Anomaly matching & S^1 compactification

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· Anomaly matching

QFT with sym. $\cancel{Z}$ G $\Rightarrow$ A : G-gauge field

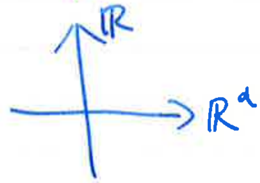
$$Z[A+d\theta] = Z[A]\, e^{i\underbrace{\mathcal{A}(\theta, A)}_{\text{anomaly}}}. \quad \Rightarrow \text{RG-inv.}$$

✓ Vacua must reproduce $\mathcal{A}$. ($\underbrace{\text{'t Hooft anomaly}}_{\mathcal{A}}$ matching).

· Volume indep. (or. Adiabatic compactification).

$$\mathbb{R}^{d+1} \Longrightarrow \mathbb{R}^d \times \underbrace{S^1}_{}$$

L: small Weak coupling (Asymptotic freedom)



$\Rightarrow$

$$\vec{\Phi}(x, x^{d+1}+L) = \Omega \cdot \vec{\Phi}(x, x^{d+1}).$$

✓ Claim: Vacuum of $\mathbb{R}^{d+1}$ = Vacuum of $\mathbb{R}^d \times S^1$.
with appropriate t.b.c.

To be consistent,

Anomaly on $\mathbb{R}^d \times S^1$ = Anomaly on $\mathbb{R}^{d+1}$.

Q. Is it possible?

· Difficulty : Thermal fluc. eliminates $\mathcal{A}$ usually.

ex. 3D free Dirac fermion : $\underset{\substack{\Downarrow \\ A.}}{U(1)}$ & T.

$$Z[T\cdot A] = Z[A] \exp\left(-\frac{i}{4\pi}\int A dA\right).$$

Set $M^3 = M^2 \times S^1_L$ & $L \ll \mathrm{size}(M^2)$.
A_2: $U(1)$-conn. on M^2 $\Rightarrow$ $A_2 dA_2 = 0$. No anomaly.

Solve this difficulty.

- Pure YM. (Gaiotto, Kapustin, Komargodski, Seiberg)
  or Theories with 1-form symm.

- Theories without 1-form symm. (our work)
  e.g. CP^{N-1} model
  $\Longrightarrow$ Massless N-flavor QCD with flavor twisted b.c.

- Pure $SU(N)$ YM @ $\theta=\pi$.

Symm: $\mathbb{Z}_N$ one-form & T.

$W(C) = \mathrm{tr}(Pe^{i\int a})$.

$\mapsto \omega\, W(C)$. $\quad (\omega = e^{2\pi i/N})$

$\mathbb{Z}_N$ two-form gauge field B: $Z[B]$

$$Z[T\cdot B] = Z[B]\, e^{\frac{iN}{4\pi}\int B\wedge B}$$

$\mathbb{R}^4 \Longrightarrow \mathbb{R}^3\times S^1$.

Polyakov loop $\mathrm{tr}(Pe^{i\int_0^L a})$.

$\mathbb{Z}_N$ 1-form $\Longrightarrow$ $\underset{B^{(1)}}{\mathbb{Z}_N \text{ 0-form}}$. $+$ $\underset{B^{(2)}}{\mathbb{Z}_N \text{ 1-form}}$.

Anomaly in 3D: $B \equiv B^{(2)} + B^{(1)}\wedge L^{-1}dx^4$,

$$Z[T\cdot(B^{(1)}, B^{(2)})] = Z[(B^{(1)}, B^{(2)})]\, e^{\frac{iN}{2\pi}\int B^{(1)}\wedge B^{(2)}}.$$

$\Rightarrow$ Good for V.I.

We can claim that both vacua are controlled

by the same anomaly matching.

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- $\mathbb{C}P^{N-1}$. (Dunne, Ünsal, ..., Sulejmanpasic)

$$S = \int \left(|(d+ia)\vec{z}|^2 + V(|\vec{z}|^2) \right) + \frac{i\theta}{2\pi}\int da .$$

Symm. : $\begin{cases} \text{flavor} & \frac{SU(N)}{\mathbb{Z}_N}, \quad \vec{z} \mapsto U\cdot\vec{z} \\ @\ \theta=\pi, & T. \end{cases}$

$\mathbb{R}^2 \Rightarrow \mathbb{R}\times S^1$.

$$\vec{z}(x^1, x^2+L) = \Omega\cdot\vec{z}(x^1,x^2). \qquad \Omega = \begin{pmatrix} 1 & & & 0 \\ & \omega & & \\ & & \ddots & \\ 0 & & & \omega^{N-1} \end{pmatrix} \in SU(N)/\mathbb{Z}_N.$$

$\Rightarrow$ Large-N V.I.

- Anomaly.

flavor gauge field

$A - SU(N)$, $B - \mathbb{Z}_N$.

$$Z_{\theta=\pi}[T\cdot(A,B)] = Z_{\theta=\pi}[(A,B)]\, e^{i\int B}.$$

$\mathbb{R}^2 \Rightarrow \mathbb{R}\times S^1$.

$\Omega = P e^{i\int_0^L A}$: Background $SU(N)$ holonomy.

$\mathbb{Z}_N$ 1-form : $\Omega \to \omega\cdot\Omega$.

B.C. is changed $\Rightarrow$ Different theories.

Shift symm.

$$\vec{z} = \begin{pmatrix} z_1 \\ \vdots \\ z_N \end{pmatrix} \longmapsto \begin{pmatrix} z_2 \\ \vdots \\ z_N \\ z_1 \end{pmatrix} \equiv S\cdot\vec{z}.$$

This also changes B.C.:

$$\Omega \to S\Omega S^{-1} = \omega\Omega.$$

Intertwined $\mathbb{Z}_N$ symm: $- B^{(1)}$.

$$\Omega \xrightarrow{\text{shift}} S\cdot\Omega\cdot S^{-1} \xrightarrow{\mathbb{Z}_N\text{-0-form}} \omega^{-1}\cdot S\cdot\Omega\cdot S^{-1} = \Omega.$$

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$B = B^{(1)} \wedge L^{-1} dx^2$: Acts on $U(1)$ Polyakov loop

$$e^{i\oint_0^L a} \to \omega\, e^{i\oint_0^L a}.$$

$$Z_\Omega[T\cdot(A,B)] = Z_\Omega[(A,B)]\, e^{i\int B^{(1)}}.$$

Th'm (Y.T.-Misumi-Sakai)

QFT. with symm. $G = \frac{SU(N)}{\mathbb{Z}_N}$ (A, B) & $H \ni h$.

Anomaly

$$Z[h\cdot(A,B)] = Z[(A,B)]\, e^{i\mathcal{A}_h[B]}.$$

$\mathbb{R}^{D+1} \Rightarrow \mathbb{R}^D \times S^1$. w/ Background $SU(N)$ hol. $\Omega = Pe^{i\int A}$,
s.t. $\exists S \in SU(N)$ $S\cdot\Omega\cdot S^{-1} = \omega\cdot\Omega$.

$\Rightarrow$ Mixed anomaly among $(\mathbb{Z}_N)_{\text{shift}}$ ($B^{(1)}$), $\frac{Ab(SU(N))}{\mathbb{Z}_N}$ (A', $B^{(2)}$), $H \ni h$

$$Z[h\cdot(A', B^{(1)}, B^{(2)})]$$
$$= Z[(A', B^{(1)}, B^{(2)})]$$
$$\times \exp(i\mathcal{A}_h[B^{(2)} + B^{(1)} \wedge L^{-1} dx^{D+1}]).$$ //

Application Massless N-flavor QCD ($N_c = N_f = N$)
(Related works: Shimizu, Yonekura ; Gaiotto, Komargodski, Seiberg.)

Symm. $SU(N)/\mathbb{Z}_N$ (A, B), $(\mathbb{Z}_{2N})_{\text{axial}}$.

$$Z[(A,B)] \xrightarrow{(\mathbb{Z}_{2N})_{\text{axial}}} Z[(A,B)]\, e^{\frac{2iN}{4\pi}\int B\wedge B}.$$

anomaly if $N \geq 3$.

Flavor twisted b.c.

$$q_n(x, x^4+L) = q_n(x,x^4)\, e^{\frac{2\pi i n}{N}}. \quad (n = 0, \ldots, N-1)$$
u d s

$q_i \to q_{i+1}$, Polyakov $\to \omega\cdot$Polyakov

Symm: $(\mathbb{Z}_N)_{\text{shift}}$ ($B^{(1)}$), $\frac{U(1)^{N-1}}{\mathbb{Z}_N}$ (A', $B^{(2)}$), $(\mathbb{Z}_{2N})_{\text{axial}}$.

$$Z[(A', B^{(1)}, B^{(2)})] \xrightarrow{(\mathbb{Z}_{2N})_{\text{axial}}} Z[(A', B^{(1)}, B^{(2)})]\, e^{\frac{2iN}{2\pi}\int B^{(1)}\wedge B^{(2)}}.$$

$\boxed{\text{No trivially gapped phase for } \mathbb{Z}_N\text{-QCD!}}$