Resurgence and Relativistic Hydrodynamics — Status Report

Michal P. Heller

aei.mpg.de/GQFI

based on **1802.08225** with Svensson, **1609.04803v2** with Kurkela, Spaliński, Svensson, **2007.05524** with Serantes, Spaliński, Svensson, Withers, as well as earlier works and papers by other people

Introduction

Motivation

Relativistic hydrodynamics is important for HEP since it encapsulates effective reduction of degrees of freedom in non-equilibrium processes in QFTs

Key applications to <u>heavy ion collisions at RHIC and LHC</u>, now also of interest in neutron star mergers, long-standing connections with black holes

The topic of my talk: **large order behaviour** of this classical dissipative EFT:

$$\langle T^{\mu\nu}\rangle = \mathcal{E}\, u^{\mu}u^{\nu} + P(\mathcal{E})\{\, g^{\mu\nu} + u^{\mu}u^{\nu}\,\} \overbrace{- \eta(\mathcal{E})\, \sigma^{\mu\nu} - \zeta(\mathcal{E})\{\, g^{\mu\nu} + u^{\mu}u^{\nu}\,\}(\nabla\cdot u) \boxed{+\ldots}}^{\pi^{\mu\nu}}$$

microscopic input:

EoS ($P(\mathcal{E}) = \frac{1}{3}\mathcal{E}$ for CFTs)

shear viscosity contribution

bulk viscosity (vanishes for CFTs)

Some developments since the KITP program

Resurgence and Hydrodynamic Attractors in Gauss-Bonnet Holography
Jorge Casalderrey-Solana (Oxford U., Theor. Phys.), Nikola I. Gushterov (Oxford U., Theor. Phys.), Ben Meiring (Oxford U., Theor. Phys.) (Dec 7, 2017)
Published in: *JHEP* 04 (2018) 042 • e-Print: 1712.02772 [hep-th]

Hydrodynamization in kinetic theory: Transient modes and the gradient expansion
Michal P. Heller (Potsdam, Max Planck Inst. and Perimeter Inst. Theor. Phys. and Warsaw, Inst. Nucl. Studies), Aleksi Kurkela (CERN and Stavanger U.), Michal Spaliński (Warsaw, Inst. Nucl. Studies and Bialystok U.), Viktor Svensson (Warsaw, Inst. Nucl. Studies and Potsdam, Max Planck Inst.) (Sep 15, 2016)
Published in: *Phys.Rev.D* 97 (2018) 9, 091503 • e-Print: 1609.04803 [nucl-th]

How does relativistic kinetic theory remember about initial conditions?
Michal P. Heller (Potsdam, Max Planck Inst. and NCBJ, Warsaw), Viktor Svensson (Potsdam, Max Planck Inst. and NCBJ, Warsaw) (Feb 22, 2018)
Published in: *Phys.Rev.D* 98 (2018) 5, 054016 • e-Print: 1802.08225 [nucl-th]

Short-lived modes from hydrodynamic dispersion relations
Benjamin Withers (Geneva U., Dept. Theor. Phys.) (Mar 21, 2018)
Published in: *JHEP* 06 (2018) 059 • e-Print: 1803.08058 [hep-th]

Hydrodynamic attractor and the fate of perturbative expansions in Gubser flow
Gabriel S. Denicol (Niteroi, Fluminense U.), Jorge Noronha (Sao Paulo U.) (Apr 12, 2018)
Published in: *Phys.Rev.D* 99 (2019) 11, 116004 • e-Print: 1804.04771 [nucl-th]

Non-perturbative rheological behavior of a far-from-equilibrium expanding plasma
Alireza Behtash (North Carolina State U.), C.N. Cruz-Camacho (Colombia, U. Natl.), Syo Kamata (North Carolina State U.), M. Martinez (North Carolina State U.) (May 20, 2018)
Published in: *Phys.Lett.B* 797 (2019) 134914 • e-Print: 1805.07881 [hep-th]

Universal behaviour, transients and attractors in supersymmetric Yang–Mills plasma
Michał Spaliński (Bialystok U. and Warsaw U.) (May 29, 2018)
Published in: *Phys.Lett.B* 784 (2018) 21-25 • e-Print: 1805.11689 [hep-th]

Gradient resummation for nonlinear chiral transport: an insight from holography
Yanyan Bu (Harbin Inst. Tech.), Tuna Demircik (Ben Gurion U. of Negev), Michael Lublinsky (Ben Gurion U. of Negev) (Jul 30, 2018)
Published in: *Eur.Phys.J.C* 79 (2019) 1, 54 • e-Print: 1807.11908 [hep-th]

Exact hydrodynamic attractor of an ultrarelativistic gas of hard spheres
Gabriel S. Denicol (Niteroi, Fluminense U.), Jorge Noronha (Illinois U., Urbana and Sao Paulo U.) (Aug 26, 2019)
Published in: *Phys.Rev.Lett.* 124 (2020) 15, 152301 • e-Print: 1908.09957 [nucl-th]

Resummed hydrodynamic expansion for a plasma of particles interacting with fields
L. Tinti (Ohio State U. and Frankfurt U.), G. Vujanovic (Ohio State U.), J. Noronha (Sao Paulo U.), U. Heinz (CERN) (Aug 20, 2018)
Published in: *Phys.Rev.D* 99 (2019) 1, 016009 • e-Print: 1808.06436 [nucl-th]

The large proper-time expansion of Yang-Mills plasma as a resurgent transseries
Inês Aniceto (Jagiellonian U. and Southampton U.), Ben Meiring (Oxford U., Theor. Phys.), Jakub Jankowski (Warsaw U.), Michał Spaliński (Bialystok U. and Warsaw U.) (Oct 16, 2018)
Published in: *JHEP* 02 (2019) 073 • e-Print: 1810.07130 [hep-th]

Dynamical systems and nonlinear transient rheology of the far-from-equilibrium Bjorken flow
Alireza Behtash (North Carolina State U.), Syo Kamata (North Carolina State U.), Mauricio Martinez (North Carolina State U.), Haosheng Shi (North Carolina State U.) (Jan 24, 2019)
Published in: *Phys.Rev.D* 99 (2019) 11, 116012 • e-Print: 1901.08632 [hep-th]

Convergence of the Gradient Expansion in Hydrodynamics
Sašo Grozdanov (MIT, Cambridge, CTP), Pavel K. Kovtun (Victoria U.), Andrei O. Starinets (Oxford U., Theor. Phys.), Petar Tadić (Trinity Coll., Dublin) (Apr 1, 2019)
Published in: *Phys.Rev.Lett.* 122 (2019) 25, 251601 • e-Print: 1904.01018 [hep-th]

The complex life of hydrodynamic modes
Sašo Grozdanov (MIT, Cambridge, CTP), Pavel K. Kovtun (Victoria U.), Andrei O. Starinets (Oxford U.), Petar Tadić (Trinity Coll., Dublin) (Apr 2019)
Published in: *JHEP* 11 (2019) 097 • e-Print: 1904.12862 [hep-th]

The hydrodynamic gradient expansion in linear response theory
Michal P. Heller, Alexandre Serantes, Michał Spaliński, Viktor Svensson, Benjamin Withers (Jul 10, 2020)
e-Print: 2007.05524 [hep-th]

Non-conformal holographic Gauss-Bonnet hydrodynamics
Alex Buchel (Western Ontario U.) (Jan 18, 2018)
Published in: *JHEP* 03 (2018) 037 • e-Print: 1801.06165 [hep-th]

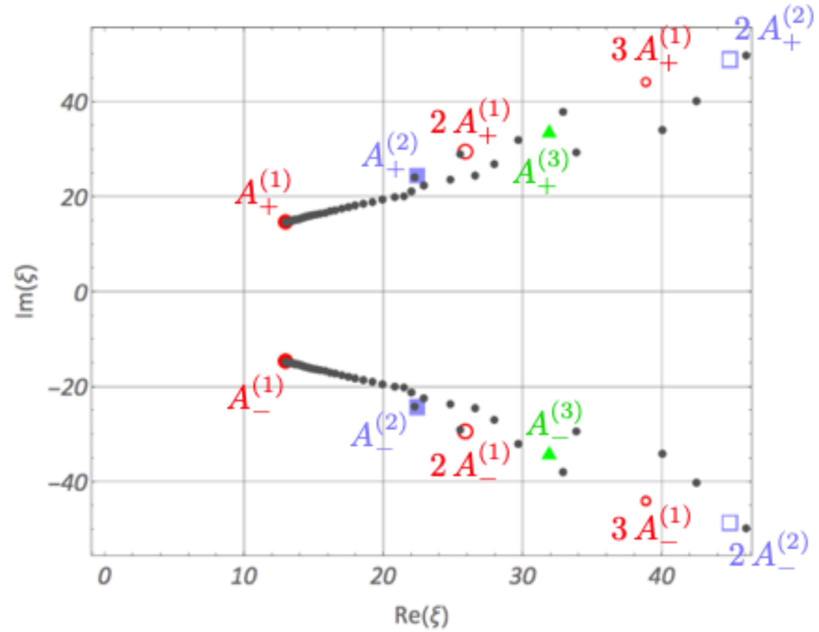
Holographic Bjorken Flow at Large-D
Jorge Casalderrey-Solana (Barcelona U. and ICC, Barcelona U. and Oxford U.), Christopher P. Herzog (King's Coll. London, Dept. Math and YITP, Stony Brook), Ben Meiring (Oxford U.) (Oct 4, 2018)
Published in: *JHEP* 01 (2019) 181 • e-Print: 1810.02314 [hep-th]

Holographic Viscoelastic Hydrodynamics
Matteo Baggioli (Crete U.), Alex Buchel (Western Ontario U. and Perimeter Inst. Theor. Phys.) (May 17, 2018)
Published in: *JHEP* 03 (2019) 146, Journal of High Energy Physics volume 2019, Article number: 146 (2019) • e-Print: 1805.06756 [hep-th]

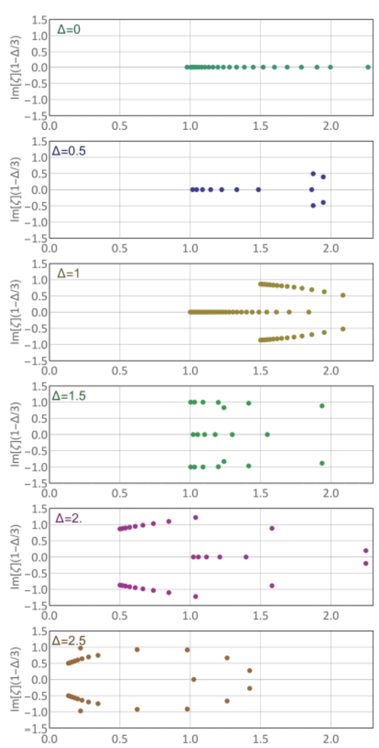
O(20) developments related to resurgence in hydrodynamics since Nov 2017. This talk covers the red ones. I am very sorry for inevitable omissions.

Goal

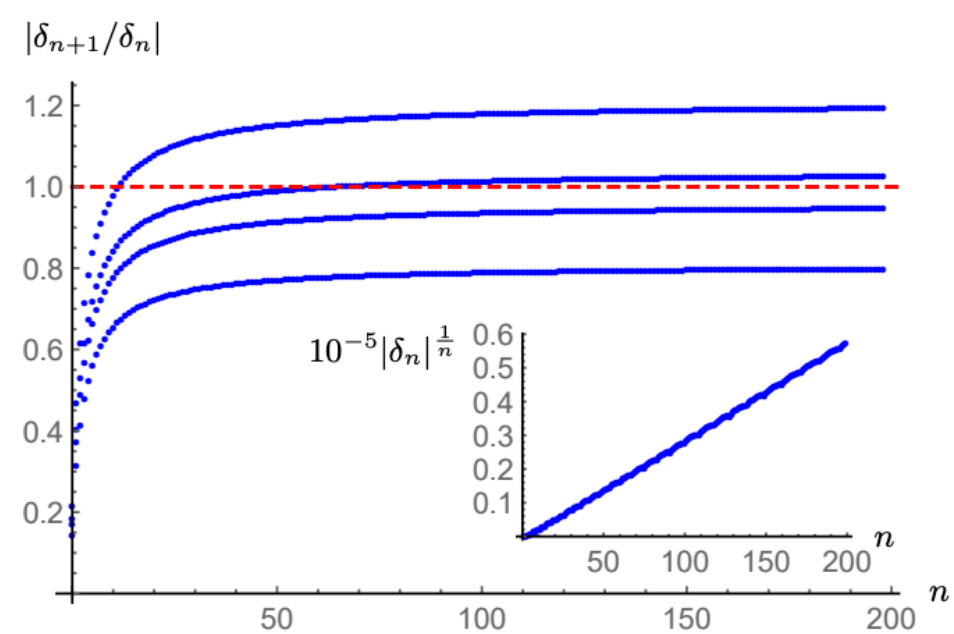
http://online.kitp.ucsb.edu/online/resurgent_c17/heller/



1802.08225 with Svensson



2007.05524 with Serantes, Spaliński, Svensson, Withers



Pre(KITP conference)history

Hydrodynamic and transient modes

see, for example, **1707.02282** with Florkowski and Spaliński

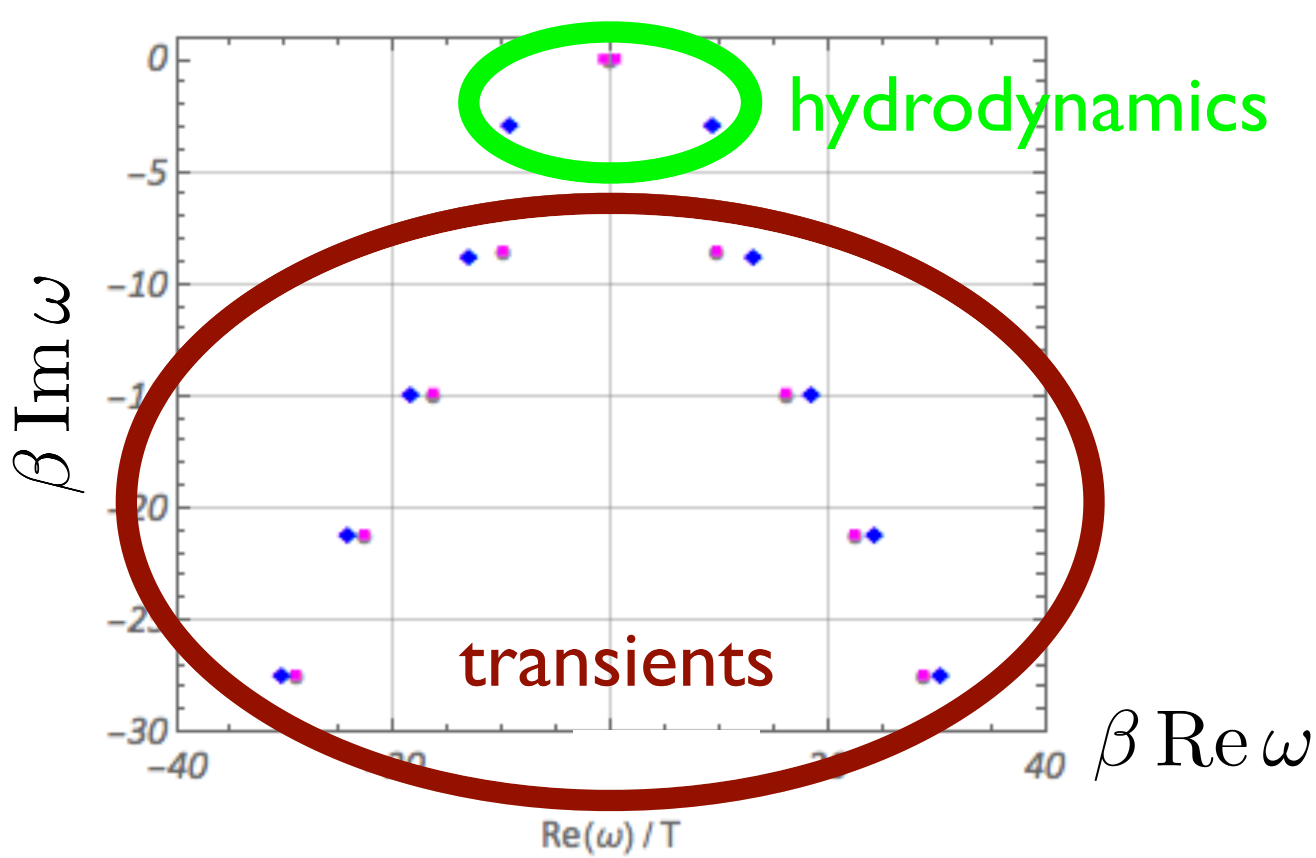
Linear response theory on top $\rho \sim e^{-\frac{1}{T}H}$ is the simplest and very powerful model of nonequilibrium physics with a hydrodynamic tail

$$\delta\langle\mathcal{O}(t,p)\rangle = \int_{-\infty}^{\infty} d\omega\, e^{-i\,\omega\,t}\, G_R^{\mathcal{O}}(\omega,p)\,\mathcal{J}(-\omega,-p)$$

with $G_R^{\mathcal{O}}(t,x) = i\,\theta(t)\,\mathrm{tr}\left(\rho\ \left[\mathcal{O}(t,x),\mathcal{O}(0,0)\right]\right)$

The response is encoded in singularities in ω of $G_R^{\mathcal{O}}(\omega,p)$ at fixed value of p

$\mathcal{O} = T_{\mu\nu}$
in a holographic
CFT:



$\beta\,p = 0.1$

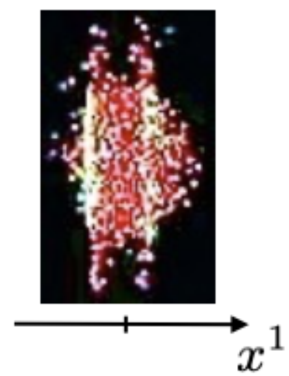
$\beta\,p = 1$

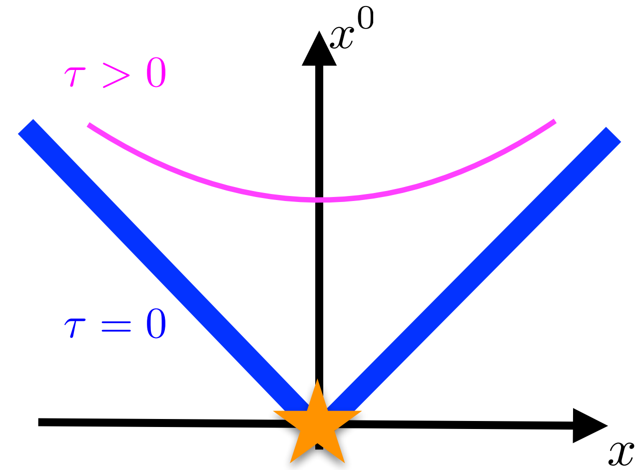
$\beta\,p = 10$

Bjorken flow in holographic CFTs

1302.0697 with Janik and Witaszczyk, see also **1810.07130** by Aniceto, Meiring, Jankowski and Spaliński

const x^0 slice:





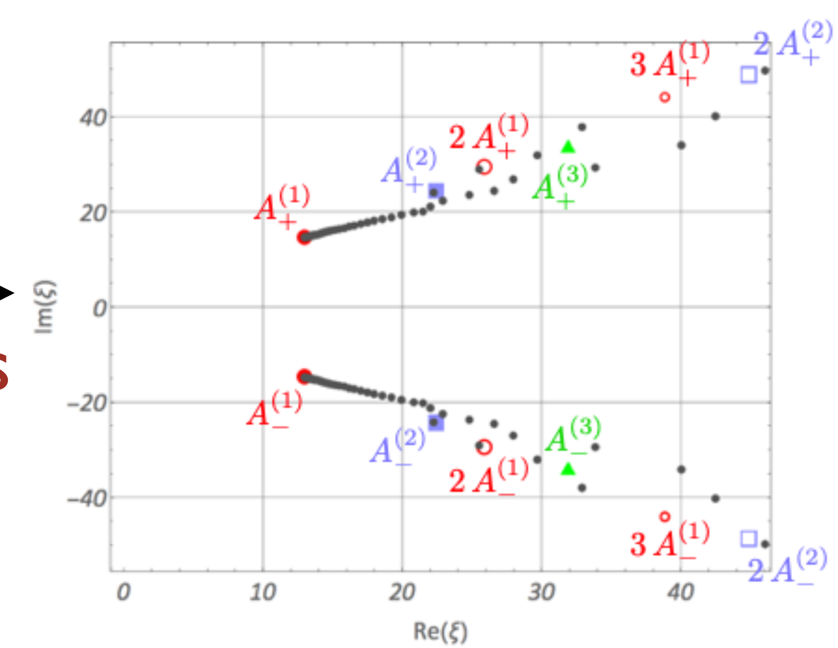
In Bjorken flow in a CFT symmetries fix $u^\mu \partial_\mu = \partial_\tau$ and the dynamics lies in a single function of a single dimensionless variable: $\mathcal{A}(w \sim \tau\, \mathcal{E}^{1/4}) \equiv \dfrac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{E}/3}$

Since $\nabla_\mu u^\mu \sim \dfrac{1}{\tau}$, on-shell gradient expansion: $\mathcal{A}(w) \approx \displaystyle\sum_{n=1}^{\infty} \frac{a_n}{w^n} = \left(\sim \frac{\eta}{s}\right) \frac{1}{w} + \ldots$

We calculated up to n = 240 terms using 2013 state of the art numerical relativity in AdS5 and found out vanishing radius of convergence: $a_n = \dfrac{\Gamma(n+\beta)}{A^n}$

Borel-Padé

reveals **transients**



Trans-series for Bjorken flow in a simple model

1503.07114 with Spaliński, see also 1509.05046 by Basar and Dunne

The key insight: in Bjorken flow the gradient expansion and transients form a trans-series in which the gradient expansion is perturbative in the "coupling constant" $\frac{1}{w}$ and transient are non-perturbative and behave as $\exp\left(-\frac{1}{\frac{1}{w}}\right)$

To conclude this, we simplified the problem and considered a model, which has only one transient and produces a <u>first order</u> ODE for $\mathcal{A}(w)$. We got:

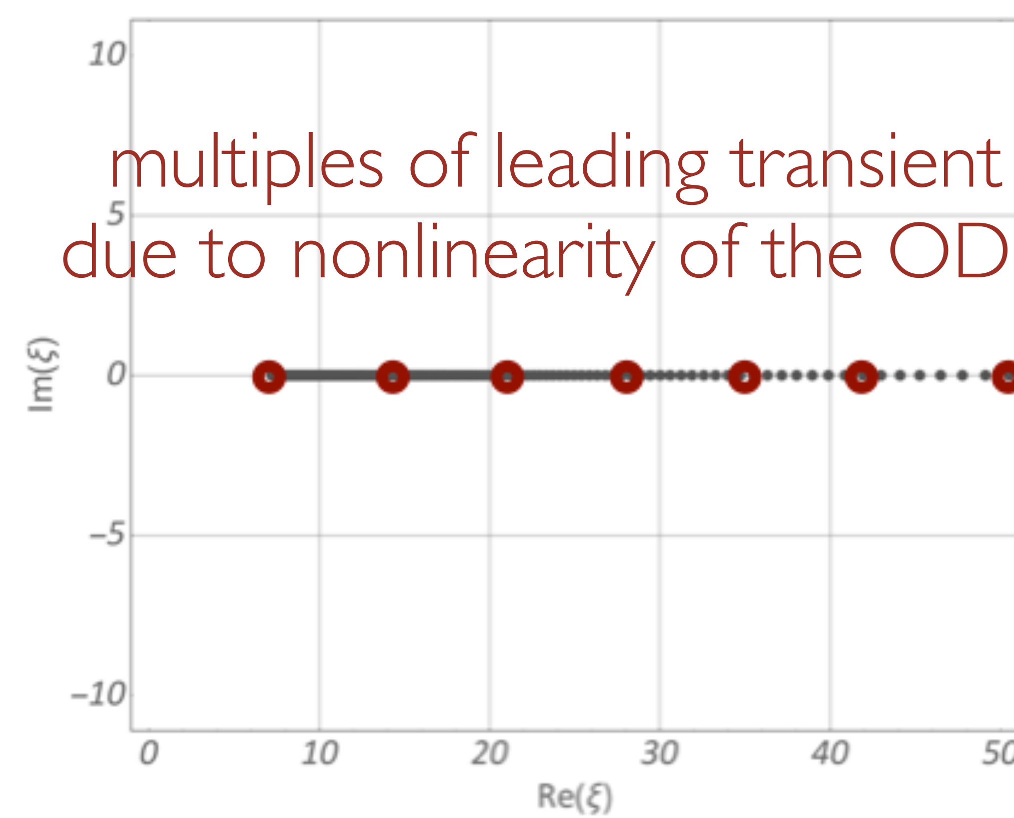
single trans-series parameter

$$\mathcal{A}(w) = \sum_{m=0}^{\infty} \overset{\downarrow}{\sigma}{}^m \, e^{-m A w} \, \Phi_m(w)$$

$$\Phi_m(w) = w^{m\,\beta} \sum_{n=0}^{\infty} \frac{a_n^{(m)}}{w^n}$$

with $a_n^{(0)} \equiv a_n$

$$\xrightarrow[\text{for } m=0]{\text{Borel-Padé}}$$



Bjorken flow in kinetic theory

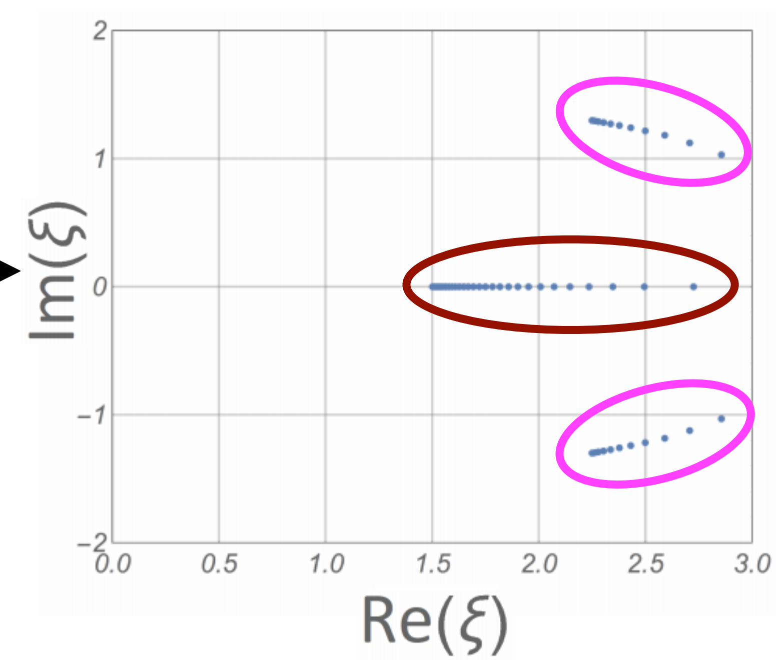
1609.04803v1 with Kurkela and Spaliński

The previous devs concerned strong coupling physics

It was then natural to ask what happens at weak coupling $\longrightarrow$ kinetic theory

In the relaxation time approximation kinetic theory for $f(x,k)$ one gets

$$k^\mu \partial_\mu f \sim \frac{1}{\tau_{rel}}(f - f_{eq}) \quad \text{with} \quad \tau_{rel} \sim \frac{1}{T} \longrightarrow \mathcal{A}(w) \approx \sum_{n=1}^{200} \frac{a_n}{w^n} \xrightarrow{\text{Borel-Padé}}$$



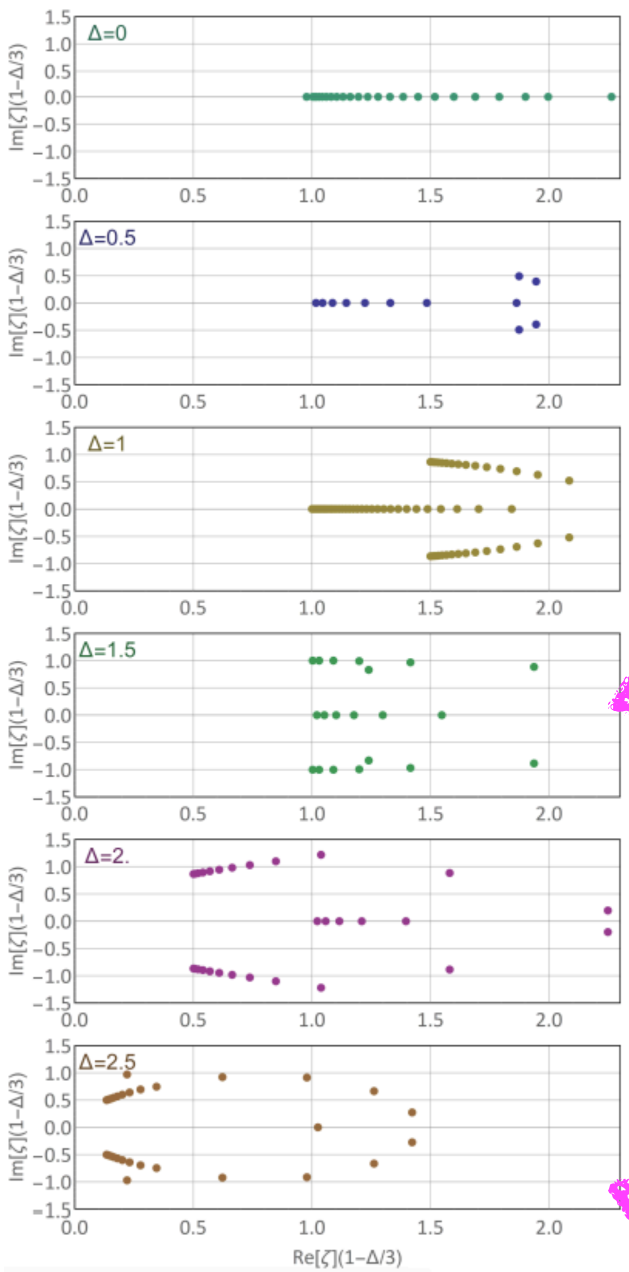
The singularities on the real axis are **transients**, but what are **the other ones**?

> 2017: “ghost modes”

Towards resolution of the kinetic theory puzzle

1802.08225 with Svensson

If the additional singularities are transients, then we should see them in the evolution. For $\tau_{rel} \sim \dfrac{1}{T}$ they would be subleading $\longrightarrow$ $\tau_{rel} \sim \dfrac{1}{T^{\Delta}}$:



in this range of Δ the additional singularity would be the dominant transient effect, but our very accurate solution of kinetic theory did not see any trace of them

Solution of the kinetic theory puzzle

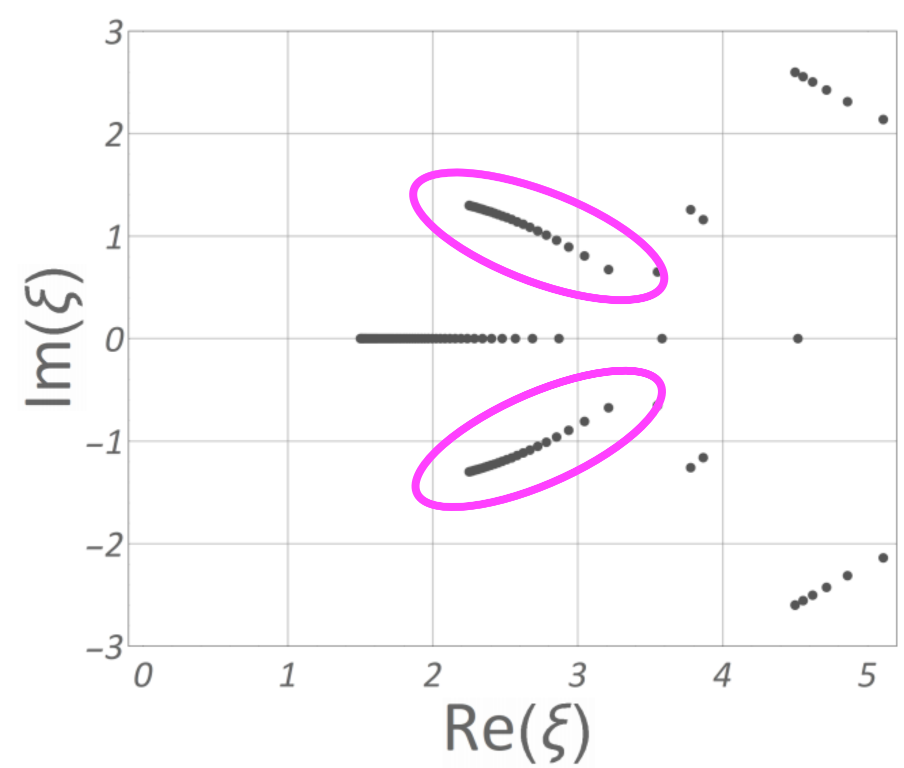
1609.04803v2 with Kurkela, Spaliński and Svensson (online on 22 Feb 2018)

Our kinetic theory admits an integral equation representation for $\mathcal{E}(\tau)$:

$$\mathcal{E}(\tau)\exp\left(\int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{\mathrm{rel}}(\tau')}\right)=\mathcal{E}_0(\tau)+\int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{\mathrm{rel}}(\tau')}\,H\left(\frac{\tau'}{\tau}\right)\mathcal{E}(\tau')\exp\left(\int_{\tau_0}^{\tau'}\frac{\mathrm{d}\tau''}{\tau_{\mathrm{rel}}(\tau'')}\right)$$

Physical contour of integration in τ is homologous to the real axis and this is how we solved the initial value problem

Resurgence knows about nontrivial contour deformations, which provide access to singularities in $H\left(\frac{\tau'}{\tau}\right)$. They lead precisely to the additional cuts:



> 2017: mode collisions

Fourier space statement: mode collisions

1803.08058 by Withers; 1904.01018 and 1904.12862 by Grozdanov, Kovtun, Starinets, Tadić

At the level of the linear response theory, hydrodynamic gradient expansion leads to the expansion of hydrodynamic mode frequencies ω_h in p, e.g.

$$\omega_h = -i\frac{\eta}{s\,T}p^2 - i\left(\frac{\eta^2\,\tau_\pi}{s^2T^2} - \frac{\theta_1}{2\,s\,T}\right)p^4 + \ldots$$

The quoted works showed that this expansion has a finite radius of convergence set by the smallest $|p_*|$ for which $\omega_h(p_*) = \omega_{some\,transient}(p_*)$

For example, in the simple model we considered in 1503. 07114 with Spaliński based on 0712.2451 by Baier, Romatschke, Son, Starinets, Stephanov with only 1 transient one has

$$\omega_h = i\frac{-1+\sqrt{1-4\,\frac{\eta}{s\,T}\,\tau_\pi\,p^2}}{2\,\tau_\pi} \quad \text{and} \quad \omega_{transient} = i\frac{-1-\sqrt{1-4\,\frac{\eta}{s\,T}\,\tau_\pi\,p^2}}{2\,\tau_\pi}$$

In this case p_* is real and equals $\pm\frac{1}{2}\sqrt{\frac{s\,T}{\eta\,\tau_\pi}}$

> 2017: gradient expansion without symmetries

Basic idea

2007.05524 with Serantes, Spaliński, Svensson, Withers

Linearized hydrodynamics, $u^\mu \partial_\mu = \partial_t + u^j \partial_j$ and $\mathcal{E} = \mathcal{E}_0 + \epsilon$, allows us to overcome the need of high symmetry, as was the case for the Bjorken flow

As we realized, in linearized hydrodynamics the hydrodynamic constitutive relations can be written as only three contributions at each order

$$\Pi_{jl} = -A(\partial^2)\,\sigma_{jl} - B(\partial^2)\,\pi^u_{jl} - C(\partial^2)\,\pi^\epsilon_{jl}$$

with

$$\sigma_{jl} = \left(\partial_j u_l + \partial_l u_j - \frac{2}{d-1}\delta_{jl}\partial_r u^r\right)$$
$$\pi^\epsilon_{jl} = \left(\partial_j \partial_l - \frac{1}{d-1}\delta_{jl}\partial^2\right)\epsilon$$
$$\pi^u_{jl} = \left(\partial_j \partial_l - \frac{1}{d-1}\delta_{jl}\partial^2\right)\partial_r u^r$$

Moreover, the properties of $A(\partial^2)$ etc determined by ω_h : $A(\partial^2) = i\,s\,T\,\omega_h\big|_{p^2=-\partial^2}$

This allows to translate the properties of the expansion of $A(\partial^2)$ <u>when acting on a solution</u> to the properties of ω_h and $|p_*|$, as well as $u^j(p)$

Real space example

2007.05524 with Serantes, Spaliński, Svensson, Withers

Let us take 0712.2451 by Baier, Romatschke, Son, Starinets, Stephanov:

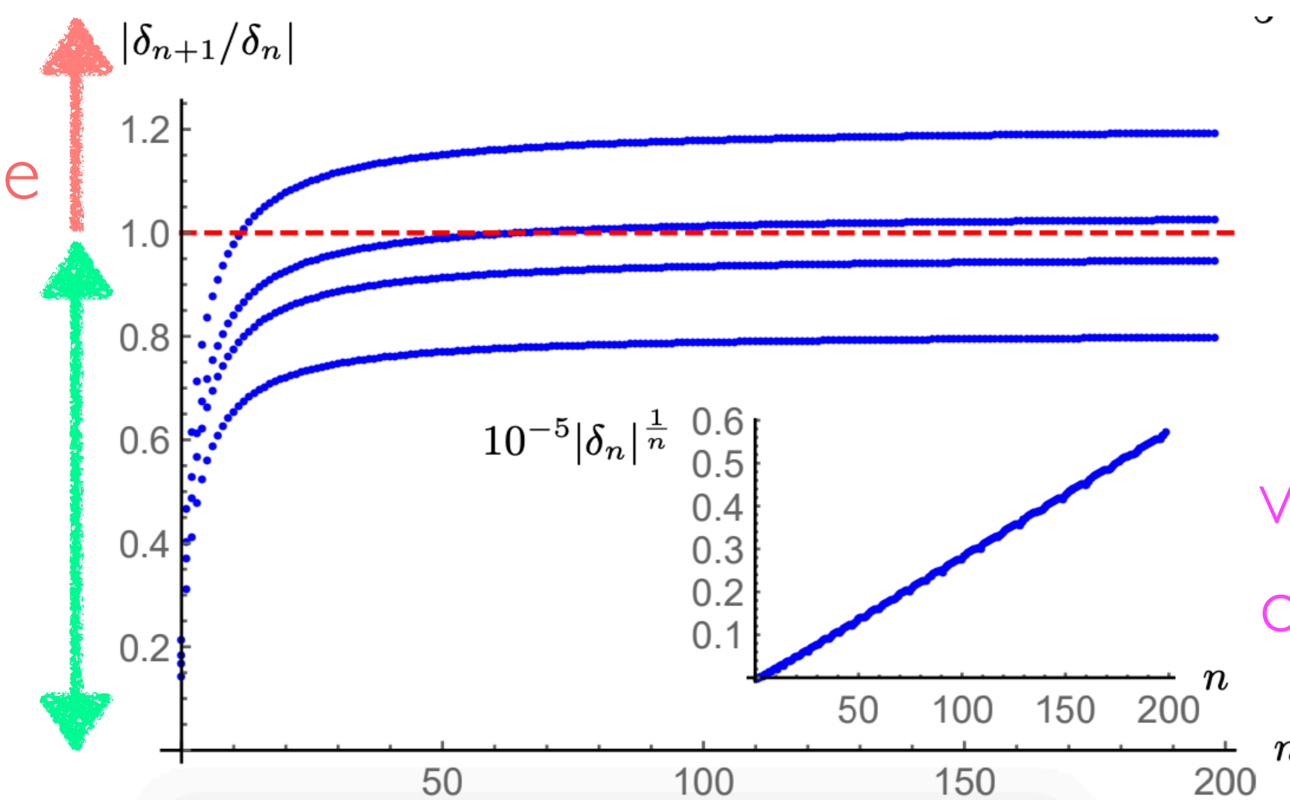
$$\omega_h = i\,\frac{-1+\sqrt{1-4\,\frac{\eta}{s\,T}\,\tau_\pi\,p^2}}{2\,\tau_\pi} \quad \text{and} \quad \omega_{transient} = i\,\frac{-1-\sqrt{1-4\,\frac{\eta}{s\,T}\,\tau_\pi\,p^2}}{2\,\tau_\pi}$$

We look at the initial conditions $u_1(0,p) = 0\,, \quad \partial_t u_1(0,p) = \frac{1}{2\pi} e^{-\frac{1}{2}\gamma^2 p^2}\,\Theta(p_{max}^2 - p^2)$

consider $\Pi_{1,3}(t,x) = A(\partial_x^2)\,\partial_x u_1(t,x) = \sum_{n=0}^{\infty} \delta_n$ with δ_n having $2n+1$ derivatives:

$|p_*| \leq p_{max} < \infty$ geometric divergence

$p_{max} < |p_*|$ convergence!

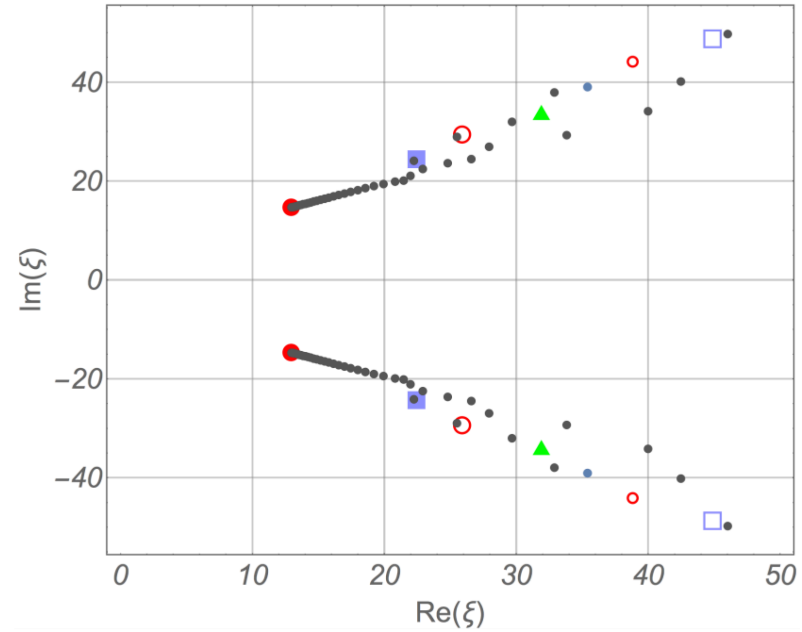


$p_{max} \to \infty$ vanishing radius of convergence

Outlook

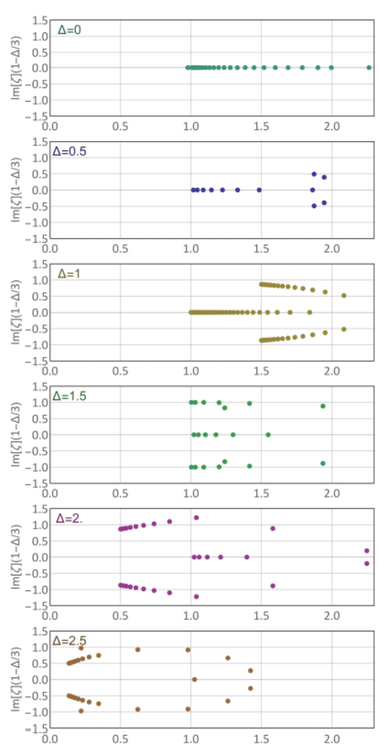
Outlook

http://online.kitp.ucsb.edu/online/resurgent_c17/heller/



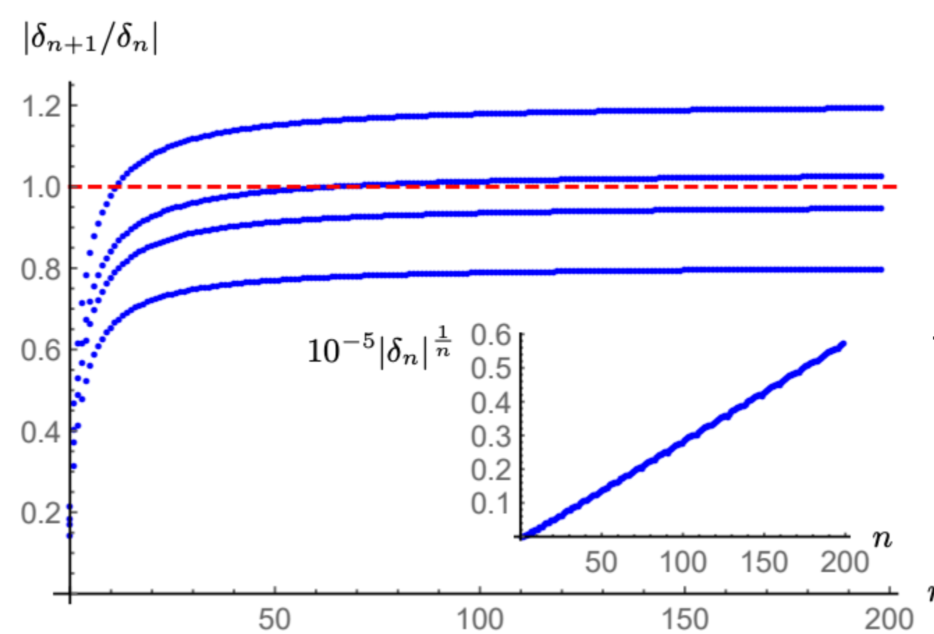
Bjorken flow:
divergent gradient
expansion + transients
= trans-series

1802.08225 with Svensson



kinetic theory:
richer mathematical
structure due to
integral eoms

2007.05524 with Serantes, Spaliński, Svensson, Withers



the first
statement
for generic
flows