Coherent majorization and a complete set of entropic conditions for **quantum thermodynamics**

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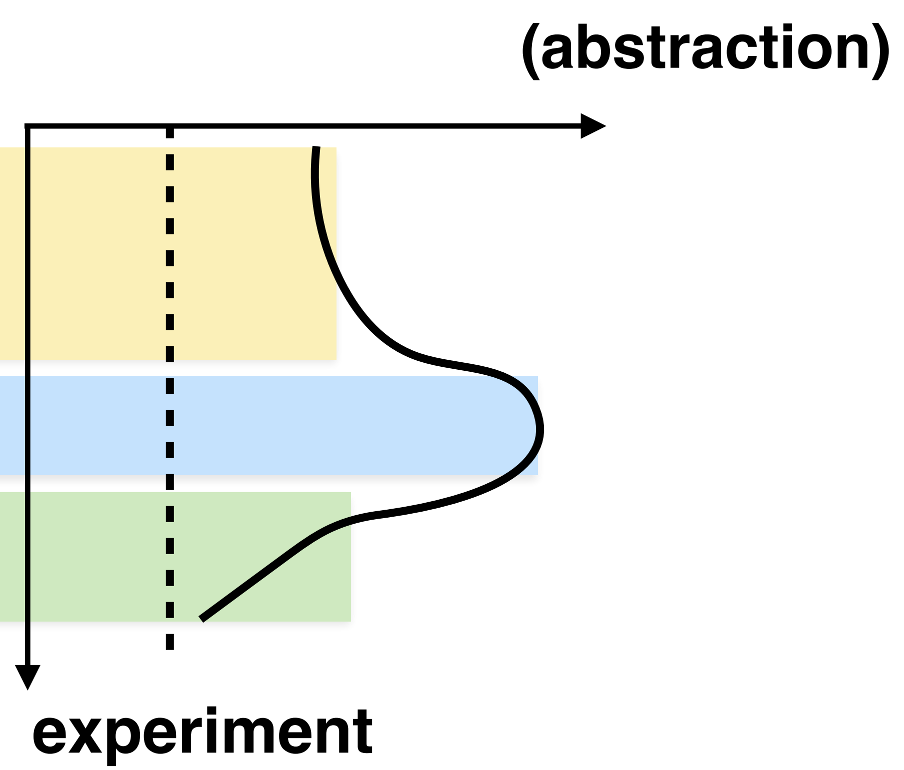
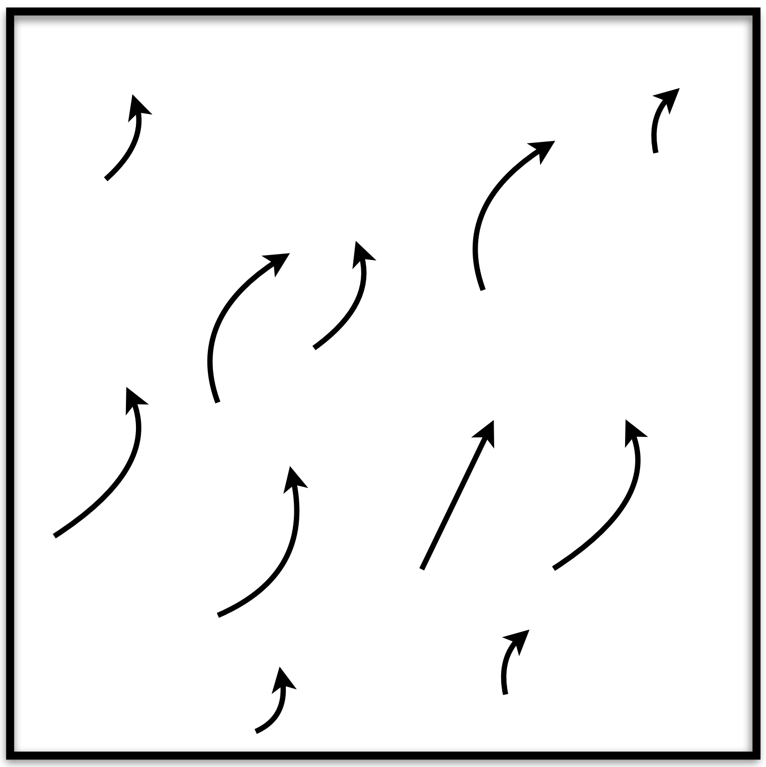
THE ROYAL SOCIETY

Outline

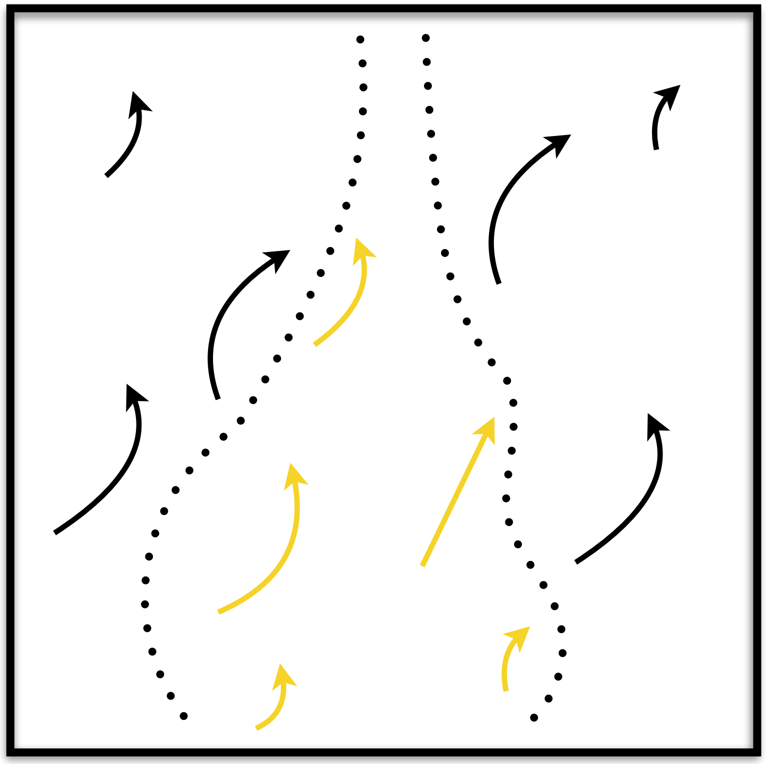
1. Background.
2. Physical principles.
3. Theoretical structure.
4. Physical consequences.



Quantum Thermodynamics?



All possible quantum processes



Distinguished processes

what physical principles?

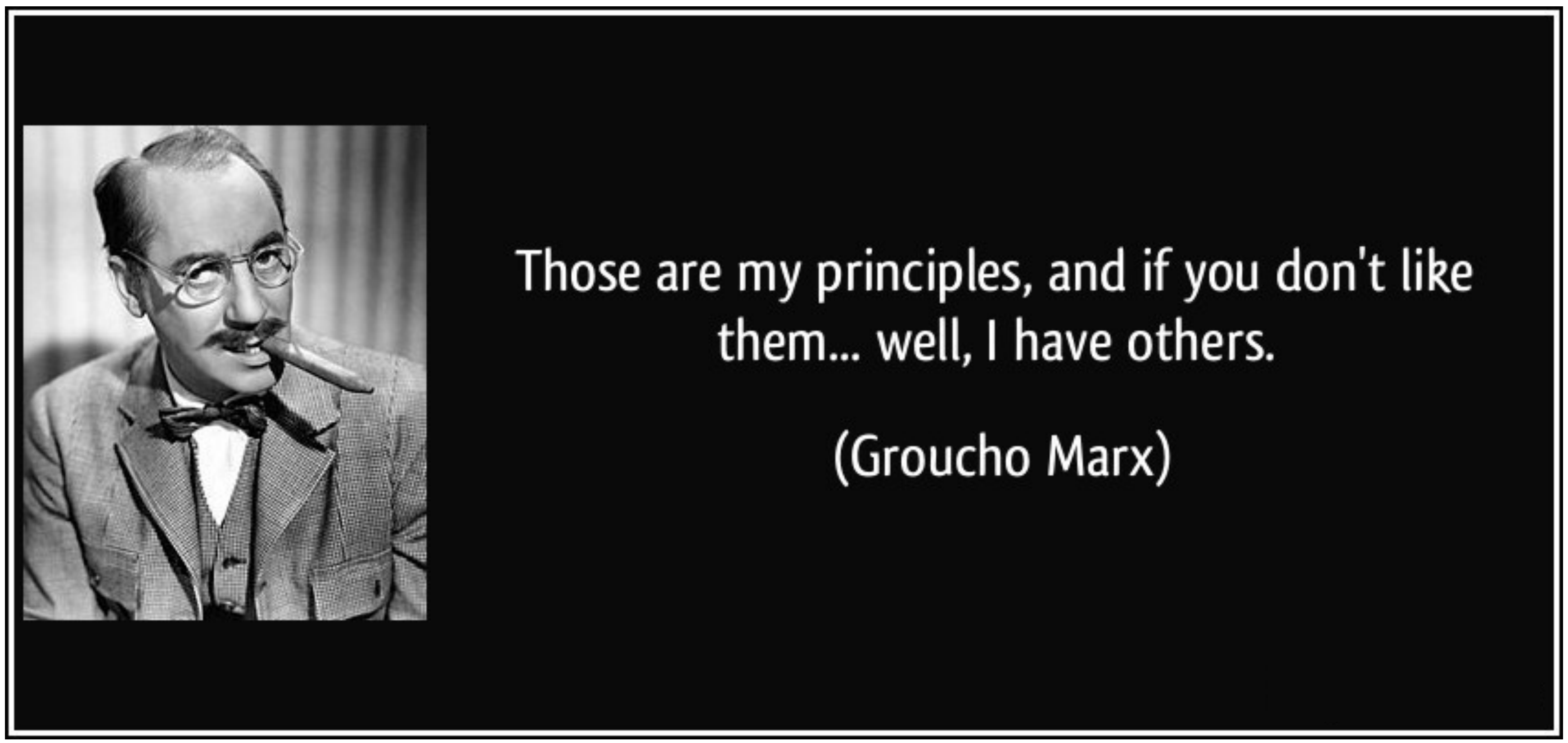
Not our starting point:

Resource theories, information

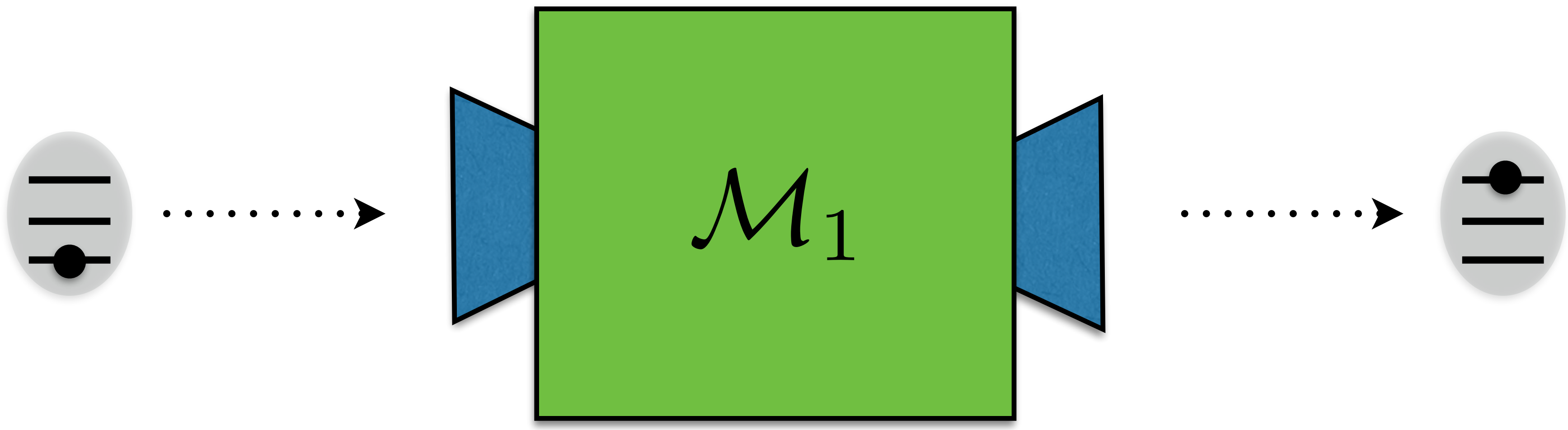
First Law of Thermodynamics

Work and Heat

Guiding principles?

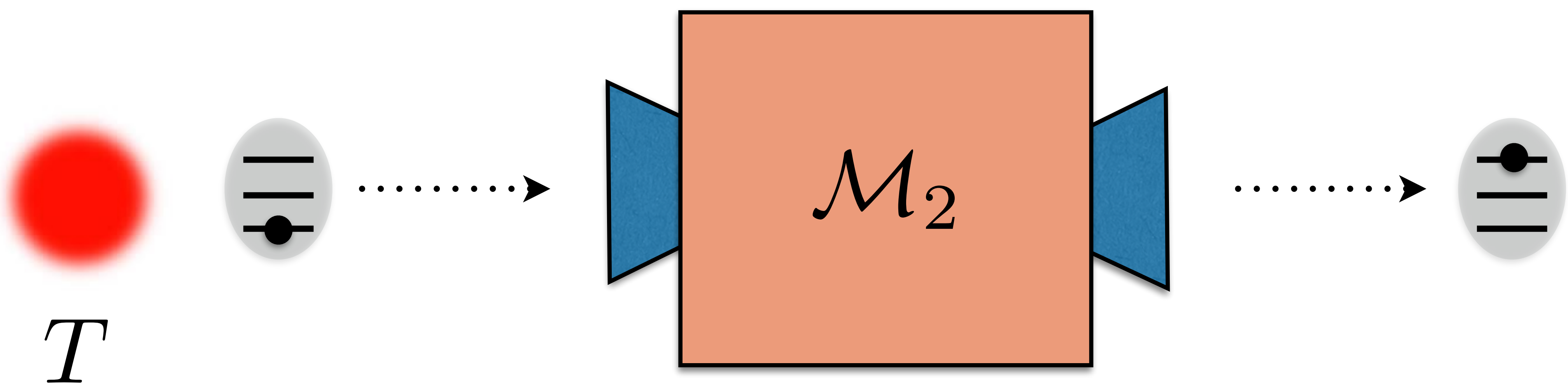


Perpetual Motion Machines



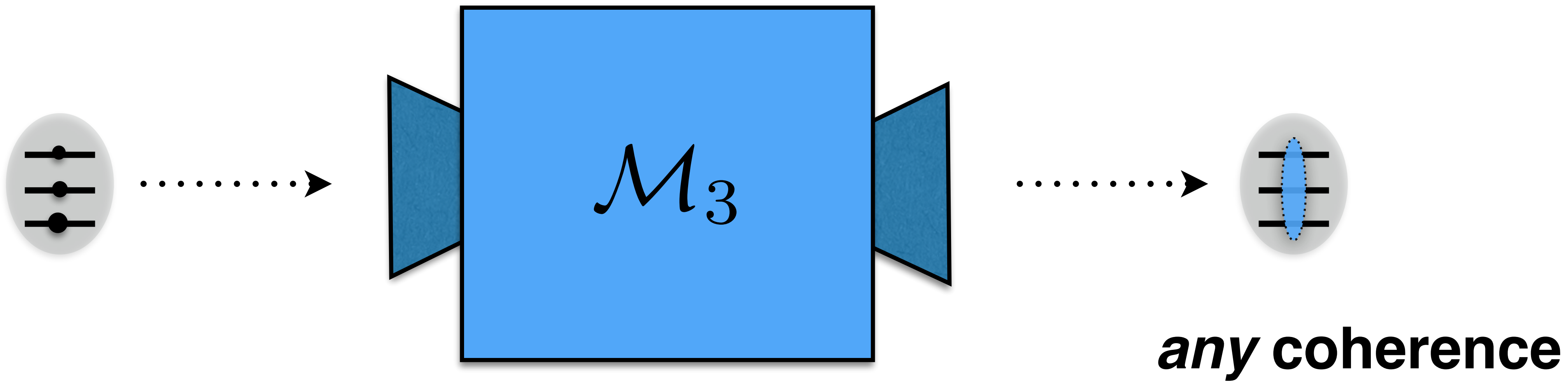
Perpetual motion machine of 1st Kind

Perpetual Motion Machines



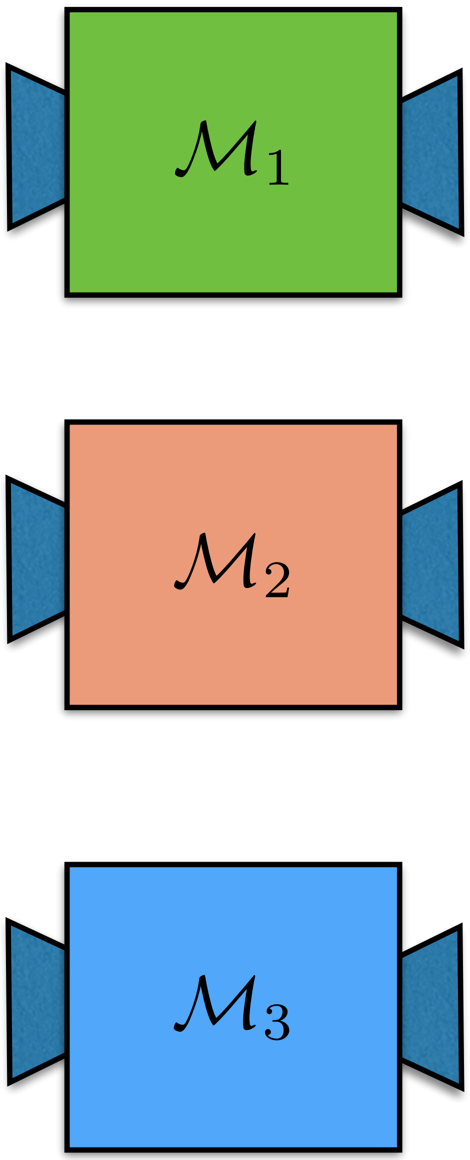
Perpetual motion machine of 2nd Kind

Perpetual Motion Machines



Define: **Coherent perpetual motion machine ("of 3rd Kind")**

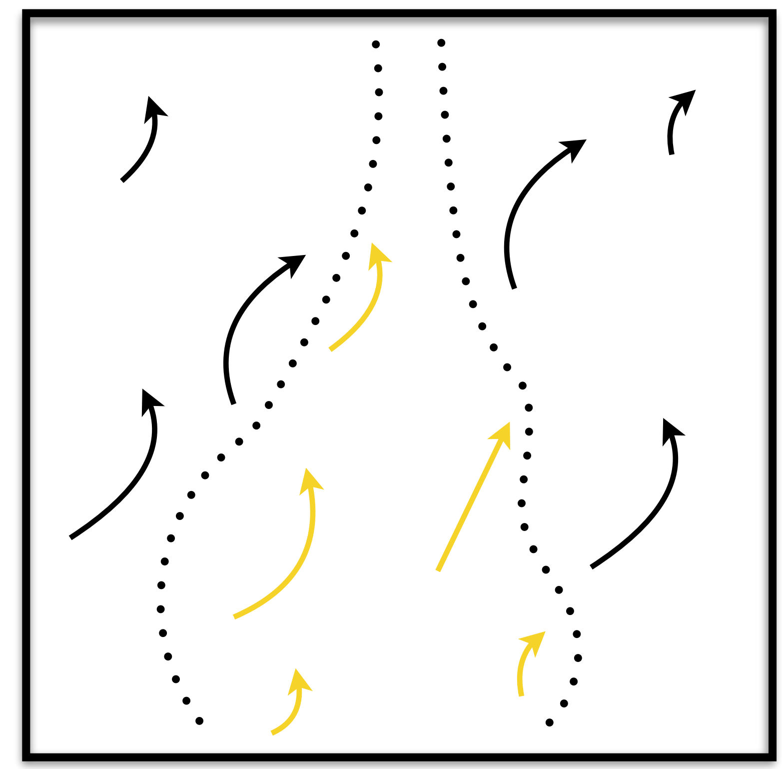
The physical assumptions



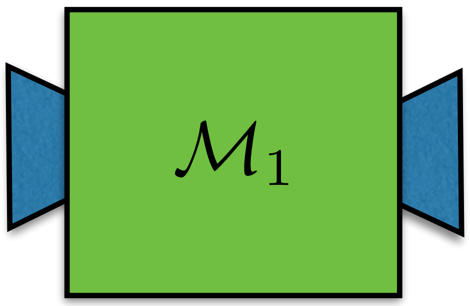
The machines

$\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$

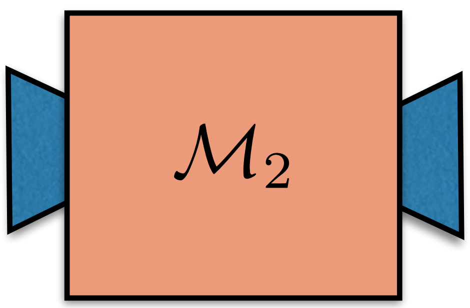
do not exist.



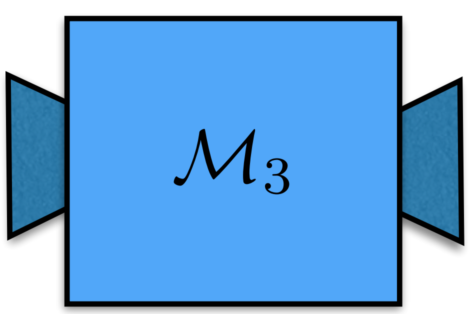
Our physical assumptions



Energy is conserved microscopically.



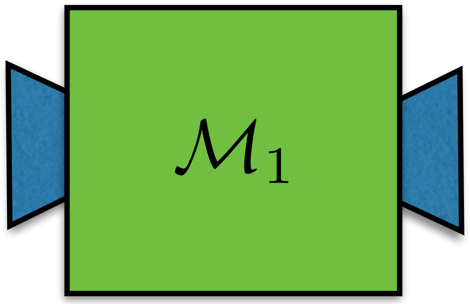
Stability of equilibrium.



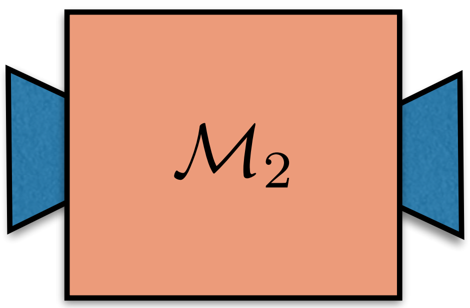
Coherence is conserved microscopically.

Aside: $\cancel{\mathcal{M}_3}$ + entanglement $\Rightarrow \cancel{\mathcal{M}_1}$

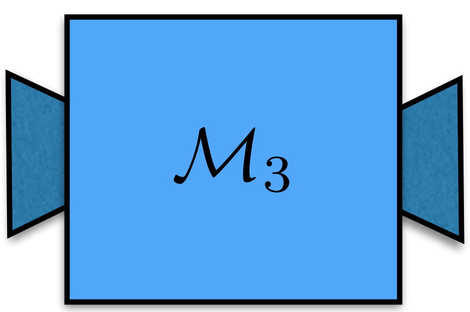
Our physical assumptions



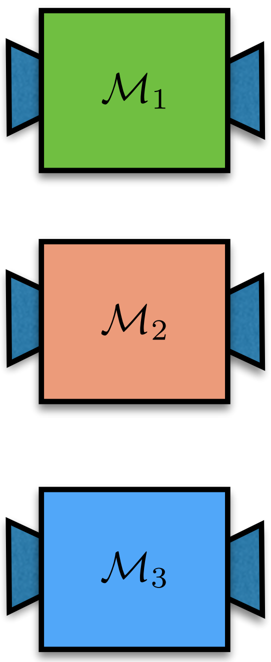
$$\mathcal{E}(\rho_A) = \mathrm{tr_C} \mathrm{V}(\rho_\mathrm{A} \otimes \sigma_\mathrm{B})\mathrm{V}^\dagger \qquad [V, H_{\mathrm{tot,micro}}] = 0$$



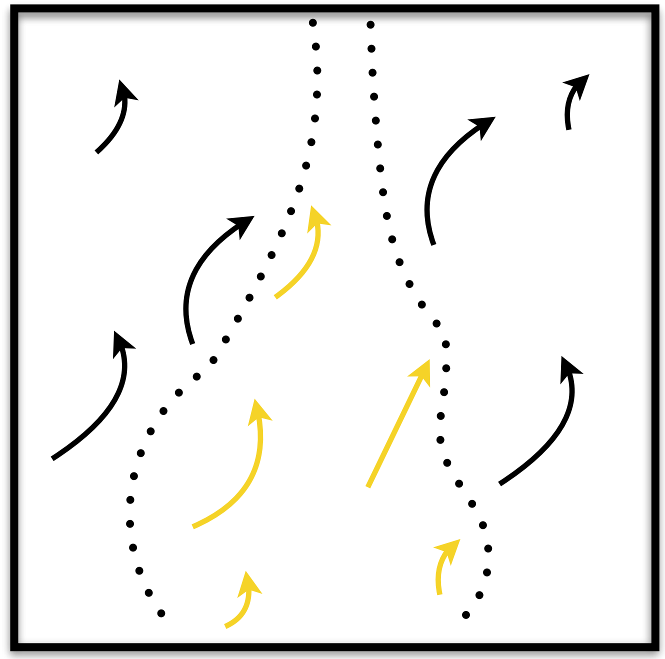
$$\mathcal{E}(\gamma) = \gamma \qquad \gamma = \frac{1}{Z}\sum_i e^{-\frac{E_i}{kT}} |E_i\rangle\langle E_i|$$



ρ diagonal in energy $\Rightarrow$ $\mathcal{E}(\rho)$ diagonal in energy



$\Leftrightarrow$



$\Leftrightarrow$

Thermodynamic entropy

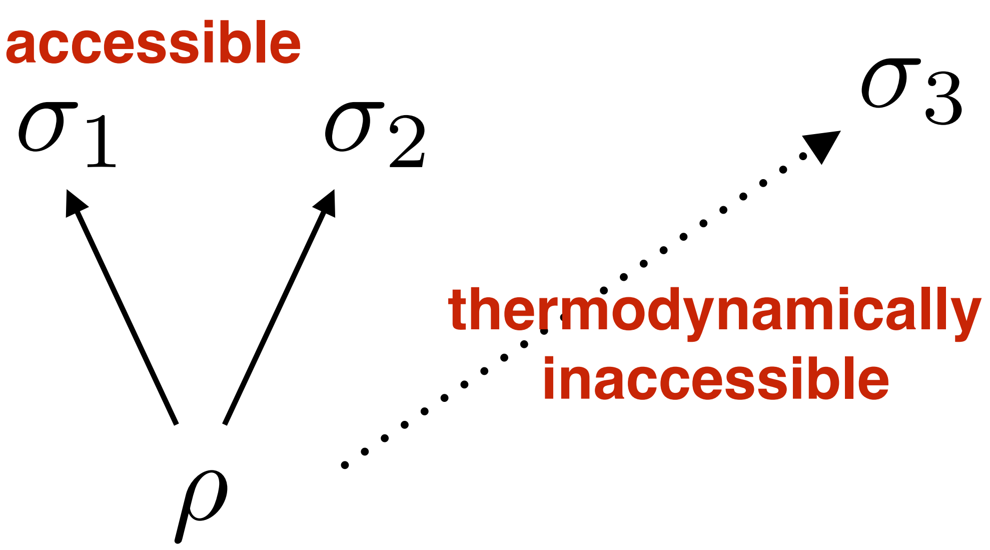
$S_\eta(\rho)$

The thermodynamic structure

[1] "The Physics and Mathematics of the Second Law of Thermodynamics", E. Lieb, J. Yngvason, Phys.Rept. 310 (1999)

[2] "The mathematical foundations of thermodynamics, R. Giles, Pergamon (1964)

Thermodynamic structure



When is

$$\rho \longrightarrow \sigma$$

thermodynamically possible?

Gory details sketched

$\mathcal{O} =$ Convex set of quantum operations

$$\mathcal{E} \in \mathcal{O}$$

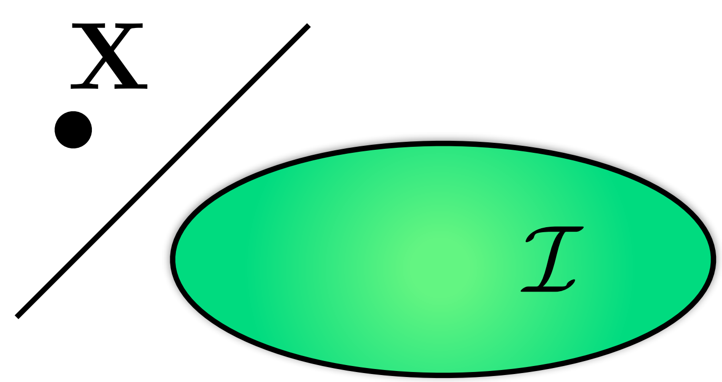
$$(\rho_1, \rho_2, \ldots, \rho_N) \longrightarrow (\sigma_1, \sigma_2, \ldots, \sigma_N) =: \mathbf{X}$$

$$\mathcal{I} := \{(\mathcal{E}(\rho_1), \mathcal{E}(\rho_2), \ldots, \mathcal{E}(\rho_N)) : \mathcal{E} \in \mathcal{O}\}$$

No such $\mathcal{E}$ exists if and only if $\mathbf{X} \notin \mathcal{I}$

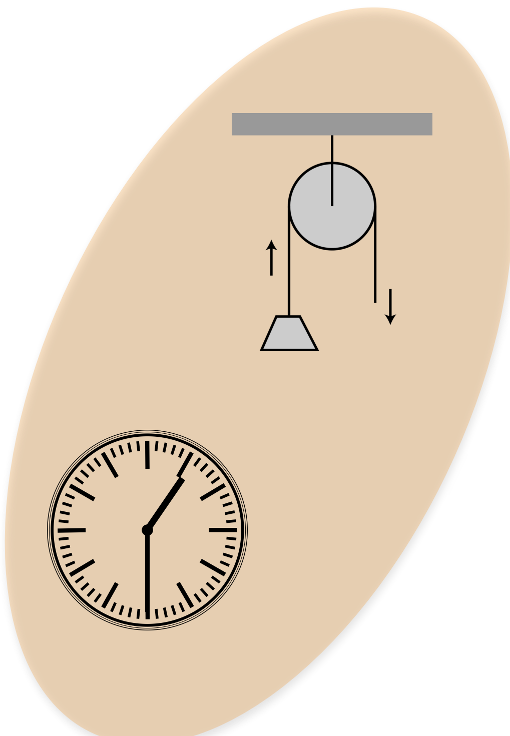
$$\mathcal{U}_t \circ \mathcal{E} = \mathcal{E} \circ \mathcal{U}_t$$
$$\mathcal{E}(\gamma) = \gamma$$

Exploit Symmetry, Convexity, single shot QI



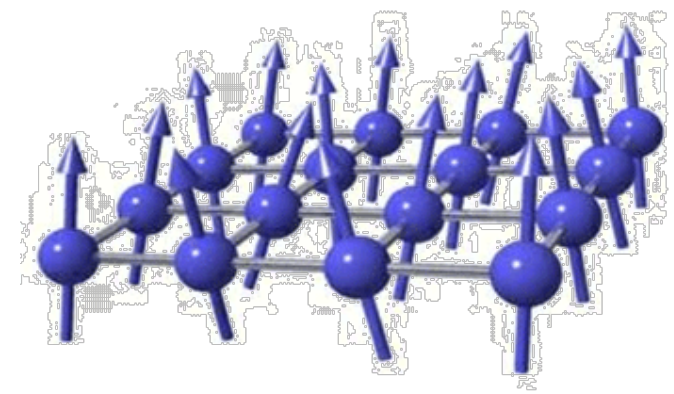
"<u>Coherent</u> Majorization"

Informal Answer:



R

S



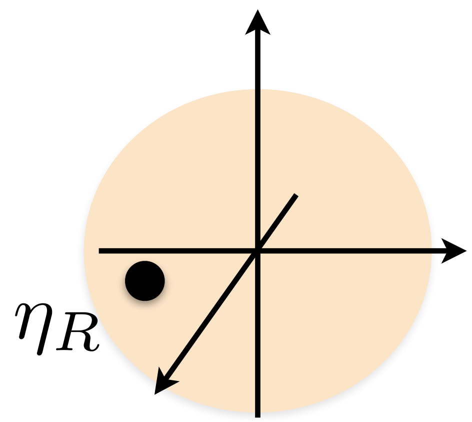
Reference frames

Quantum system

*"The coherent correlations **S** has with **R** cannot increase"*

Entropy conditions

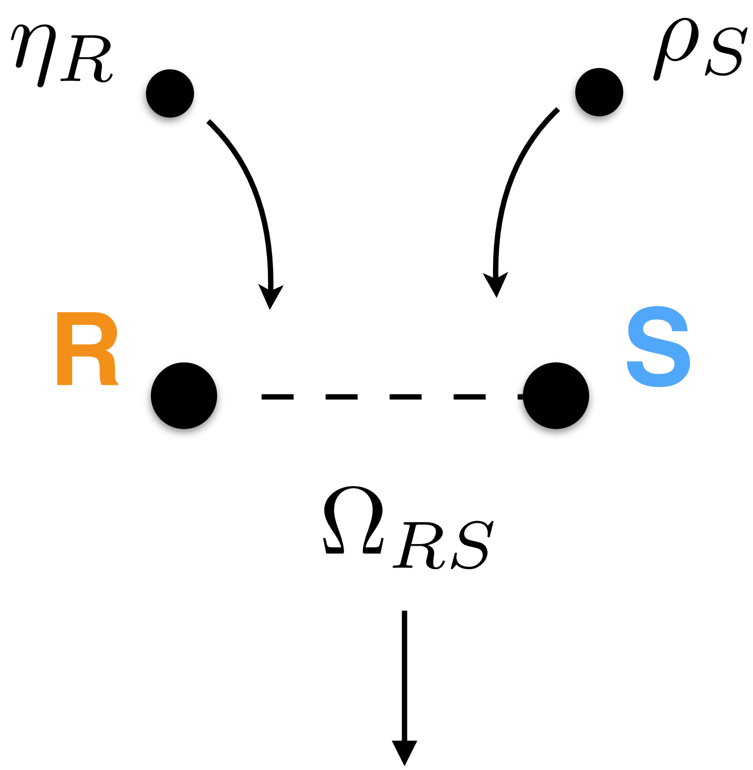
Pick any reference state



$$\rho_S \to \rho'_S$$

if and only if

$$\Delta S_\eta \geq 0$$

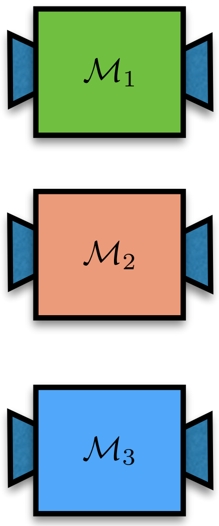


measure entropy/ correlations

$$\Omega_{RS} = \lim_{T\to\infty} \frac{1}{2T}\int_0^T dt[\eta_1(t)\otimes\rho_S(t)] + \frac{1}{2}\eta_2\otimes\gamma_S$$

$$S_\eta = H_{\min}(R|S) = -\log[\min\ \mathrm{tr}(\tau) : \mathbb{I}\otimes\tau \geq \Omega_{RA}, \tau \geq 0]$$

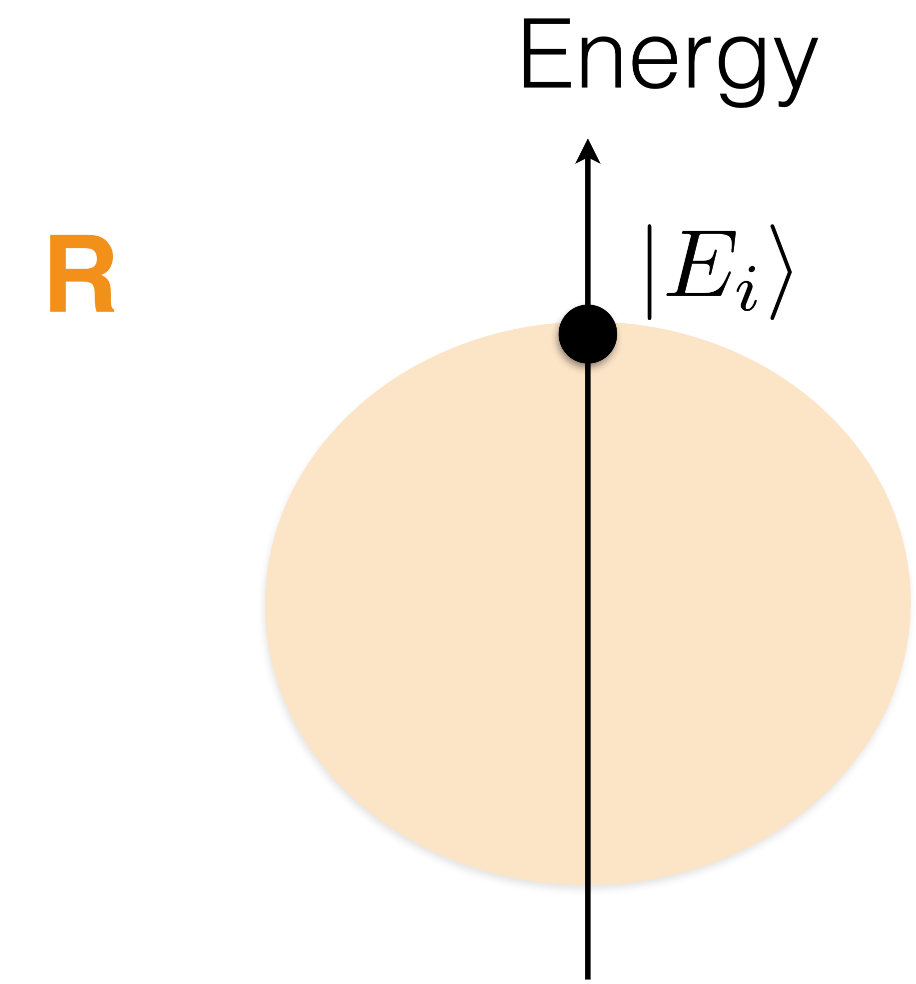
[1] Lostaglio, DJ, Rudolph Nature Comm. 6, 6383 (2015).
[2] Lostaglio, Korzekwa, DJ, Rudolph, Phys. Rev. X 5, 021001 (2015)
[3] Gour, DJ, Buscemi, Duan, Marvian, arXiv:1708.04302 (2017)



$\Leftrightarrow$ $\Delta S_\eta \geq 0$ → **Physical Consequences?**

Pros:
- Clear physics involved.
- Well-known entropy.
- Generalized Symmetry/ Noether Principles.

Cons:
- Full conditions are tough to use.
- Infinitely many conditions (unavoidable?).



$$\Delta S_\eta \geq 0$$

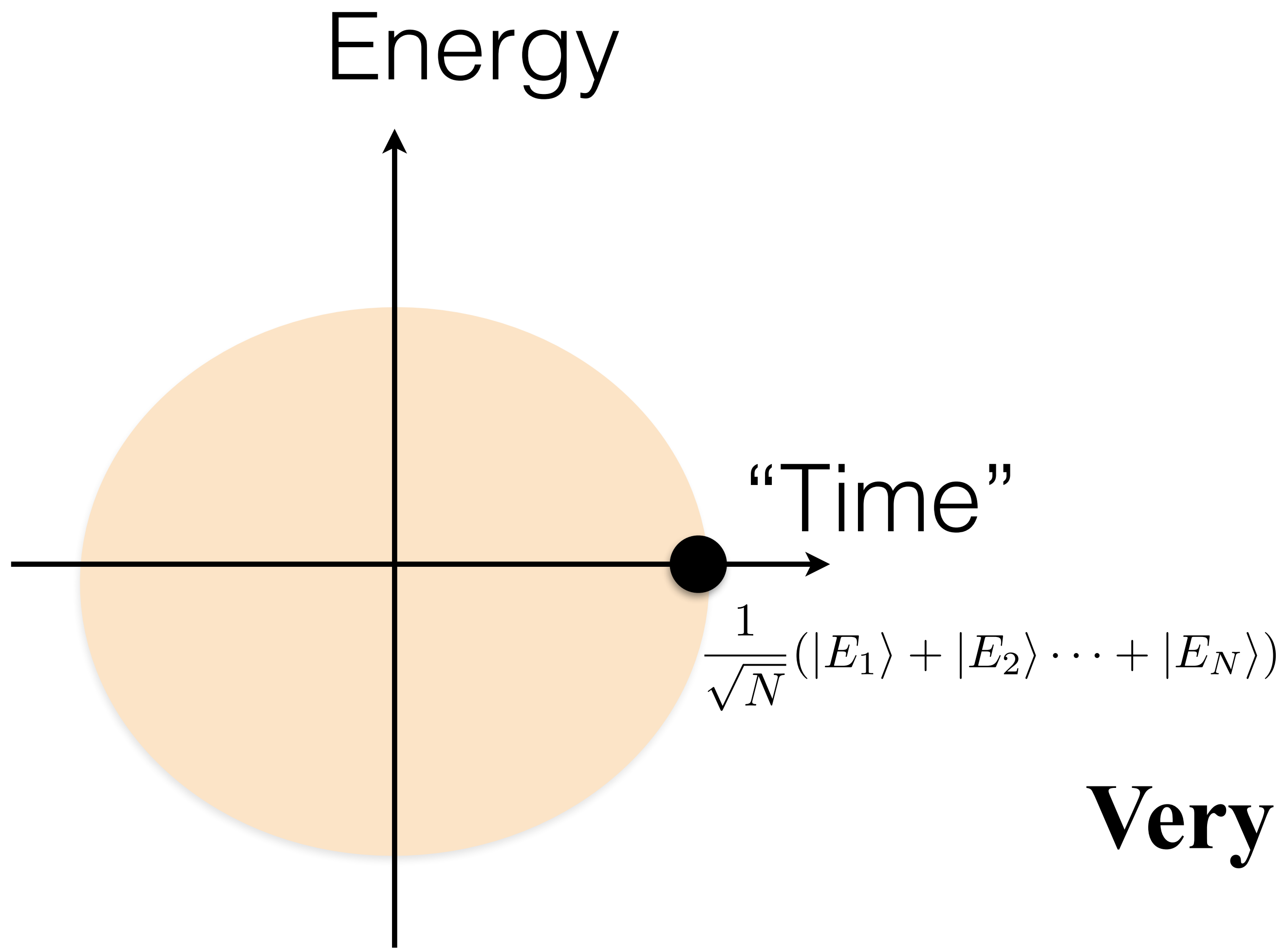
thermo-majorization

Classical stochastic thermo

$(N \to \infty)$

$$S(\rho) = -\mathrm{Tr}[\rho \log \rho]$$

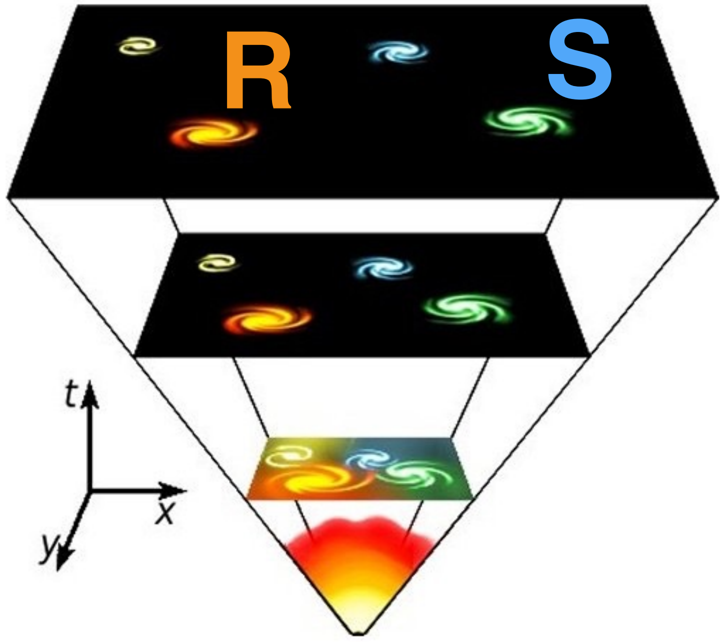
[1] Ruch, Mead, Theoret. Chim. acta, 41, 95117 (1976)
[2] Oppenheim, Horodecki Nature Comm. 4, 2059 (2013)



$$\Delta S_\eta \geq 0$$

Very high coherence regime

Ω_{RS}



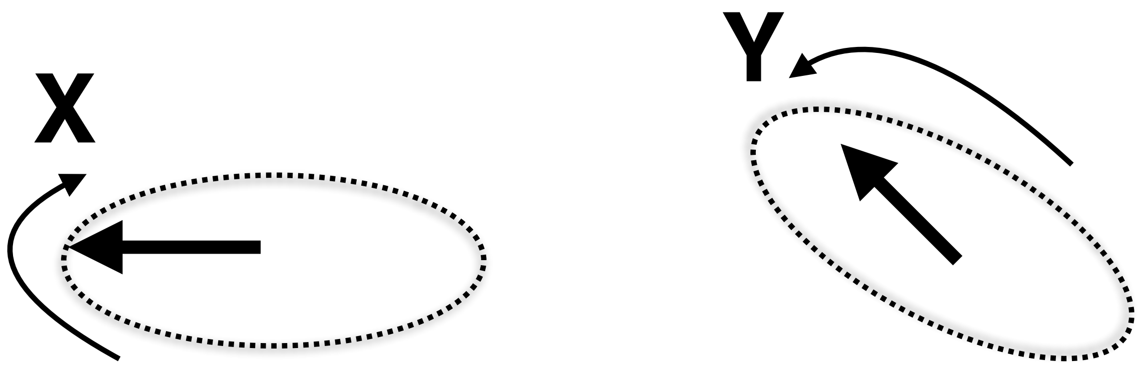
ρ

Observation:
Ω_{RS} has time-translation symmetry

Page-Wootters Time

Wheeler-de Witt:

$$H_{tot}|\Psi\rangle = 0$$



$$P(Y = y|X = x)$$

"time ~ relational correlations"

The Conditional Probability Interpretation:

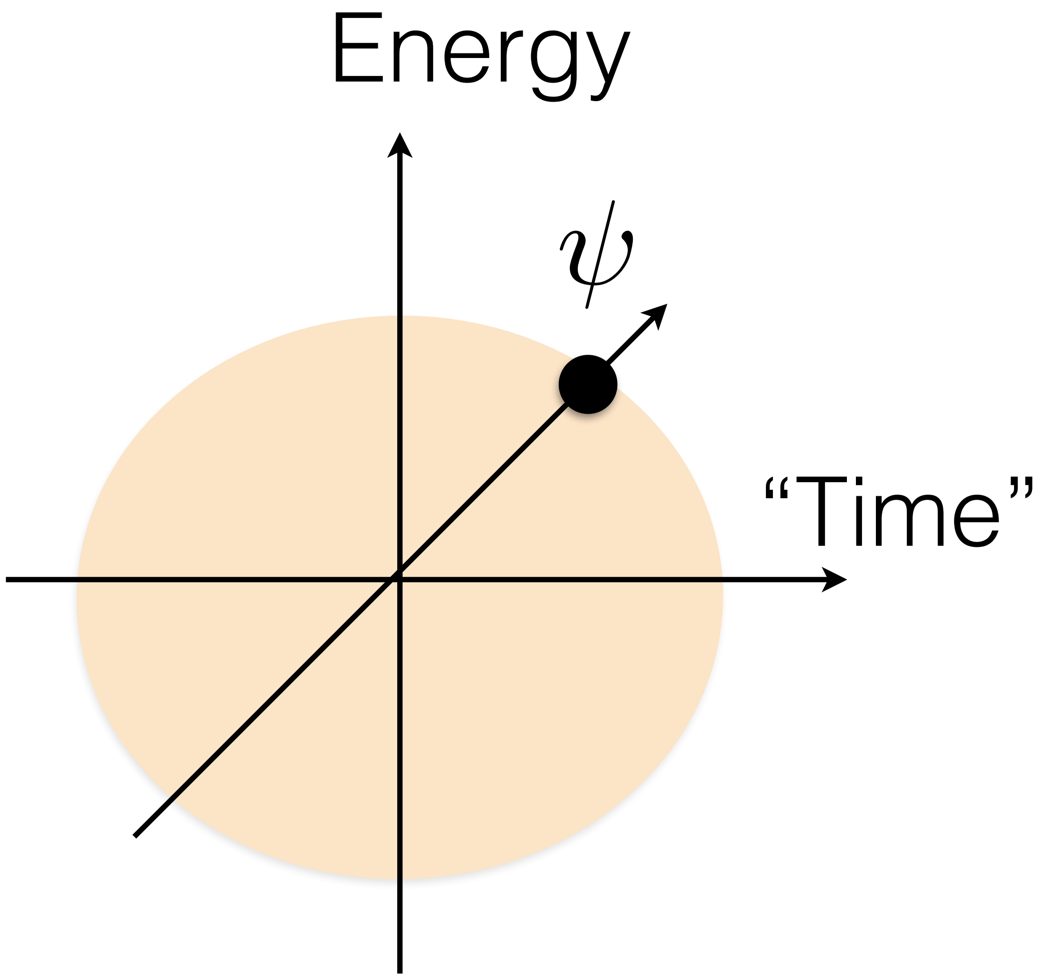
$$S_\eta(\rho) = -\log p_{\text{guess}}(t|\rho(t))$$

$$\Delta S_\eta \geq 0$$

"reading the time from S gets harder"

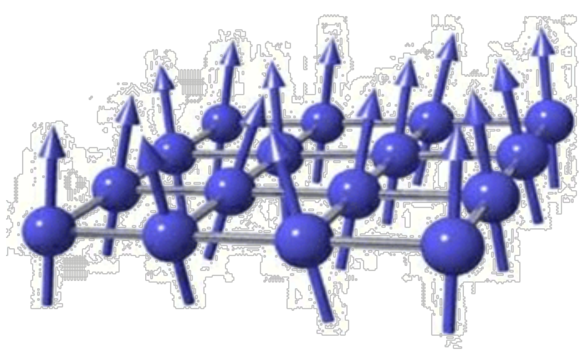
[1] Page and Wootters, Phys. Rev. D 27, 2885 (1983).
[3] Gour, DJ, Buscemi, Duan, Marvian, arXiv:1708.04302 (2017)

R



Time-energy trade-offs

S



ρ

$$W + kT\left(\frac{I(\rho,H)}{2N^2\Delta^2}\right) \leq kT\log d$$

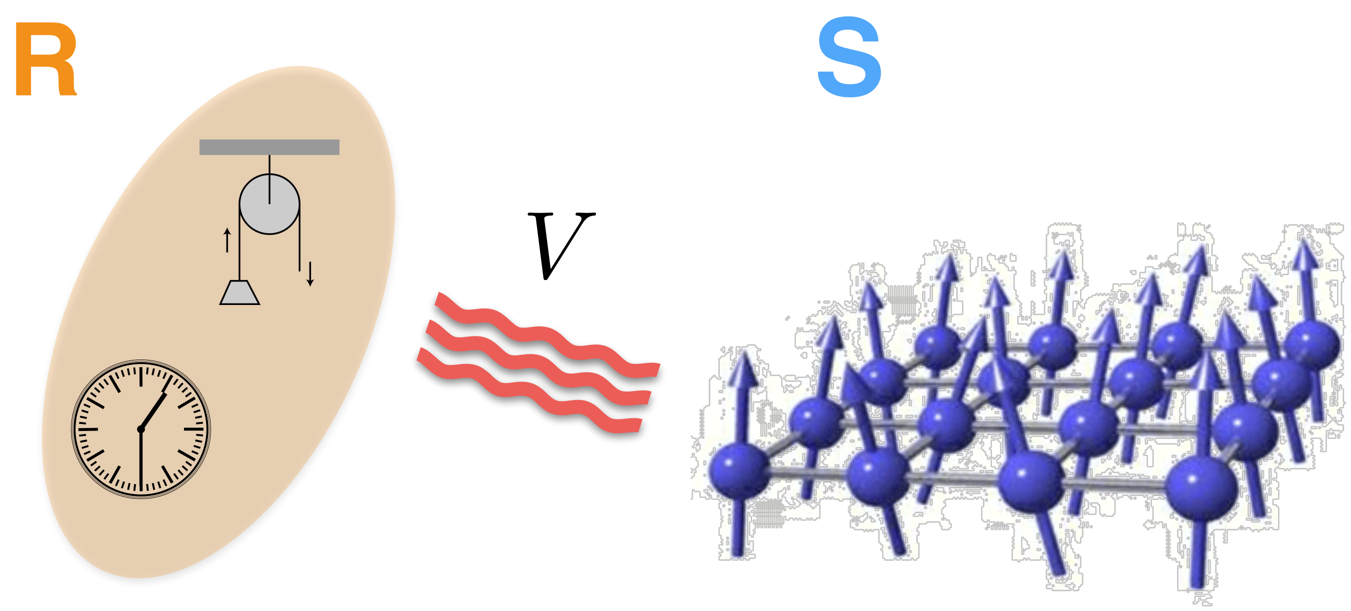
[1] Kwon, Jeong, DJ, Yadin, Kim, Phys. Rev. Lett. 120, 150602 (2018)

Open Access

Fully Quantum Fluctuation Theorems

Johan Åberg

Phys. Rev. X **8**, 011019 – Published 6 February 2018



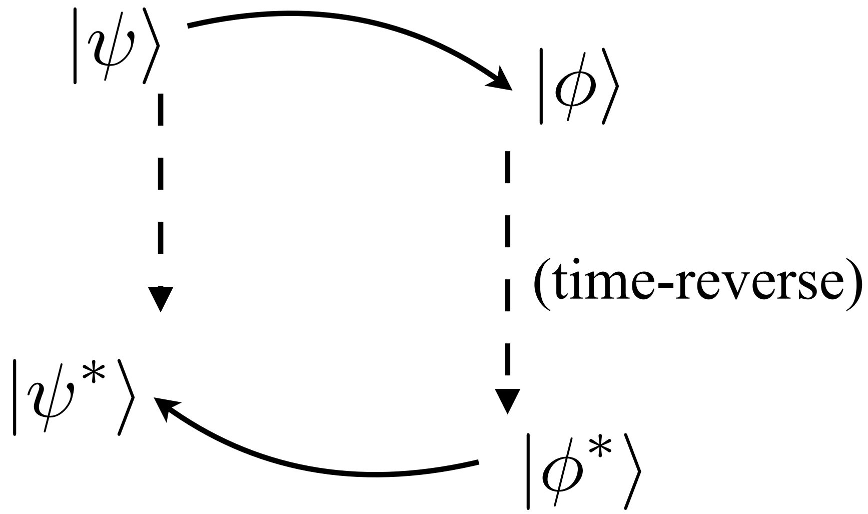
$$[V, H_{\text{tot}}] = 0$$

$\sigma_R(0)$ arbitrary

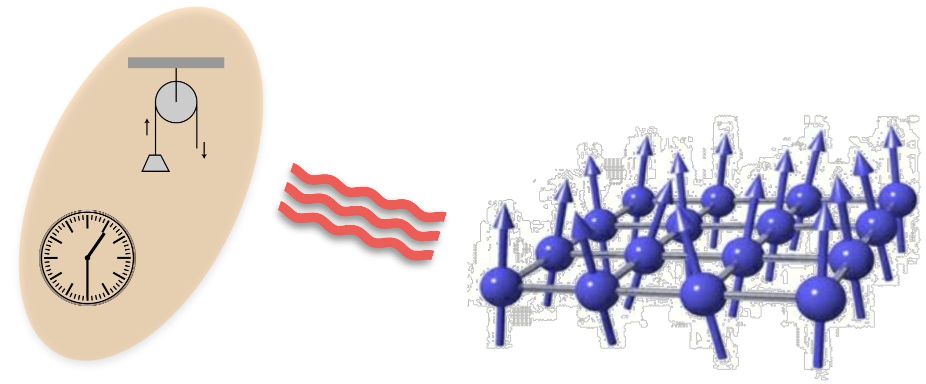
t=1 : Do *any* measurement on R

$$\rho_S(0) = \frac{1}{Z} e^{-\beta H_S(0)}$$

$$H_S(0) \rightarrow H_S(1)$$



$|\psi\rangle$



$|\phi^*\rangle \propto \Theta(e^{-\frac{1}{2}\beta H_{RS}}|\phi\rangle)$

$|\psi^*\rangle \propto \Theta(e^{\frac{1}{2}\beta H_{RS}}|\psi\rangle)$

$$\frac{P(\phi|\psi)}{P(\psi^*|\phi^*)} = e^{-\beta(\Delta F - \Delta\Lambda)}$$

effective potential

$$\Lambda(\rho) = -\log \mathrm{tr}[\mathrm{e}^{-\beta \mathrm{H}}\rho]$$

$$= e^{-\beta(\Delta F - W)}\left[e^{-\frac{1}{2}\beta^2\Delta I}\right]\left[e^{-\sum_{n\geq 3}\frac{\beta^n \Delta\kappa_n}{n!}}\right]$$

Classical Crooks

semi-classical

fully quantum

[1] Erick Hinds Mingo, DJ, in preparation, ***see Erick's poster.***

Resource theory of coherence

$$|\psi\rangle \xrightarrow{\mathcal{E}} |\phi\rangle$$

deterministically!

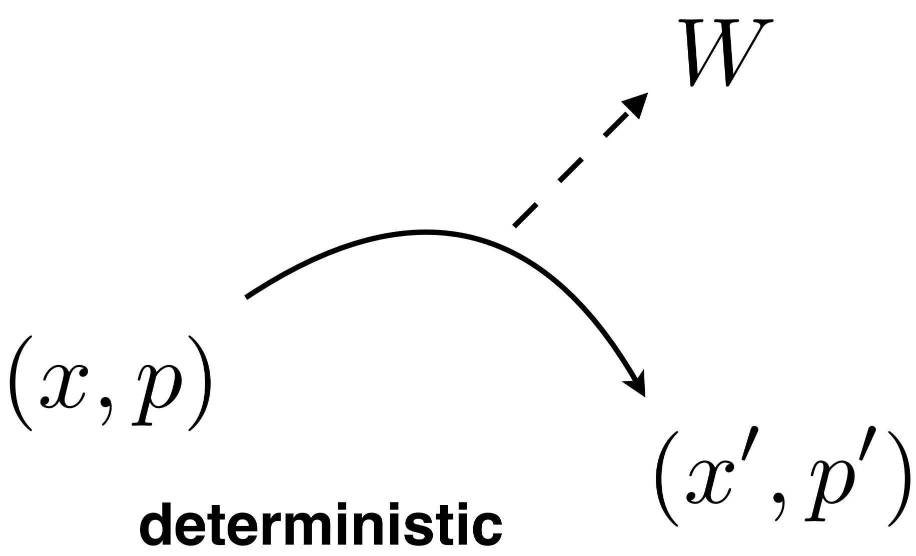
$\mathcal{E}$: covariant

if and only if

There exists $\mu(w)$ in FT setting s.t.

$$\frac{P(\phi|\psi)}{P(\psi^*|\phi^*)} = \int d\mu(w) e^{-\beta(\Delta F - w)}$$

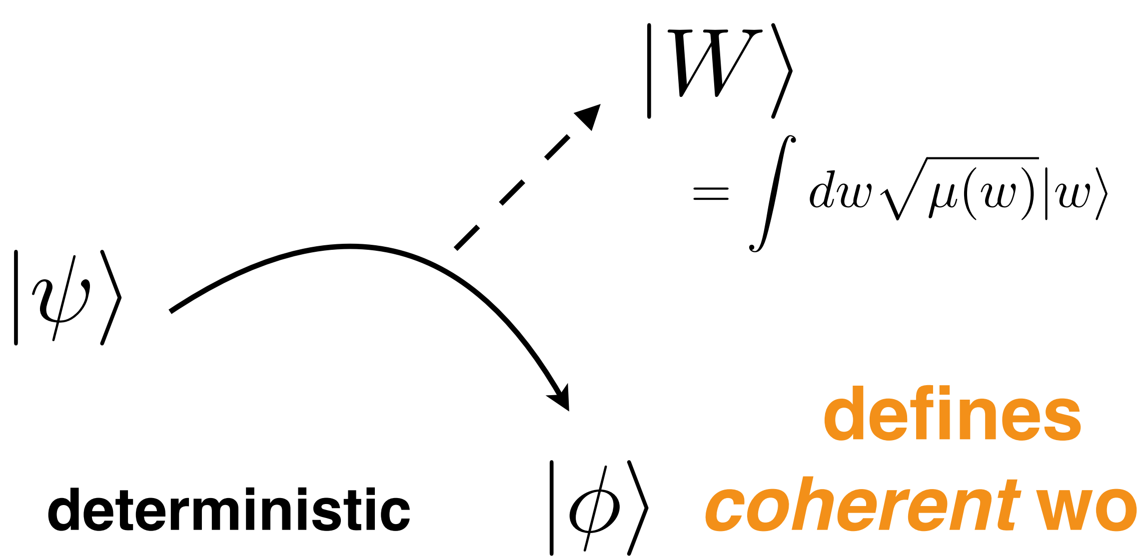
Newtonian mechanics



$$\frac{P_+}{P_-} = e^{-\beta(\Delta F - W)}$$

- -

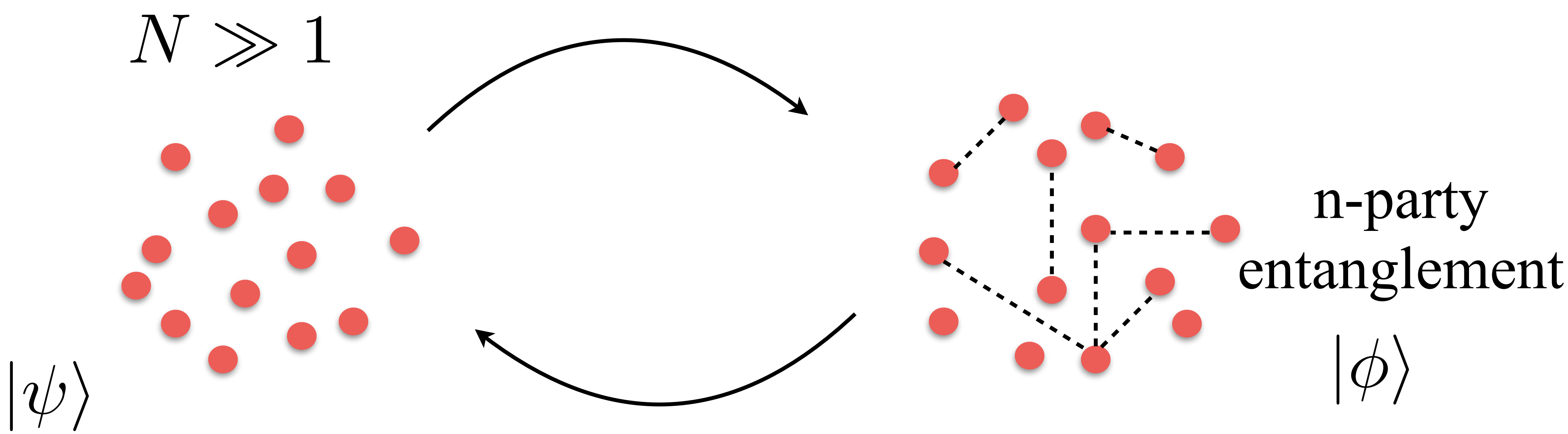
Quantum mechanics



defines
***coherent* work**

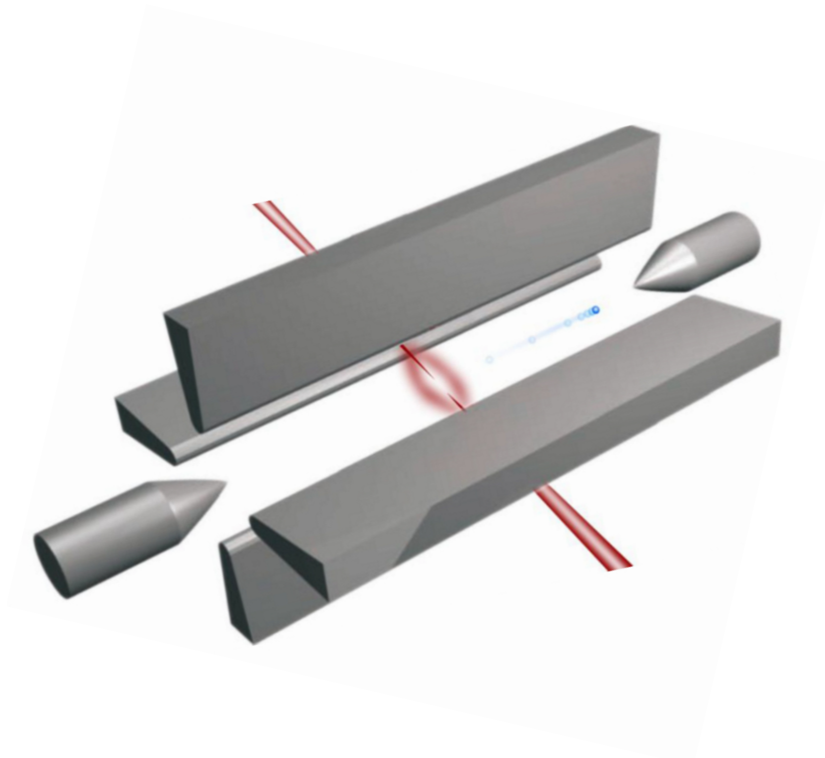
$$\frac{P(\phi|\psi)}{P(\psi^*|\phi^*)} = e^{-\beta(\Delta F - \Delta\Lambda)}$$

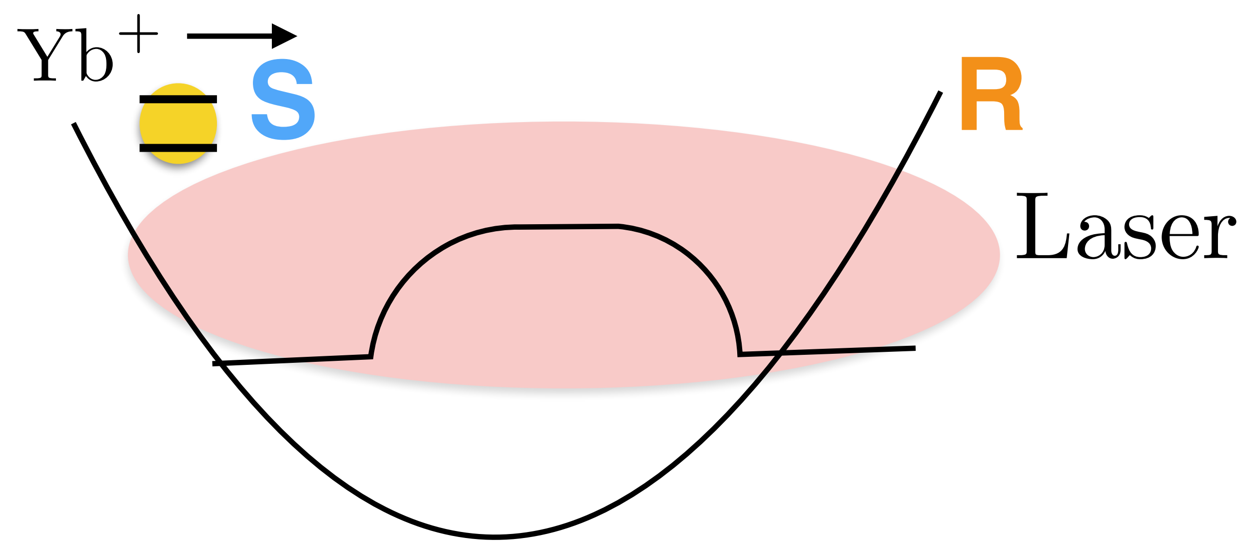
Application: multipartite entanglement



$$\frac{P(\text{create})}{P(\text{destroy})} \leq e^{-\frac{\Delta F}{kT}} e^{-\frac{sn^2 + (N-sn)^2}{(kT)^2}}$$

Trapped Ion proposal



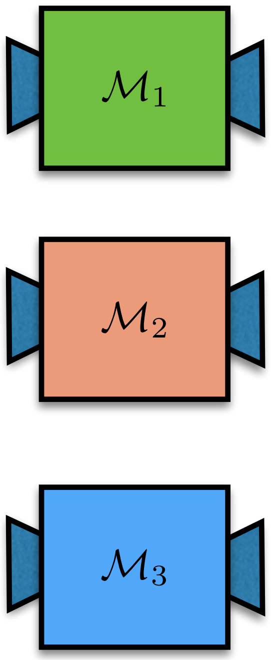


$$\frac{P(\phi|\psi)}{P(\psi^*|\phi^*)} = e^{-\left(\frac{\Delta F}{kT} - \frac{W}{\hbar\omega(T)}\right)}$$

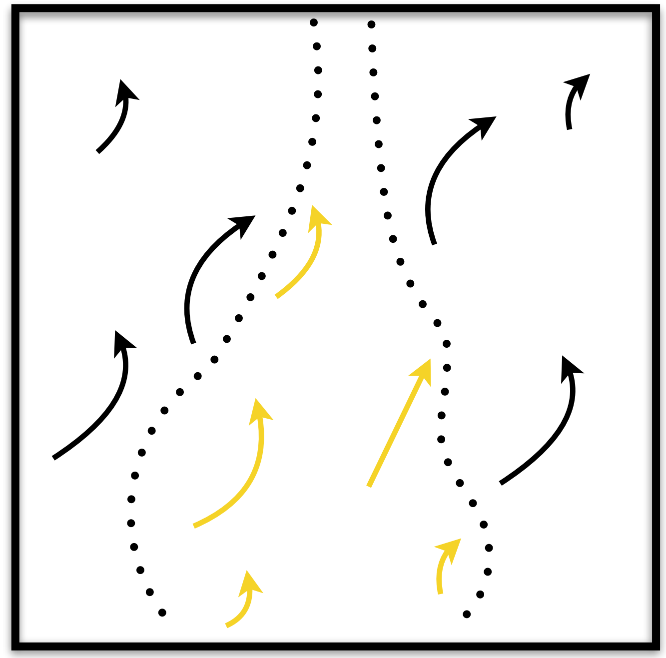
de Broglie thermal frequency!

$$\hbar\omega(T) = \frac{h^2}{m\lambda_{dB}(T)^2} + \frac{1}{2}\hbar\omega$$

[1] Zoë Holmes, Anders, DJ, Mintert, Weidt in preparation, ***see Zoë's poster.***



$\Leftrightarrow$



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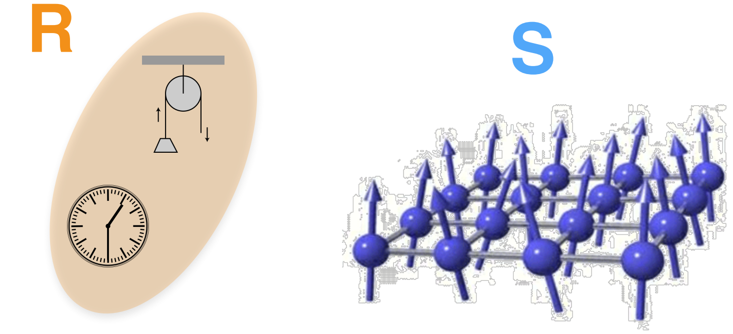


$$\Delta S_\eta \geq 0$$

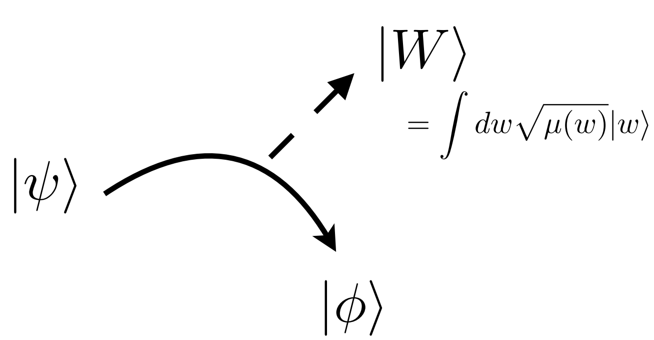
(energy)

thermo-majorization

$$W + kT\left(\frac{I(\rho, H)}{2N^2\Delta^2}\right) \leq kT \log d$$

(coherence)

Page-Wootters



$$\frac{P(\phi|\psi)}{P(\psi^*|\phi^*)} = e^{-\beta(\Delta F - \Delta\Lambda)}$$