Information gain and loss for a quantum Maxwell's demon

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Naghiloo et al arXiv:1802.07205 (to appear in PRL)

John Templeton Foundation

Where in the world is St. Louis?

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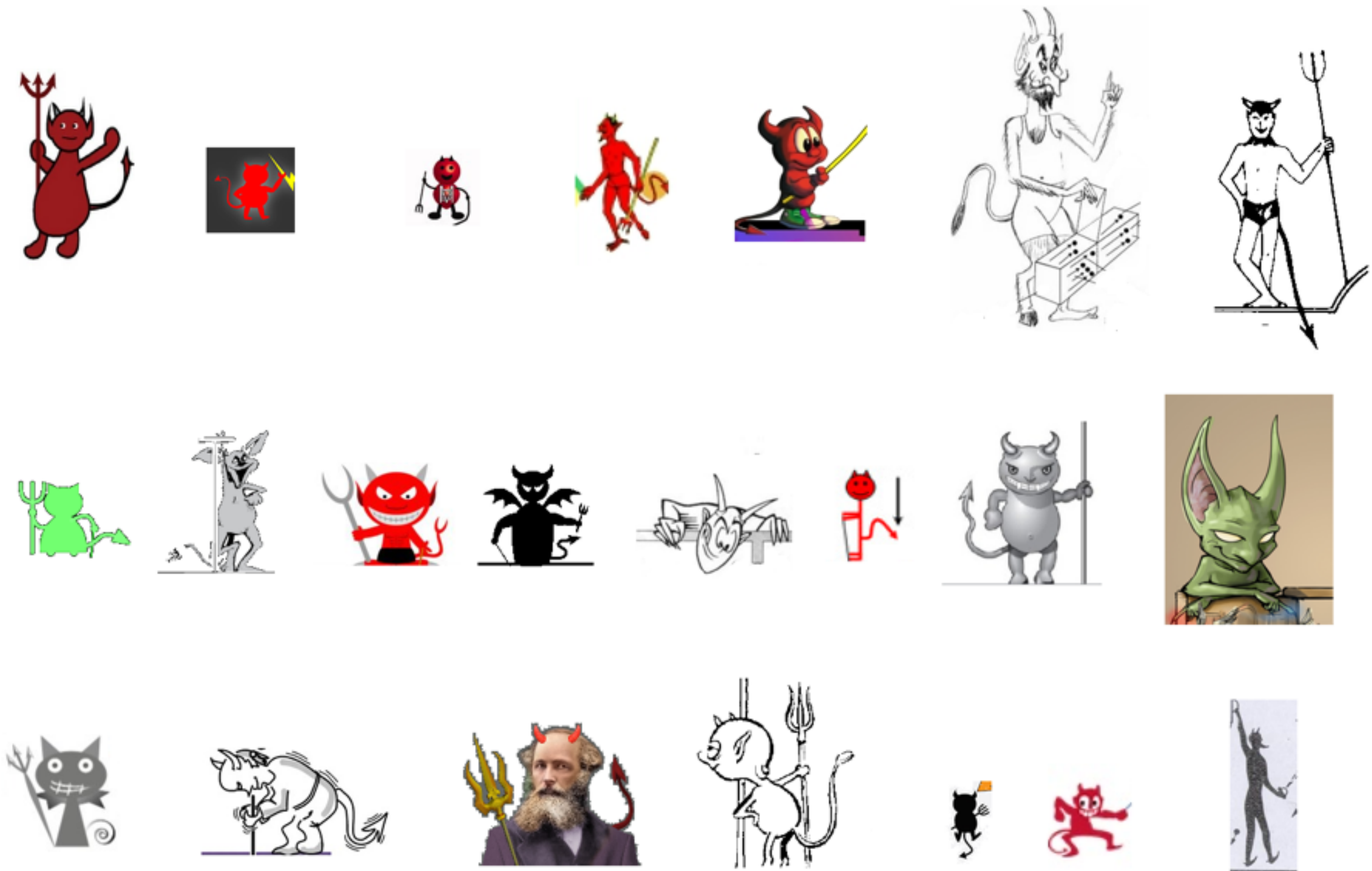
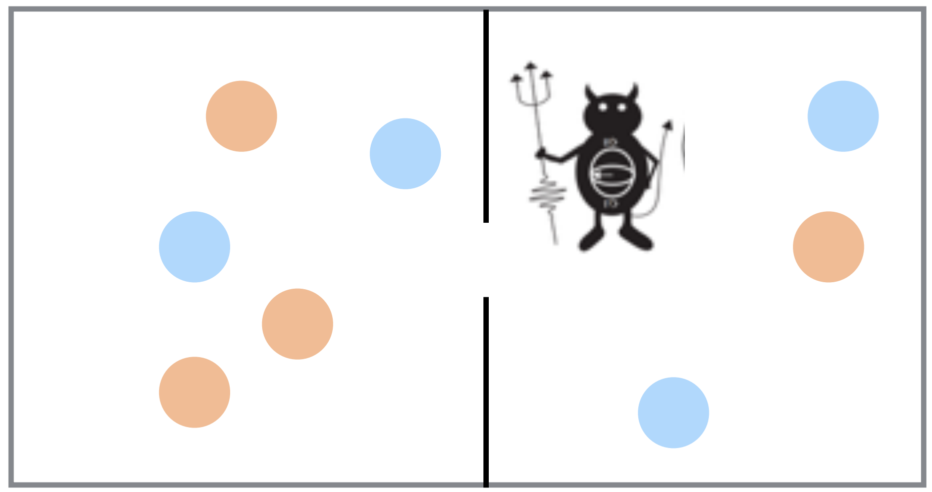
Experimental research with superconducting qubits.

Quantum Measurement: Zeno effects, quantum trajectories
State smoothing and post-selection: weak values, retrodiction, optimal routes
Metrology: frequency metrology, Axion dark matter search
Quantum Thermodynamics: heat, work, entropy, heat engines

Maxwell's demon in google images

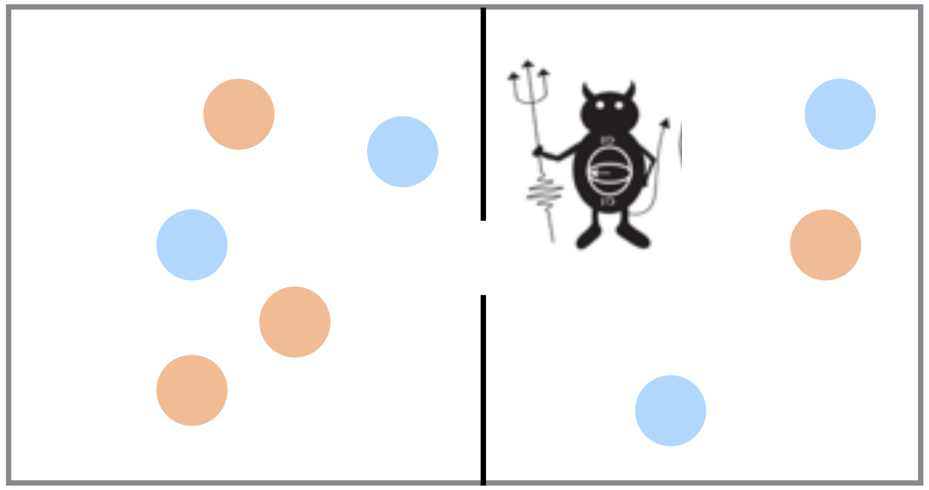


A quantum Maxwell's demon

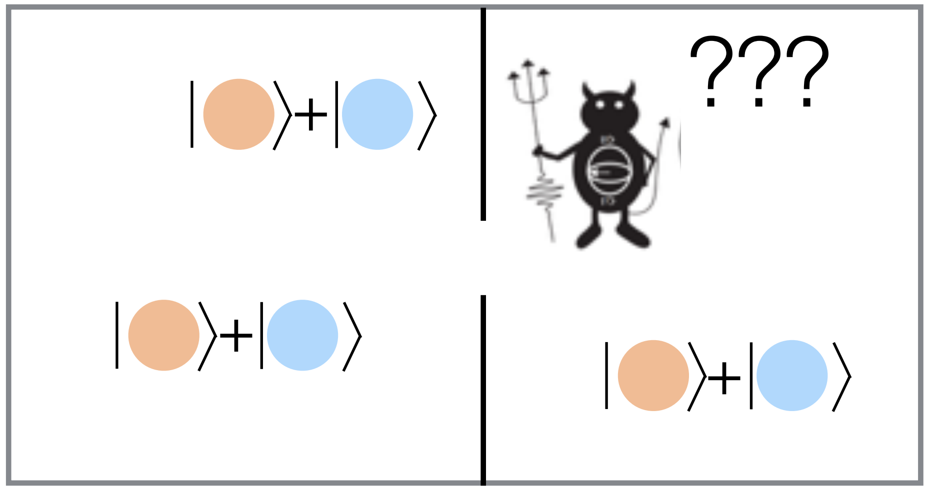


Sorts "swift" from "slow"

A quantum Maxwell's demon

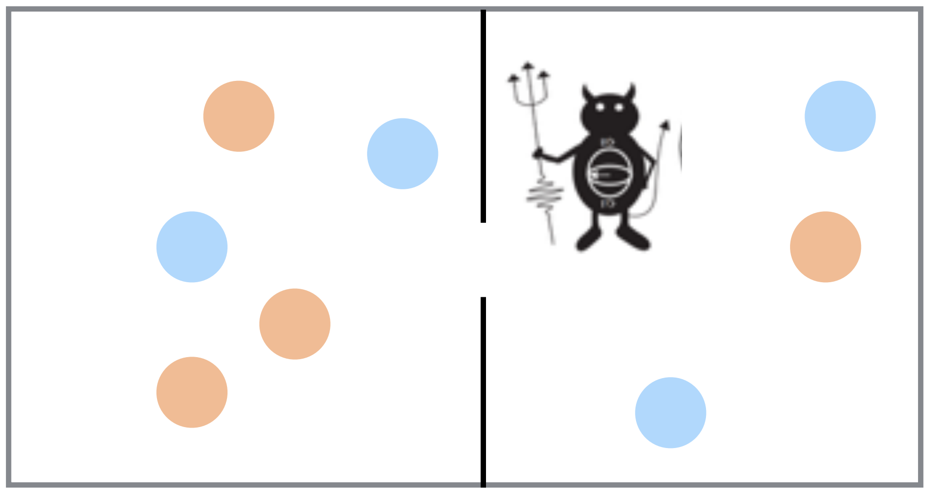


Sorts "swift" from "slow"



Sort "swift" **and** "slow"?

A quantum Maxwell's demon



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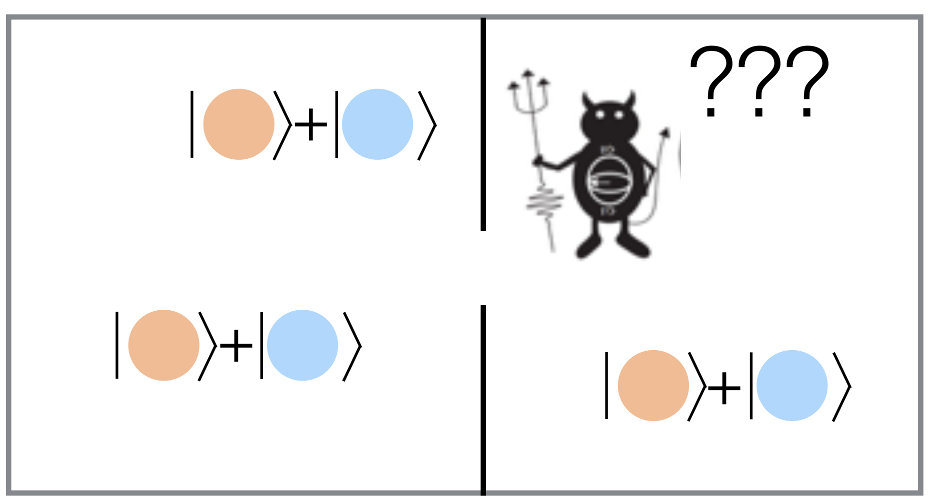
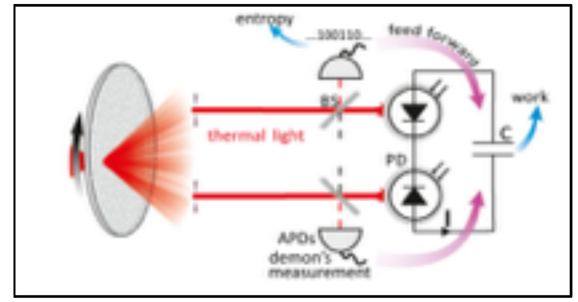


Sort "swift" **and** "slow"?

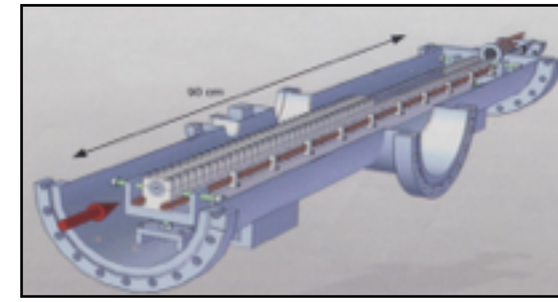
Demon uses weak measurements…

Maxwell’s demon at the level of single quanta

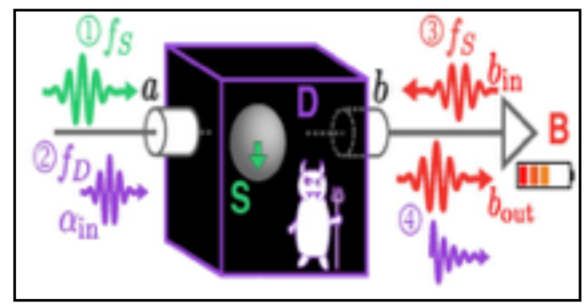
- Cold atoms
- Molecular ratchet
- Colloidal particles
- Single electrons
- Photons



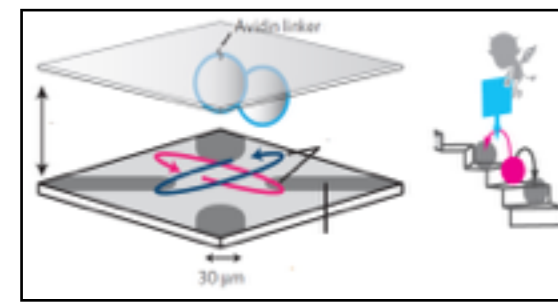
PRL, 116, 050401 (2016)



Science 324, 1403 (2009)



PNAS, 114, 7561 (2017)



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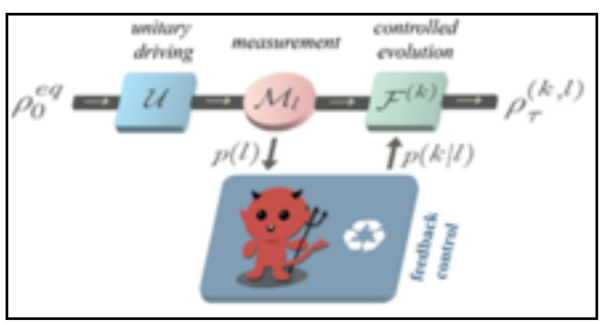
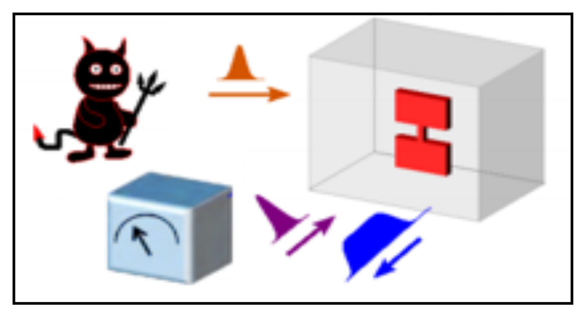
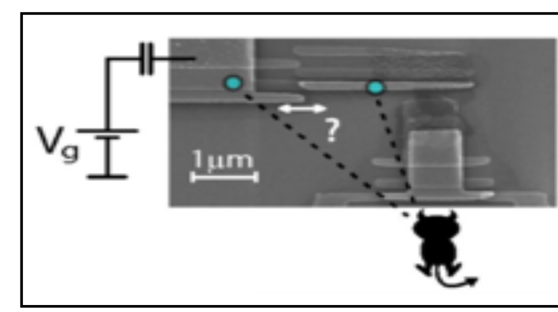


PRL 117, 240502 (2016)



arXiv, 1709.00548 (2017)



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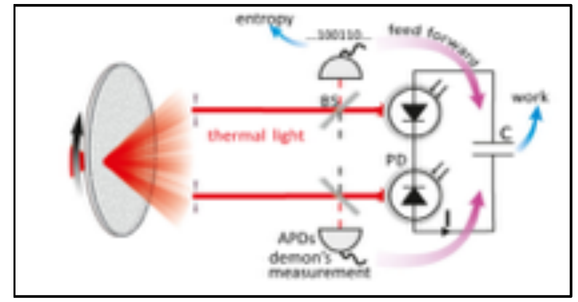

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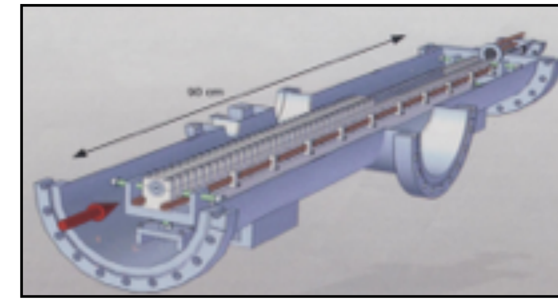

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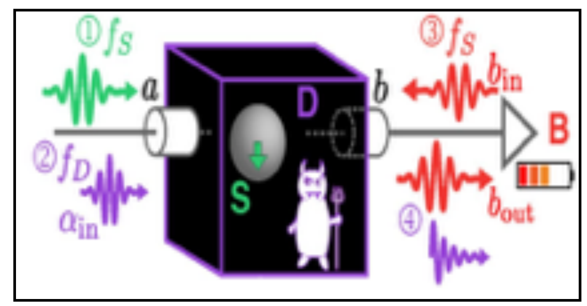

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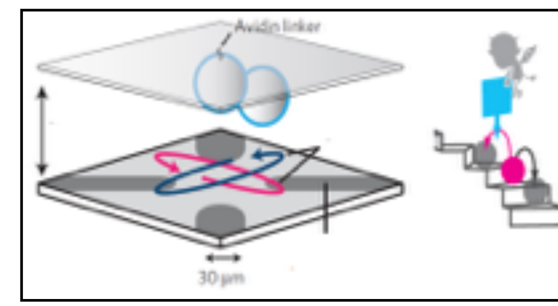

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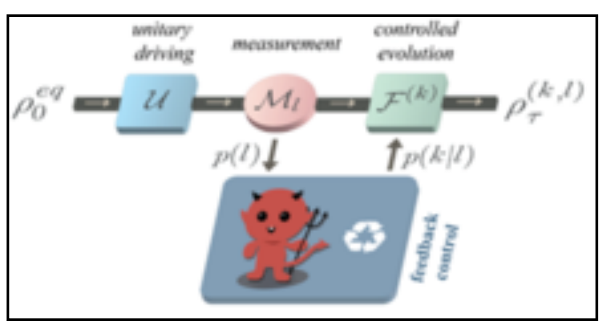

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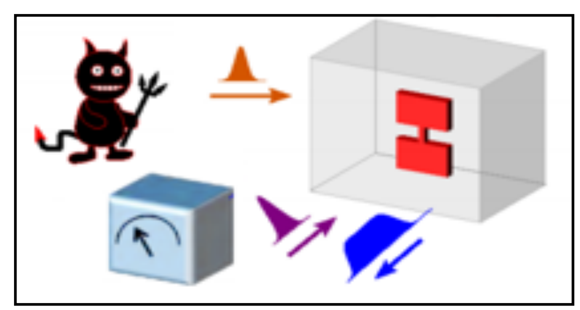

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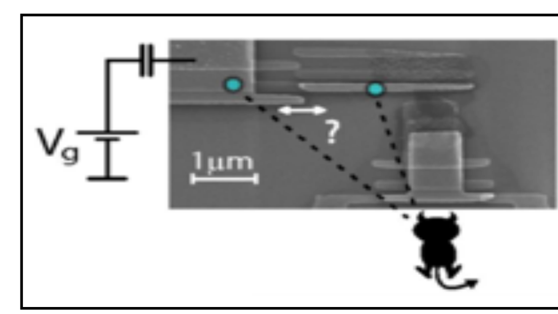

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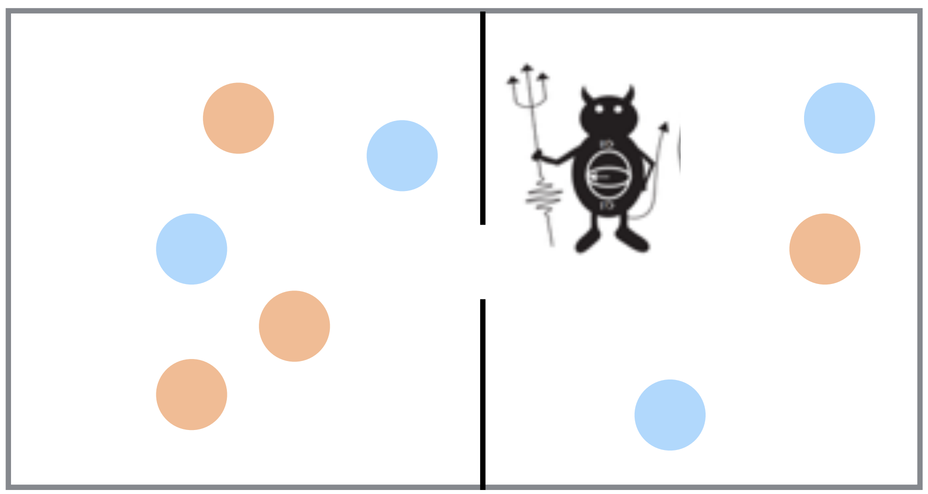
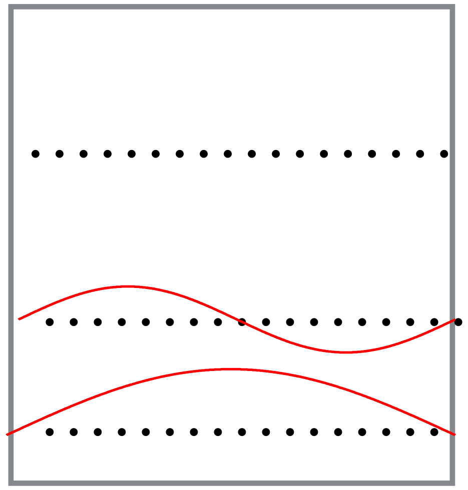
Dynamics does not include coherences or the demon destroys coherence with measurement.

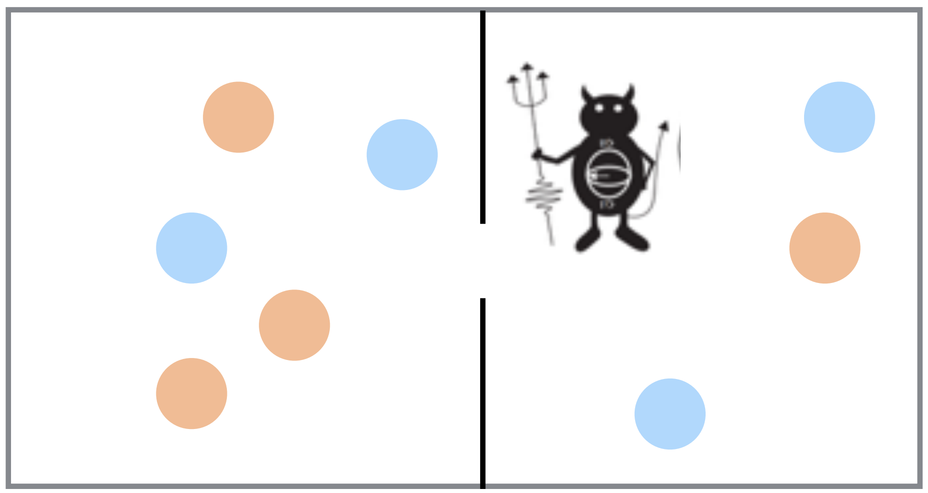
A quantum Maxwell's demon

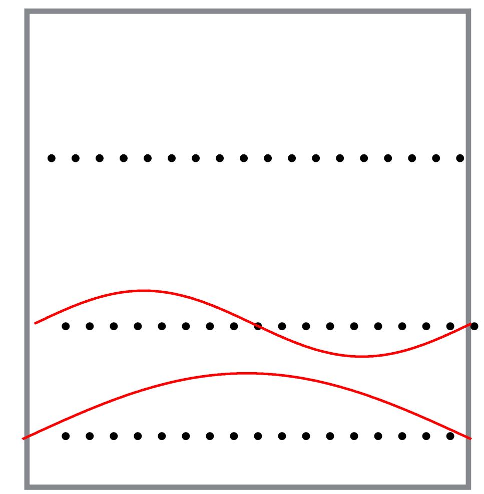




one particle in a box

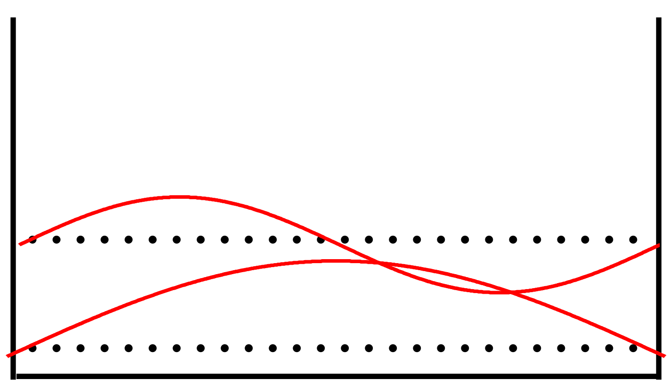
A quantum Maxwell's demon





one particle in a box

Two lowest energy levels



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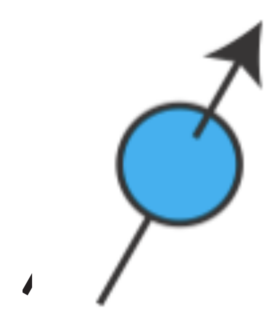
Pseudo spin-1/2



Information and Measurement in quantum mechanics

A sequence of measurements in time…

Z X Z Z X X Z X

Information and Measurement in quantum mechanics

A sequence of measurements in time…

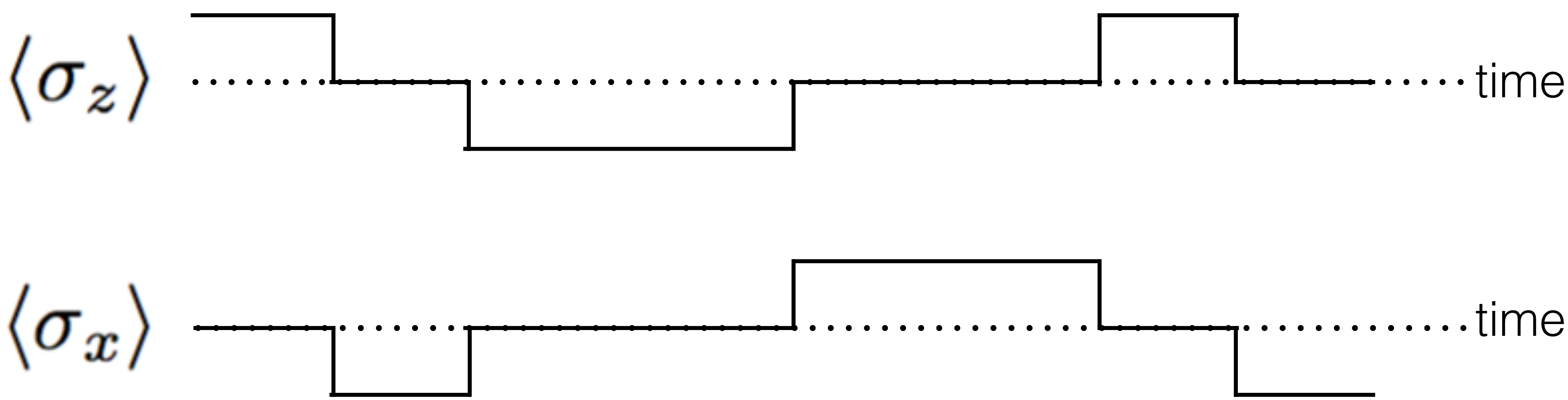
Z	X	Z	Z	X	X	Z	X
+1	-1	-1	-1	+1	+1	+1	-1

Information and Measurement in quantum mechanics

A sequence of measurements in time…

Z X Z Z X X Z X

+1 -1 -1 -1 +1 +1 +1 -1



Information and Measurement in quantum mechanics

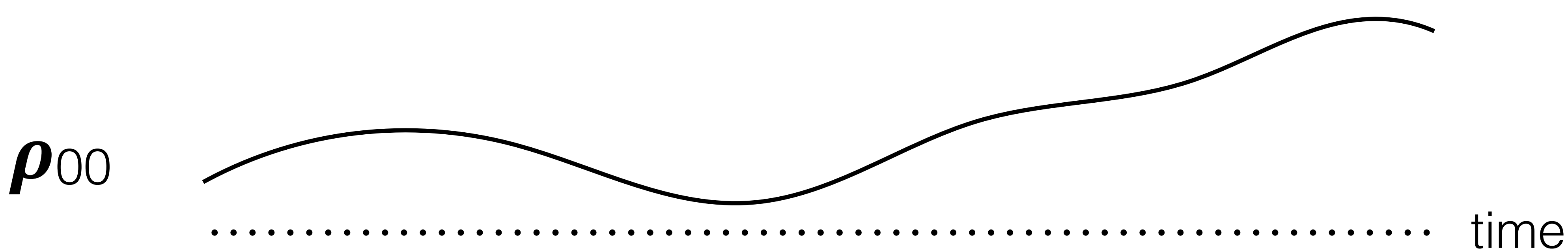
More generally…

M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11} M_{12}….

Information and Measurement in quantum mechanics

More generally…

$M_1\ M_2\ M_3\ M_4\ M_5\ M_6\ M_7\ M_8\ M_9\ M_{10}\ M_{11}\ M_{12}\ldots.$

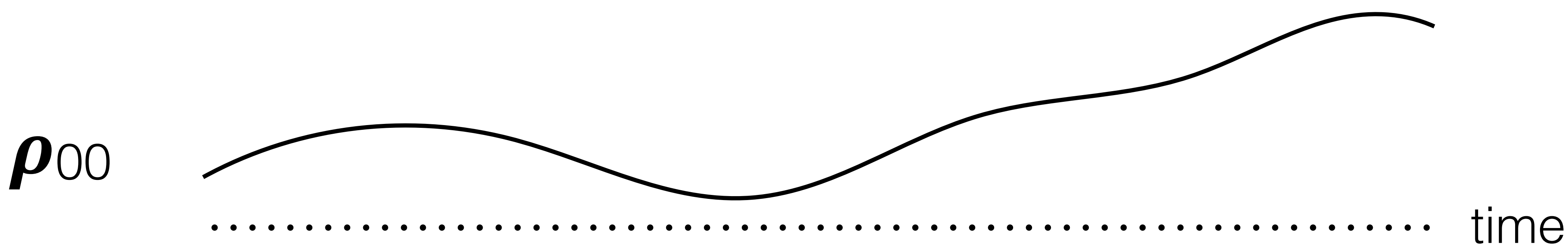


Time evolution of a quantum state = "quantum trajectory"

Information and Measurement in quantum mechanics

More generally…

$M_1\ M_2\ M_3\ M_4\ M_5\ M_6\ M_7\ M_8\ M_9\ M_{10}\ M_{11}\ M_{12}\ldots.$



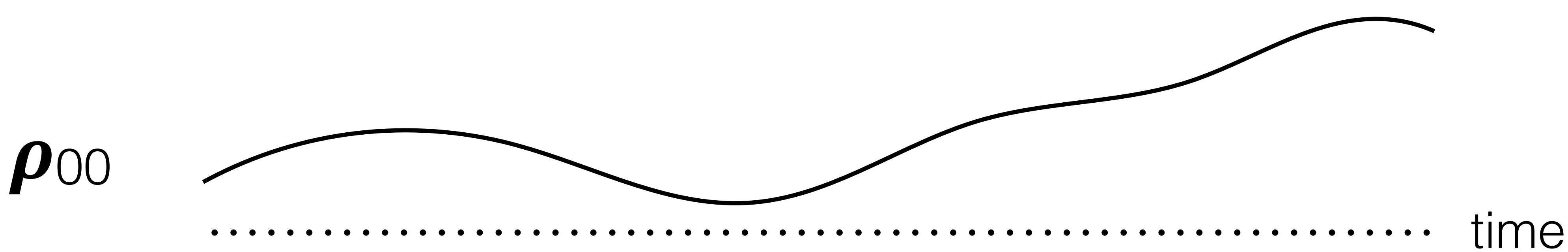
Time evolution of a quantum state = "quantum trajectory"

Quantum Maxwell's Demon:

Information and Measurement in quantum mechanics

More generally…

M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11} M_{12}….



Time evolution of a quantum state = "quantum trajectory"

Quantum Maxwell's Demon:

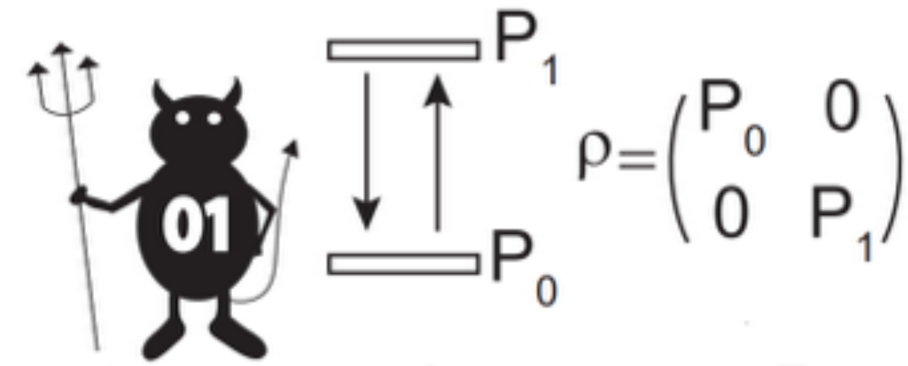
Track quantum trajectories and do something with that information.

A quantum Maxwell's demon

- Classical demon:

✓ Evolution of populations - definite states

✓ Measurement without disturbance

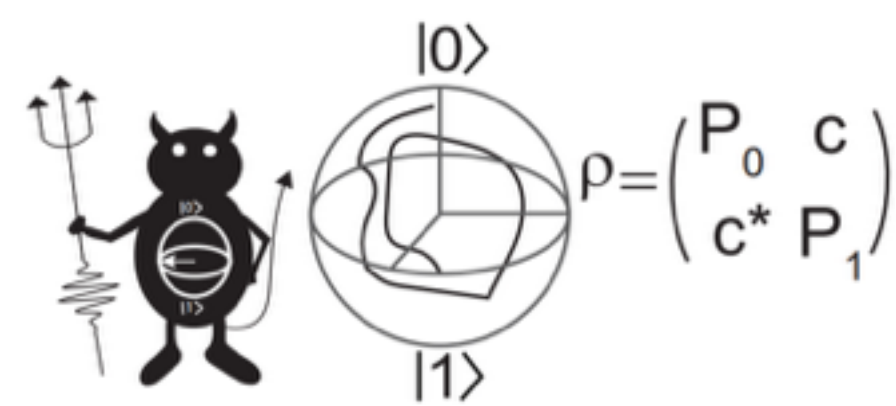


- Quantum demon:

✓ Measurement with disturbance

✓ Role of coherence and entanglement

✓ Information can be lost



TPM: Two projective measurement.

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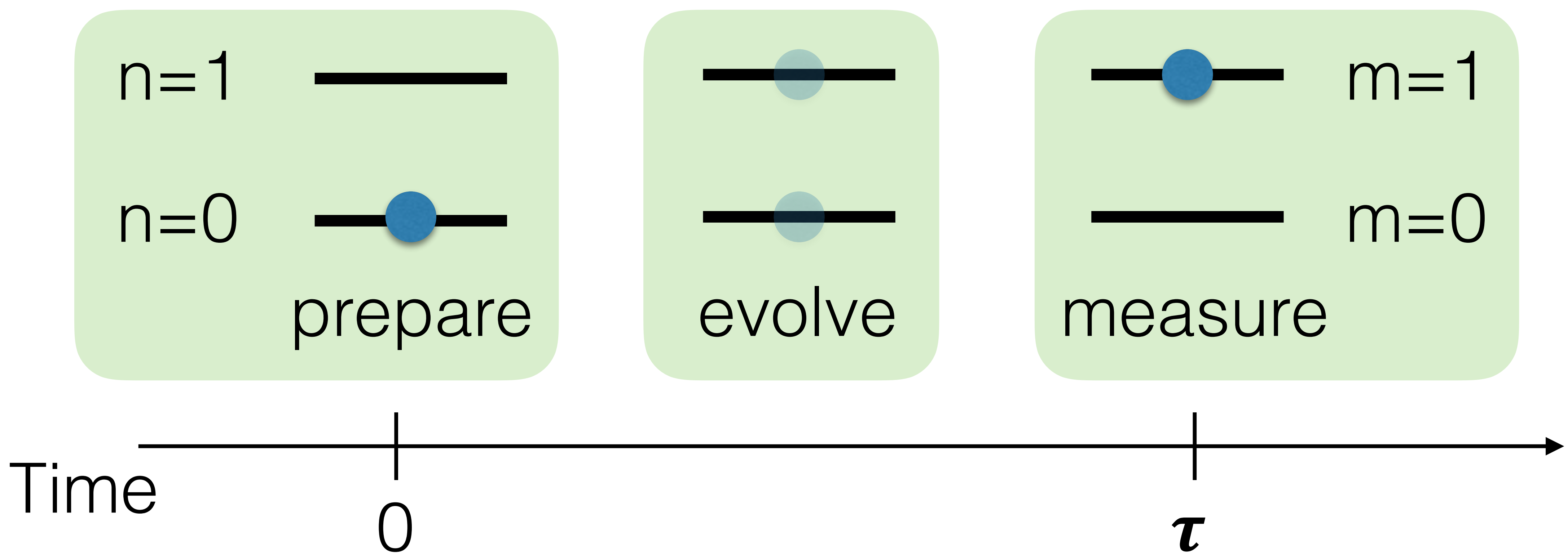
TPM: gives the distribution of total energy change from transition probabilities



$$P(\Delta U) = \sum_{m,n} P^{\tau}_{m,n} P^{0}_{n} \delta(\Delta U - (E^{\tau}_{m} - E^{0}_{n}))$$

Quantum Maxwell's demon protocol

Initial quantum system
in thermal equilibrium

Quantum Maxwell's demon protocol

Initial quantum system in thermal equilibrium

Demon makes Q. measurements on system

Quantum Maxwell's demon protocol

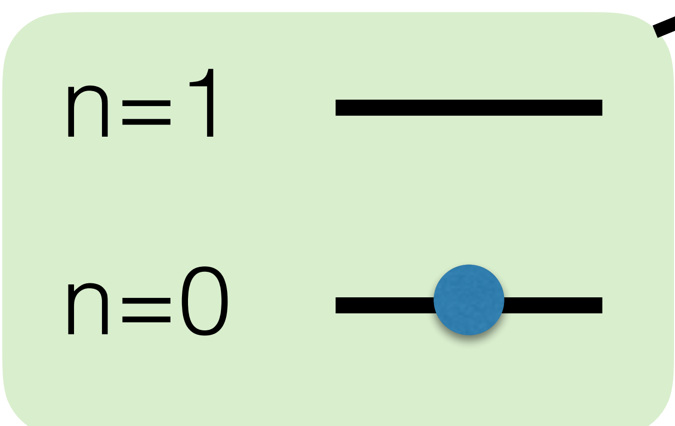
Initial quantum system in thermal equilibrium

Demon makes Q. measurements on system

Demon extracts work from Q. System

Quantum Maxwell's demon protocol

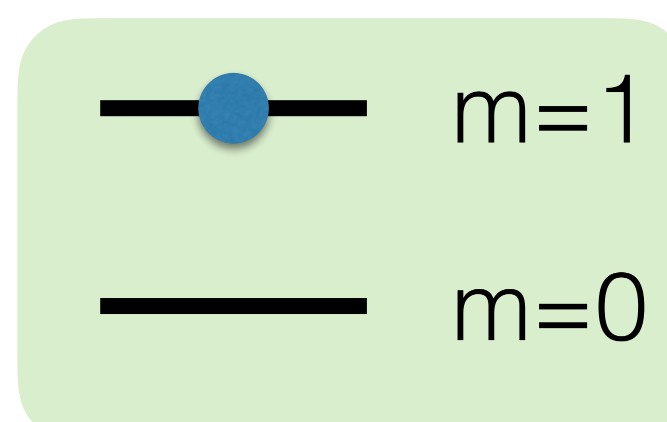
Initial quantum system in thermal equilibrium



Demon makes Q. measurements on system

Demon extracts work from Q. System

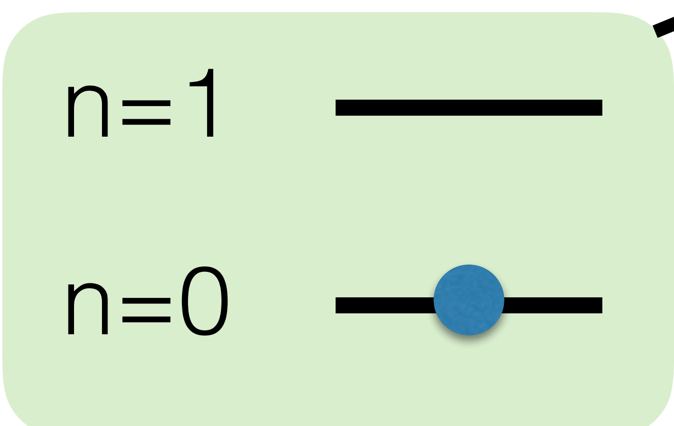
Projective measurements to characterize work distribution



Quantum Maxwell's demon protocol

Initial quantum system in thermal equilibrium

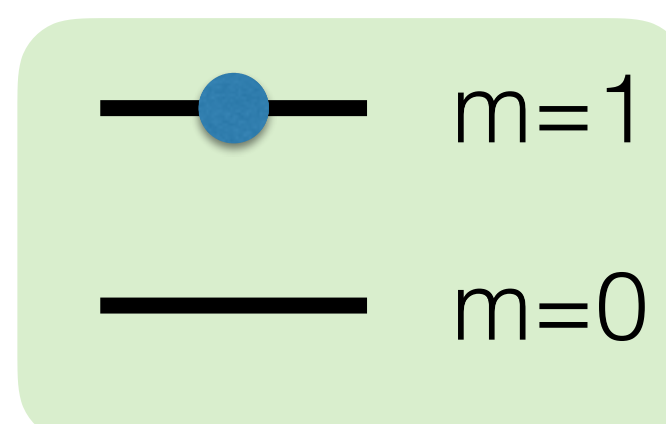
- Superconducting qubit



Demon makes Q. measurements on system

Demon extracts work from Q. System

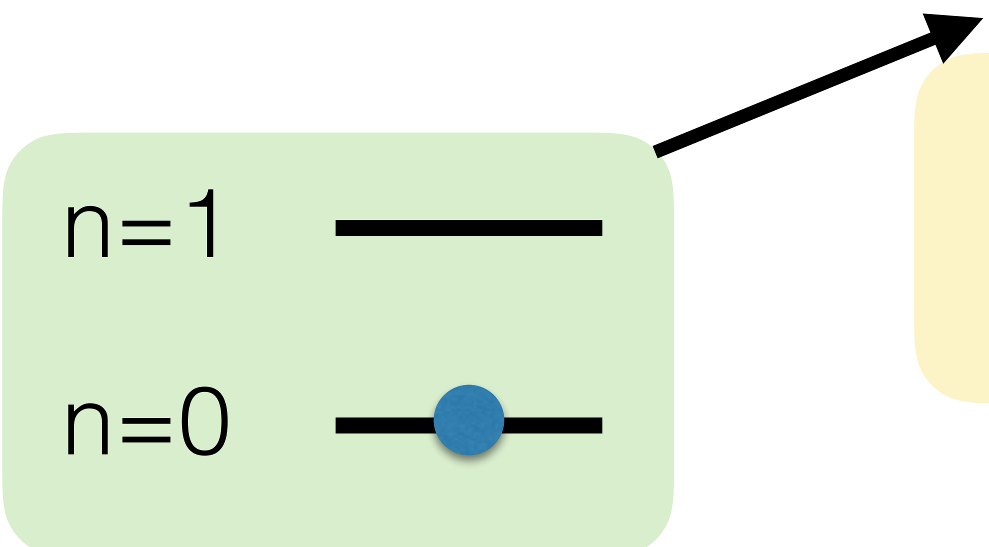
Projective measurements to characterize work distribution



Quantum Maxwell's demon protocol

Initial quantum system in thermal equilibrium

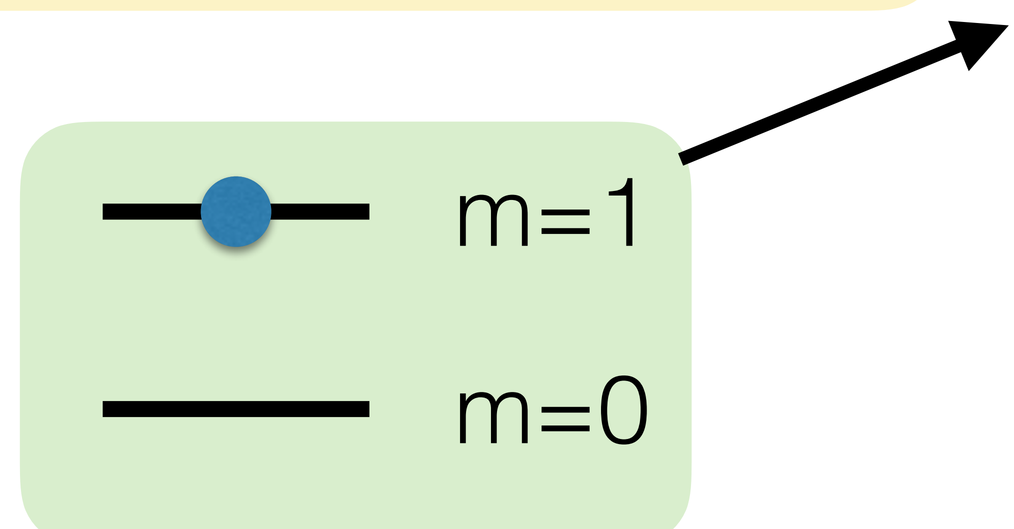
- Superconducting qubit
- Quantum trajectories



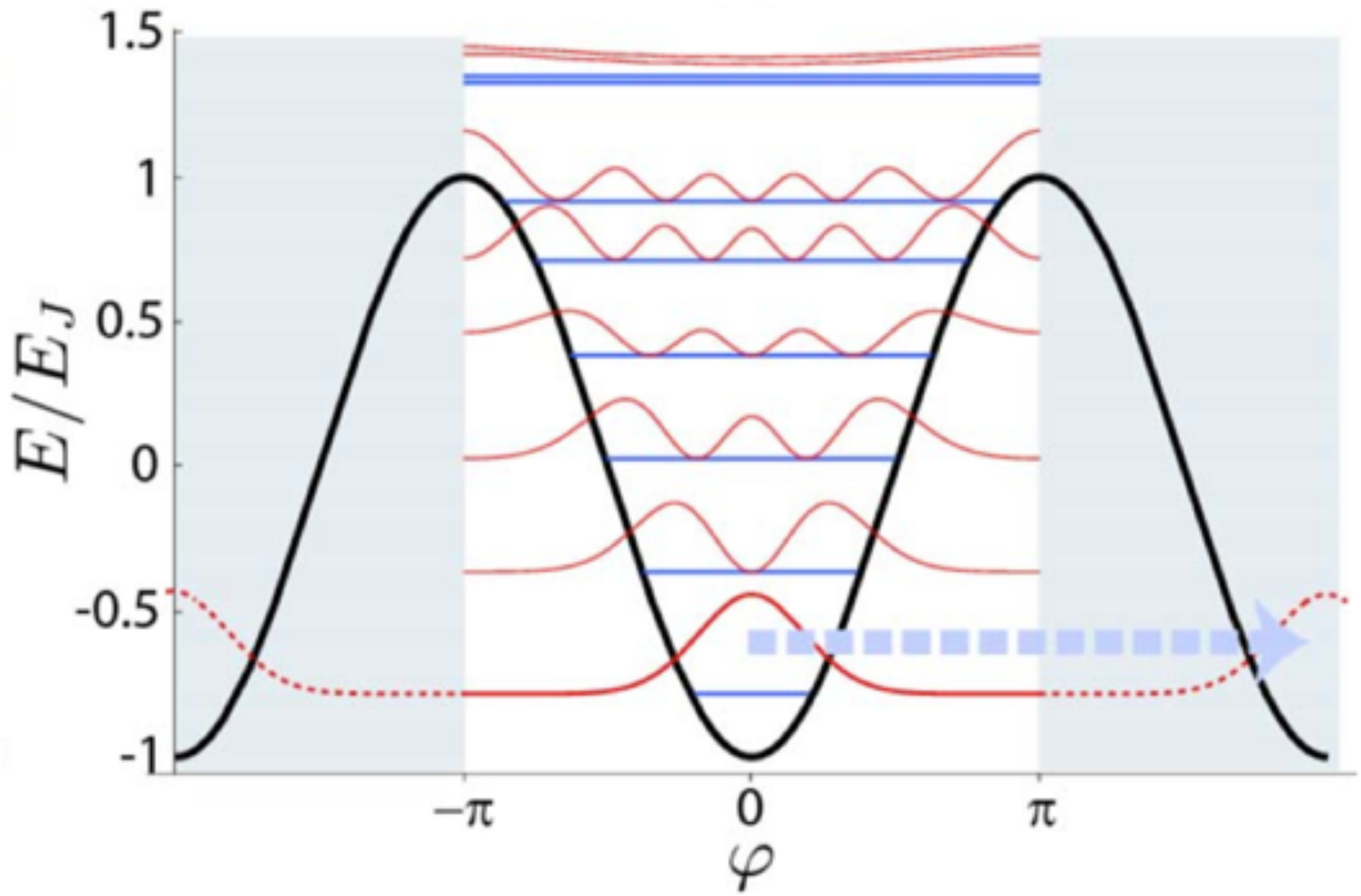
Demon makes Q. measurements on system

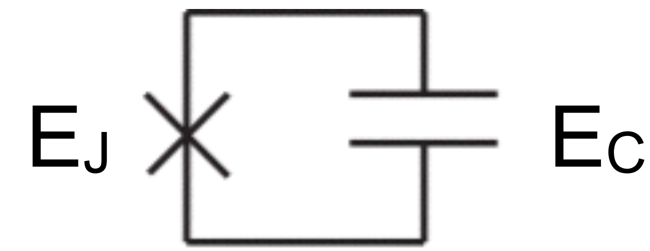
Demon extracts work from Q. System

Projective measurements
to characterize work distribution



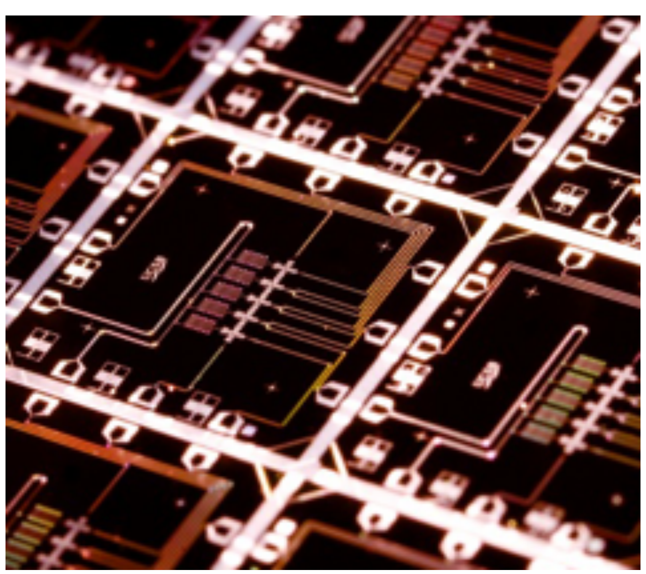
Our spin half system: a "transmon" circuit



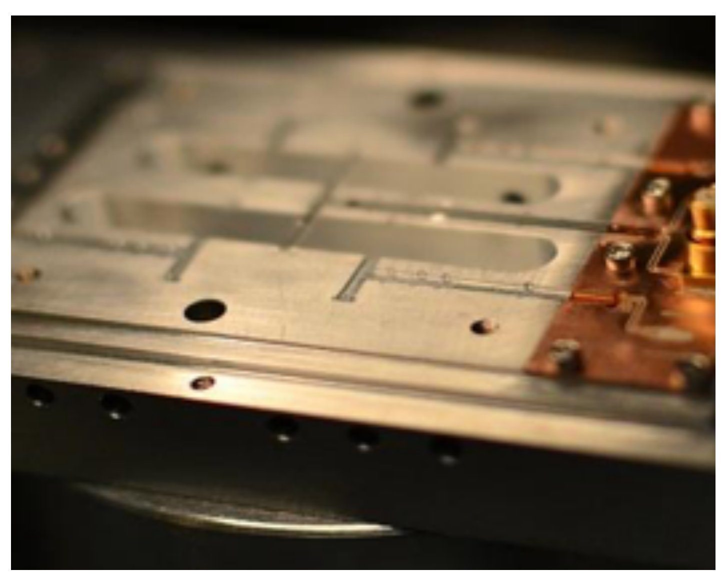


$E_J >> E_C$

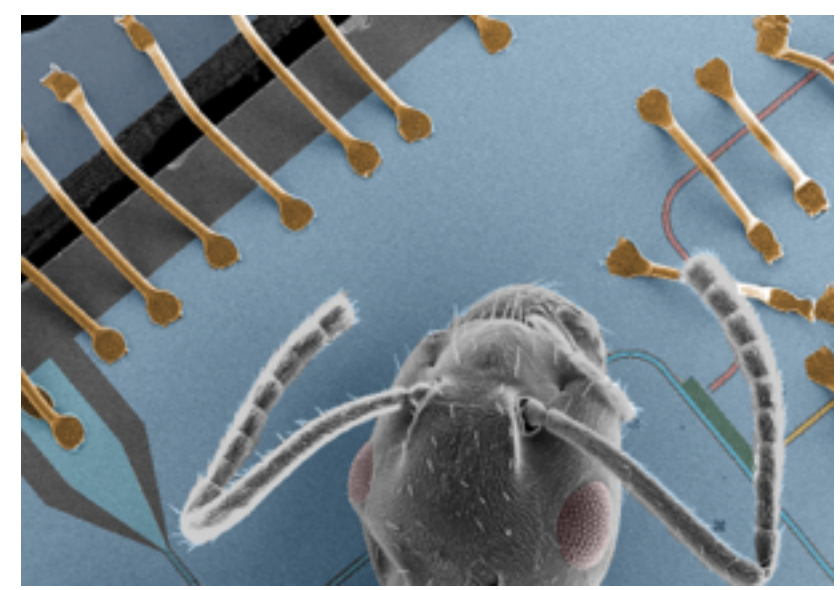
$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi}.$$



UCSB/Google



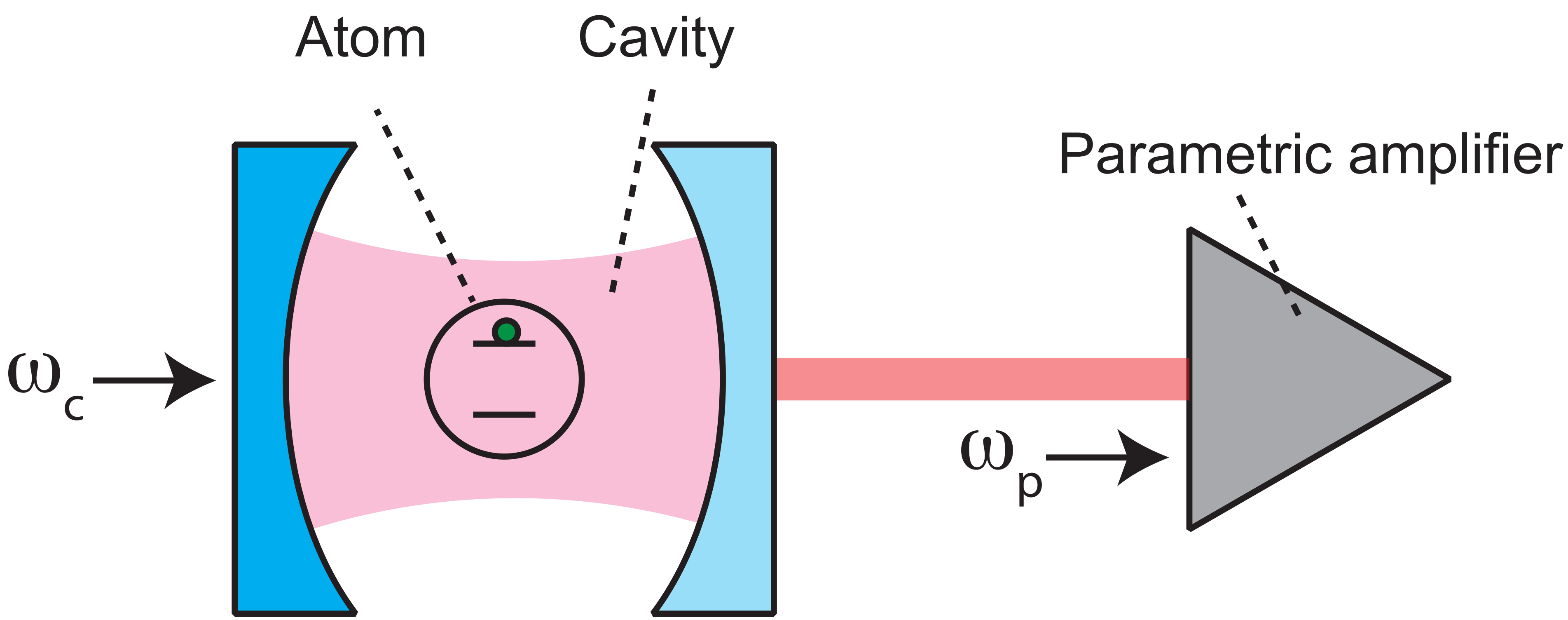
Yale

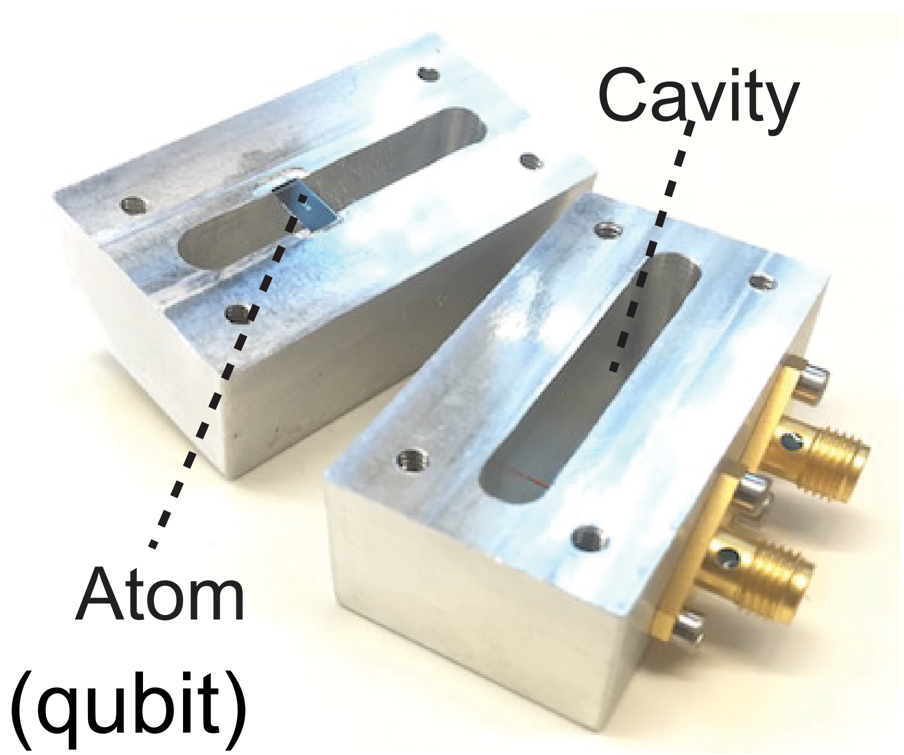


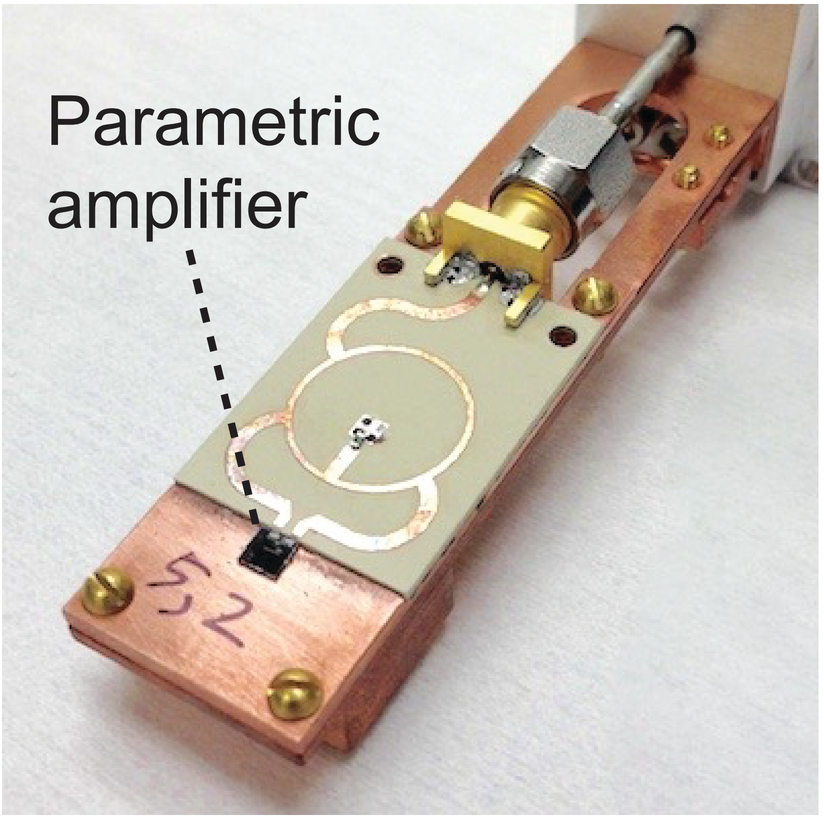
ETH Zurich

+ many other groups

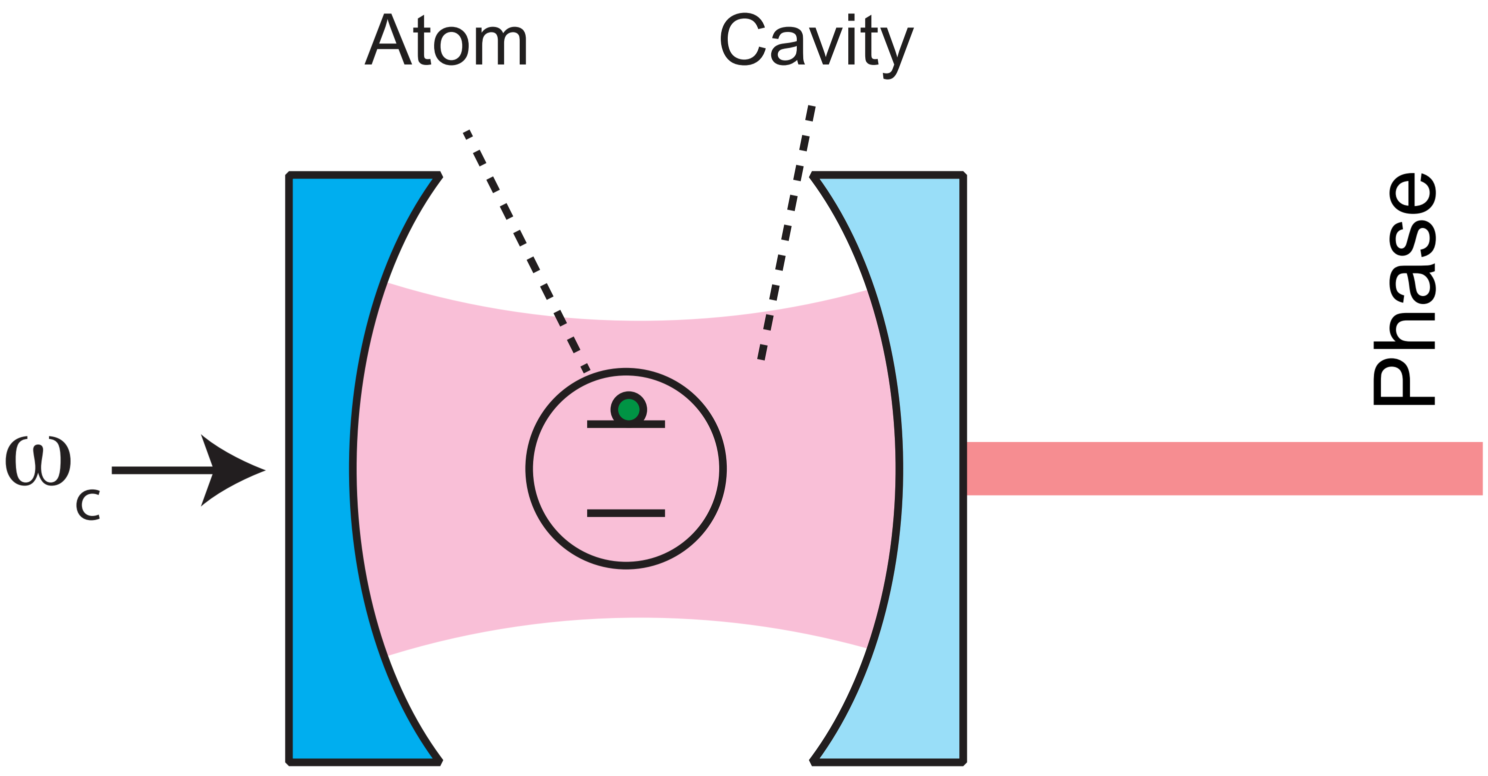
Dispersive measurement interaction







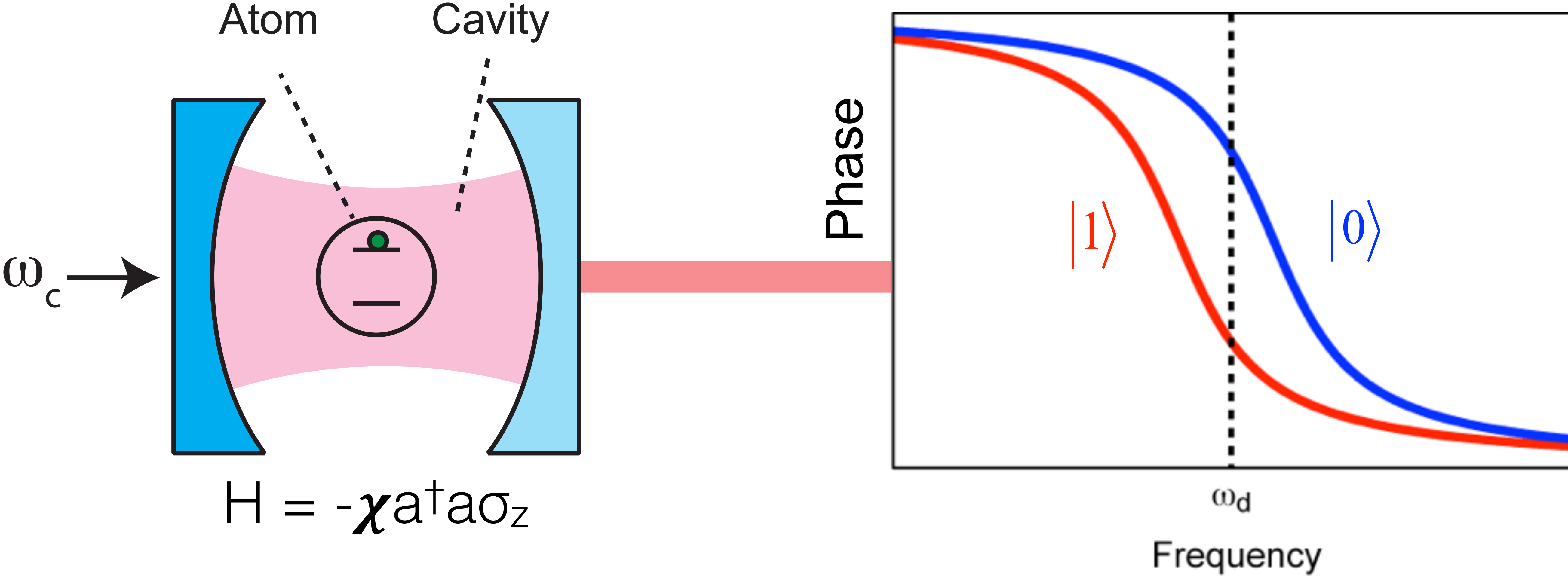
State dependent cavity shift



$$H = -\chi a^\dagger a \sigma_z$$

Vijay et al 2012, Murch 2013, Hatridge 2013

State dependent cavity shift



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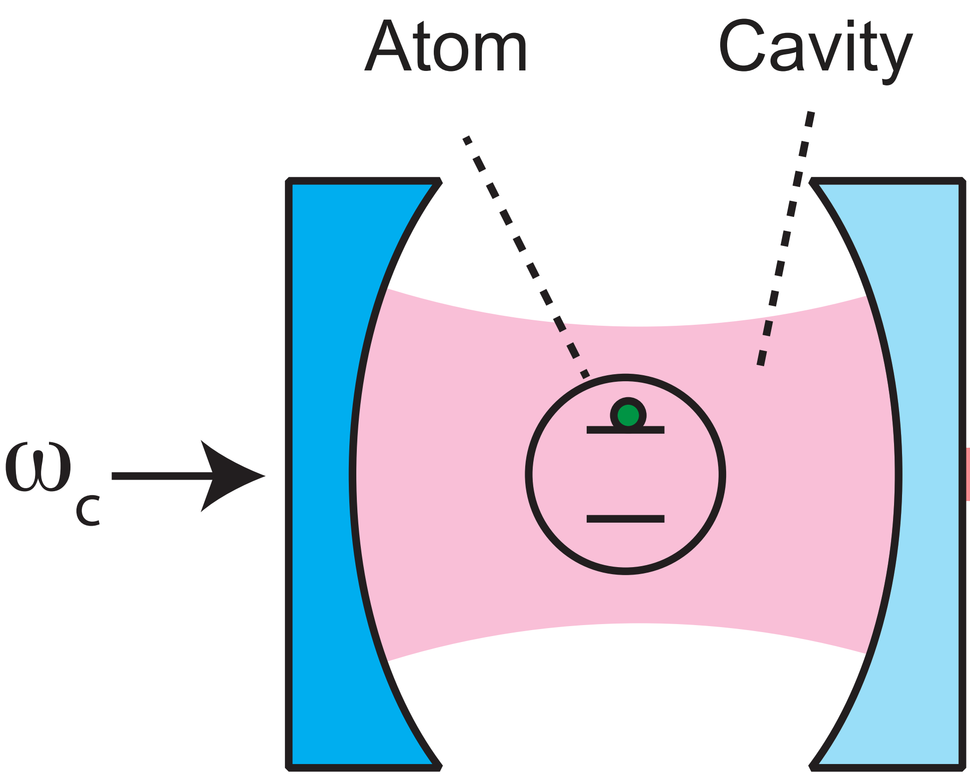
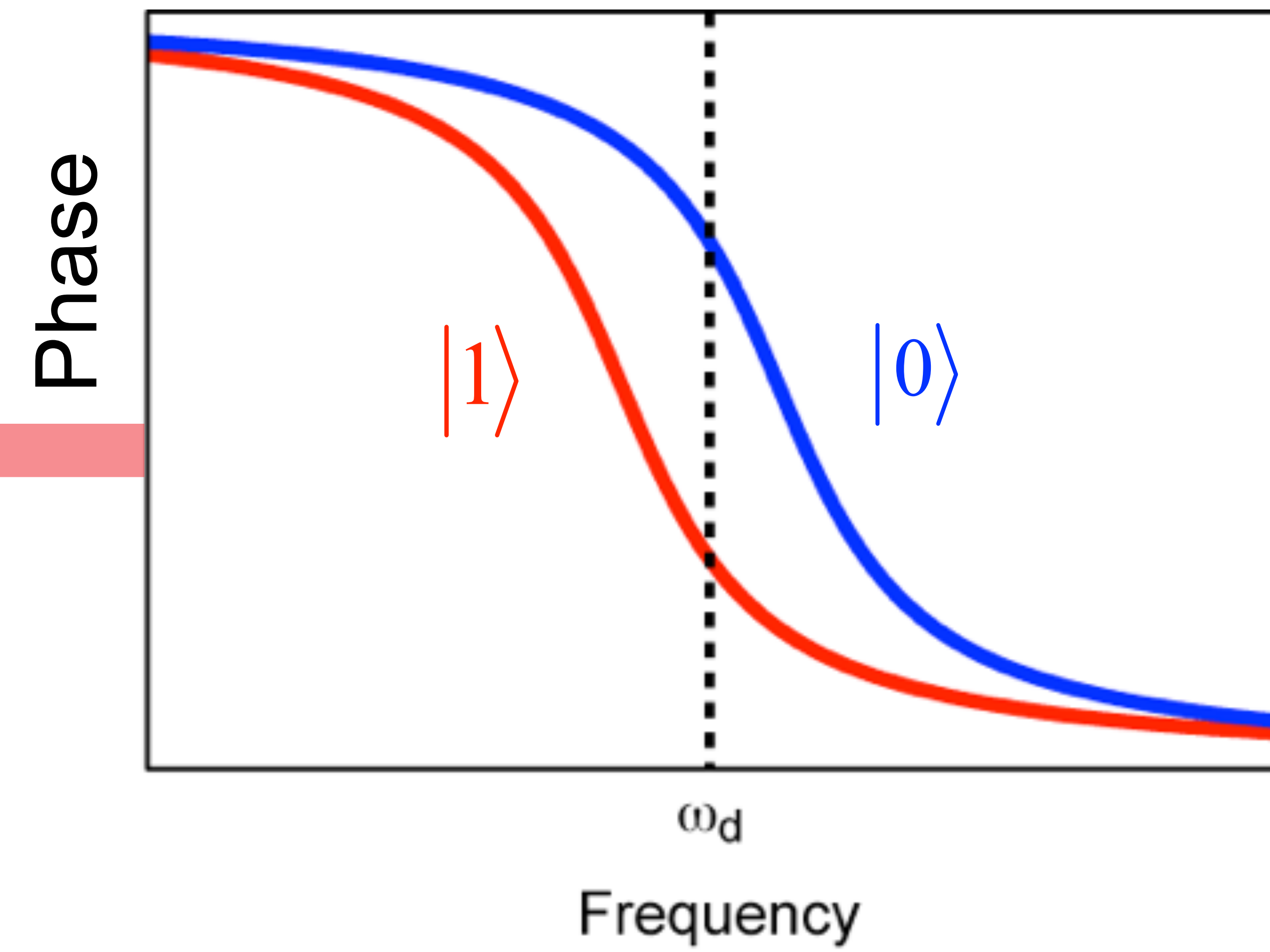
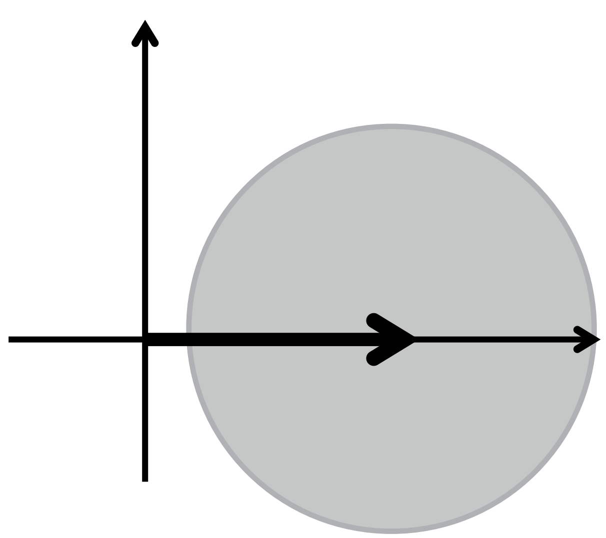




$$H = -\chi a^\dagger a \sigma_z$$

Input coherent state



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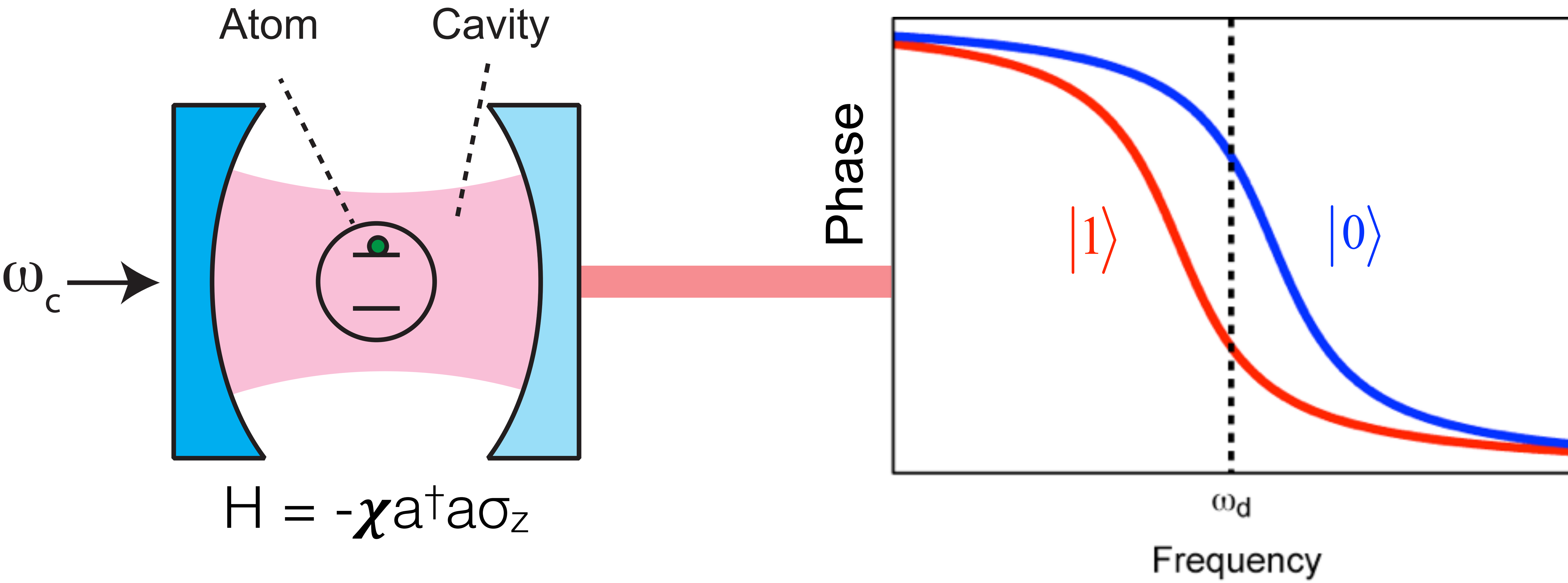

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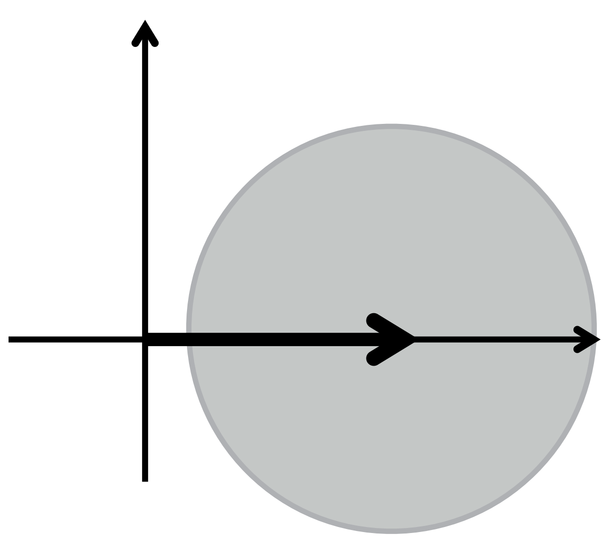
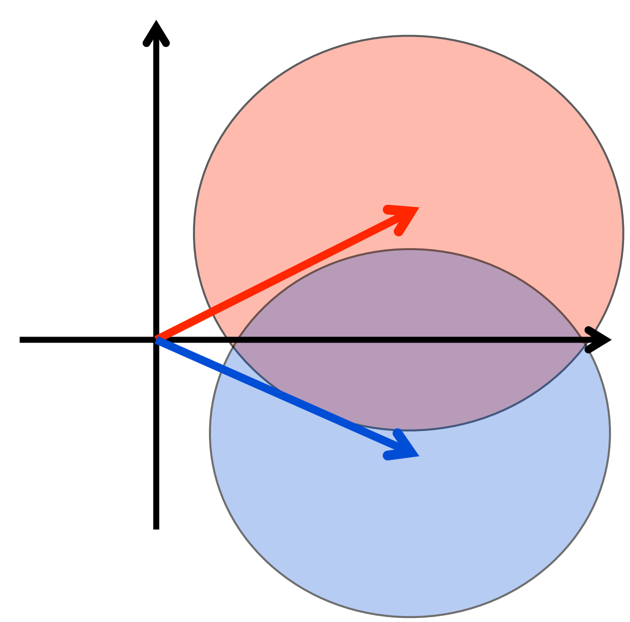
Input coherent state

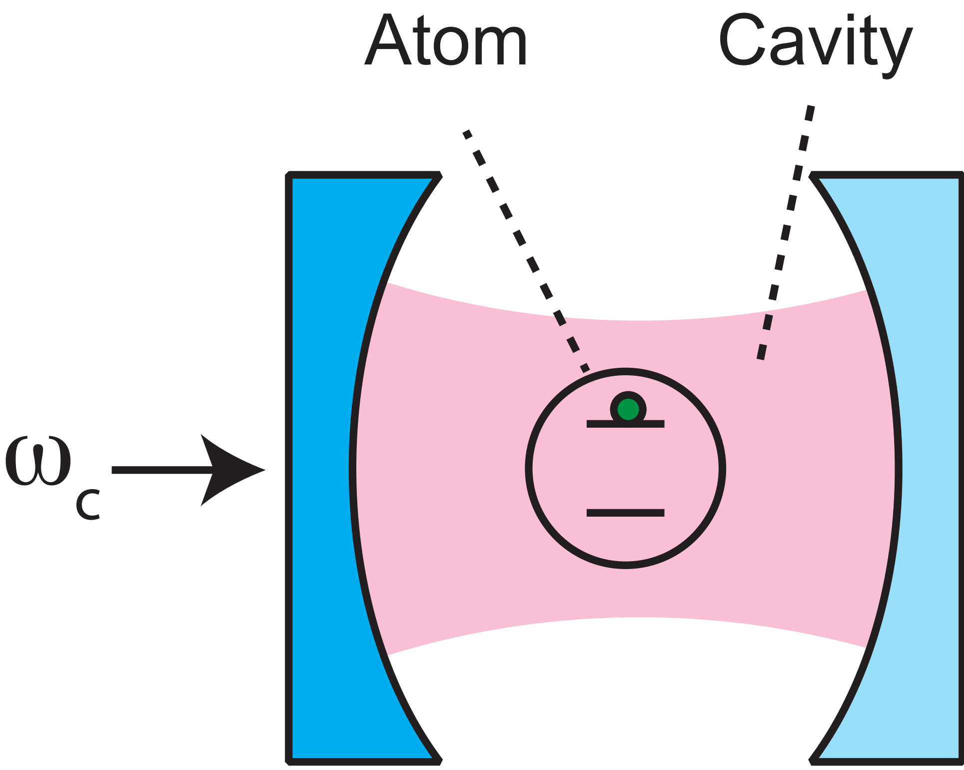


State dependent phase shift

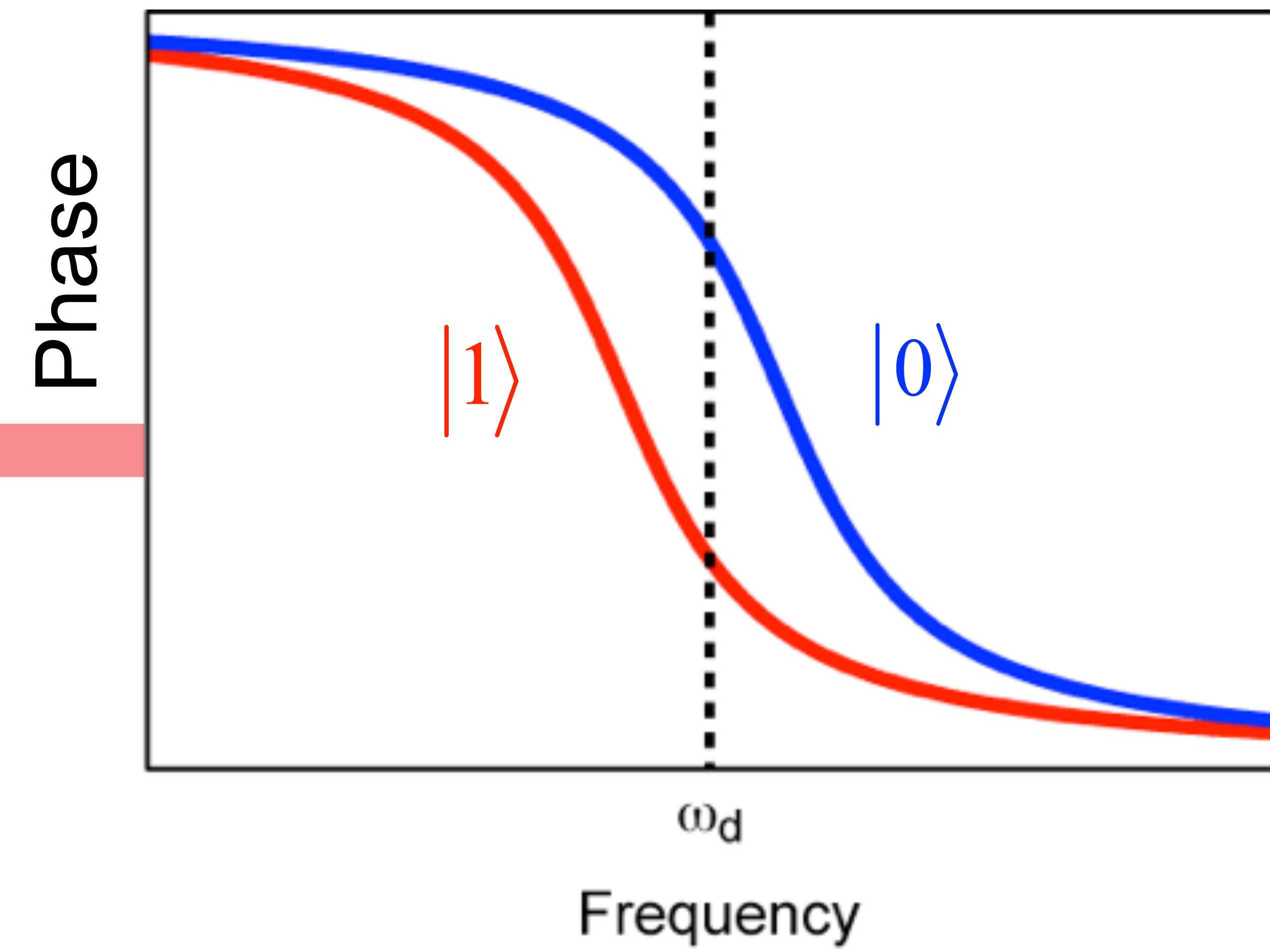


Vijay et al 2012, Murch 2013, Hatridge 2013

State dependent cavity shift



$$H = -\chi a^\dagger a \sigma_z$$



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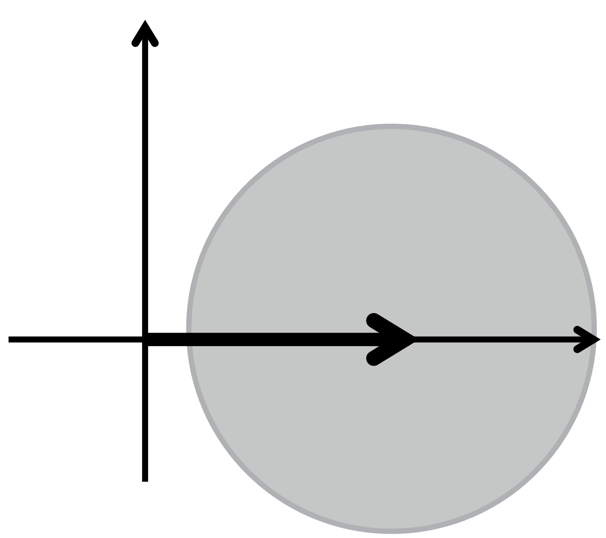
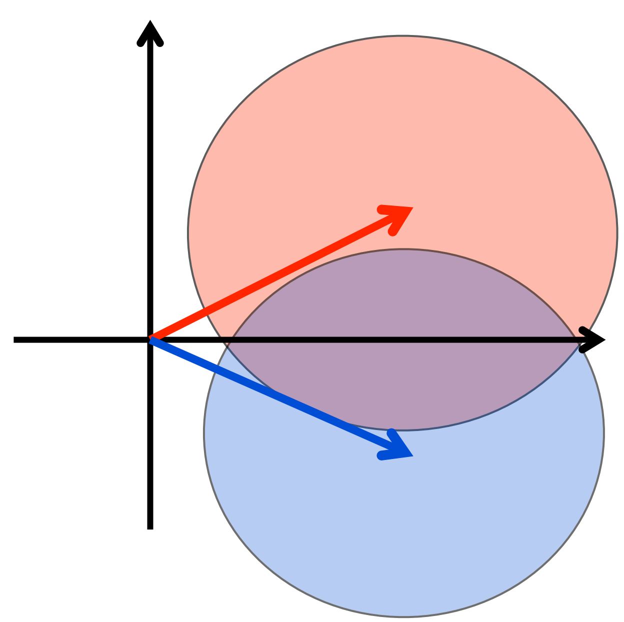
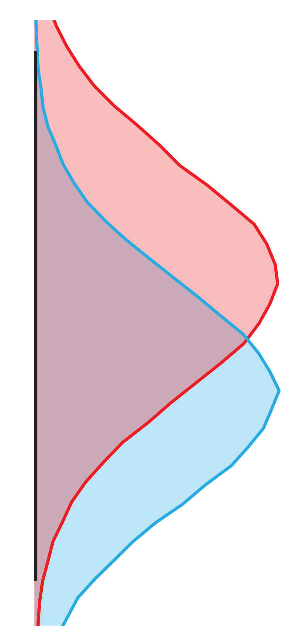


State dependent phase shift



Use amplifier to measure phase

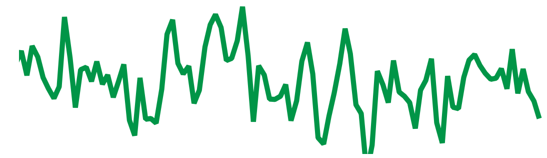


Vijay et al 2012, Murch 2013, Hatridge 2013

Partial and projective measurements

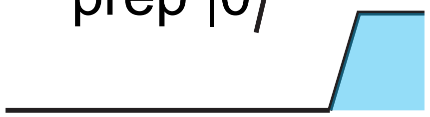
Partial and projective measurements

Measure
quadrature

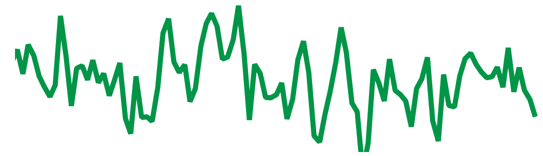


Partial and projective measurements

prep $|0\rangle$

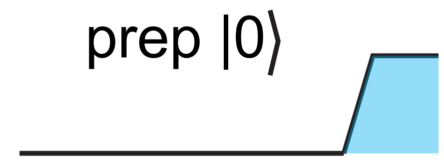


Measure quadrature

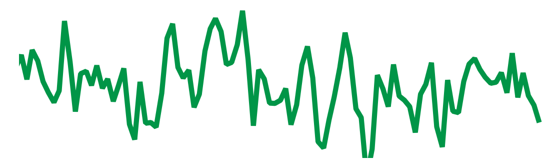


Average value V_m

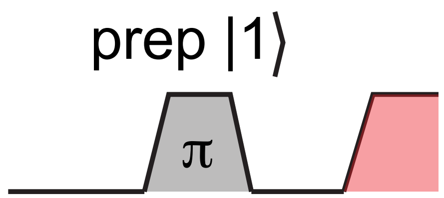
Partial and projective measurements



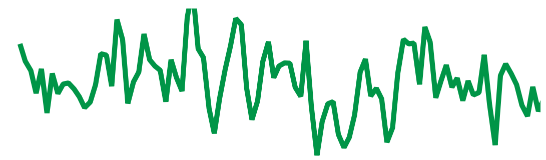
Measure quadrature



Average value
V_m



Measure quadrature

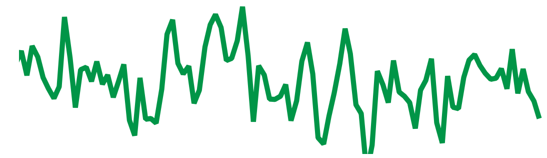


Average value
V_m

Partial and projective measurements

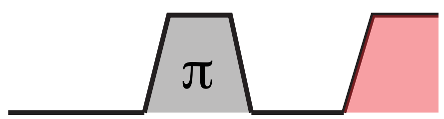
prep $|0\rangle$

Measure quadrature

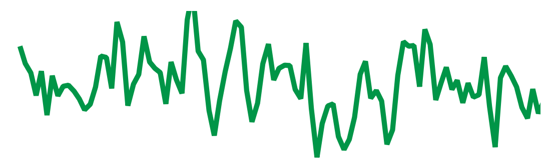


Average value V_m

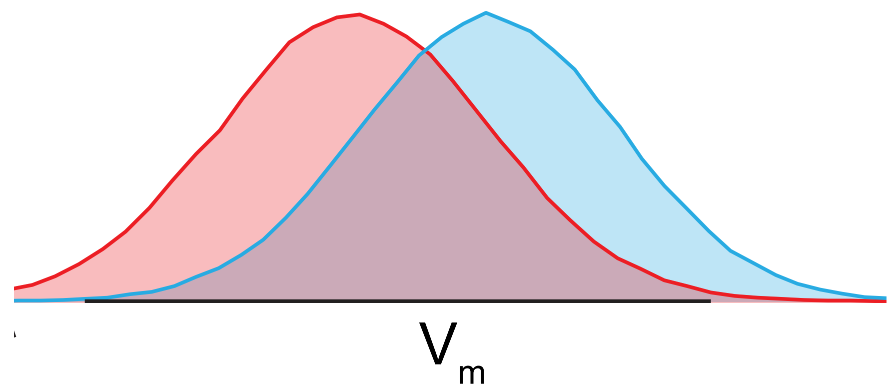
prep $|1\rangle$



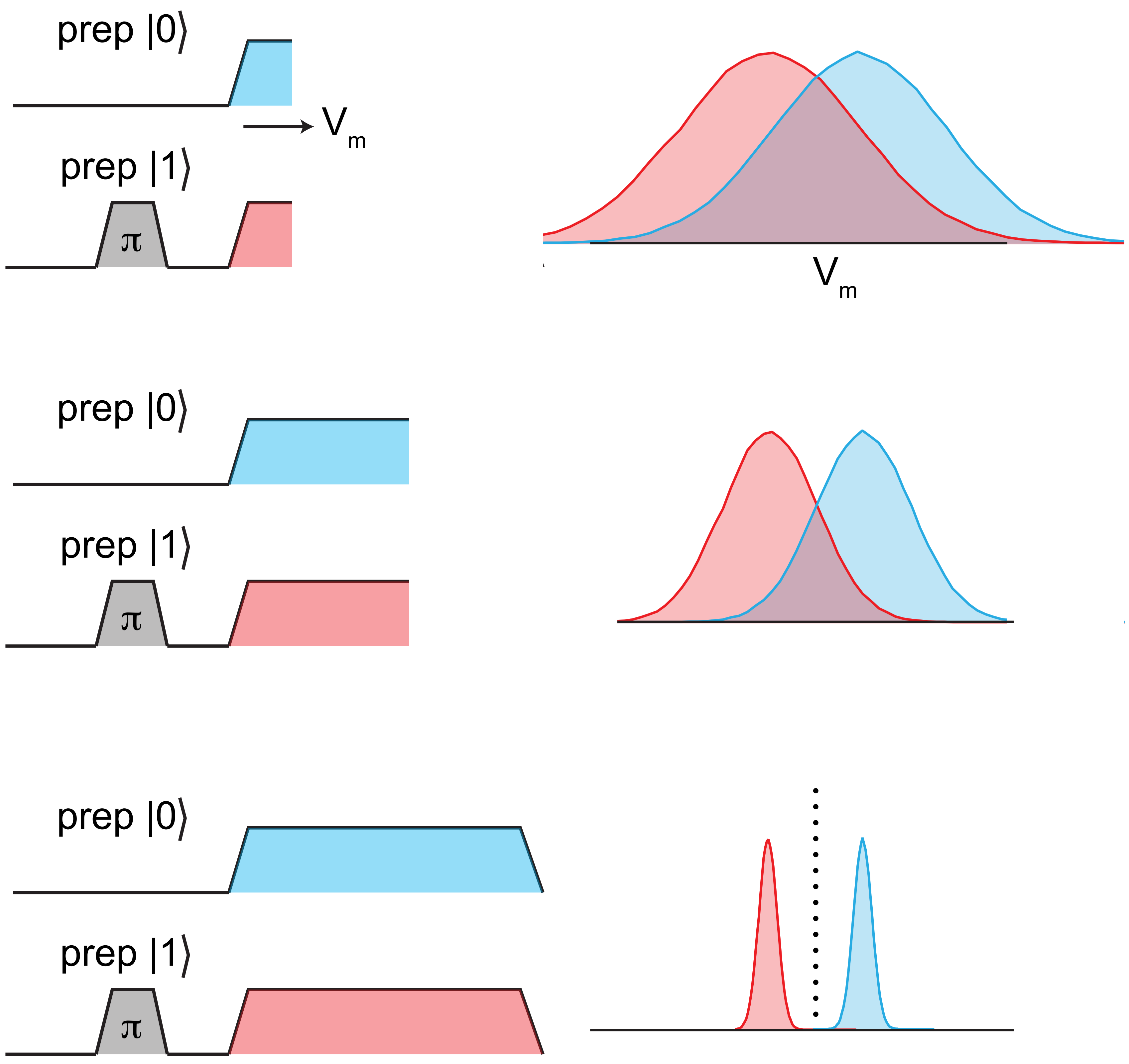
Measure quadrature



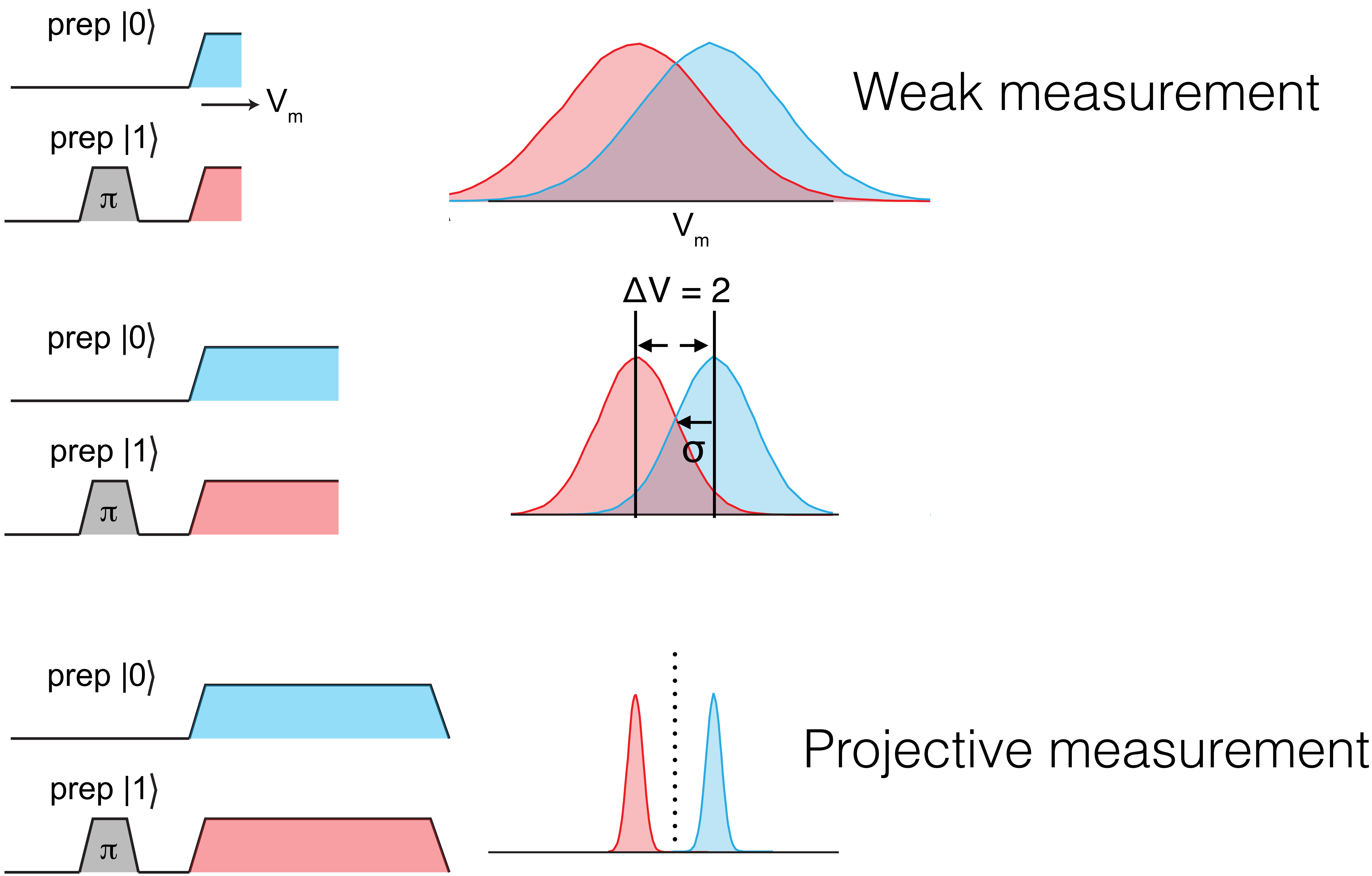
Average value V_m



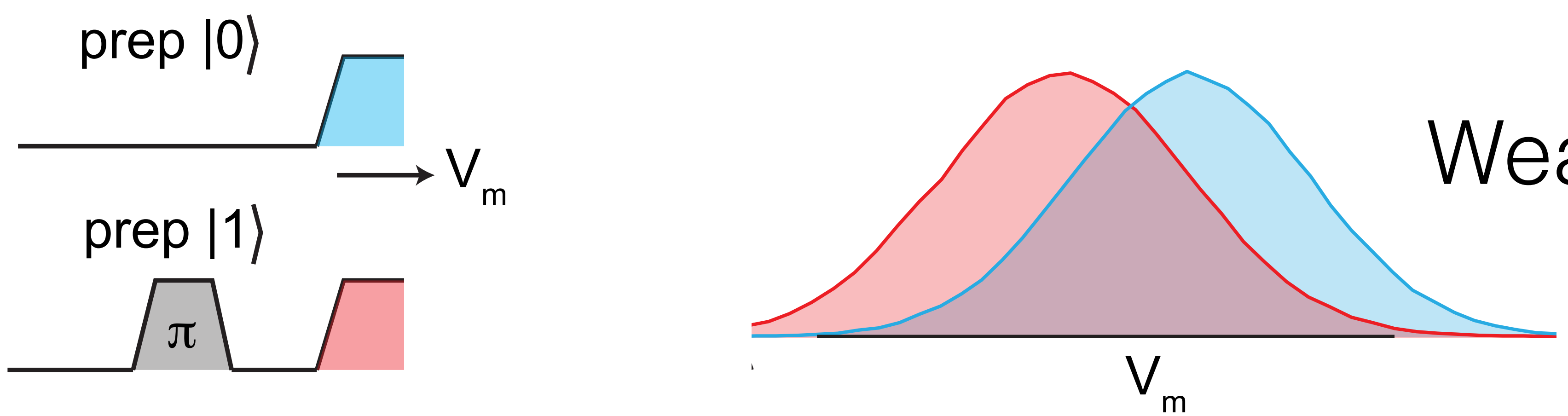
Partial and projective measurements



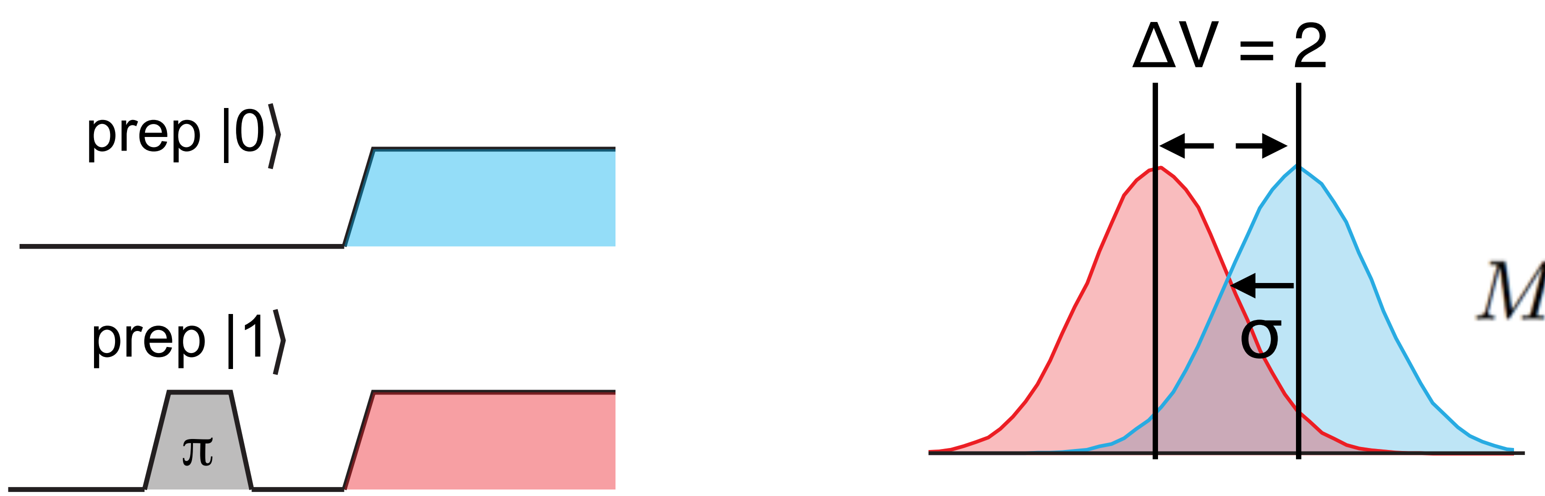
Partial and projective measurements



Partial and projective measurements

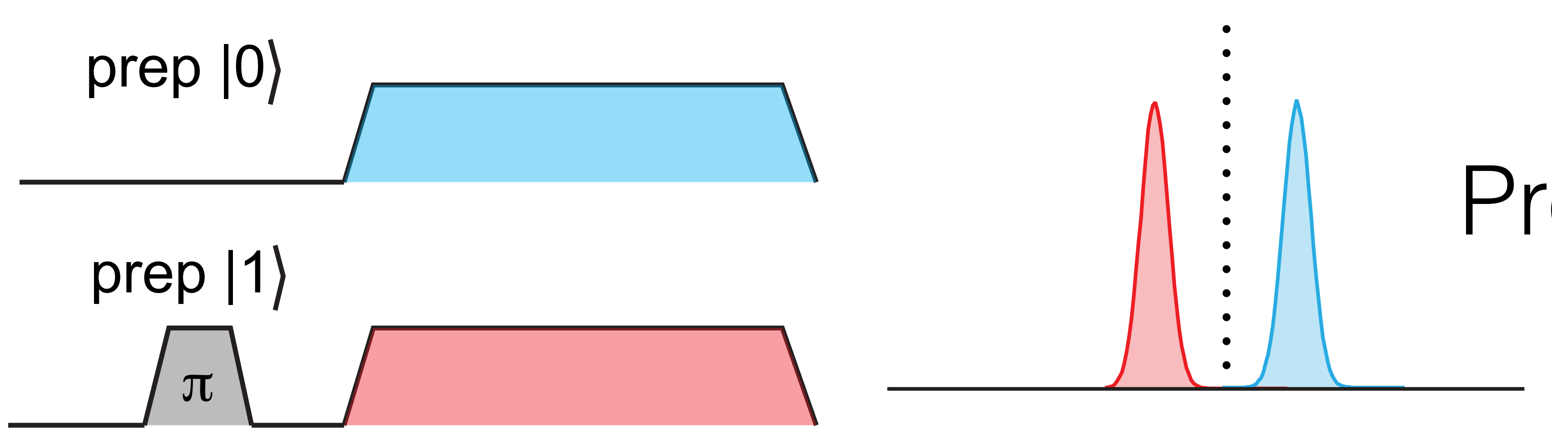


Weak measurement



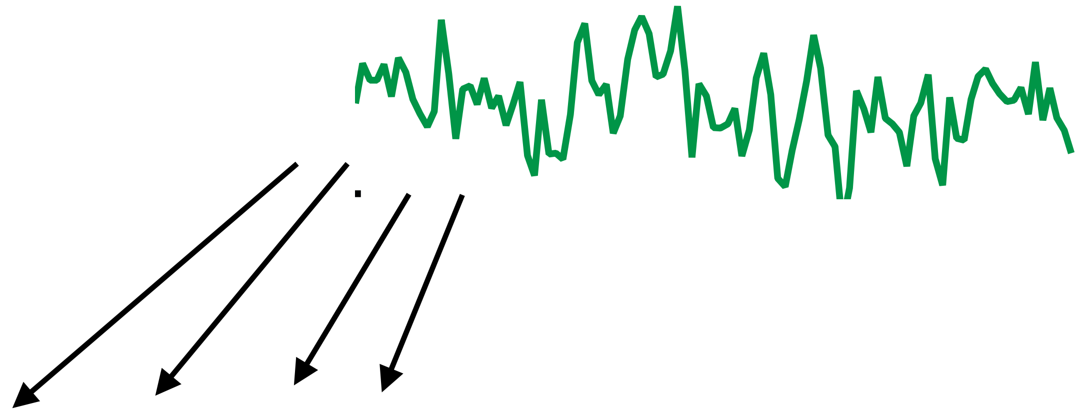
POVM:

$$M_V = (2\pi\sigma^2)^{-1/4} e^{-(V-\sigma_z)^2/4\sigma^2}$$



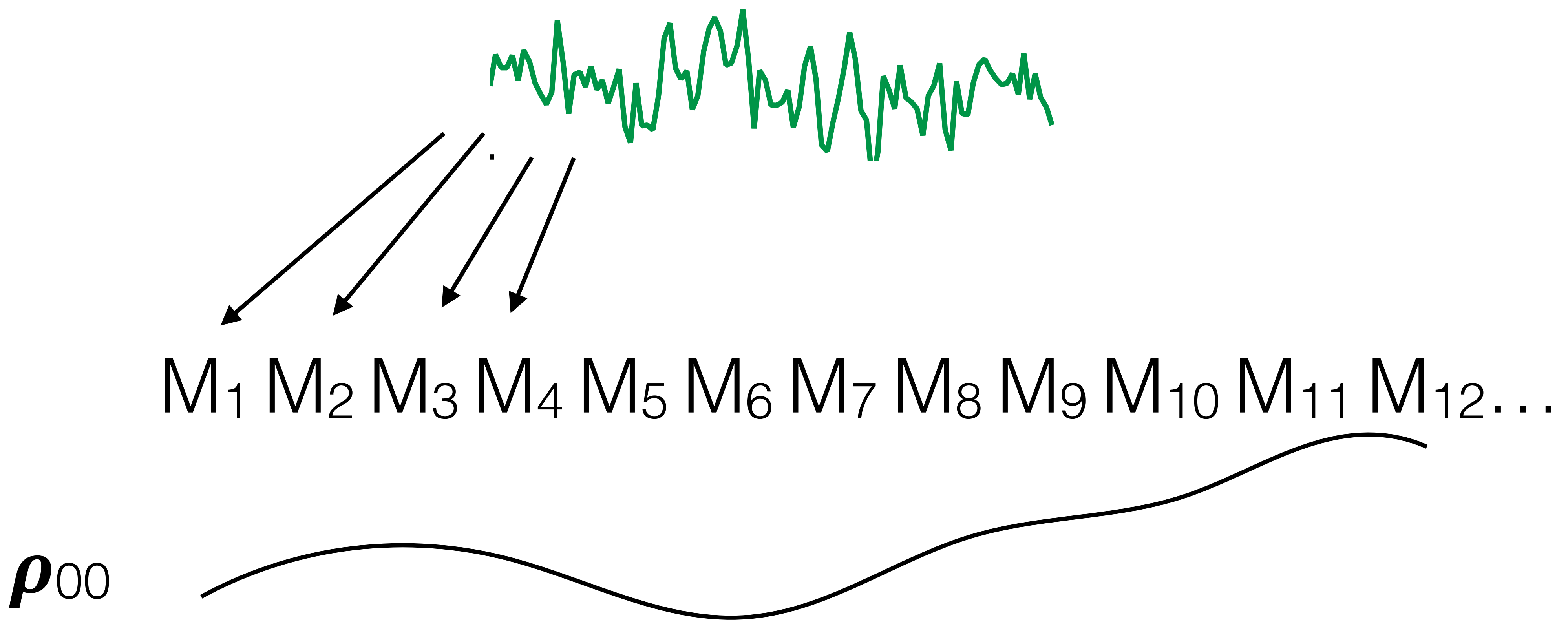
Projective measurement

Quantum trajectories

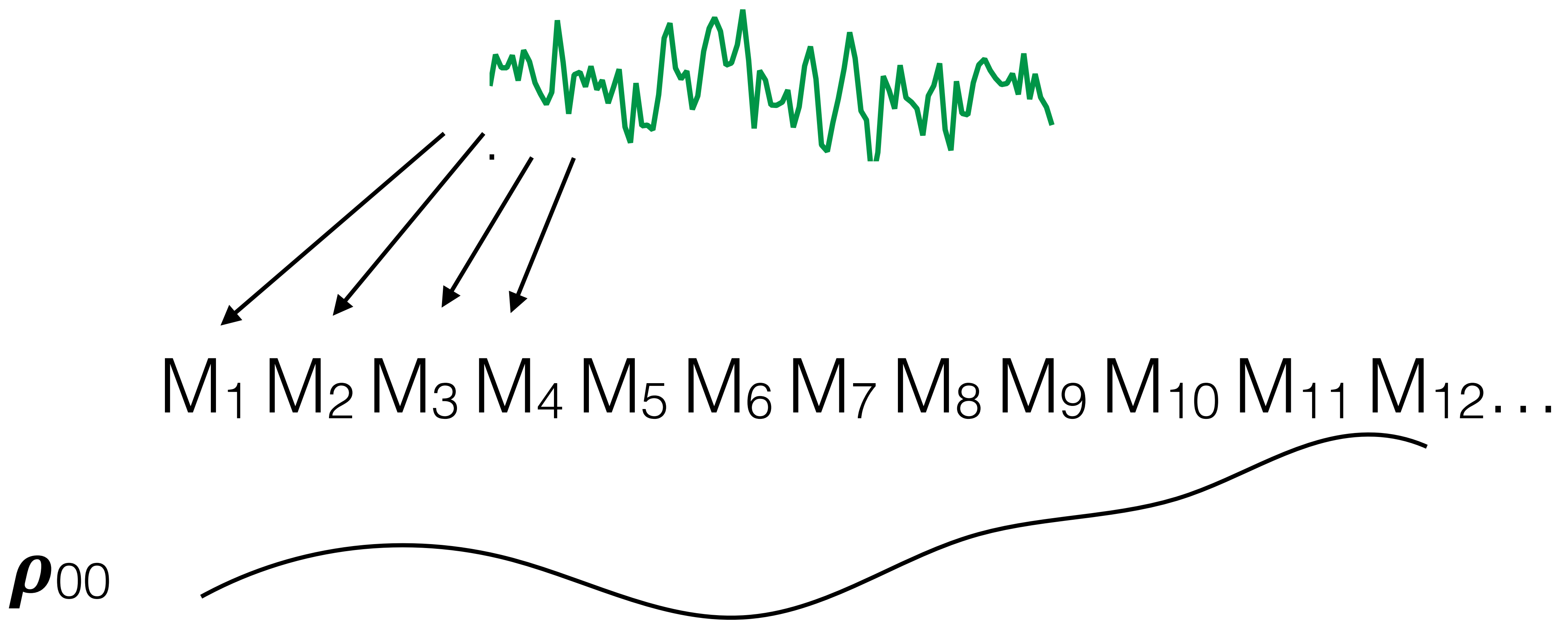


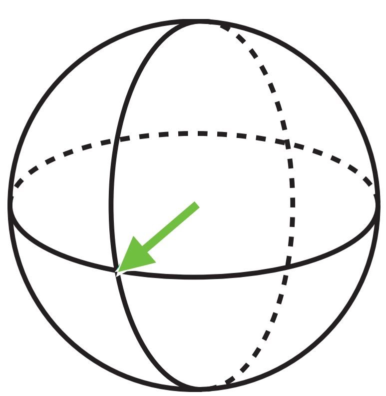
$M_1\ M_2\ M_3\ M_4\ M_5\ M_6\ M_7\ M_8\ M_9\ M_{10}\ M_{11}\ M_{12}\ldots.$

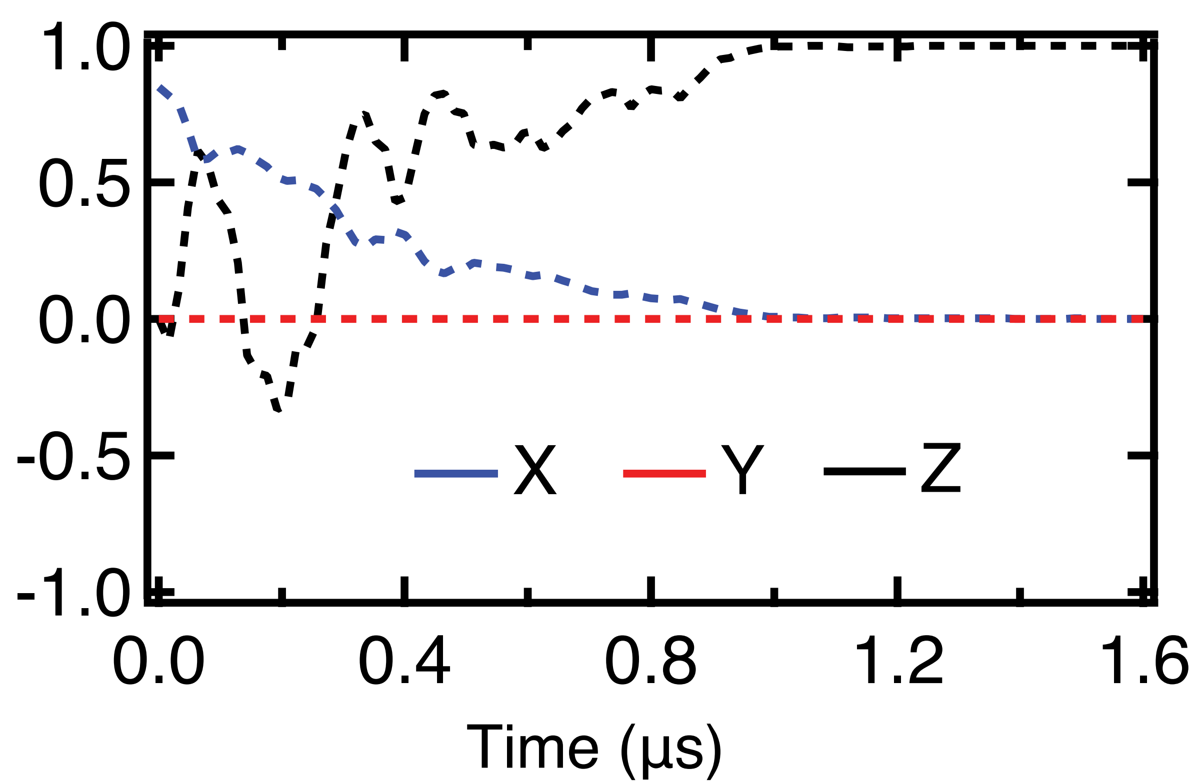
Quantum trajectories



Quantum trajectories



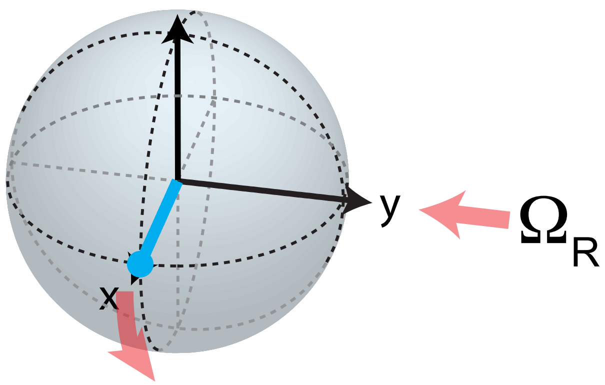




Single quantum trajectory based on one run of the experiment

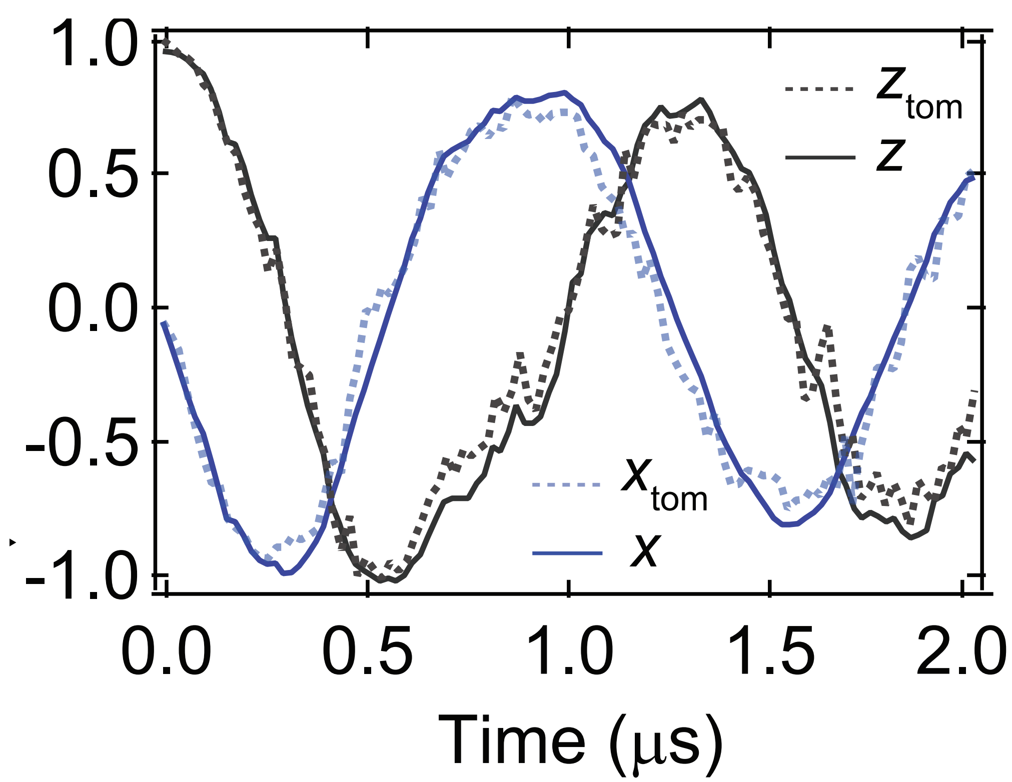
Drive and measurement

Unitary rotation



Stochastic master equation

$$\dot{\rho} = \frac{1}{i\hbar}[H_R, \rho] + k(\sigma_z \rho \sigma_z - \rho) + 2\eta k(\sigma_z \rho + \rho \sigma_z - 2\mathrm{Tr}(\sigma_z \rho)\rho) r(t)$$



Oscillatory trajectories acquire a coherence from drive.

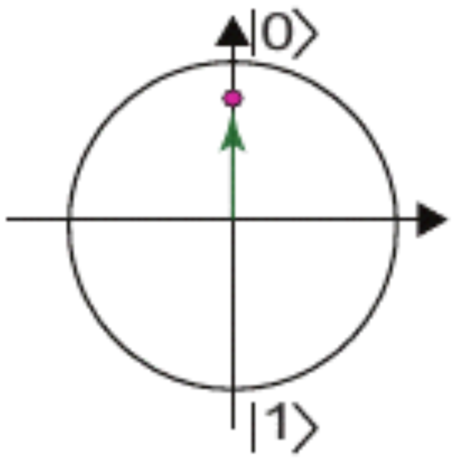
The demon experiment

The demon experiment

The demon experiment

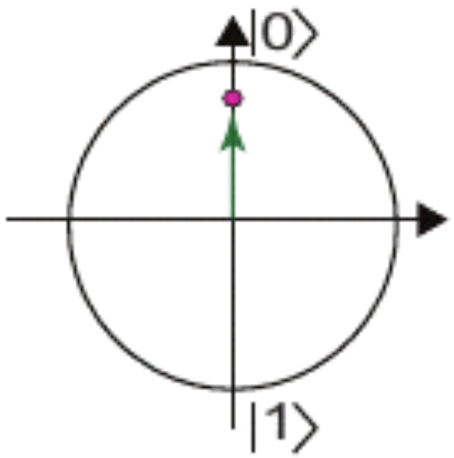
Step 1: initial thermal state

The demon experiment



Step 1: initial thermal state

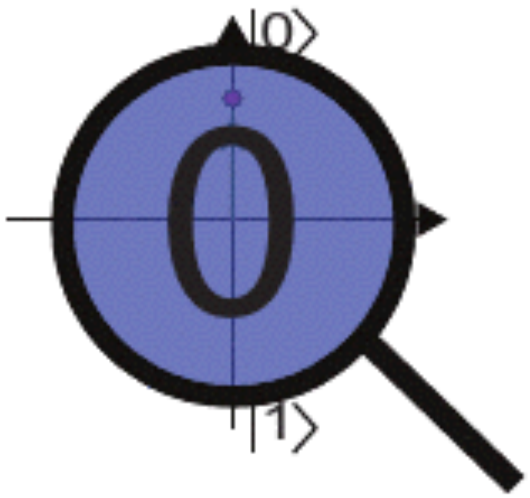
The demon experiment



Step 1: initial thermal state

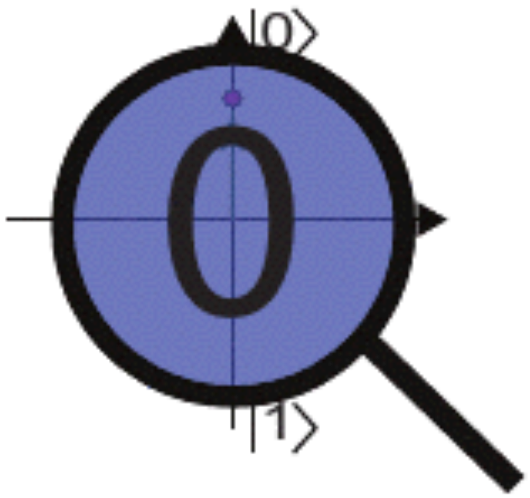
Step 2: first projective measurement

The demon experiment



Step 1: initial thermal state

The demon experiment

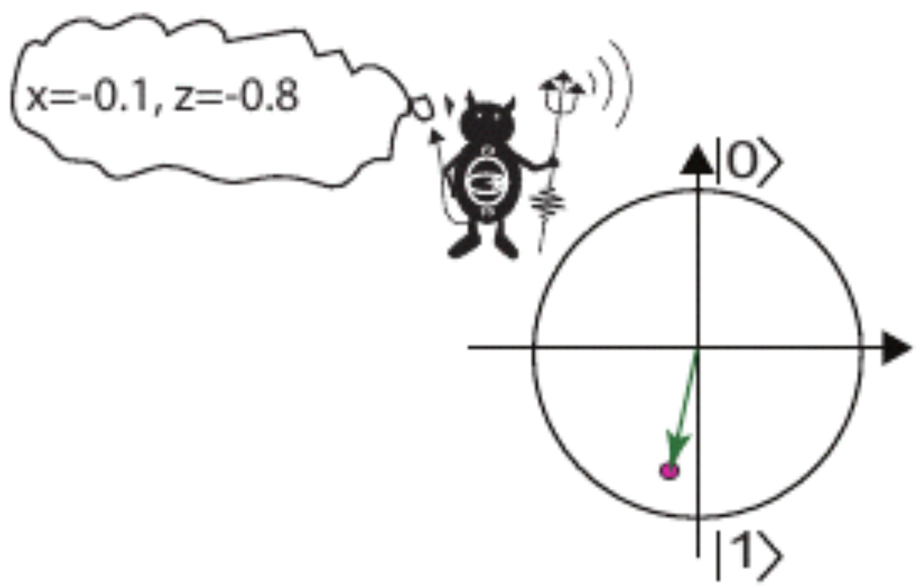


Step 1: initial thermal state

Step 2: first projective measurement

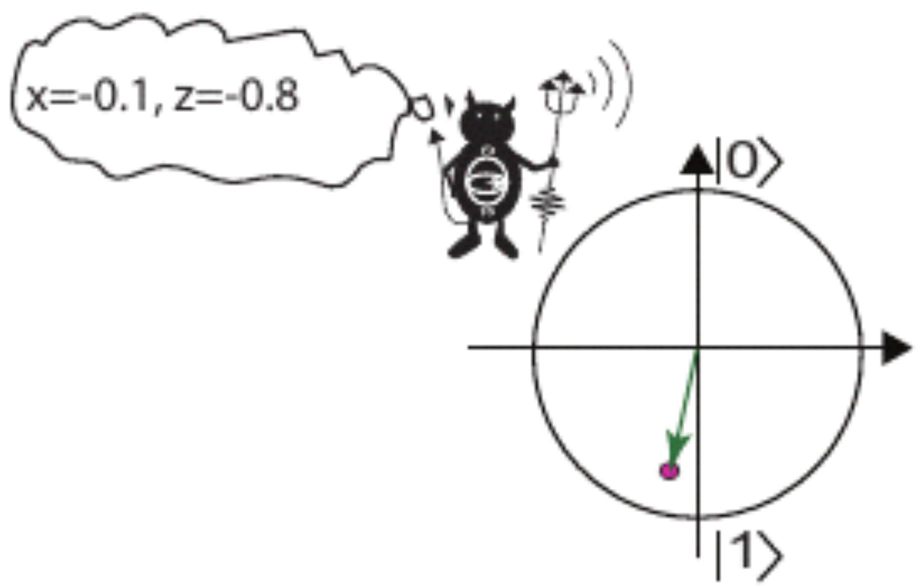
Step 3: demon tracks state

The demon experiment



Step 1: initial thermal state

The demon experiment



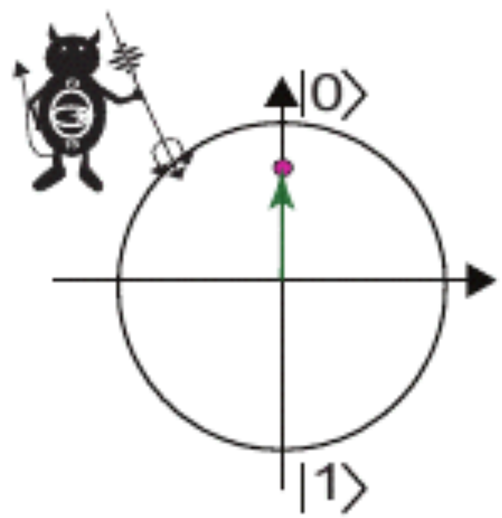
Step 1: initial thermal state

Step 2: first projective measurement

Step 3: demon tracks state

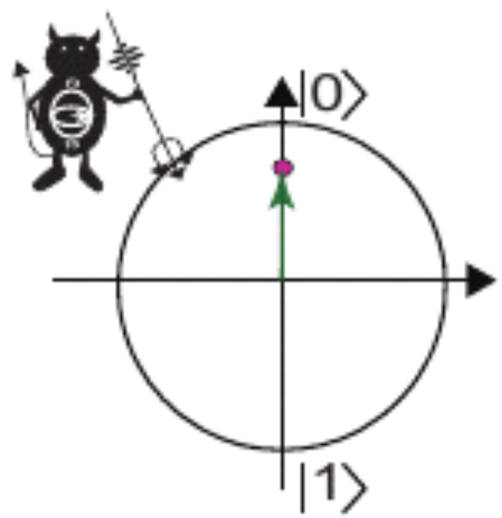
Step 4: demon extracts work

The demon experiment



Step 1: initial thermal state

The demon experiment



Step 1: initial thermal state

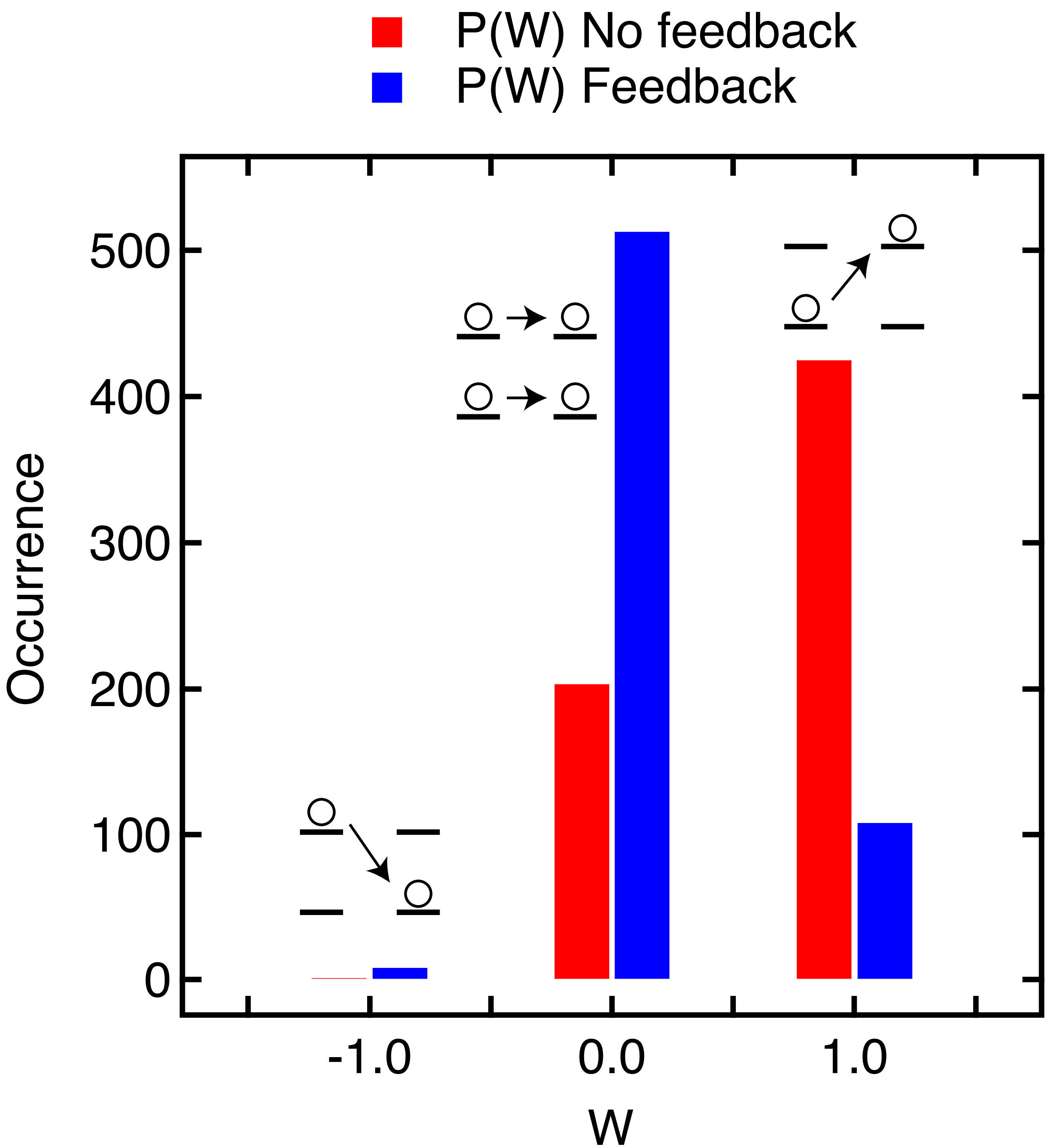
Step 2: first projective measurement (TPM)

Step 3: demon tracks state

Step 4: demon extracts work

Step 5: second projective measurement (TPM)

“Work” distribution

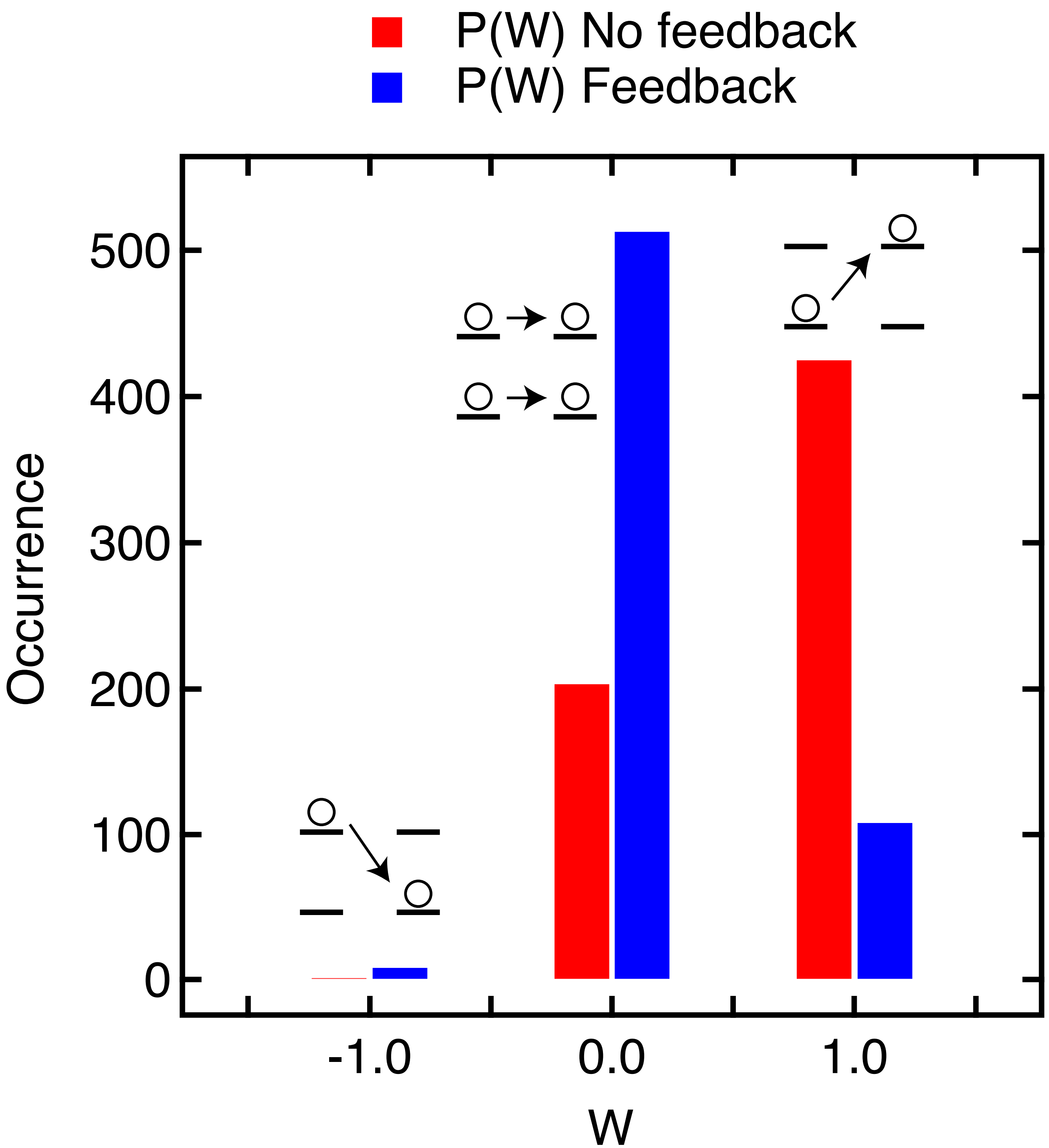


Includes:

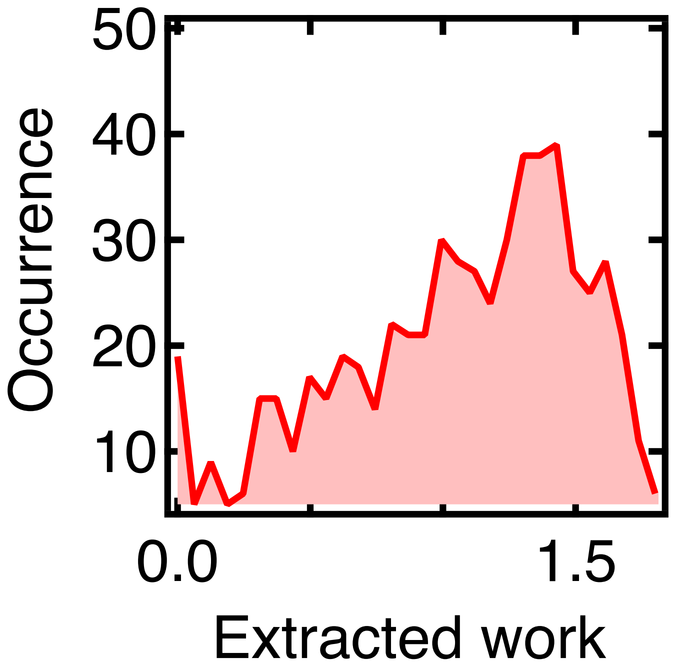
- work due to coherent drive
- quantum heat due to continuous measurement
- work due to feedback rotation
- quantum work due to final projective measurement.

Feedback extracts work compared to no feedback

“Work” distribution

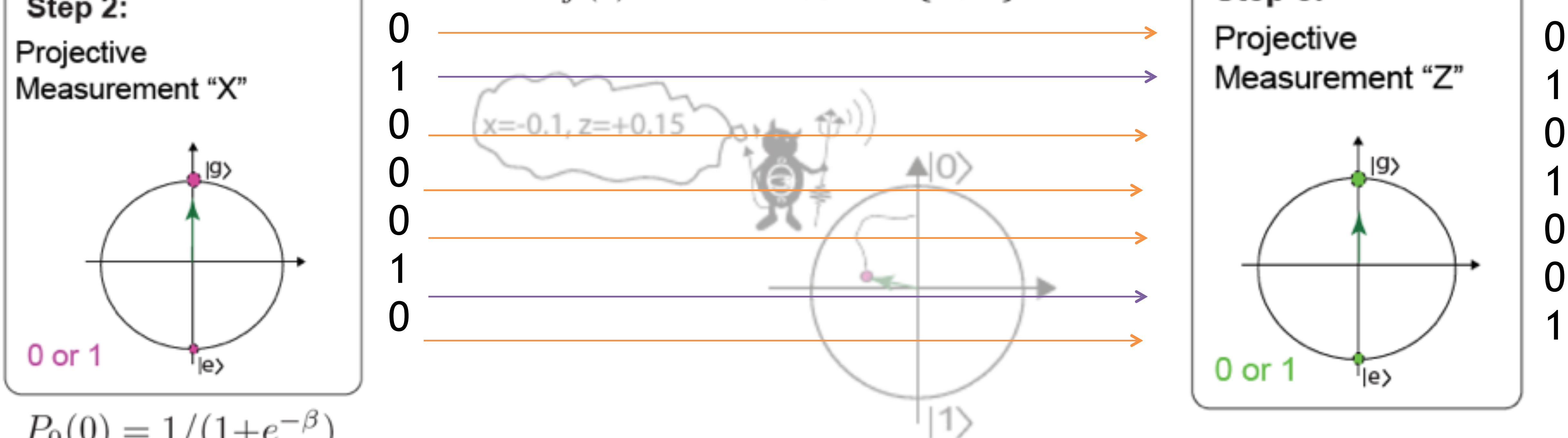


Work extracted in feedback step



Demon violates 2nd Law

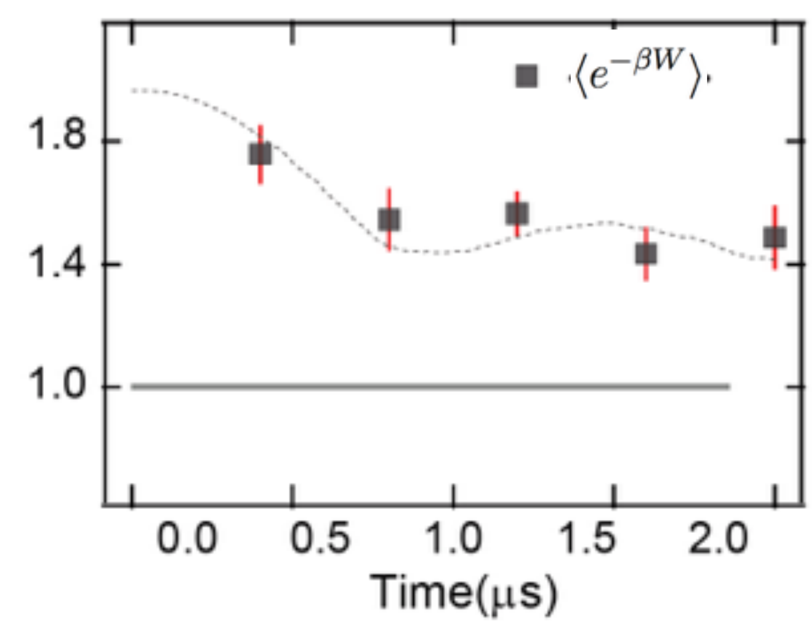
$P_{ij}(t)$ where $i, j = \{0, 1\}$



$$P_0(0) = 1/(1+e^{-\beta})$$
$$P_1(0) = e^{-\beta}/(1+e^{-\beta})$$

Jarzynski equality:

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F} = 1$$



Generalized Jarzynski Equality

$$\langle e^{-\beta W - I} \rangle = e^{-\beta \Delta F} = 1$$

Mutual information: $I(\rho_{t|r}, r) \; = \; \ln P_z \, (\rho_{t|r}) - \ln P_z(\rho_0)$

T. Sagawa and M. Ueda (2008)

Generalized Jarzynski Equality

$$\langle e^{-\beta W - I} \rangle = e^{-\beta \Delta F} = 1$$

Mutual information: $I(\rho_{t|r}, r) \;=\; \ln P_{z'}(\rho_{t|r}) - \ln P_{z}(\rho_0)$

(Information exchange)

T. Sagawa and M. Ueda (2008)

Generalized Jarzynski Equality

$$\langle e^{-\beta W - I} \rangle = e^{-\beta \Delta F} = 1$$

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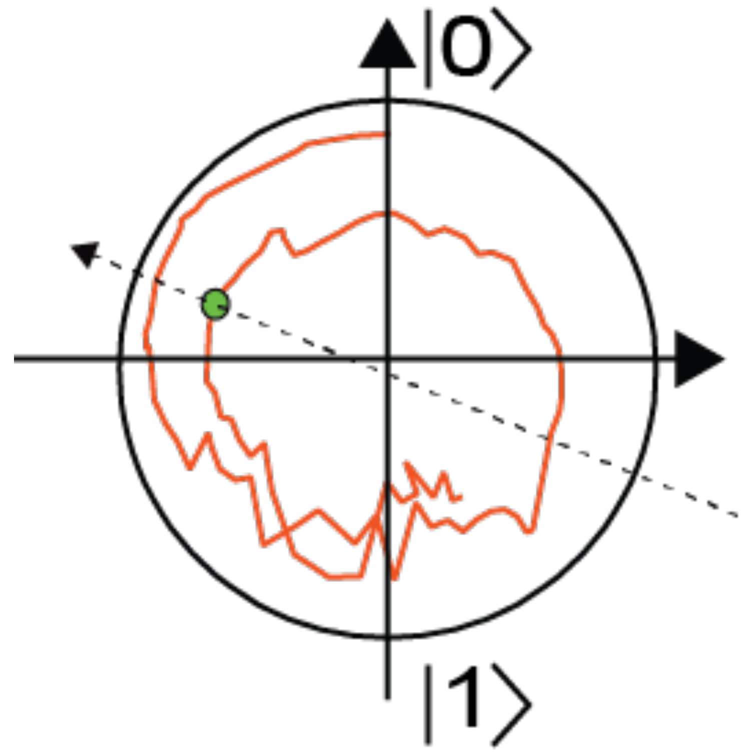
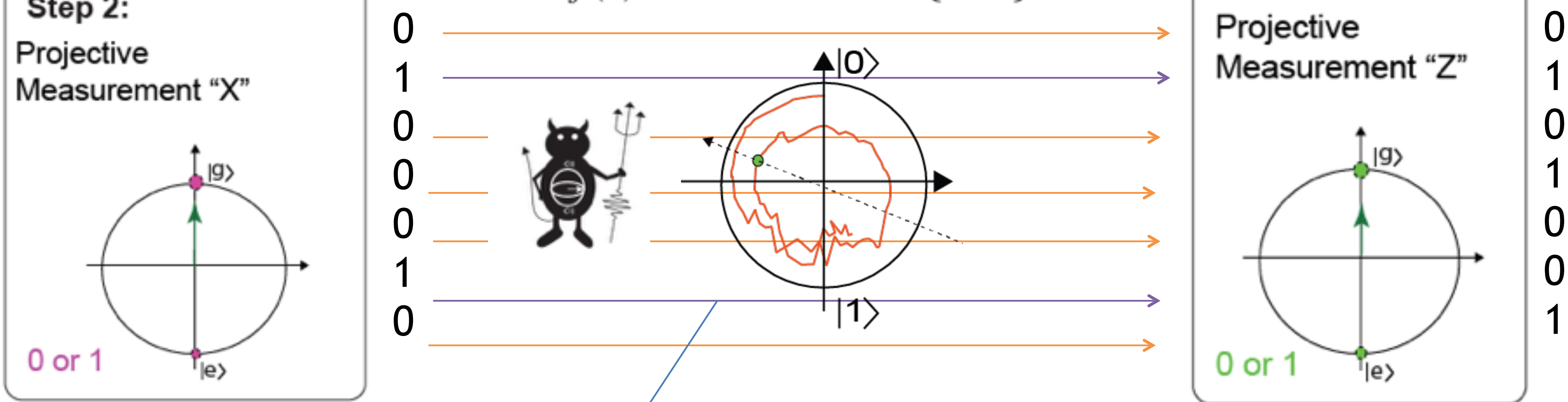
(Information exchange)



T. Sagawa and M. Ueda (2008)

Demon's information saves 2nd Law

$P_{ij}(t)$ where $i, j = \{0, 1\}$

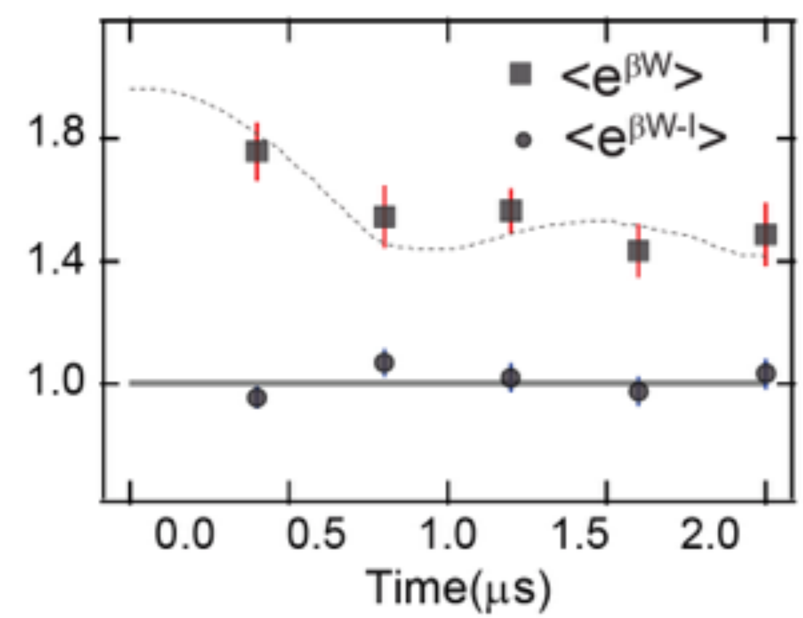


$$P_0(0) = 1/(1+e^{-\beta})$$

$$P_1(0) = e^{-\beta}/(1+e^{-\beta})$$

$$I(\rho_{t|r}, r) \;=\; \ln P_{z'}(\rho_{t|r}) - \ln P_z(\rho_0)$$

$$\langle e^{-\beta W - I}\rangle \;=\; P_0(0)P_{00}(\tau)e^{-I_{00}} + P_1(0)P_{11}(\tau)e^{-I_{11}} + P_0(0)P_{10}(\tau)e^{+\beta - I_{10}} + P_1(0)P_{01}(\tau)e^{-\beta - I_{01}}$$



Information dynamics along a single trajectory

Information exchange

$$I(\rho_{t|r}, r) \;=\; \ln P_{z'}(\rho_{t|r}) - \ln P_z(\rho_0)$$

- depends on z, z’ probabilities
- depends individual trajectories

K. Funo, Y. Watanabe, and M. Ueda, (2013).

Information dynamics along a single trajectory

Information exchange

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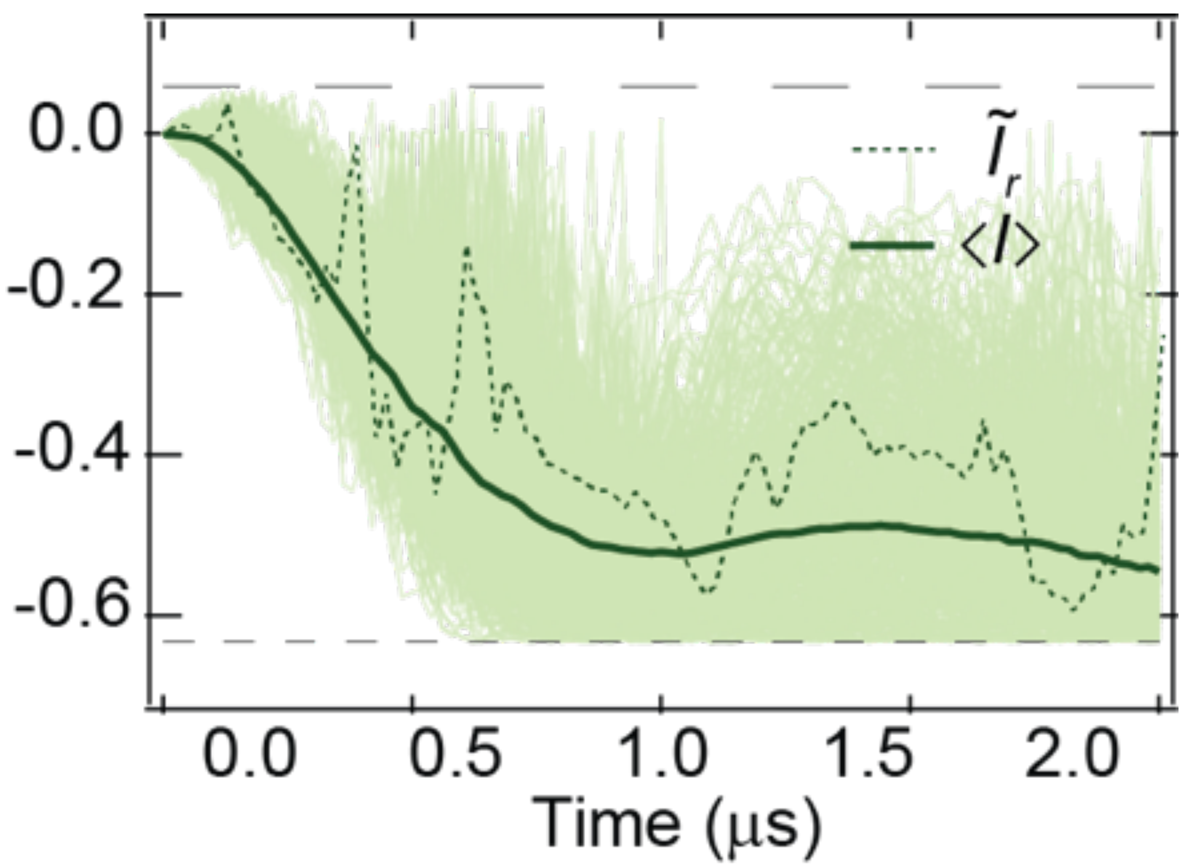

K. Funo, Y. Watanabe, and M. Ueda, (2013).

Information dynamics along a single trajectory

Information exchange

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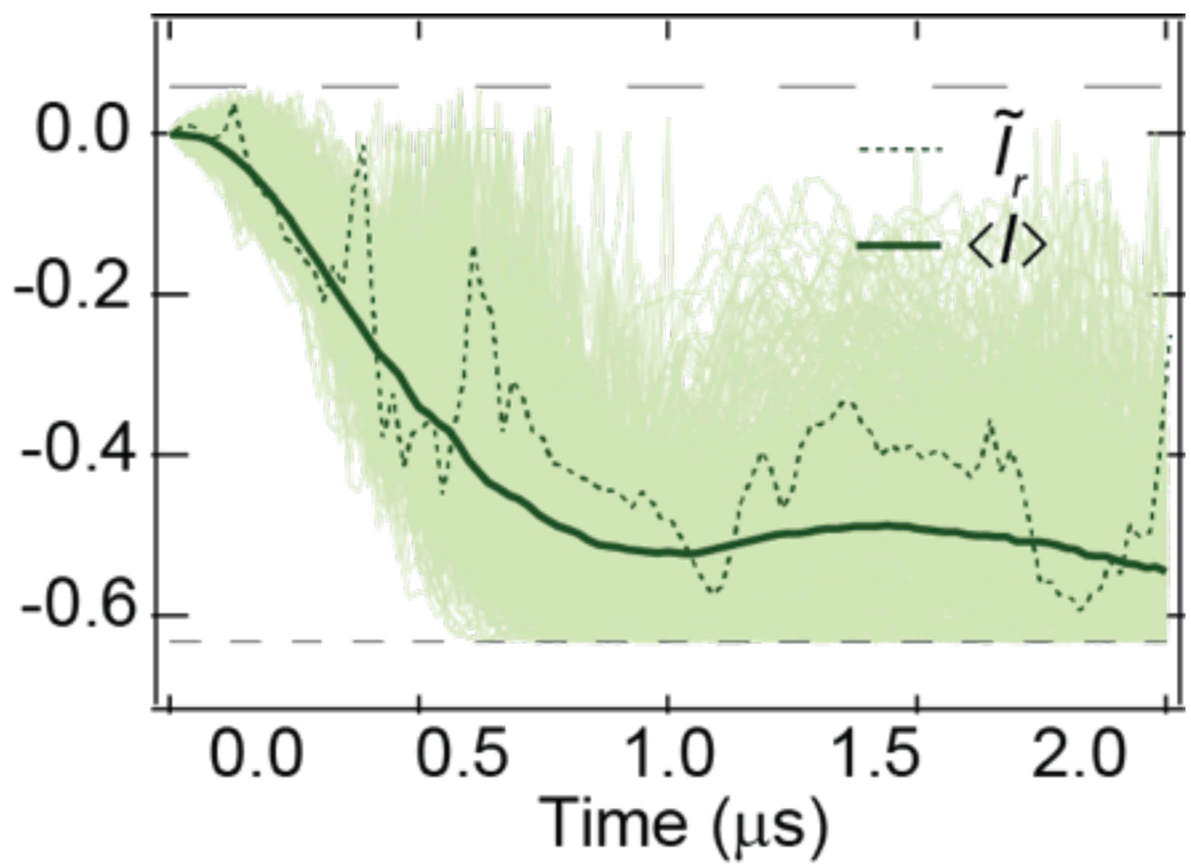
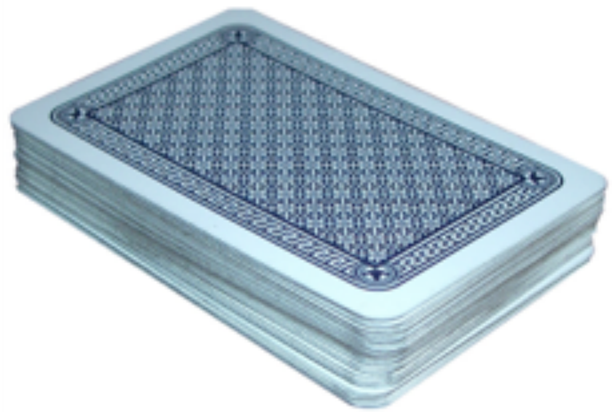


Classical mutual information is always positive

K. Funo, Y. Watanabe, and M. Ueda, (2013).

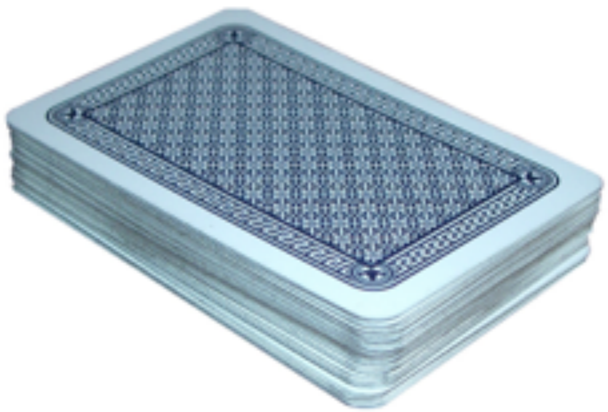
Information gain and loss



Classical case:

P(bottom card = 2♣) = 1/52

Information gain and loss



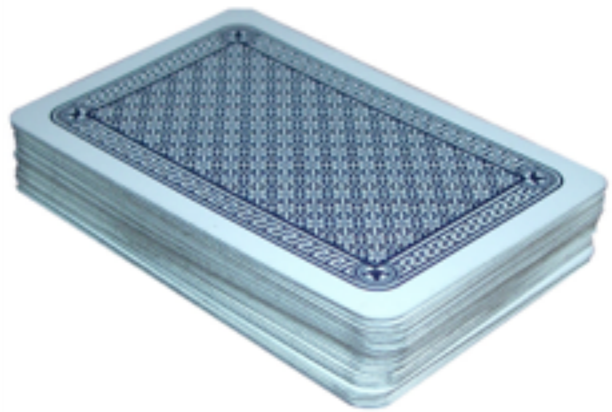
Classical case:

P(bottom card = 2♣) = 1/52

Look at the top card
P(bottom card = 2♣) = 1/51

Information gain and loss

Classical case:

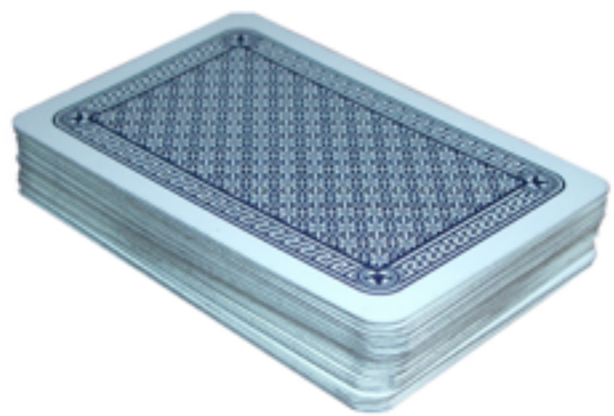


P(bottom card = 2♣) = 1/52

Look at the top card
P(bottom card = 2♣) = 1/51

Someone else looks at a card…
(it doesn't matter)

Information gain and loss



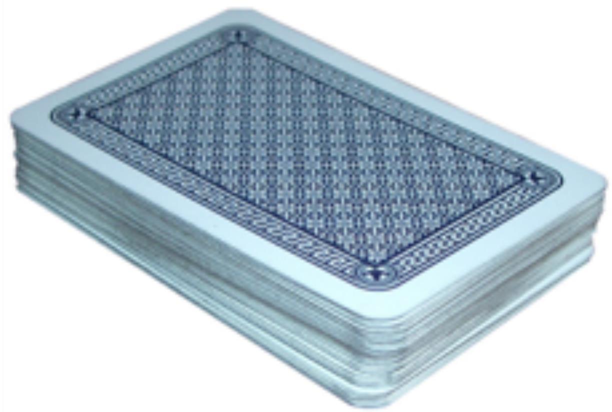
Classical case (stacked deck):

P(bottom card = 2♣) = 1

Look at the top card
P(bottom card = 2♣) = 1

Someone else looks at a card…
(it doesn't matter)

Information gain and loss



Quantum case (stacked deck):

Bottom card =
$(1/\sqrt{52})\,(2♣ + 3♣ + 4♣ + \ldots)$

Someone else looks at the card…

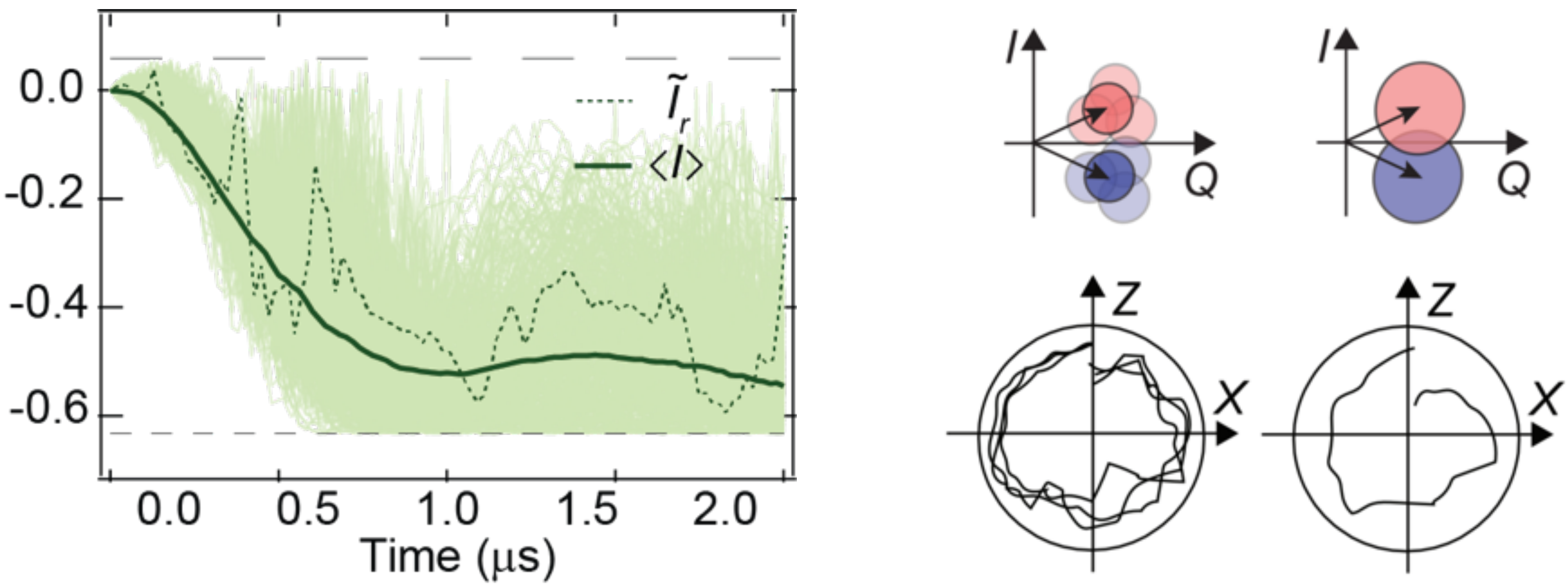
It changes the bottom card!

Information dynamics along a single trajectory

Information exchange

$$I(\rho_{t|r}, r) = \ln P_{z'}(\rho_{t|r}) - \ln P_z(\rho_0)$$

- depends on z, z' probabilities
- depends individual trajectories

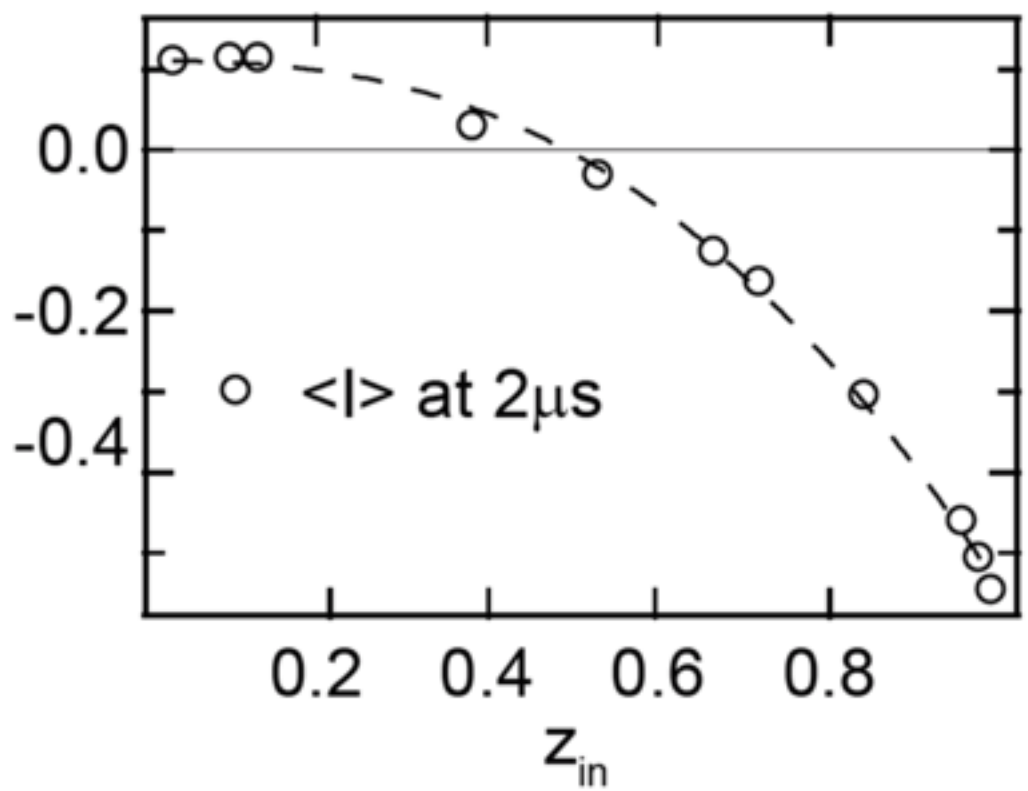


$$I_{\text{gain}} = S(\rho_0) - \sum_a p(a,r) S(\rho_{t|r,a})$$

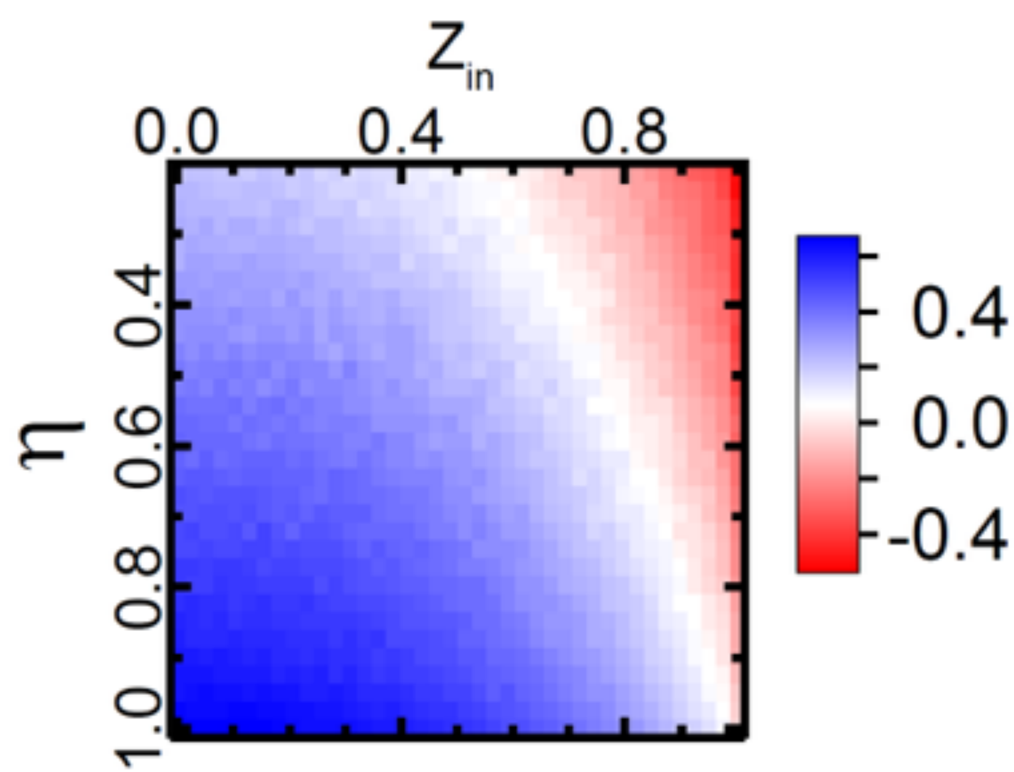
$$I_{\text{loss}} = \sum_r S(\rho_{t|r}) - \sum_a p(a,r) S(\rho_{t|r,a})$$

K. Funo, Y. Watanabe, and M. Ueda, (2013).

Information gain and loss



Vary the initial purity (effective temperature)

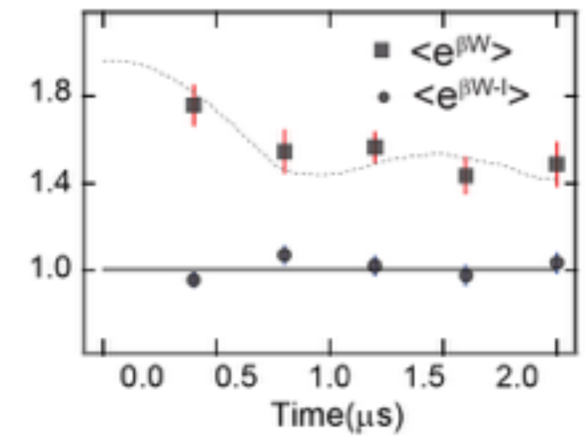


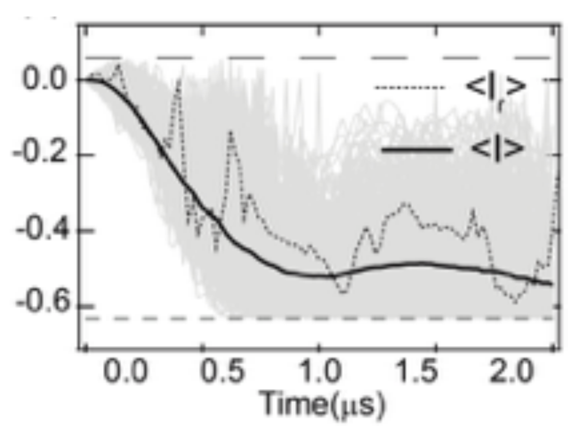
Good agreement with simulation for η=0.4

K. Funo, Y. Watanabe, and M. Ueda, (2013).

Summary

Generalized quantum fluctuation theorem in the weak measurement regime.





Information dynamics along a single quantum trajectories

Gain to loss transition