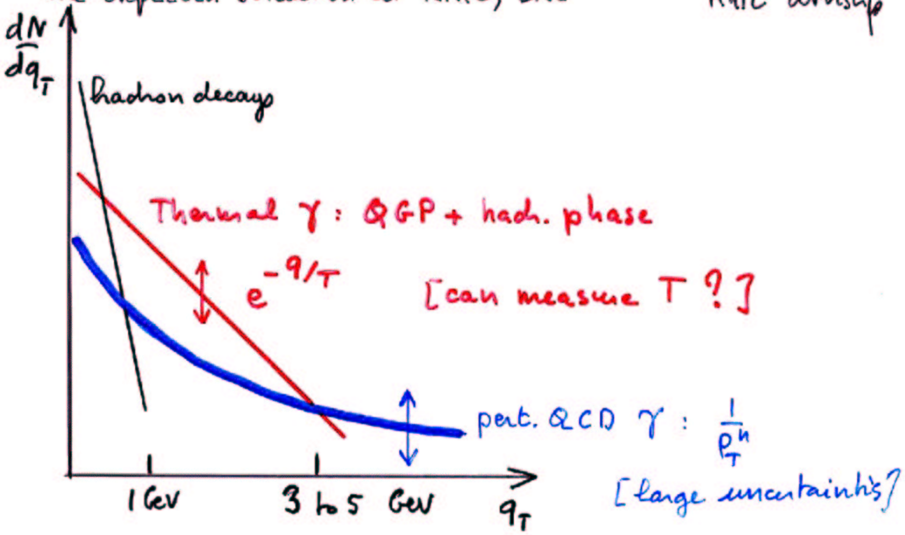
HARD PHOTON AND DILEPTON PRODUCTION IN A QUARK-GLUON PLASMA

P. Aurenche
RHIC workshop

The expected situation at RHIC, LHC



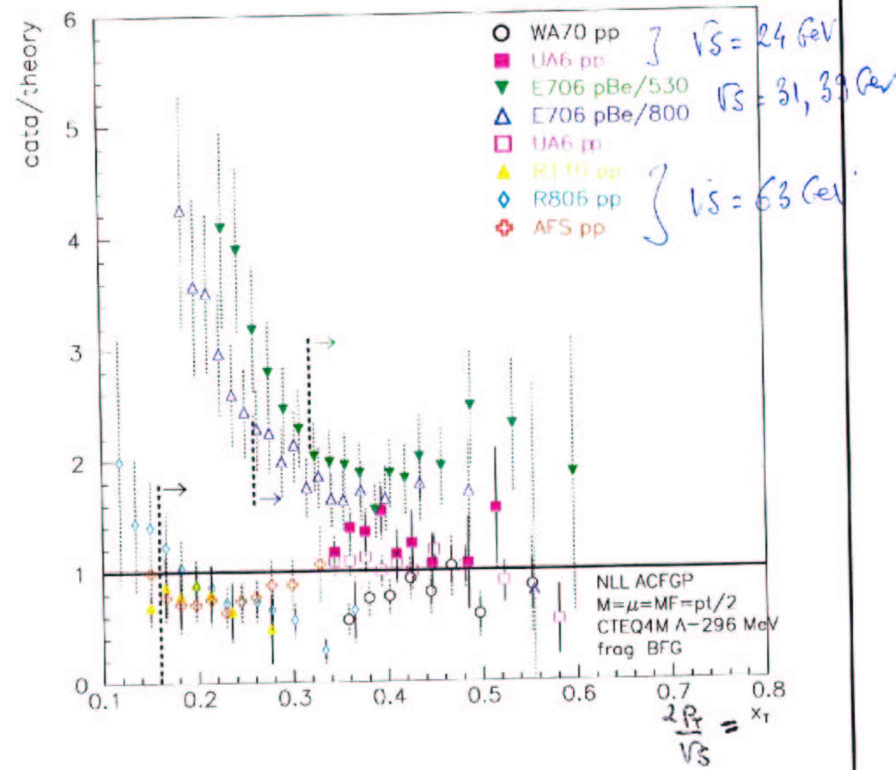
- One expects excess of γ in $2T < q_T < 5T$
  [based on equil. calcul. of rates + hydro evolution: Huovinen, Rasanen, Ruuskanen; Srivastava, ..]

- Also look at $\ell^+\ell^-$ pairs: in $M_{ee} \simeq 200 - 500$ MeV
  $2T < q_T < 5T$

  Backgrd should be smaller [P. Stankus]

Data/Theory CTEQ4M
scales $p_T/2$

$3.5 < p_T$ [GeV/c] < 10 GeV



E706 vs ISR: large discrepancies
WA70, UA6, ISR : approximate agreement with theory
E706: underestimated by a factor 2 to 4

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Calculate rate of γ, γ^*, in effective theory of Braaten-Pisarski : Hard Thermal Loop resummed

Assume equilibrium, T, $\mu = 0$ [$\mu \neq 0$ more complic.]

$g \ll 1$

1 loop

2 loop

∞ loop [cf Jaffe's talk]

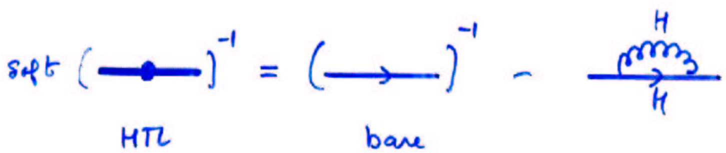
Why going to many loops in HTL? 2 puzzles.

— soft virtual γ^* at rest [B.P.Y. (1990)]

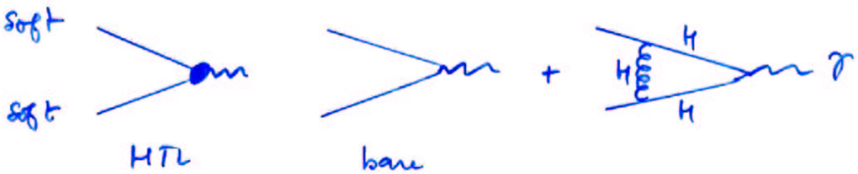
Rate ~ Im  $(\sqrt{Q^2}, \vec{0})$

soft ~ gT

hard ~ T



Hard particles in loop



1 loop HTL contains many processes

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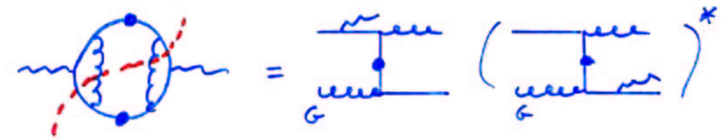
Rate $\supset$ 

Bremsβ in backward gq scatt.

but  never appears!

Cleymans, Goloviznin, Redlich (1993)

— Relation with bare theory

$$\mathrm{Im}\,\Pi^{pert}(Q^2)\Big|_{2\,loop} \sim e^2 g^2 T^2 \left(\boxed{\frac{\pi^2}{4}} + 2 \right) \ln \frac{T^2}{Q^2}$$

HTL $\qquad g^2T^2 \ll Q^2 \ll T^2$

$\Rightarrow$ go to 2-loop HTL (at least!).

Rates:

$$\gamma : \quad \frac{q_0\, dN}{d^3q\, d^4x} = -\frac{1}{2\pi^3}\, n_B(q_0)\, \mathrm{Im}\,\Pi(q_0 + i\varepsilon, \vec{q})$$

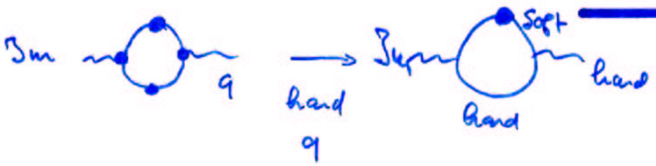
$$\gamma^* : \quad \frac{dN}{dq_0\, d^3q\, d^4x} = -\frac{\alpha}{12\pi^4}\, \frac{1}{Q^2}\, n_B(q_0)\, \mathrm{Im}\,\Pi(q_0 + i\varepsilon, \vec{q})$$

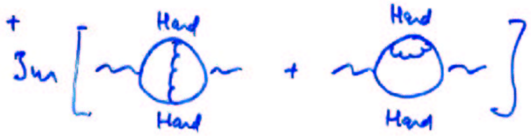
retarded 2 pt. function

$Q^2 = M^2_{e^+e^-}$: integrate over lepton angles.

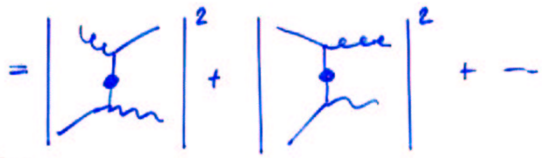
(4) – 1 loop results: review

q hard, $\frac{Q^2}{q_0^2} \ll 1$





Shuryak (1978)
Kajantie, Miettinen ('81)
" Ruuskanen ('83)
used hand made regulator



Kapusta, Lichard, Seibert ('91)
Baier, Nakkagawa, Niegawa, Redlich ('92)
used HTL regulator

soft exchanges — hard exch

$$\boxed{m_q^2 = C_F \frac{g^2T^2}{4} \sim \frac{g^2T^2}{3}}$$

Result:

$$\boxed{\frac{q_0\, dN}{d^3q\, d^4x} \sim e^2g^2T^2 e^{-\frac{q_0}{T}}\left[\ln\frac{Tq_0}{m_q^2} - \frac{1}{2} + \ln\frac{2}{3} - \gamma + \frac{\zeta'(2)}{\zeta(2)}\right]}$$

For γ^*

$$\frac{dN}{d^4q\, d^4x} \sim \frac{e^4g^2}{Q^2} T^2 e^{-\frac{q_0}{T}}\left[\ln\frac{TE}{Q^2} + c_1\right] \quad \text{Altherr-Ruuskanen ('92)}$$

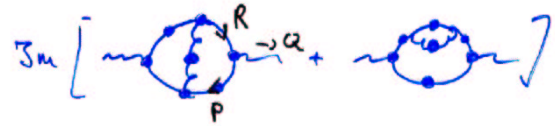
$$\sim \text{———} \left[\ln\frac{T f(E)}{m_q^2} + c_2\right] \quad \text{Thoma-Traxler ('97)}$$

$$\sim \text{———} \left[\ln\frac{TE}{f(m_q^2, Q^2)} + c_3\right] \quad \text{M. Carrington, F. Gelis ('03)}$$

(5) – 2 loop calculation

with F. Gelis
H. Zaraket
R. Kobes

In principle should calculate



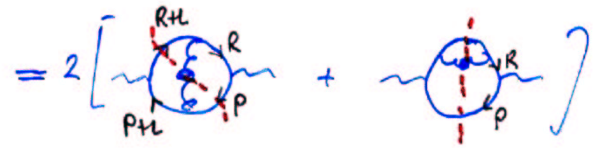
$R = P + Q$.

- Fermion loop integral dominated by hard momenta P, R

$$\frac{i}{\not P - \Sigma} = \text{———} \to \frac{\not{\tilde P}\, i}{P^2 - m_q^2 - i p_0 \varepsilon} \qquad \tilde P = (p_0, \sqrt{m_q^2 + p^2}\, \hat p)$$

$$= \pi\, \varepsilon(p_0)\, \delta(p^2 - m_q^2)\, \not{\tilde P}$$

Since p hard, R hard can (almost) neglect HTL corrections to vertices, so $[P, R \sim T, q^0]$



for real γ
or γ^*: $Q^2 < 4m_q^2$

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• For the gluon

$$G^{\mu\nu}(L) = \frac{P_T^{\mu\nu}}{L^2-\Pi_T} + \frac{P_L^{\mu\nu}}{L^2-\Pi_L} + G.T.$$

$$= \rho_T(\ell,x)\, P_T^{\mu\nu} + \rho_L(\ell,x)\, P_L^{\mu\nu}$$

spectral function : $L=(\ell_0,\vec{\ell}) = \ell(x,\hat{\ell})$, $x=\frac{\ell_0}{\ell}$

$$\boxed{\rho(\ell,x) = \frac{-2\,\mathrm{Im}\,\Pi(x)}{\left(\ell^2(x^2-1)-\mathrm{Re}\,\Pi(x)\right)^2+\left(\mathrm{Im}\,\Pi(x)\right)^2}}$$

Ex: $\mathrm{Im}\,\Pi_T = \frac{3\pi}{4} m_g^2\, x(1-x^2)\,\theta(1-|x|)$

$\mathrm{Re}\,\Pi_T = 3m_g^2\left[x^2+\frac{x}{4}(1-x^2)\ln\left|\frac{1+x}{1-x}\right|\right]$

$$\boxed{m_g^2 = \frac{g^2T^2}{9}\left(N_c+\frac{N_F}{2}\right)}\ ;$$

• Putting all together [use R/A formalism]

$$\mathrm{Im}\,\Pi^\gamma_{ret}(Q) \sim e^2g^2\sum_{T,L}\int\frac{d^4P}{(2\pi)^4}(n_F(r_0)-n_F(p_0))\int\frac{d^4L}{(2\pi)^4}\, n_B(\ell_0)\,\rho_{T,L}(\ell,x)$$

$$P^{\mu\nu}_{T,L}\,.\,\mathrm{Tr}^V_{\mu\nu}\ \mathrm{Re}\ \Delta_{ret}(P)\,\Delta_{ret}(P+L)\,\Delta_{adv}(R)\,\Delta_{adv}(R+L) + \text{self}.$$

$$\Delta_{ret}(P) = \frac{1}{P^2-m_q^2+ip_0\varepsilon}\ ,\quad \Delta_{adv}(P) = \frac{1}{P^2-m_q^2-ip_0\varepsilon}$$

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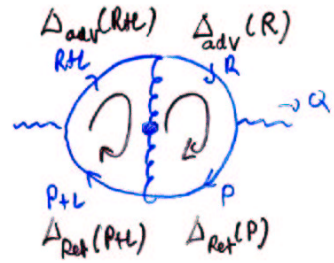

Consider pairs of poles

$\Delta_{Ret}(P)\ \Delta_{Adv}(R)$

Pick up poles in $\Delta_{Ret}(P)$

$$\Delta_{Adv}(R)\Big|_{p^2} = \frac{p_0}{q_0}\,\frac{1}{p_T^2+M_{eff}^2-4i\frac{p_0r_0}{q_0}\epsilon}$$

small denominator !

divergence if m_q, $Q^2=0$.

$$\boxed{M_{eff}^2 = m_q^2+\frac{Q^2}{q_0^2}p_0r_0}$$

$\sim g^2T^2$

More precisely

$$\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\,\Delta_{Ret}(P)\,\Delta_{adv}(R) = \frac{1}{2iq_0}\,\frac{1}{p_T^2+M_{eff}^2-4i\frac{p_0r_0}{q_0}\epsilon}$$

$$\int_{-\infty}^{\infty}\frac{d\ell_z}{2\pi}\,\Delta_{ret}(P+L)\,\Delta_{adv}(R+L) = \frac{1}{2iq_0}\,\frac{1}{(\vec{p}_T+\vec{\ell}_T)^2+M_{eff}^2-4i\frac{p_0r_0}{q_0}\epsilon}$$

with $\boxed{p_z\sim p_0\ ,\quad \ell_z\sim\ell_0}$

$$\Rightarrow \mathrm{Im}\,\Pi^\gamma_{Ret}(Q) \sim -e^2g^2\frac{1}{q_0^2}\int_{-\infty}^{\infty}dp_0\,(n_F(r_0)-n_F(p_0))\ d\vec{p}_T\ d\vec{\ell}_T$$

$\int d\ell_0\, n_B(\ell_0) \approx T\int\frac{dx}{x}$

$x=\ell_0/\ell$

$$T\int\frac{dx}{x}\,\rho_{T,L}(\ell,x)(-1)^L\,\frac{\ell_T^2\,(p_0^2+r_0^2)}{(p_T^2+M_{eff}^2)\left((\vec{p}_T+\vec{\ell}_T)^2+M_{eff}^2\right)}$$

$+$ self

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- $$\rho_{T,L}(\ell_T, x) = \frac{-2\,\mathrm{Im}\,\Pi_{T,L}(x)}{(\ell_T^2 + \underbrace{\mathrm{Re}\,\Pi_{T,L}(x)}_{g^2T^2})^2 + (\underbrace{\mathrm{Im}\,\Pi_{T,L}(x)}_{g^2T^2})^2}$$

All transverse variables constrained to be $\mathcal{O}(g^2T^2)$

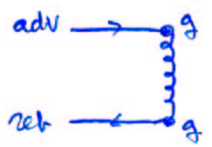
Easy to see:

$$\int d\vec{p}_T \frac{1}{p_T^2 + M^2_{eff}} \frac{1}{(\vec{p}_T + \vec{\ell}_T)^2 + M^2_{eff}} \sim \frac{1}{g^2T^2}$$

↳ kills g^2 in numerat.

Note:

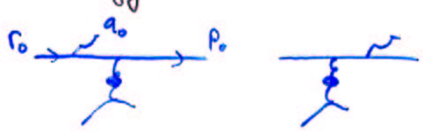
Insertion of



$$\sim g^2 \times \underbrace{\frac{1}{g^2}}_{} \sim \mathcal{O}(1)$$

from "pinching" singularities

- Energy variables.



$r_0 = p_0 + q_0$

$p_0 > 0 \Rightarrow r_0 > 0$ quark bremß

$p_0 < -q_0 \Rightarrow r_0 < 0$ quark bremß

: $\int dp_0 (n_F(r_0) - n_F(p_0))$
$\sim T$

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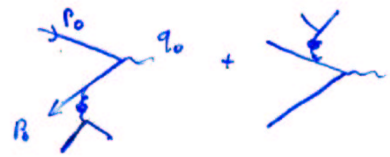

$-q_0 < p_0 < 0 \Rightarrow r_0 > 0$

off-shell annihilation

$\int dp_0 (n_F(p_0 + q_0) - n_F(p_0)) \sim q^0$

this process dominates at large q_0

- Results:

For real γ, $N_c = 3$, $N_F = 2$ integrals done exactly

$$\boxed{\mathrm{Im}\,\Pi^{\gamma}(q_0) = -\frac{2e^2g^2}{3\pi^3}\frac{5}{9}\Big(\underbrace{\pi^2\frac{T^3}{q_0}}_{\text{Bremß}} + \underbrace{q_0 T}_{\text{off-shell anni.}}\Big)}$$

Note $\mathrm{Im}\,\Pi^{\gamma}$ antisym in $q_0 / -q_0$ as it should.

- Sum rule: a small miracle.

$$\boxed{\int_{-1}^{1} \frac{dx}{2\pi}\frac{1}{x}\rho = \int_{-1}^{1}\frac{dx}{2\pi}\frac{1}{x}\frac{-2\,\mathrm{Im}\,\Pi(x)}{(\ell_T^2 + \mathrm{Re}\,\Pi(x))^2 + (\mathrm{Im}\,\Pi(x))^2} = \left[\frac{1}{\ell_T^2 + \Pi(0)} - \frac{1}{\ell_T^2 + \Pi(\infty)}\right]}$$

$\Pi_T(0) = 0$

$\Pi_L(0) = M_D^2 = 3m_g^2$

$\Pi_{T,L}(\infty) = m_g^2$

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What is needed is

$$\boxed{C(\ell_T) \ni \int_{-1}^{1} \frac{dx}{x} \frac{1}{2\pi} \left(\rho_T - \rho_L \right) = \Big[\underbrace{\frac{1}{\ell_T^2}}_{\text{no mag. mass}} - \frac{1}{\ell_T^2 + \underbrace{3m_g^2}_{\text{Debye mass}}} \Big]}$$

The plasmon mass drops out.

$C(\ell_T)$: kernel in Arnold, Moore, Yaffe integral

S.R. simplifies solution of int. equation.

- Details of sum rule:

Main difference: usually in sum rule

$$\int_{-\infty}^{\infty} \frac{dE}{2\pi} \frac{E\ \rho(\ell, \frac{E}{\ell})}{\ell_0^2 - E^2 + i\ell_0\varepsilon} = \frac{1}{\ell_0^2 - \ell^2 - \pi(\frac{\ell_0}{\ell})}$$

$$\rho(\ell, \tfrac{E}{\ell}) = \frac{-2\, \mathrm{Im}\, \pi(E/\ell)}{\left(\ell^2(1-x^2) - \mathrm{Re}\,\pi(E/\ell) \right)^2 + \left(\mathrm{Im}\,\pi(E/\ell) \right)^2}$$

usually : ℓ fixed, $\int dx$

we need $\ell^2(1-x^2) = \text{fixed} = -\ell_T^2$, $\int dx$

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Consider: $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{2\pi}\frac{1}{x} \frac{-2\,\mathrm{Im}\,\pi}{(z + \mathrm{Re}\,\pi)^2 + (\mathrm{Im}\,\pi)^2}$

$$= \int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{(1-x^2)}{x} \underbrace{\frac{-2\,\mathrm{Im}\,\bar{\pi}(x)}{(z(1-x^2) + \mathrm{Re}\,\bar{\pi})^2 + (\mathrm{Im}\,\bar{\pi})^2}}_{\bar{\rho}(z,x)}$$

$$\bar{\pi}(x) = (1-x^2)\, \pi(x)$$

and write usual S.R. with $\bar{\pi}$

$$\int_{-\infty}^{\infty} \frac{dE}{2\pi} E \frac{\bar{\rho}(z, E/\sqrt{z})}{\ell_0^2 - E^2 + i\ell_0\varepsilon} = \frac{1}{\ell_0^2 - z - \bar{\pi}}$$

$\ell_0^2 = z y^2$, $E^2 = z x^2$ and take real part

$$\int_{-\infty}^{\infty} \frac{dx}{2\pi} x \frac{\bar{\rho}(z,x)}{y^2 - x^2} = \frac{z(y^2-1) - \mathrm{Re}\,\bar{\pi}(y)}{(z(y^2-1) - \mathrm{Re}\,\bar{\pi}(y))^2 + (\mathrm{Im}\,\bar{\pi}(y))^2}$$

$y = 0$ limit $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{\bar{\rho}}{x} = \frac{1}{z + \bar{\pi}(0)}$

$y \to \infty$ limit $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{2\pi} x\, \bar{\rho} = \frac{1}{z + \bar{\pi}(\infty)}$

$$\boxed{\int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{(1-x^2)}{x}\, \bar{\rho} = \frac{1}{z + \bar{\pi}(0)} - \frac{1}{z + \bar{\pi}(\infty)}}$$

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For $|x|>1$ in HTL $\bar{\rho} = 2\pi\, \delta\big((1-x^2)z - \bar{\Pi}(x)\big)$

$$\Rightarrow (1-x^2)\big(z + \bar{\Pi}(x)\big) = 0$$

$x=\pm 1$ no solution $\bar{\Pi}(x) > 0$

Residues at $|x|=1$ are 0.

$$\boxed{\int_{-1}^{-1} \frac{dx}{2\pi}(1-x^2)\,\bar{\rho} = \frac{1}{z+\bar{\Pi}(0)} - \frac{1}{z+\bar{\Pi}(\infty)}}$$

- Result for γ, γ^* : general case

One has to do

$$\int d\vec{p}_T\, d\vec{\ell}_T\; C(\ell_T^2)\, \frac{\ell_T^2}{(p_T^2+M_{eff}^2)\big((\vec{p}_T+\vec{\ell}_T)^2+M_{eff}^2\big)} = F\left(\frac{m_g^2}{M_{eff}^2}\right)$$

contains m_g^2

dimensionless fn of ratio of scales

$$\mathrm{Im}\,\Pi^{\gamma}_{ret}(Q) = -\frac{e^2 g^2 N_c C_F}{(2\pi)^4}\, \frac{T}{q_0} \int_{-\infty}^{\infty} dp_0\, \big(n_F(r_0) - n_F(p_0)\big)$$

$$r_0 = p_0 + q_0 \qquad \left\{ (p_0^2+r_0^2)\, J\left(\frac{m_g^2}{M_{eff}^2}\right) + \frac{2Q^2 p_0 r_0 + m_g^2(p_0^2+r_0^2)}{M_{eff}^2}\, K\left(\frac{m_g^2}{M_{eff}^2}\right) \right\}$$

hard scales

soft scale

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- case $Q^2 > 4 m_q^2$

$$M_{eff}^2 = m_q^2 + \frac{Q^2 p_0 r_0}{q_0^2}$$

can vanish when $p_0 < 0,\ r_0 > 0$ i.e. off shell annihil.

$\Rightarrow$ I.R. cut-off vanishes et may be negative!

Related to fact that

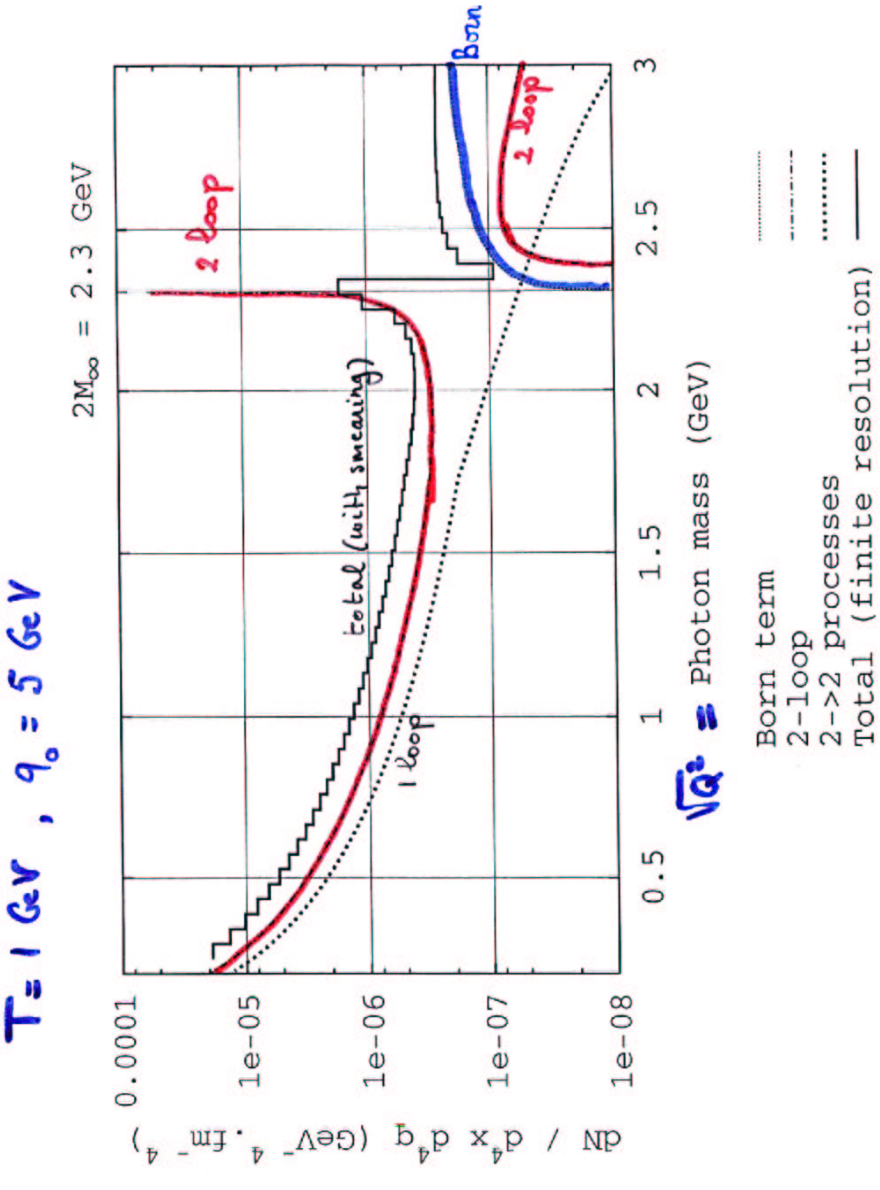
previously ignored

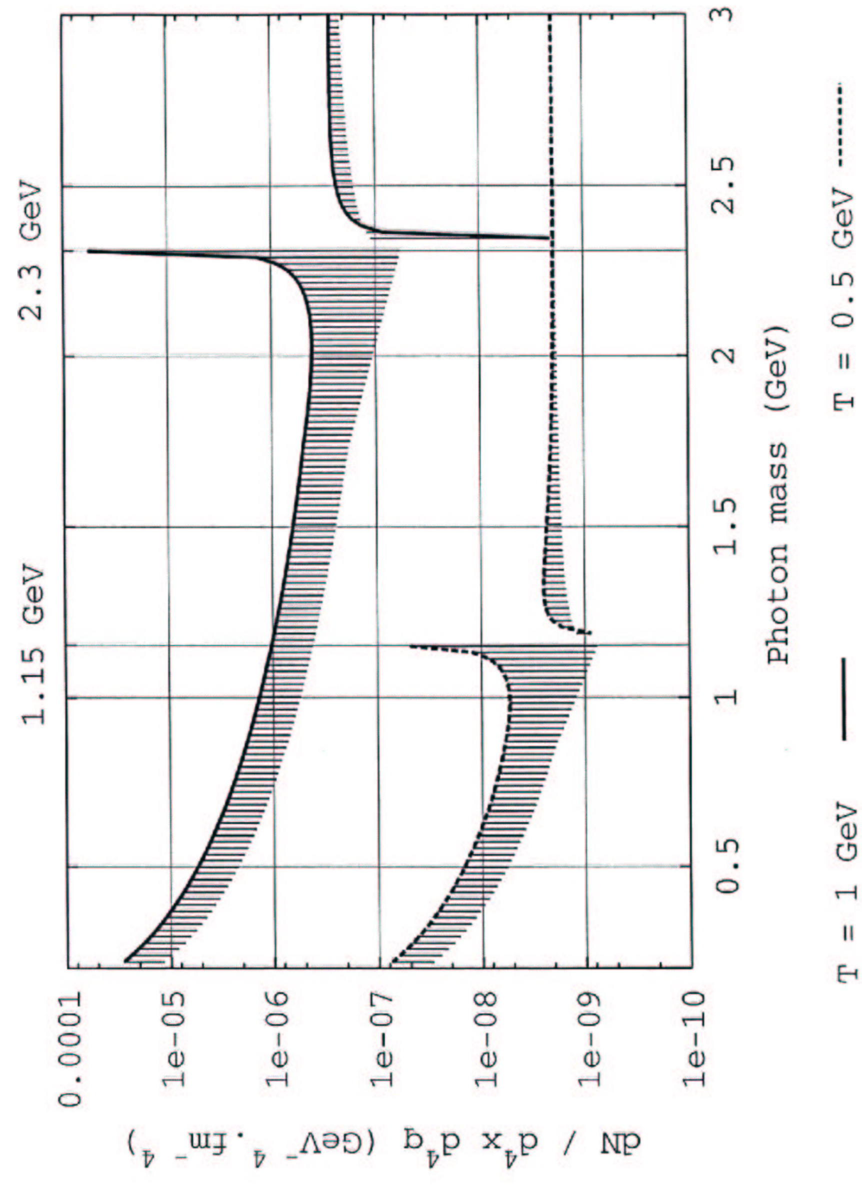
$\gamma^* \to q\bar{q}$

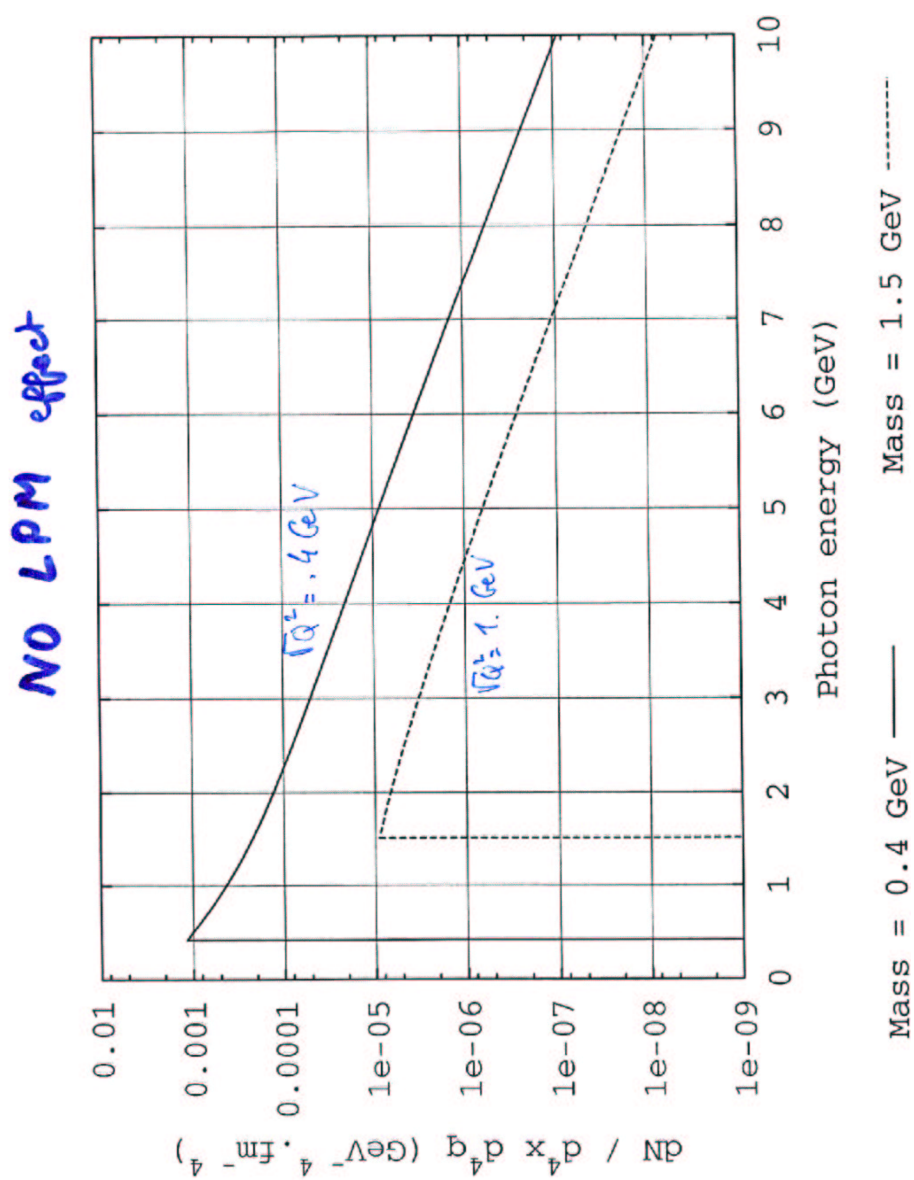
Each type of cut has I.R. singul. but cancel in sum.

Left with integrable singularities at threshold (as usual)

Figs


T = 1 GeV , q0 = 5 GeV
2M∞ = 2.3 GeV
2 loop
Born
2 loop
total (with smearing)
1 loop
√Q² ≡ Photon mass (GeV)
Born term
2-loop
2->2 processes
Total (finite resolution)
dN / d4x d4q (GeV-4.fm-4)


2.3 GeV
1.15 GeV
Photon mass (GeV)
T = 1 GeV
T = 0.5 GeV
dN / d4x d4q (GeV-4.fm-4)



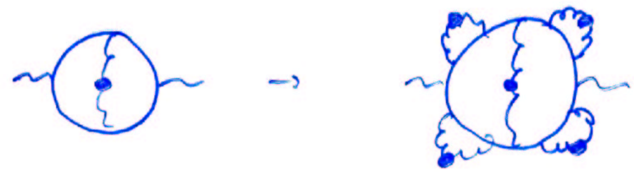
(14)

- Qualitative considerations on LPM effect.

We had to evaluate

$$\int dp_T^2 \, \frac{1}{p_T^2+M_{eff}^2 - i\frac{4p_0r_0}{q_0}\epsilon} \, \frac{1}{(\vec{p}_T+\vec{\ell}_T)^2+M_{eff}^2 - i\frac{4p_0r_0}{q_0}\epsilon}$$

ϵ from ret/adv. prescription



allow for rescattering of q in plasma

$$\int dp_T^2 \, \frac{1}{p_T^2+M_{eff}^2 - i\frac{4p_0r_0}{q_0}\Gamma} \, \frac{1}{(\vec{p}_T+\vec{\ell}_T)^2+M_{eff}^2 - i\frac{4p_0r_0}{q_0}\Gamma}$$

damping rate $\boxed{\Gamma \sim g^2 T}$

Competition between 2 cut-offs

$M_{eff}^2 \sim m_q^2$ and $\frac{p_0 r_0}{q_0}\Gamma$

q_0 small, bremß dominate $p_0 \sim r_0 \sim T$ $\frac{T^2}{q_0}\Gamma > m_q^2$, q_0 small

rescattering corrections
hence LPM effect important

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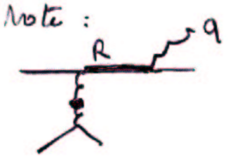
q_0 very large, annih. dominates

$p_0 \sim r_0 \sim q_0$ $\qquad \frac{p_0 r_0}{q_0}\Gamma \sim \underline{q_0 \Gamma > m_q^2}$

LPM important at large q_0

Note :



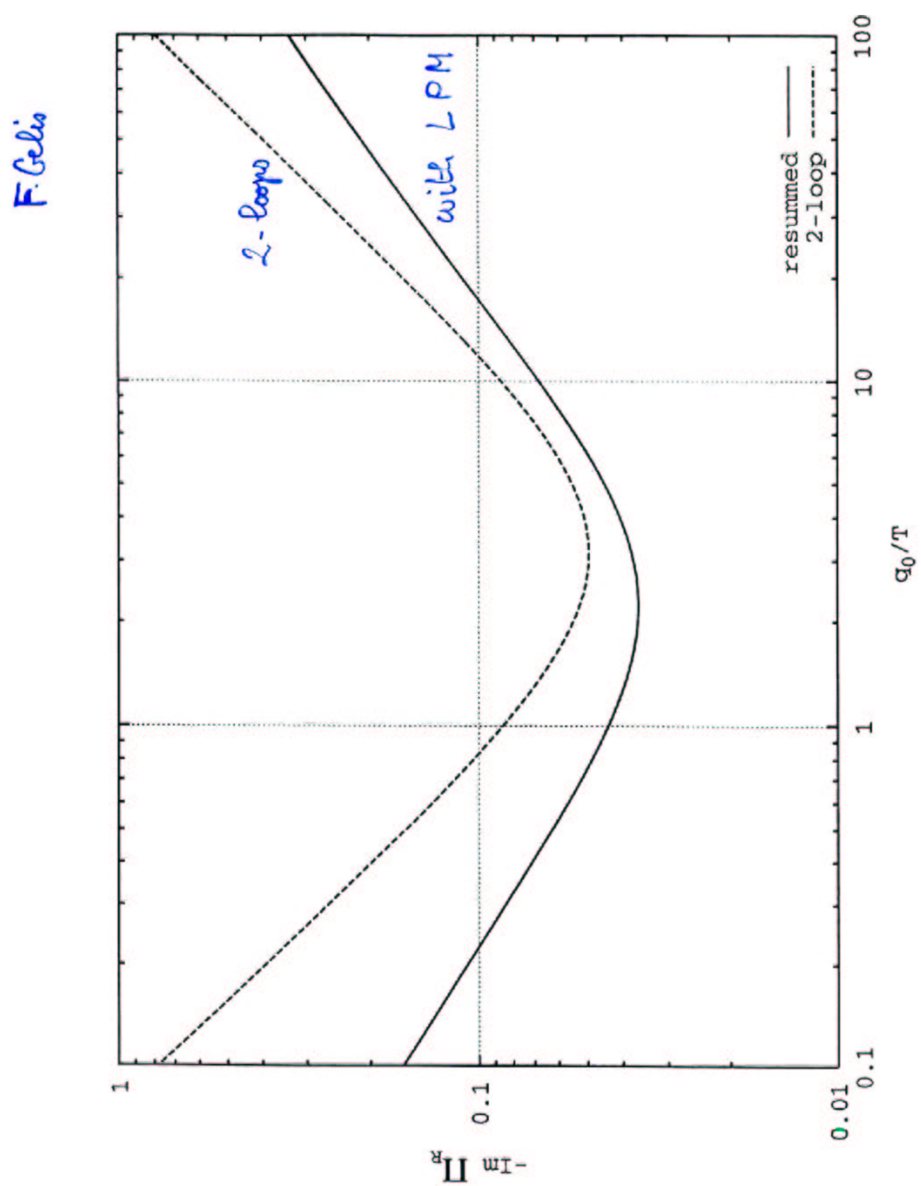
γ formation time $\tau_{form} \sim \frac{r_0}{R^2} \simeq \frac{p_0 r_0}{q_0 M^2_{eff}}$

Discussion above : $\boxed{\tau_{form} > \frac{1}{\Gamma} = \tau_{scatt}}$

LPM effect important

small $q_0 \ll T$

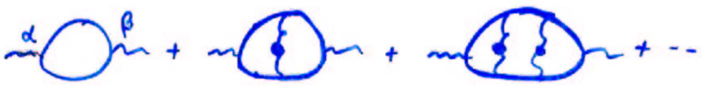
large $q_0 \gg T$



(16)

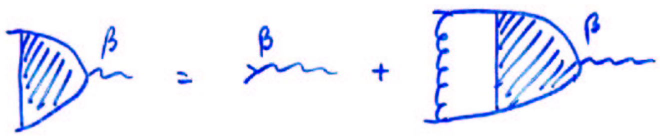
- **LPM effect : Arnold, Moore, Yaffe (2001)**

Ladder diagr. only ones with enhancement due to "pinch" singularities.

$\Pi_L^{\alpha\beta}(Q) =$ 

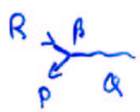
$=$ 

Vertex satisfies



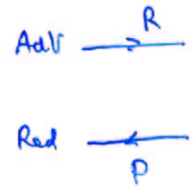
Rescatt corrections are summed on 

Building blocks

 $= \boxed{I^\beta(P,Q) = (P+R)^\beta}$

(17)

- Pair of fermion propagators (pinching)



$= F(P,Q)\quad , \quad R = P+Q.$

Recall : $\displaystyle\int \frac{dp_0}{2\pi}\, \Delta_{Ret}(P)\, \Delta_{Adv}(R) = \frac{-1}{2iq_0\left(p_T^2 + M_{eff}^2 - 2i\frac{p_0 r_0}{q_0}\Gamma\right)}$

$\displaystyle = -\frac{1}{4p_0 r_0}\;\frac{1}{\Gamma + i\delta E}$

$\displaystyle \delta E = q_0\,\frac{p_T^2 + M_{eff}^2}{p_0 r_0}$

$\displaystyle = \frac{1}{\tau_{formation}}$

$\displaystyle \Gamma = \frac{1}{\tau_{scat}}$

Undoing $\displaystyle\int \frac{dp_z}{2\pi}$, AMY defines

$$\boxed{F(P,Q) = -\frac{1}{4p_0 r_0}\;\frac{2\pi\,\delta(p_0 - p_z)}{\Gamma + i\delta E}}$$

- gluon exchange

$= g^2 C_F\, \mathrm{Im}\, iG_{\mu\nu}(L)\; \underbrace{2R^\mu\, 2P^\nu}_{\text{from quark couplings}} \qquad L \ll P$

$= g^2 C_F\, \frac{T}{l_0}\, \rho_{T,L}(l_\perp, x)\; \underbrace{2\pi\,\delta(l_0 - l_z)}_{\text{from pinch constraints}}\; \underbrace{2p^0\, 2r^0}_{\text{from collinear kinematics}}$

$= 2p_0\, 2r_0\; M(L)$

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The rate is

$$\mathrm{Im}\,\Pi^{\alpha\beta}(Q) = \int \frac{d^4P}{(2\pi)^4} \underbrace{\left(n_F(r_0)-n_F(p_0)\right)}_{\sim\, n_F(r_0)(1+n_F(p_0))} I^{\alpha}(P,Q)\,\mathrm{Re}\Big(\underbrace{F(P,Q)\,D^{\beta}(P,Q)}\Big)$$

$$D^{\beta}(P,Q) = I^{\beta}(P,Q) + \int\frac{d^4L}{(2\pi)^4}\, M(L)\, 2p_0 2r_0\, F(P+L,Q)\, D^{\beta}(P+L,Q)$$

Write an integral eq for $\underbrace{F(P,Q)\,D^{\beta}(P,Q)}$, so

multiply $\int \frac{dp_3}{2\pi} F(P,Q)\, D^{\beta}(P,Q)$

$$\int\frac{dp_z}{2\pi} F(P,Q)\,D^{\beta}(P,Q) = \frac{(P\cdot R)^{\beta}}{-4p_0r_0(\Gamma+i\delta E)} - \int\frac{d^4L}{(2\pi)^4} M(L)\,\frac{1}{\Gamma+i\delta E}\, F(P+L,Q)\,D^{\beta}(P+L,Q)\Big|_{p_0=p_z}$$

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AMY define

$$f^{\beta}(p_T,p_0) = -4p_0r_0\int\frac{dp_z}{2\pi} F(P,Q)\,D^{\beta}(P,Q)$$

Integral equation becomes

$$(P\cdot R)^{\beta} = (\Gamma+i\delta E)\, f^{\beta}(\vec p_T,p_0) - \int\frac{d^4L}{(2\pi)^4} M(L)\, f^{\beta}(\vec p_T+\vec\ell_T,p_0)$$

Last trick: absorb Γ in integral

Im  $\sim p^0\,\Gamma = g^2C_F\int\frac{d^4L}{(2\pi)^4}\, 2P^{\mu}2P^{\nu}\; \mathrm{Im}\, i\,G^{T,L}_{\mu\nu}(L)\;\; \pi\,\delta((P+L)^2)$

$(P+L)^2=0 \Leftrightarrow P\cdot L=0$

$\Leftrightarrow p(\ell_0-\ell_z)=0$

$$p_0\,\Gamma = g^2C_F\int\frac{d^4L}{(2\pi)^4}\, 2p^0\, 2r^0\, \frac{T}{\ell_0}\rho_{T,L}\;\frac{2\pi\,\delta(\ell_0-\ell_z)}{4p_0}$$

$$\Gamma = \int\frac{d^4L}{(2\pi)^4}\, M(L)$$

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$$(P+R)^\beta = i\,\delta E\, f^\beta(\vec{p}_T, p_0) + \int \frac{d^4L}{(2\pi)^4}\, M(L)\left[f^\beta(\vec{p}_T, p_0) - f^\beta(\vec{p}_T + \vec{\ell}_T, p_0)\right]$$

$$\mathrm{Im}\, \Pi^{\alpha\beta}(Q) \cong e^2 \int \frac{dp_0}{2\pi}\frac{d\vec{p}_T}{(2\pi)^4} \left(n_F(r_0) - n_F(p_0)\right) \frac{p_0^2 + r_0^2}{p_0^2 r_0} \mathrm{Re}\left[(P^\alpha + R^\alpha)\, f^\beta(\vec{p}_T, p_0)\right]$$

spin $\frac{1}{2}$ fermions

Only transverse part contribute to production rate

$$\boxed{\mathrm{Im}\, \Pi^T(Q^2) = 2e^2 \int \frac{dp_0}{2\pi}\left(n_F(r_0) - n_F(p_0)\right) \frac{p_0^2 + r_0^2}{p_0^2 r_0^2} \int \frac{d\vec{p}_T}{(2\pi)^2}\, \mathrm{Re}\, \vec{p}_T \cdot \vec{f}(\vec{p}_T)}$$

$$\boxed{2\vec{p}_T = i\,\delta E\, \vec{f}(\vec{p}_T) + \int \frac{d\vec{\ell}_T}{(2\pi)^2}\, C(\ell_\perp)\left[\vec{f}(\vec{p}_T) - \vec{f}(\vec{p}_T + \vec{\ell}_T)\right]}$$

since we have proven, by sum rule, that

$$\int \frac{d\ell_0}{2\pi}\frac{d\ell_3}{2\pi}\, M(L) = g^2 C_F T\left[\frac{1}{\ell_T^2} - \frac{1}{\ell_T^2 + 3m_g^2}\right]$$

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By Fourier transforming

$$f(\vec{p}_T) = \int d^2b\; e^{-i\vec{p}_T\cdot\vec{b}}\, g(b)$$

one obtains a differential equation rather than an integral equation.

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To be done :

- Use more realistic m_g, m_q from lattice rather than HTL parameters.
- LPM for virtual photon : should be less important
- Fugacities.