

# Stochastic thermodynamics in single electron circuits



**Prof.  
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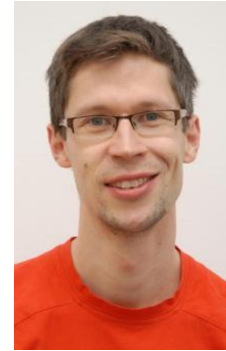
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[arXiv:1712.01693](https://arxiv.org/abs/1712.01693)



**Edgar  
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Quantum thermodynamics,  
KITP, June 26, 2018

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MAX-PLANCK-GESELLSCHAFT

# Outline

- **General introduction:**
  - ✓ Definition of entropy. Fluctuation relations
  - ✓ Coulomb blockade and thermodynamics
  - ✓ Jarzynski and Crooks relations in single-electron circuits
  - ✓ Realization of Maxwell's Demons
  - ✓ Quantum calorimetry and heat transport
- **Nonequilibrium steady state**
  - ✓ Negative entropy events
  - ✓ Statistics of finite-time minima of entropy production
- **Experimental verification of theoretical results**
  - ✓ Double-dot structure as a minimal physical model
  - ✓ Boundaries for entropy production records
  - ✓ Relation with the heat absorption
- **Summary and outlook**

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# Stochastic thermodynamics

## Gibbs-Shanon system entropy

$$S_{\text{sys}} = -\ln P(n(t); t)$$

$$\langle S_{\text{sys}} \rangle = -\langle \ln P(n(t); t) \rangle = S_{\text{Gibbs}}(t) = \frac{U(t) - F(t)}{T}$$

$$\Delta S_{\text{Gibbs}} = \frac{\Delta U - \Delta F}{T}$$

## Heat and microscopic reversibility (local detailed balance)

$$Q(\tau) = \int_0^\tau \frac{\partial H}{\partial t} dt \quad \frac{p_{i \rightarrow j}}{p_{j \rightarrow i}} = e^{-(U_j - U_i)/T}$$

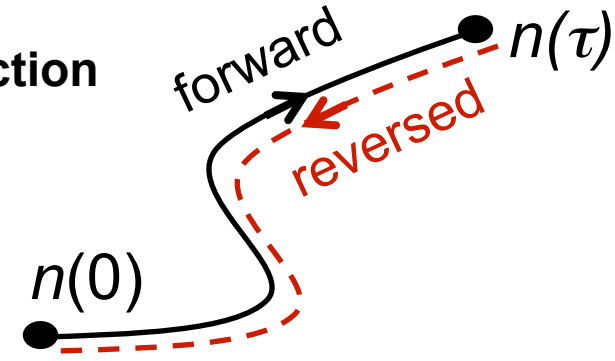
$$\frac{P(\{n(t)\} | n(0))}{P_R(\{n(\tau - t)\} | n(\tau))} = e^{-Q(\tau)/T}$$

## Stochastic total entropy production

$$\Delta S = \Delta S_{\text{sys}} + Q(\tau)/T$$

$$\Delta S = \ln \frac{P(\{n(t)\})}{P_R(\{n(\tau - t)\})}$$

Measure of irreversibility



## Crooks relation

$$\frac{P(\Delta S)}{P_R(-\Delta S)} = e^{\Delta S}$$

## Jarzynski equality

$$\langle e^{-\Delta S} \rangle = 1$$

## 2<sup>nd</sup> law of thermodynamics

$$\langle \Delta S \rangle \geq 0$$

# Fluctuation theorem

$$\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_B}$$

U. Seifert, Rep. Prog. Phys.  
**75**, 126001 (2012)

$$\langle e^{-\Delta S/k_B} \rangle = 1$$

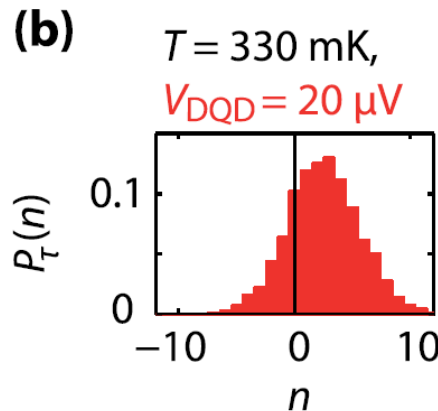
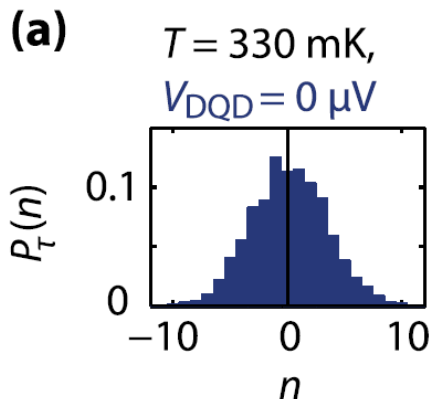
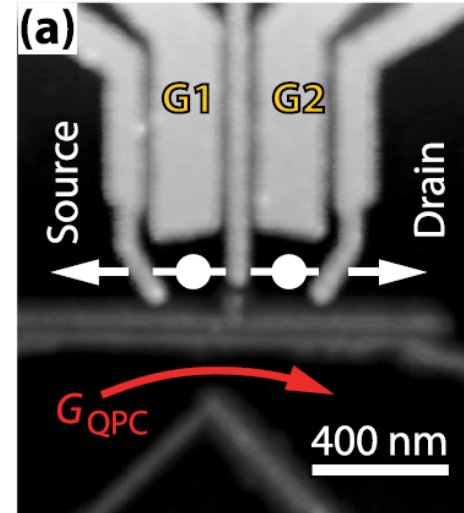
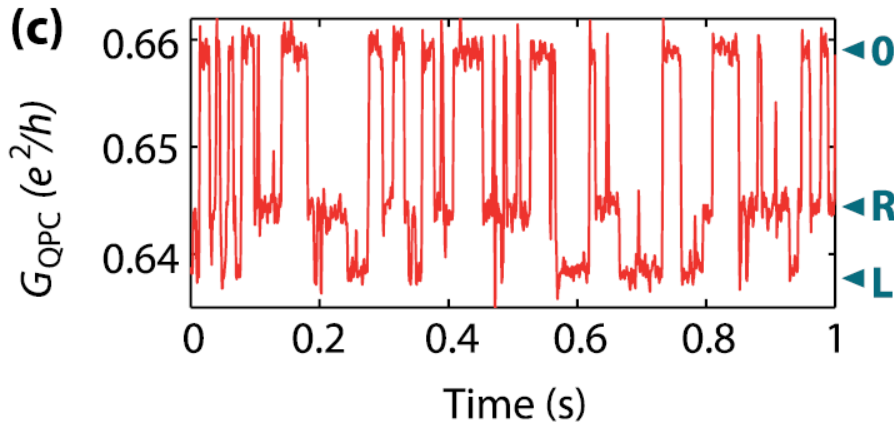
G. Bochkov, Yu. Kuzovlev, JETP 1977, Physica A 1981

*Electric circuits: Experiment on a double quantum dot*

Y. Utsumi et al. PRB **81**, 125331 (2010),

B. Kung et al. PRX **2**, 011001 (2012)

Nonequilibrium steady state



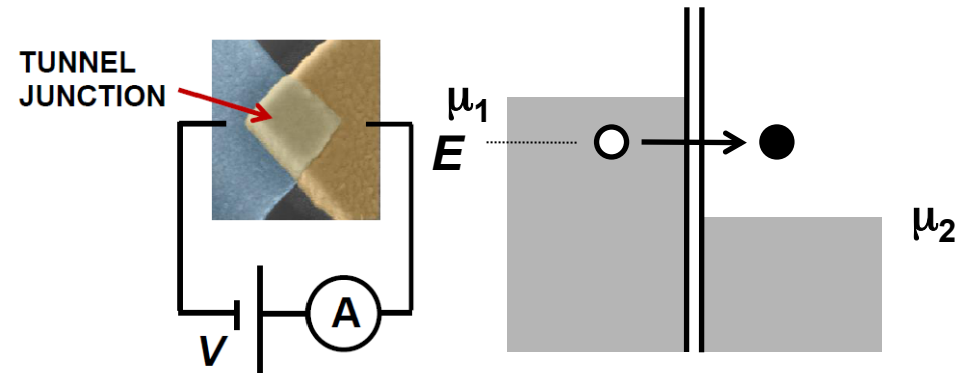
Crooks as detailed balance

$$\frac{P_\tau(n)}{P_\tau(-n)} = e^{neV_{\text{DQD}}/k_B T}$$

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# Dissipation in circuit transport through a barrier - tunneling



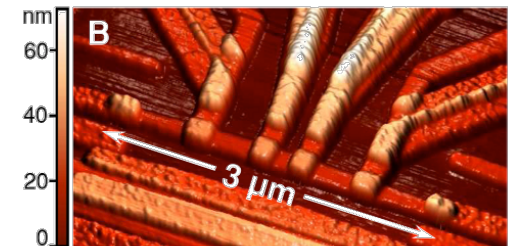
Dissipation generated by a tunneling event in a junction biased at voltage  $V$

$$\Delta Q = (\mu_1 - E) + (E - \mu_2) = \mu_1 - \mu_2 = eV$$

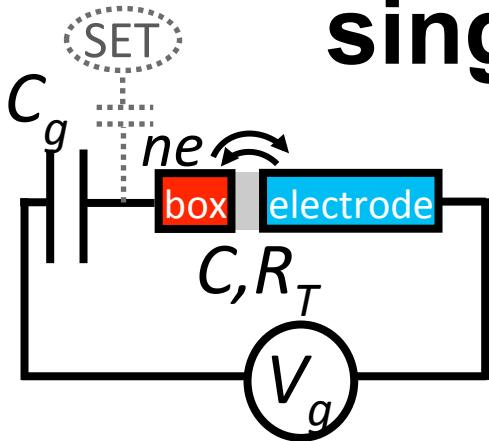
$\Delta Q = T\Delta S$  is first quickly distributed among the electron system, then - to the lattice by electron-phonon scattering

For average current  $I$  through the junction, total average power dissipated is naturally Joule heating power

$$P = (I/e)\Delta Q = IV$$



# Thermodynamics and dissipation in single-electron transitions



Heat generated in a tunneling event  $i$ :

$$Q_i = \pm 2E_C(n_{g,i} - 1/2)$$

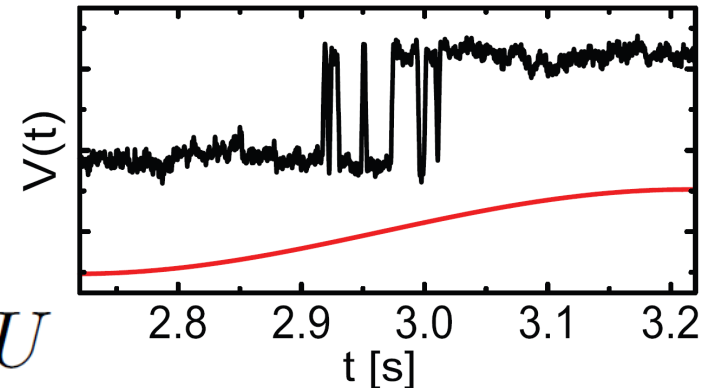
Total heat generated in a process:

$$Q = \sum_i Q_i$$

Work in a process:

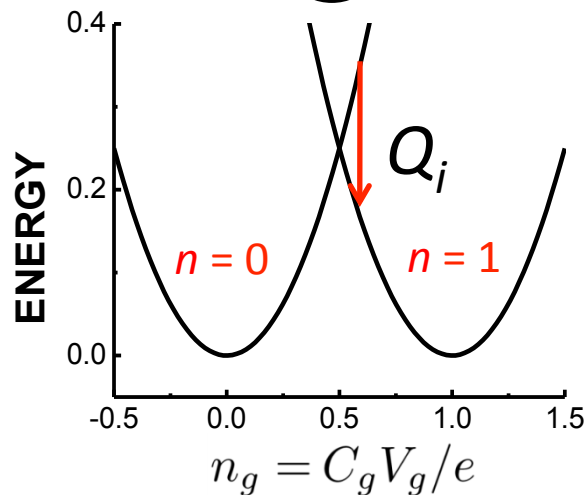
$$W = Q + \Delta U$$

Change in internal (charging) energy



| - charge state  $n$

| - gate voltage  $n_g$



$$H = E_C(n - n_g)^2$$

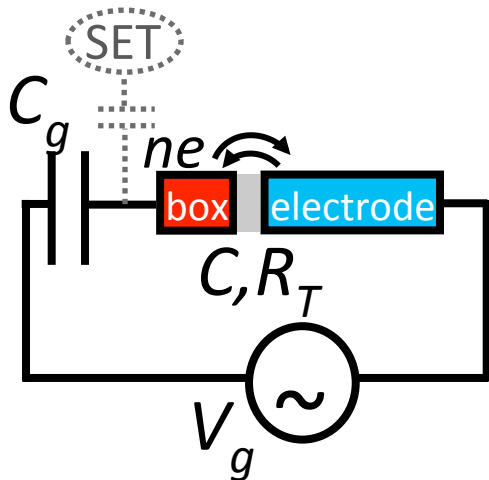
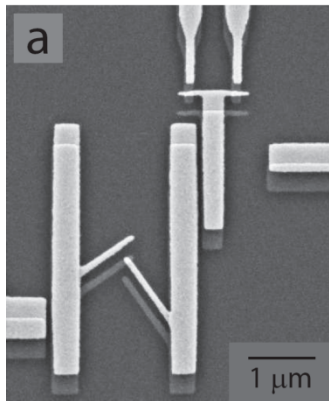
D. Averin and J. Pekola EPL **96**, 67004 (2011);  
J. Pekola and O.-P. Saira JLTP **169**, 70 (2012).

$$U = H(n(t), n_g(t)) - H(n(0), n_g(0))$$

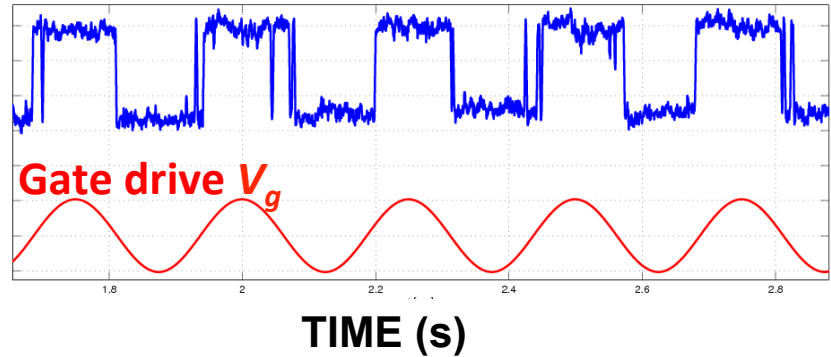


# Experiment on a single-electron box

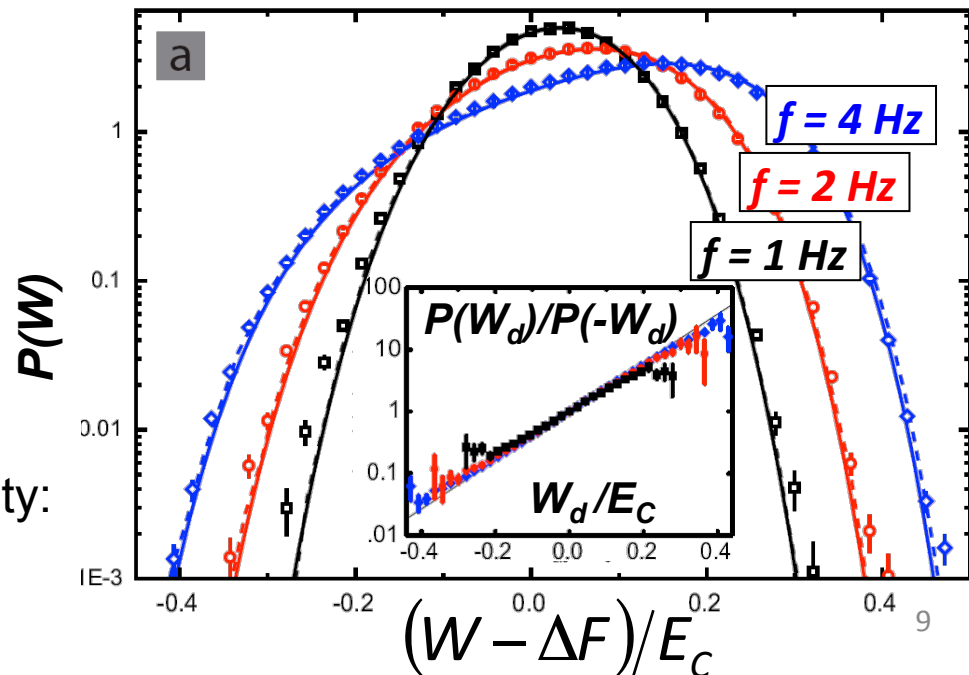
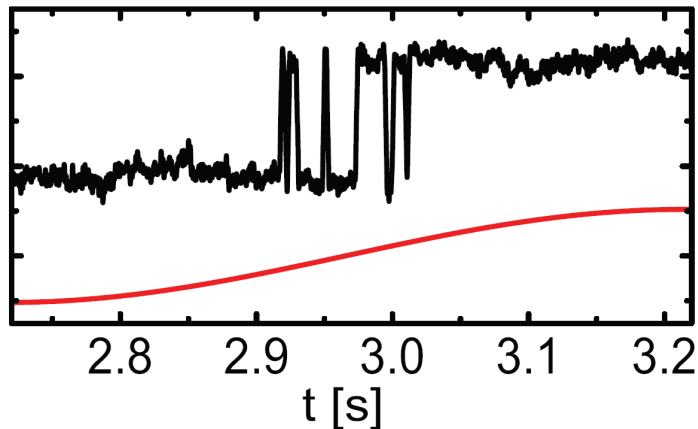
O.-P. Saira et al., PRL **109**, 180601 (2012); J.V. Koski et al., Nat. Phys. **9**, 644 (2013).



Detector current  $I_{\text{det}} \sim n$



$$T\Delta S = W - \Delta F$$



The distributions satisfy Jarzynski equality:

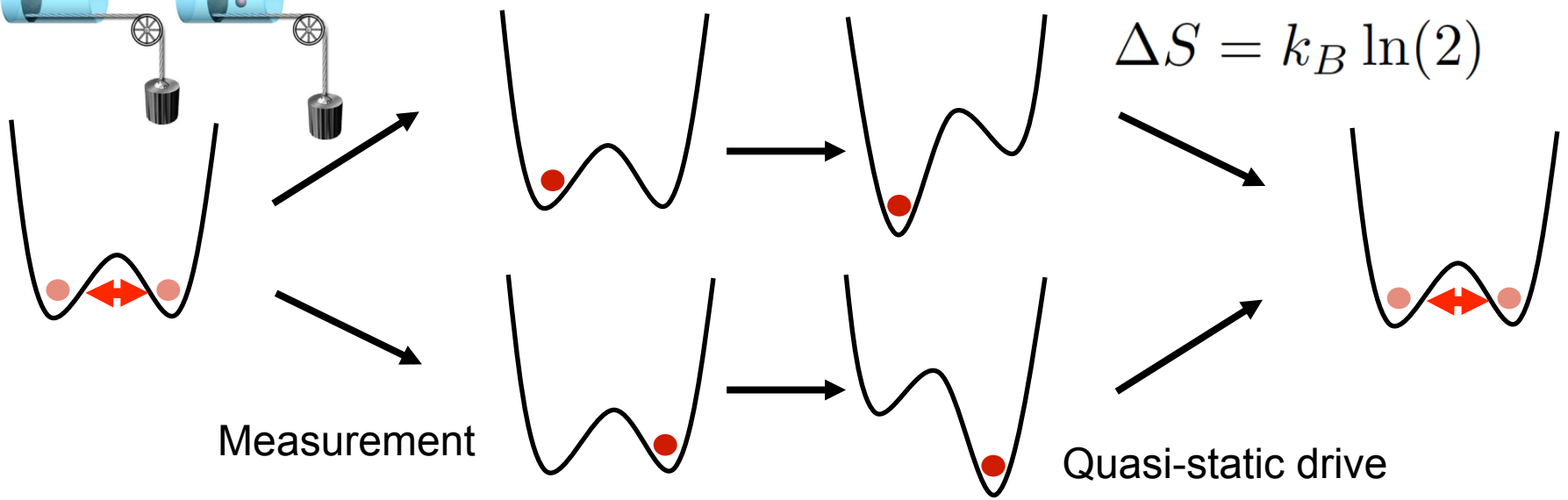
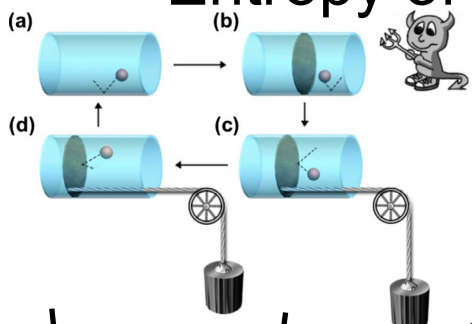
$$\langle e^{-\beta(W - \Delta F)} \rangle = 1.03 \pm 0.03$$

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# Maxwell's demon for single electrons

Entropy of the charge states:  $S = -k_B \sum_{i=0,1} p(i) \ln[p(i)]$



Measurement

Fast drive after the decision

Quasi-static drive

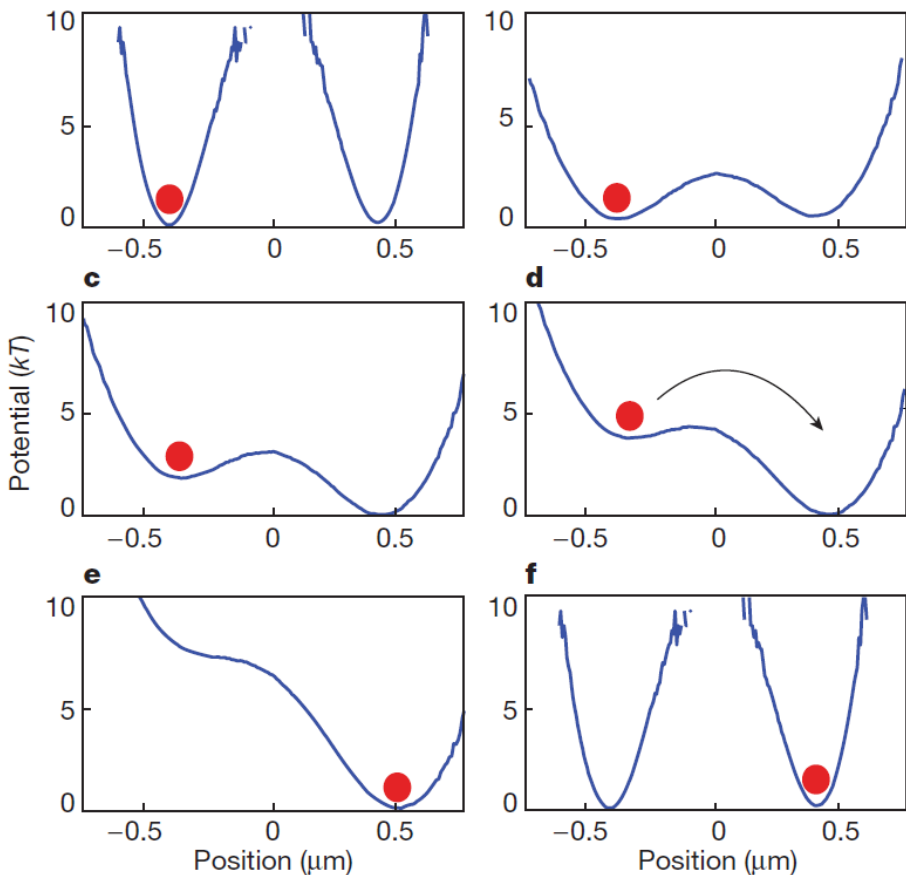
In the full cycle (ideally):  $Q_{sys} = W_{sys} = -k_B T \ln 2 < 0$

# Erasure of information

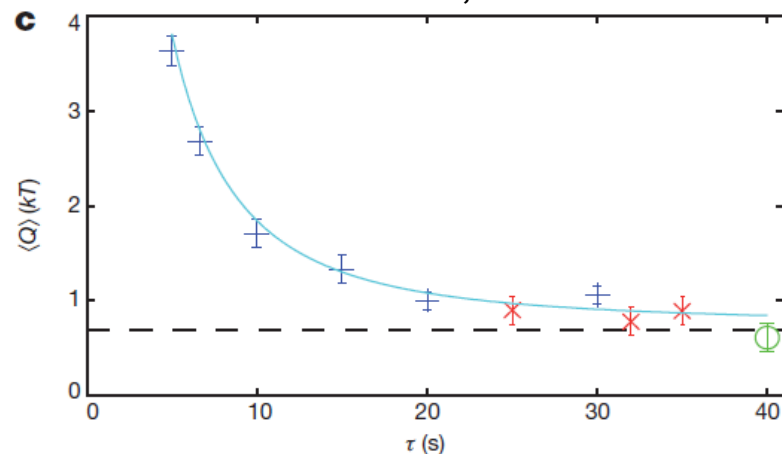
R. Landauer, IBM J. Res. Dev. 1961

**Landauer principle: erasure of a single bit costs energy of at least  $k_B T \ln(2)$**

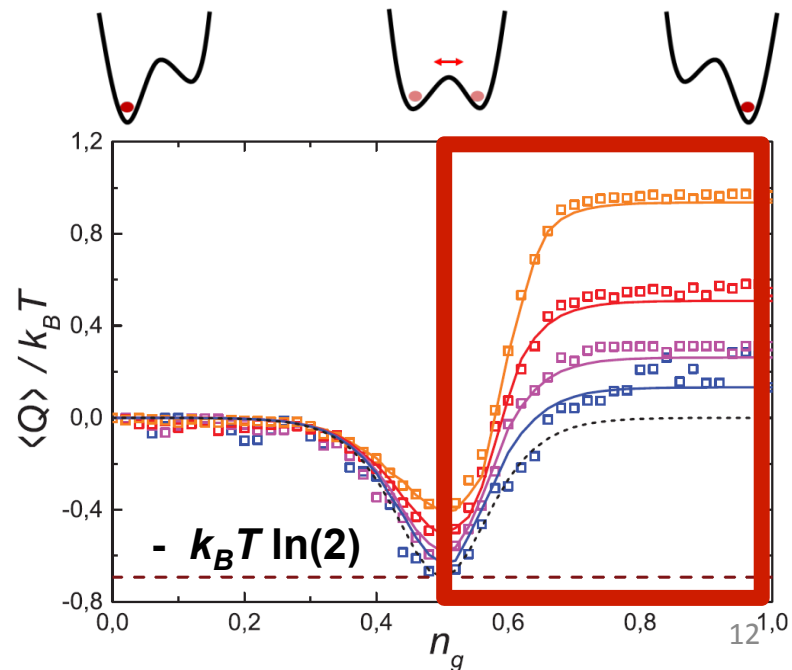
Experiment on a colloidal particle:



A. Berut et al., Nature 2012

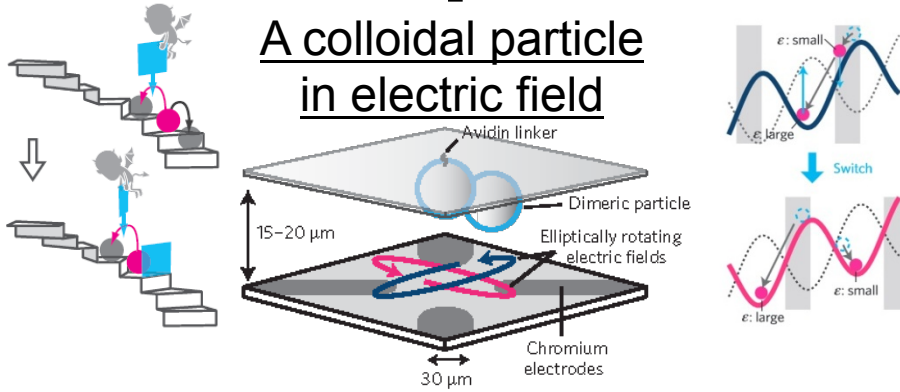


Corresponds to MD experiment:



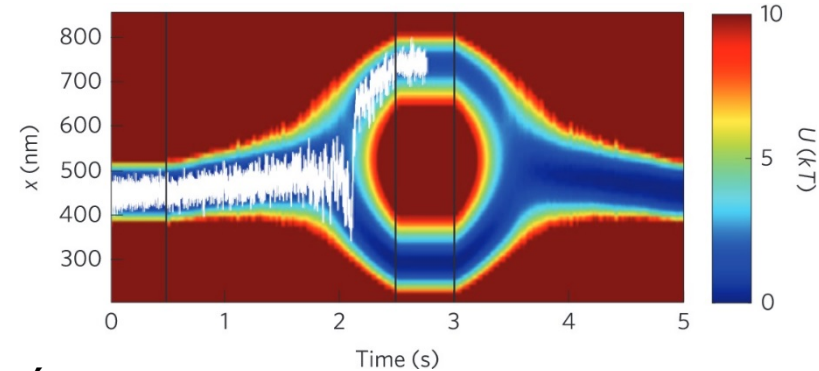
# Examples of Maxwell's demon

## A colloidal particle in electric field



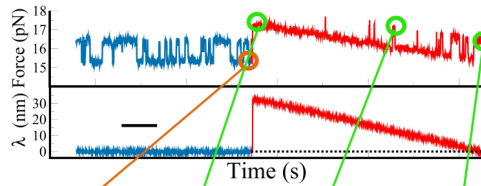
S. Toyabe et al., *Nature Physics* 2010

## A colloidal particle in laser tweezers



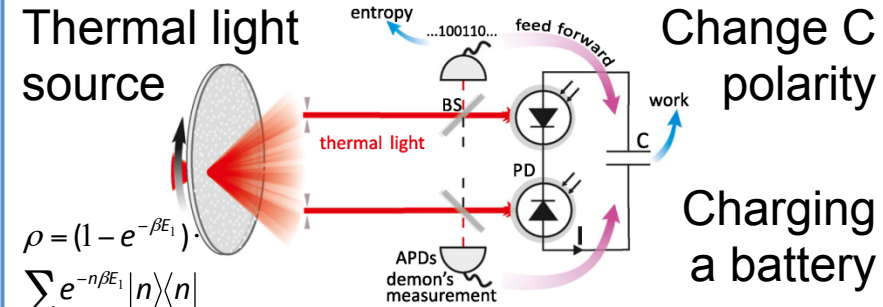
É. Roldán et al., *Nature Phys.* 2014

## DNA molecule



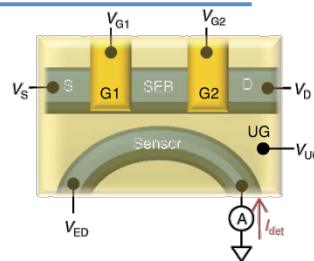
M. Ribezzi-Crivellari et al., in preparation

## Photonic MD



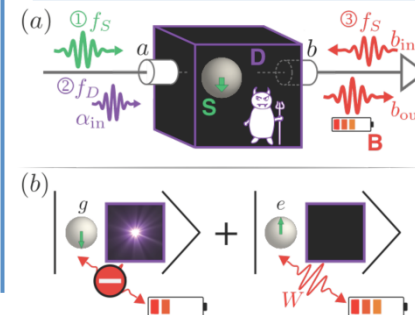
M. D. Vidrighin et al PRL 2016

## MD electronic pump



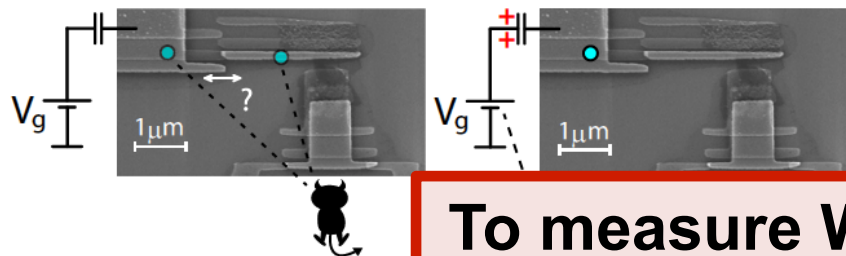
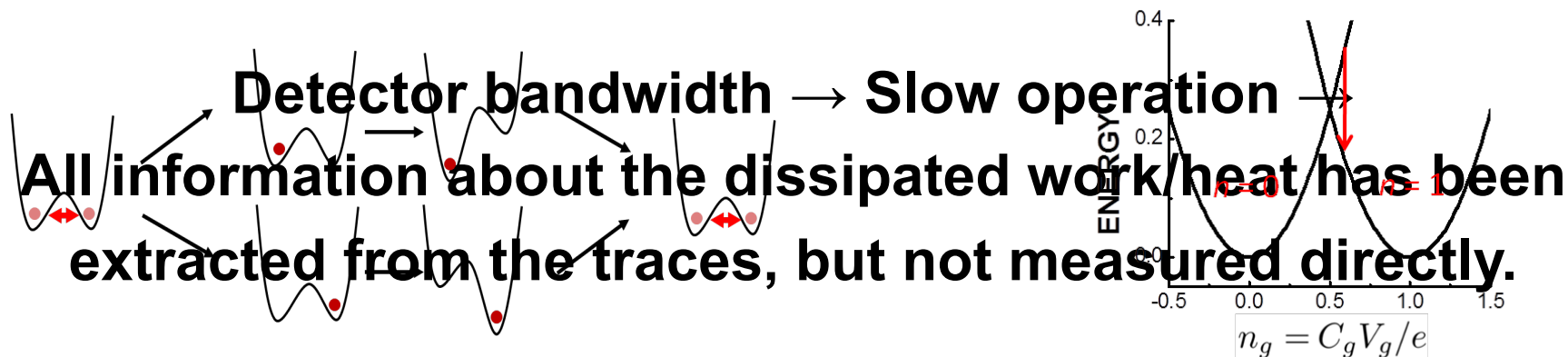
K. Chida et al, *Nat. Comm.* 2017

## Quantum MD

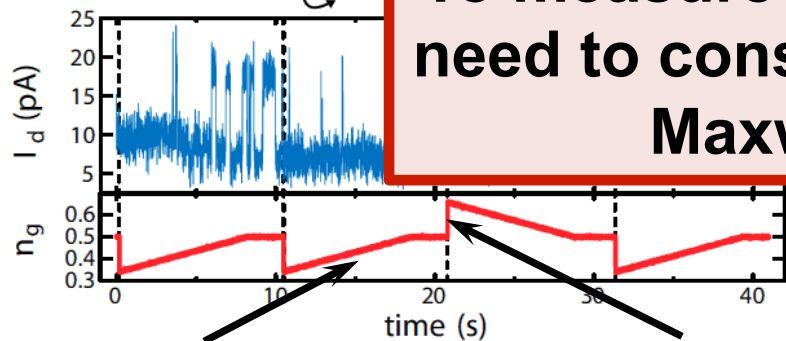


N. Cottet et al, *PNAS* 2017

# Maxwell's demon with SEB



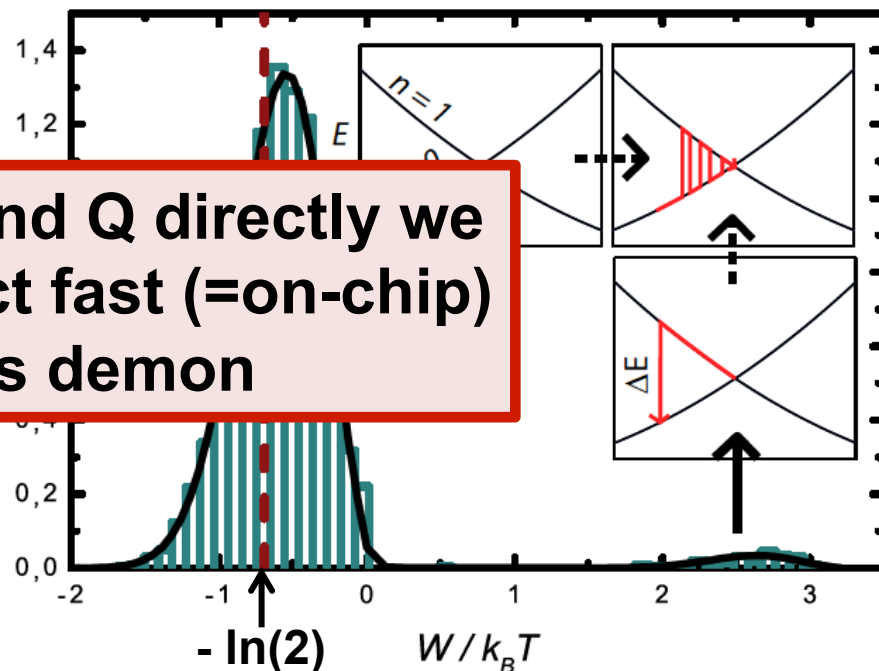
To measure  $W$  and  $Q$  directly we need to construct fast (=on-chip) Maxwell's demon



Quasi-static ramp

Measurement and decision

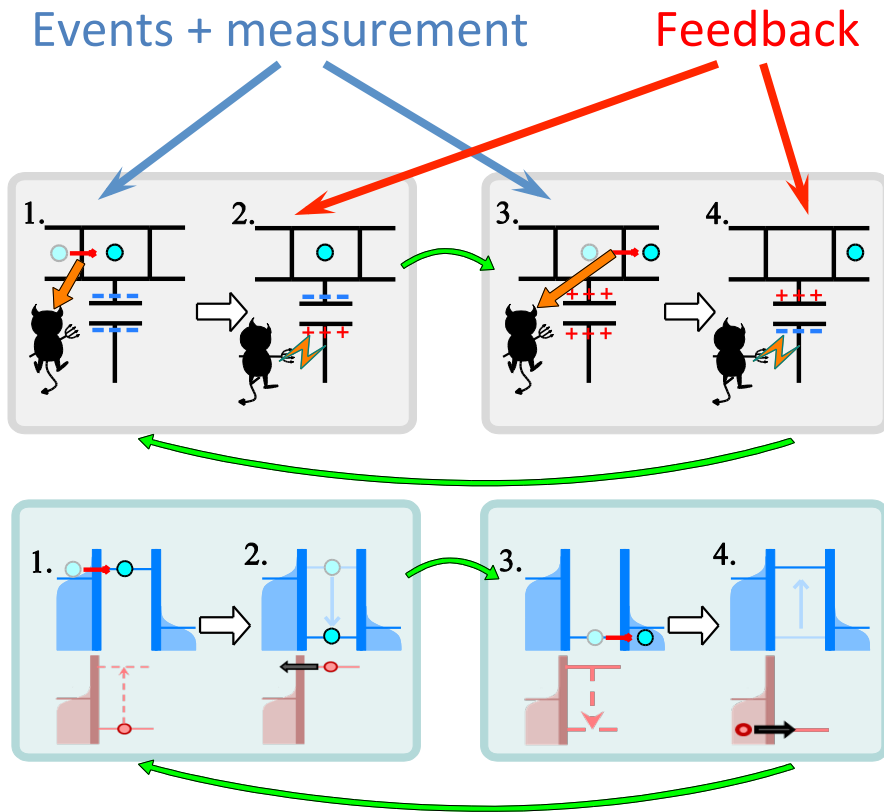
| - charge state | - gate voltage



Efficiency ( $\sim 3000$  repetitions):  
 $\langle W \rangle \approx -0.75 k_B T \ln 2$

# Autonomous Maxwell's demon.

## Operation principle



1. or 3. Demon detects that electron enters or leaves SET island (Event + measurement)

2. or 4. Feedback:

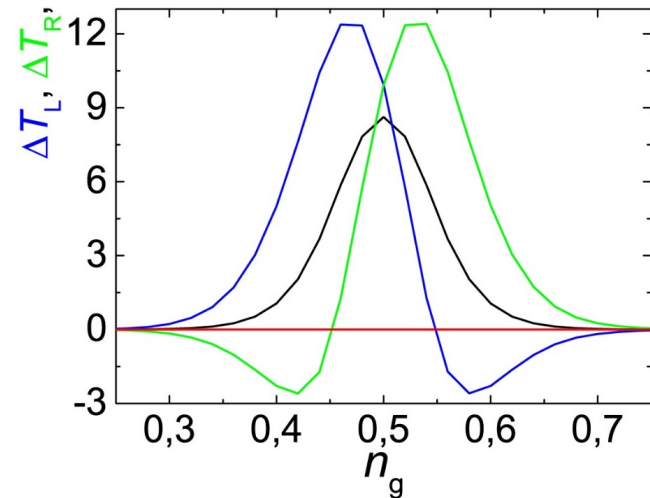
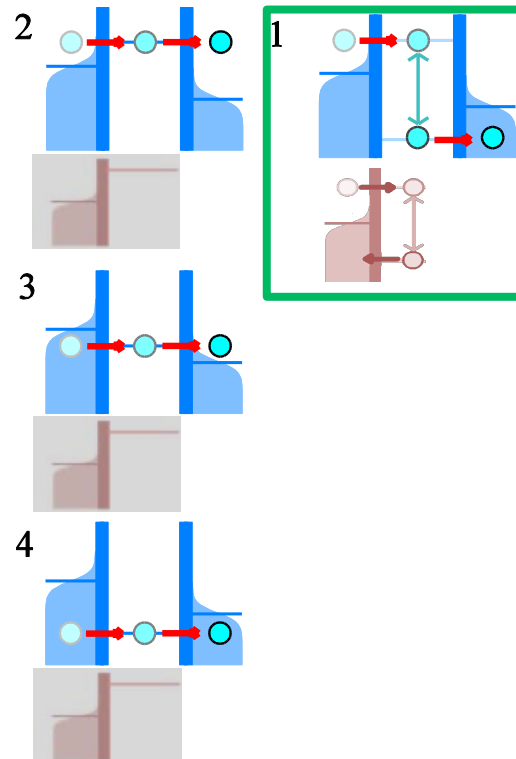
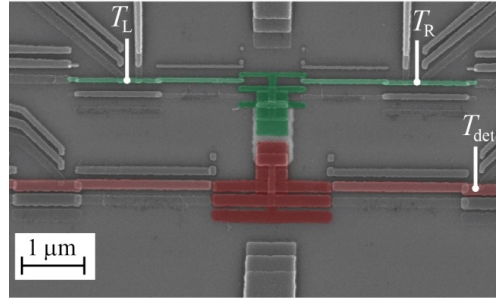
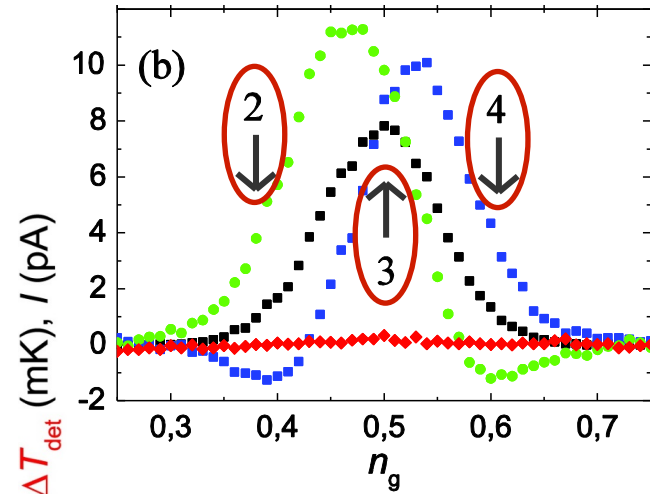
-Enter: trap with + charge

-Leave: block with - charge

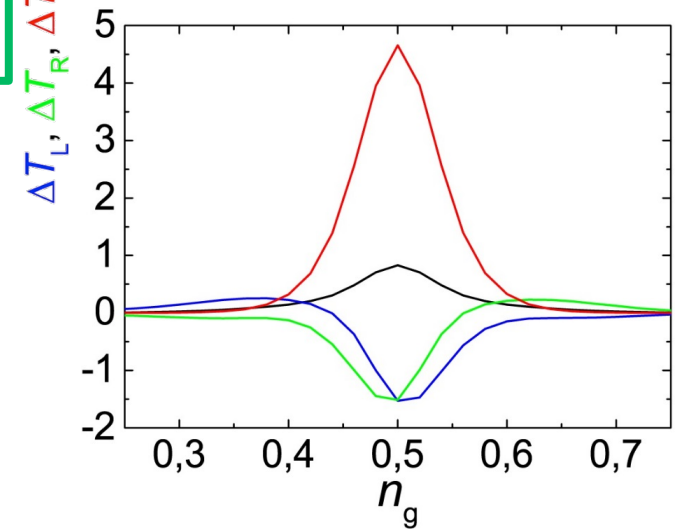
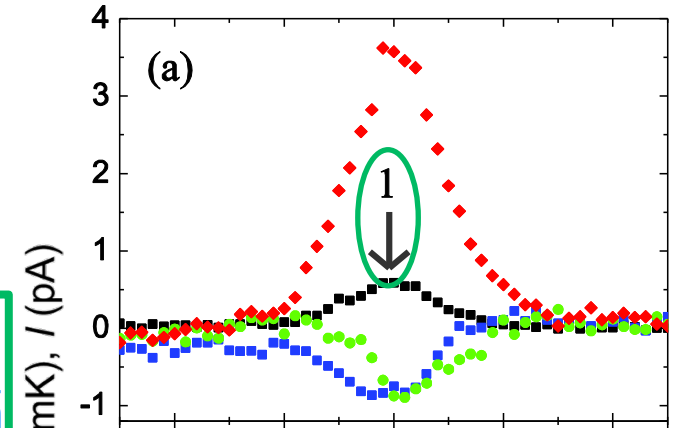
Electrons tunnel through the system more slowly and 'cool' down

# Results. $N_g = 1/2$ : feedback control

$N_g = 1$ : no feedback control



$N_g = 1/2$ : feedback control



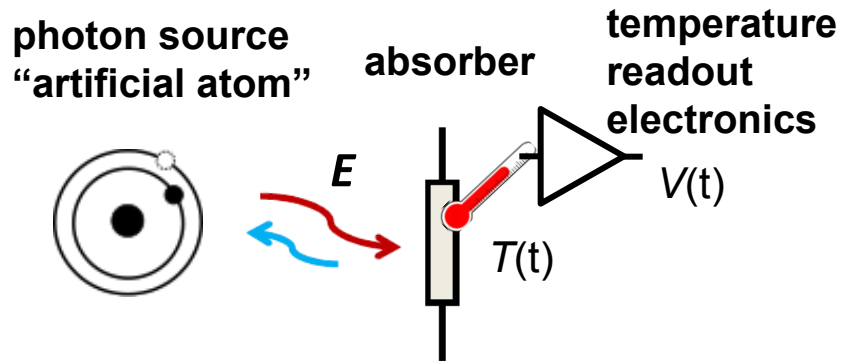


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# Calorimetry for measuring MW photons

Requirements for calorimetry on single microwave quantum level. Photons from relaxation of a superconducting qubit.



Typical parameters:

Operating temperature

$$T_0 = 0.1 \text{ K}$$

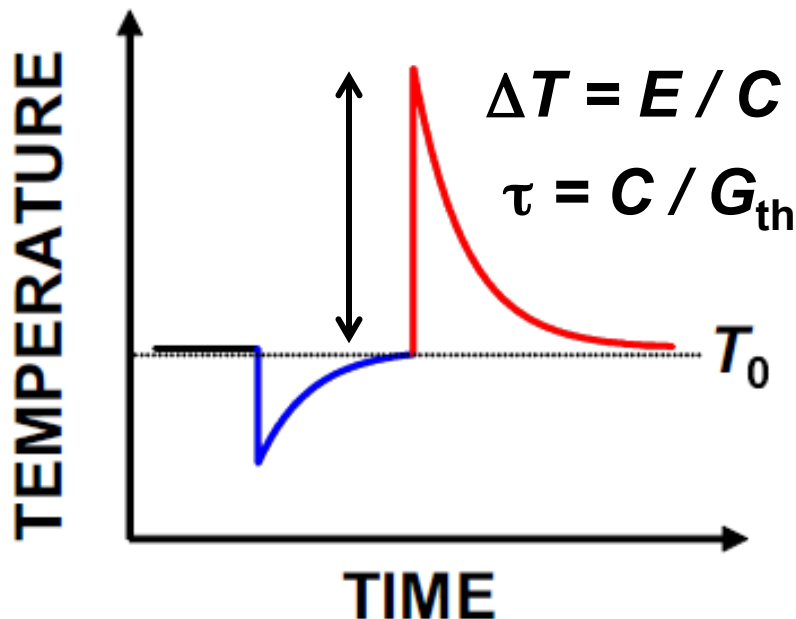
$$E/k_B = 1 \text{ K}, C = 300 \dots 1000 k_B$$

$$\Delta T \sim 1 - 3 \text{ mK}, \tau \sim 0.01 - 1 \text{ ms}$$

$T_{\text{NET}} = 10 \mu\text{K}/(\text{Hz})^{1/2}$  is  
sufficient for single photon  
detection

$$\delta E = T_{\text{NET}} (C G_{\text{th}})^{1/2}$$

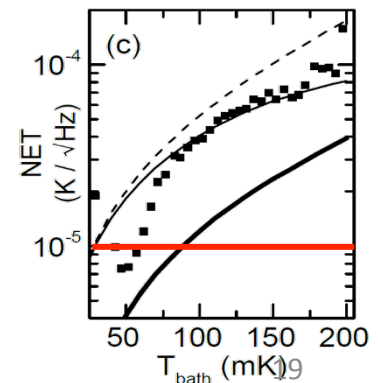
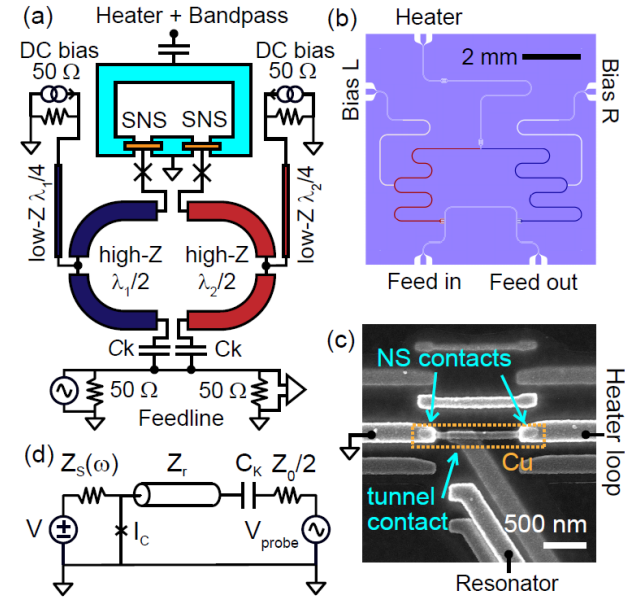
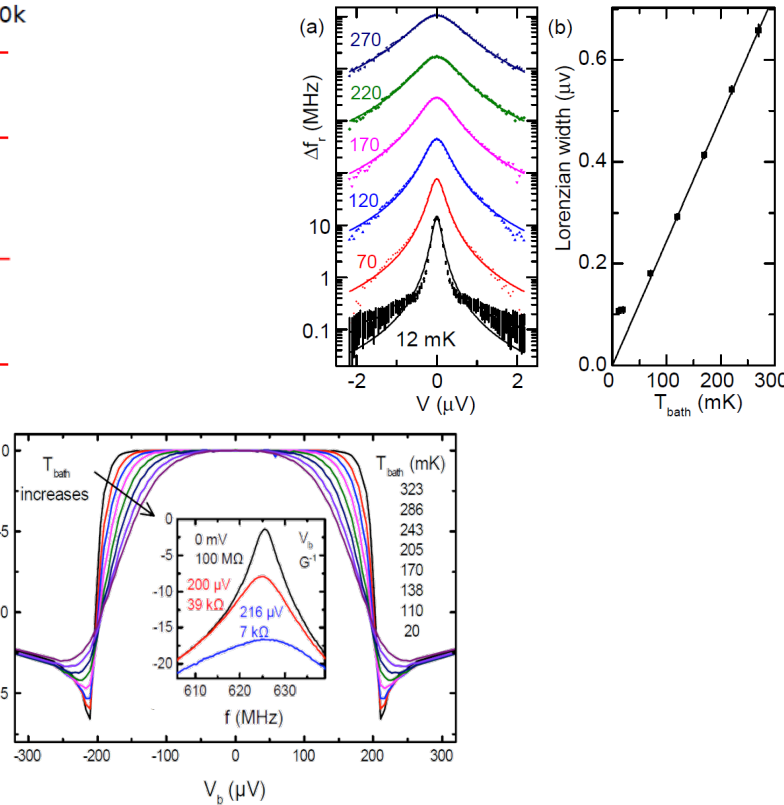
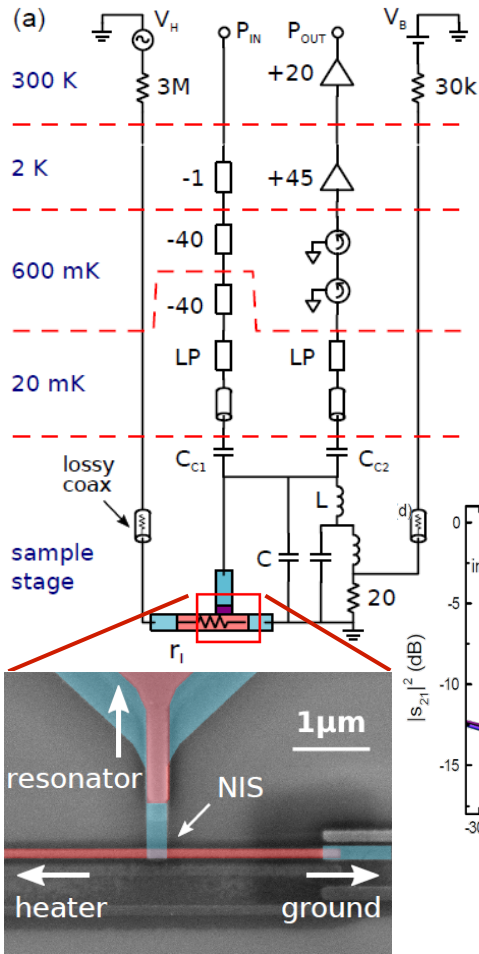
J. Pekola, P. Solinas, A. Shnirman, D. V. Averin.,  
NJP **15**, 115006 (2013).



# Fast NIS thermometry on electrons

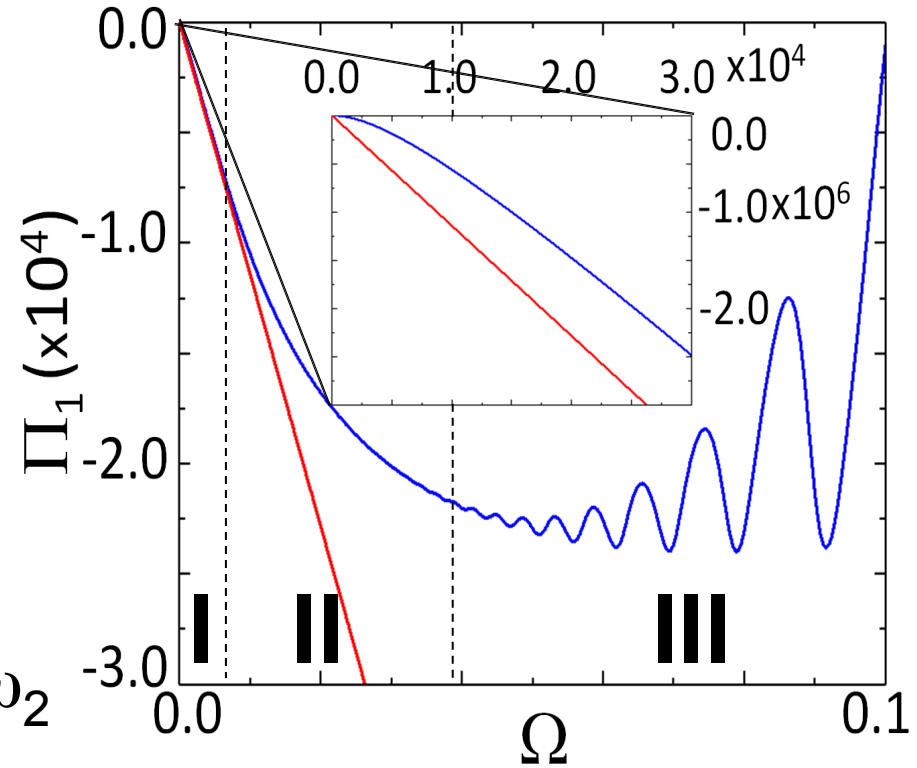
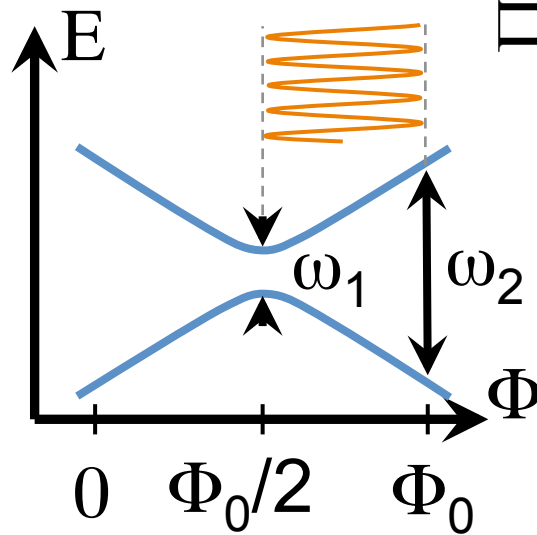
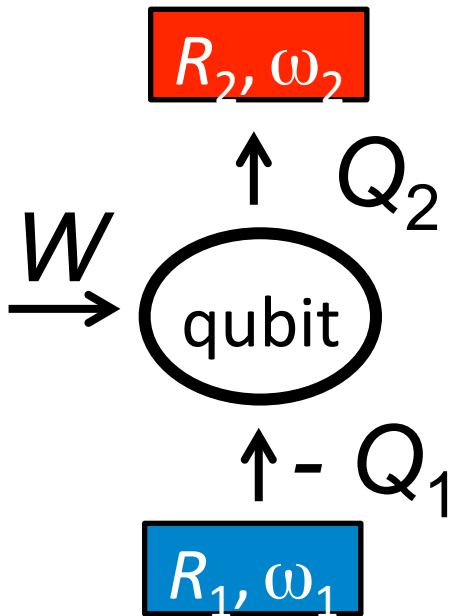
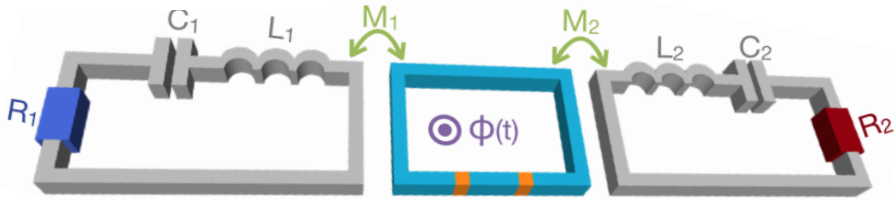
Read-out at 600 MHz of a NIS junction,  
10 MHz bandwidth

O.-P. Saira et al. PRAppI. 6, 024005 (2016)  
M. Zgirski et al. arXiv:1704.04762,  
L. Wang et al., arXiv: 1710.10082



S. Gasparinetti et al., Phys. Rev. Applied 3, 014007 (2015);  
K. L. Viisanen et al., New J. Phys. 17, 055014 (2015).  
Proof of the concept: Schmidt et al., 2003;

# Quantum Otto refrigerator

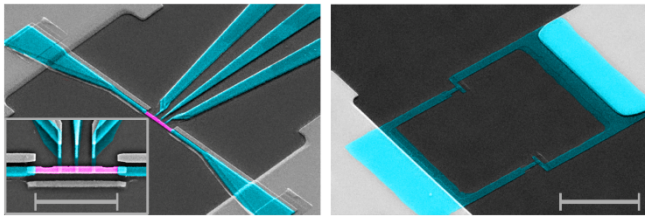
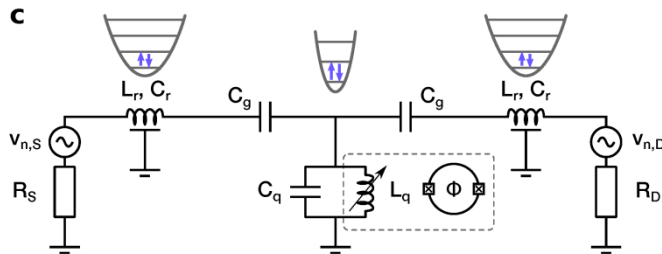
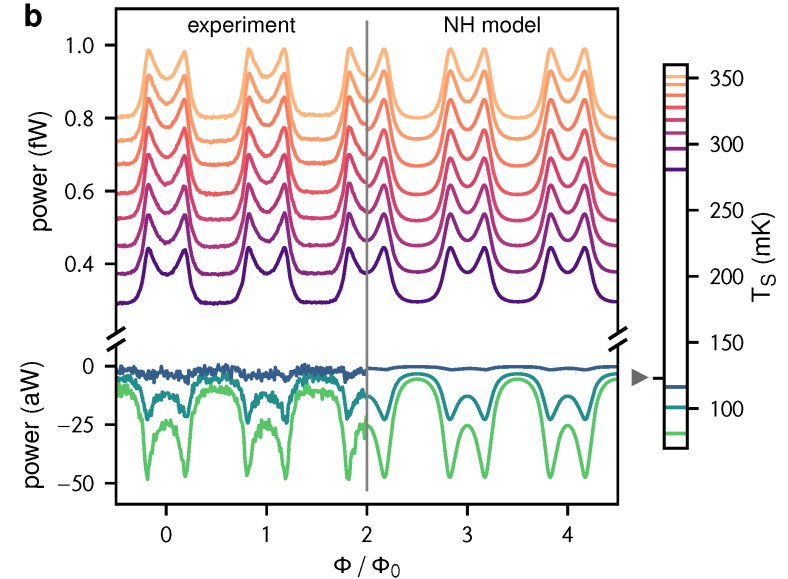
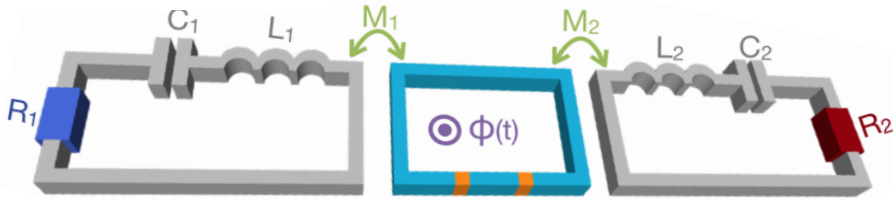


- I. Nearly adiabatic regime
- II. Ideal Otto cycle
- III. Coherent oscillations at high frequencies

Niskanen, Nakamura, Pekola, PRB 76, 174523 (2007)

B. Karimi and Pekola, Phys. Rev. B 94, 184503 (2016).

# Quantum Heat Valve

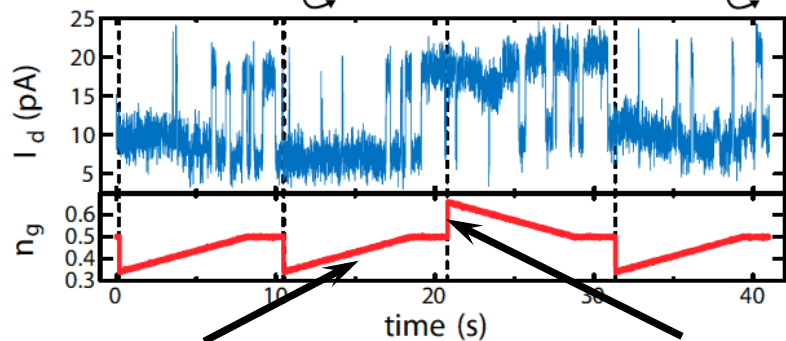
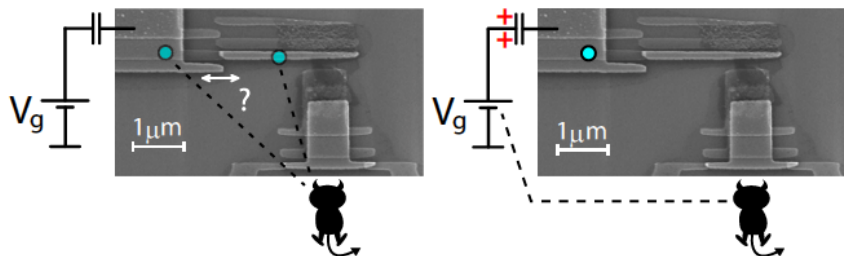
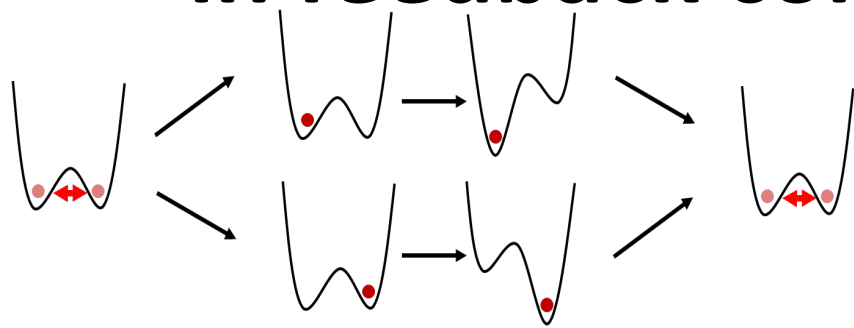


Tunable photonic heat transport (both cooling and heating) between QED resonators via a qubit

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# Negative entropy production events in feedback controlled systems

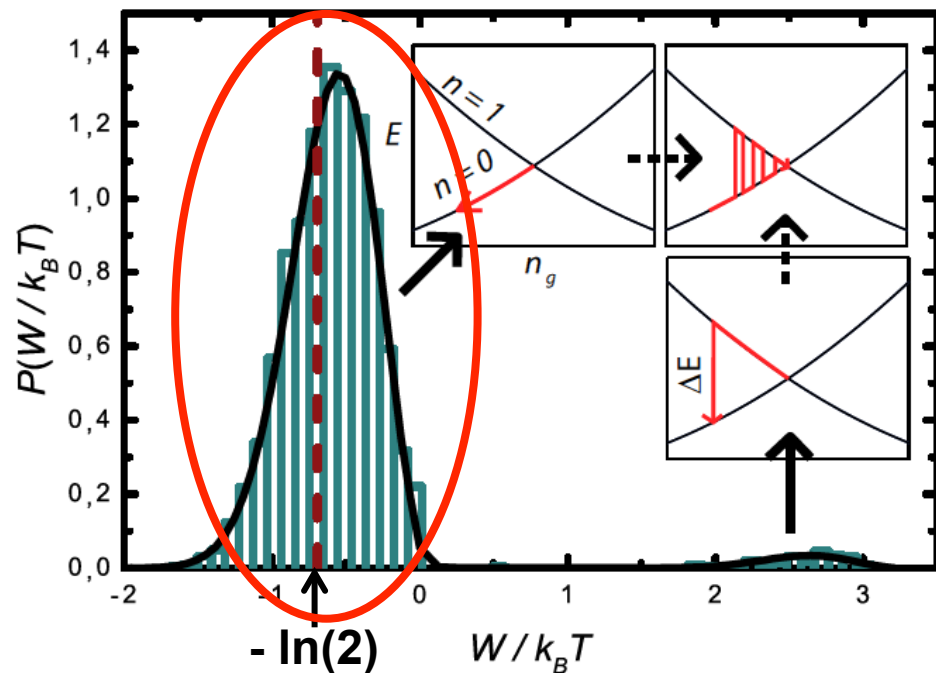


Quasi-static ramp

Measurement and decision

| - charge state

| - gate voltage



Efficiency ( $\sim 3000$  repetitions):  
 $\langle W \rangle \approx -0.75 k_B T \ln 2$

# Negative entropy production events in NESS

$$\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_B}$$

U. Seifert, Rep. Prog. Phys.  
**75**, 126001 (2012)

$$\langle e^{-\Delta S/k_B} \rangle = 1$$

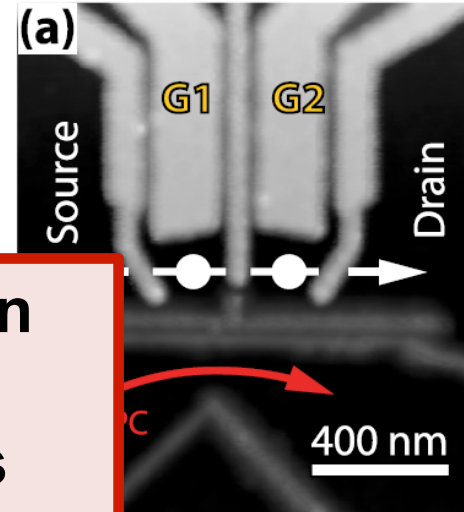
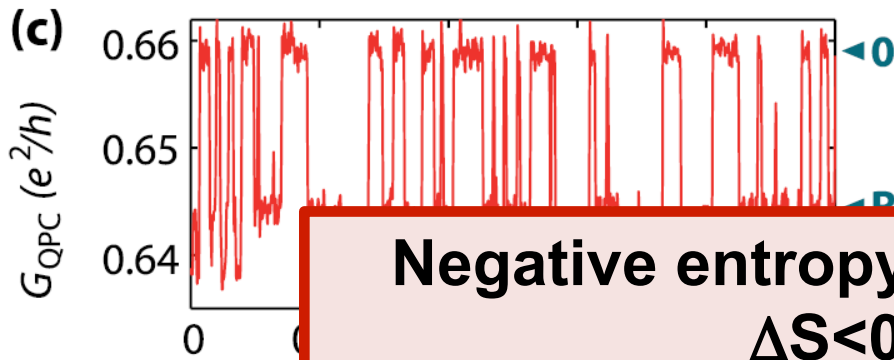
G. Bochkov, Yu. Kuzovlev, JETP 1977, Physica A 1981

Electric circuits: Experiment on a double quantum dot

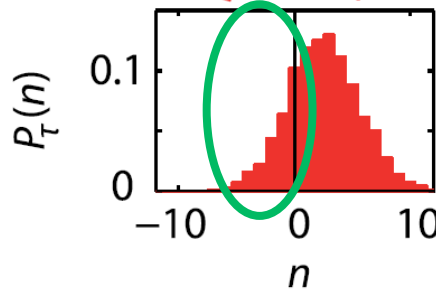
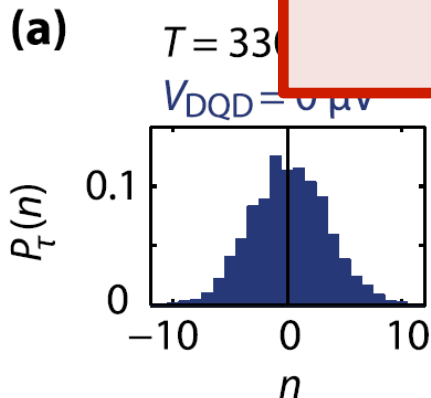
Y. Utsumi et al. PRB **81**, 125331 (2010),

B. Kung et al. PRX **2**, 011001 (2012)

Nonequilibrium steady state



**Negative entropy production**  
 $\Delta S < 0$   
**due to thermal fluctuations**

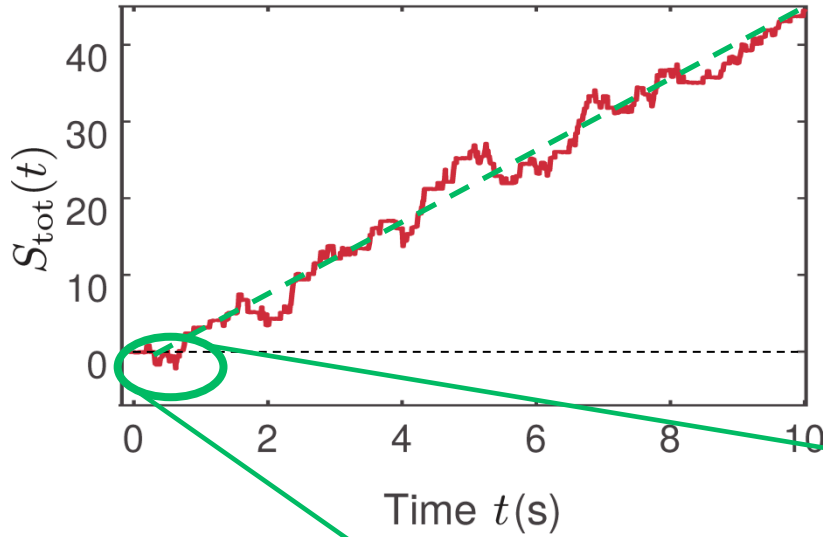


Fluctuation theorem as detailed balance

$$\frac{P_\tau(n)}{P_\tau(-n)} = e^{neV_{DQD}/k_B T}$$



# NESS. Negative events of entropy



$$\frac{P(\Delta S)}{P_R(-\Delta S)} = e^{\Delta S} \quad \langle e^{-\Delta S} \rangle = 1$$

$$\Delta S = \ln \frac{P(\{n(t)\})}{P_R(\{n(\tau - t)\})}$$

Single bath

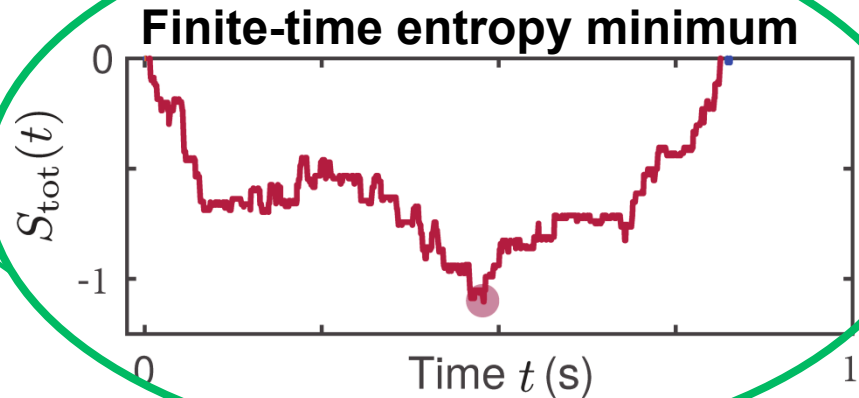
$$T\Delta S = W - \Delta F$$

2<sup>nd</sup> law of thermodynamics

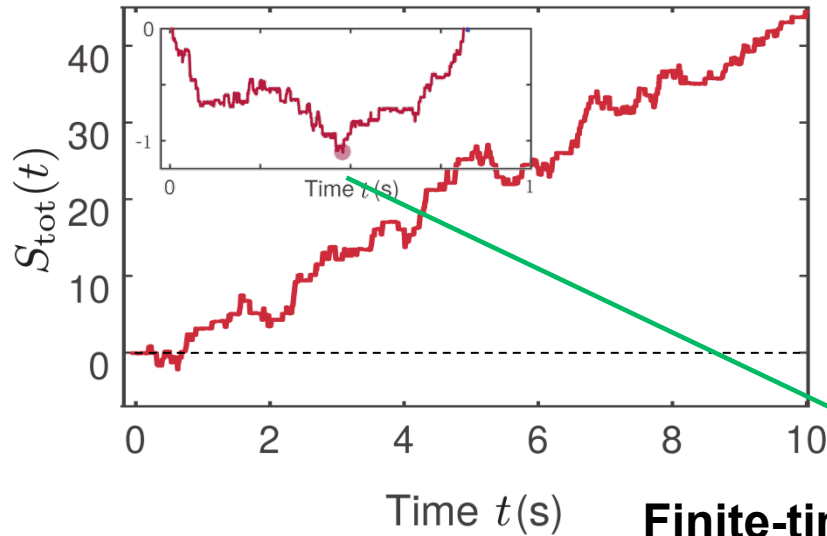
$$\langle \Delta S \rangle \geq 0$$

Nonequilibrium steady state

$$\left\langle \frac{d\Delta S}{dt} \right\rangle (t) \equiv \sigma = \text{const} > 0$$



# NESS. Negative records of entropy



$$\Delta S = \ln \frac{P(\{n(t)\})}{P_R(\{n(\tau - t)\})}$$

$e^{-\Delta S}$  is a **martingale** variable  
(J. L. Doob, *Stochastic Processes*, 1953)

$$\langle e^{-\Delta S(t)} \mid e^{-\Delta S(t_1)}, \dots, e^{-\Delta S(t_N)} \rangle = e^{-\Delta S(t_N)}$$

**Finite-time entropy minimum**

$$P(\Delta S \geq -s) \geq 1 - e^{-s} \quad \Delta S_{\min} = \min_{0 < t < \tau} \Delta S(t) \leq 0$$

**Doob's Maximal inequality**  
(cumulative distribution function)

$$P(\Delta S_{\min} \geq -s) \geq 1 - e^{-s}$$

**Infimum law**

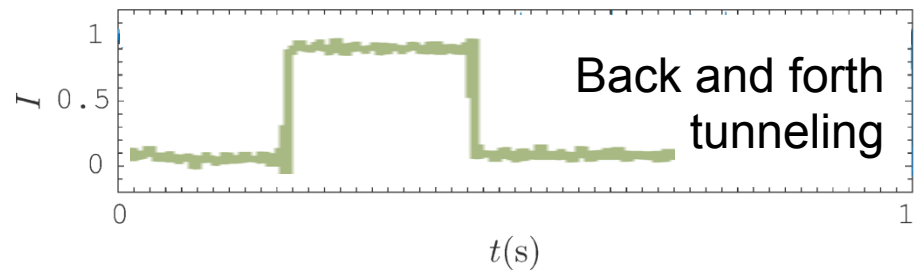
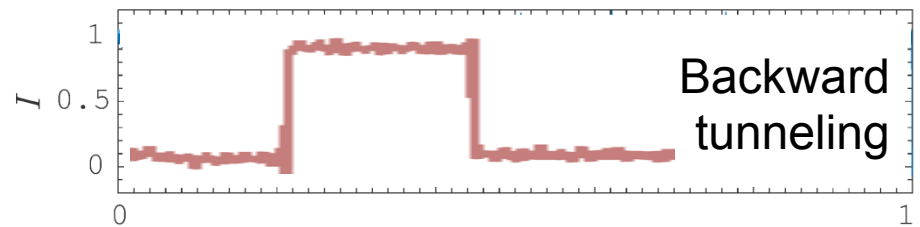
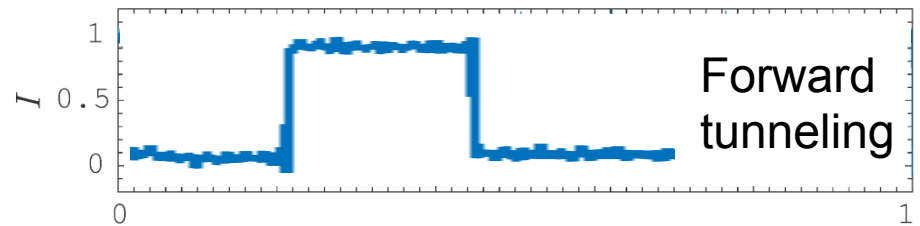
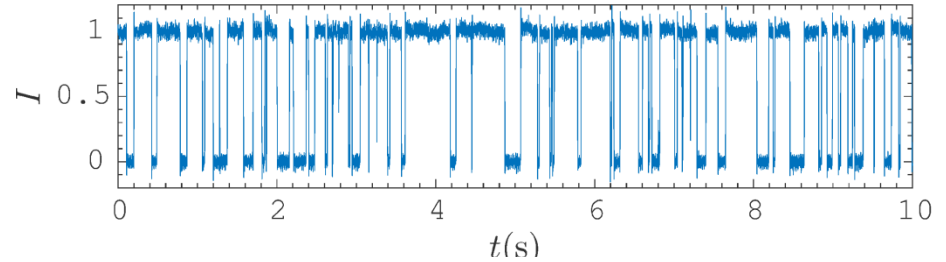
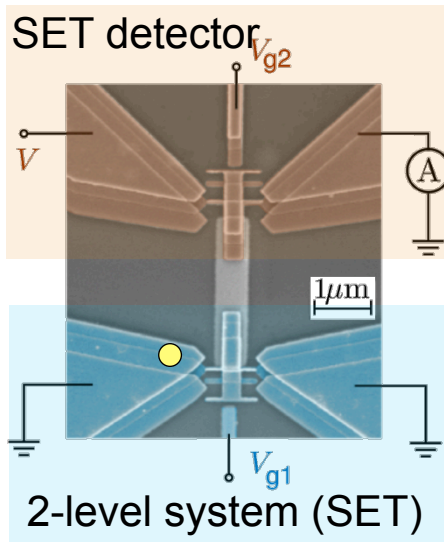
Average minimum entropy production is above -1

$$\langle \Delta S_{\min} \rangle \geq -1$$

# Outline

- General introduction:
  - ✓ Definition of entropy. Fluctuation relations
  - ✓ Coulomb blockade and thermodynamics
  - ✓ Jarzynski and Crooks relations in single-electron circuits
  - ✓ Realization of Maxwell's Demons
  - ✓ Quantum calorimetry and heat transport
- Nonequilibrium steady state
  - ✓ Negative entropy events
  - ✓ Statistics of finite-time minima of entropy production
- **Experimental verification of theoretical results**
  - ✓ **Double-dot structure as a minimal physical model**
  - ✓ **Boundaries for entropy production records**
  - ✓ **Relation with the heat absorption**
- **Summary and outlook**

# Single-electron transistor

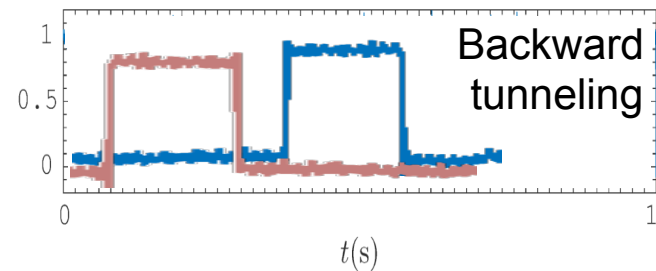
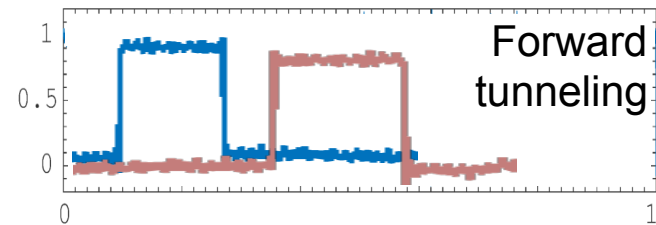
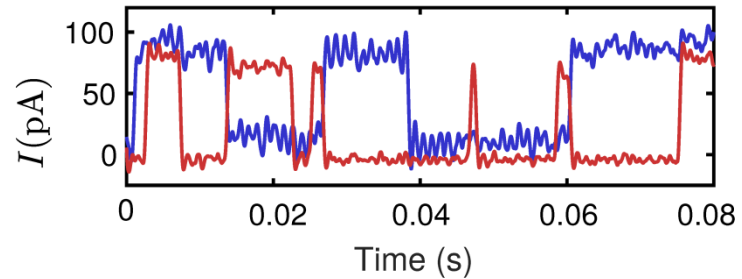
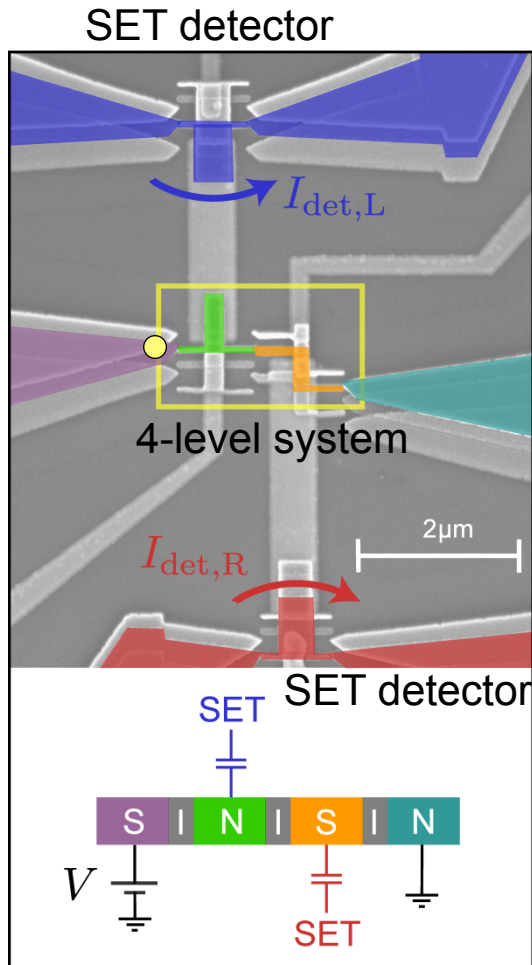


**Insesitive to a direction of current**

$R_s \sim 54 \text{ M}\Omega$   
 $E_s \sim 138 \text{ meV}$

$R_D \sim 12.5 \text{ M}\Omega$   
 $E_D \sim 137 \text{ meV}$

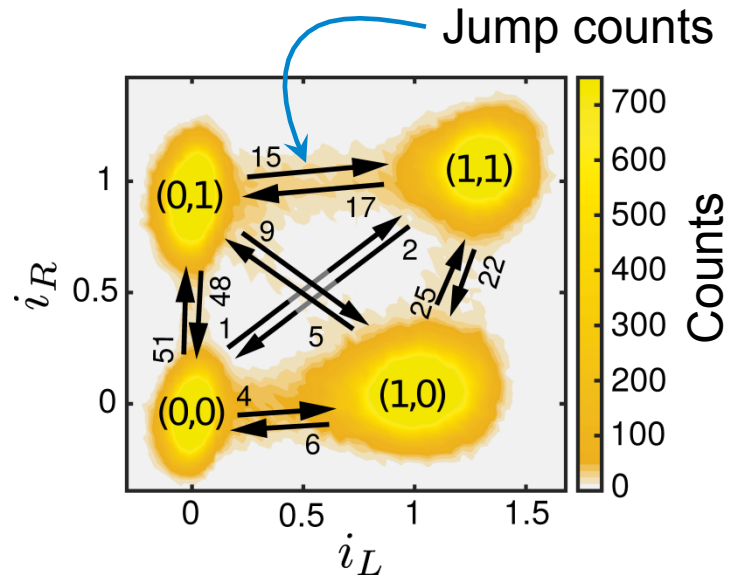
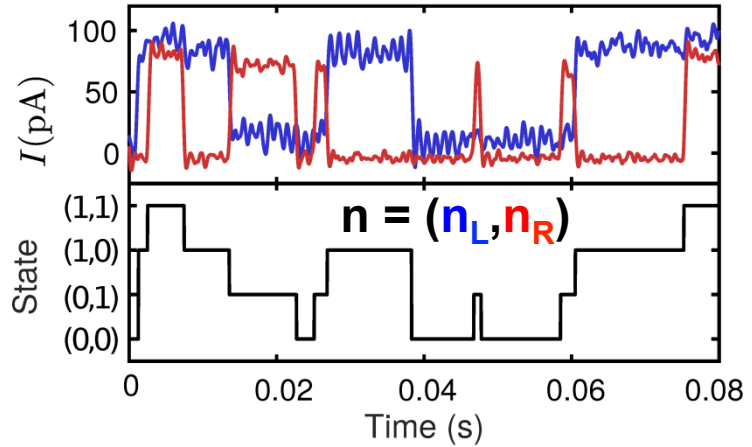
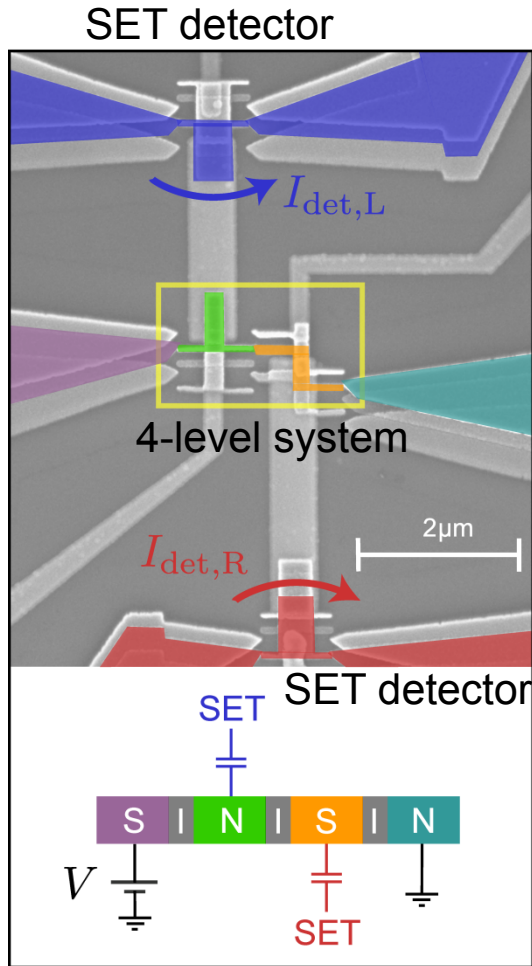
# Multi-island structure



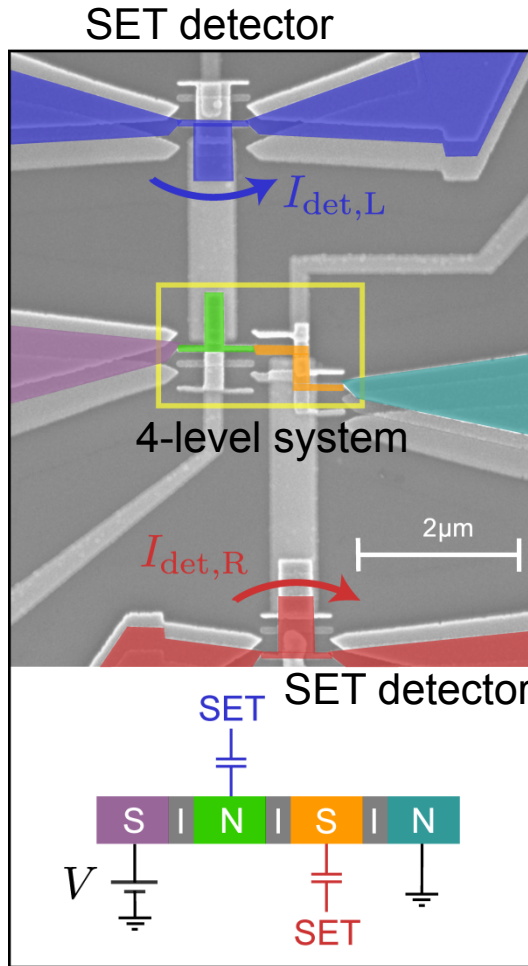
Direction of mesoscopic currents characterizes entropy production

$$\Delta S(\tau) = \ln \frac{P(\{n(t)\})}{P_R(\{n(\tau - t)\})}$$

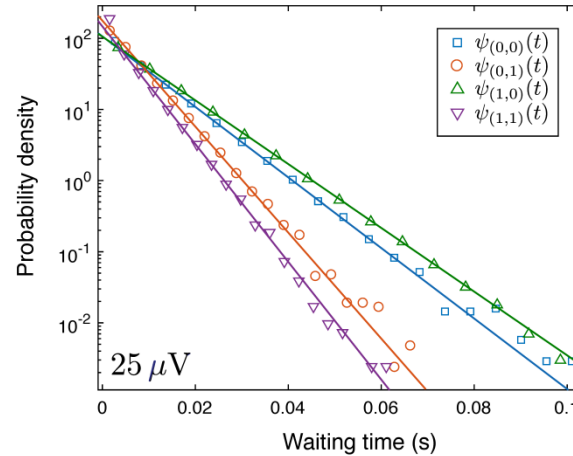
# Classical 4-level system



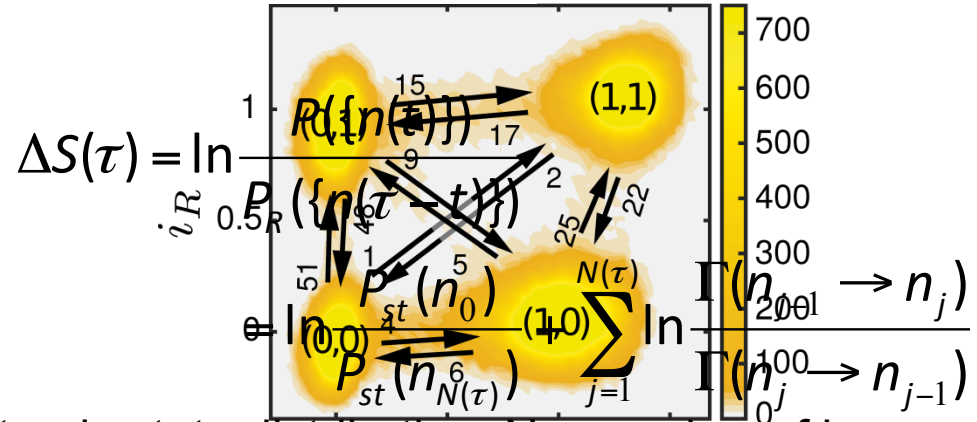
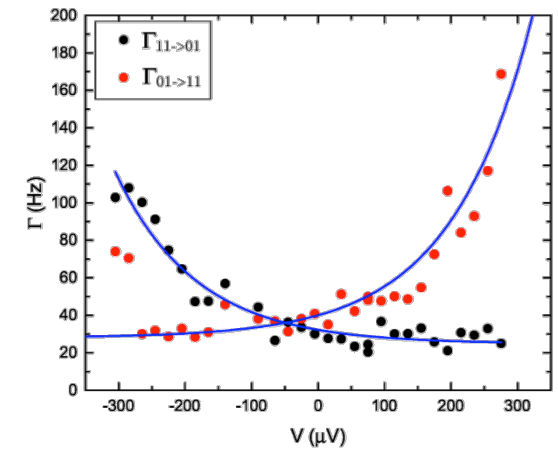
# Markovian 4-level system



Exponential waiting time distributions = Markovian dynamics



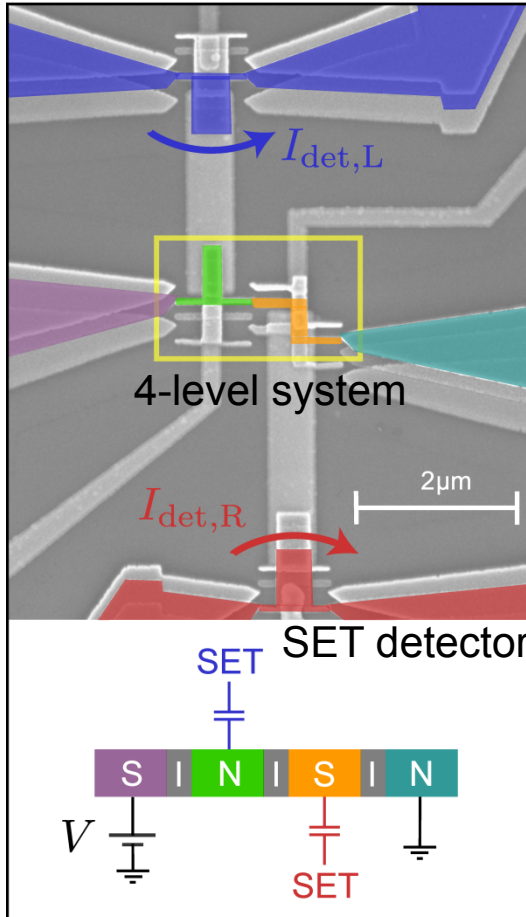
Thermal tunneling rates = Quasiequilibrium regime



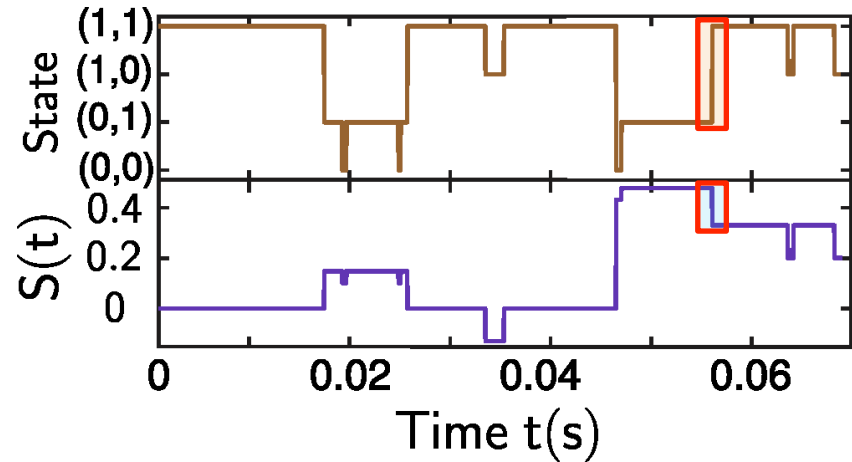
$P_{st}(n)$  – steady state distribution;  $N$  – number of jumps on the trajectory;  $\Gamma(n \rightarrow m)$  – tunneling rates between states  $n$  and  $m$ .

# Stochastic entropy on trajectory

SET detector



State trajectory unambiguously determines finite-time stochastic entropy production



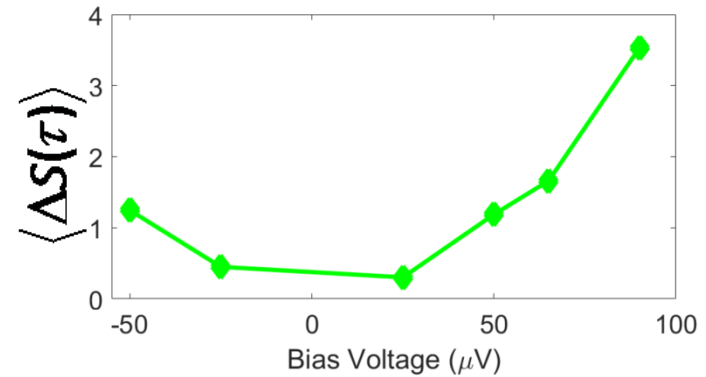
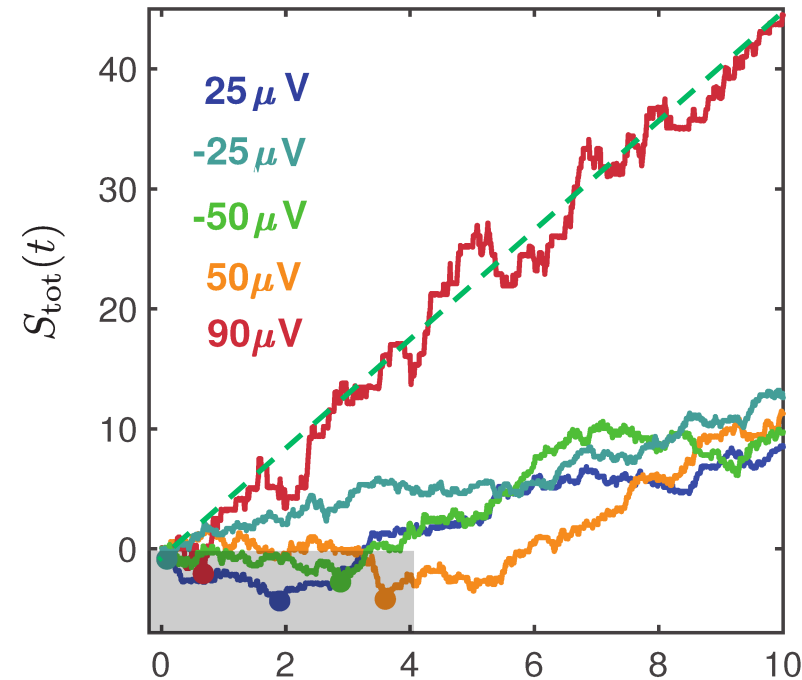
$$\Delta S(\tau) = \ln \frac{P(\{n(t)\})}{P_R(\{n(\tau-t)\})}$$

$$= \ln \frac{P_{st}(n_0)}{P_{st}(n_{N(\tau)})} + \sum_{j=1}^{N(\tau)} \ln \frac{\Gamma(n_{j-1} \rightarrow n_j)}{\Gamma(n_j \rightarrow n_{j-1})}$$

$P_{st}(n)$  – steady state distribution;  $N$  – number of jumps on the trajectory;  $\Gamma(n \rightarrow m)$  – tunneling rates between states  $n$  and  $m$ .

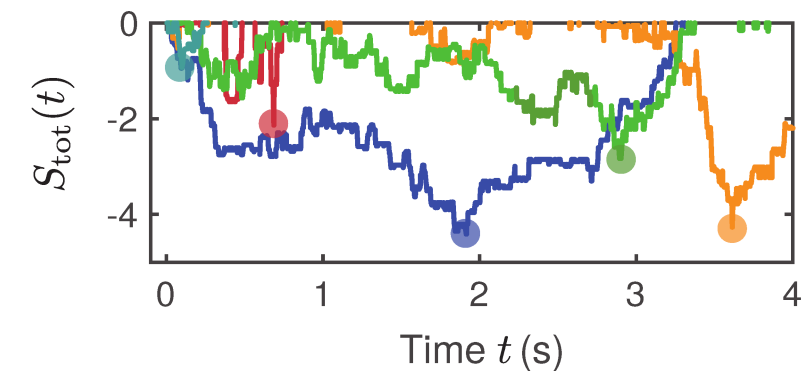


# Stochastic entropy on trajectory. Mean



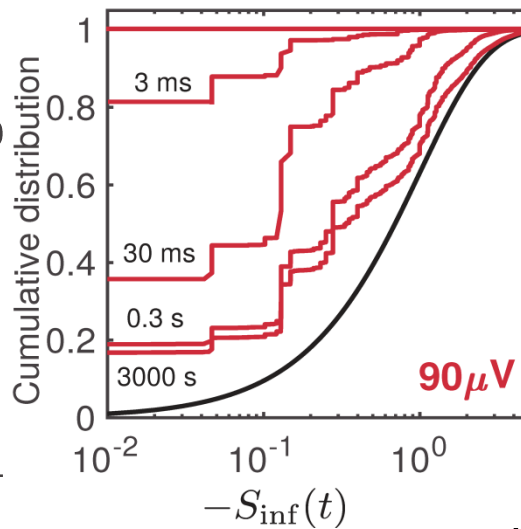
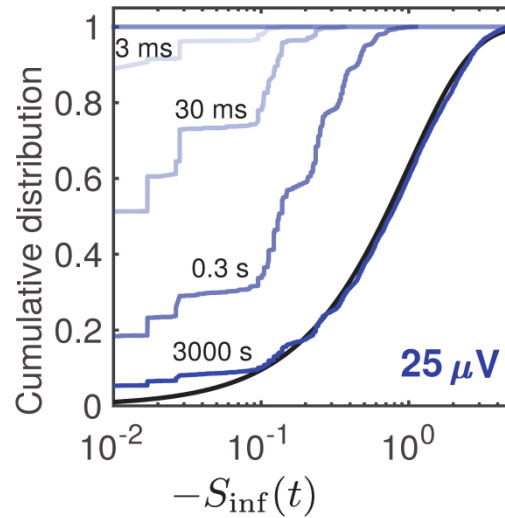
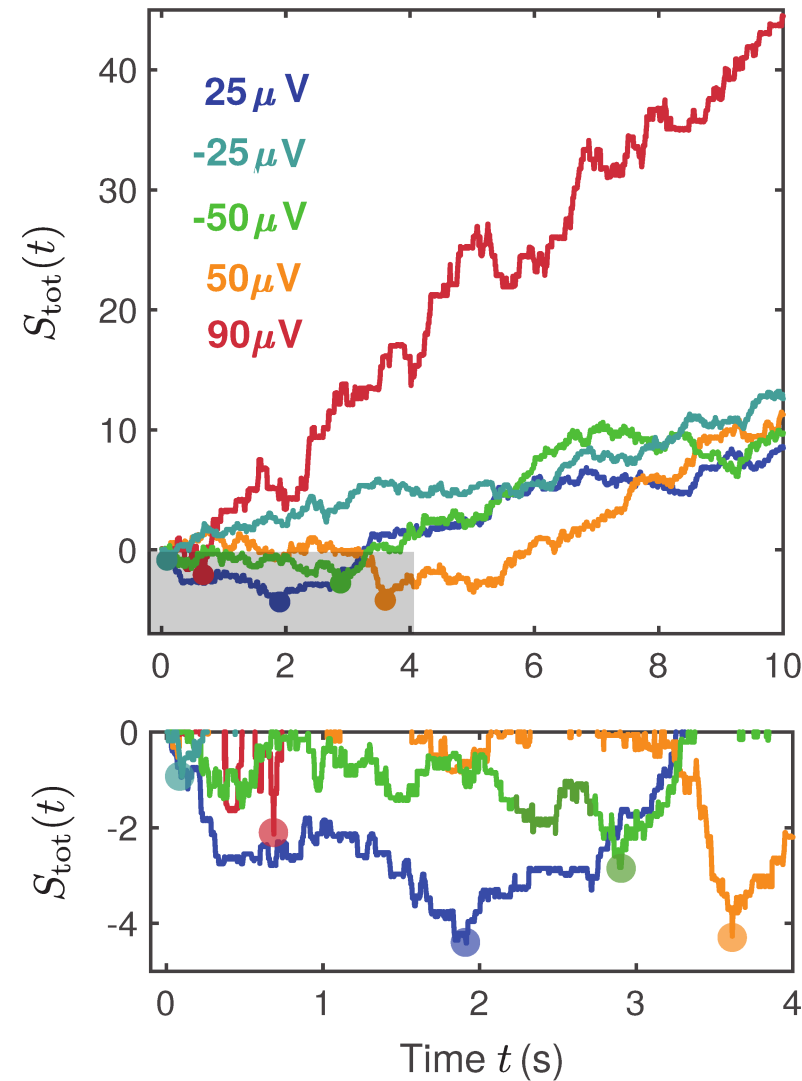
$$\langle \Delta S(\tau) \rangle = IV / T \propto V^2$$

Average entropy production is mostly due to Joule heating



$$\Delta S(\tau) = \ln \frac{P_{st}(n_0)}{P_{st}(n_{N(\tau)})} + \sum_{j=1}^{N(\tau)} \ln \frac{\Gamma(n_{j-1} \rightarrow n_j)}{\Gamma(n_j \rightarrow n_{j-1})}$$

# Stochastic entropy on trajectory. CDF



$$\Delta S_{\text{min}} = \min_{0 < t < \tau} \Delta S(t)$$

**Doob's Maximal inequality**  
(cumulative distribution function)

$$P(\Delta S_{\text{min}} \geq -s) \geq 1 - e^{-s}$$

Cumulative distribution function for the finite-time minimal entropy production is limited by exponent'l distribution.

$$\Delta S(\tau) = \ln \frac{P_{st}(n_0)}{P_{st}(n_{N(\tau)})} + \sum_{j=1}^{N(\tau)} \ln \frac{\Gamma(n_{j-1} \rightarrow n_j)}{\Gamma(n_j \rightarrow n_{j-1})}$$

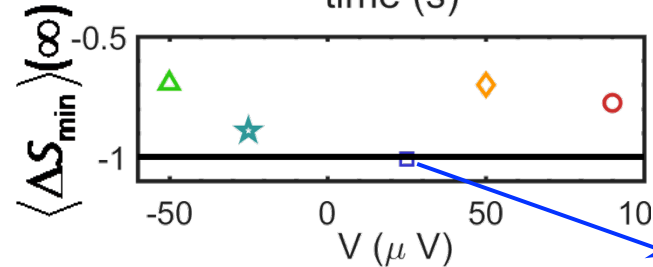
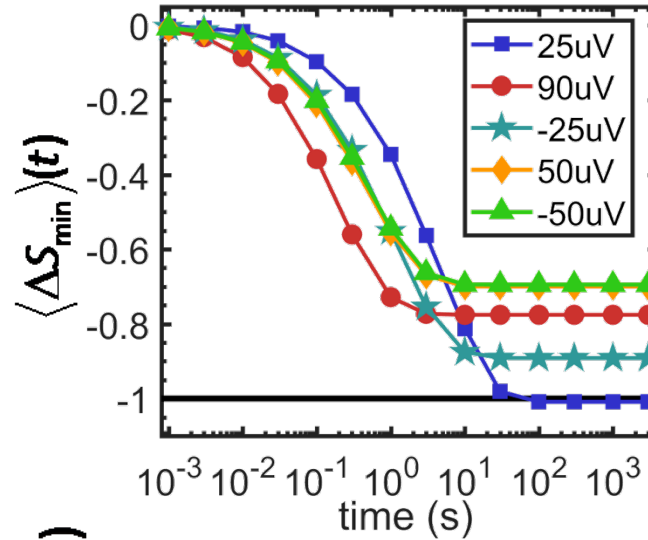
# Stochastic entropy on trajectory. $\langle \Delta S_{\min} \rangle$

$$\Delta S_{\min} = \min_{0 < t < \tau} \Delta S(t)$$

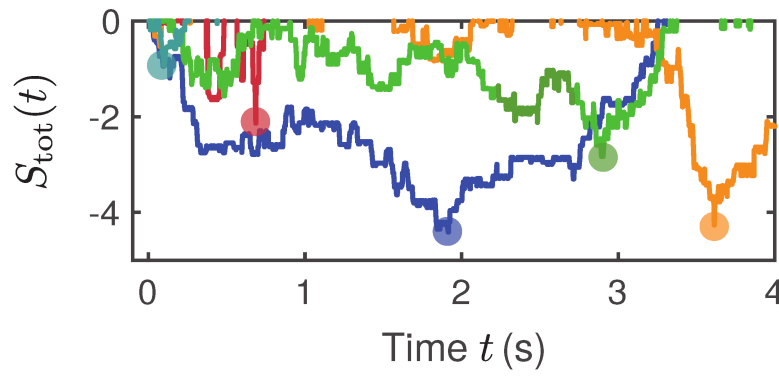
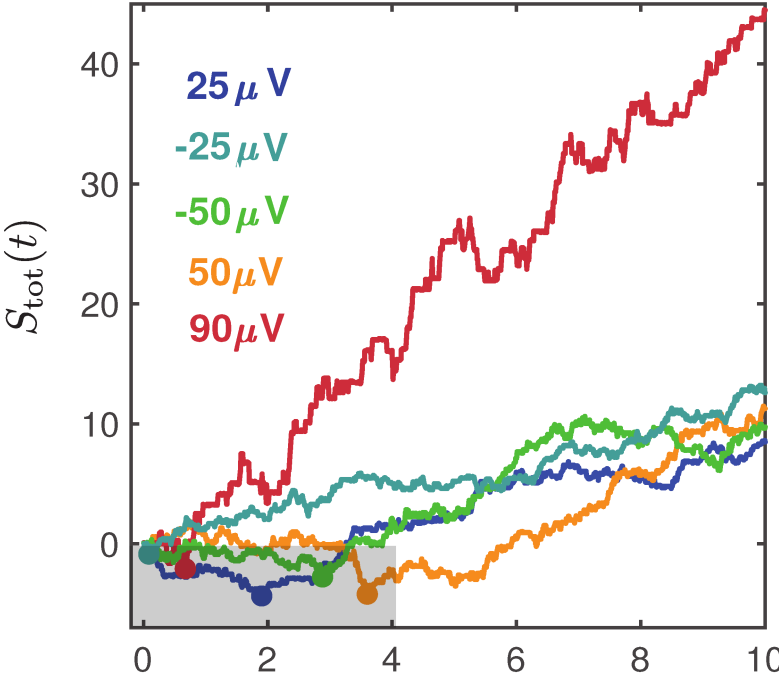
## Inimum law

Average min.entropy production is above -1

$$\langle \Delta S_{\min} \rangle \geq -1$$

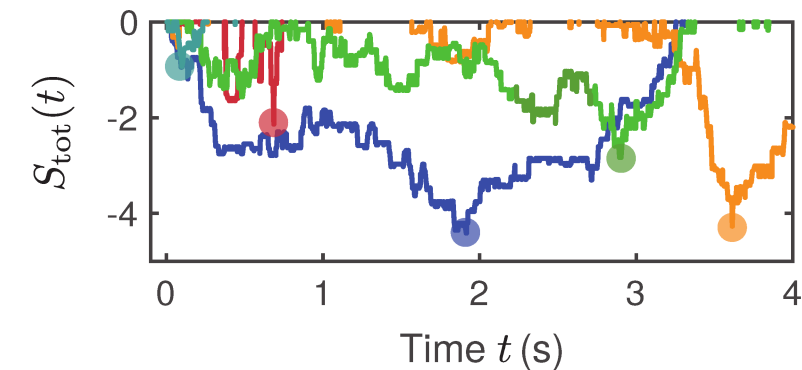
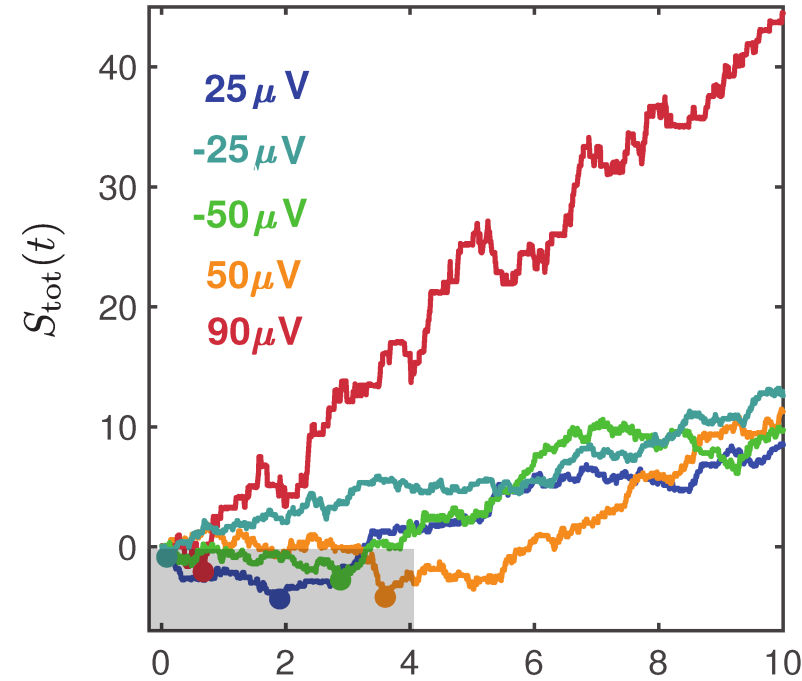


Close to equilibrium  $\langle \Delta S_{\min} \rangle(\infty) = -1.01 \pm 0.02$

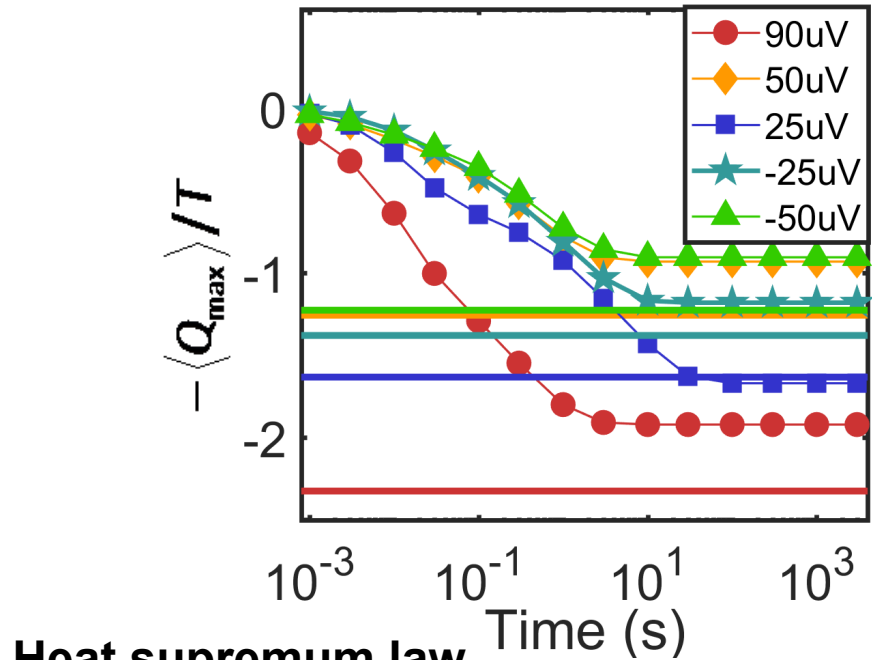


$$\Delta S(\tau) = \ln \frac{P_{st}(n_0)}{P_{st}(n_{N(\tau)})} + \sum_{j=1}^{N(\tau)} \ln \frac{\Gamma(n_{j-1} \rightarrow n_j)}{\Gamma(n_j \rightarrow n_{j-1})}$$

# Stochastic entropy on trajectory. $\langle Q_{\max} \rangle$



$P_{st}(n)$  – steady state distribution;



**Heat supremum law**

Average max. absorbed heat is below

$$\frac{\langle Q_{\max} \rangle}{T} \leq 1 + \sum_n P_{st}(n) \ln \frac{P_{st}(n)}{P_{st}^{\min}}$$

$$\Delta S(\tau) = \ln \frac{P_{st}(n_0)}{P_{st}(n_{N(\tau)})} + \sum_{j=1}^{N(\tau)} \ln \frac{\Gamma(n_{j-1} \rightarrow n_j)}{\Gamma(n_j \rightarrow n_{j-1})}$$

# Summary

Realization of a Coulomb blockaded device with single-electron counting sensitive to the direction of the current.

First experimental study of extreme-value statistics of stochastic entropy production in nonequilibrium steady states.

Statistics of  $\sim 10^6$  records of negative entropy production in an electronic double dot is in agreement with the universal bounds.

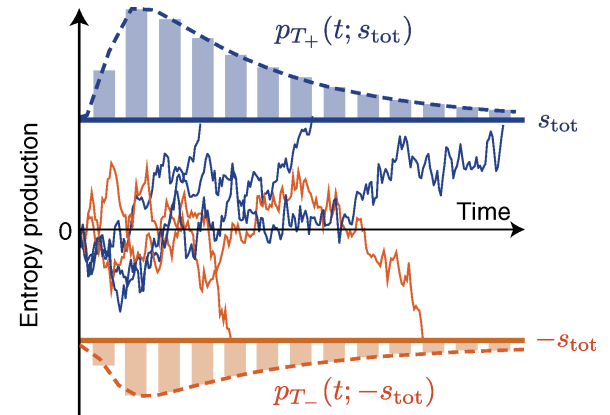
The bound for the average maximal amount of absorbed heat is derived theoretically and verified experimentally.

# Outlook

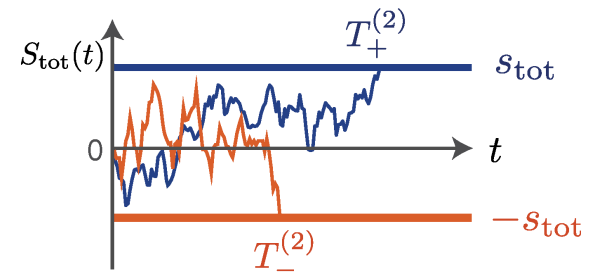
Relations between first passage time distributions

Stopping time distributions

Moments and cumulants of minimal entropy production. Relation with full-counting statistics



Two absorbing boundaries



$$\langle \Delta S_{\text{min}}^n \rangle \geq ???$$
$$\langle \langle \Delta S_{\text{min}}^n \rangle \rangle \geq ???$$

# Contributions

## Experiment (PICO)

Department of Applied Physics,  
Aalto University, Finland

- **Prof. Jukka Pekola**
- **Doc. Matthias Meschke**  
(now @Hoch Schule Munich)
- **Olli-Pentti Saira** (now @Caltech)
- **Ville Maisi** (now @Lund)
- **Jonne Koski** (now @ETH Zurich)
- **Shilpi Singh**
- **Joonas Peltonen**
- **Simone Gasparinetti** (now @ETH Zurich)
- **Klaara Viisanen**
- **Bayan Karimi**
- **Alberto Ronzani** (@ VTT, Finland)
- **Jorden Senior**

## Theory

Max Planck Institute for Physics of  
Complex Systems, Dresden, Germany

- **Prof. Frank Julicher**
- **Edgar Roldan** (now@ICTP)
- **Izaak Neri** (moving to King's college)

Department of Applied Physics,  
Aalto University, Finland

- **Dmitry Golubev**



In **A!** f



MAX-PLANCK-GESELLSCHAFT

