From Scattering Amplitude to Gravitational Waves

August 3, 2020

Scattering Amplitudes and Beyond Online Reunion Conference

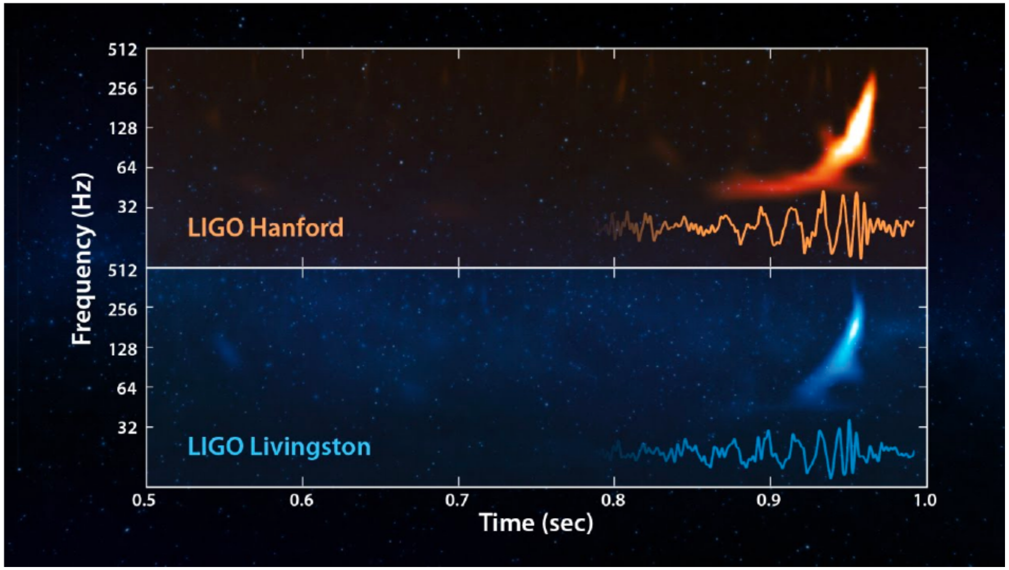
Zvi Bern

ZB, C. Cheung, R. Roiban, C. H. Shen, M. Solon, M. Zeng, arXiv:1901.04424 and arXiv:1908.01493.
ZB, A. Luna, R. Roiban, C. H. Shen, M. Zeng, arXiv:2005.03071

UCLA Mani L. Bhaumik Institute for Theoretical Physics

Overview

Era of gravitational wave astronomy has begun.



Feb 11, 2016

For an instant brighter in gravitational radiation than all the stars in the visible universe are in EM radiation!

How can we, who work on scattering amplitudes, help out with core mission of LIGO/Virgo?

The Seeds Planted at KITP

Possibility of applying amplitudes methods to LIGO physics was excitedly discussed at the workshop

June 2017

Some events:

- **Walter Goldberger: "Classical gravitational radiation and the double copy"**
- **Donal O'Connell: "Perturbative black holes from the double copy"**
- **Many discussions, private and public, on possibility of applying amplitude to gravitational radiation problem.**

Double copy: See Henrik's talk

KITP is where ideas really started going on shell

While we were still far from directly helping LIGO theorists, these talks and discussions convinced us that amplitudes + double copy was promising.

One effect: People in the GR community heard the excitement.

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bod-

"… and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian."

tum gravi… amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

…10599v1 [gr-qc] 29 Oct 2017

Hard to resist an invitation with this kind of clarity!

- **Very difficult using standard methods.**
- **Of direct importance to LIGO/Virgo theorists.**
- **Can in principle enter LIGO/Virgo analysis pipeline.**

The recent observation [1–4] of gravitational wave sig- nals from inspiralling and coalescing binary black holes has been significantly helped, from the theoretical side, by the availability of a large bank of waveform templates, defined [5, 6] within the analytical effective one-body (EOB) formalism [7–11]. The EOB formalism combines … mostly based on the post-Newtonian (PN) approach to the general relativistic two-body interaction. The con- servative two-body dynamics was derived, successively, at the second post-Newtonian (2PN) [14, 15], third post-

…ntly intro- … to derive … from the (gauge-invariant) *scattering function* Φ linking (half) the center of mass (c.m.) classical gravitational scattering angle χ to the total energy, $E_{\rm real} \equiv \sqrt{s}$, and the total angular momentum, J, of the system[1]

(1.1)

(1.2)

(1.3)

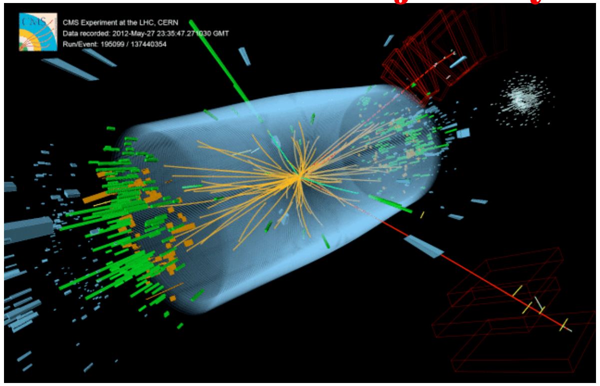
with

$M \equiv m_1 + m_2$; $\mu \equiv \frac{m_1 m_2}{\ldots}$; $\nu \equiv \frac{\mu}{M} = \frac{m_1 m_2}{\ldots}$.

Can Particle Theory Help with Gravitational Waves?

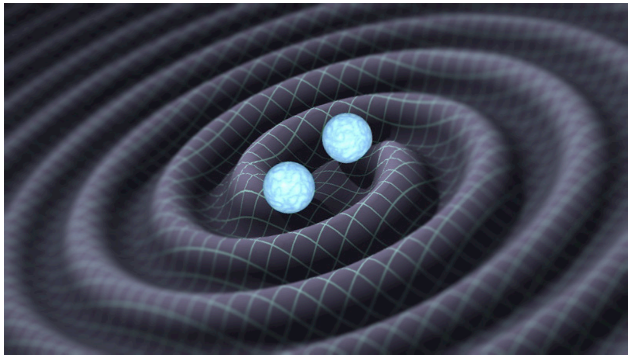
What does particle physics have to do with classical dynamics of astrophysical objects?

unbounded trajectory



gauge theories, QCD, electroweak
quantum field theory

bounded orbit



General Relativity
classical physics

Black holes and neutron stars are point particles as far as long wavelength radiation is concerned.

Iwasaki (1971); Goldberger, Rothstein (2006), Porto; Vaydia, Foffa , Porto, Rothstein, Sturani; Kol; Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Levi, Steinhoff; Vines etc

Will explain that amplitudes are well suited to push state-of-the-art perturbative calculations for gravitational-wave physics.

Can Quantum Scattering Help with Gravitational Waves?

In amplitudes community we are very very good at gravitational perturbation theory.

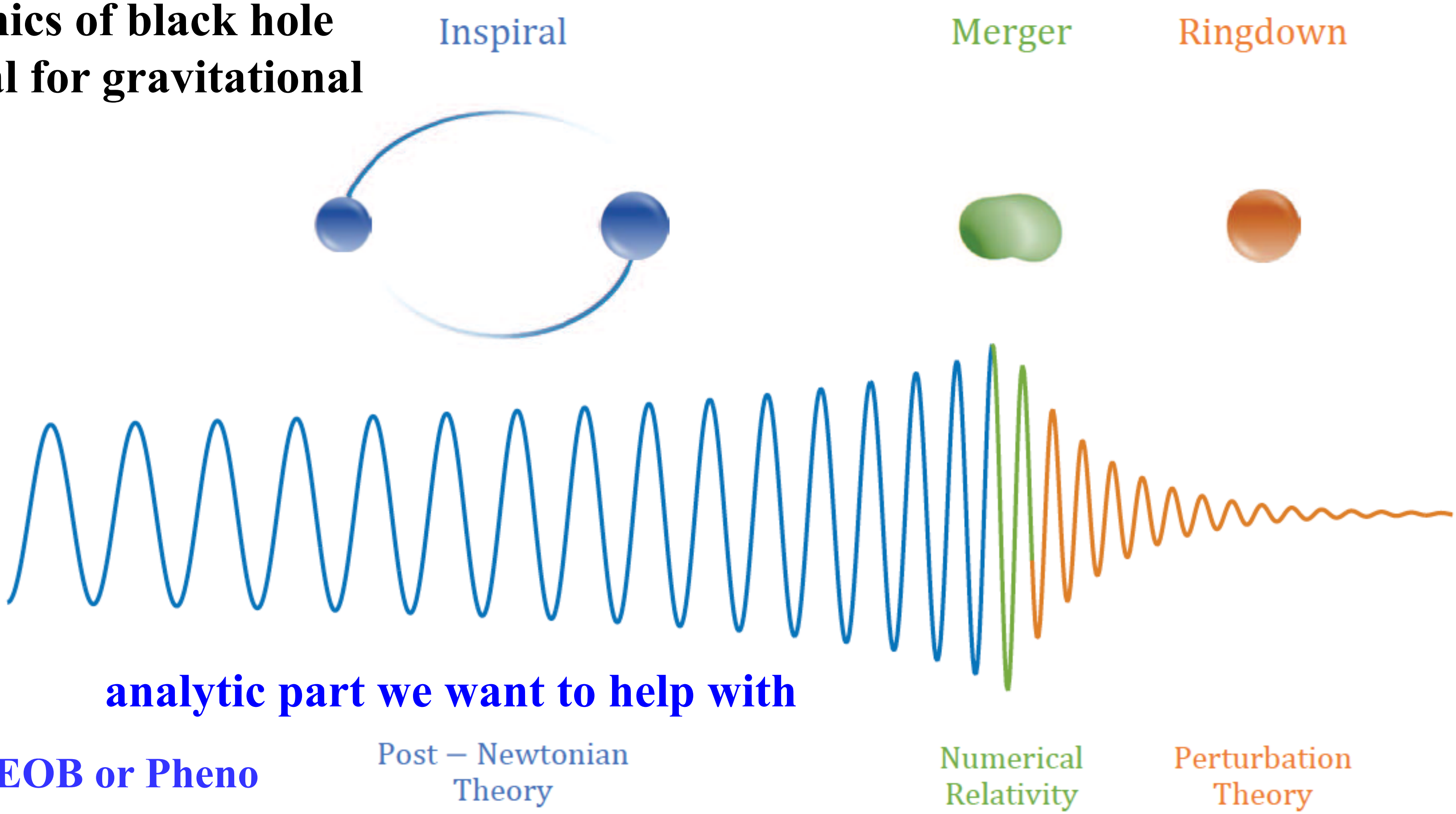
However, two serious issues discussed at KITP workshop:

1. **We do quantum, *not* classical perturbation theory.**
2. **Scattering process unbounded orbit. Want bounded one for binary black hole gravitational wave emission.**

Will explain in this how we deal with these issues.

Goal: Improve on post-Newtonian Theory

Dynamics of black hole inspiral for gravitational waves.



analytic part we want to help with

PN + EOB or Pheno

Small errors accumulate. Need for high precision.

From Antelis and Moreno, arXiv:1610.03567

Post Newtonian Approximation

For orbital mechanics:

$$v^2 \sim \frac{GM}{R} \ll 1$$

Expand in *G* and v^2

virial theorem

In center of mass frame:

$$m = m_A + m_B, \quad \nu = \mu/M,$$

$$\mu = m_A m_B/m, \quad P_R = P \cdot \hat{R}$$

$$\frac{H}{\mu} = \frac{P^2}{2} - \frac{Gm}{R} \quad \leftarrow \text{Newton}$$

$$+ \frac{1}{c^2}\left\{ -\frac{P^4}{8} + \frac{3\,\nu\, P^4}{8} + \frac{Gm}{R}\left(-\frac{{P_R}^2\,\nu}{2} - \frac{3\,P^2}{2} - \frac{\nu\,P^2}{2} \right) + \frac{G^2 m^2}{2R^2} \right\}$$

$$+ \dots$$

1PN: Einstein, Infeld, Hoffmann; Droste, Lorentz

Hamiltonian known to 4PN order.

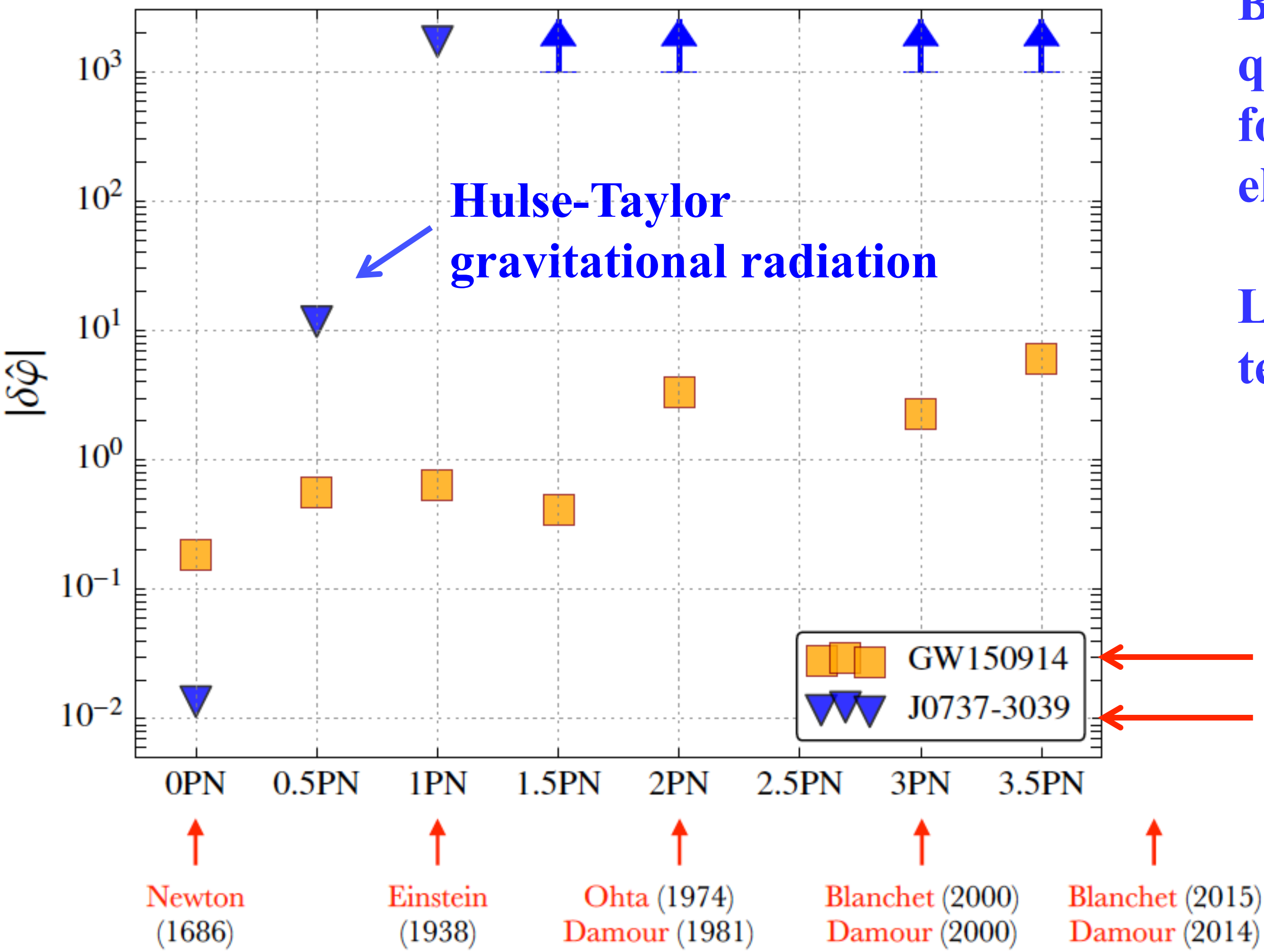
2PN: Ohta, Okamura, Kimura and Hiida.

3PN: Damour, Jaranowski and Schaefer; L. Blanchet and G. Faye.

4PN: Damour, Jaranowski and Schaefer (2017); Foffa, Porto, Rothstein, Sturani (2019).

Importance of higher orders for LIGO/Virgo

LIGO/Virgo Collaboration arXiv:1602.03841



Binary pulsar confirms quadrupole radiation formula and not much else.

LIGO/Virgo tests PN terms from GR

LIGO

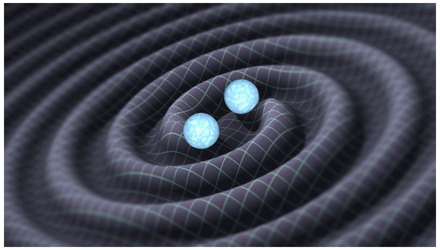
Binary pulsar

LIGO/Virgo sensitive to high PN orders.

Which problem to solve?

ZB, Cheung, Roiban, Shen, Solon, Zeng

Some problems for (analytic) theorists:



1. **Spin.**
2. **Finite size effects.**
3. **New physics effects.**
4. **Radiation.**
5. → **High orders in perturbation theory.** ←

Which problem should we solve?

- **Needs to be extremely difficult using standard methods.**
- **Needs to be of direct importance to LIGO theorists.**
- **Needs to be in a form that can in principle enter LIGO analysis pipeline.**

2-body Hamiltonian at 3rd post-Minkowskian order

Clear, given Damour's paper and talks from Buonanno

PN versus PM expansion for conservative two-body dynamics

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}} + \frac{1}{c^2}\left[\dots\right] + \frac{1}{c^4}\left[\dots\right] + \dots$$

From Buonanno Amplitudes 2018

$$E(v) = -\frac{\mu}{2} v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

(credit: Justin Vines)

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results — current known PM results — overlap between PN & PM results — unknown

- PM results (Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)

The Double Copy

See Henrik Johansson's talk

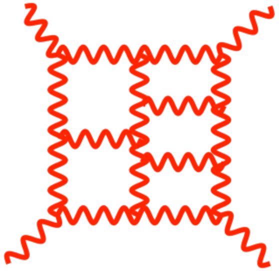
In a very precise sense:

Gravity ~ (gauge theory) x (gauge theory)

- Very general: "web of theories".
- Gives us awesome calculational power.
- Use it to do impossible looking calculations, to answer questions of physical interest.

Recent Review: ZB, Carrasco, Chiodaroli, Johansson, Roiban

Examples:

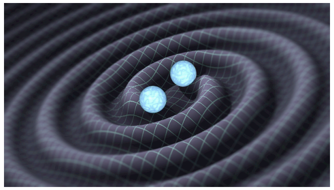


- 5 loop supergravity to study nonrenormalizability of gravity theories.

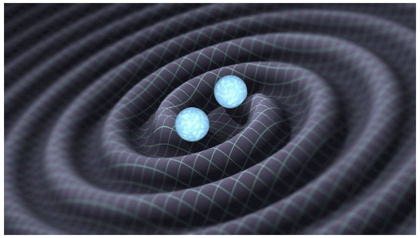
  ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

- G^3 corrections to Newton's potential from GR.

  ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)



Scattering Amplitudes and Gravitational Radiation



A small industry has developed to study this.

- **Connection to scattering amplitudes.**

  Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Luna, Nicholson, O'Connell, White; Guevara; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon; Damour; Bautista, Guevara; Kosower, Maybee, O'Connell; Plefka, Steinhoff, Wormsbecher; Foffa, Mastrolia, Sturani, Sturm; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee; etc.

- **Worldline approach for radiation and double copy.**

  Goldberger and Ridgway; Goldberger, Li, Prabhu, Thompson; Chester; Shen.

- **Technical issues having to do with keeping right physical states.**

  Luna, Nicholson, O'Connell, White; Johansson, Ochirov; Johansson, Kalin; Henrik Johansson, Gregor Kälin, Mogull.

Key Question: Can we calculate something of direct interest to LIGO/Virgo, decisively *beyond* previous state of the art?

What are we after?

- **Replace scattering in General Relativity with a two body potential that is easy to use in bound-state problem.**
- **Extract physics juice, leaving behind complexity of General Relativity.**

$$V(\mathbf{r}, \mathbf{p}) = -\frac{G m_1 m_2}{r} + \cdots$$

Just like Newton's potential, except:

- **Compatible with special relativity (all orders in velocity)**
- **Valid through O(G^3).**

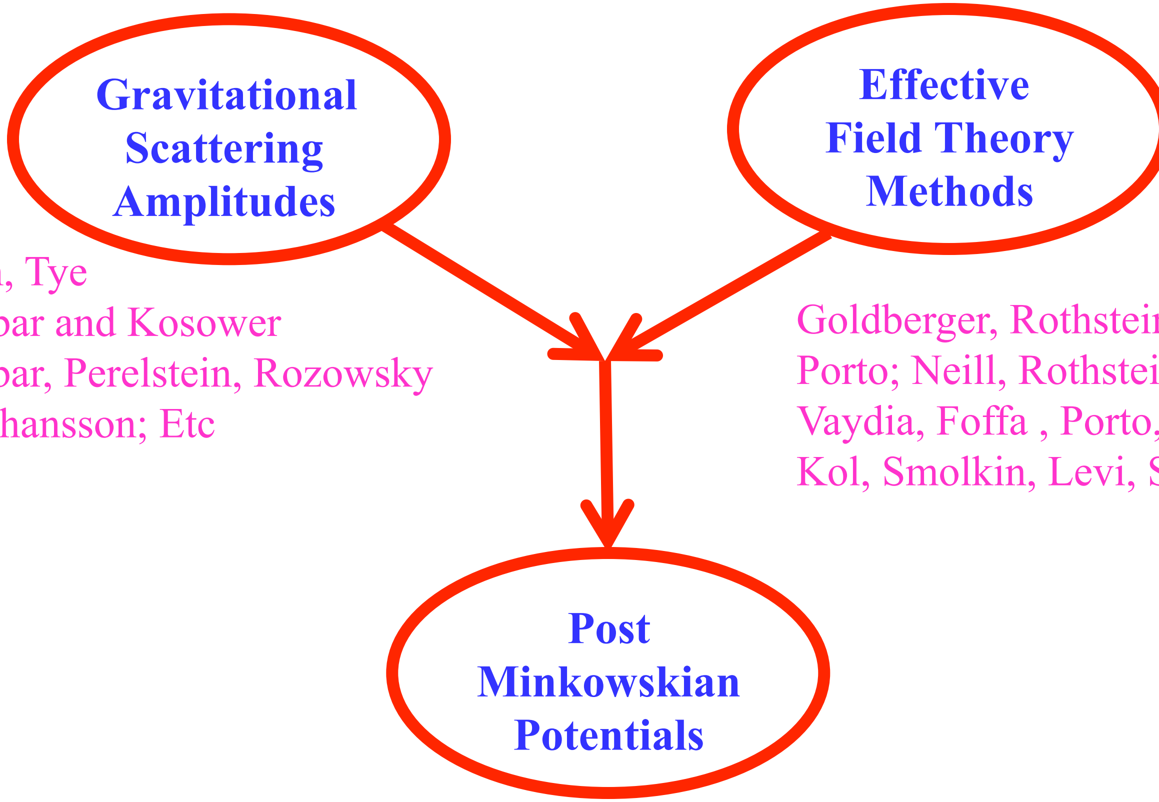
Effective Field Theory Approach

ZB, Cheung, Roiban, Shen, Solon, Zeng

Cheung, Rothstein, Solon (2018)

Amplitudes community

EFT community



Kawai, Lewellen, Tye
ZB, Dixon, Dunbar and Kosower
ZB, Dixon, Dunbar, Perelstein, Rozowsky
ZB, Carrasco, Johansson; Etc

Goldberger, Rothstein;
Porto; Neill, Rothstein;
Vaydia, Foffa , Porto, Rothstein, Sturani;
Kol, Smolkin, Levi, Steinhoff, etc.

Inefficient: **Start with quantum theory and take $\hbar \to 0$**

Efficient: **Almost magical simplifications for gravity amplitudes.**
EFT methods efficiently target pieces we want.

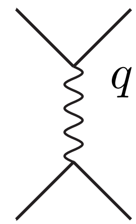
Efficiency wins

2 Body Potentials and Amplitudes

Iwasaki; Gupta, Radford; Donoghue; Holstein, Donoghue; Holstein and A. Ross; Bjerrum-Bohr, Donoghue, Vanhove; Neill, Rothstein; Bjerrum-Bohr, Damgaard, Festuccia, Planté. Vanhove; Chueng, Rothstein, Solon; Chung, Huang, Kim, Lee; etc.

Tree-level: Fourier transform gives classical potential.

$$V(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} A^{\text{tree}}(\mathbf{q})$$



Newtonian potential follows from Feynman diagrams

Beyond 1 loop things quickly become much less obvious:

- **What I learned in grad school on $\hbar$ and classical limits is wrong! Loops have classical pieces.**
- **$1/\hbar^L$ scaling of at L loop.**
- **Double counting and iteration.**
- **Cross terms between $1/\hbar$ and $\hbar$.**

$$e^{iS_{\text{classical}}/\hbar}$$

Piece of loops are classical: Our task is to efficiently extract these pieces.

We harness EFT to clean up confusion

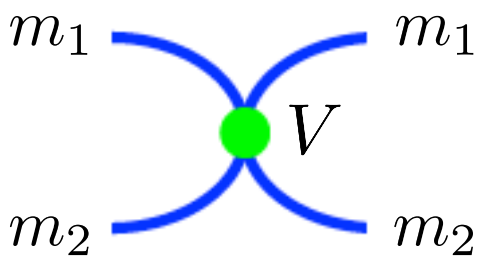
Effective Field Theory is a Clean Approach

Build EFT from which we can read off potential.
Want a Newtonian-like potential,
with GR corrections

Goldberger and Rothstein
Neill, Rothstein
Cheung, Rothstein, Solon (2018)

$$L_{\text{kin}} = \int_{\boldsymbol{k}} A^\dagger(-\boldsymbol{k}) \left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_A^2} \right) A(\boldsymbol{k}) + \int_{\boldsymbol{k}} B^\dagger(-\boldsymbol{k}) \left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_B^2} \right) B(\boldsymbol{k})$$

***A*, *B* scalars represents spinless black holes**



$$L_{\text{int}} = - \int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k}, \boldsymbol{k}')\, A^\dagger(\boldsymbol{k}') A(\boldsymbol{k}) B^\dagger(-\boldsymbol{k}') B(-\boldsymbol{k})$$

potential we want to obtain

$$H(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{r})$$

2 body Hamiltonian in c.o.m. frame.

Match amplitudes of this theory to the full theory in classical limit to extract a classical potential of the type Newton would like.

Our LIGO/Virgo theory friends want Hamiltonians.

EFT Matching

Cheung, Rothstein, Solon

full general relativity
(complicated)

Amplitude methods
double copy

tree amplitude

generalized
unitarity

$\hbar \to 0$

loop integrand

loop
integration

GR loop amplitude

identical physics

$=$

effective theory
(simpler)

build
ansatz

potential

Feynman
diagrams

loop integrand

loop
integration

EFT loop amplitude

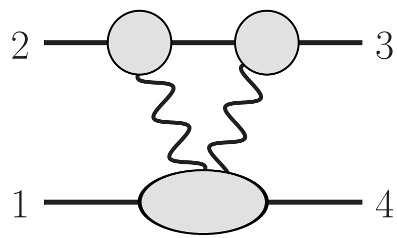
Roundabout, but efficiently extracts potential.

General Relativity: Unitarity + Double Copy

- **Long-range force: Two matter lines must be separated by on-shell propagators.**
- **Classical potential: 1 matter line per loop is cut (on-shell).**

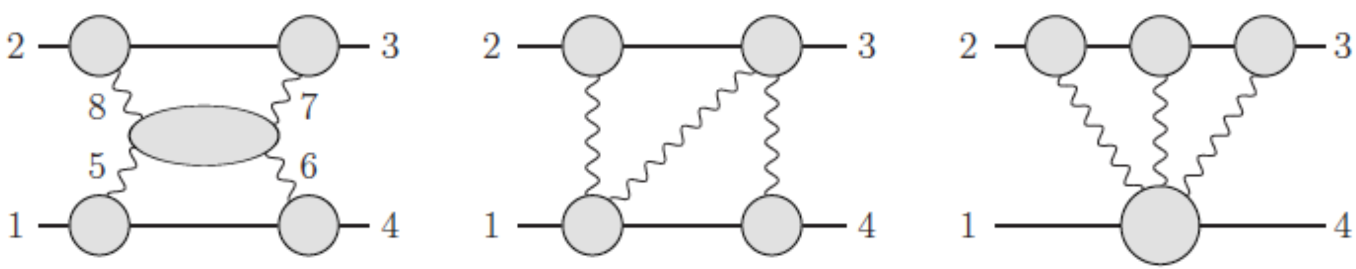
Neill and Rothstein ; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon

Only independent unitarity cut for 2 PM.

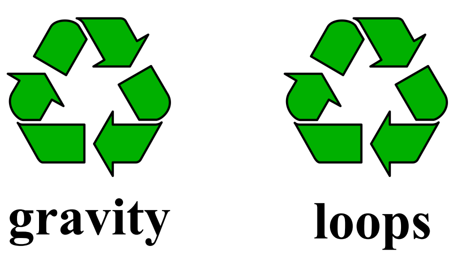


Treat exposed lines on-shell (long range).
Pieces we want are simple!

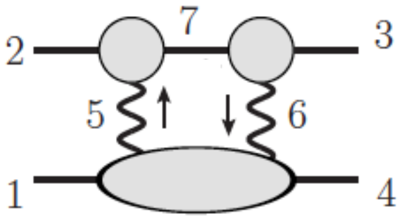
Independent generalized unitarity cuts for 3 PM.



Our amplitude tools fit perfectly with extracting pieces we want.



Generalized Unitarity Cuts



2nd post-Minkowkian order

KLT relations

$$
\begin{aligned}
C_{\rm GR} &= \sum_{h_5,h_6=\pm} M_3^{\rm tree}(3^s, 6^{h_6}, -7^s)\, M_3^{\rm tree}(7^s, -5^{h_5}, 2^s)\, M_4^{\rm tree}(1^s, 5^{-h_5}, -6^{-h_6}, 4^s) \\
&= \sum_{h_5,h_6=\pm} it[A_3^{\rm tree}(3^s, 6^{h_6}, -7^s)\, A_3^{\rm tree}(7^s, -5^{h_5}, 2^s)\, A_4^{\rm tree}(1^s, 5^{-h_5}, -6^{-h_6}, 4^s)] \\
&\qquad \times [A_3^{\rm tree}(3^s, 6^{h_6}, -7^s)\, A_3^{\rm tree}(7^s, -5^{h_5}, 2^s)\, A_4^{\rm tree}(4^s, 5^{-h_5}, -6^{-h_6}, 1^s)]
\end{aligned}
$$

Problem of computing the generalized cuts in gravity is reduced to multiplying and summing gauge-theory tree amplitudes.

This is then fed into integration and EFT matching.
Follow Cheung, Rothstein and Solon's paper.

Amplitude in Classical Potential Limit

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

Classical limit. The O(G^3) or 3PM terms are:

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2\xi}\Bigg[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)(1+\sigma)}\Bigg] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}\Big[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)F_1 - 32 m^2\nu^2\left(1-2\sigma^2\right)^3 F_2\Big]$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2/m_1 m_2,$$

Amplitude containing classical potential is surprisingly simple!

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The 3PM Hamiltonian:

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

Newton in here

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\Bigg[&\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ &- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ &+ \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg],\end{aligned}$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

This is high order general relativity.

How do we know it is right?

Primary check: ZB, Cheung, Roiban, Shen, Solon, Zeng

Compare to 4PN Hamiltonian of Damour, Jaranowski, Schäfer, used in high precision template constructions.

Need canonical transformation: **preserve Poisson bracket**

$$(\boldsymbol{r},\boldsymbol{p}) \to (\boldsymbol{R},\boldsymbol{P}) = (A\,\boldsymbol{r} + B\,\boldsymbol{p}, C\,\boldsymbol{p} + D\,\boldsymbol{r})$$

$$A = 1 - \frac{Gm\nu}{2|\boldsymbol{r}|} + \cdots\,, \quad B = \frac{G(1-2/\nu)}{4m|\boldsymbol{r}|}\boldsymbol{p}\cdot\boldsymbol{r} + \cdots$$

$$C = 1 + \frac{Gm\nu}{2|\boldsymbol{r}|} + \cdots\,, \quad D = -\frac{Gm\nu}{2|\boldsymbol{r}|^3}\boldsymbol{p}\cdot\boldsymbol{r} + \cdots\,,$$

Our Hamiltonian equivalent to 4PN Hamiltonian on overlap.

Additional tests:

1. **Classical scattering angle matches 4PN result in overlap** Bini and Damour
2. **In test mass limit, $m_1 \ll m_2$, matches Schwarzschild Hamiltonian.**
3. **Recent paper confirms our result also at 6PN order.** Wex and Schaefer

Bini, Damour, Geralico

4 PN Hamiltonian

Damour, Jaranowski, Schaefer

$$\mathbf{n} = \hat{\mathbf{r}}$$

$$\widehat{H}_{\mathrm{N}}(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r},$$

$$c^2\,\widehat{H}_{1\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\Big\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r} + \frac{1}{2r^2},$$

$$c^4\,\widehat{H}_{2\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\Big\{\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r}$$
$$+ \frac{1}{2}\Big\{(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^2} - \frac{1}{4}(1+3\nu)\frac{1}{r^3},$$

$$c^6\,\widehat{H}_{3\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4 + \frac{1}{16}\Big\{\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3$$
$$+ (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\Big\}\frac{1}{r}$$
$$+ \Big\{\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r^2}$$
$$+ \Big\{\Big(-\frac{25}{8} + \Big(\frac{\pi^2}{64} - \frac{335}{48}\Big)\nu - \frac{23\nu^2}{8}\Big)\mathbf{p}^2 + \Big(-\frac{85}{16} - \frac{3\pi^2}{64} - \frac{7\nu}{4}\Big)\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^3} + \Big\{\frac{1}{8} + \Big(\frac{109}{12} - \frac{21}{32}\pi^2\Big)\nu\Big\}\frac{1}{r^4},$$

G^4

4 PN Hamiltonian

Damour, Jaranowski, Schaefer

$$\mathbf{n} = \hat{\mathbf{r}}$$

$$
\begin{aligned}
c^8\,\widehat{H}^{\text{local}}_{4\text{PN}}(\mathbf{r},\mathbf{p}) = {} & \left(\frac{7}{256} - \frac{63}{256}\nu + \frac{189}{256}\nu^2 - \frac{105}{128}\nu^3 + \frac{63}{256}\nu^4\right)(\mathbf{p}^2)^5 \\
& + \left\{\frac{45}{128}(\mathbf{p}^2)^4 - \frac{45}{16}(\mathbf{p}^2)^4\,\nu + \left(\frac{423}{64}(\mathbf{p}^2)^4 - \frac{3}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 - \frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2\right)\nu^2\right. \\
& + \left(-\frac{1013}{256}(\mathbf{p}^2)^4 + \frac{23}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 + \frac{69}{128}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2 - \frac{5}{64}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2 + \frac{35}{256}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^3 \\
& \left. + \left(-\frac{35}{128}(\mathbf{p}^2)^4 - \frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 - \frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2 - \frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2 - \frac{35}{128}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^4\right\}\frac{1}{r} \\
& + \left\{\frac{13}{8}(\mathbf{p}^2)^3 + \left(-\frac{791}{64}(\mathbf{p}^2)^3 + \frac{49}{16}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 - \frac{889}{192}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 + \frac{369}{160}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu\right. \\
& + \left(\frac{4857}{256}(\mathbf{p}^2)^3 - \frac{545}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + \frac{9475}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - \frac{1151}{128}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^2 \\
& \left. + \left(\frac{2335}{256}(\mathbf{p}^2)^3 + \frac{1135}{256}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 - \frac{1649}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 + \frac{10353}{1280}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^3\right\}\frac{1}{r^2} \\
& + \left\{\frac{105}{32}(\mathbf{p}^2)^2 + \left(\left(\frac{2749\pi^2}{8192} - \frac{589189}{19200}\right)(\mathbf{p}^2)^2 + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu\right. \\
& + \left(\left(\frac{18491\pi^2}{16384} - \frac{1189789}{28800}\right)(\mathbf{p}^2)^2 + \left(-\frac{127}{3} - \frac{4035\pi^2}{2048}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \left(\frac{57563}{1920} - \frac{38655\pi^2}{16384}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^2 \\
& \left. + \left(-\frac{553}{128}(\mathbf{p}^2)^2 - \frac{225}{64}(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - \frac{381}{128}(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^3\right\}\frac{1}{r^3} \\
& + \left\{\frac{105}{32}\mathbf{p}^2 + \left(\left(\frac{185761}{19200} - \frac{21837\pi^2}{8192}\right)\mathbf{p}^2 + \left(\frac{3401779}{57600} - \frac{28691\pi^2}{24576}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu\right. \\
& \left. + \left(\left(\frac{672811}{19200} - \frac{158177\pi^2}{49152}\right)\mathbf{p}^2 + \left(\frac{110099\pi^2}{49152} - \frac{21827}{3840}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu^2\right\}\frac{1}{r^4} \quad \longleftarrow G^4 \\
& + \left\{-\frac{1}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169199}{2400}\right)\nu + \left(\frac{7403\pi^2}{3072} - \frac{1256}{45}\right)\nu^2\right\}\frac{1}{r^5}. \quad \longleftarrow G^5
\end{aligned}
$$

After canonical transformation we match all but G^4 and G^5 terms

Mess is partly due to their gauge choice.

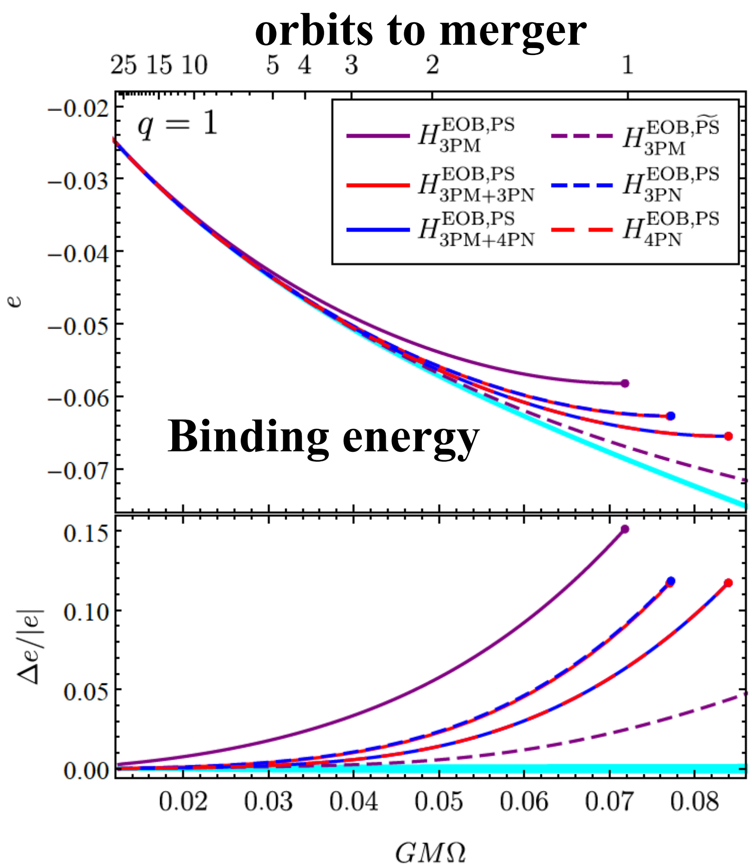
Ours is all orders in p at G^3

Our goal is to not only find new powerful methods but to clean up results

Tests of Our 3PM Hamiltonian for LIGO/Virgo

Antonelli, Buonanno, Steinhoff, van de Meent, and Vines, arXiv:1901.07102

(8 days after our paper!)



Test against numerical relativity.

Fed into EOB models, which are needed for good agreement.

Note: Not conclusive, e. g. radiation not taken into accounted

numerical relativity taken as truth

"This rather encouraging result motivates a more comprehensive study…"

We definitely have the attention of our GR friends

Outlook

- **Most exciting part is that methods are far from exhausted.**
- **Working on 4th post-Minkowskian order. Methods certainly up to the task.**
- **Much greater improvements on horizon.**

Parra-Martinez, Ruf and Zeng

Obvious topics to investigate:

- **Higher orders. Resummation in *G*.**
- **Radiation.**
- **Finite size effects.**
- **Spin — a lot of activity**

Geuvara, O'Connell, Vines; Chung, Huang, Kim, Lee; ZB, Luna, Roiban, Shen and Zeng, etc

Expect many more advances in coming years.

Summary

- **Amplitude methods give us new ways to think about problems of current interest in general relativity.**
- **Double-copy idea gives a unified framework for gravity and gauge theory.**
- **The 2017 KITP workshop is where ideas for gravitational-wave applications were planted.**
- **Combining with EFT methods gives a powerful tool for gravitational-wave physics in language LIGO/Virgo can use.**
- **Obtained a state of the art result: O(G^3) Hamiltonian.**
- **Higher orders in G, resummations in G, spin, finite-size effects, radiation obvious directions.**

Expect many more advances in coming years, not only for gravitational-wave physics, but more generally for understanding gravity and its relation to the other forces via double copy.

Definitely time to plan a new KITP workshop