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Irreversibility and the quantum arrow of time

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- 1 Thermodynamic arrow of time
 - Characterization of irreversibility
 - Observation in a driven quantum system

- 2 Reversing the arrow of time
 - Thermodynamics with initial correlations
 - Observation in a bipartite quantum system

Arrow of time

Macroscopic processes have a preferred *direction in time*

"Heat flows from *hot* to *cold*"

→ reversed process does not spontaneously occur

→ *irreversibility*

In thermodynamics: mean entropy production is *positive*

irreversible if $\langle \Sigma \rangle > 0$ reversible if $\langle \Sigma \rangle = 0$ impossible if $\langle \Sigma \rangle < 0$

→ *arrow of time*

(Eddington 1927)

Two asymmetries: that of a *process in time* and that of *time itself*

Simple example: *reversed movie*

(Jarzynski 2011)

Arrow of time

(Quasi) reversible processes

Here: entropy production $\langle \Sigma \rangle \simeq 0$ (during duration of experiment)

"Lectures on thermodynamics" George Porter 1965 (Nobel 1968)

Arrow of time

Irreversible processes

Here: entropy production $\langle \Sigma \rangle > 0$ (reversal not observed)

"Lectures on thermodynamics" George Porter 1965 (Nobel 1968)

Apparent paradox

Macroscopic systems made of microscopic particles (atoms)

Microscopic laws of physics are reversible

Example: Newton's law of motion

$$m \frac{d^2 x}{dt^2} = F$$

Reversal: $t \rightarrow t' = \tau - t$ $dt' = -dt$ $dt'^2 = dt^2$

→ also true for Maxwell, Schrödinger,

→ Question: how to *explain* macroscopic irreversibility?

Apparent paradox

Processes in nature are described by:

- i) **laws** of physics → **fixed**
- ii) **initial (boundary)** conditions → **random**

Newton 1687, Wigner 1963

Examples:

- planetary orbits = ellipses (law)
- quasi circular in our solar system (initial condition)
- all planets orbit in the same direction (initial condition)

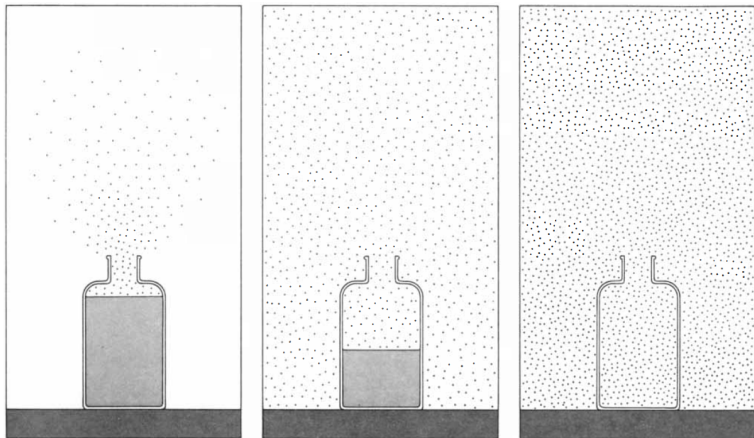
Resolution:

→ initial conditions break time reversal (Boltzmann 1877)

"From the fact that the differential equations of mechanics are left unchanged by reversing the sign of time ..., Herr Ostwald concludes that the mechanical view of the world cannot explain why natural processes run preferentially in a definite direction. But such a view appears to me to overlook that mechanical events are determined not only by differential equations but also by initial conditions."

Irreversibility in a many-particle system

Bottle of perfume:



Initial state breaks time reversal — reversal unlikely for 10^{24} particles

Arrow of time in the 21st century

PRL **119**, 220507 (2017)

PHYSICAL REVIEW LETTERS

week ending
1 DECEMBER 2017

Arrow of Time for Continuous Quantum Measurement

Justin Dressel,^{1,2} Areeya Chantasri,^{3,4,5} Andrew N. Jordan,^{3,4,1} and Alexander N. Korotkov⁶

PRL **115**, 250602 (2015)

PHYSICAL REVIEW LETTERS

week ending
18 DECEMBER 2015

Decision Making in the Arrow of Time

Édgar Roldán,^{1,5} Izaak Neri,^{1,2,5} Meik Dörringhaus,^{3,5} Heinrich Meyr,^{3,4,5} and Frank Jülicher^{1,5,*}

PRL **103**, 080401 (2009)


PHYSICAL REVIEW LETTERS

week ending
21 AUGUST 2009

Quantum Solution to the Arrow-of-Time Dilemma

Lorenzo Maccone^{*}

PRL **115**, 190601 (2015)

 Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS

week ending
6 NOVEMBER 2015

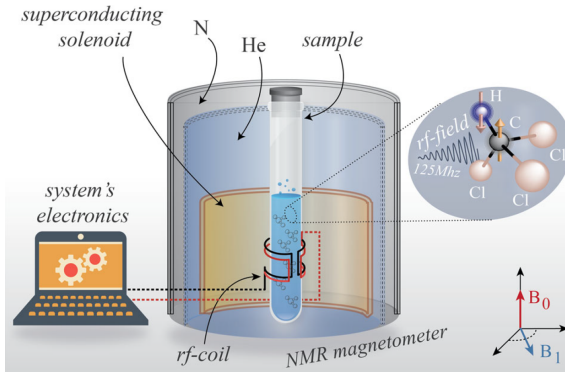


Irreversibility and the Arrow of Time in a Quenched Quantum System

T. B. Batalhão,^{1,2} A. M. Souza,³ R. S. Sarthour,³ I. S. Oliveira,³ M. Paternostro,⁴ E. Lutz,⁵ and R. M. Serra^{1,6}

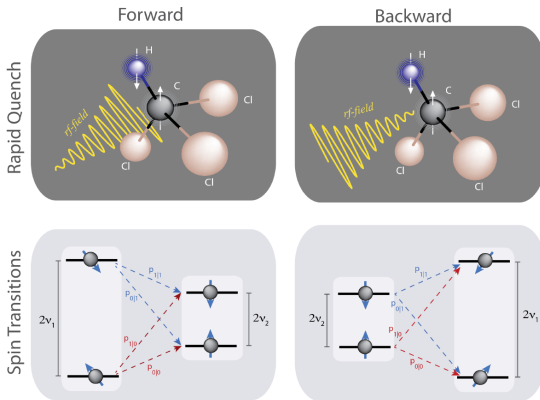
Experimental observation

Spin-1/2 driven by an external magnetic field: (Batalhão PRL 2015)



Experimental observation

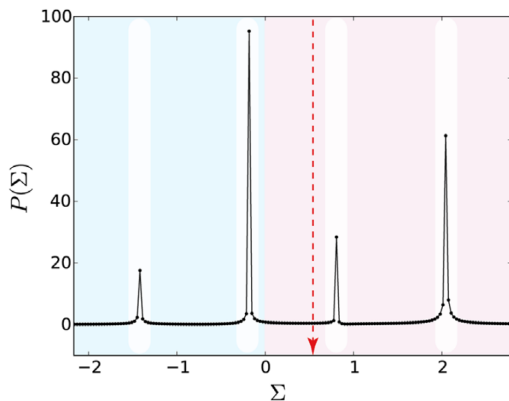
Spin-1/2 driven by an external magnetic field: (Batalhão PRL 2015)



Unitary dynamics with $\mathcal{H}_t^F = 2\pi\hbar\nu(t) [\sigma_x^C \cos \phi(t) + \sigma_y^C \sin \phi(t)]$

Experimental observation

Nonequilibrium entropy production: $\Sigma = \beta(W - \Delta F)$



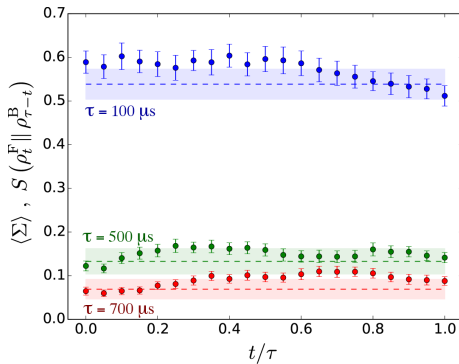
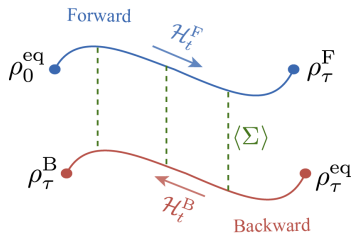
→ experimental proof of $\langle \Sigma \rangle \geq 0$ for driven quantum system

Experimental observation

Mean entropy production:

(Deffner-Lutz PRL 2011)

$$\langle \Sigma \rangle = S(\rho_t^F || \rho_{\tau-t}^B) = \text{tr}[\rho_t^F \ln \rho_t^F - \rho_t^F \ln \rho_{\tau-t}^B]$$



→ experimental demonstration of the arrow of time

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Reversing the arrow of time

Conventional thermodynamics:

systems are assumed to be initially **uncorrelated**

→ preferred direction of the arrow of time

Theoretical suggestion:

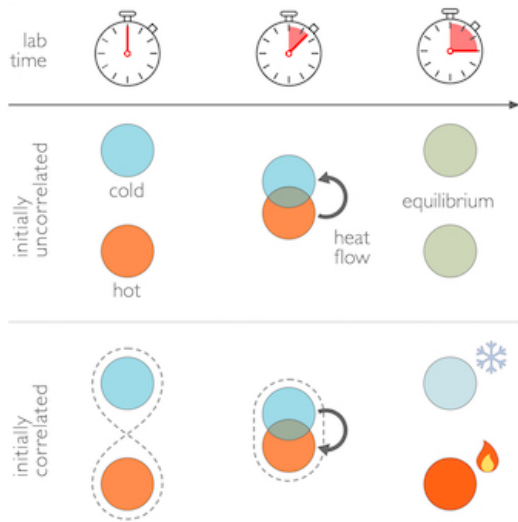
consider systems that are initially **correlated**

→ direction of the arrow of time could be reversed

that is, heat could **spontaneously** flow from cold to hot

(Partovi PRE 2008, Jennings-Rudolph PRE 2010, PRL 2012)

Reversing the arrow of time



Reversing the arrow of time

Concrete example: two qubits A and B

$$\rho_{AB} = \rho_A \otimes \rho_B + \alpha (|01\rangle\langle 10| + |10\rangle\langle 01|)$$

Properties:

- $\rho_{A,B}$ are **thermal** states at inverse temperature $\beta_{A,B}$
- for $\alpha = 0$ uncorrelated and for $\alpha \neq 0$ correlated
- reduced qubit states **always locally thermal**
- thermal contact via **random partial swaps** in Z direction,
 $S(\theta) = \exp[i\theta(X_A Y_B + Y_A X_B)/2]$ (Scarani PRL 2002)

Thermodynamics of initially correlated systems

Heats for the bipartite systems:

$$\beta_A Q_A + \beta_B Q_B \geq \Delta I(A : B)$$

where $I(A : B) = S_A + S_B - S_{AB} \geq 0$ is the mutual information

Initially uncorrelated spins: $\Delta I(A : B) \geq 0$

$$Q_B(\beta_B - \beta_A) \geq 0 \quad \implies \quad Q_B \geq 0 \quad (T_A \geq T_B)$$

→ standard arrow of time

Initially correlated spins: $\Delta I(A : B) \leq 0$

$$Q_B(\beta_B - \beta_A) \leq 0 \quad \implies \quad Q_B \leq 0 \quad (T_A \geq T_B)$$

→ reversal of the arrow of time

Thermodynamics of initially correlated systems

Heat for spin B:

$$\Delta\beta Q_B = \Delta I(A : B) + \mathcal{S}(\rho_A^\tau || \rho_A) + \mathcal{S}(\rho_B^\tau || \rho_B)$$

where $\mathcal{S}(\rho_i^\tau || \rho_i) = \text{Tr}_i \rho_i^\tau (\ln \rho_i^\tau - \ln \rho_i) \geq 0$ is the relative entropy that is, the **entropy production** in each spin

Criterion for reversal:

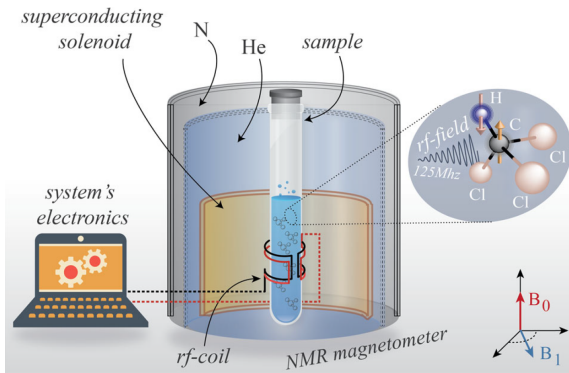
$$\Delta I(A : B) \leq -\mathcal{S}(\rho_A^\tau || \rho_A) - \mathcal{S}(\rho_B^\tau || \rho_B)$$

- decrease of mutual information compensates entropy production
- trade off between entropy and information

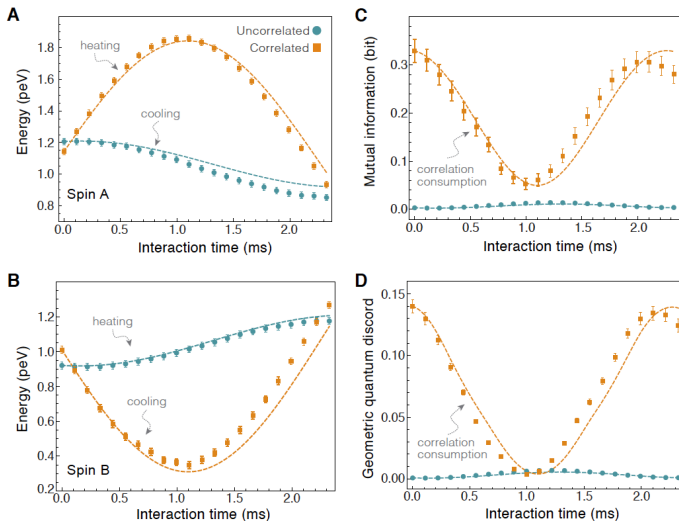
(Lloyd PRE 1989, Sagawa-Ueda PRL 2012, Koski PRL 2014)

Experimental observation

Two coupled thermal spins-1/2



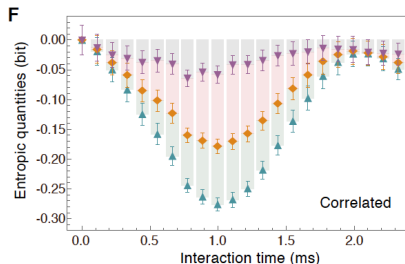
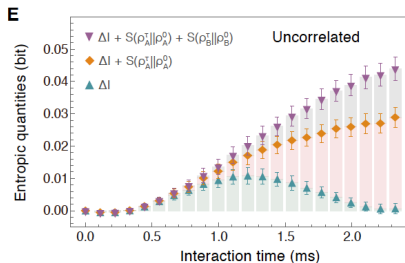
Experimental observation



→ local second law does not apply when (nonlocal) correlations

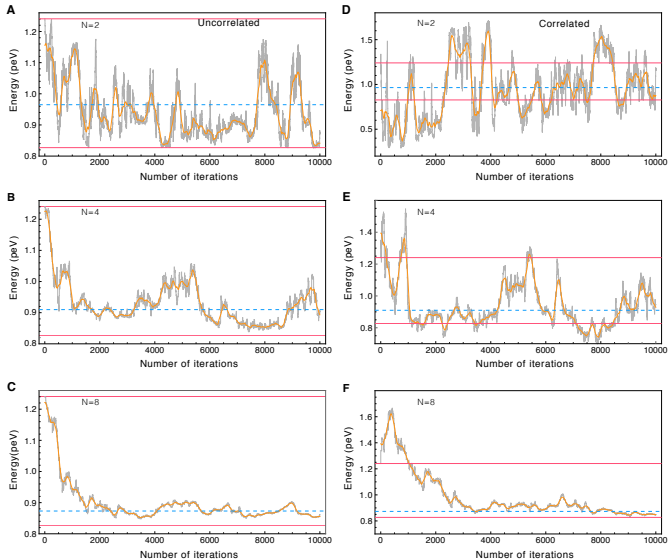
Experimental observation

$$\text{Heat for spin B: } \Delta\beta Q_B = \Delta I(A : B) + \mathcal{S}(\rho_A^\tau || \rho_A) + \mathcal{S}(\rho_B^\tau || \rho_B)$$



→ trade off between information and entropy

Numerical simulation for larger systems $1 \times N$



→ reversals still occur

Summary

- **arrow of time** is not an abstract, philosophical concept
 - it can be **quantified** and **observed** in the lab
- may be **reversed** for quantum **correlated** systems
 - trade off between **information** and **entropy**
 - is a **relative** concept and allows **control** of heat flow
- local second law **fails** in presence of (nonlocal) correlations
 - subtle interplay of **quantum** and **thermodynamics**