Non-Perturbative Effects in G_N and the Information Paradox

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based on work with
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What Paradoxes?

Qualitative disagreement between AdS gravitational field theory/perturbative string theory and exact, unitary Conformal Field Theory.

Two classes:

Easier = unambiguous discrepancies with CFT

Hard = potentially ambiguous questions about AdS observables

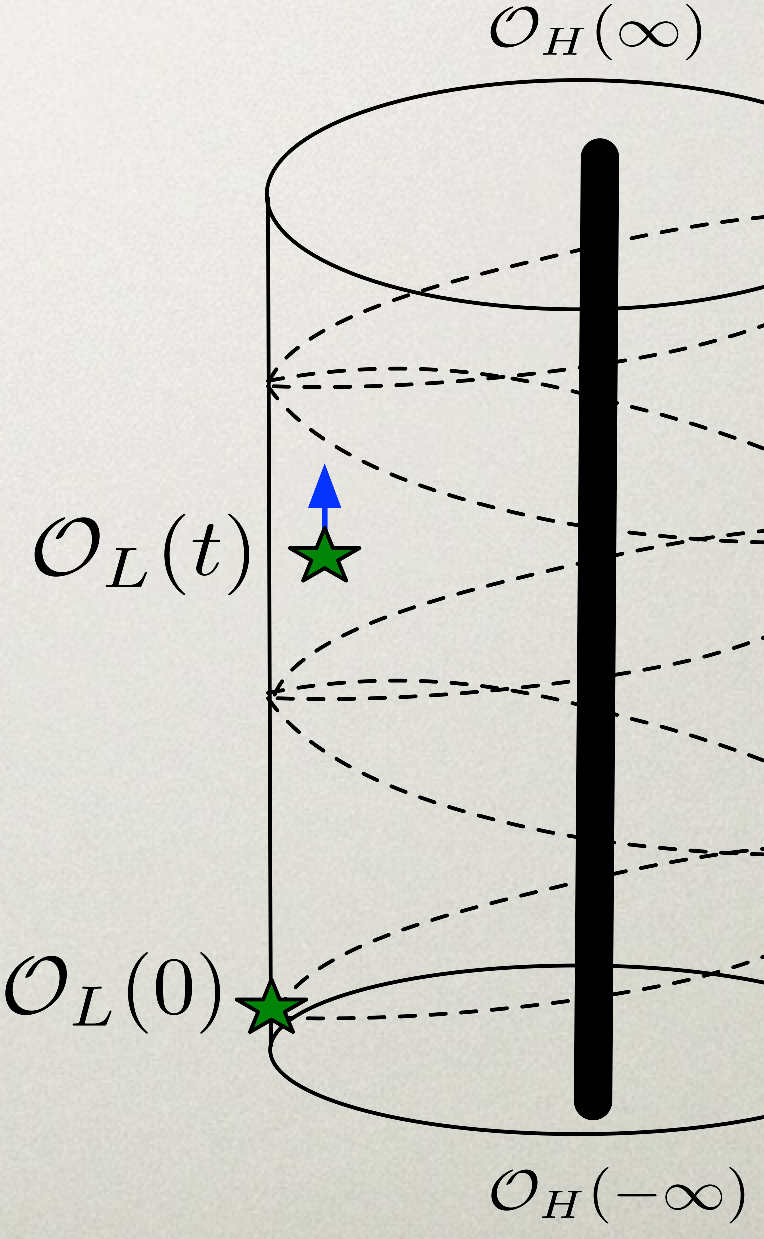
Unambiguous Disagreement: Late Time Correlations

Late time behavior of correlation functions in an AdS black hole background.

{Maldacena]

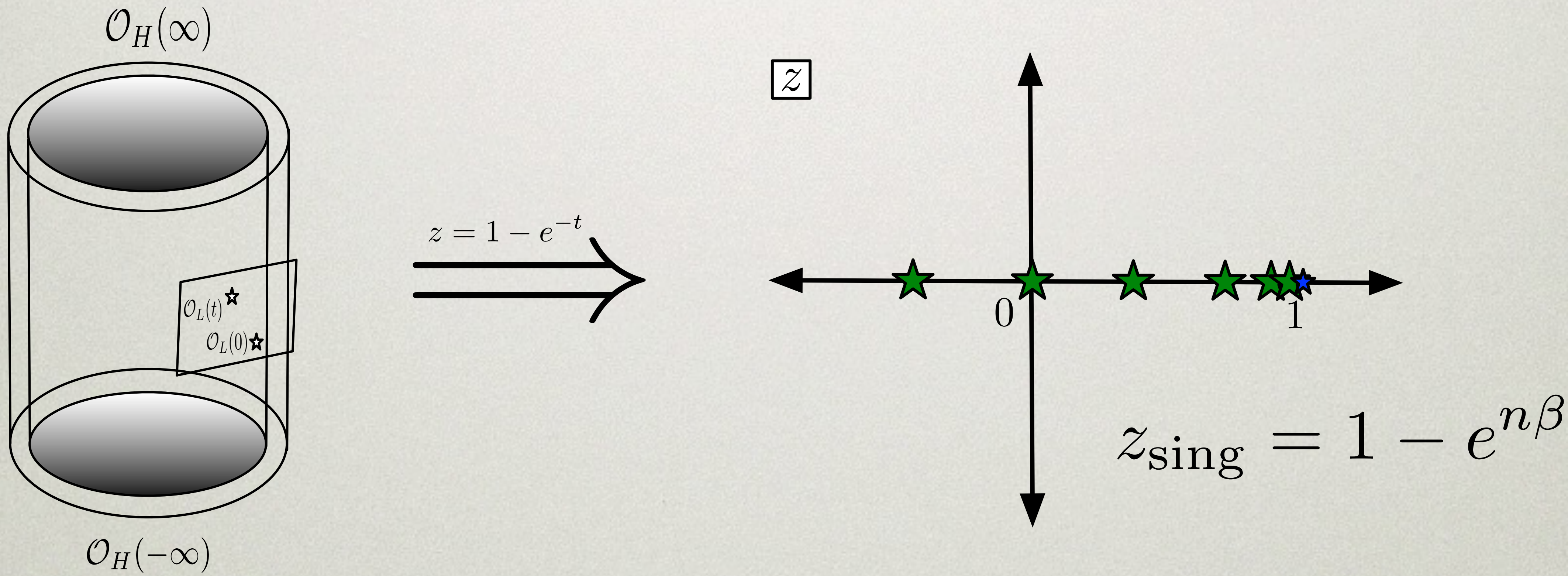
Does $\langle \mathcal{O}_L(t)\mathcal{O}_L(0)\rangle_{BH}$ decay forever?

Is the spectrum discrete?



Unambiguous Disagreement: Forbidden Singularities

Forbidden Singularities due to Euclidean-time periodicity (KMS) in **pure state** black holes:



How well do high-energy **pure states** **mimic** the canonical ensemble?

"Hard" = Ambiguous (?) Problems

→

I. How ambiguous is bulk reconstruction? When/where/why/to what extent?

II. What do observers see near and across black hole horizons?

Our Approach

1) Identify the approximation within the CFT that agrees with the perturbative bulk description and produces (information loss) problems.

2) From the vantage point of this approximation, identify and compute the non-perturbative effects that resolve information loss problems.

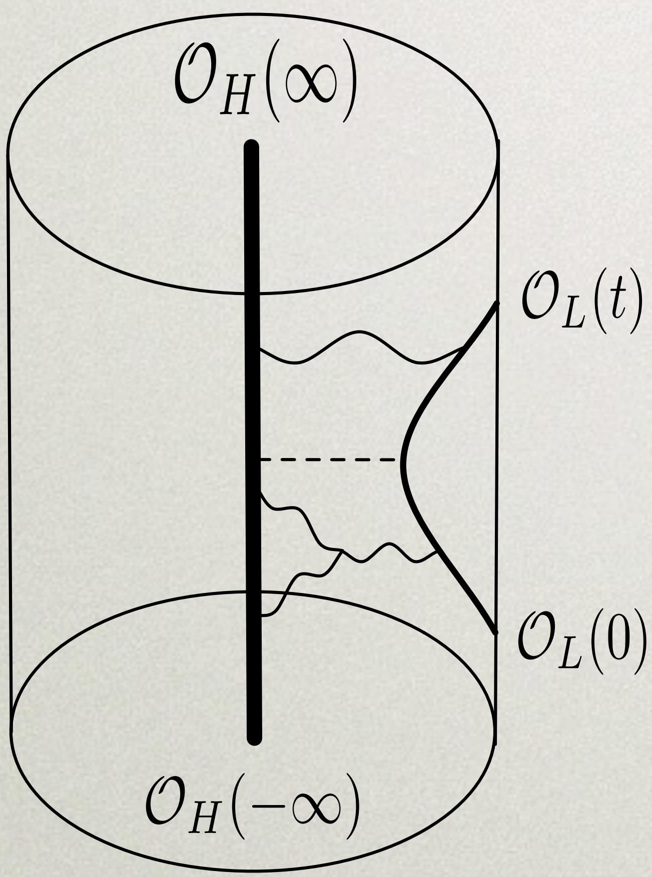
3) Comparing 1 & 2, what are the bulk implications?

In the approximation of (1), expect no ambiguities or firewalls. Both should come from effects in (2).

Old News about Heavy-Light Virasoro Blocks

What Observable (in AdS/CFT)?

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_H(1)\mathcal{O}_L(z)\mathcal{O}_L(0)\rangle$$



Recall:

$$G_N = \frac{3}{2c},\ m_{AdS} \sim h$$

We'll study light probes of heavy pure states.

Always expand in the $\mathcal{O}_L(z)\mathcal{O}_L(0)$ OPE channel.

Building Blocks for Correlators

Natural to **organize amplitudes** into **blocks**,
ie irreducible representations of the **symmetry**.

Flat space with Poincare symmetry, find partial waves.

In d>2 CFT, we have SO(d+1,1) conformal blocks
or conformal partial waves.

Building Blocks for 2d CFT Correlators

Virasoro conformal blocks encapsulate contributions from all states related by Virasoro:

$$\langle \mathcal{O}_1(\infty)\mathcal{O}_2(1)\mathcal{O}_3(z)\mathcal{O}_4(0)\rangle = \sum_{h,\bar{h}} P_{h,\bar{h}} \mathcal{V}_{h_i,h,c}(z)\mathcal{V}_{\bar{h}_i,\bar{h},c}(\bar{z})$$

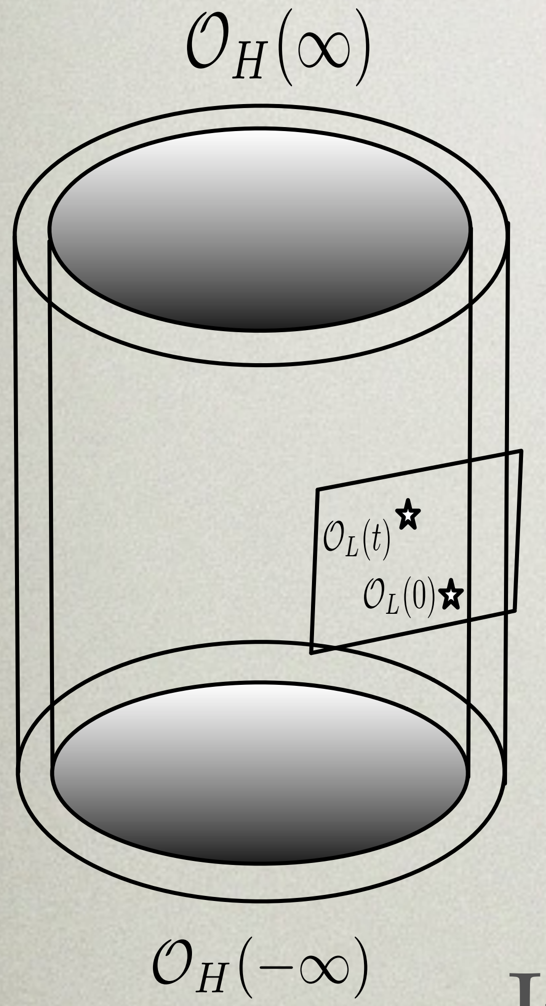
This is interesting because: $g_{\mu\nu}(X) \leftrightarrow T_{\mu\nu}(x)$

and in 2d CFTs: $T(z) = \sum_n z^{-2-n} L_n$

Virasoro blocks know about quantum gravity.

Example: Heavy-Light Vacuum Block as $c \to \infty$.

Semiclassical heavy-light Virasoro vacuum block:



$$\mathcal{V}(t) = \left(\frac{\pi T_H}{\sin(\pi T_H t)} \right)^{2h_L}$$

on the Euclidean cylinder, with

$$T_H = \frac{1}{2\pi}\sqrt{24\frac{h_H}{c} - 1}$$

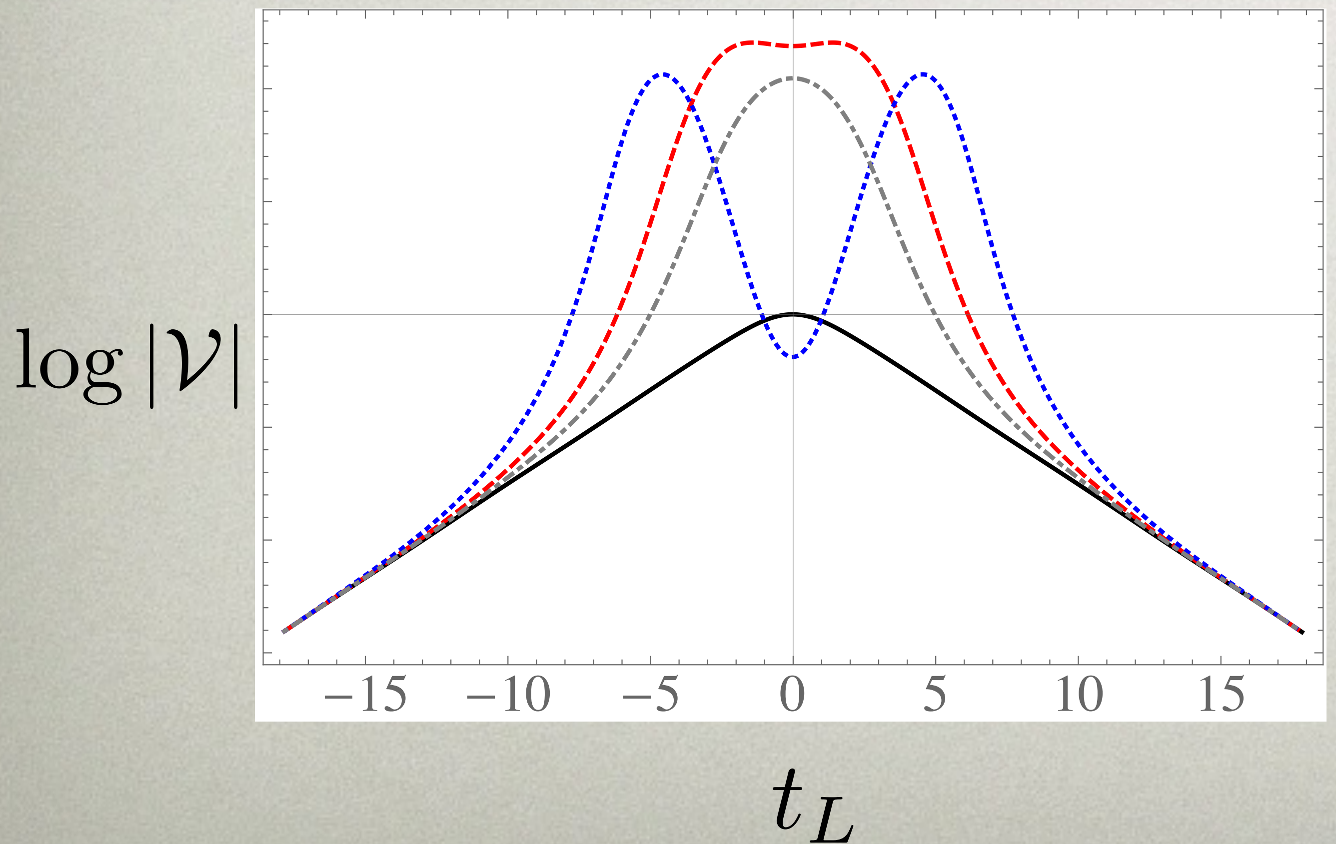
It knows the BTZ black hole temperature!

Let's note two features of this semiclassical result…

Late Time Information Loss from Virasoro Blocks

$$\mathcal{V}(t_L) = \left(\frac{\pi T_H}{\sinh(\pi T_H t_L)} \right)^{2h_L}$$

It decays exponentially at late **Lorentzian** times.



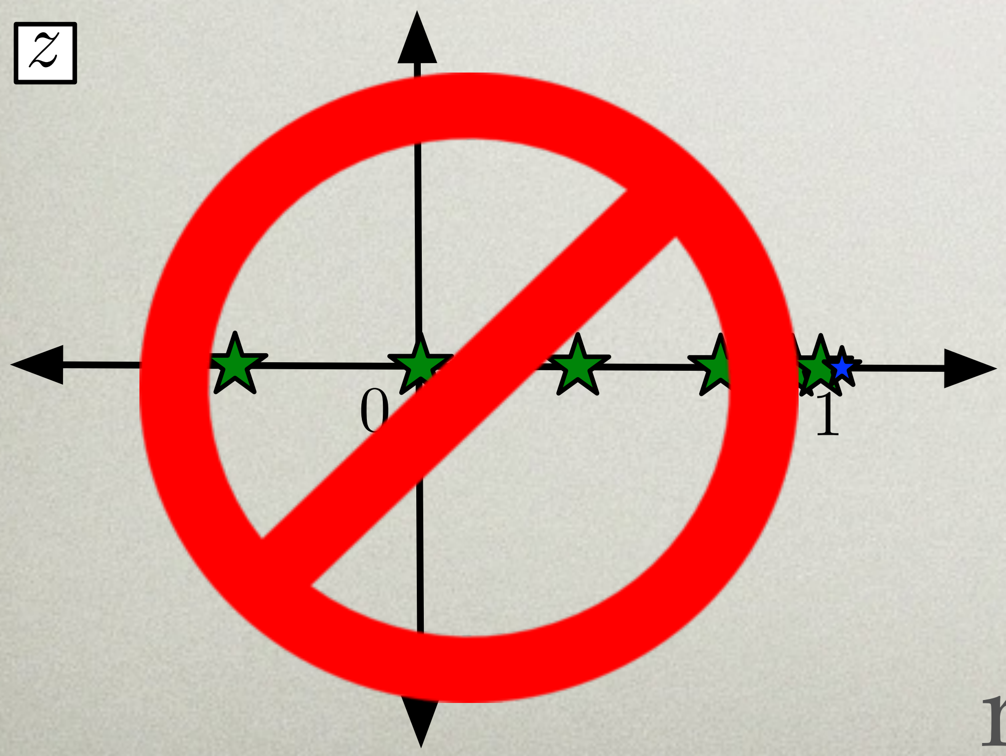
All semiclassical blocks decay at the same exponential rate:

$$\mathcal{V}(t_L) \propto e^{-2\pi h_L T_H t_L}$$

Forbidden Singularities from Virasoro Blocks

$$\mathcal{V}(t) = \left(\frac{\pi T_H}{\sin(\pi T_H t)} \right)^{2h_L}$$

It is **periodic in Euclidean time**, ie satisfies KMS.



This means that it has **forbidden singularities**:

$$z_{\text{sing}} = 1 - e^{n\beta}$$

representing Unitarity violation

Virasoro Blocks Encapsulate Quantum Gravity and Info Loss

- BTZ Black Hole Thermodynamics from Blocks
- Information loss comes from the blocks, and occurs block-by-block, largely independent of CFT data (ie spectrum and OPE coefficients)

So gravity is very robust/generic.

And we may not have to solve any particular theory.

Resolving Information Loss Problems

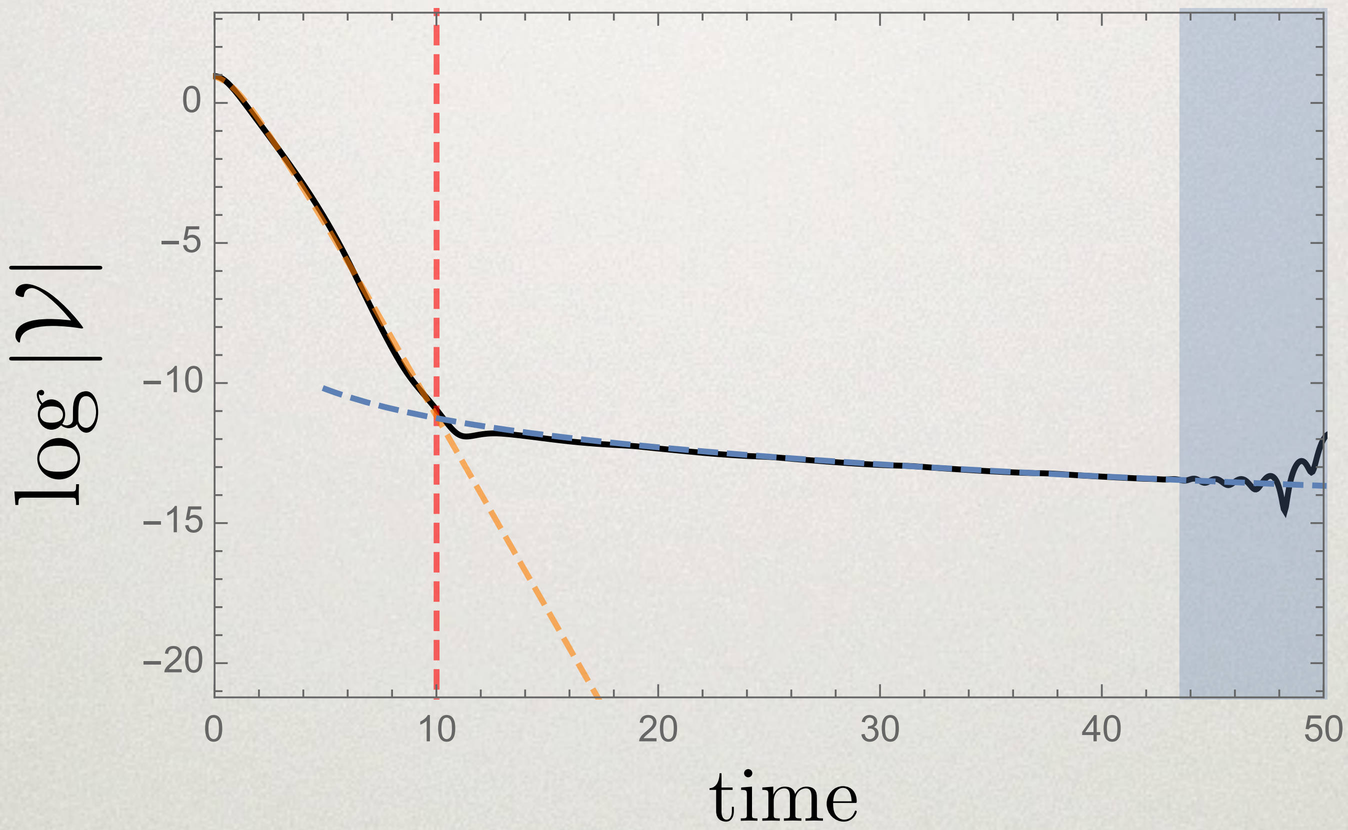
Exact Information about Virasoro Blocks and Gravity?

It is possible to get exact information using analytic continuation of degenerate states.

A simpler approach is to just evaluate the blocks numerically to very high precision using the Zamolodchikov recursion relations.

Exact vs Semiclassical: Late Time Behavior

At late times the gravity prediction breaks down:



Early: $e^{-2\pi h_L T_H t_L}$ Late: $t_L^{-3/2}$

Transition at $t_D = \dfrac{\pi c}{6 h_L}$ predicted analytically.

Late Time Punchlines

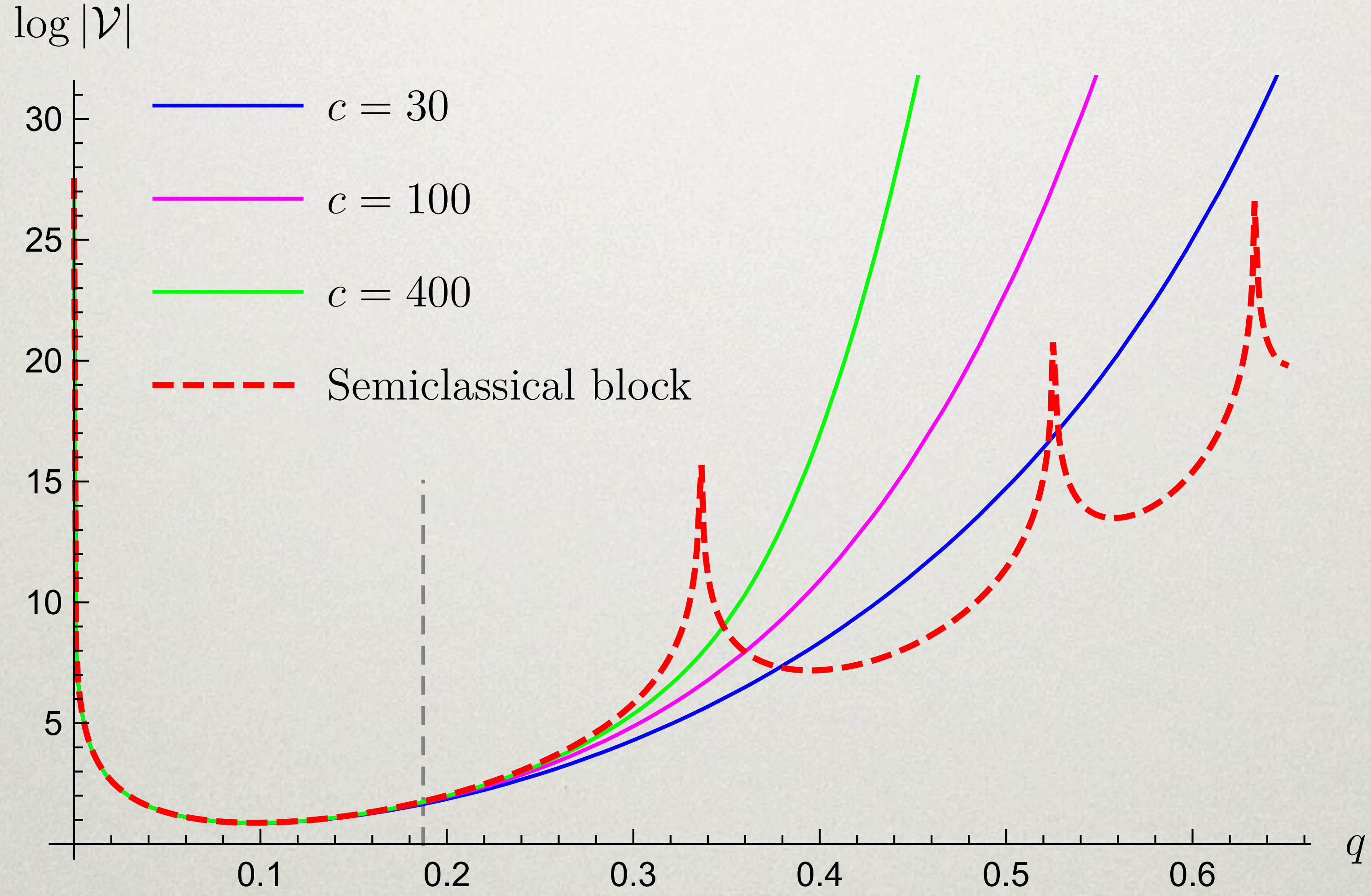
- Non-perturbative corrections are universal and qualitatively change late-time behavior of blocks
- Behavior of individual Virasoro blocks **ameliorates but does not resolve** late time decay

We understand the exponential early decay, but we do not have an analytic derivation of the universal late-time power-law, or a detailed understanding of the transition region.

Resurgence???

Euclidean-Time Periodicity and Forbidden Singularities

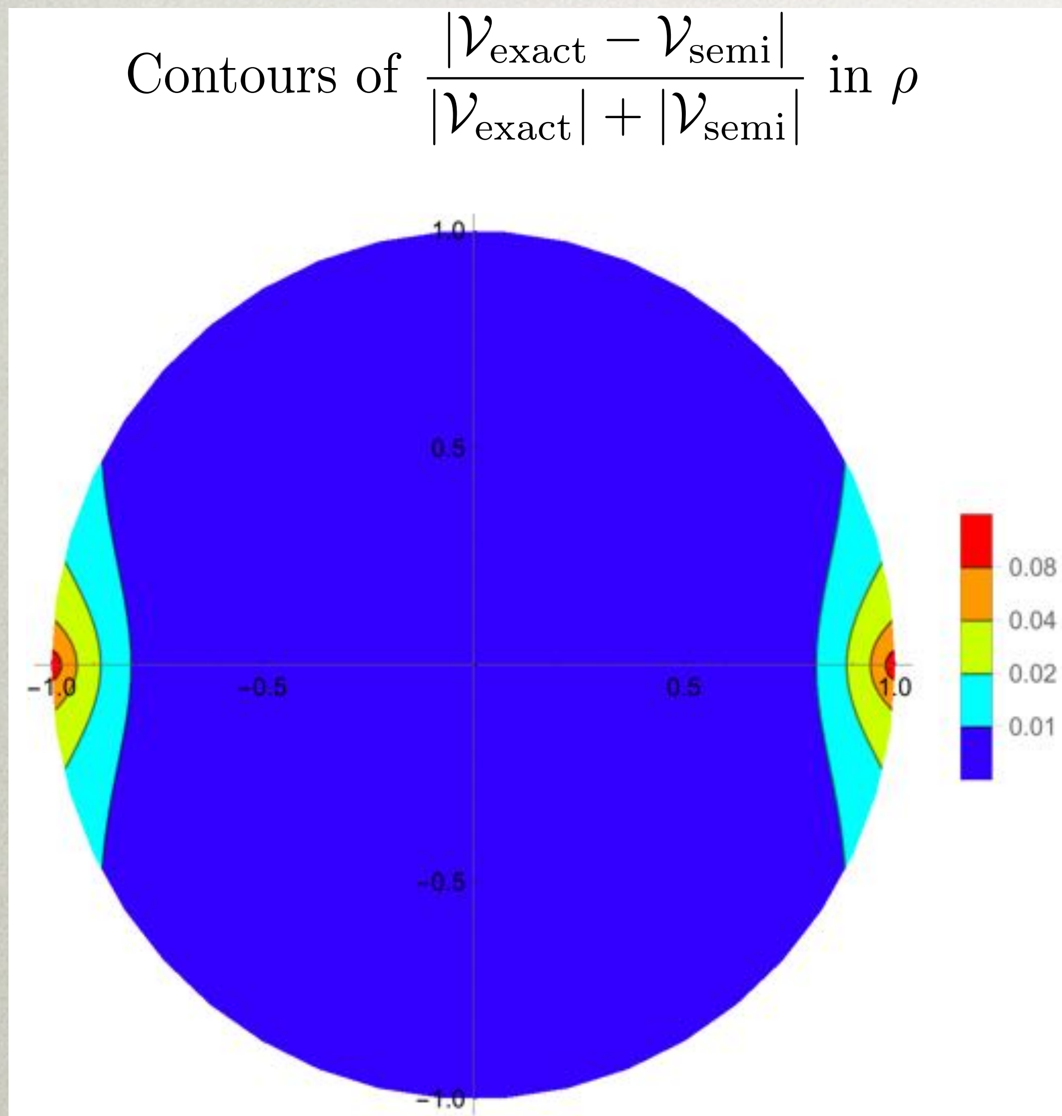
Semiclassical and exact blocks on the real axis:



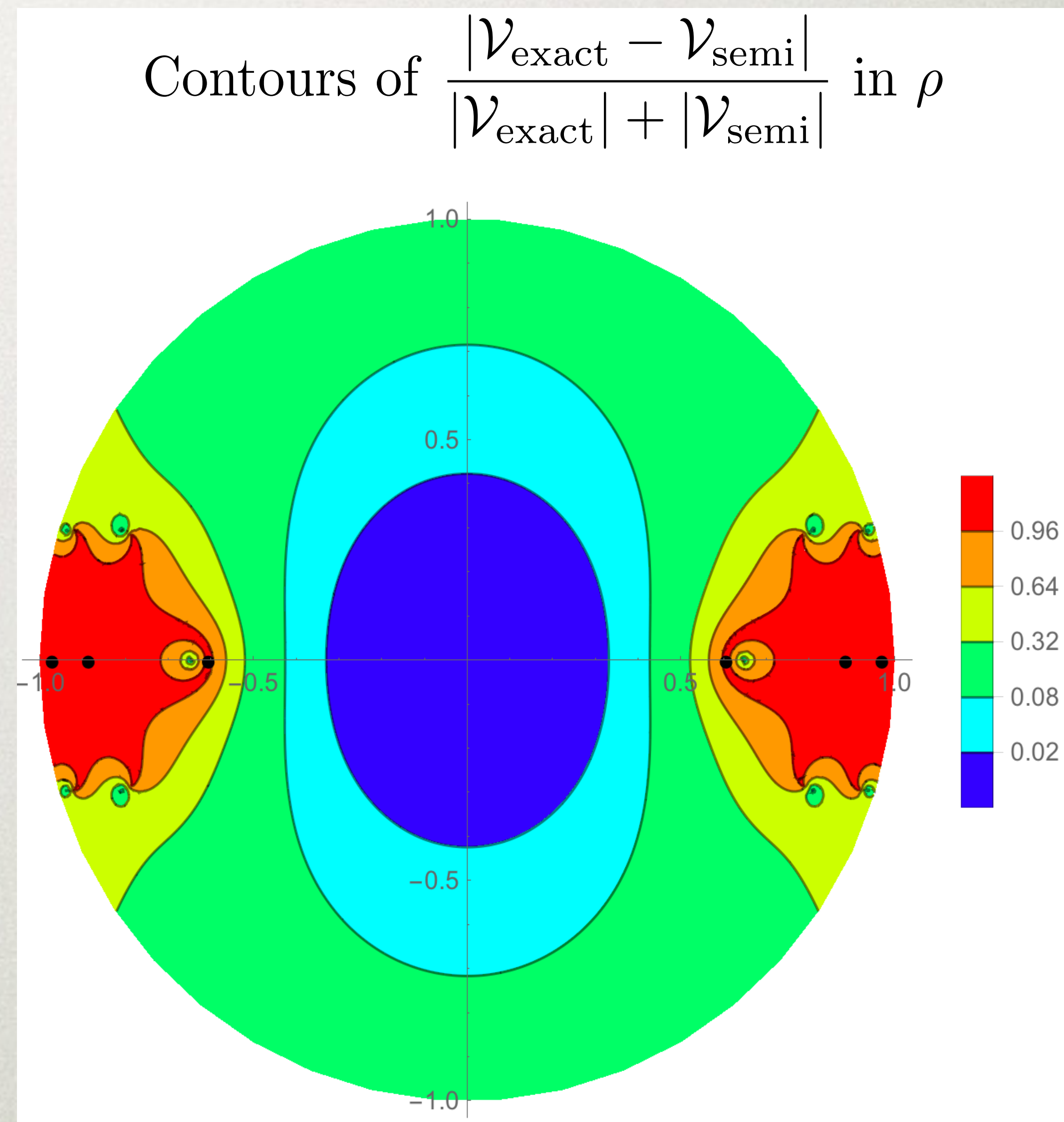
Semiclassical approximation **breaks down completely** beyond $t \approx \dfrac{1}{T_H}$ due to Stokes phenomena.

Semiclassical Approximation in the Euclidean Region

Without a Black Hole



With a Black Hole



Contour plots in $\rho = \dfrac{z}{(z + \sqrt{1-z})^2}$

How can we obtain an Analytic Undersanding of Virasoro Blocks?

Degenerate Operators

Special primaries have null Virasoro descendants, e.g.

$$\left(L_{-1}^2 + b^2 L_{-2}\right)|h_{1,2}\rangle = 0$$

with dimension

$$h_{1,2} = -\frac{1}{2} - \frac{3}{4b^2} \quad \text{where} \quad c = 1 + 6\left(b + \frac{1}{b}\right)^2$$

You could discover by studying the determinant of:

$$\begin{pmatrix} \langle h|L_1^2 L_{-1}^2|h\rangle & \langle h|L_1^2 L_{-2}|h\rangle \\ \langle h|L_2 L_{-1}^2|h\rangle & \langle h|L_2 L_{-2}|h\rangle \end{pmatrix}$$

Degenerate Operators

Via stress tensor Ward identities, Virasoro generators act as differential operators inside correlators:

$$L_{-m} \to \sum_{i \geq 1} \left(\frac{(r-1)h_i}{(z_i - z)^m} - \frac{1}{(z_i - z)^{m-1}} \partial_{z_i} \right)$$

In the simplest case, the null state correlators satisfy:

$$\left(\partial_z^2 + \left(2\frac{1+b^2}{z} + \frac{b^2}{1-z} \right) \partial_z - \frac{b h_H}{(1-z)^2} \right) \langle \mathcal{O}_H(\infty) \mathcal{O}_H(1) \mathcal{O}_{1,2}(z) \mathcal{O}_{1,2}(0) \rangle = 0$$

Solution is hypergeometric; interesting non-perturbative effects in large $b^2 \propto c$ expansion.

Degenerate States at Large Central Charge

Let's study the general case:

$$h_{r,s} \overset{c\to\infty}{\approx} \frac{c}{24}(1-r^2) + \frac{1-s}{2} + \frac{(r-1)(13+13r-12s)}{24} + \frac{3\left(r^2-s^2\right)}{2c} + \cdots$$

So there are **heavy** and **light** examples:

$$h_{r,1} \approx -\frac{c}{24}(r^2-1) \qquad \text{and} \qquad h_{1,s} \approx \frac{1-s}{2}$$

Infinite class of examples where we can obtain exact information (r^{th} order differential equation) about the Virasoro vacuum block.

Connecting Degenerate State Correlators to Large C Results

Heavy Degenerate Blocks at Large Central Charge

This case will be more relevant for information loss.

In general, at large c we obtain a differential equation:

$$\left(\partial_t - h_L g_r(t)\right) \mathcal{V}(t) = 0$$

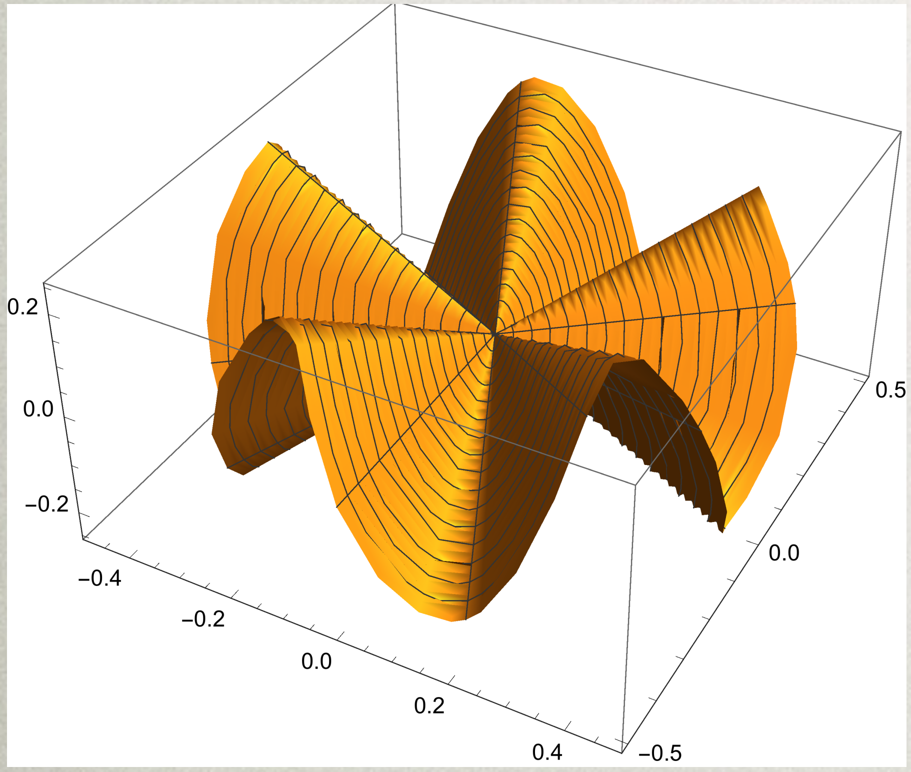
solved by heavy-light result, since we have:

$$g_r(t) = \coth\left(\frac{t}{2}\right) - r\coth\left(\frac{rt}{2}\right)$$

An r^{th} order equation has become first order!

Heavy Degenerate States

But $h_{r,1} \approx -\frac{c}{24}(r^2 - 1)$ is negative…



Heavy degenerate states have **additional angle**, total is:

$$\frac{\Theta_{r,1}}{2\pi} = r$$

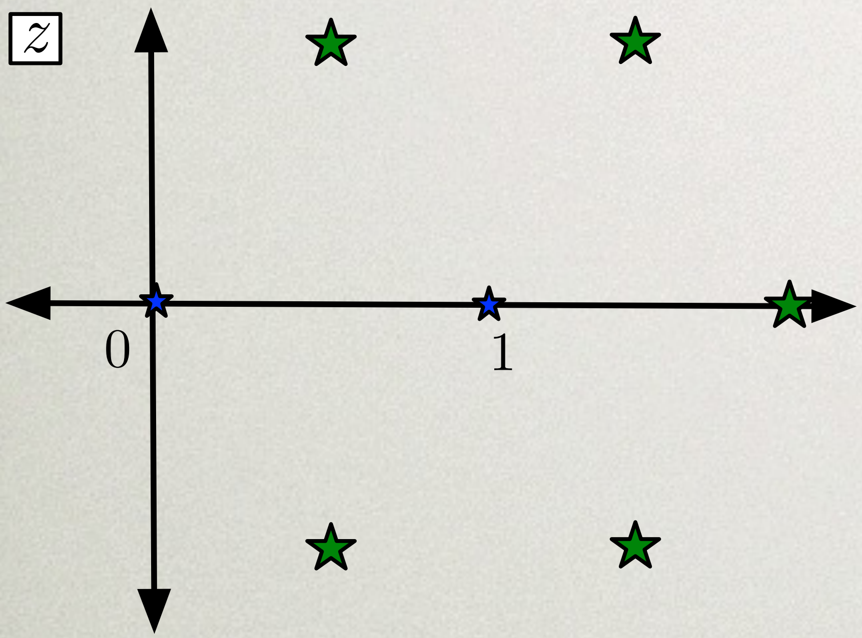
for r = 2,3,4,…

Can be seen in the large central charge block:

$$\mathcal{V}(t) = \frac{1}{\left[\sinh\left(\frac{r}{2}t\right)\right]^{2h_L}}$$

Heavy Degenerate States

Forbidden singularities at large central charge:



r=6 case

$$z = 1 - e^{\frac{2\pi i k}{r}}$$

for integers:

$$k = 1, \cdots, r-1$$

So it's interesting to see how they are **resolved**, and to **connect** to the **general** case with **Black Holes**.

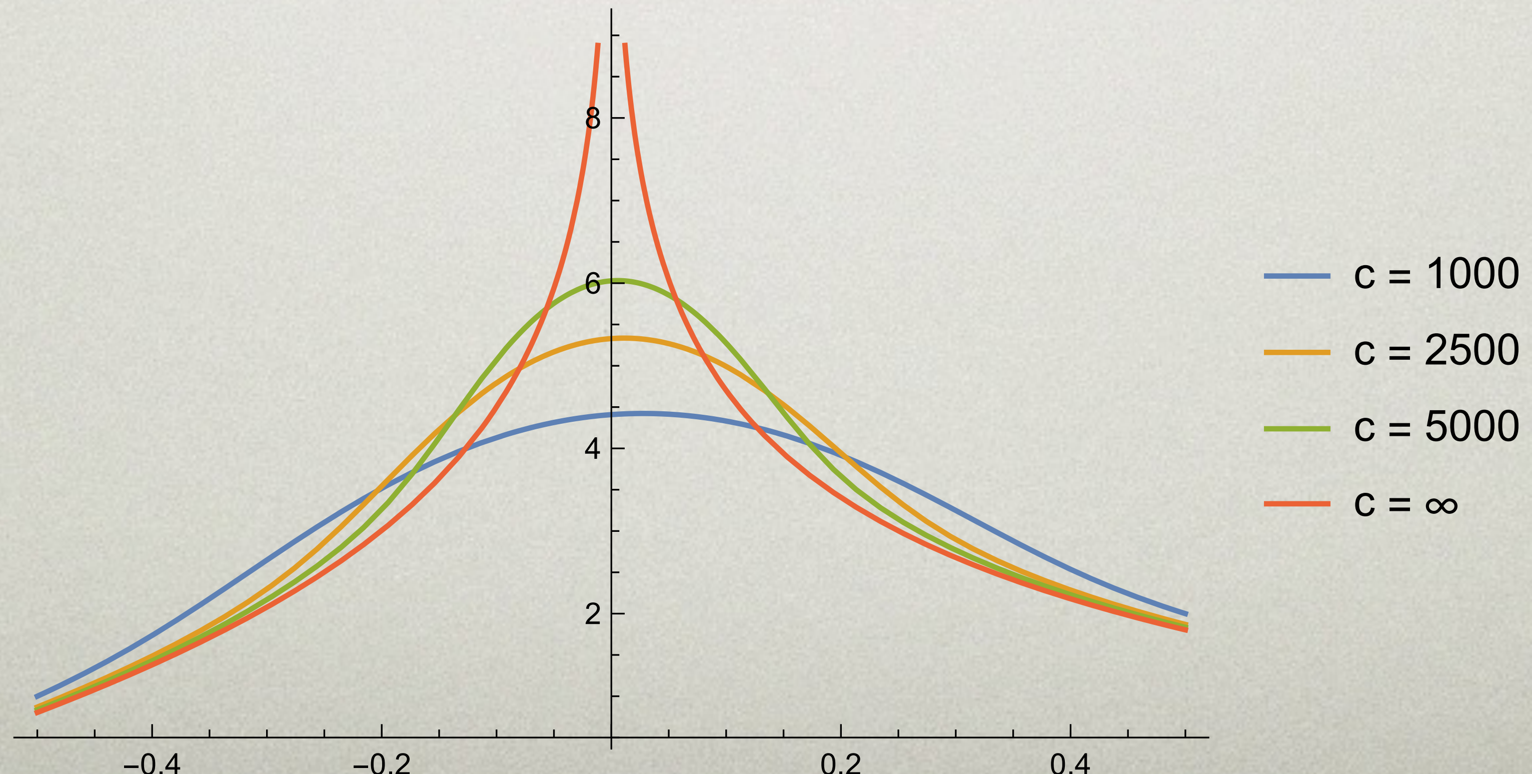
Resolution of Forbidden Singularities

Singularity Resolution: Some Data

Simplest example is the (2,1) case, with exact block:

$$\mathcal{V}_{2,1} = \frac{{}_2F_1(2, b^2+1, 2b^2+2, z)}{z^2} \qquad \xrightarrow{c\to\infty} \qquad \frac{1}{\sinh^2(t)}$$

where $h_L = 1$. In the vicinity of z = 2



Singularity Resolution in General

Singularities are **resolved in a universal way**.
Let's scale towards a singularity at large b as:

$$z_n(x) = 1 - e^{-\frac{2\pi i n}{r} + \frac{x}{b}}$$

We obtain a universal differential equation:

$$\sigma_n^2(r)\mathcal{V}''(x) - x\mathcal{V}'(x) - 2h_L\mathcal{V}(x) = 0$$

where the coefficient can be computed:

$$\sigma_n^2(r) \equiv 4\sum_{j=1}^{r-1} \frac{\sin^2\left(\frac{jn\pi}{r}\right)}{rj(r-j)}$$

Singularity Resolution in General

Differential equation for correlators near singularities:

$$\sigma_n^2(r)\mathcal{V}''(x) - x\mathcal{V}'(x) - 2h_L\mathcal{V}(x) = 0$$

Putting c back, it can be solved by the function:

$$S(x) = \int_0^\infty dp\, p^{2h_L-1} e^{-px - \frac{\sigma_n^2(r)}{2c}p^2}$$

It's a natural toy model for an entire function that has a singularity at large c… but this is a physical result.

A Prediction for 1/c Perturbation Theory

Consider expanding this function in 1/c:

$$S(x) = \int_0^\infty dp\, p^{2h_L - 1} e^{-px - \frac{\sigma_n^2(r)}{2c} p^2}$$

$$\propto \frac{1}{x^{2h_L}} + \frac{\sigma_n^2(r)(2h_L+1)h_L}{c\, x^{2h_L+2}} + \cdots$$

If this resolves the singularities, it makes a **prediction** about 1/c corrections for general heavy-light blocks.

We computed them, and they match.

What about the HARD problem: Bulk Reconstruction?

Exact Bulk Reconstruction in d=2

In $\mathrm{AdS}_3/\mathrm{CFT}_2$ Virasoro acts as asymptotic symmetry.

If we fix the gauge, we fix the symmetry generators. Then demanding scalar field transformations uniquely determines the bulk field:

$$\phi(y,0,0) = \sum_N y^{2h+2N} \lambda_N \mathcal{L}_N \bar{\mathcal{L}}_N \mathcal{O}(0)$$

with $L_m \left|\phi\right\rangle_N = 0, \quad \overline{L}_m \left|\phi\right\rangle_N = 0, \qquad \text{for } m \geq 2$

in the gauge (coordinate system):

$$ds^2 = \frac{dy^2 + dz d\bar{z}}{y^2} - \frac{6T(z)}{c} dz^2 - \frac{6\bar{T}(\bar{z})}{c} d\bar{z}^2 + y^2 \frac{36 T(z)\bar{T}(\bar{z})}{c^2} dz d\bar{z}$$

Non-Perturbative Fate of Locality

Let's study (work in progress) the propagator.
Lore - no local observables in quantum gravity…

We find a divergent expansion at short distances:

$$\langle \phi(X)\phi(Y)\rangle \approx \sum_n \frac{(4n-1)!!}{n!}\left(\frac{12}{c\,\sigma^4}\right)^n$$

Bulk locality appears to breakdown at short distances due to non-perturbative effects…
we also have significant **numerical evidence**.

Summary

I. Information loss in $\mathrm{AdS}_3/\mathrm{CFT}_2$ arises from the semiclassical expansion of Virasoro blocks

II. Many information loss problems are ameliorated or resolved by computable non-perturbative effects within Virasoro blocks; a better understanding of resurgence very useful here!

III. These non-perturbative effects have implications for the breakdown of bulk locality and (presumably) the physics near & beyond horizons