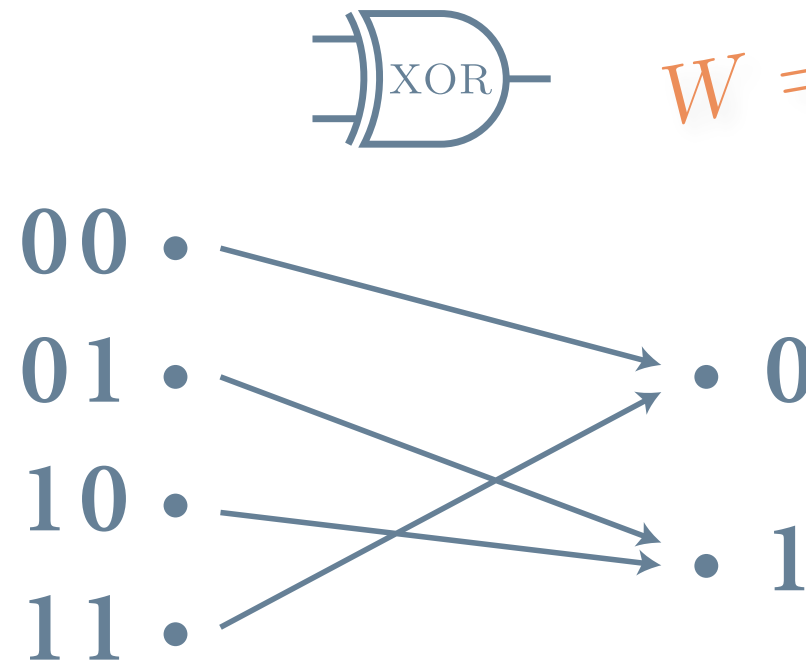

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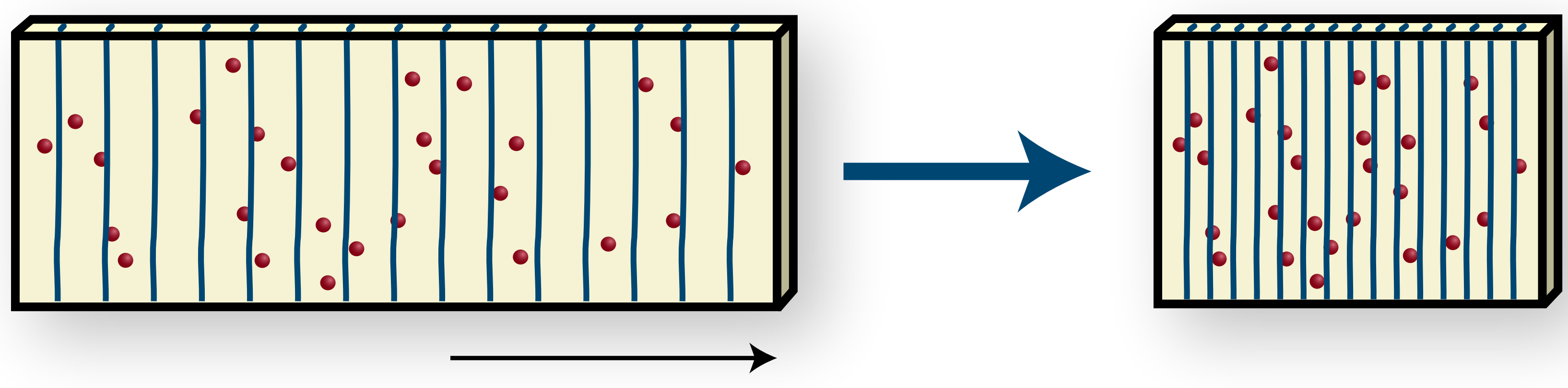
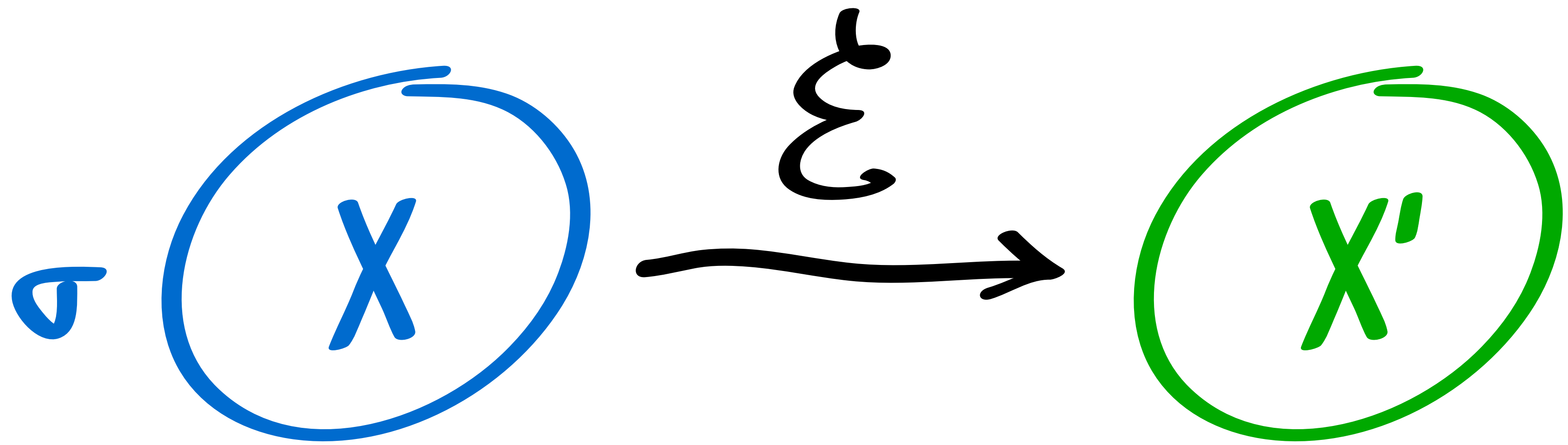


same input/output states, very different processes

Work cost of a process?



- mapping of input states to output states

→ AND, XOR, … gate

→ any classical or quantum computation

→ any physical process (completely positive, trace-preserving map)

Cost of process: Known input state

Case input state known: Fundamental limit given by the coherent relative entropy

$$W = -kT \cdot \hat{D}^{\epsilon}_{X \to X'}\left(\mathcal{E}(\sigma_{X R_X}) \,\middle\|\, e^{-\beta H_X}, e^{-\beta H_{X'}}\right)$$

units of work
[$\beta = 1/(kT)$]

tolerate small
inaccuracy

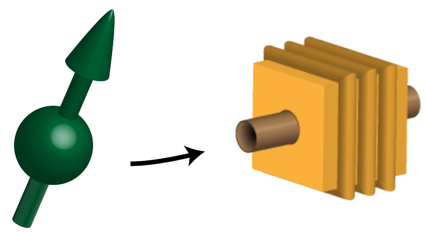
quantum
process

input state,
incl. reference
system

H_X,$H_{X'}$ = in/out
Hamiltonian

PhF & Renner, PRX, 2018

Macroscopic i.i.d. limit



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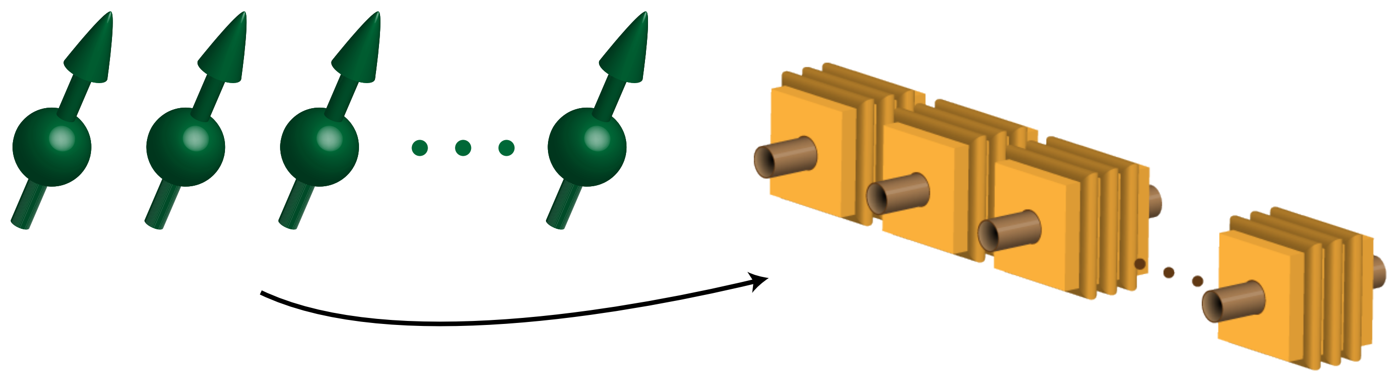
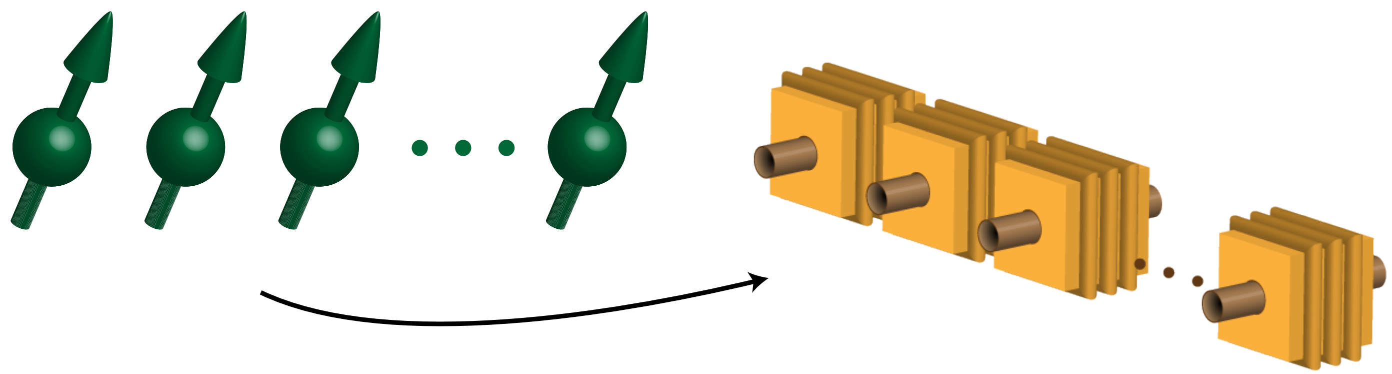


$$H_{X^n} = \sum_{i=1}^{n} H_{X_i}$$

$$\frac{1}{n} W = -\frac{kT}{n} \cdot \hat{D}^{\epsilon}_{X^n \to X^n}\left(\mathcal{E}^{\otimes n}(\sigma^{\otimes n}_{X R_X}) \,\|\, e^{-\beta H_{X^n}}, e^{-\beta H_{X^n}}\right)$$

PhF & Renner, PRX, 2018

Macroscopic i.i.d. limit



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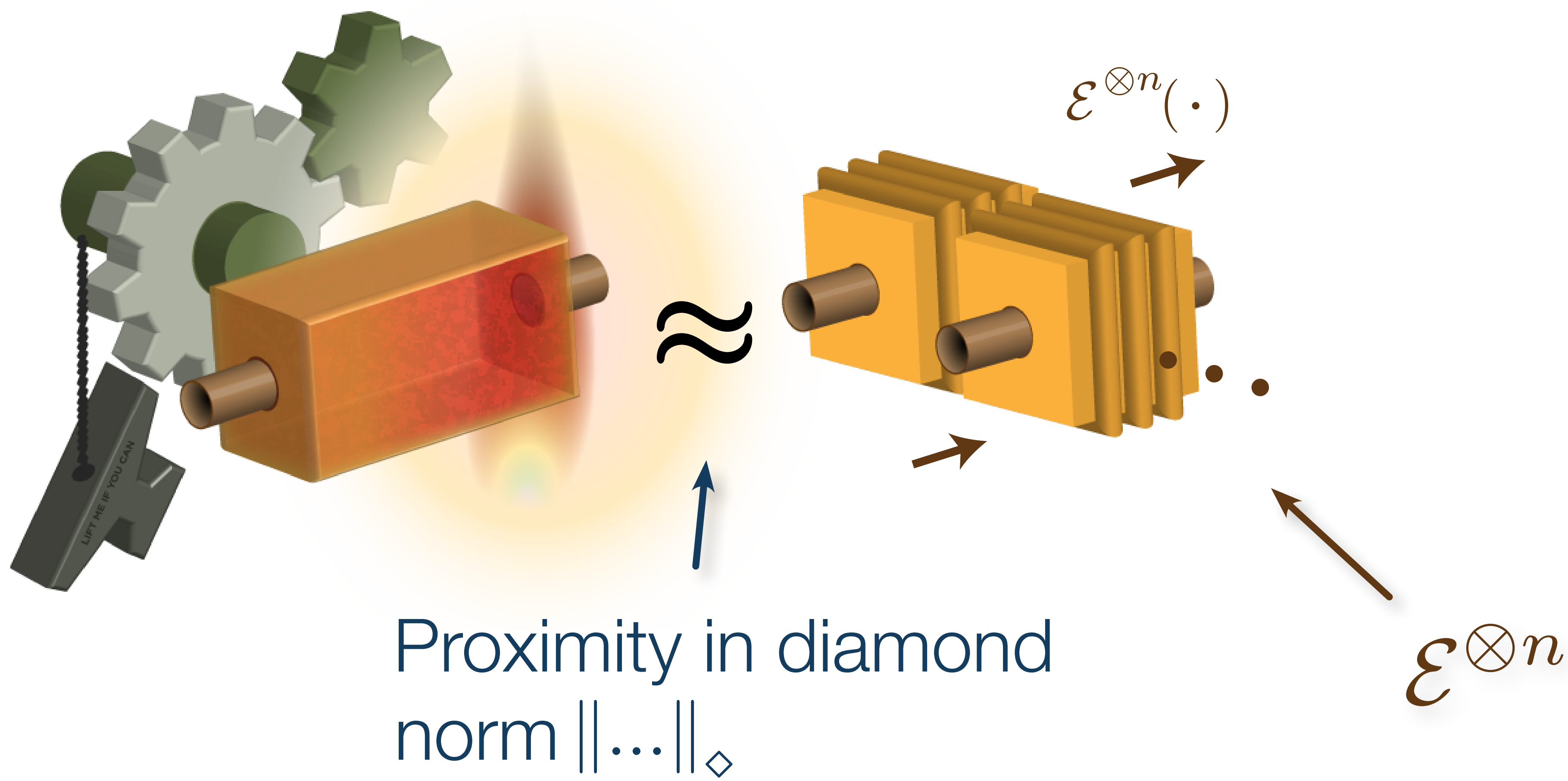
$$\xrightarrow{n \to \infty} \quad F(\mathcal{E}(\sigma)) - F(\sigma)$$

$$F(\rho) = -\beta^{-1} S(\rho) + \langle H \rangle_{\rho}$$
$$= \beta^{-1} D(\rho \,\|\, e^{-\beta H})$$

PhF & Renner, PRX, 2018

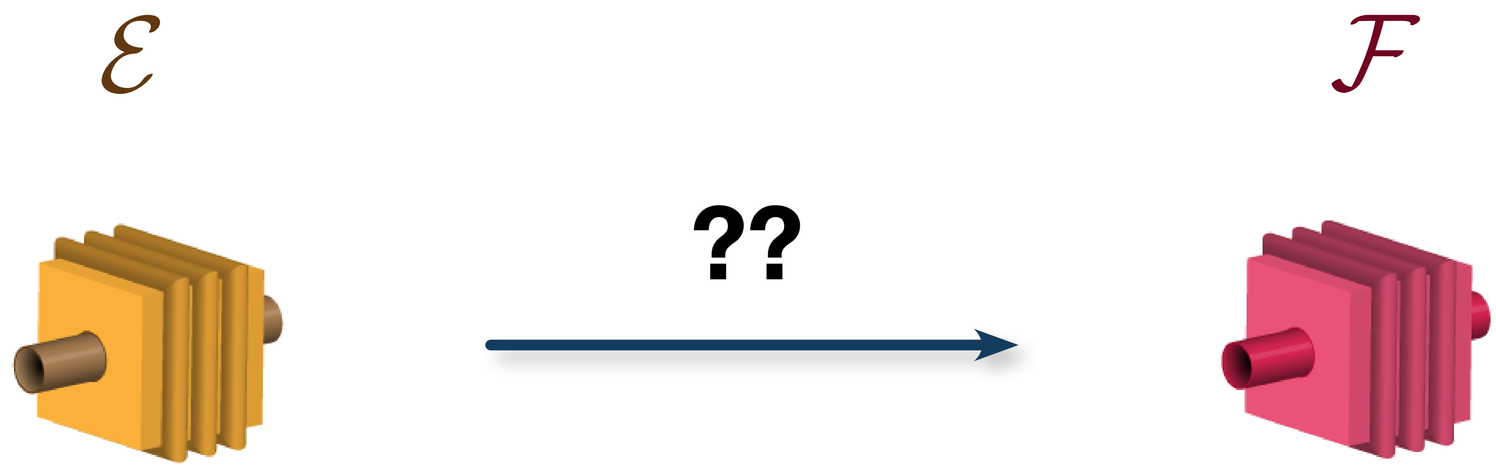
Universal implementation of a process

- New result: Implementation of an i.i.d. process $\mathcal{E}$ that works for all input states



Work cost $= \max_{\sigma} \left[F(\mathcal{E}(\sigma)) - F(\sigma) \right] =: T(\mathcal{E})$ *thermodynamic capacity*

Resource theory for channels



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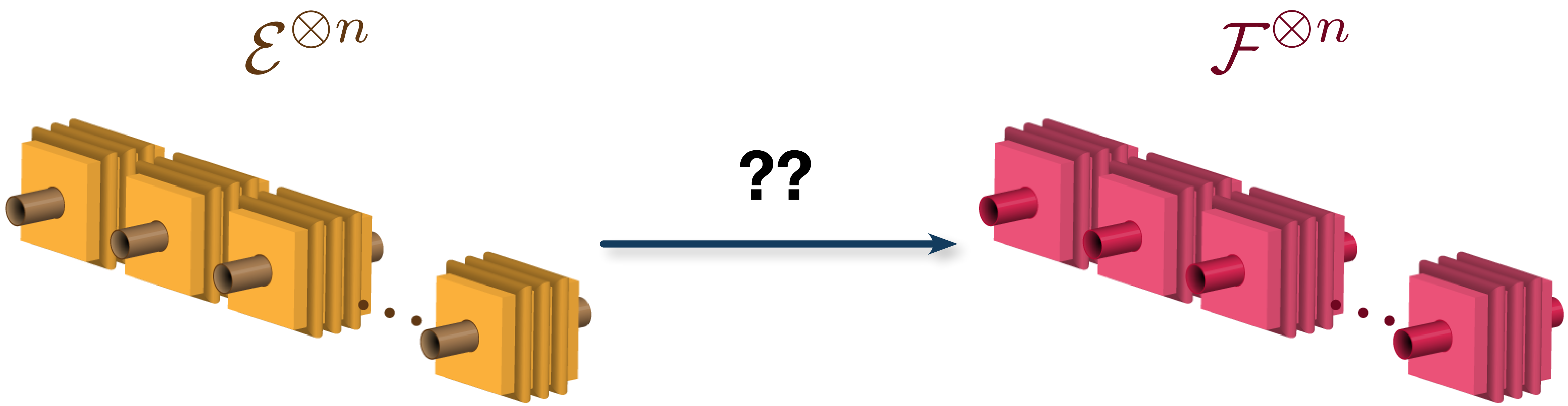
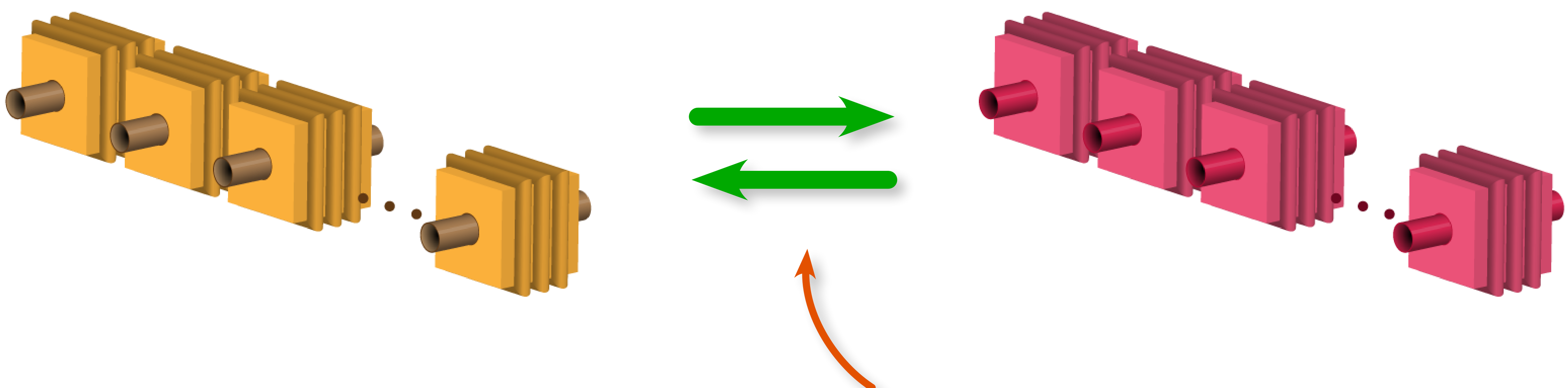


Resource theory for channels

$\mathcal{E}^{\otimes n}$ $\mathcal{F}^{\otimes n}$



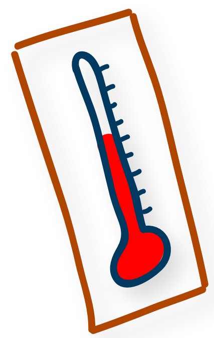
Reversible cost per copy
$T(\mathcal{F}) - T(\mathcal{E})$

Can extract $T(\mathcal{E})$ work per copy
from an implementation of $\mathcal{E}$

Navascues & García-Pintos, PRL, 2015

Gibbs-preserving maps, really?

More physical framework $\rightarrow$ thermal operations

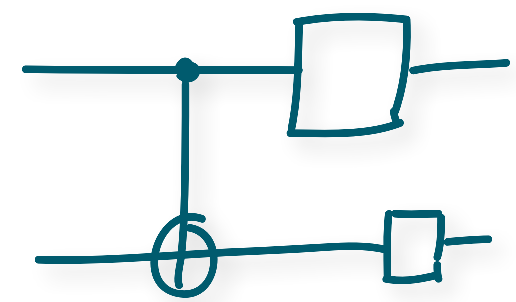


- Allowed any ancilla in a Gibbs state

$$\gamma_B = e^{-\beta H_B} / \operatorname{tr}(e^{-\beta H_B})$$

- Allowed any energy-conserving unitaries:

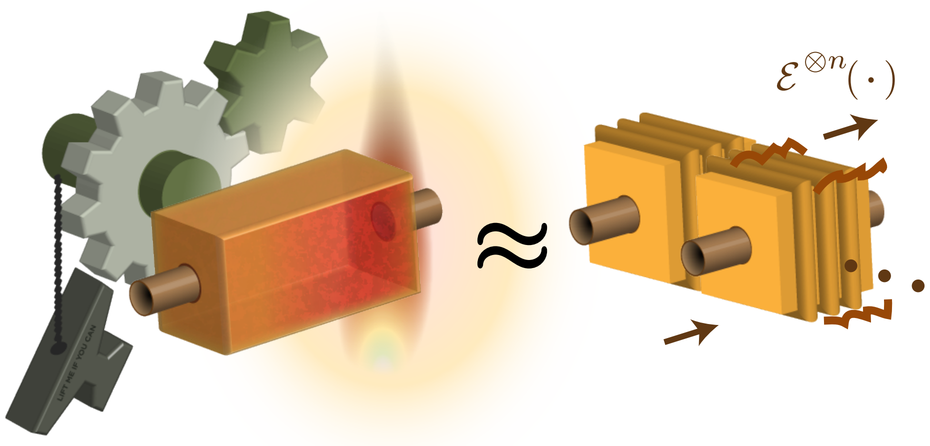
$$[U, H_{\text{total}}] = 0$$



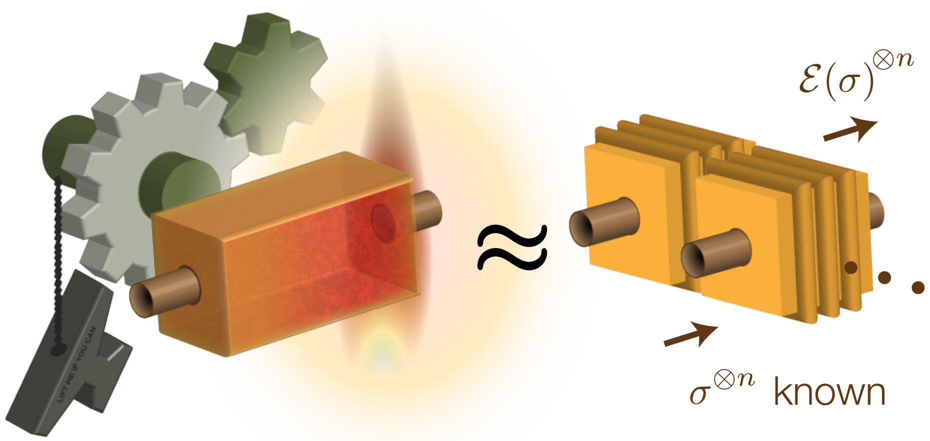
- Allowed to discard any system

Results using thermal operations

- Universal implementation of i.i.d. process [time-covariant only]



- Implementation of any i.i.d. process for fixed i.i.d. input + small amount of coherence



Gibbs-preserving maps are not that powerful in the i.i.d. regime

Anshu et al., arxiv:1702.019402

Outlook

- Thermodynamic resource theory of channels reversible (i.i.d.), like for states
- Thermodynamic capacity $T(\mathcal{E})$ = "value" of the channel
- Our result is analogous to the reverse Shannon theorem for communication

  Bennett et al., 2014; Berta et al., 2011

- New information-theoretic tools "typical universal conditional relative projectors"

  Bennett et al., 2014; Haah, IEEE TIT 2017; Bjelakovic et al., arXiv:quant-ph/0307170

Thank you for
your attention!