Generalized Double Copies and the Five-Loop Four-Point Integrand of $\mathcal{N} = 8$ Supergravity

KITP

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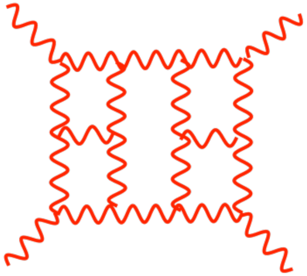
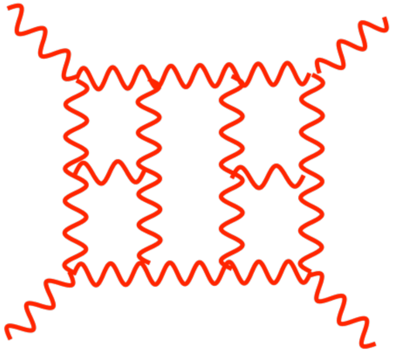


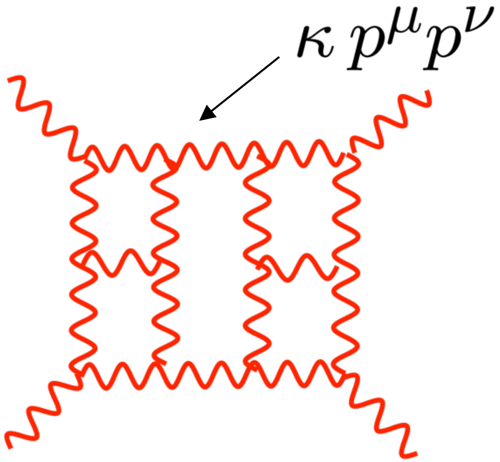
Outline

1. Why we care about supergravity amplitudes at high-loop orders. Understand structure of scattering. UV.
2. Color-kinematics duality and double copy.
3. Difficulties at 5 loops, causing multi-year delay.
4. Striking structure at 5 loops.
5. Exploiting the structure: Converting gauge theory to gravity.
6. Construction of 5-loop integrand of $N = 8$ supergravity.



UV Behavior of Gravity?

$\kappa = \sqrt{32\pi G_N}$ ⟵ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**
- ***N* = 8 supergravity best theory to study.**

UV Behavior of Gravity

What is the actual UV behavior of $N = 8$ supergravity?

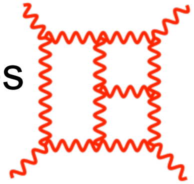
Not a philosophical question, but a technical question.

By trying to answer this question we learn a lot about gravity

Feynman Diagrams for Gravity

Suppose we want to check UV properties of gravity theories:

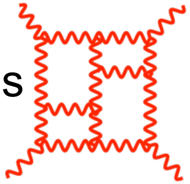
3 loops



~10^{20} TERMS

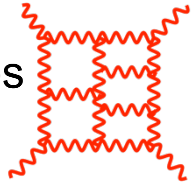
No surprise it has never been calculated via Feynman diagrams.

4 loops



~10^{26} TERMS

5 loops



~10^{31} TERMS

More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

Supersymmetry helps, but not enough to make a difference.

$N = 8$ supergravity: Where is First $D = 4$ UV Divergence?

3 loops **$N = 8$**	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops **$N = 8$**	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops **$N = 8$**	Howe and Stelle (2003)	✗
7 loops **$N = 8$**	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
3 loops **$N = 4$**	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops **$N = 5$**	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops **$N = 4$**	Vanhove and Tourkine (2012)	✓
9 loops **$N = 8$**	Berkovits, Green, Russo, Vanhove (2009)	✗

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

← **Don't bet on divergence**

← **Weird structure. Anomaly-like behavior of divergence.**

← **retracted**

- **Conventional wisdom holds that it will diverge sooner or later.**
- **But every detailed prediction either wrong or misleading.**

Divergences in Pure Gravity

Goroff and Sagnotti, ZB, Cheung, Chi, Davies, Dixon and Nohle; ZB, Chi, Dixon, Edison

Even in pure gravity, UV divergences don't work the way you were taught.

$$D = 4 - 2\epsilon$$

$$\mathcal{M}_4 = \left[\frac{1}{\epsilon}\left(\frac{209}{24} - \frac{15}{2}n_3\right) - \frac{1}{4}\ln\mu^2\right]\mathcal{K} + \text{finite} + \text{IR}$$

Renormalization scale

robust

number of 3 forms

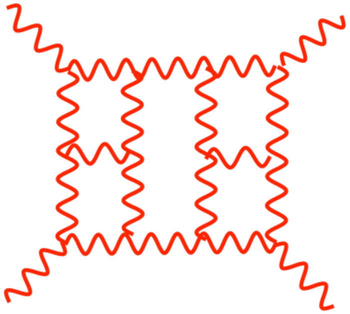
divergence

$$\mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

- **Weird that renorm. scale and UV divergence not linked.**
- **The UV divergence depends on the details of the regularization prescriptions. 3 forms not dynamical.**
- **The divergence itself comes from anomaly-like behavior:** $\epsilon/\epsilon \times 1/\epsilon$
- **Only other pure (super)gravity theory with known divergences is $N = 4$ supergravity: Similar anomaly-like behavior.**
- **No such anomaly expected in $N \geq 5$ sugra.**

Carrasco, Kallosh, Tseytlin and Roiban; Kallosh; Freedman, Kallosh, Murli, Van Proeyen, Yamada

New Structures?



Might there be a new unaccounted structure in gravity theories that suggests the UV might be is tamer than conventional arguments suggest?

Yes!

Duality Between Color and Kinematics

Consider five-point tree amplitude:

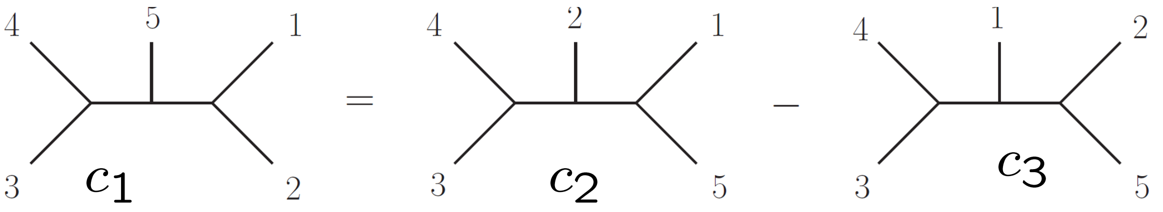
ZB, Carrasco, Johansson (BCJ)

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_i^2}$$

color factor

kinematic numerator factor

Feynman propagators



$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 + c_2 + c_3 = 0 \;\Leftrightarrow\; n_1 + n_2 + n_3 = 0}$$

Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.

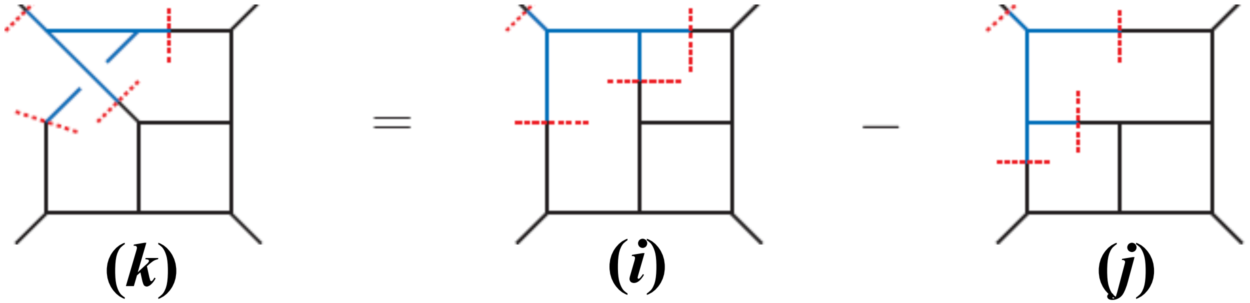
Progress on unraveling relations.

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer;
Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer
O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White, etc.

BCJ

Gravity Loop Integrands from YM

Ideas conjectured to generalize to loops:



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory → gravity theory

simply take

color factor → kinematic numerator

$$c_k \longrightarrow n_k$$

Gravity loop integrands follow from gauge theory!

Gravity From Gauge Theory

$$-i\left(\frac{2}{\kappa}\right)^{(n-2)}\mathcal{M}_n^{\text{tree}}(1,2,\ldots,n)=\sum_i\frac{n_i\,\tilde{n}_i}{\prod_{\alpha_i}p^2_{\alpha_i}}$$

BCJ

Here we consider only simplest constructions:

Johansson's talk

$\mathcal{N}=8$ sugra: $(\mathcal{N}=4$ sYM$)\times(\mathcal{N}=4$ sYM$)$
$\mathcal{N}=5$ sugra: $(\mathcal{N}=4$ sYM$)\times(\mathcal{N}=1$ sYM$)$
$\mathcal{N}=4$ sugra: $(\mathcal{N}=4$ sYM$)\times(\mathcal{N}=0$ sYM$)$

Spectrum controlled by simple tensor product of gauge theories.

More sophisticated lower-susy cases: QCD, magical supergravities, Einstein-YM with and without Higgsing, twin supergravities.

Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle; Nohle; Chiodaroli, Günaydin, Johansson, Roiban. A. Anastasiou, L. Borsten, M.J. Duff, M.J. Hughes, Marrani, Nagy, Zoccali.

Many other theories in double-copy story, including open and closed string theory, NLSM, (Dirac)Born Infeld, Galileon and Z theory.

Cachazo, He, Yuan; Chen Du, Broedel, Schlotterer and Stieberger; Carrasco, Mafra, Schlotterer;

Supergravity and Ultraviolet Divergences

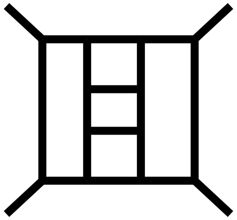
Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

- **First quantized formulation of Berkovits' pure-spinor formalism.** Bjornsson and Green
- **Unitarity method.** ZB, Davies, Dennen

Key point: *all* supersymmetry cancellations are exposed.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense" Bjornsson and Green

- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ? ← **Want to check this.**
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.** ?
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗



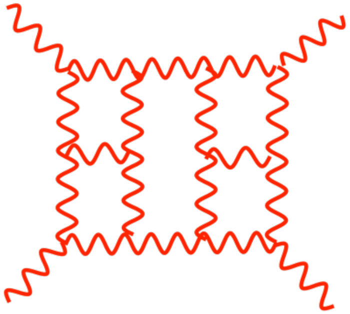
Consensus agreement from all methods

These new types of cancellations do exist: "enhanced cancellations".

ZB, Davies, Dennen

$N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Chen Johannson, Roiban



~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same.

Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

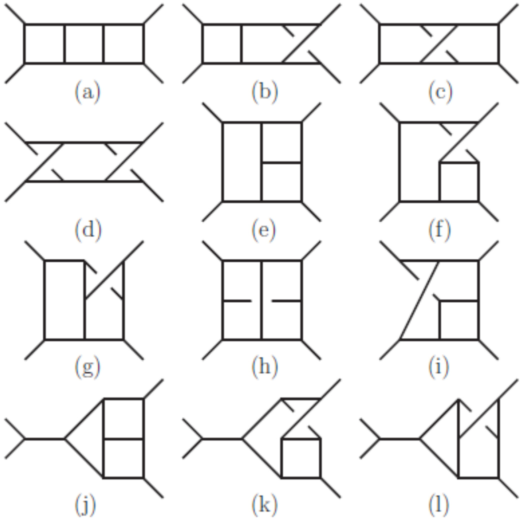
5 loops

Kelly Stelle:
English wine
"It will diverge"

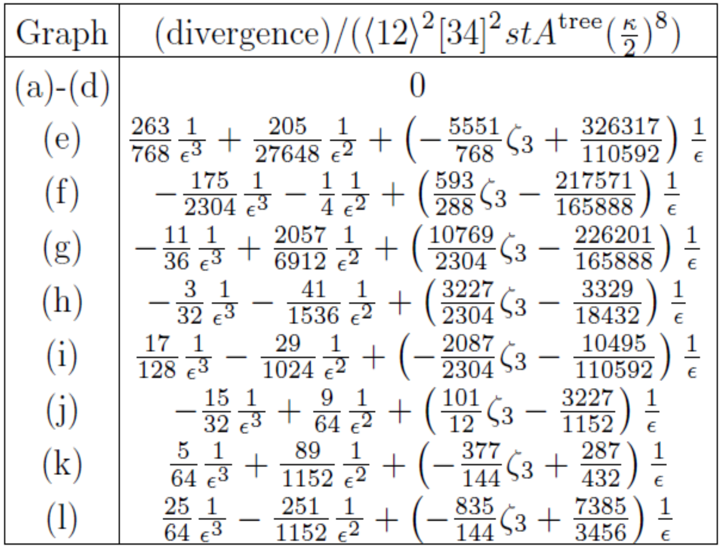
Zvi Bern:
California wine
"It won't diverge"

$N = 4$ Supergravity UV Cancellation

BCJ duality works easily



$D = 4 - 2\epsilon$ ZB, Davies, Dennen, Huang



Graph	$(\text{divergence})/(\langle 12\rangle^2 [34]^2 st A^{\text{tree}} (\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

($N = 4$ sugra) = ($N = 4$ sYM) x ($N = 0$ YM)

***All* three-loop divergences and subdivergences cancel completely!**

Still no standard-symmetry explanation, despite valiant attempt.

Bossard, Howe, Stelle; ZB, Davies, Dennen

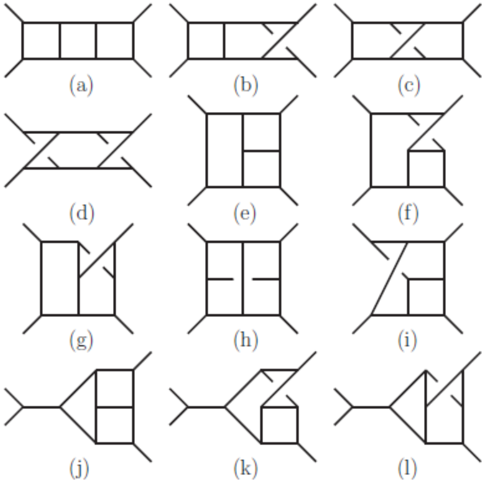
A nontrivial example of "enhanced cancellations"

Enhanced UV Cancellations

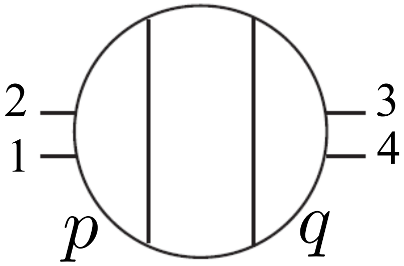
ZB, Davies, Dennen

Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.

- **By definition this is an enhanced cancellation.**
- **Not the way nonabelian gauge theory works.**



$N = 4$ sugra



already log divergent

$N = 4$ sugra: pure YM x $N = 4$ sYM

$$n_i \sim s^3 t A_4^{\text{tree}} (p \cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

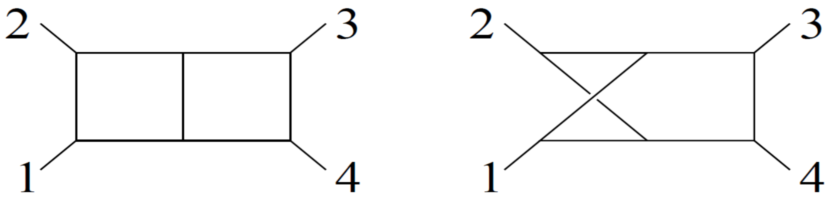
- **3 loop UV finiteness of $N = 4$ supergravity proves existence of “enhanced cancellation” in supergravity theories.**
- **No known standard symmetry explanation.**

Where does new magic come from?

ZB, Davies, Dennen, Huang; Bossard, Howe, Stelle

To analyze we need a simpler example: Half-maximal supergravity in *D* = 5 at 2 loops. No known symmetry explanation in this case.

Similar to *N* = 4, *D* = 4 sugra at 3 loops, except much simpler.



***D* = 5 half max sugra**
***N* = 4 sYM x *N* = 0 YM**

Quick summary:

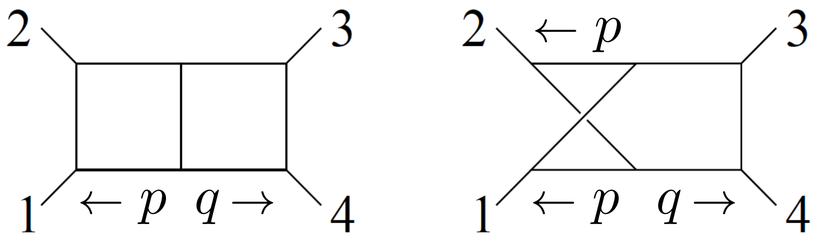
- **Finiteness in *D* = 5 tied to double-copy structure.**
- **Cancellations in certain forbidden gauge-theory color structures imply hidden UV cancellations in supergravity.**

Double-copy structure implies extra cancellations!

Unfortunately, argument relies on special two-loop property: integrals of *N* = 4 sugra are identical to those of QCD.

Need a more general approach

Multiloop Cancellations



ZB, Enciso, Parra-Martinez, Zeng

route momenta in different ways

- **Demonstrated enhanced cancellations require integration properties. Not like non-abelian gauge theory.**
- **Standard proofs of UV properties are ruled out.**

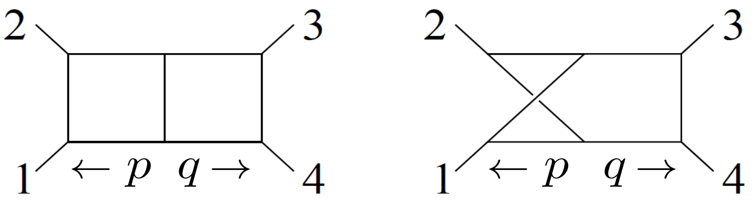
We make use of enormous advances in understanding relations between integrals based on IBP technology.

Gluza, Kajda, Kosower ; Johansson, Kosower and Larsen; Ita; Larsen and Zhang

Allows us to write integrands manifestly UV finite up to terms that integrate to zero.

Is there a generic structure for the enhanced cancellations?

Multiloop Enhanced Cancellations



ZB, Enciso, Parra-Martinez, Zeng (2017)

Analysis of cancellations in half-maximal supergravity in *D* = 5 reveals following interesting pattern.

Conjecture: At large loop momentum enhanced cancellations follow from Lorentz symmetry and SL(*L*) relabeling symmetry.

Lorentz symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D \ell_a\Big) \sum_{a=1}^{L} \Big(\ell_{a\mu} \frac{\partial}{\partial \ell_a^\nu} - \ell_{a\nu} \frac{\partial}{\partial \ell_a^\mu}\Big) \frac{\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

SL(*L*) relabeling symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D \ell_a\Big) \sum_{a=1}^{L} \frac{\partial}{\partial \ell_a^\nu} \frac{\omega_{ab} \ell_{b\mu} \mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

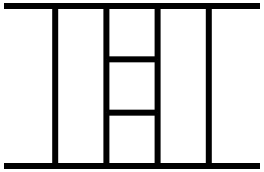
L loops

Symmetries generate a generic set of identities between integrals

Finding BCJ Forms Nontrivial

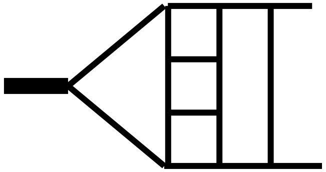
Gravity integrands might be free, but gauge-theory ones are not. Trouble beyond four loops.

5-loop 4-pt $\mathcal{N} = 4$ sYM amplitude:



Despite considerable effort no one has succeeded in finding a BCJ form.

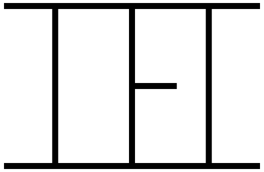
$\mathcal{N} = 4$ sYM 5 loop form factor:



On other hand, no trouble with form factor.

Gang Yang (2016)

Two-loop five-point QCD identical helicity:



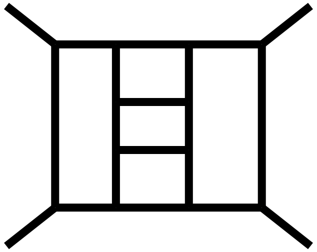
This required an ansatz with curiously high power counting.

O'Connell and Mogull (2015)

It can be difficult to find BCJ representations.

Five-Loop $N = 8$ Supergravity

$N = 4, 5$ supergravity complicated. $N = 8$ supergravity simpler.



Are the expected enhanced cancellations actually present?

Turns out to be quite nontrivial to find BCJ representations:

- Need ansatz (guess) for numerators.
- Large linear systems (up to 10^6 parameters).
- No one has succeeded as yet, despite considerable effort.

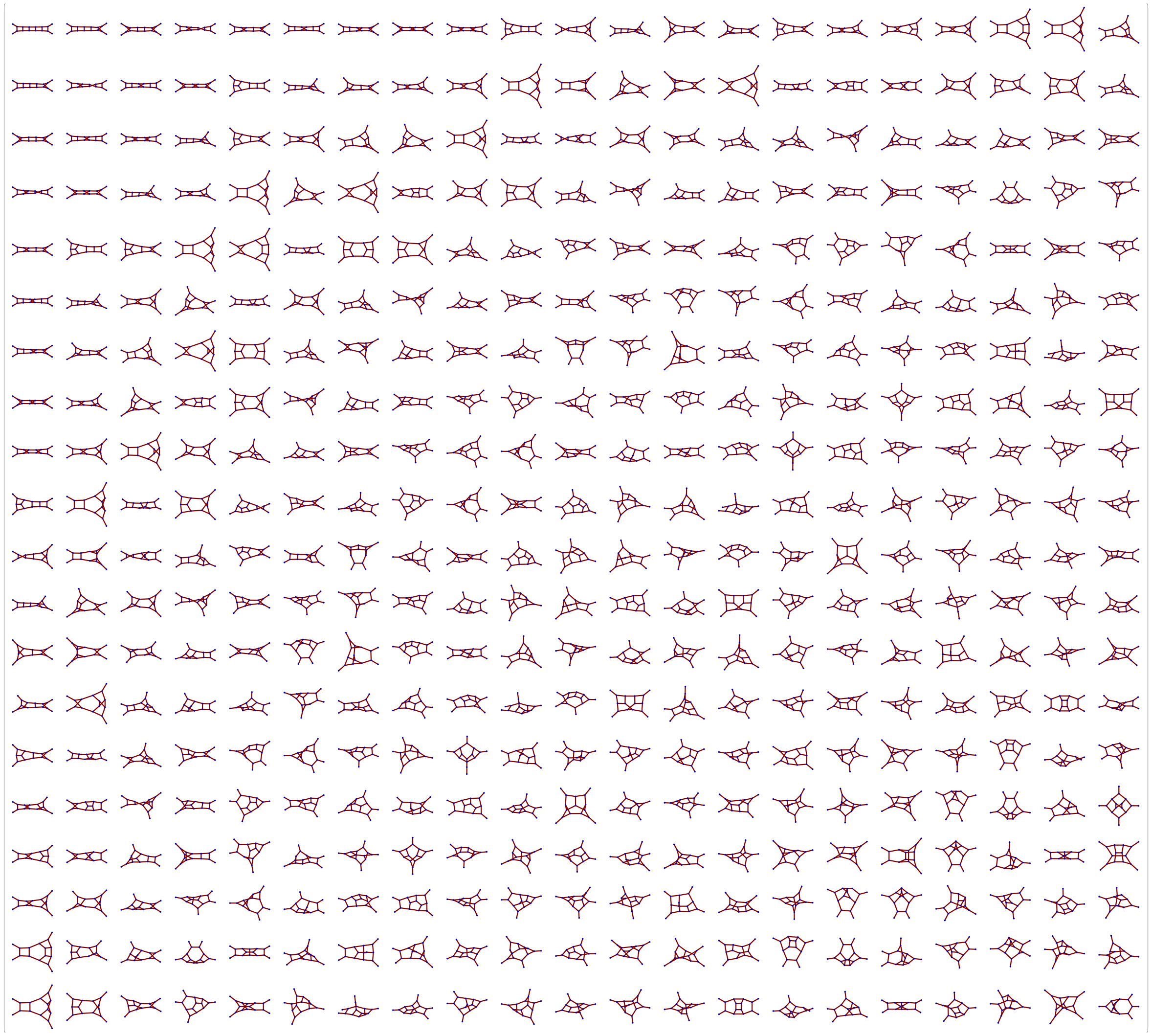
Need a better way:

- Banish large Ansatze or guessing.
- No large linear systems. Get amplitude one piece at a time.
- Stick to sYM as long as possible and convert to sugra at end.
- Keep double-copy idea. Essential!

The $\mathcal{N} = 4$ sYM 5 Loop Integrand

ZB, Carrasco Johansson, Roiban, arXiv:1207.6666

410 diagrams



Want to convert this to $\mathcal{N} = 8$ supergravity.

Contact Term Method

ZB, Carrasco, Chen, Johansson, Roiban

Task is to convert $\mathcal{N} = 4$ sYM 5-loop integrand into $\mathcal{N} = 8$ sugra.

Start with "naïve double copy" of *any* correct sYM integrand:

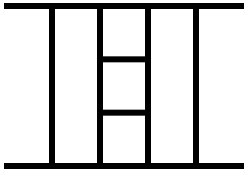
$\mathcal{N} = 4$ sYM $\displaystyle\sum_{\text{diags } i} \frac{c_i n_i}{D_i}$ **$\mathcal{N} = 8$ sugra** $\displaystyle\sum_{\text{diags } i} \frac{n_i n_i}{D_i}$

φ^3 **diagrams**

Without BCJ duality, not the correct $\mathcal{N} = 8$ integrand

$\mathcal{N} = 8$ cuts:

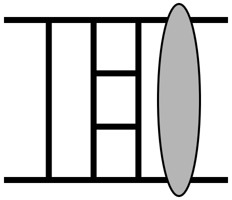
Max cuts:



Automatic

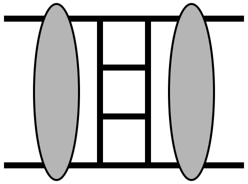
Generalized Unitarity
All exposed legs
on shell

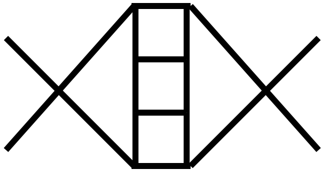
Nmax cuts:



Automatic via BCJ

N^{2}max cuts:



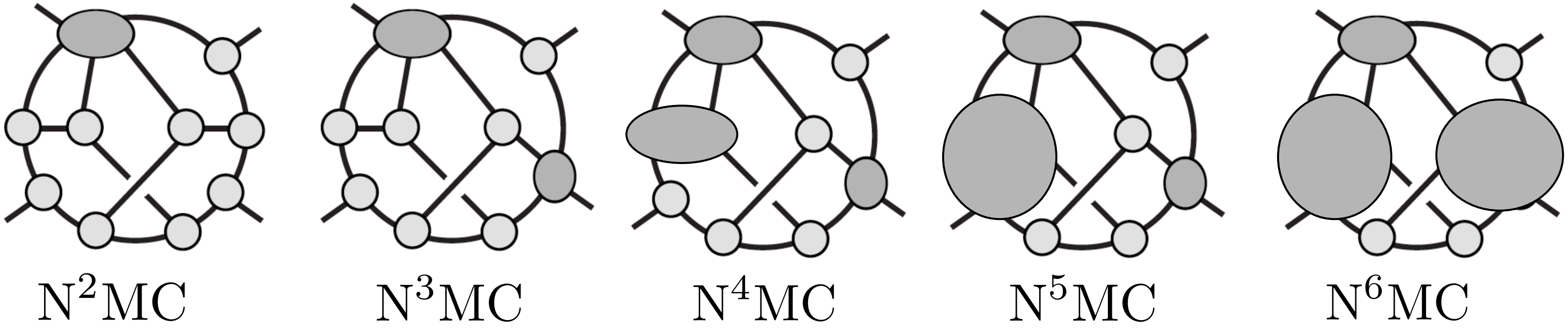


Add contact term to make it work

Blobs interfere: not automatic

Contact Term Method

contact = (gravity cut) – (cut of incomplete amplitude)



Cuts become complicated

analytical → **numerical**

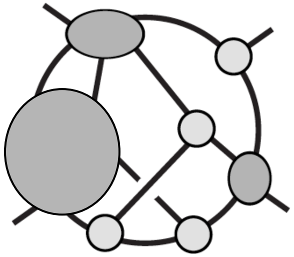
Missing contact terms become simple for complicated cuts

N^6 MC contact numerator: $a_1 s^2 + a_2 st + a_3 t^2$

- **Contact associated with cut directly giving missing piece of amplitude.**
- **75K cuts need to be evaluated.**
- **Sounds daunting. Not for faint of heart.**

Game for optimists: "Simplifying miracle is around the corner"

Generating SUGRA Cut



N^{5}MC

$$(\text{SUGRA cut}) = \sum_{\substack{N=8\\ \text{states}}} M_1^{\text{tree}} M_2^{\text{tree}} \cdots M_m^{\text{tree}}$$

Gravity tree ampl

KLT kernel

Color order YM tree

KLT Relation:

$$M_n^{\text{tree}} = \sum_{i,j} K_{i,j} (A_n^{\text{tree}})_i (\tilde{A}_n^{\text{tree}})_j$$

Kawai, Lewellen and Tye

$$(N = 8 \text{ state sum}) = (N = 4 \text{ sYM state sum}) \times (N = 4 \text{ sYM state sum})$$

$$(\text{gravity cut}) = \sum_{\substack{i1\cdots i_k\\ j1\cdots j_k}} K_{\substack{i1\cdots i_k\\ j1\cdots j_k}} \sum_{N=4 \text{ states}} A_1^{\text{tree}} \cdots A_k^{\text{tree}} \sum_{N=4 \text{ states}} \tilde{A}_1^{\text{tree}} \cdots \tilde{A}_k^{\text{tree}}$$

$$= (\text{gauge theory cut}) \times (\text{gauge theory cut})$$

Gravity cut generated directly from known 5 loop sYM result.

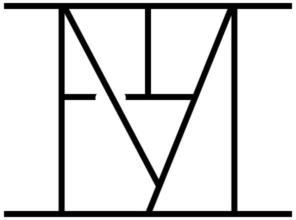
Apply KLT to cuts of known $N = 4$ sYM loop amplitudes.
Fast, but complicated analytic structure.

A Simplifying Miracle

ZB, Carrasco, Chen, Johansson, Roiban

contact = (gravity cut) – (cut of incomplete amplitude)

1. **Most contact terms vanish! Why?**
2. **In general gravity contacts far simpler than expected.**
3. **Four-point double-contacts factorize. Extremely striking.**



double 4pt contact

$$\left[2s^3 - s^2 u + 4s^2(2k_1 \cdot l_6) + \cdots\right]$$
$$\times \left[s^2 u + 2su^2 - s^2(2k_1 \cdot l_6) + \cdots\right]$$

Each factor looks like gauge theory

Reminds us of KLT factorization:

$$M^{\text{tree}}(1,2,3,4) = s_{12} A^{\text{tree}}(1,2,3,4) \times A^{\text{tree}}(1,2,4,3)$$

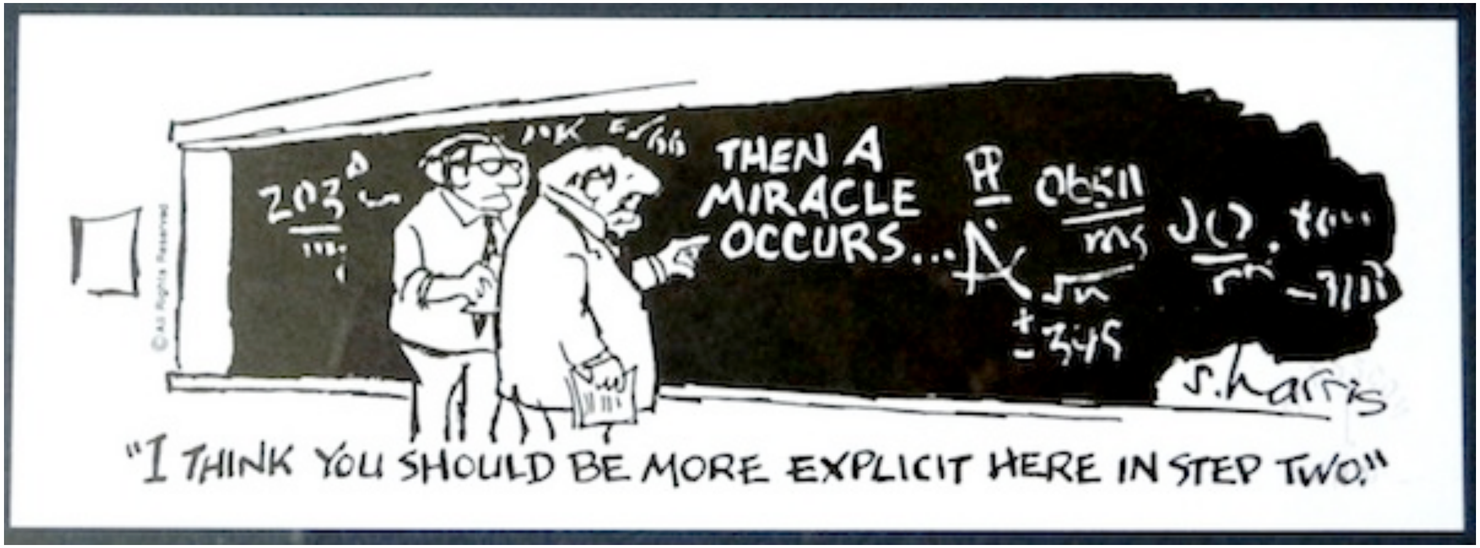
For 5 or higher-point contacts no overall factorization, as with KLT.

Can we write down formulas that give missing gravity pieces directly from gauge theory, bypassing gravity cuts?

A Miracle

1. Start from gauge-theory loop amplitude.
2. Construct naïve double copy.
3. Compute cut of naïve double copy.
4. Compute gravity cut from gauge-theory cuts via KLT.
5. Subtract and shake hard (nontrivial).
6. Extract surprisingly simple gravity contact.

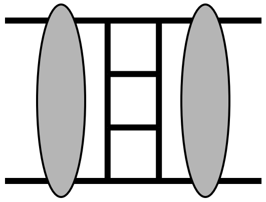
Miracle: The contact terms are so simple we should be able write down missing gravity contacts directly from gauge theory.



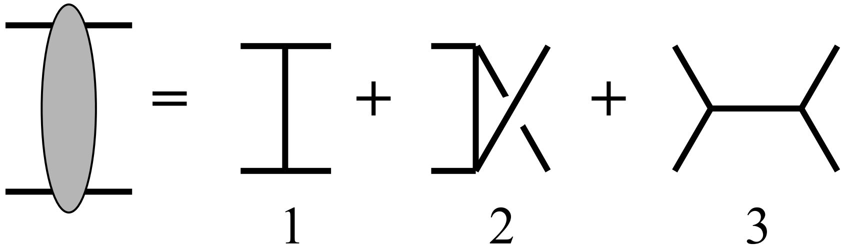
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

BCJ Discrepancy Functions

Need a function defined purely in gauge theory as building block for missing gravity pieces.



Inside multiloop diagram



kinematic numerators

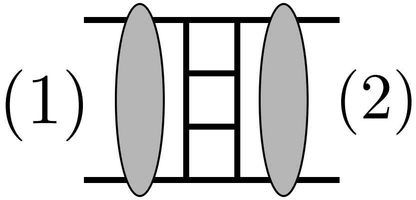
BCJ discrepancy function: $$J \equiv \sum_{i=1}^{3} n_i$$

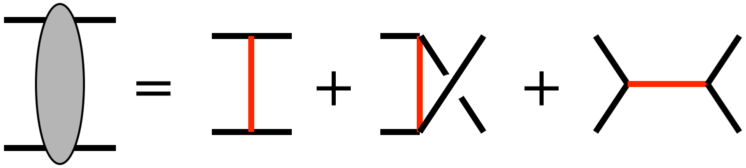
Vanishes if we have BCJ form of gauge theory.

Obvious guess is these are building blocks for missing gravity pieces.

Missing pieces: $$\sim \sum J \times J$$

Deriving Gravity Contact Formulas





color **numerator**

$$\mathcal{C}^{4\times4}_{\rm YM} = \sum_{i_1,i_2} \frac{c_{i_1 i_2} n_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

propagator

Generalized gauge transformation

$$\delta_{i1,i2} \equiv n_{i1i2} - n^{\rm BCJ}_{i1,i2} = d^{(1)}_{i1} k^{(2)}(i_2) + d^{(2)}_{i_2} k^{(1)}(i1)$$

Generalized gauge invariance:

$$\sum_{i_1,i_2} \frac{c_{i_1 i_2} \delta_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} = 0 = \sum_{i_1,i_2} \frac{n^{\rm BCJ}_{i_1 i_2} \delta_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

BCJ discrepancy function:

$$J^{(1)}_{i_2} \equiv \sum_{i_1}^{3} n_{i_1 i_2} = d^{(1)}_{i_2} \sum_{i_1}^{3} k^{(1)}(i_1) \qquad J^{(2)}_{i_1} \equiv \sum_{i_2}^{3} n_{i_1 i_2} = d^{(2)}_{i_1} \sum_{i_2}^{3} k^{(2)}(i_2)$$

$$\mathcal{C}^{4\times4}_{\rm SG} = \sum_{i_1,i_2} \frac{n^{\rm BCJ}_{i_1 i_2} n^{\rm BCJ}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

cross term between numerators and discrepancy vanishes.

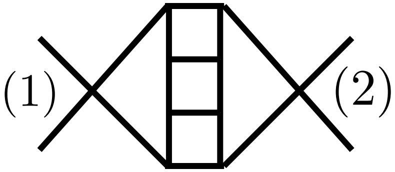
Formula for missing contact:

$$\mathcal{C}^{4\times4}_{\rm SG} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} n_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \frac{2}{d^{(1)}_{1} d^{(2)}_{1}} J^{(1)}_{1} J^{(2)}_{1}$$

Gravity from Gauge Theory

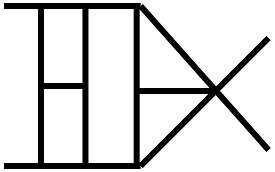
ZB, Carrasco, Chen, Johansson, Roiban

Missing gravity from any gauge theory representation



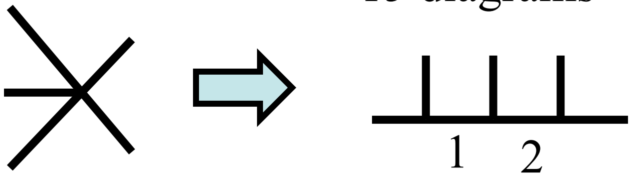
$$-\frac{2}{d_1^{(1)} d_1^{(2)}} J_1^{(1)} J_1^{(2)}$$

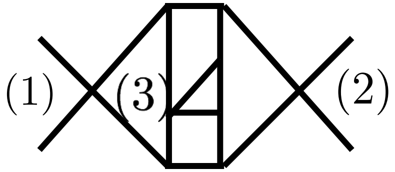
BCJ discrepancy functions



$$-\frac{1}{3}\sum_{i=1}^{15} \frac{J_{i,1}J_{i,2}}{d_{i,1}d_{i,2}}$$

propagators cancel trivially





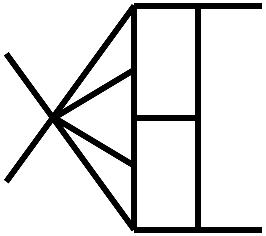
$$2\,\frac{J_{1,1}^{(1)}J_1^{(2,3)} + J_{1,1}^{(2)}J_1^{(1,3)} + J_{1,1}^{(3)}J_1^{(1,2)}}{d_1^{(1)}d_1^{(2)}d_1^{(3)}}$$

$$-\sum_{i_3}\frac{2J_{1,i_3}^{(1)}J_{1,i_3}^{(2)}}{d_1^{(1)}d_1^{(2)}d_{i_3}^{(3)}} - \sum_{i_2}\frac{2J_{i_2,1}^{(1)}J_{1,i_2}^{(3)}}{d_1^{(1)}d_{i_2}^{(2)}d_1^{(3)}} - \sum_{i_1}\frac{2J_{i_1,1}^{(2)}J_{i_1,1}^{(3)}}{d_{i_1}^{(1)}d_1^{(2)}d_1^{(3)}}$$

Etc.

- **Applies to *any* adjoint gauge theory, not just $N = 4$ sYM.**
- **Simple generalization for asymmetric double copies.**
- **Same constructions work at tree level! Five-point formula similar to known tree formula.** Bjerrum-Bohr, Damgaard, Søndergaad, Vanhove

5 Loop $N = 8$ supergravity



The generalized double copy enormously simplifies the computation of missing gravity contact terms. The impossible becomes doable!

N^2 – N^3maximal cuts: use formulas.
N^4 – N^6maximal cuts: numerical analysis more efficient.

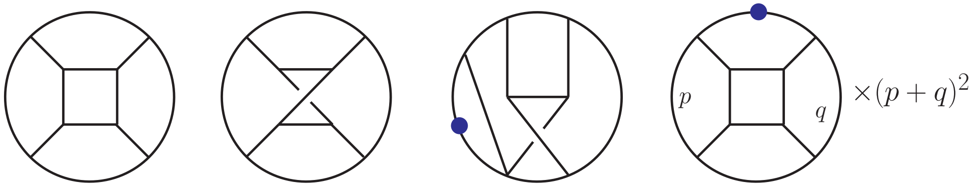
We have the five-loop integrand!

- **Of 76K potential contact terms 60K vanish.**
- **Details depends on arbitrary choices starting from naïve double copy.**
- **Contact-term representation is poor. Terms quartically divergent in $D = 24/5$ instead of log divergent.**
- **Dropped scale free tadpoles which integrate zero.**
- **Have confirmed our construction using nearly all N^7maximal and 70/638 N^8maximal cuts.**

Integrating 5 Loop $N = 8$ supergravity

Next step is to integrate the expression for large loop momenta.

Constructed representation is poor: need to series expand in four powers of external momenta. Get vacuum diagrams.



+ many thousands

Lorentz symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D\ell_a\Big) \sum_{a=1}^{L} \Big(\ell_{a\mu}\frac{\partial}{\partial \ell_a^\nu} - \ell_{a\nu}\frac{\partial}{\partial \ell_a^\mu}\Big)\frac{\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

SL(5) relabeling symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D\ell_a\Big) \sum_{a=1}^{L} \frac{\partial}{\partial \ell_a^\nu}\frac{\omega_{ab}\ell_{b\mu}\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

Should be sufficient for finding enhanced cancellations.

ZB, Enciso, Parra-Martinez, Zeng

Looks possible to find a much better representation with clever BCJ trickery.

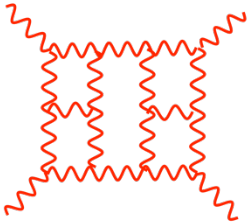
Alex Edision and Julio Parra-Martinez

Some Remaining Challenges

- Want complete set of formulas for converting *any* gauge-theory loop amplitude to corresponding gravity ones.
- Want better 5-loop sYM starting point to make UV extraction easier.
- Systematic and complete understanding of "enhanced cancellations" still needed.
- Can we carry over ideas to general classical solutions, without starting from special gauges? Applications to gravitational radiation?

Goldberger, Ridgway (2016); Luna, Monterio, Nicholson, O'Connell, Ochirov, Westerberg, White (2016)

Summary

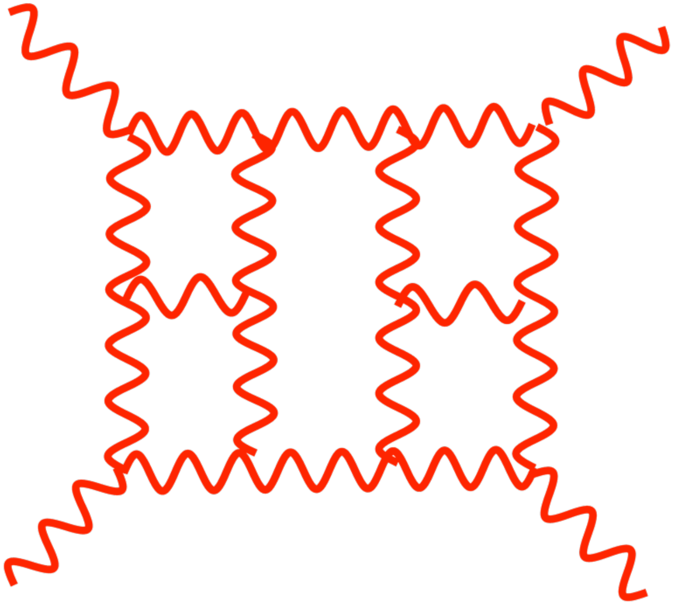


1. Duality between color and kinematics.
2. Double copy idea offer nontrivial insight into gravity.
   - — Gravity loops from gauge theory.
   - — Classical solutions.
3. Even when duality not manifest, new ideas allow us to extract gravity generalized double copy from gauge theory.
4. Applied these ideas to obtain 5 loop integrand.
5. We have new ideas for speeding up integration. Lorentz and SL(*L*) relabeling symmetry. Stay tuned!

In the coming months we hope report on whether English wine is any good…

We have a powerful new way to construct multiloop gravity amplitudes from gauge theory. Expect to learn a lot more about gravity in the coming years.

Extra



Recent Related Activities

- **Examples of exact classical solutions, including black holes.**
  Monteiro, O'Connell, White; Luna,Monteiro, O'Connell, White (2015)
- **Perturbative constructions of general classical solutions, including gravitational radiation problems (LIGO)**
  Goldberger, Ridgway (2016); Luna, Monterio, Nicholson, O’Connell, Ochirov, Westerberg, White (2016)
- **Loop level KLT and BCJ, using CHY, ambitwistor string, Q-cuts** Song He, Oliver Schlotterer (2016), Tourkine, Vanhove (2016)
- **Analytic properties of gravity integrands.** Herrmann and Trnka (2016)
- **Kinematic algebra behind BCJ duality.**
  Monteiro and O’Connell; Cheung and Shen (2016).
- **Simplified gravity Lagrangian from left-right factorization.**
  Cheung and Remmen (2016)
- **Double copy as consequence of gauge invariance.**
  Chiodaroli; Boels, Medina (2016), Arkani-Hamed, Rodina, Trnka (2016)
- **Applications in string theory.** Steiberger; Vahhove
  Carrasco, Mafra, Schlotterer, (2016); Mafra and Schlotterer (2015,2016)