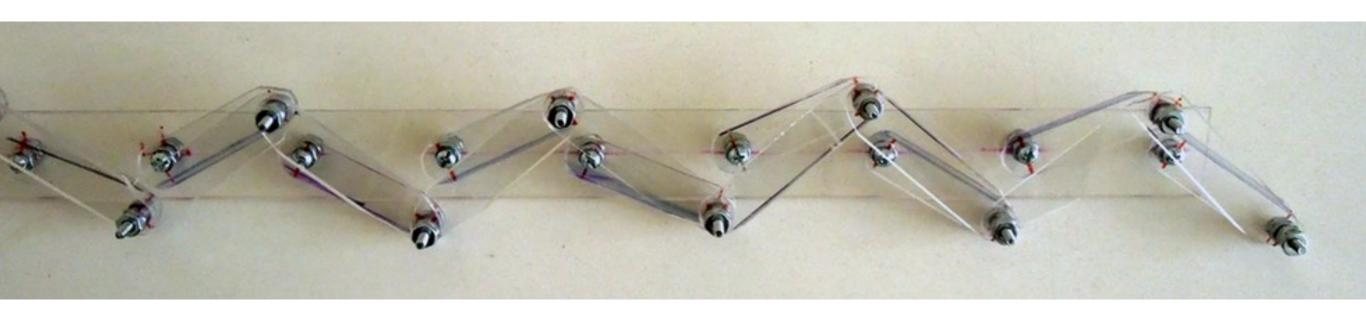
Mechanisms and nonlinear waves from topological modes

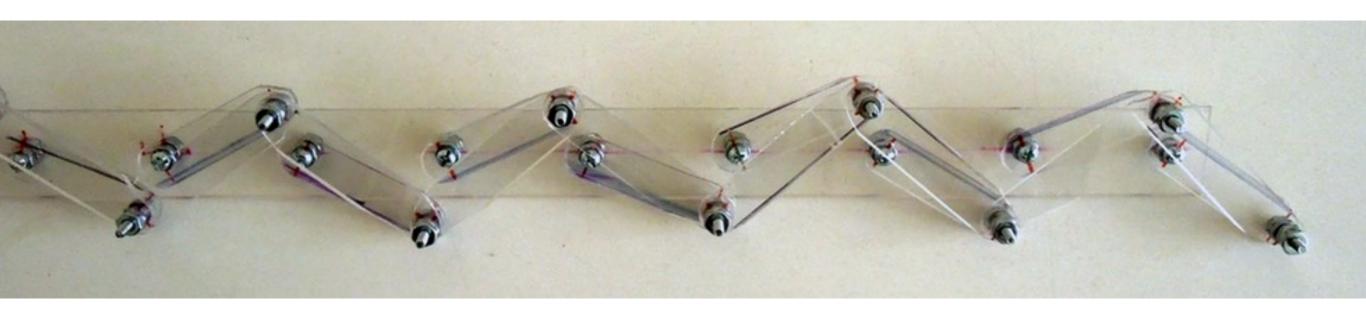


Bryan Gin-ge Chen

UMass Amherst Physics



How I got my boss to buy me \$600 of LEGO

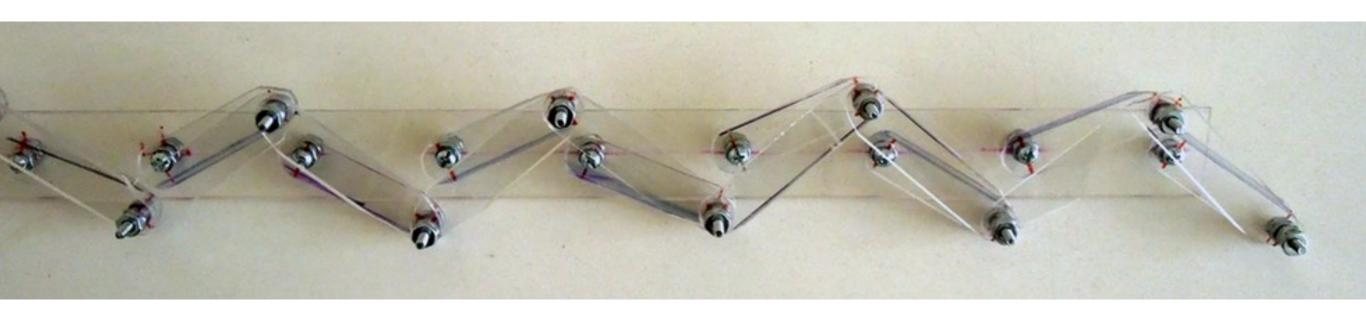


Bryan Gin-ge Chen

UMass Amherst Physics



Mechanisms and nonlinear waves from topological modes

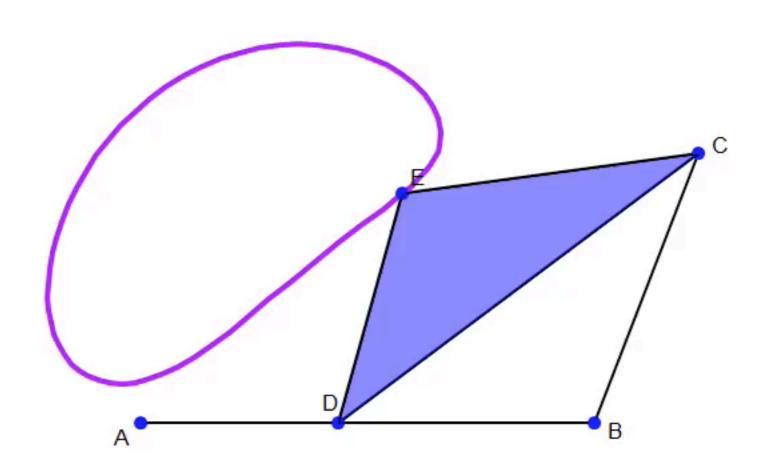


Bryan Gin-ge Chen

UMass Amherst Physics

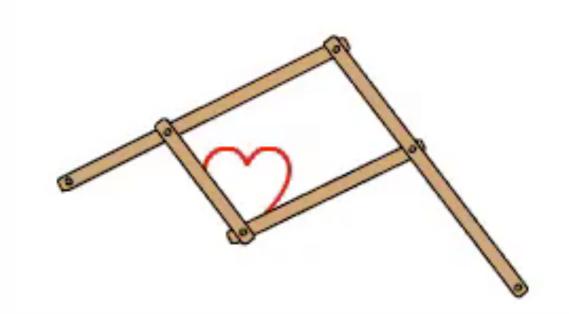
















Ron Resch, "The Paper and Stick Film" 1971



From mechanisms to **metamaterials** ...



Ron Resch, "The Paper and Stick Film" 1971

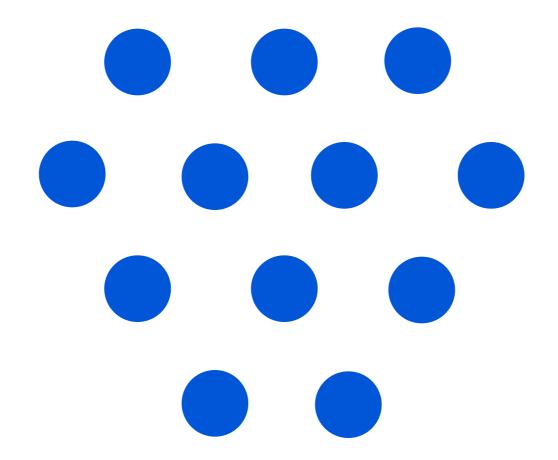


Can we understand these materials and design their motions?

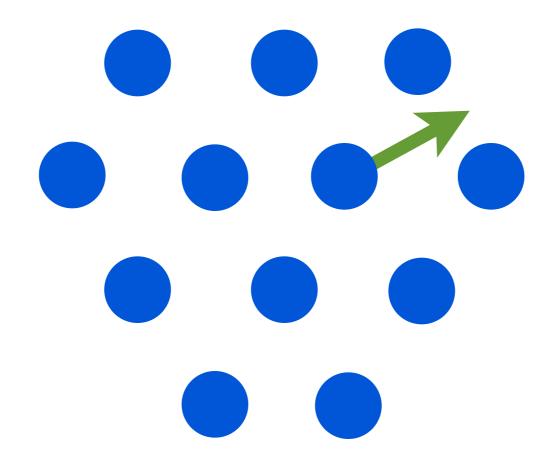


Ron Resch, "The Paper and Stick Film" 1971

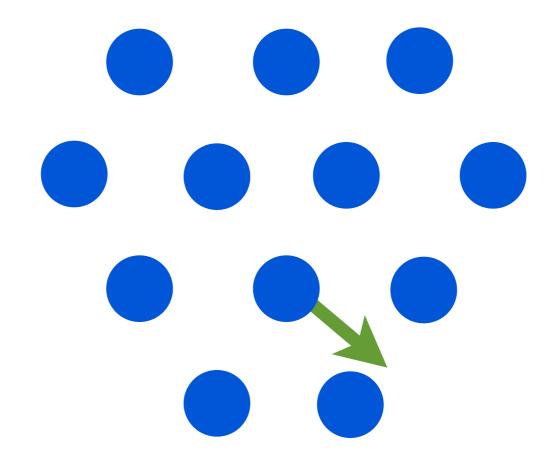




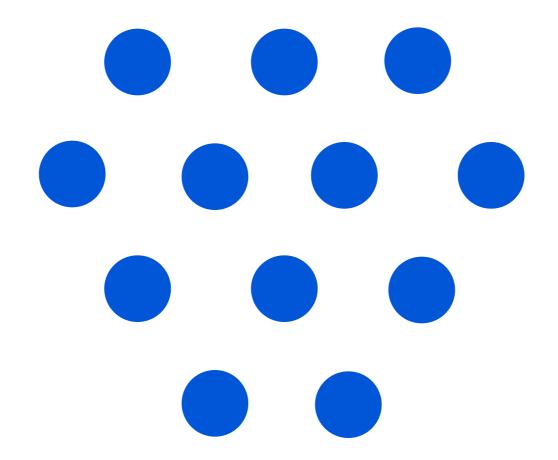
Degrees of freedom >> constraints Under-coordinated



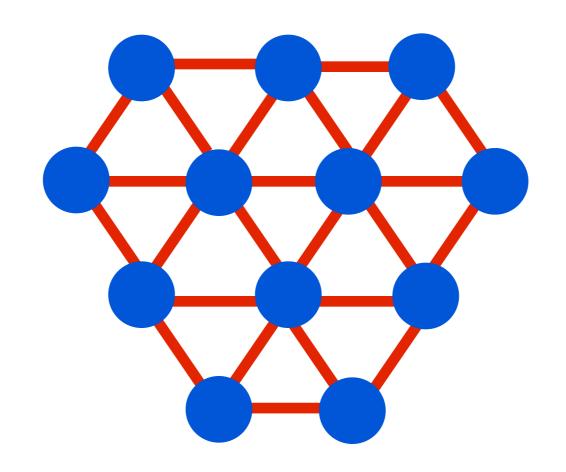
Degrees of freedom >> constraints Under-coordinated



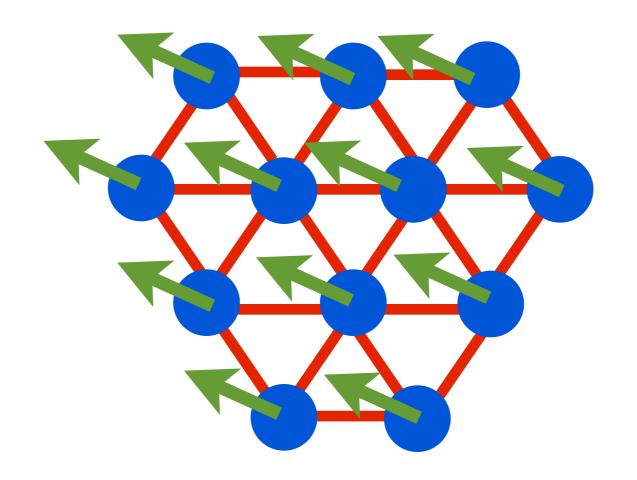
Degrees of freedom >> constraints Under-coordinated



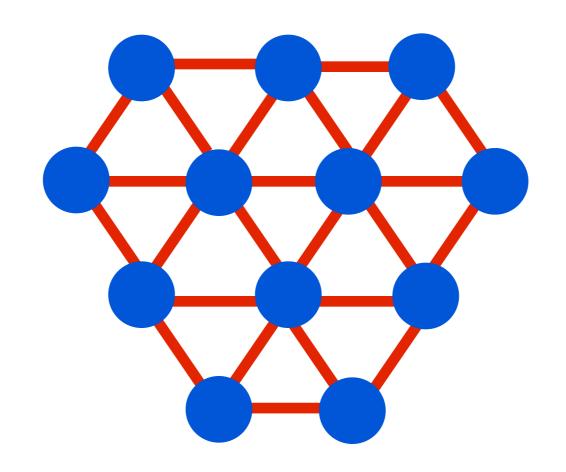
Degrees of freedom >> constraints Under-coordinated



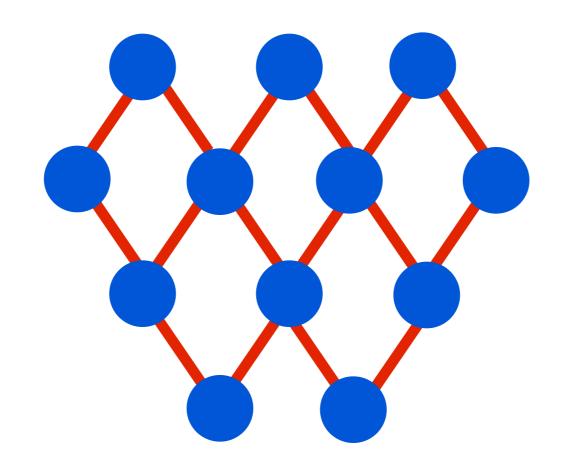
Degrees of freedom << constraints Over-coordinated



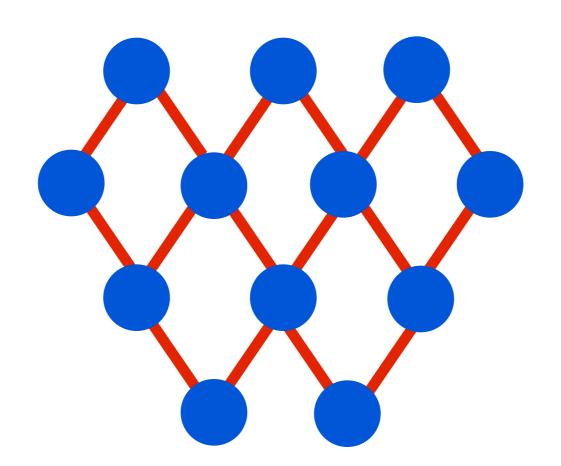
Degrees of freedom << constraints Over-coordinated

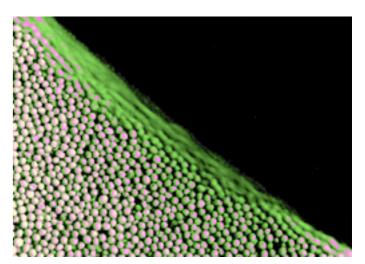


Degrees of freedom << constraints Over-coordinated



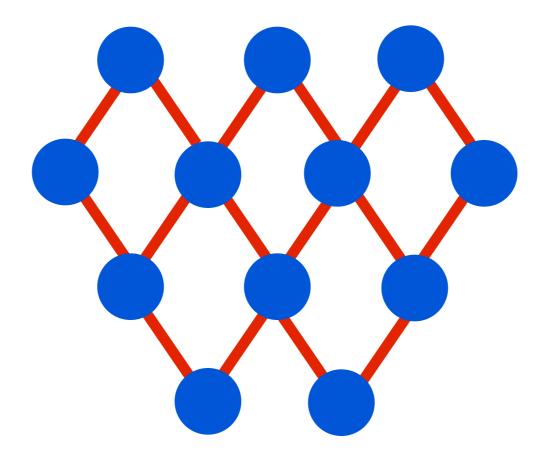
Degrees of freedom ≈ constraints (bulk) **Isostatic**

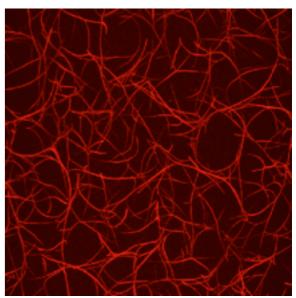




Jaeger and Nagel 1992

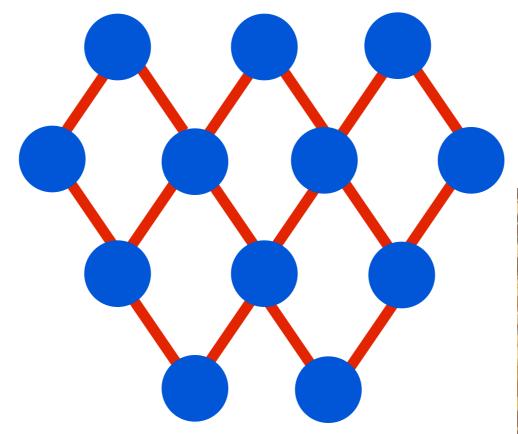
Degrees of freedom ≈ constraints (bulk) **Isostatic**





Andreas Bausch, TUM, http://bio.ph.tum.de/home/e27-prof-dr-bausch/bausch-home.html

Degrees of freedom ≈ constraints (bulk) **Isostatic**





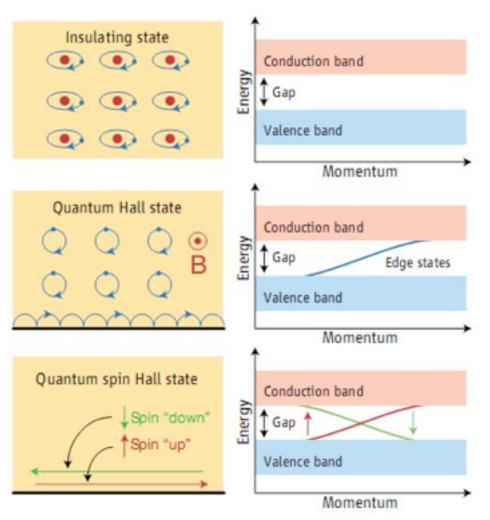
Curious Expeditions, "Gear Work 2", https://flic.kr/p/KikA9

Degrees of freedom ≈ constraints (bulk) **Isostatic**

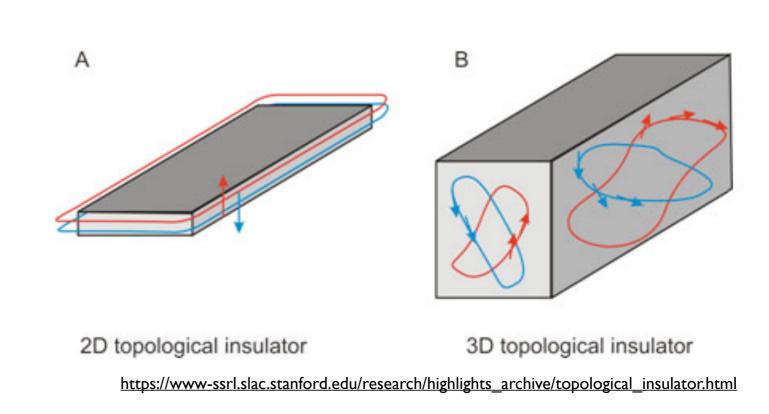
What can **theoretical physics** say about linkages?



Topological Matter



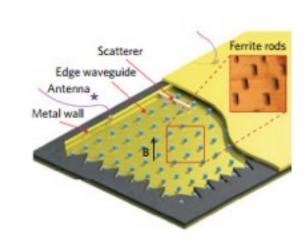
Kane, Mele, Science 2006



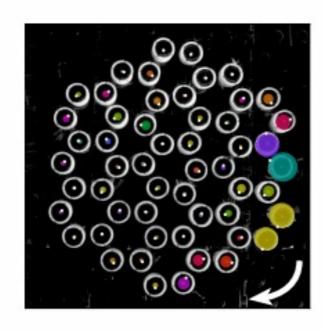
Electronic properties insensitive to smooth changes in material parameters



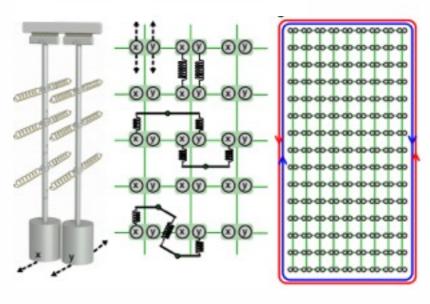
Topological matter without QM



Photonic crystals Lu, Joannopoulos and Soljačić (2014)



Gyroscopes Nash et al. (2015)

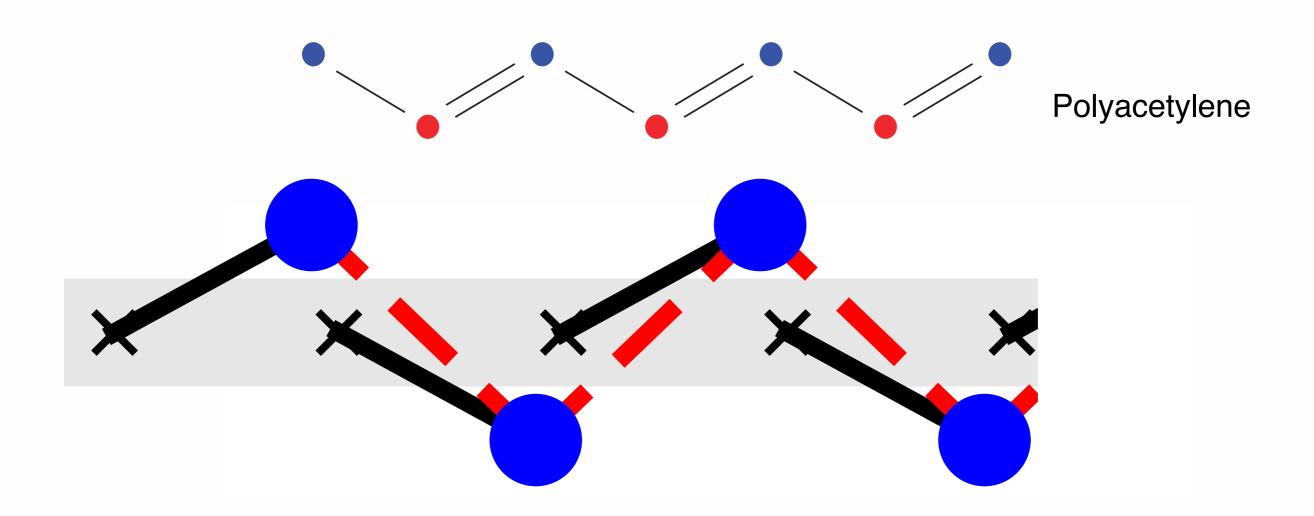


Coupled pendula Susstrunk and Huber (2015)

Physical properties insensitive to smooth changes in material parameters

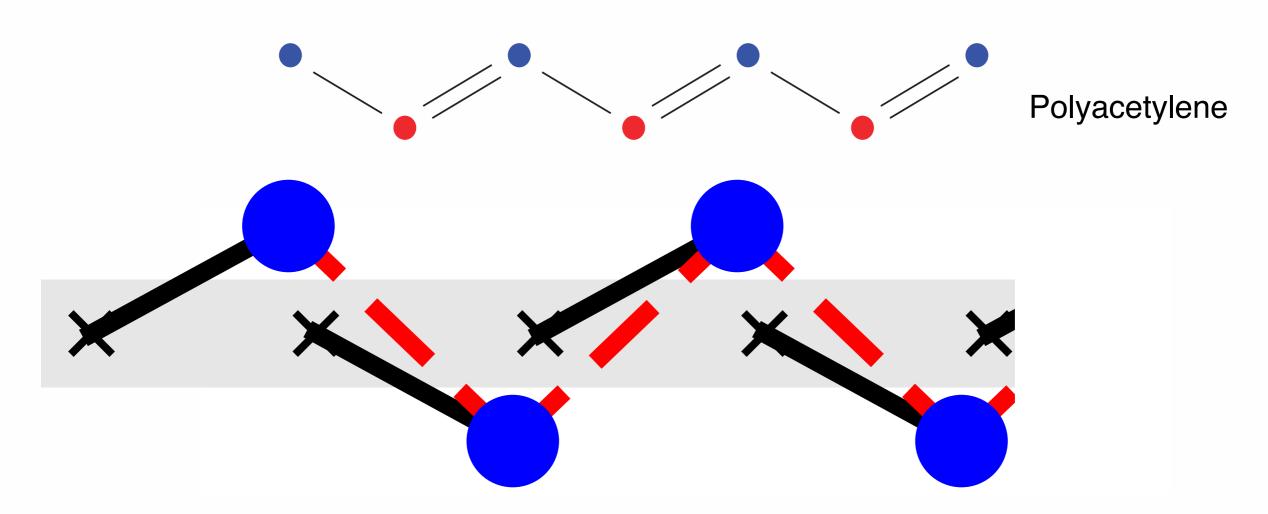


A simple topological mechanical system





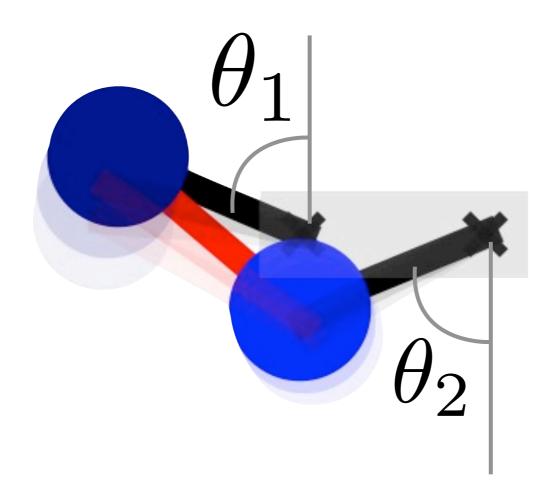
A simple topological mechanical system



The vibrations of an isostatic mechanical system may be mapped to the electronic states in a topological superconductor

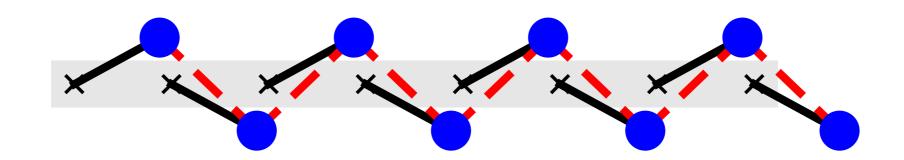


The humble four-bar linkage:





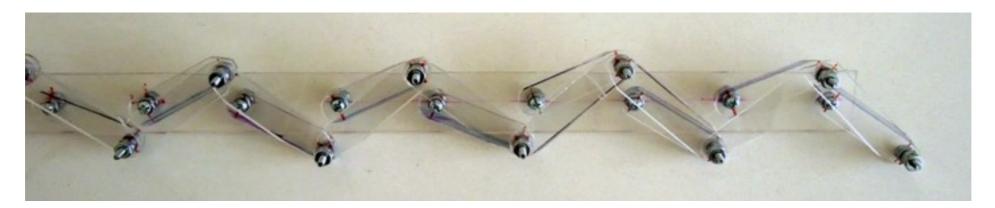
rotors



springs



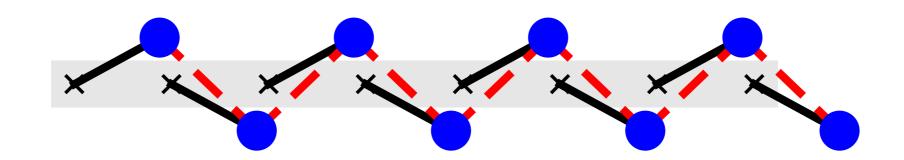
rotors



bars



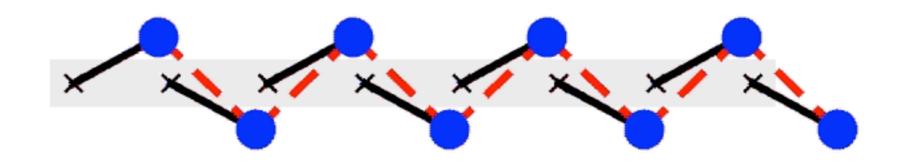
rotors



springs



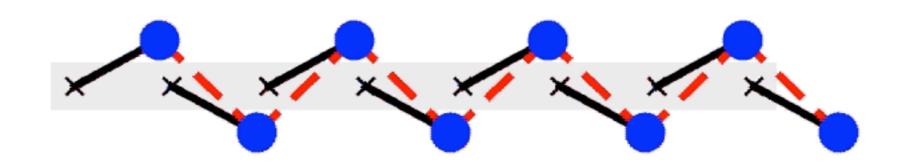
rotors



springs



rotors

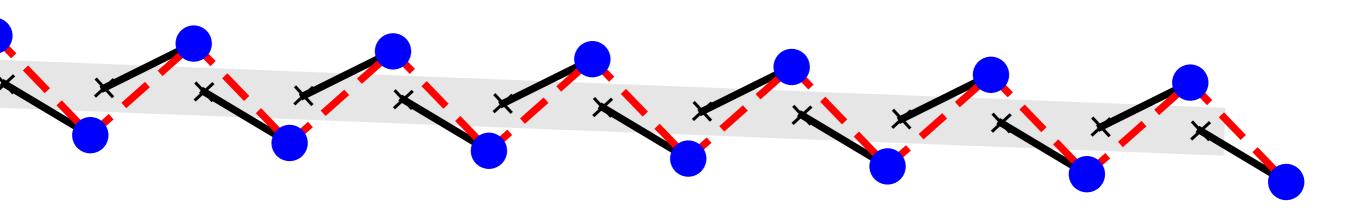


springs

2 uniform stable states!



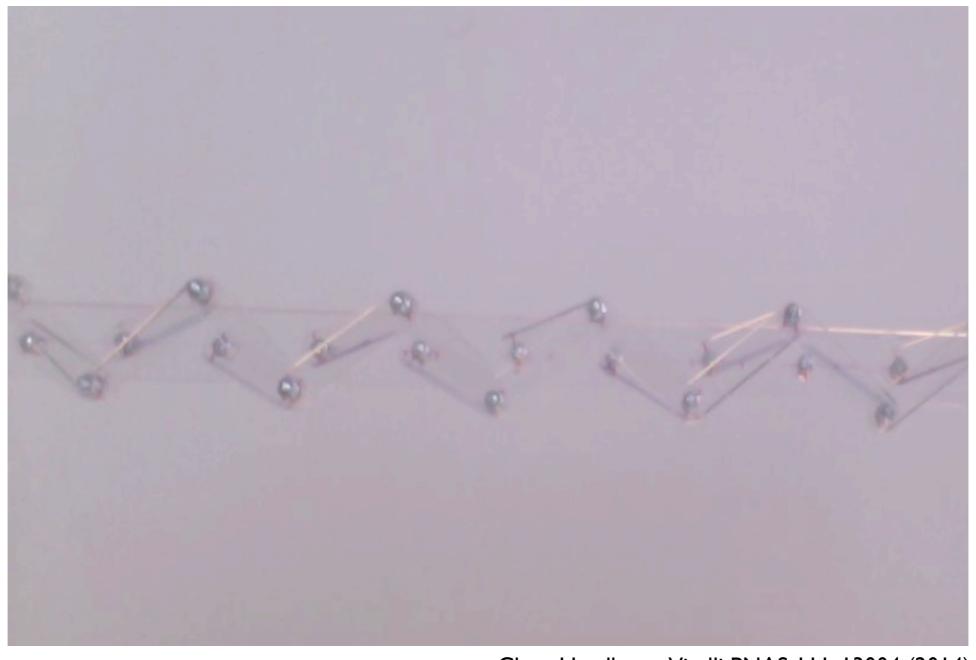
Rigid in the bulk



Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)



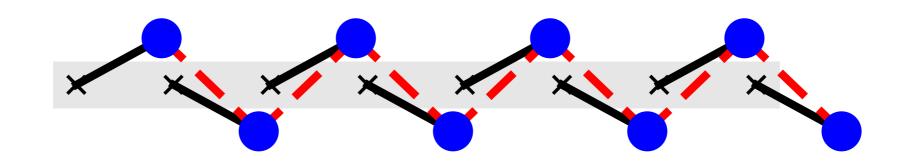
Rigid in the bulk



Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)



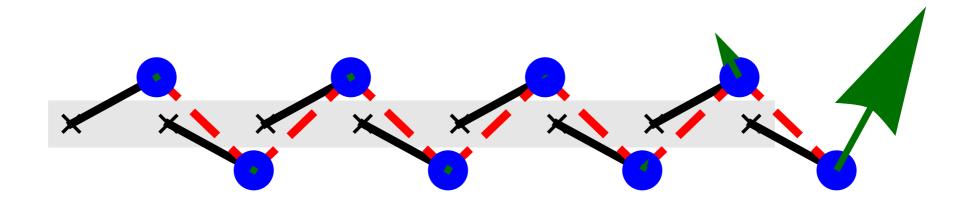
Exponentially localized zero mode



This chain has 8 rotors and 7 springs; 8-7=1 unconstrained degree of freedom



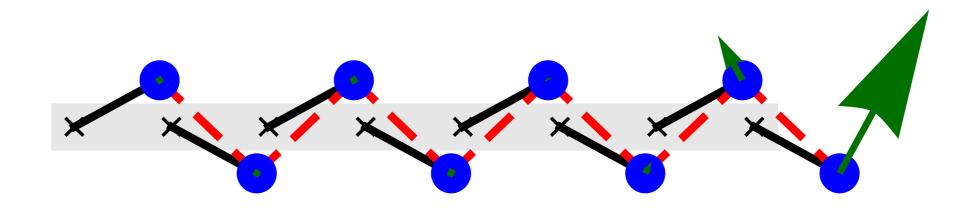
Exponentially localized zero mode



This chain has 8 rotors and 7 springs; 8-7=1 unconstrained degree of freedom



Exponentially localized zero mode

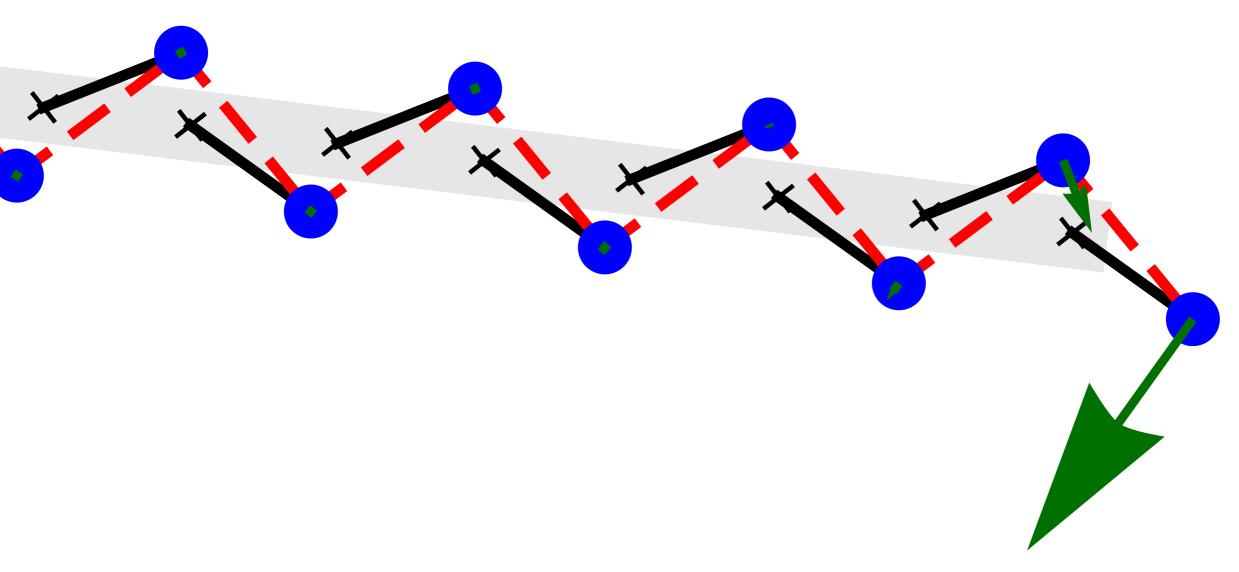


This chain has 8 rotors and 7 springs; 8-7=1 unconstrained degree of freedom

The zero mode is exponentially localized at the end.



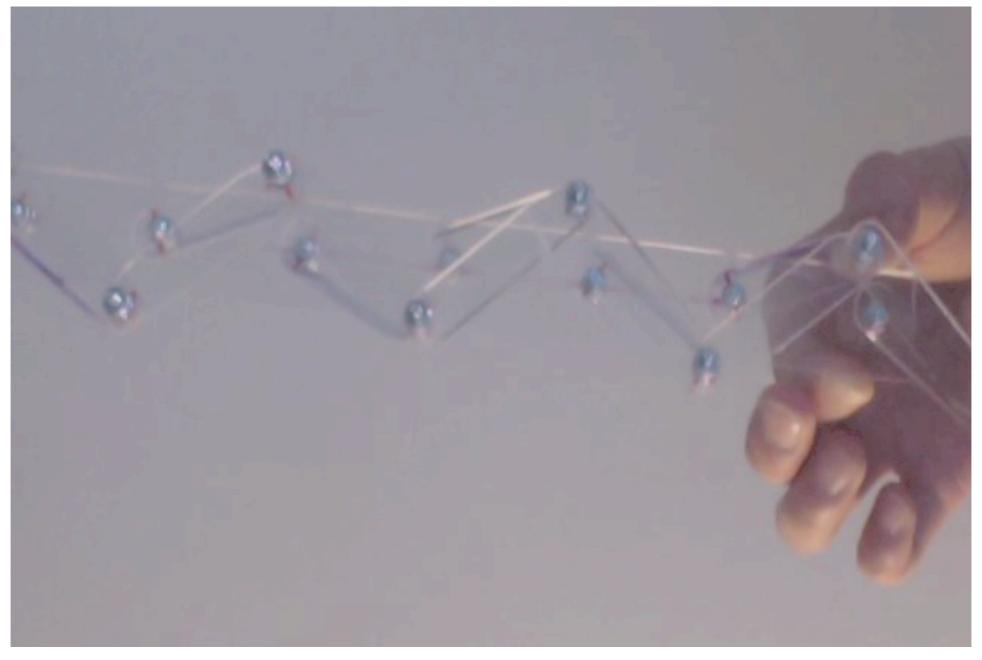
Exponentially localized zero mode



Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

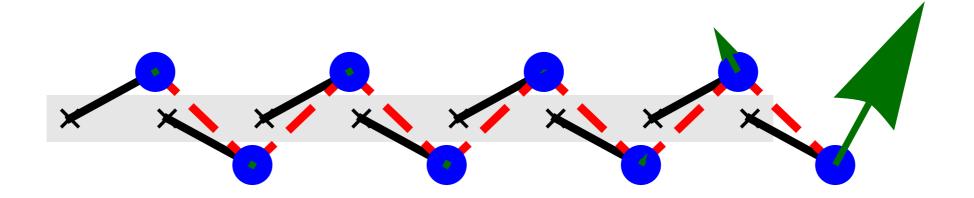


Exponentially localized zero mode

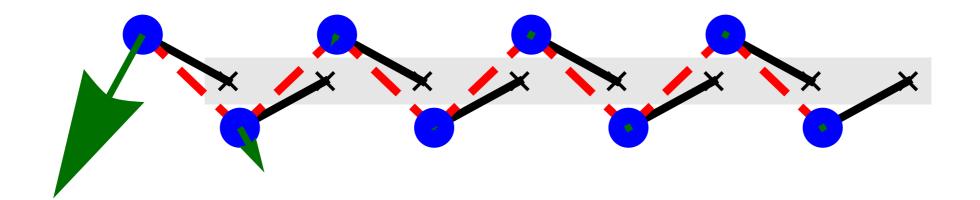


Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

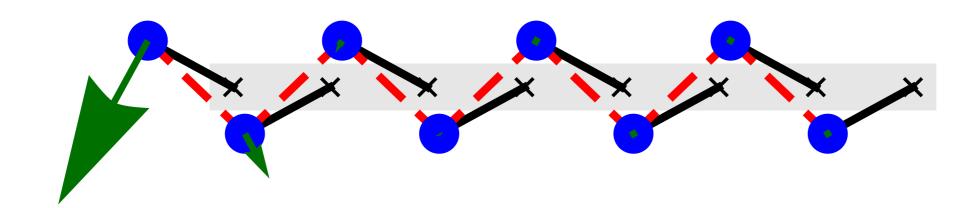




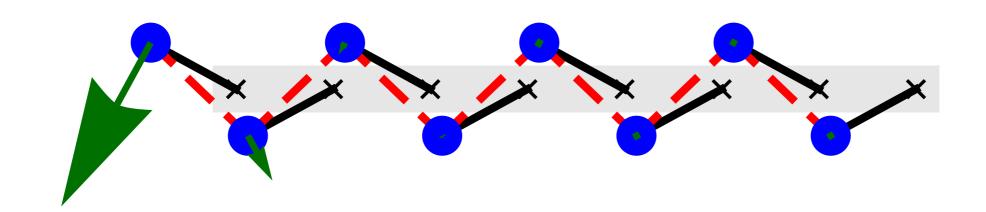


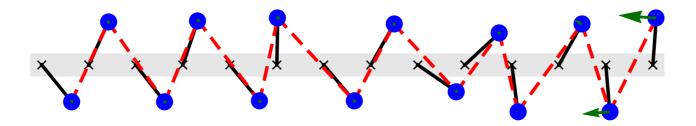




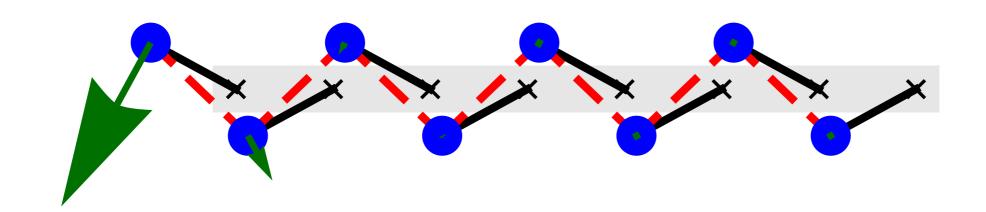


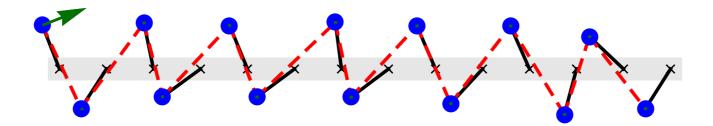




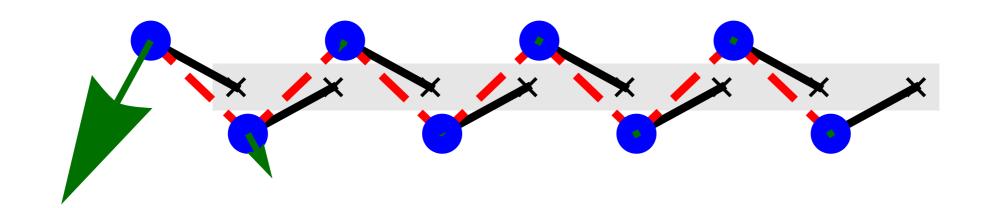


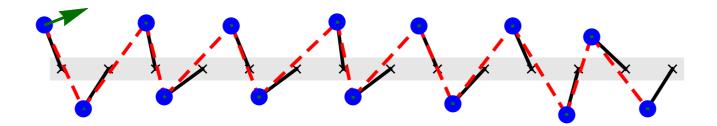






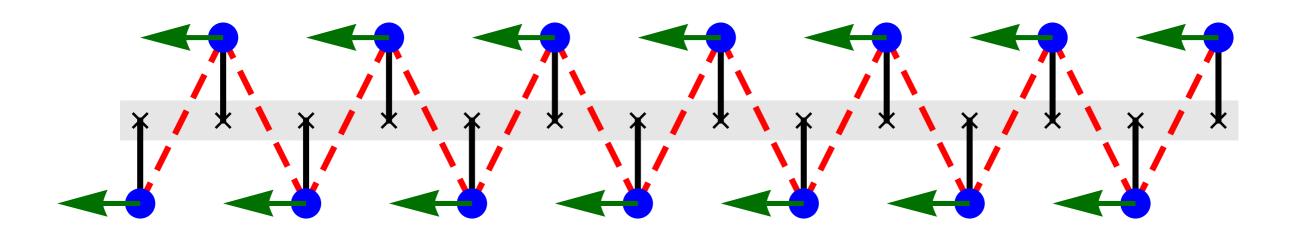


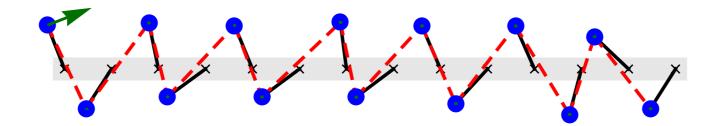




Topologically protected

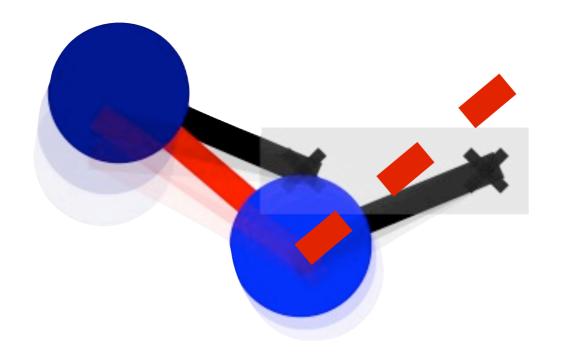




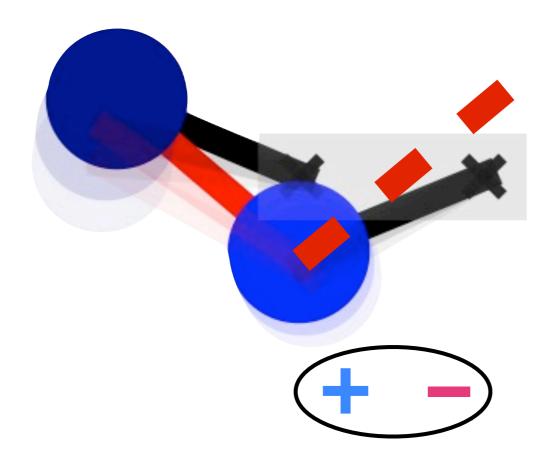


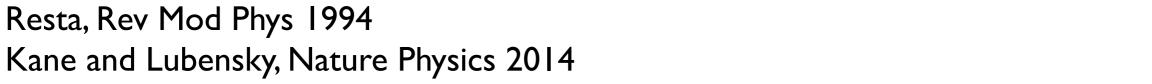
Topologically protected



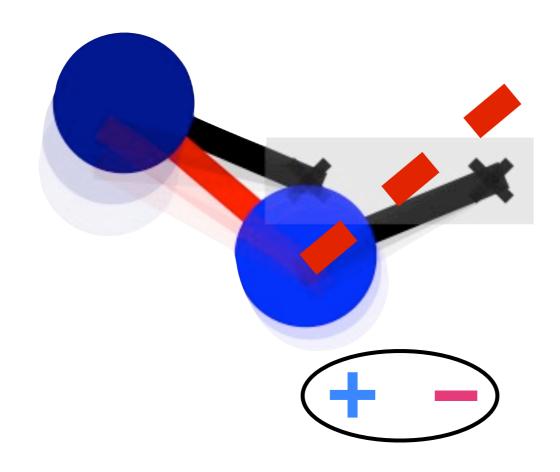






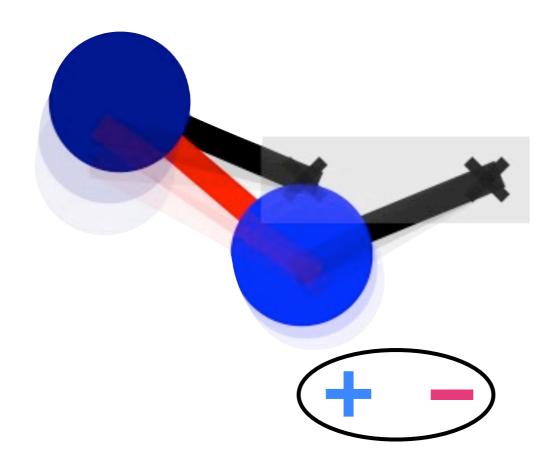






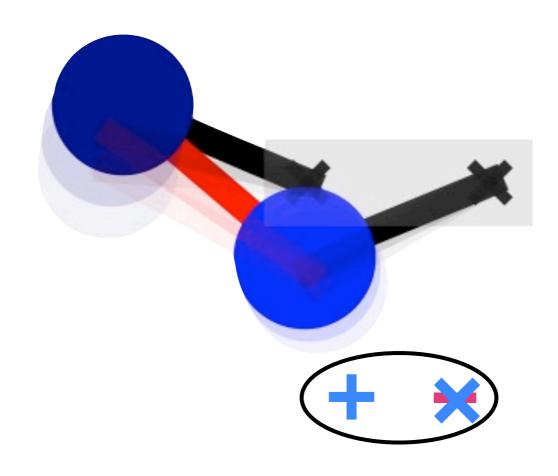
- + charges: degrees of freedom
- charges: constraints





- + charges: degrees of freedom
- charges: constraints

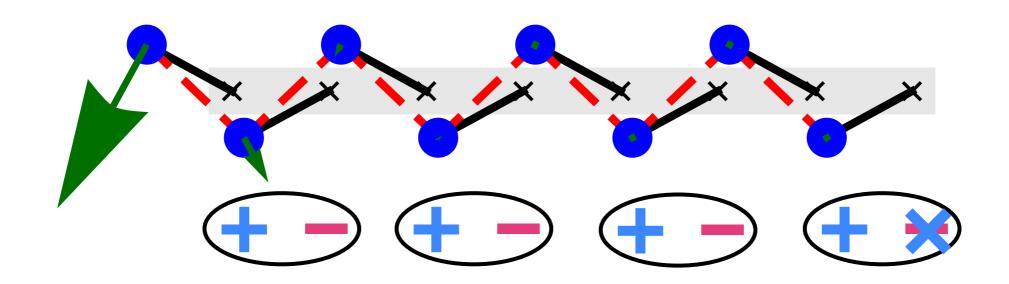




- + charges: degrees of freedom
- charges: constraints



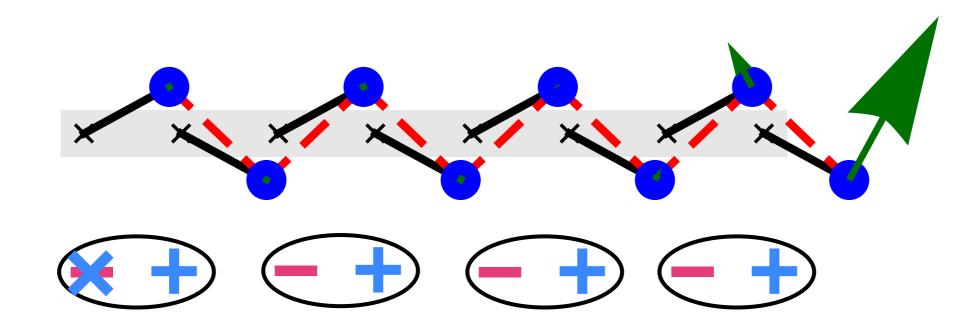
Bulk / Boundary



- + charges: degrees of freedom
- charges: constraints



Bulk / Boundary



- + charges: degrees of freedom
- charges: constraints



Mechanical energy **cannot** be transmitted across the chain via **linear** vibrations!



Mechanical energy **cannot** be transmitted across the chain via **linear** vibrations!

OUR QUESTION:

What happens when we excite the zero mode beyond the linear regime?







The chain conducts mechanical energy!





The chain conducts mechanical energy!



An insulator at harmonic level has become a conductor in non-linear theory



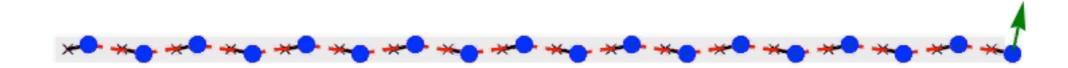
The chain conducts mechanical energy!



An insulator at harmonic level has become a conductor in non-linear theory

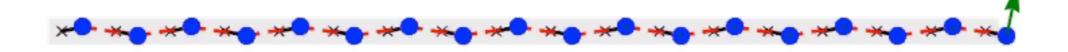
Aside: "Jacob's ladder" toy shares this mechanism



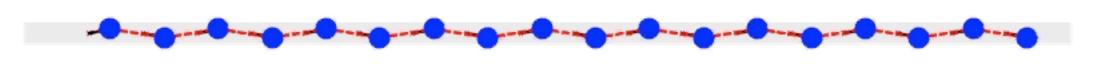




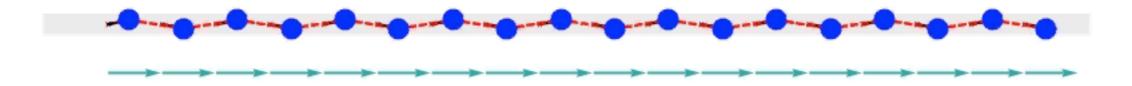
localized zero mode is transported from end to end (and back)!





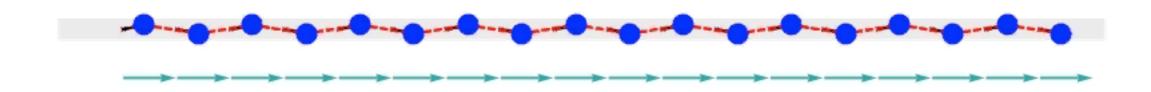






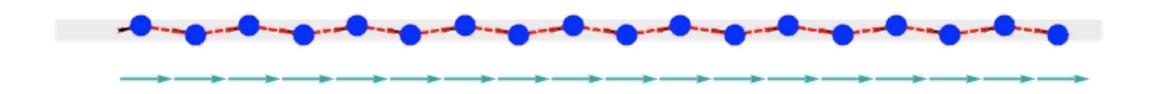


The Flipper





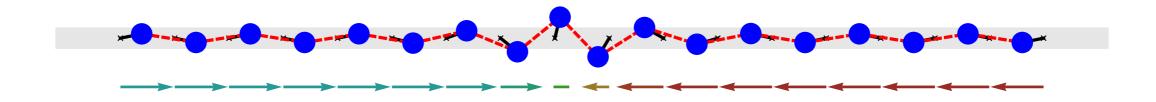
The Flipper



Beyond linear order, the zero mode becomes a moving **domain wall** converting left-leaning to right-leaning and vice versa!

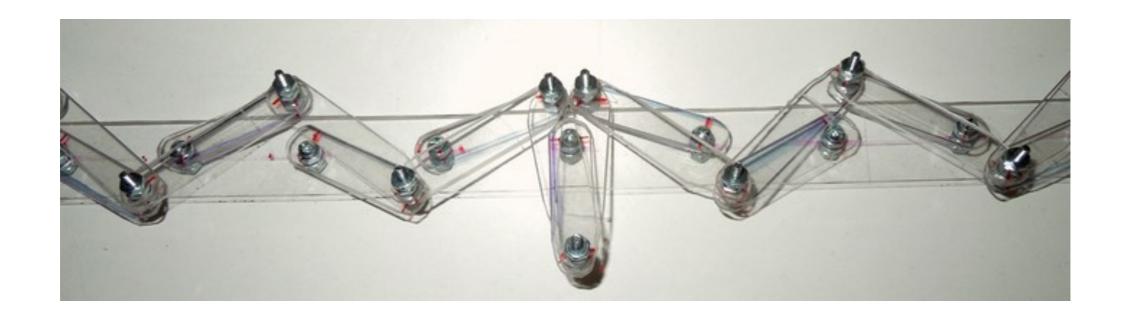


The Flipper

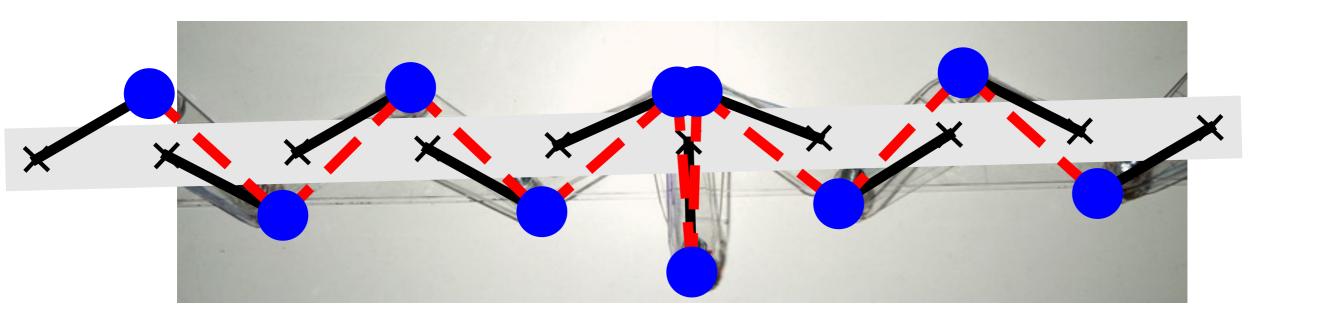


Beyond linear order, the zero mode becomes a moving **domain wall** converting left-leaning to right-leaning and vice versa!

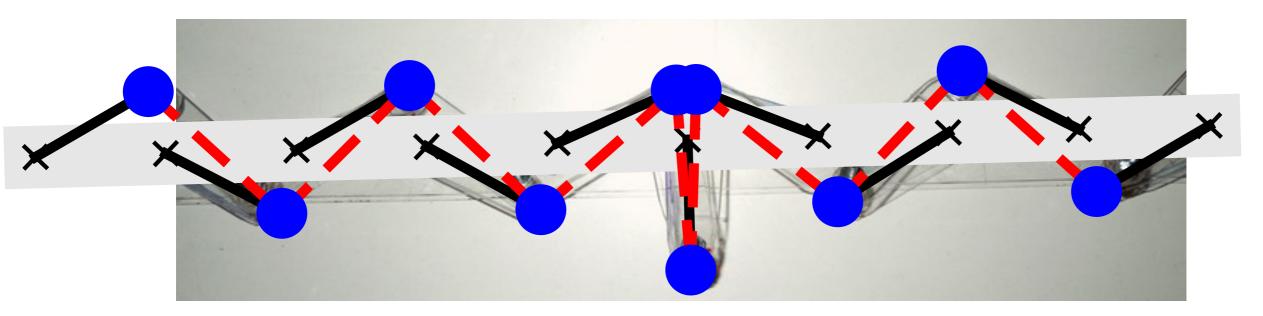






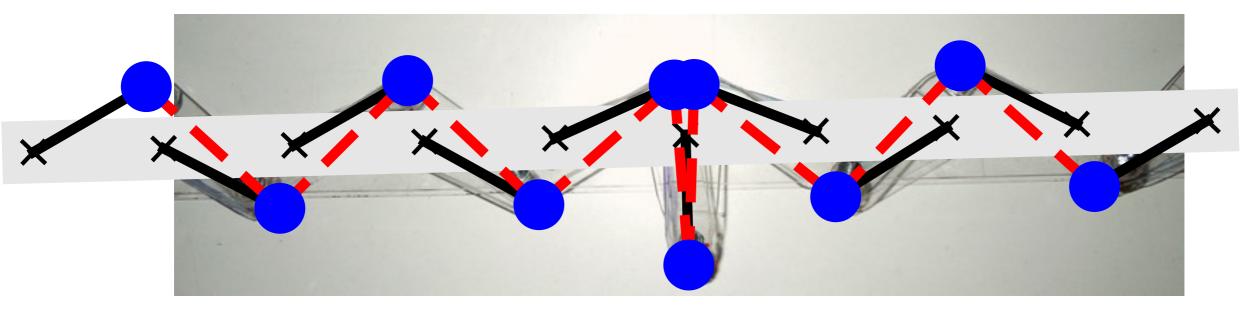






left-leaning



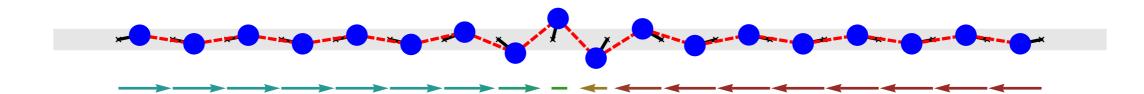


right-leaning

left-leaning

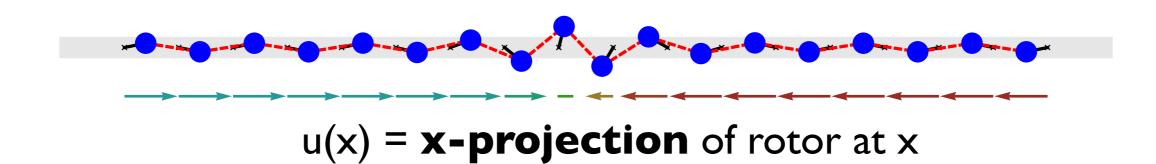


what soliton?



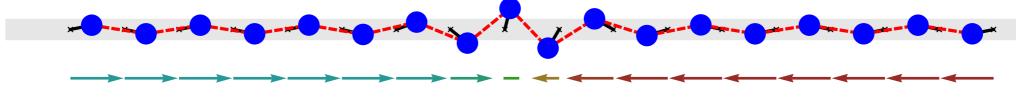


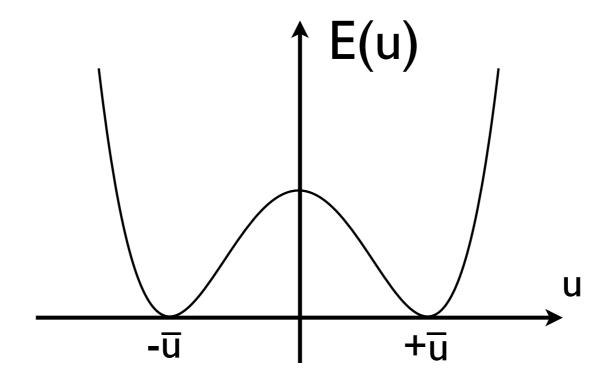
what soliton?



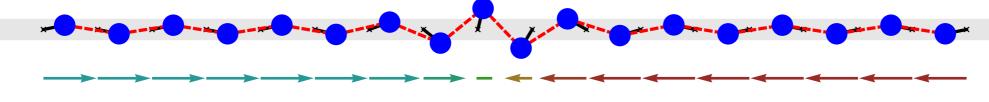


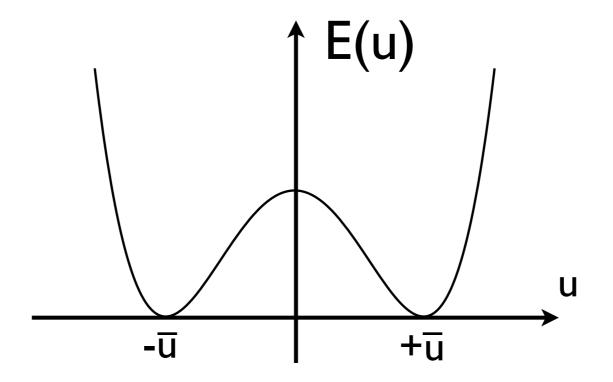
what soliton?



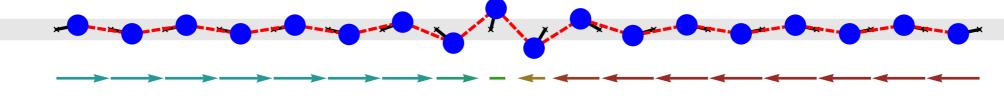


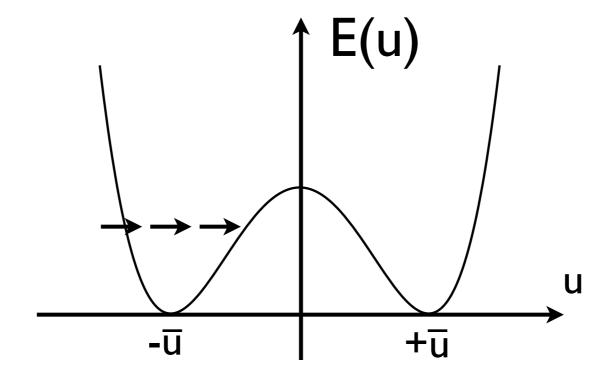




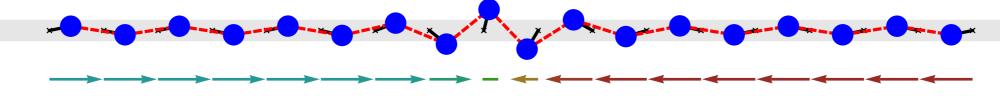


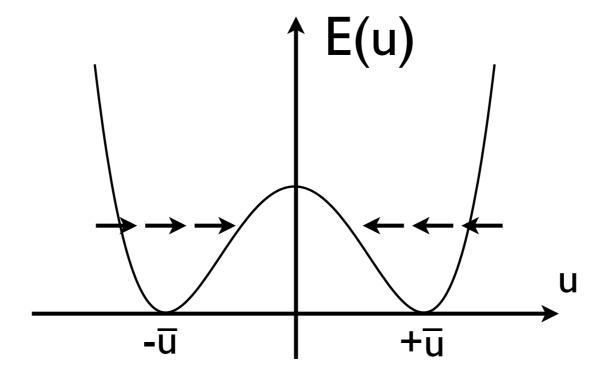




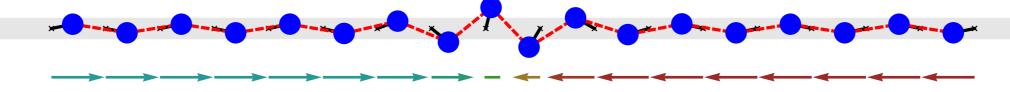




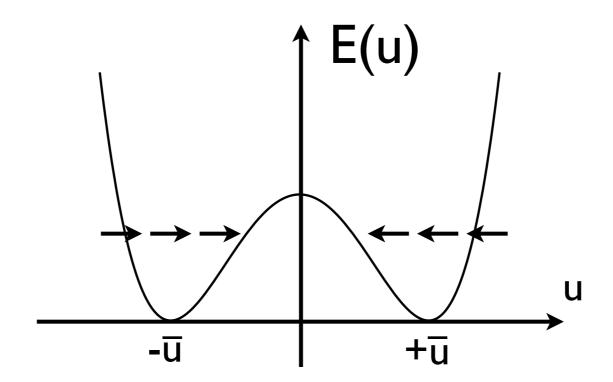






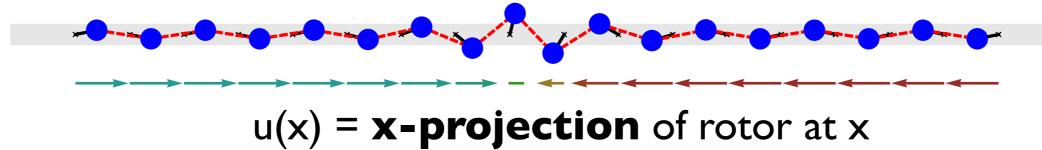


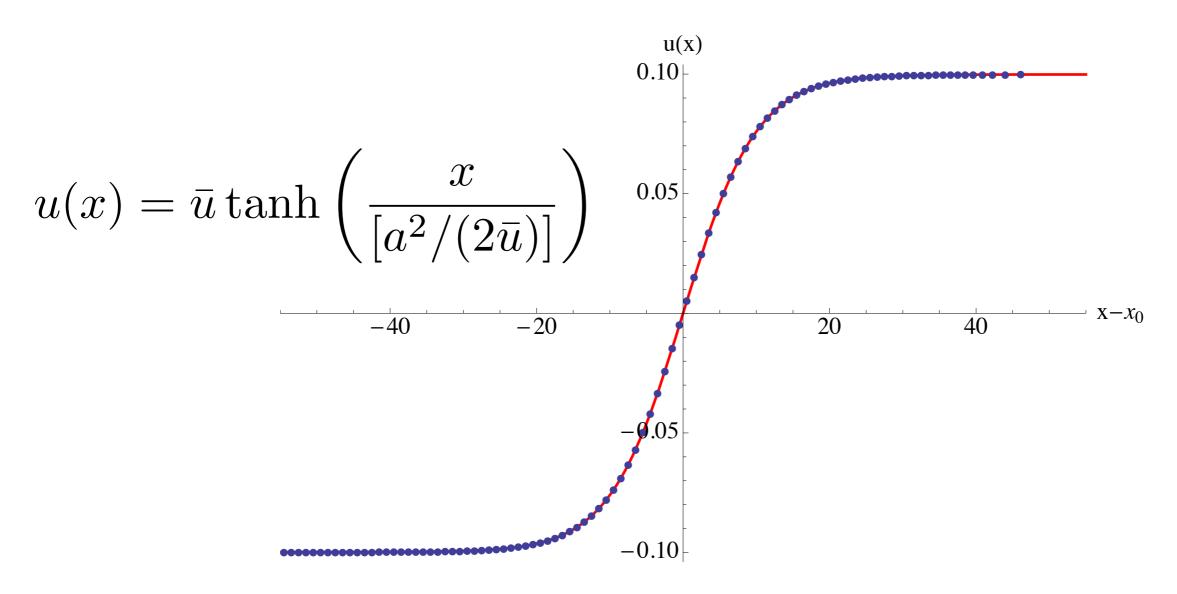
u(x) = x-projection of rotor at x



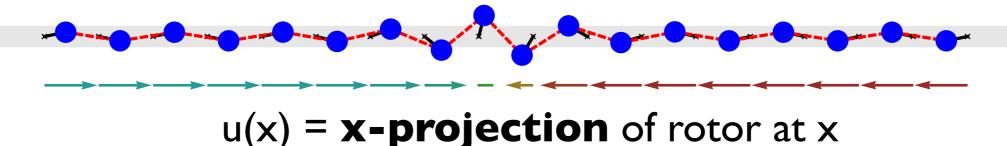
 $\rightarrow \rightarrow \rightarrow \leftarrow \leftarrow$ "kink" = domain wall profile

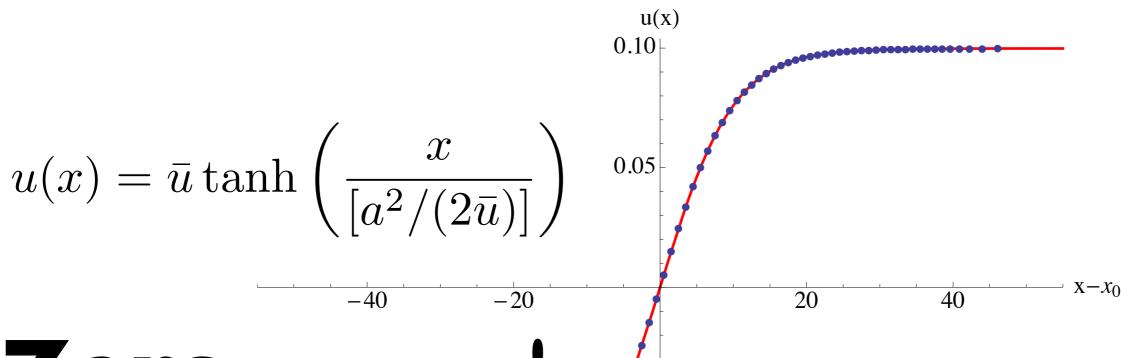








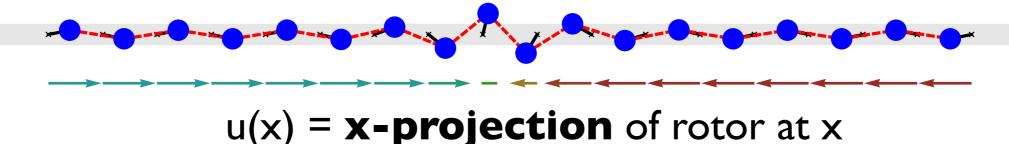


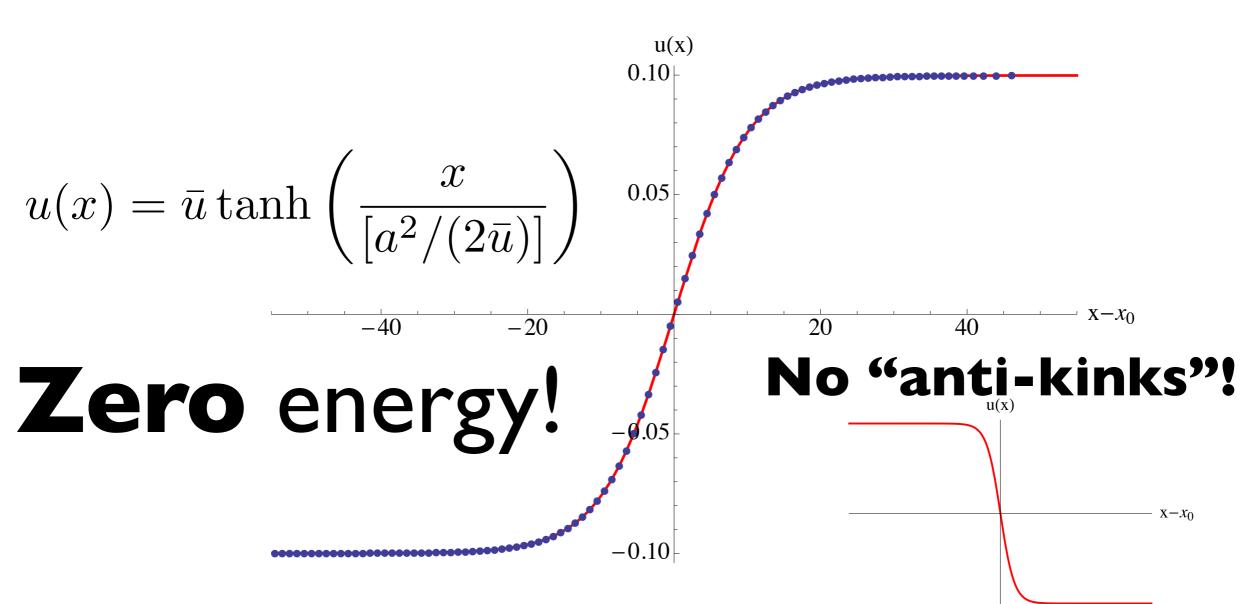


-0.10

Zero energy!

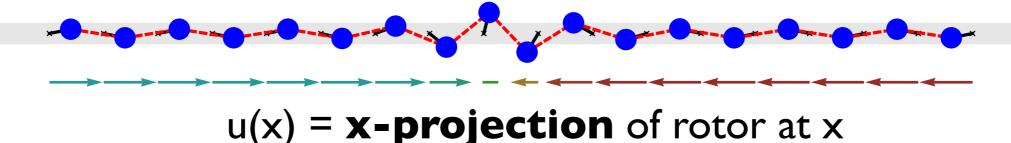


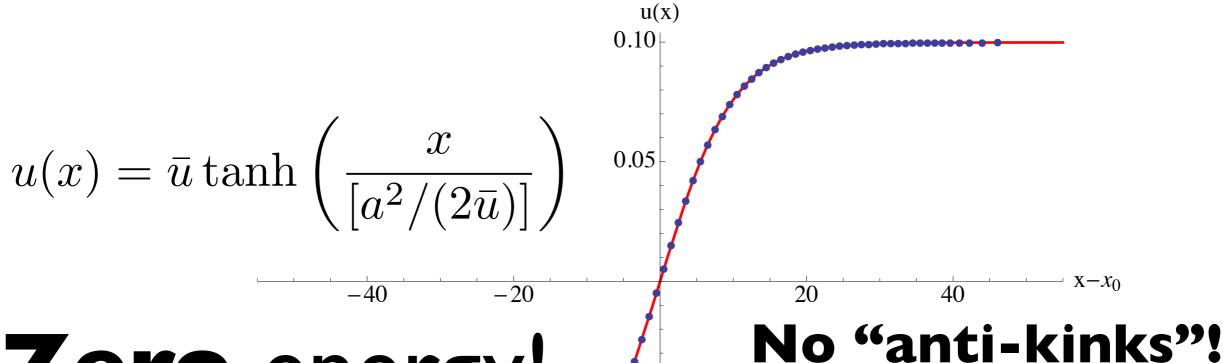




Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)





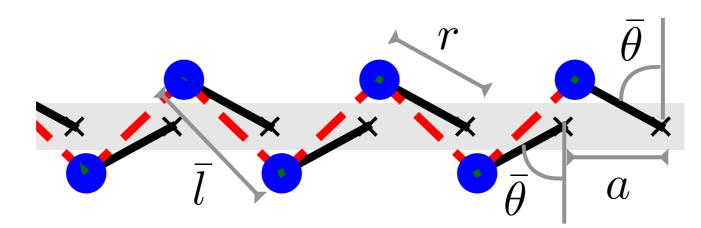


-0.10

Zero energy!

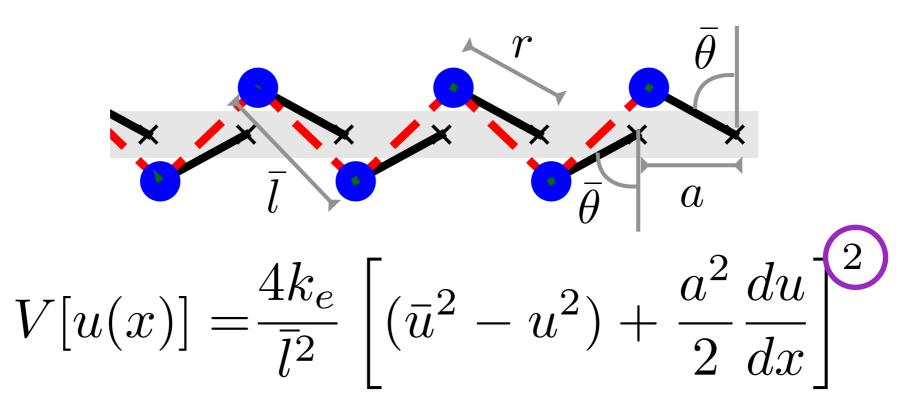
anti-Kinks in the second secon

Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)



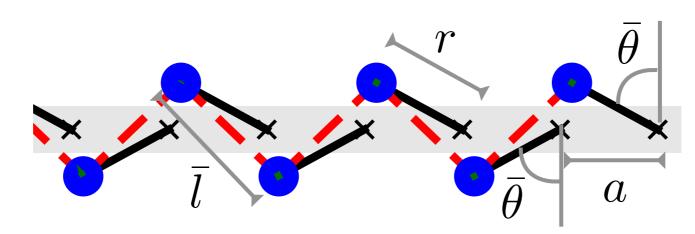
$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[(\bar{u}^2 - u^2) + \frac{a^2}{2} \frac{du}{dx} \right]^2$$





Expand the **perfect square**



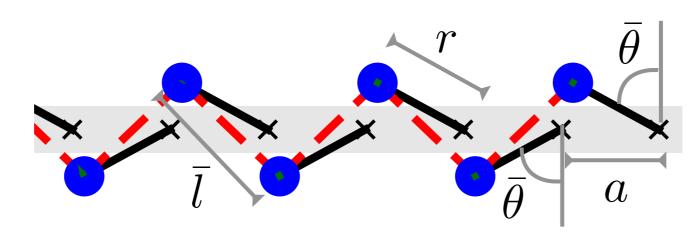


$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[(\bar{u}^2 - u^2) + \frac{a^2}{2} \frac{du}{dx} \right]^2$$

Expand the perfect square

$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[\frac{a^4}{4} \left(\frac{du}{dx} \right)^2 + (\bar{u}^2 - u^2)^2 + \frac{a^2}{2} \frac{du}{dx} (\bar{u}^2 - u^2) \right]$$



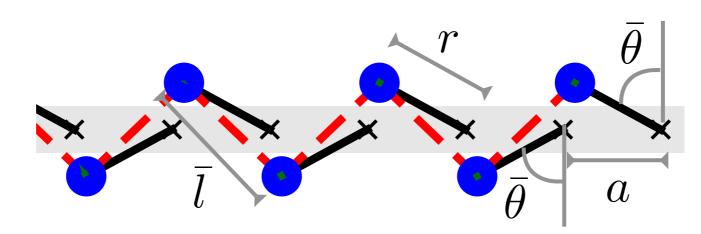


$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[(\bar{u}^2 - u^2) + \frac{a^2}{2} \frac{du}{dx} \right]^2$$

φ⁴ theory

$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[\frac{a^4}{4} \left(\frac{du}{dx} \right)^2 + (\bar{u}^2 - u^2)^2 + \frac{a^2}{2} \frac{du}{dx} (\bar{u}^2 - u^2) \right]$$





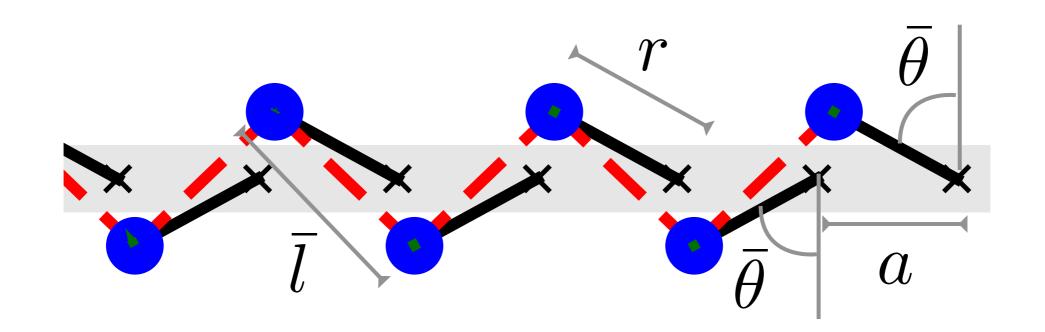
$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[(\bar{u}^2 - u^2) + \frac{a^2}{2} \frac{du}{dx} \right]^2$$

 Φ^4 theory + topological term!

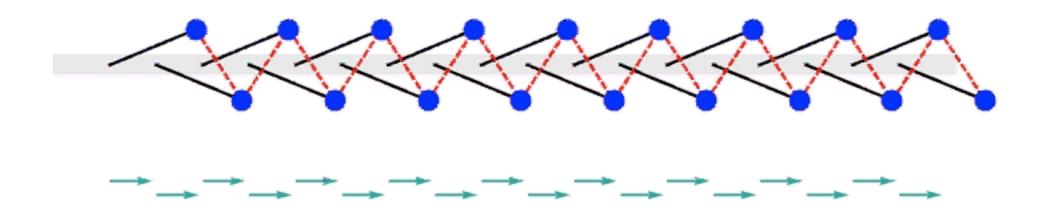
$$V[u(x)] = \frac{4k_e}{\bar{l}^2} \left[\frac{a^4}{4} \left(\frac{du}{dx} \right)^2 + (\bar{u}^2 - u^2)^2 + \left(\frac{a^2}{2} \frac{du}{dx} (\bar{u}^2 - u^2) \right) \right]$$



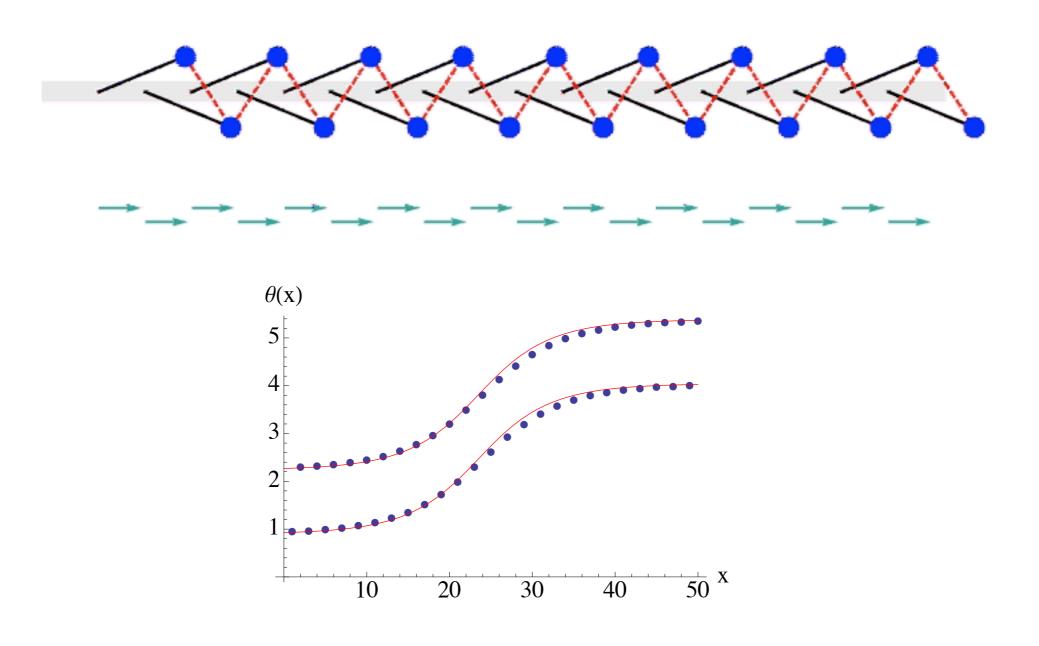
What properties are robust? What can be tuned?



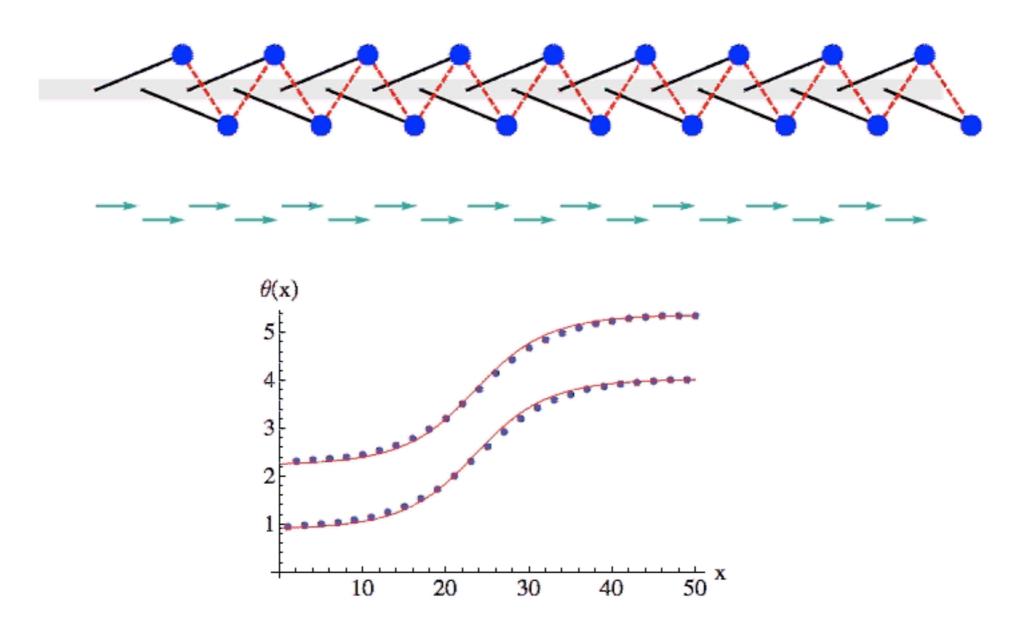




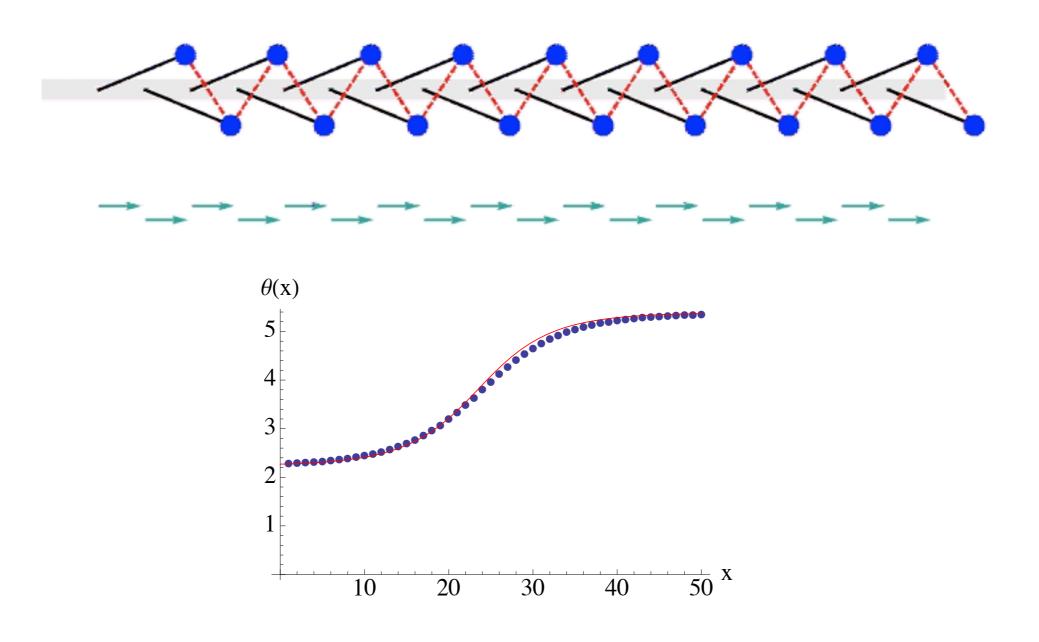




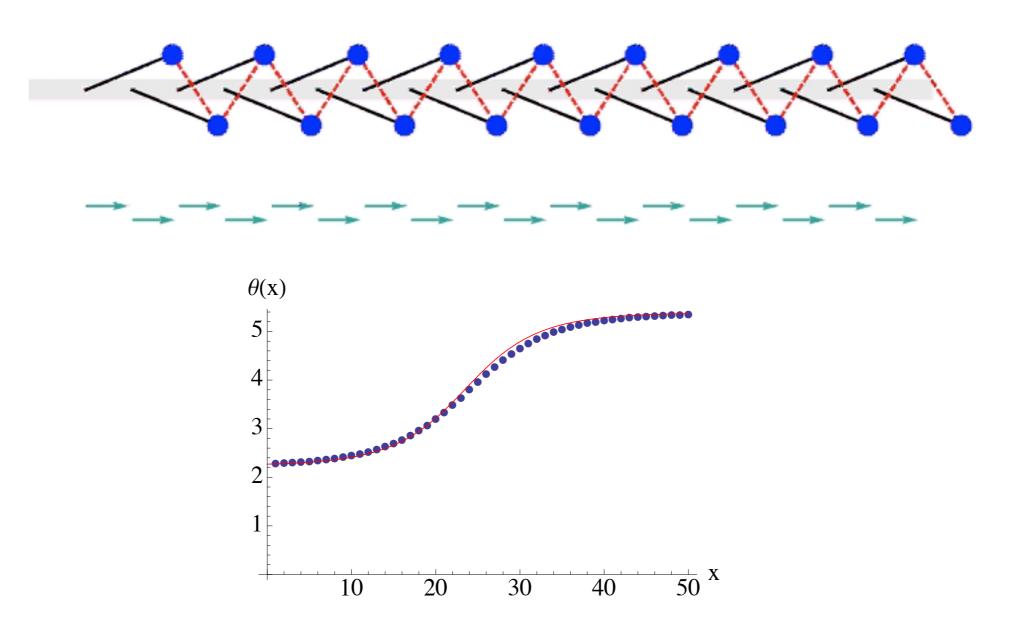








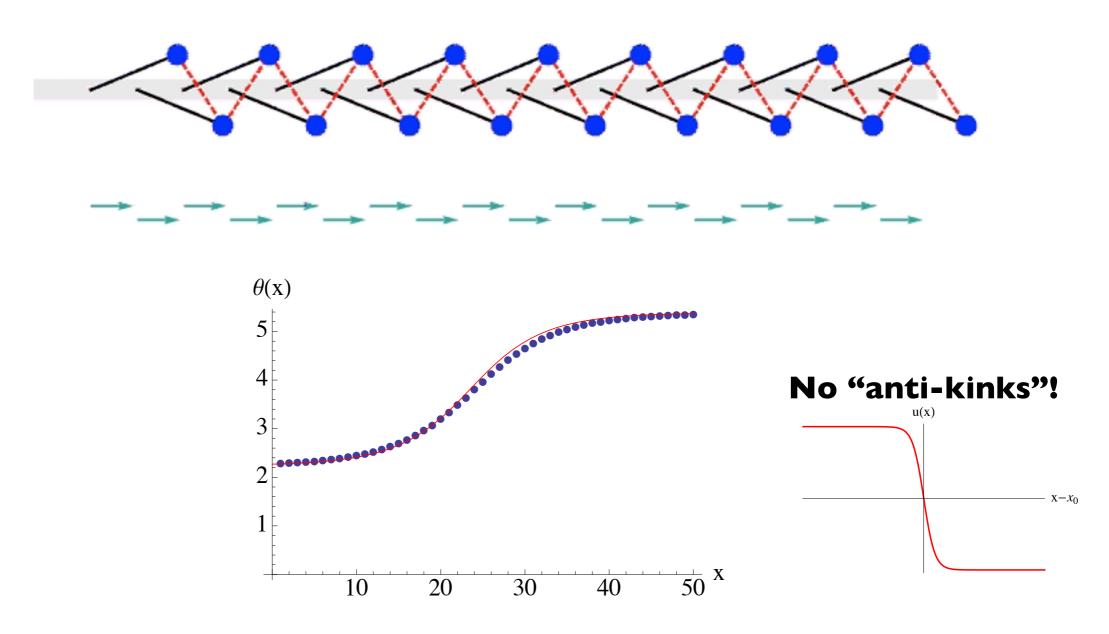




continuum limit: 2 coupled copies of sine-Gordon kink!

Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

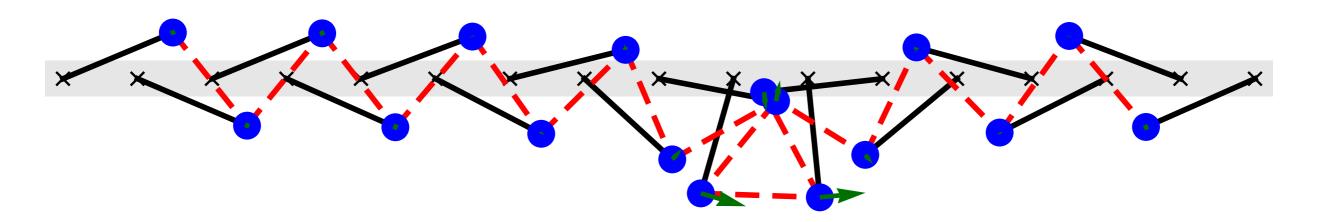




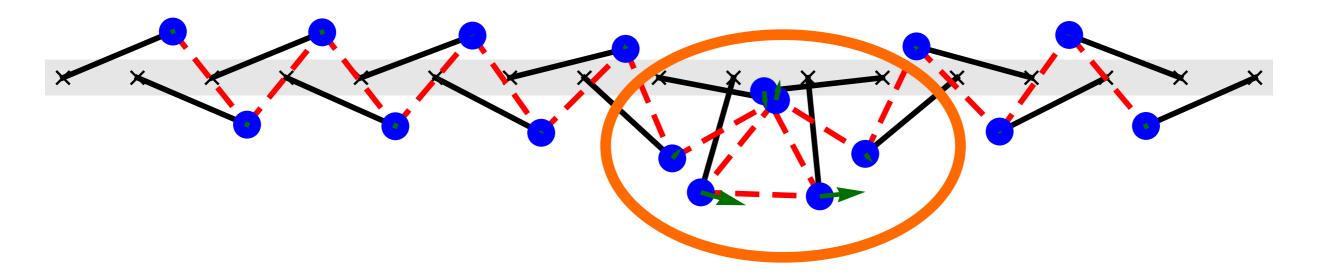
continuum limit: 2 coupled copies of sine-Gordon kink!

Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

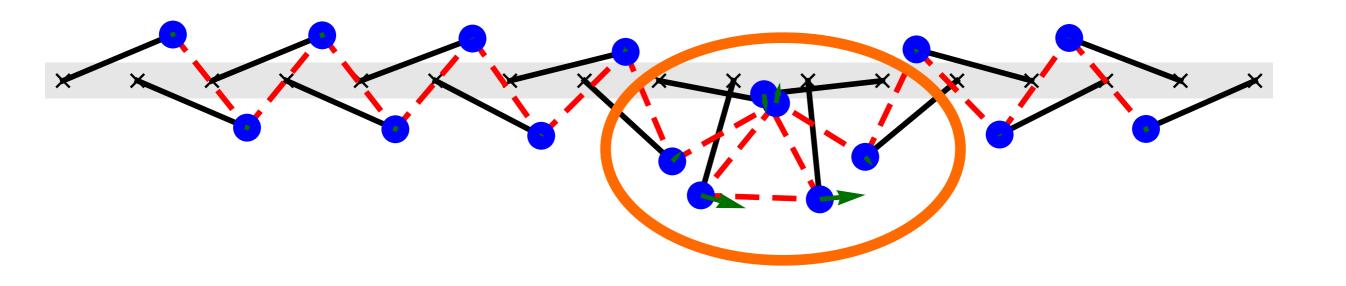






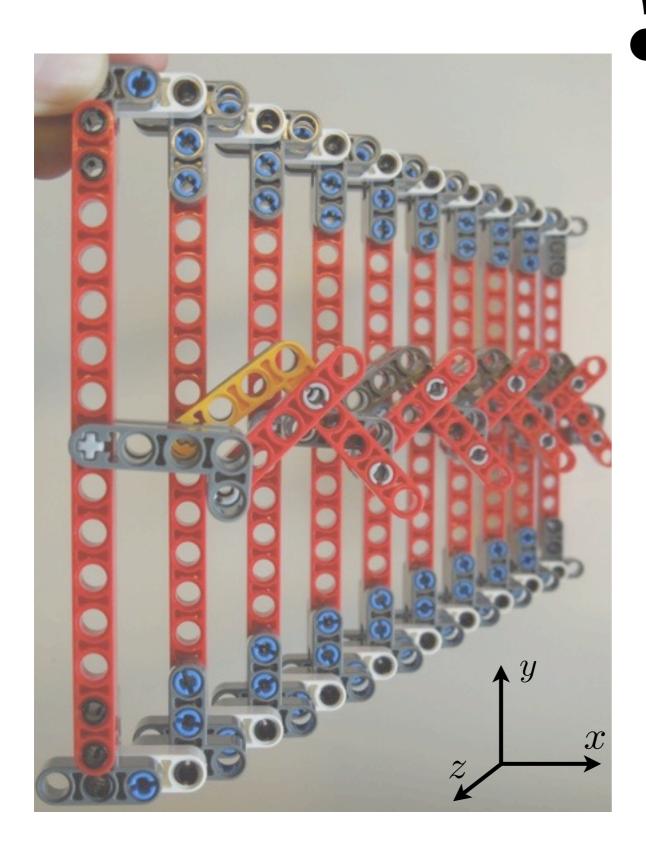


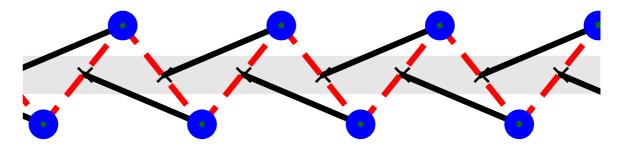






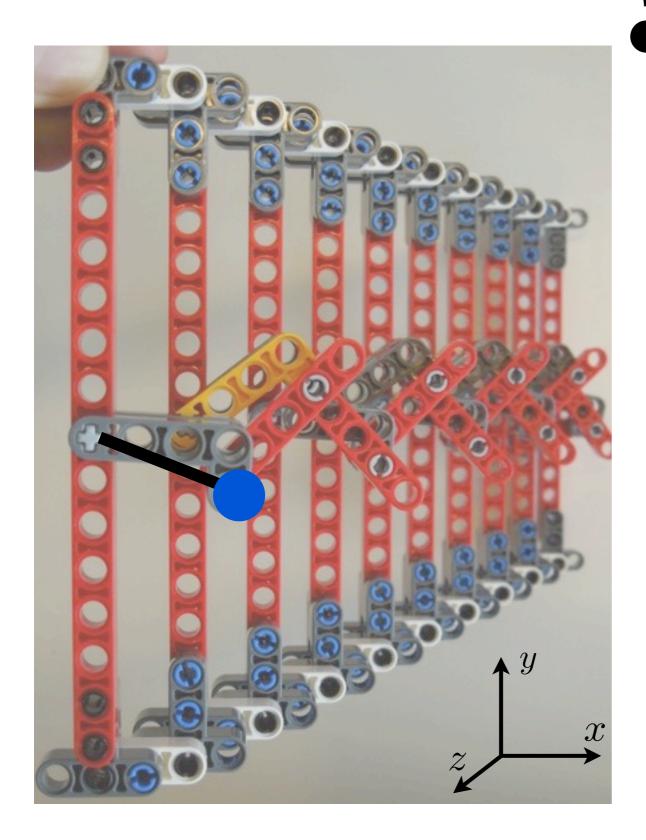


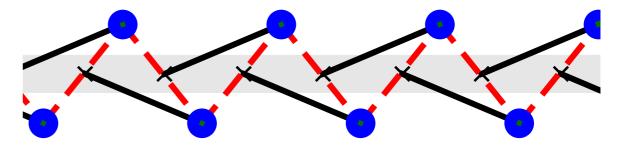




Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

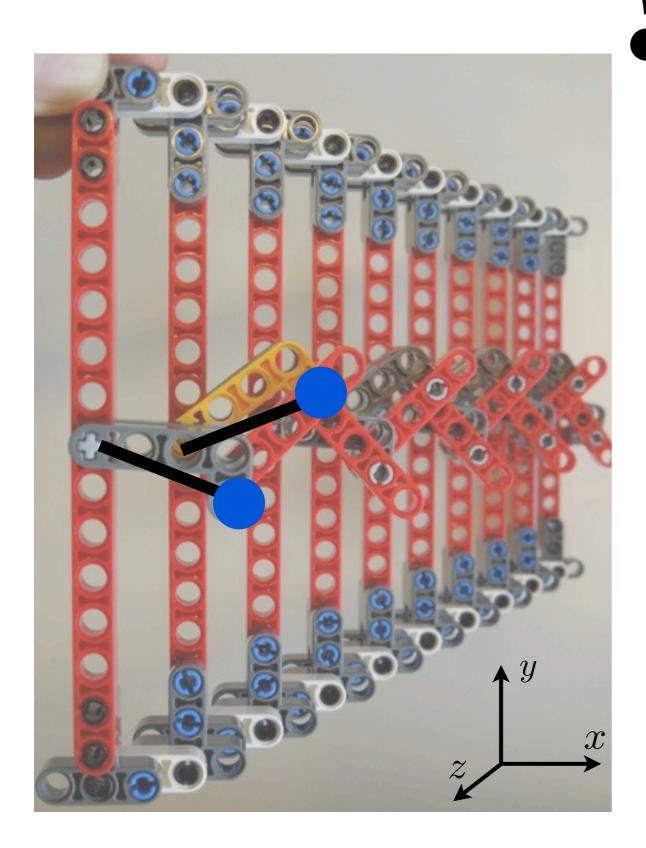


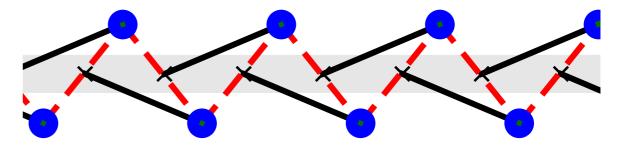




Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

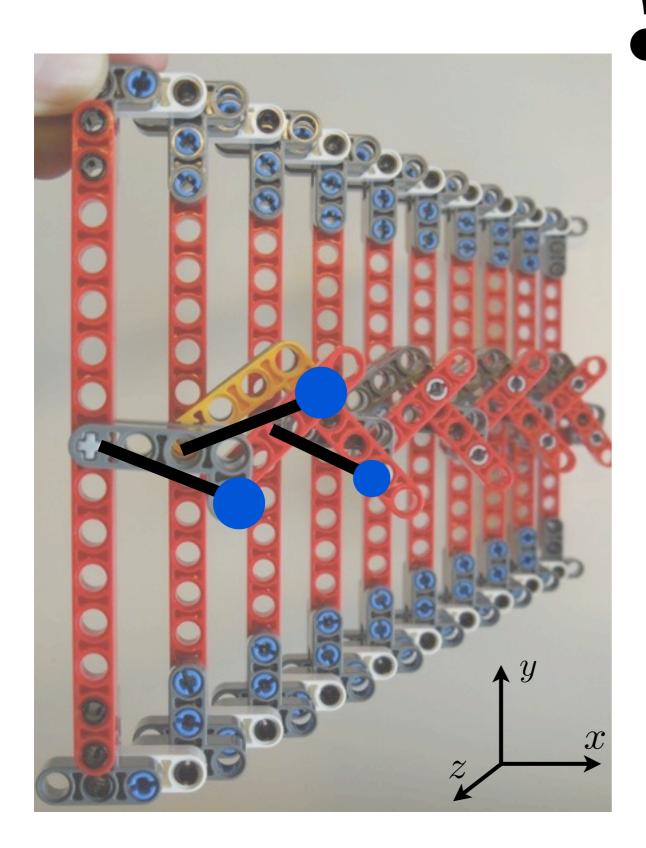


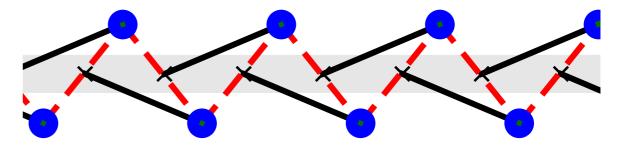




Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

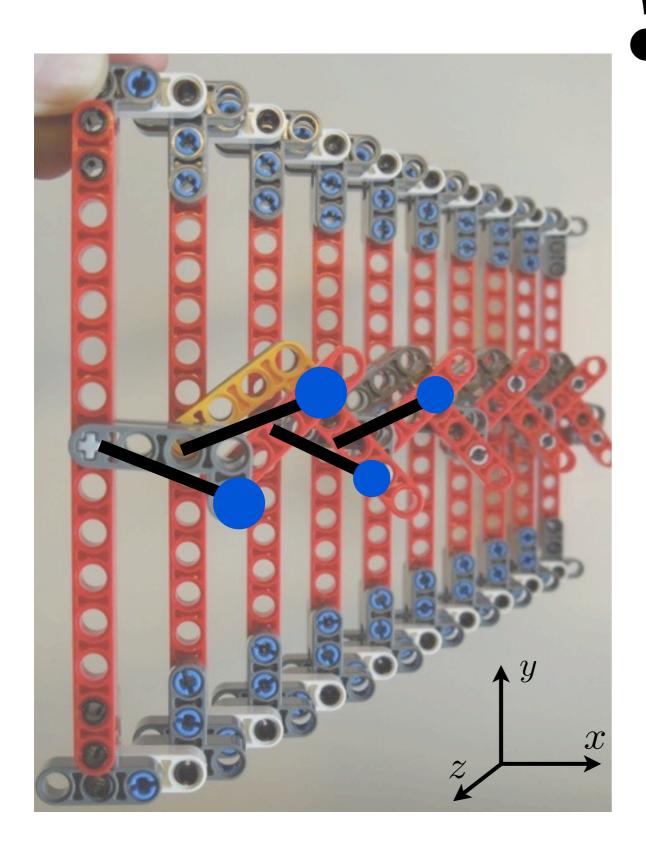


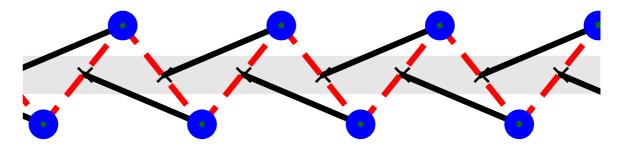




Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)

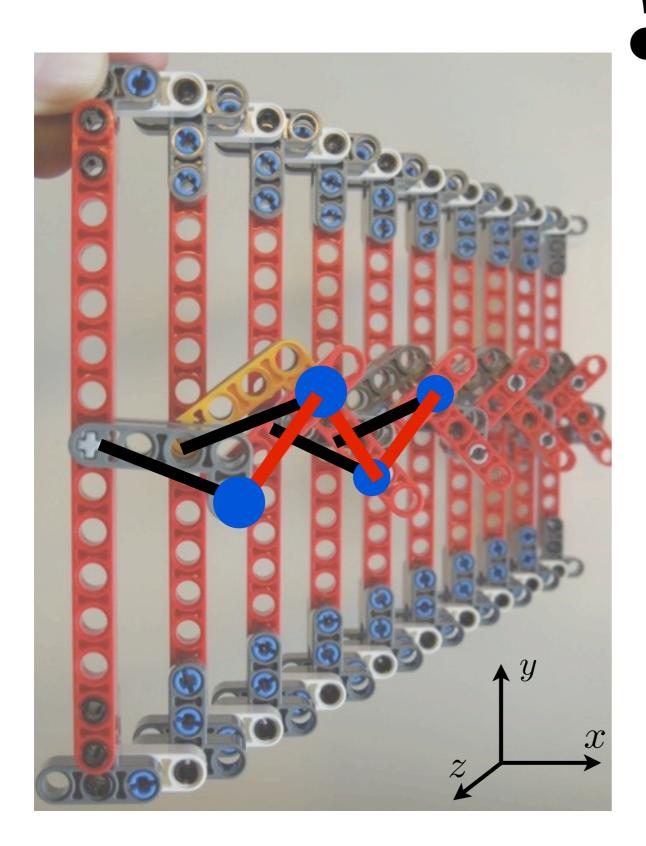


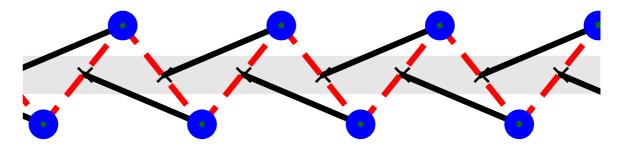




Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)







Chen, Upadhyaya, Vitelli PNAS 111, 13004 (2014)



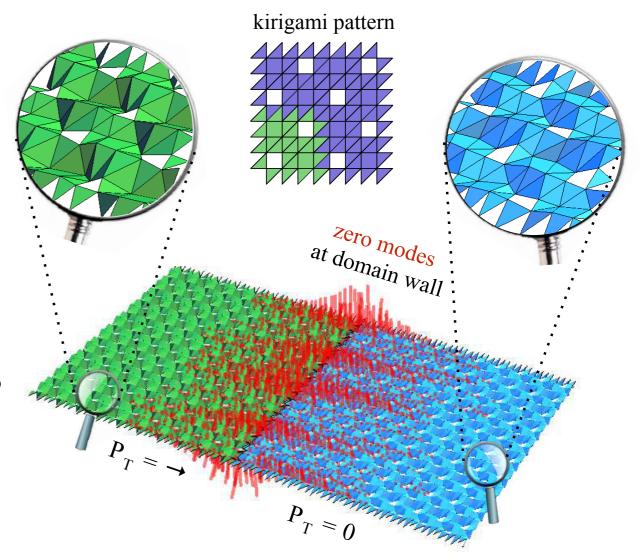
Beyond beads and springs

soft edge localized deformation

stiff edge

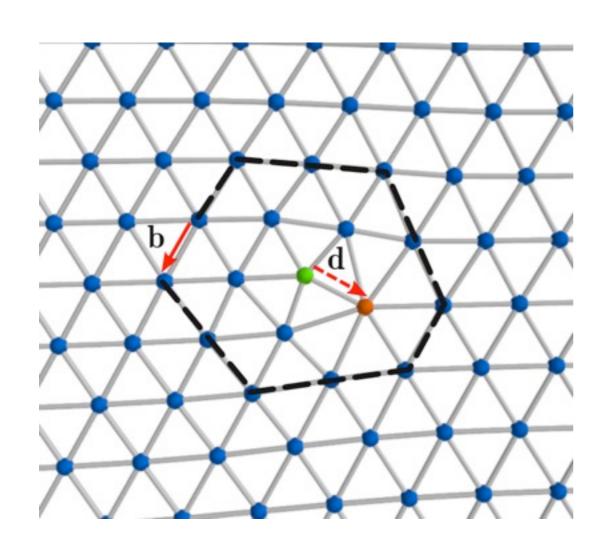
Topologically polarized origami and kirigami

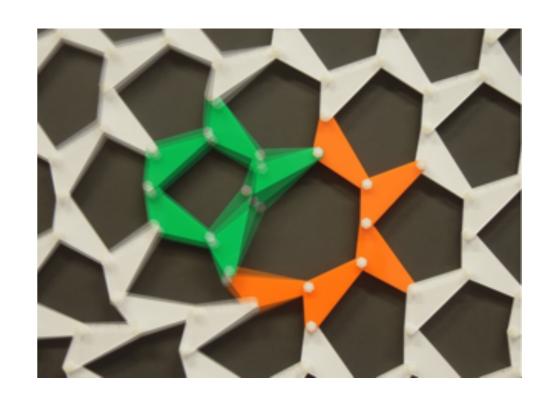
Conjecture: origami (without holes) always has zero polarization*



BGC, Liu, Evans, Paulose, Cohen, Vitelli, Santangelo. arXiv: I 508.00795

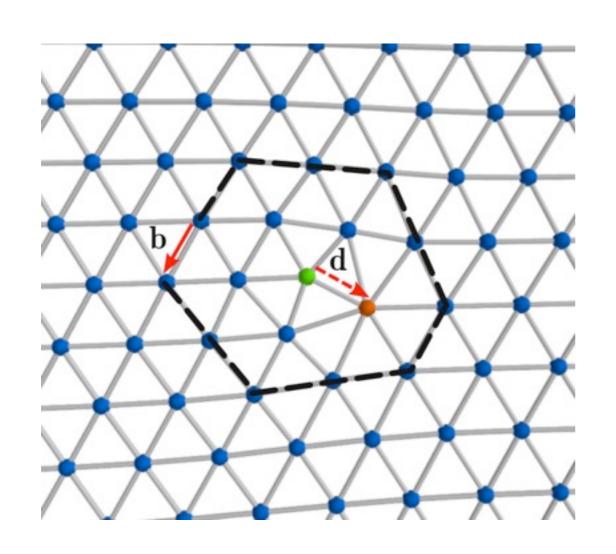


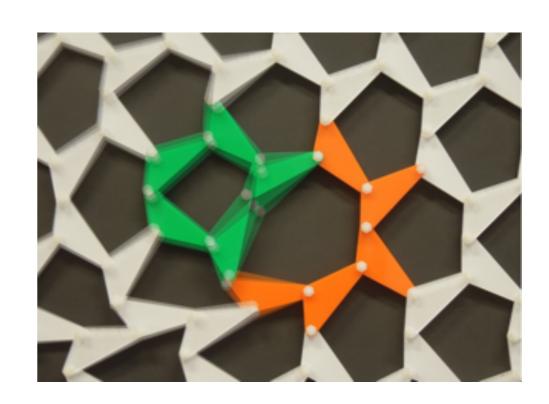






Topological modes at **point**-like defects called **dislocations**





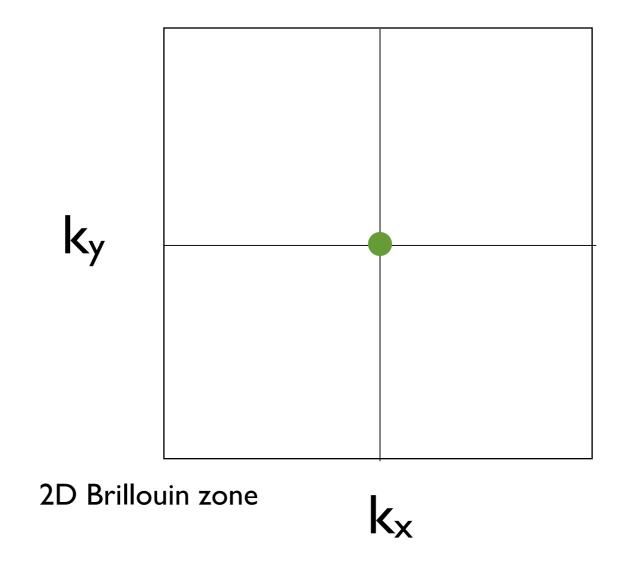


What about modes localized in reciprocal space?

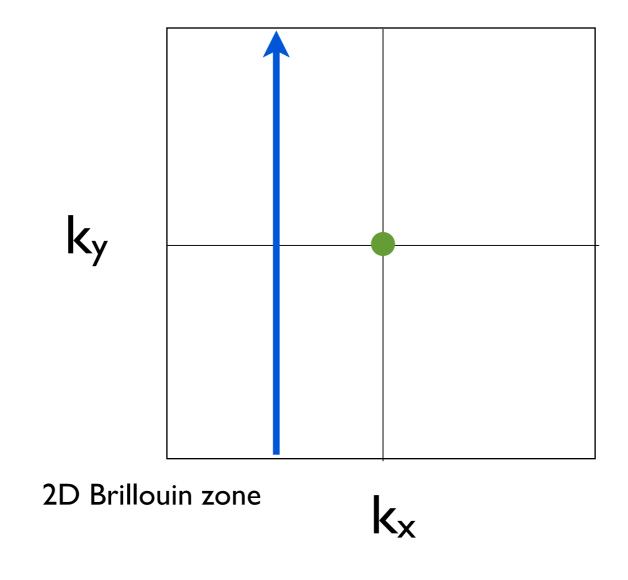
Go to 2D! winding of Arg det C e^{ik} 2D Brillouin zone k_{x}



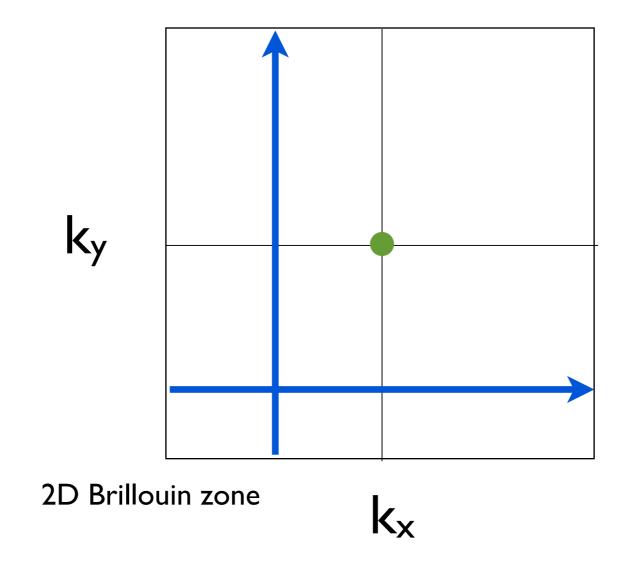
Go to 2D!



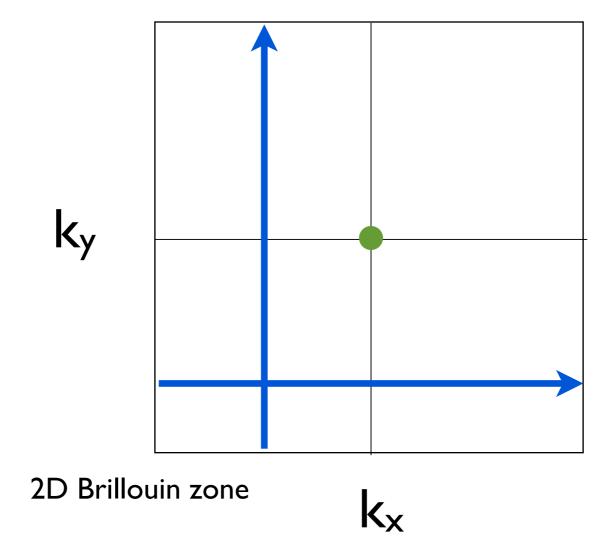






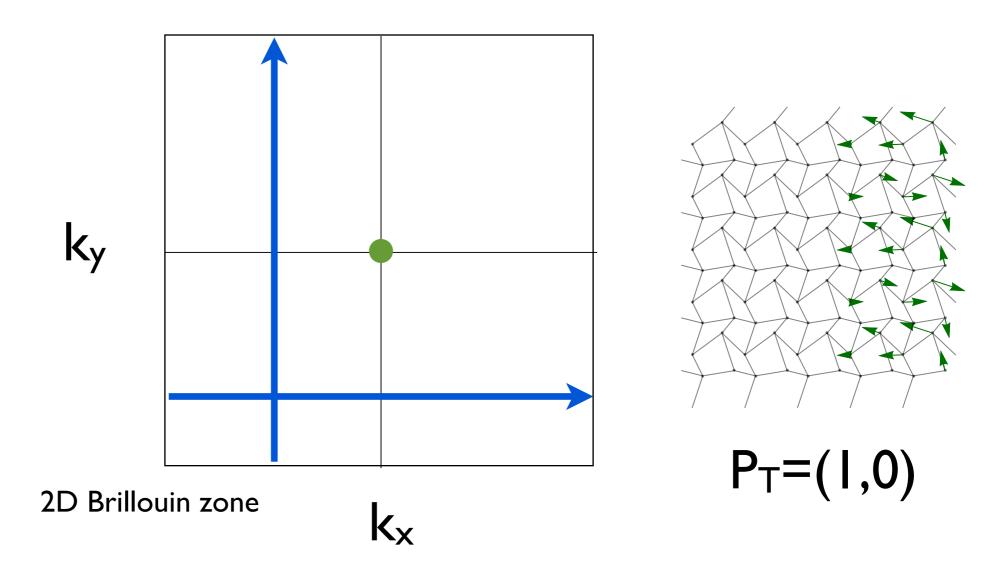






A pair of winding numbers gives a "topological polarization" **vector** that points towards floppy edge



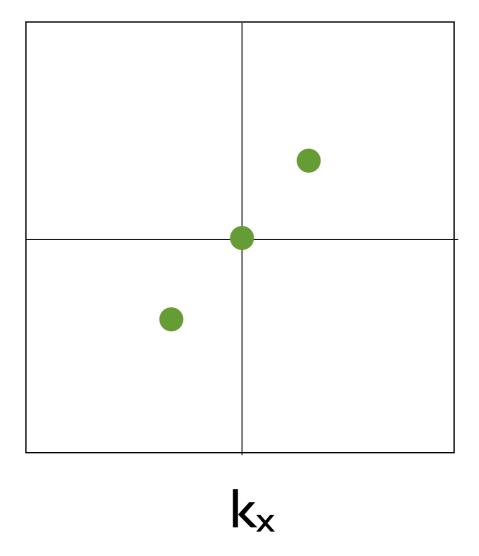


A pair of winding numbers gives a "topological polarization" **vector** that points towards floppy edge



If we have **bulk** zero modes...

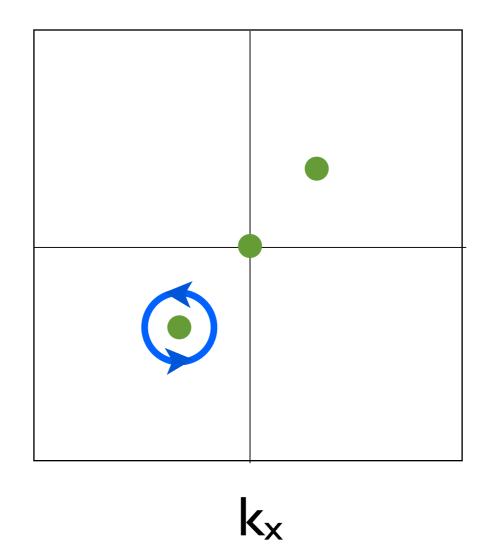
Ky





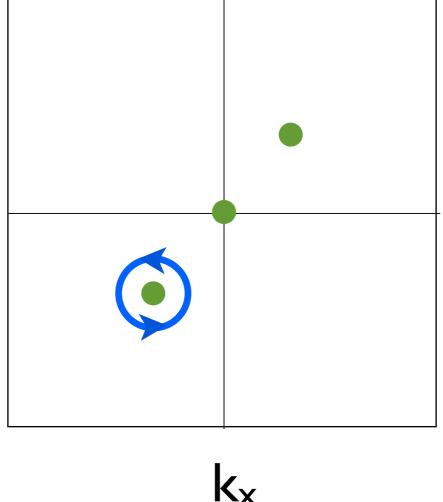
If we have **bulk** zero modes...

Ky





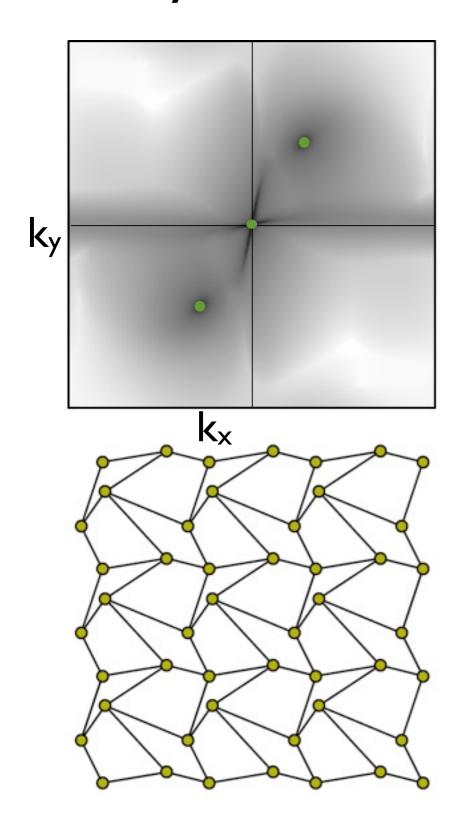
If we have **bulk** zero modes...



Winding in the phase of det R around the bulk mode protects it!

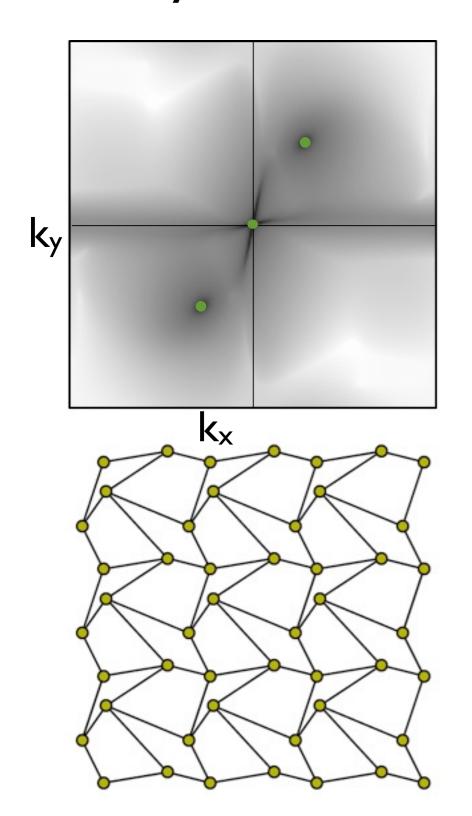


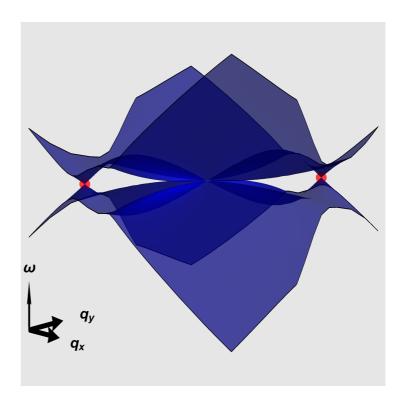
Weyl modes in deformed square lattices





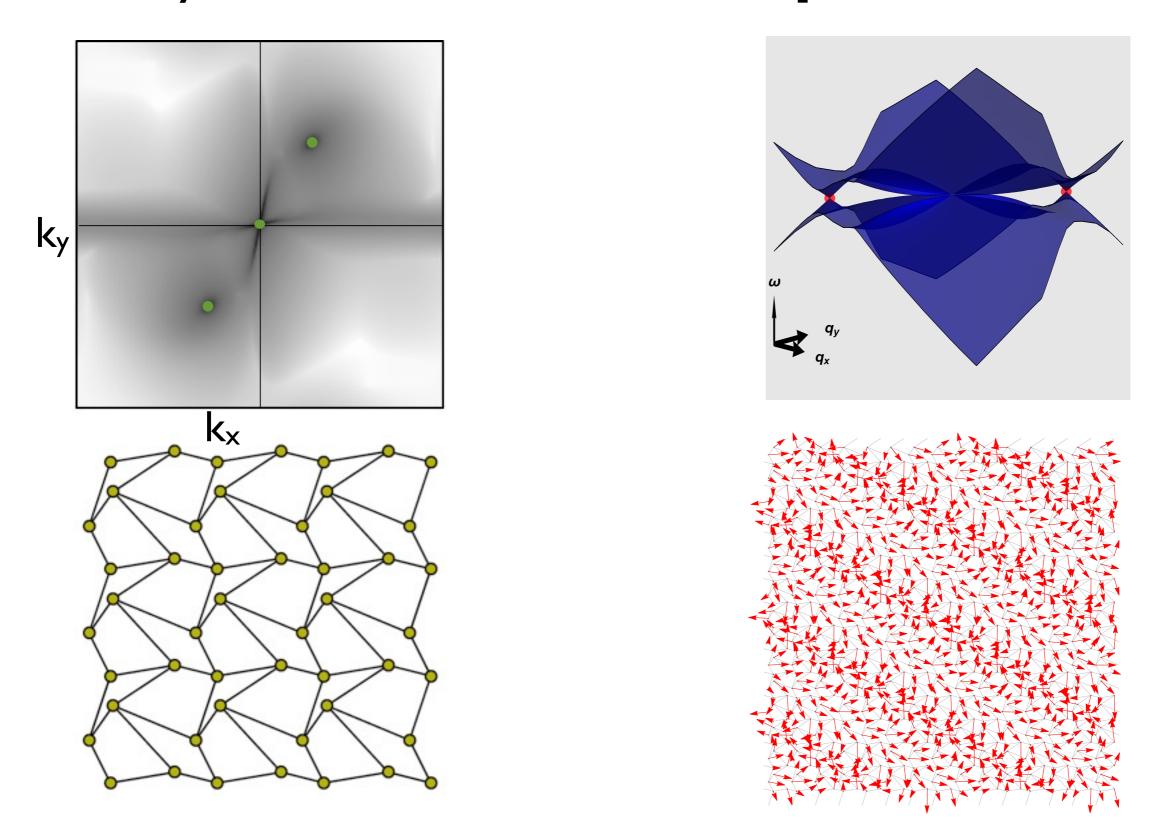
Weyl modes in deformed square lattices





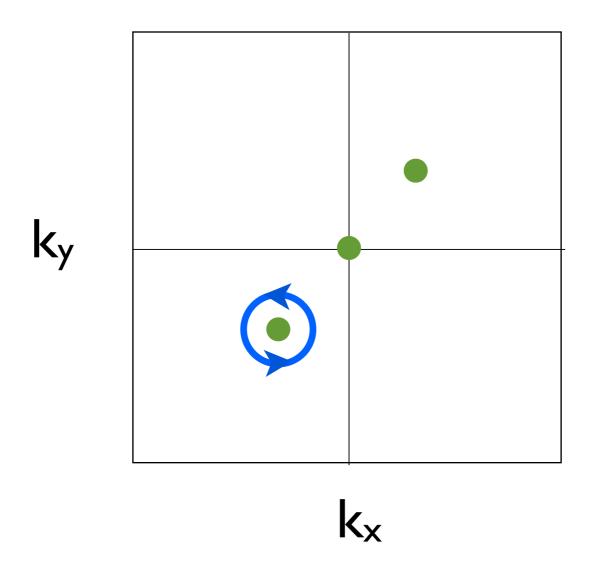


Weyl modes in deformed square lattices

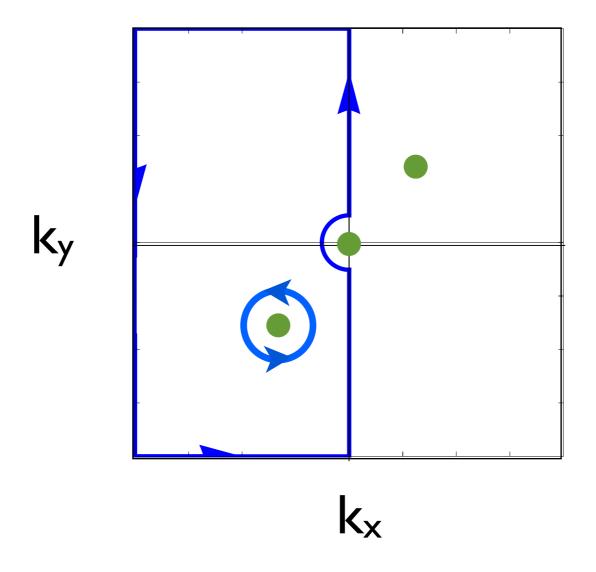




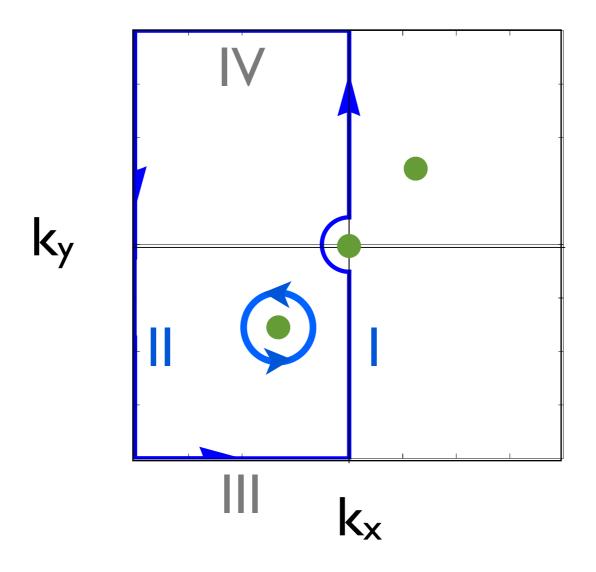
Rocklin, Chen, Falk, Vitelli, Lubensky. arXiv: 1510.04970



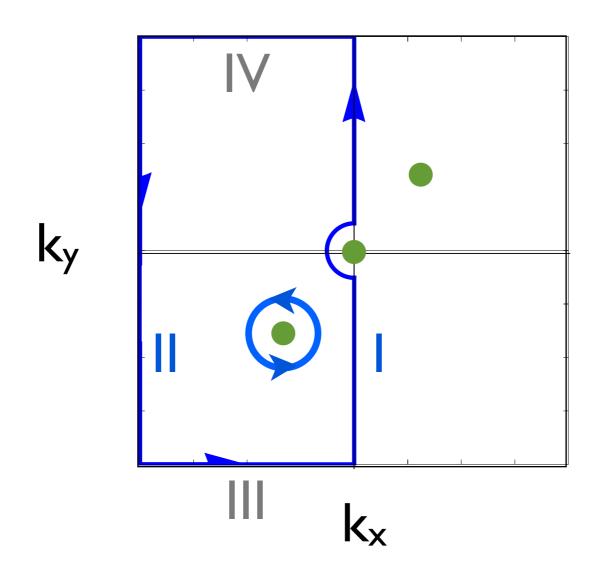






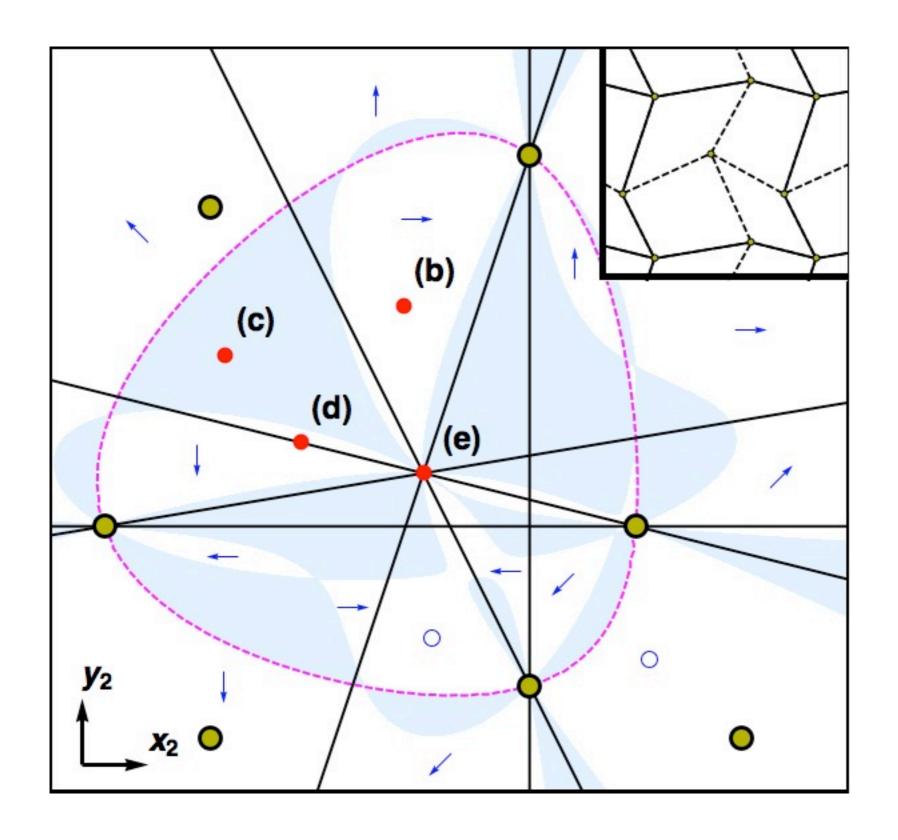




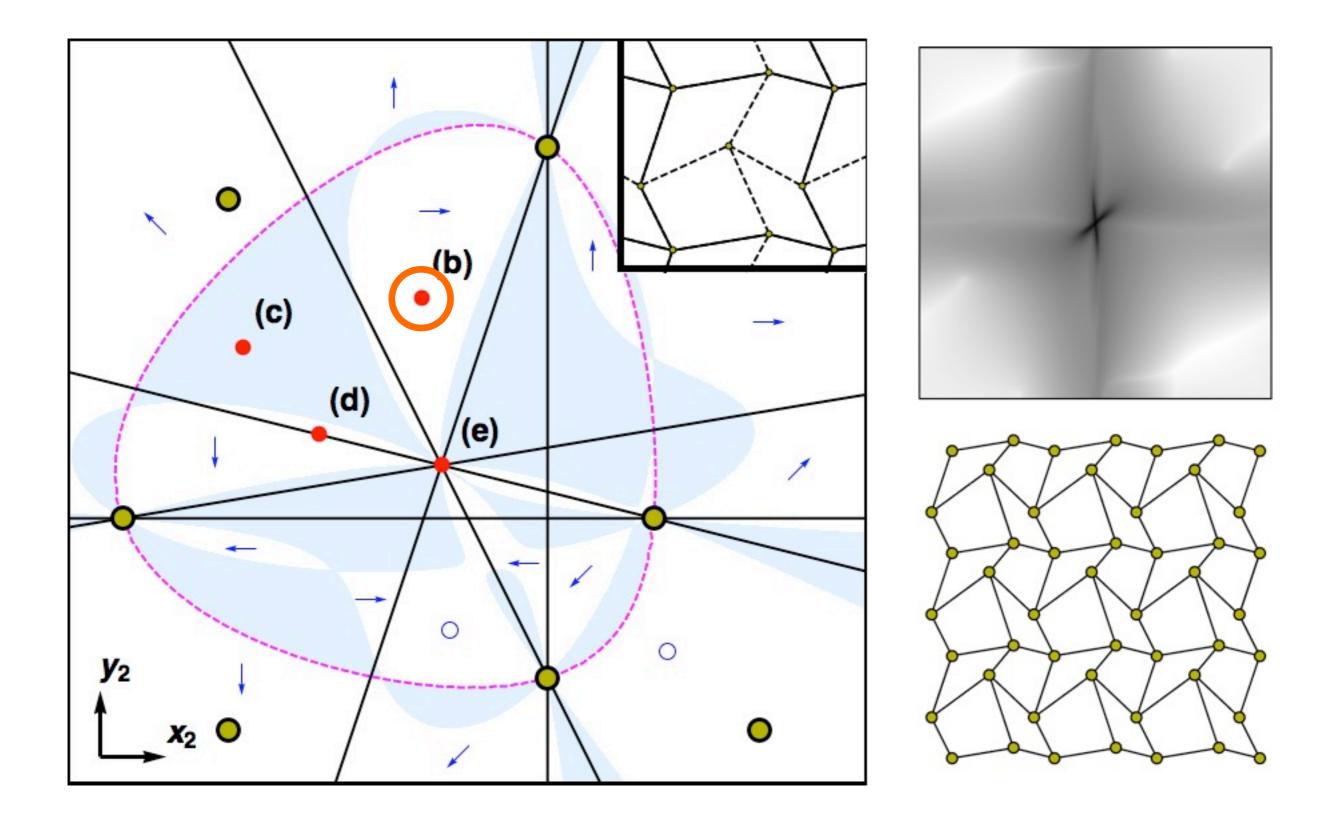


The winding around the bulk mode is the difference between contour I and contour II

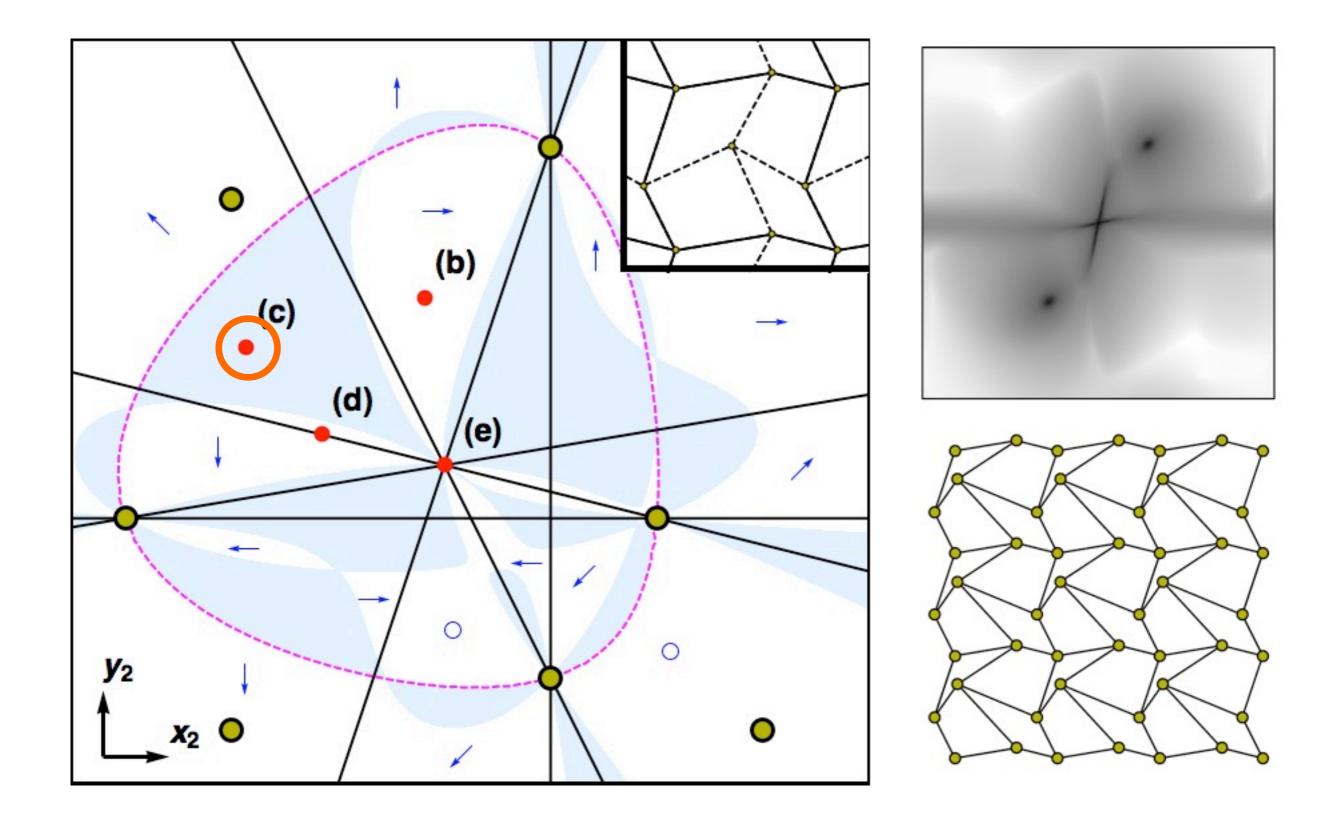




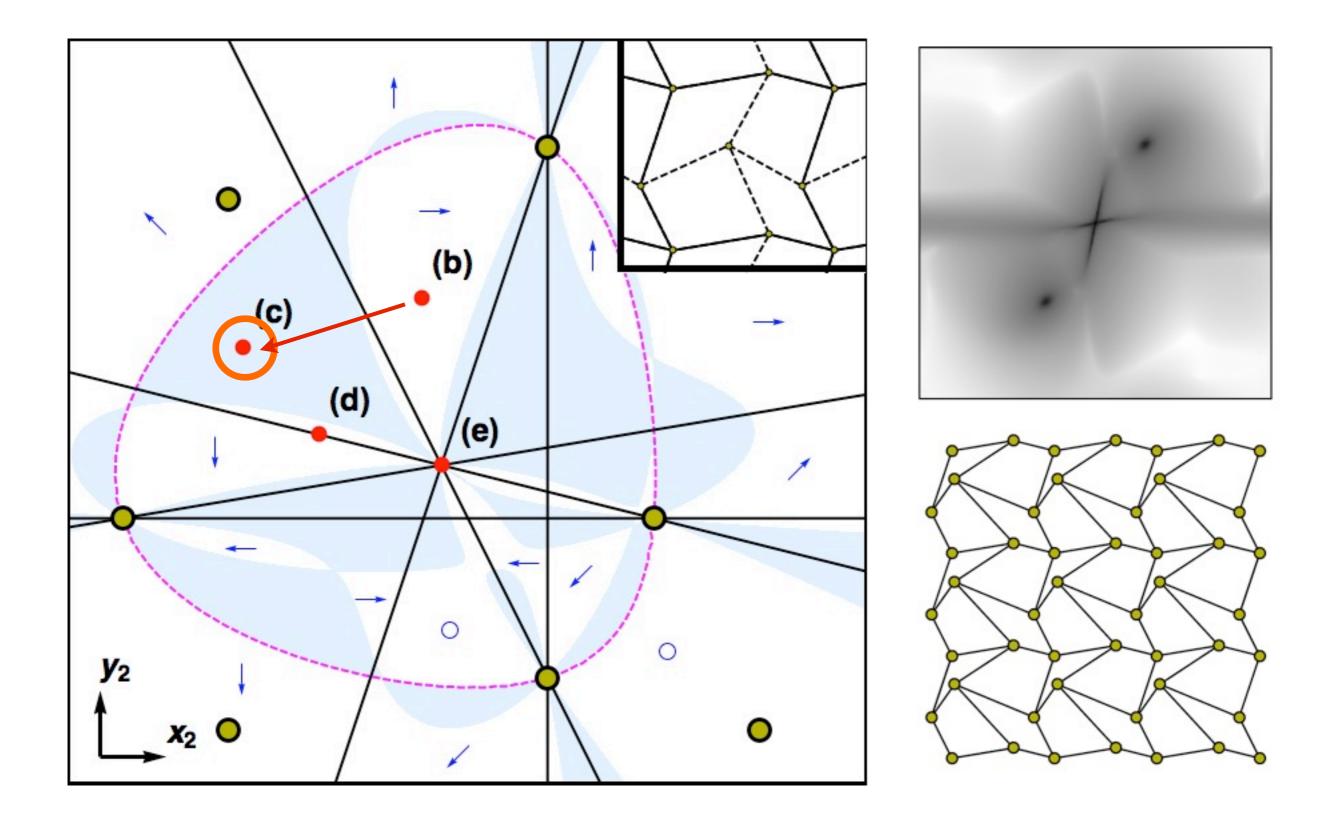




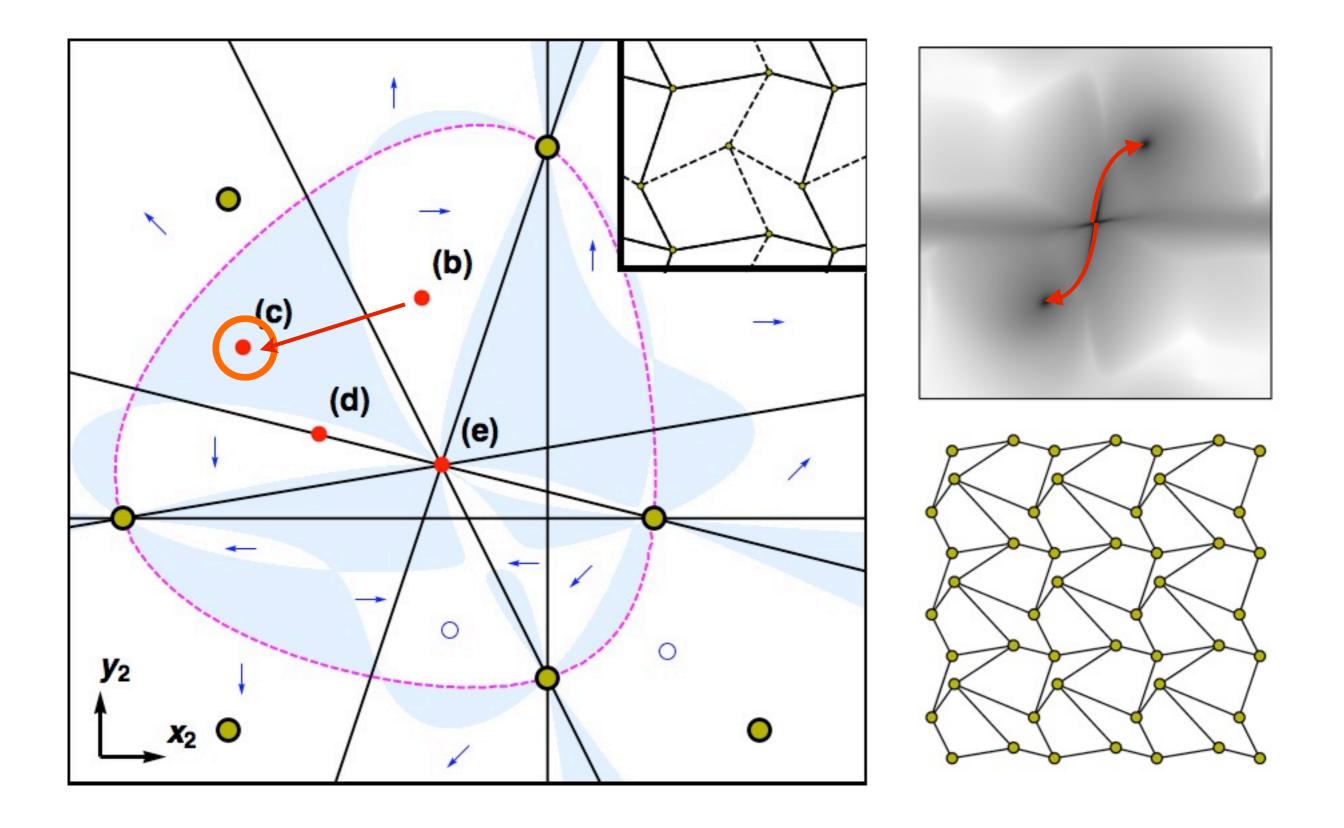




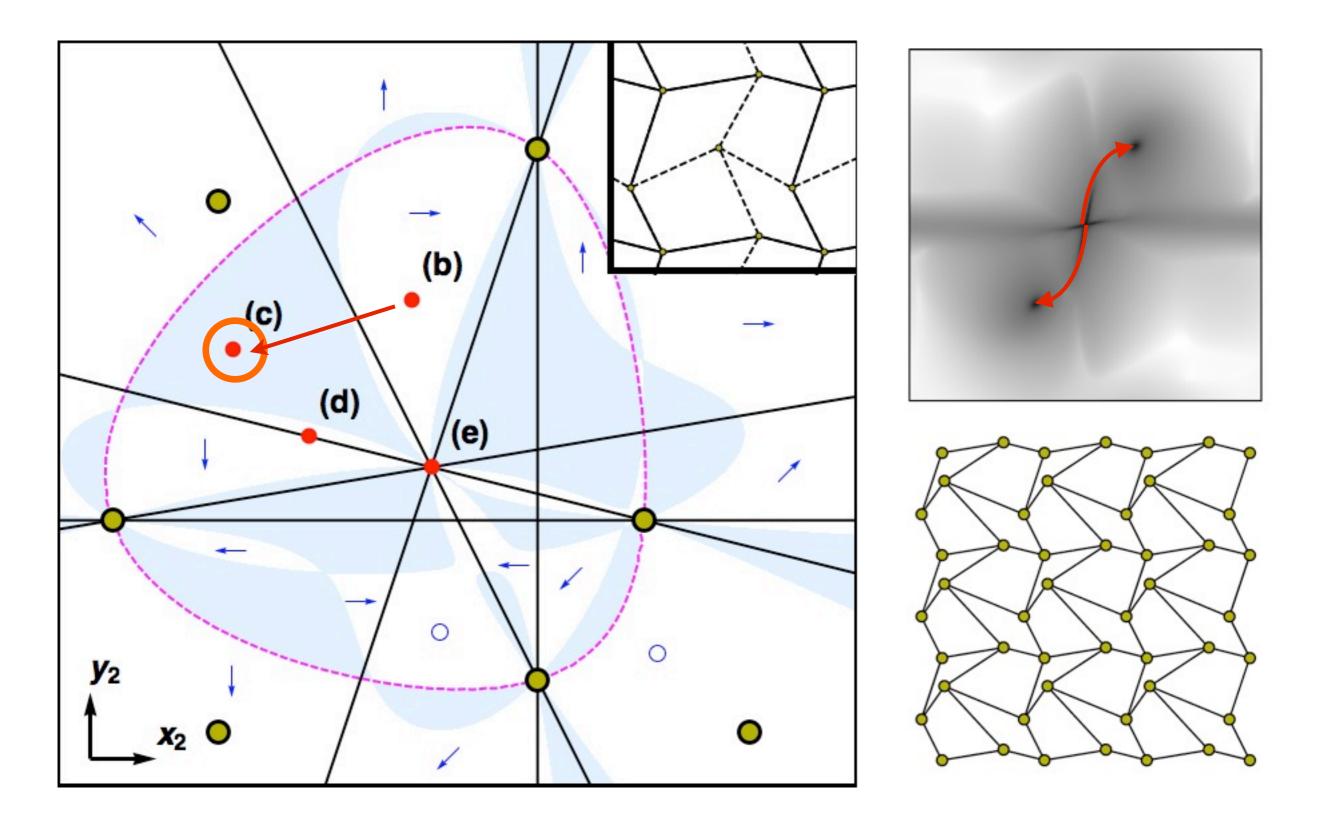






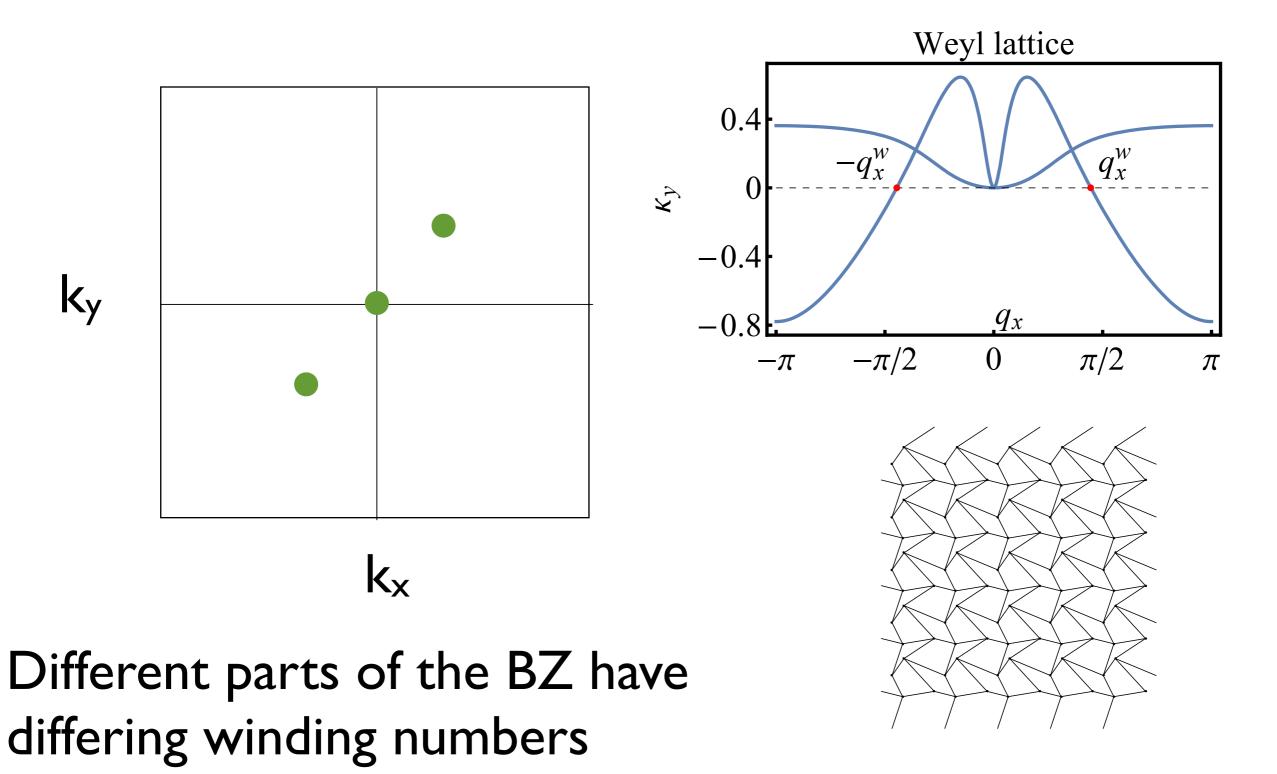




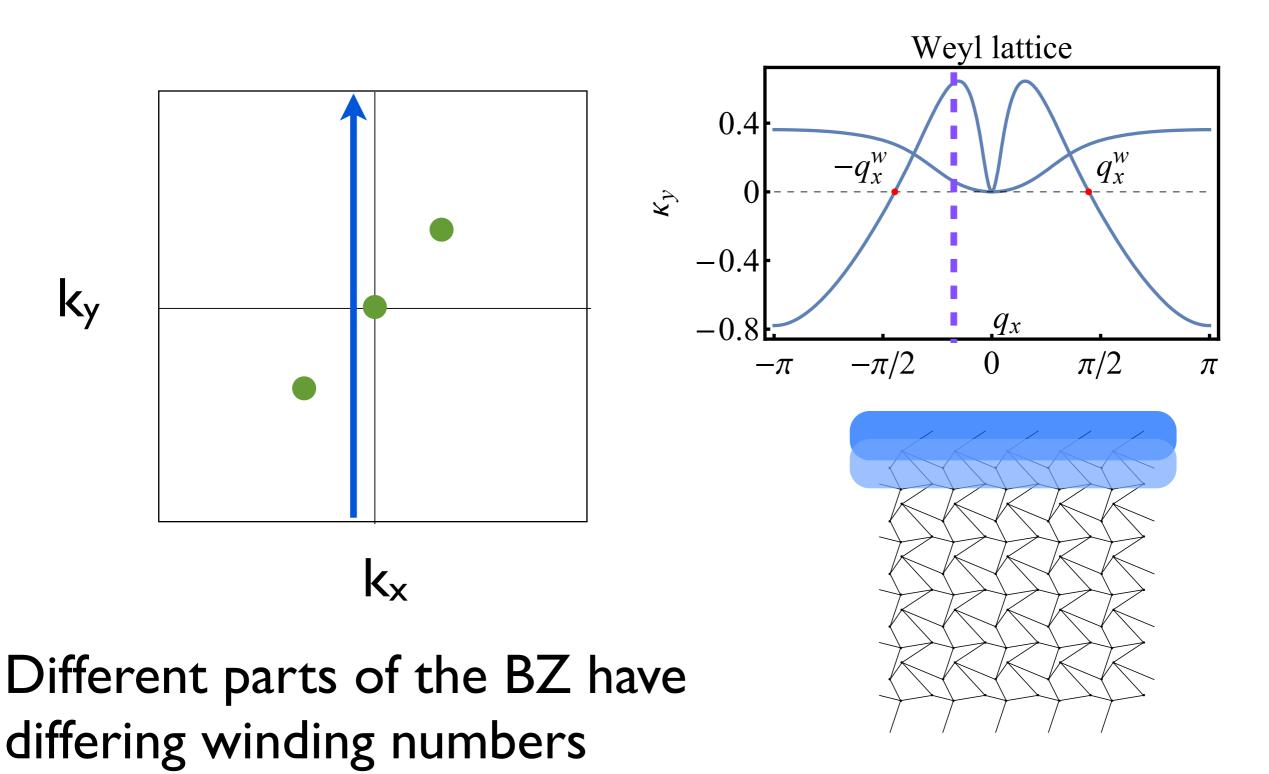


Weyl "phases" are generic in isostatic lattices

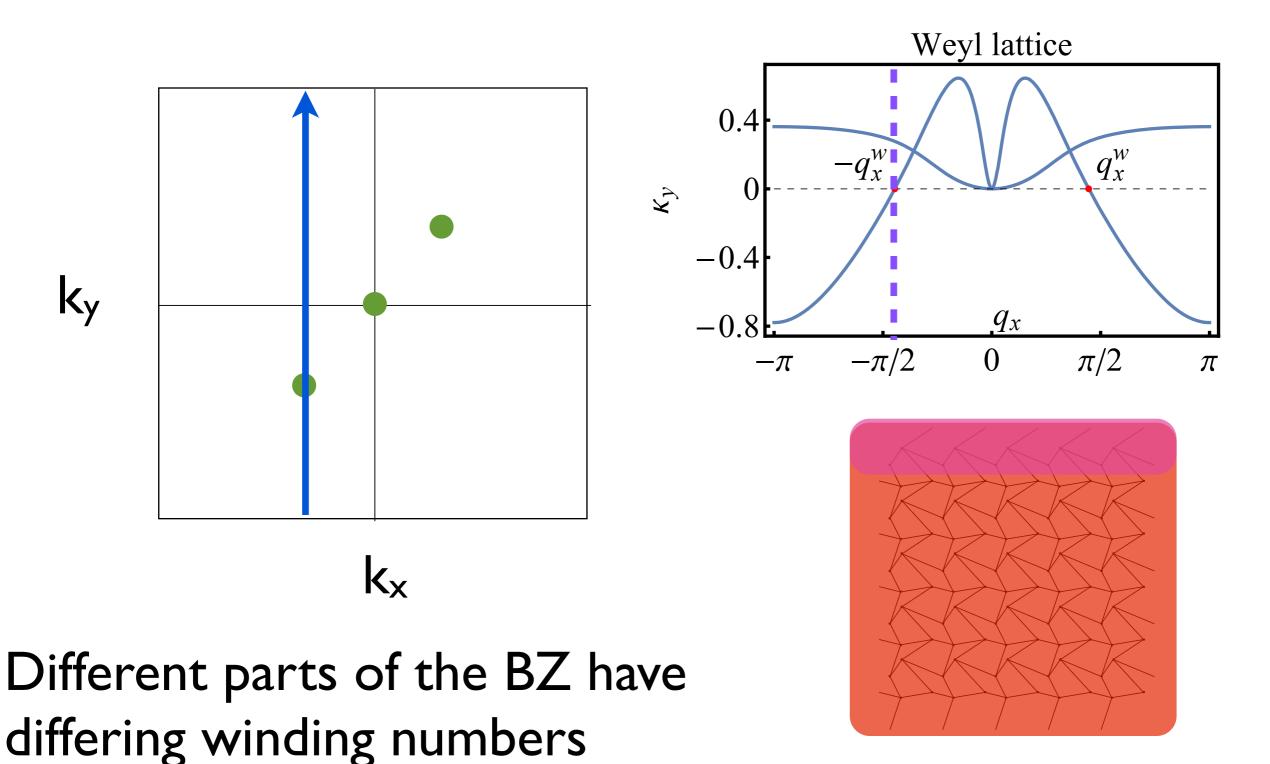




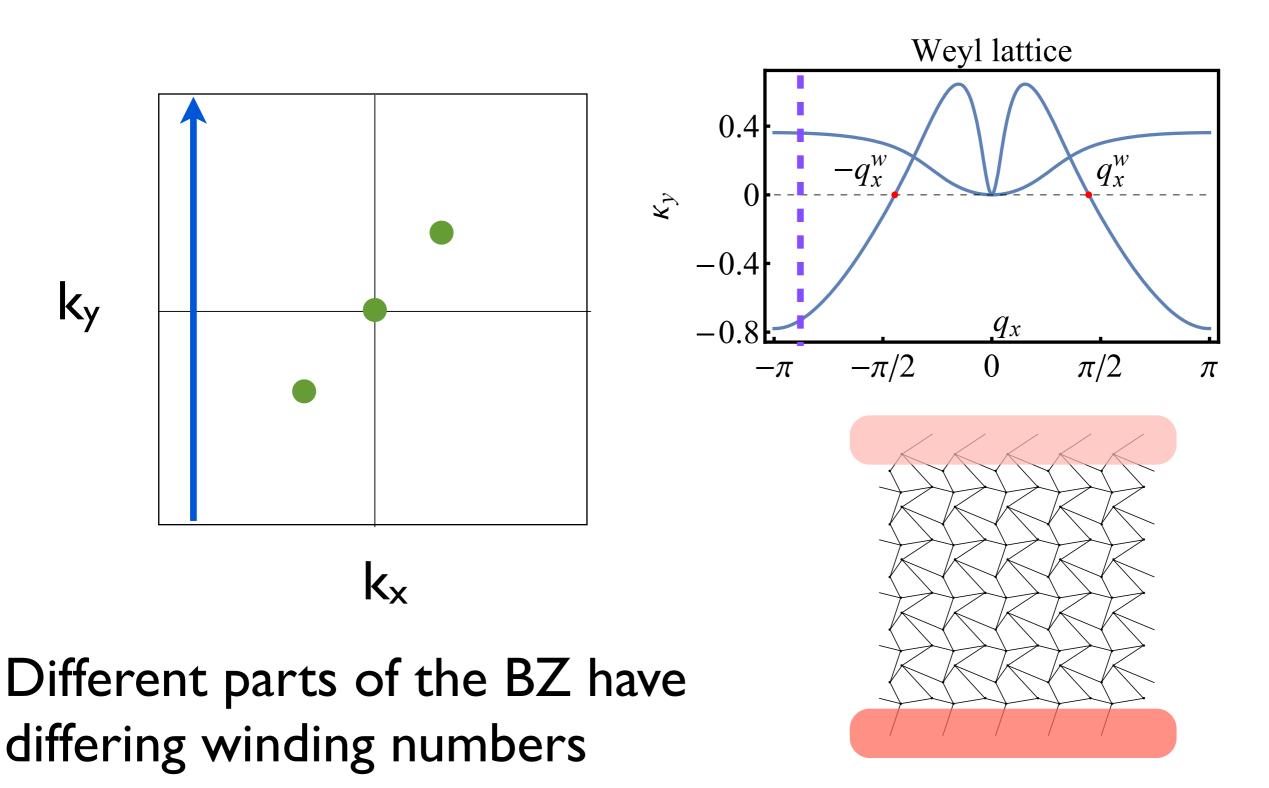




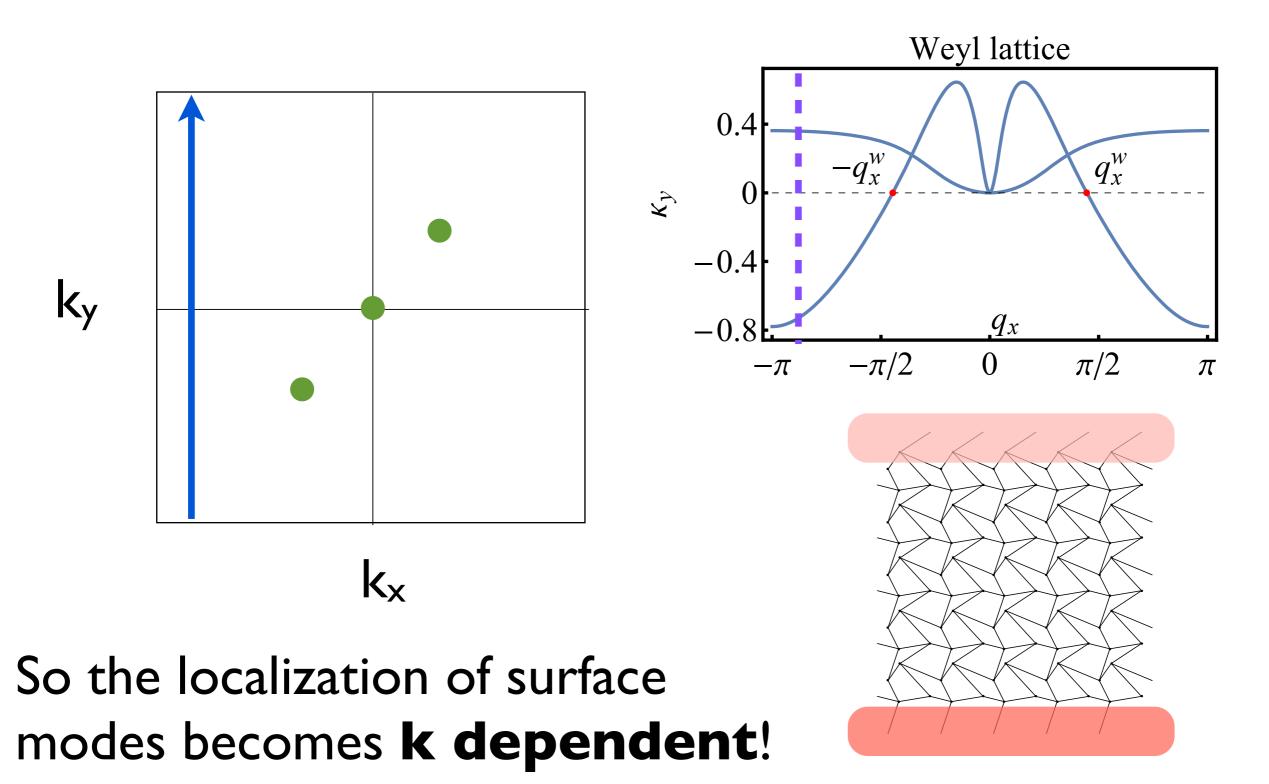






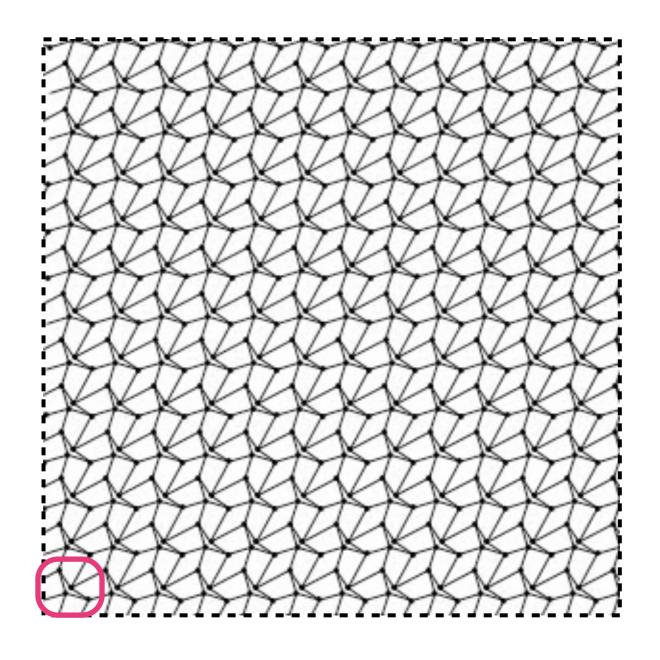




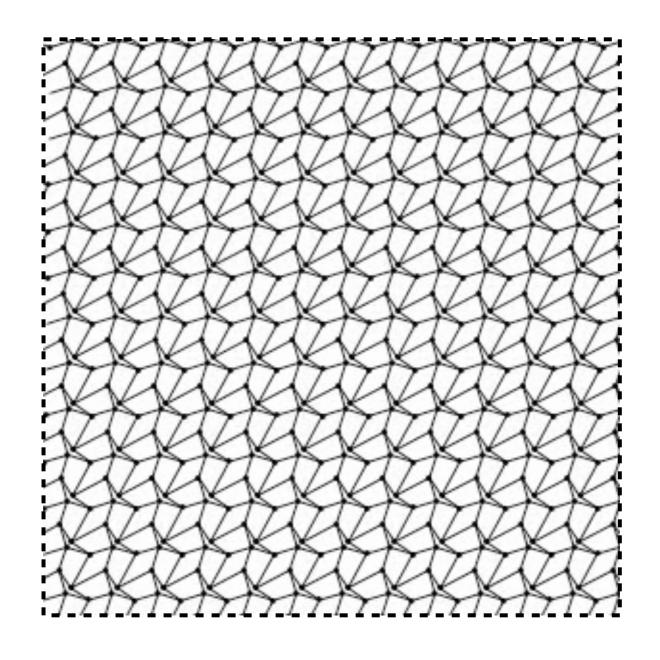




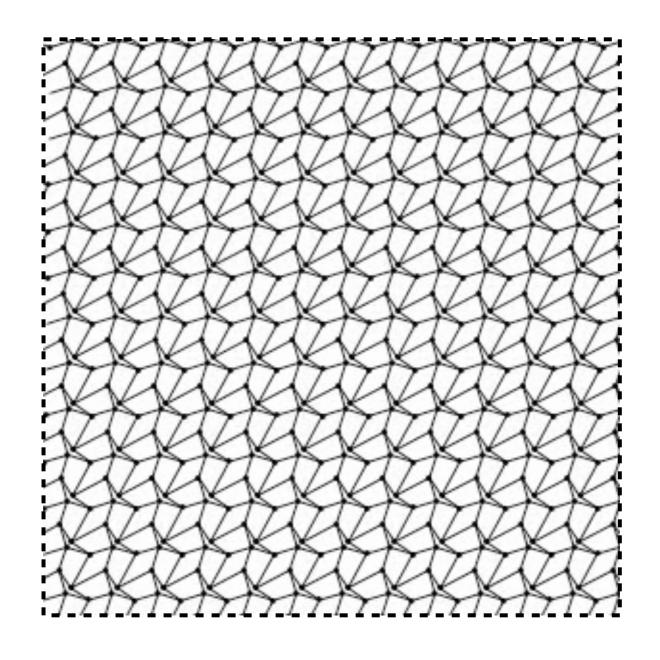




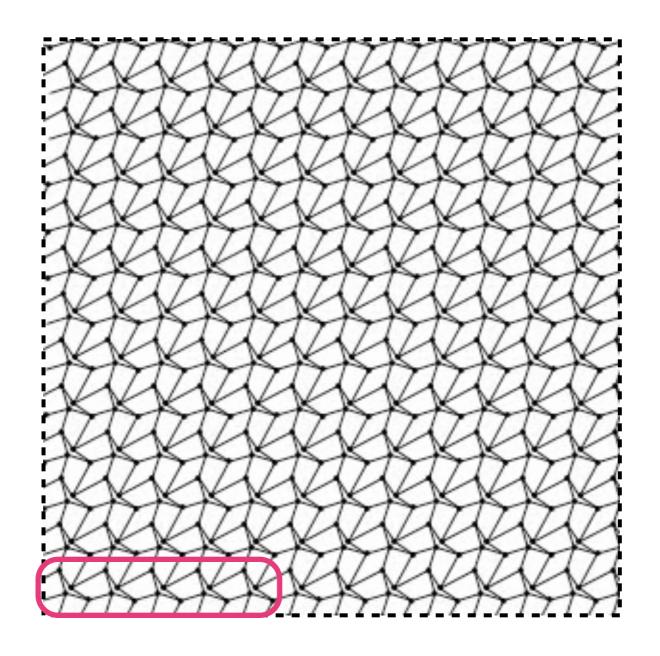










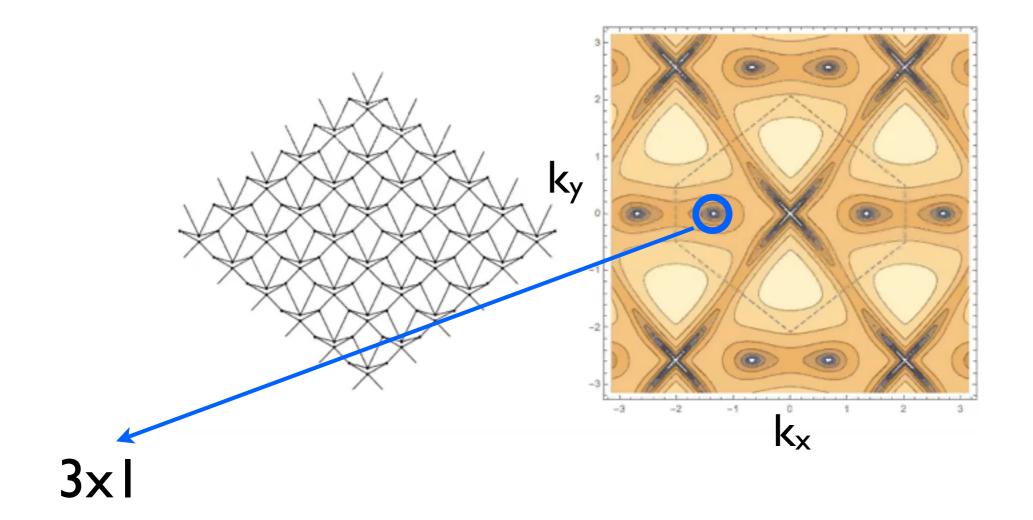




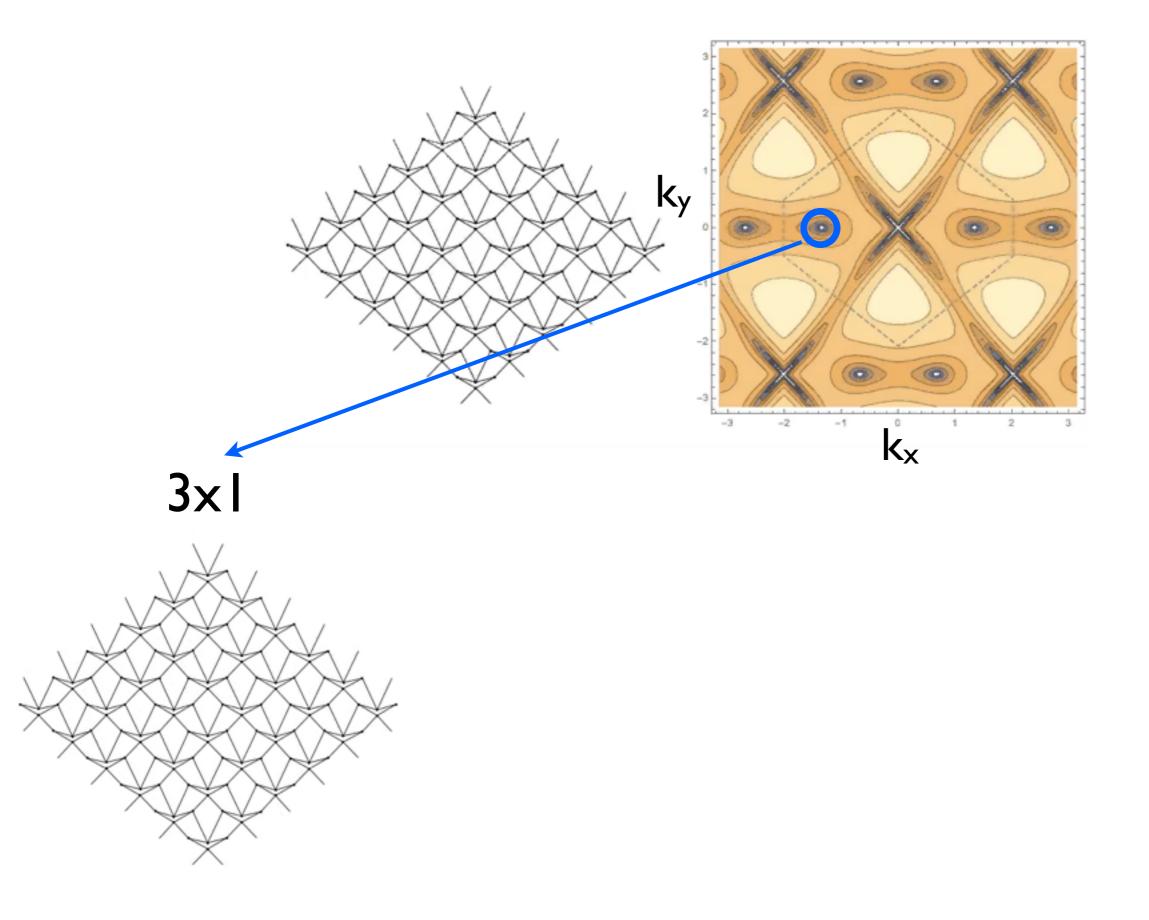
 k_{y}

 k_{x}



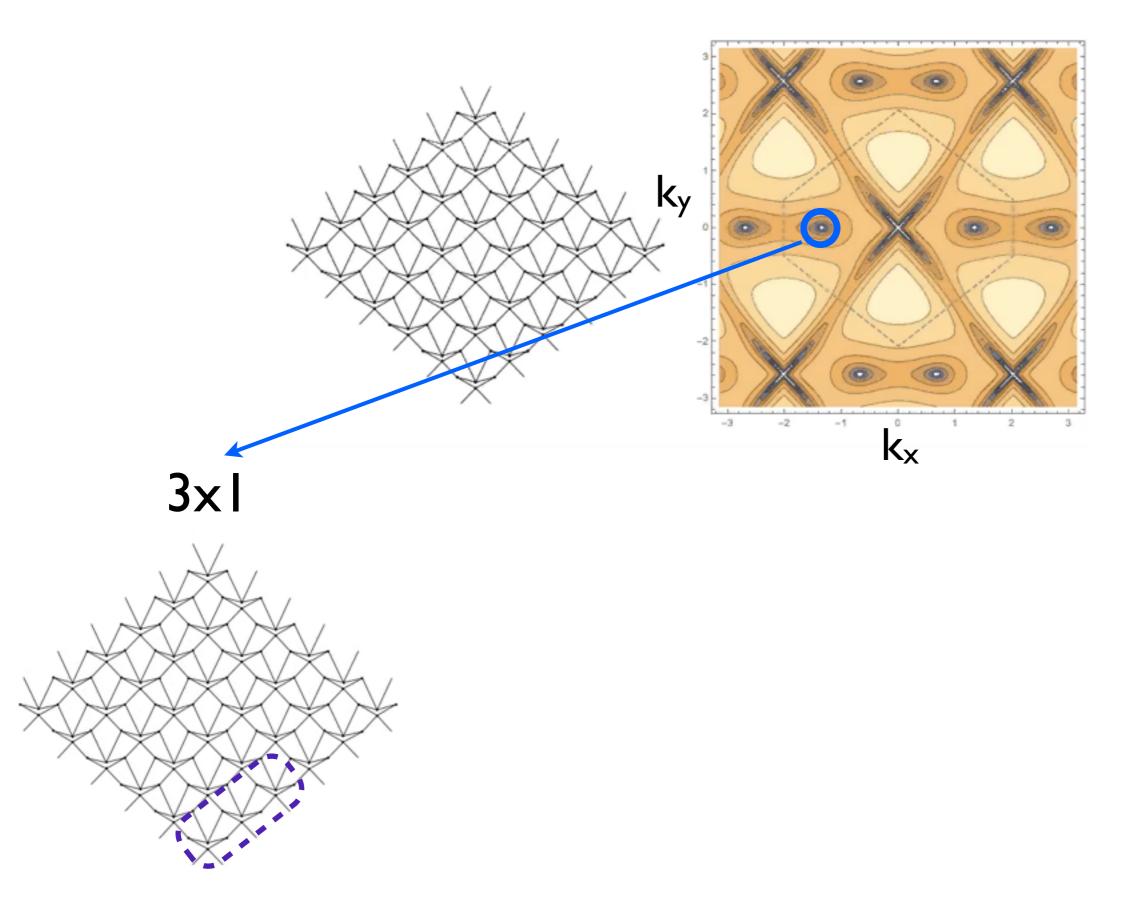






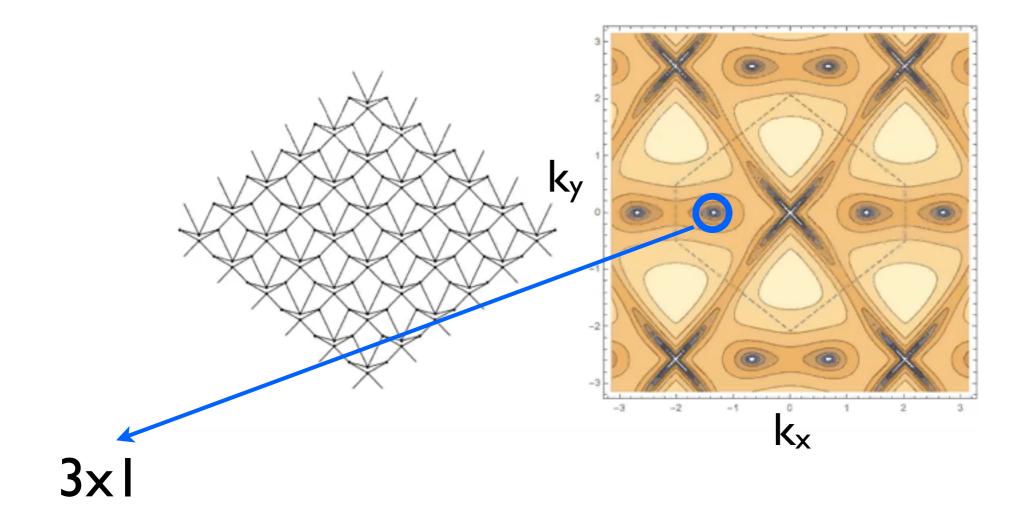
UMASS AMHERST

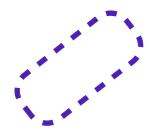
BGC, Rocklin, Falk, Vitelli, Lubensky. In preparation.



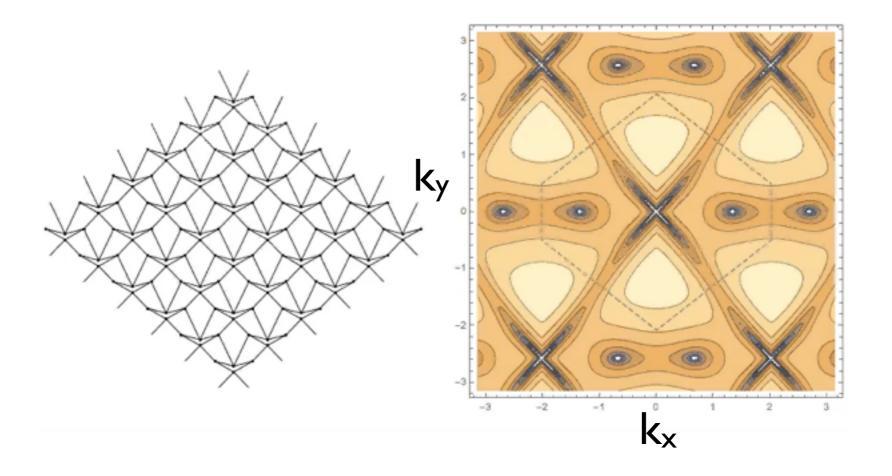
UMASS AMHERST

BGC, Rocklin, Falk, Vitelli, Lubensky. In preparation.

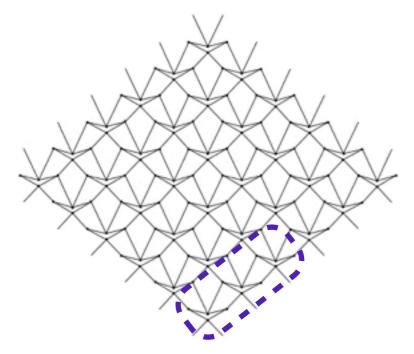






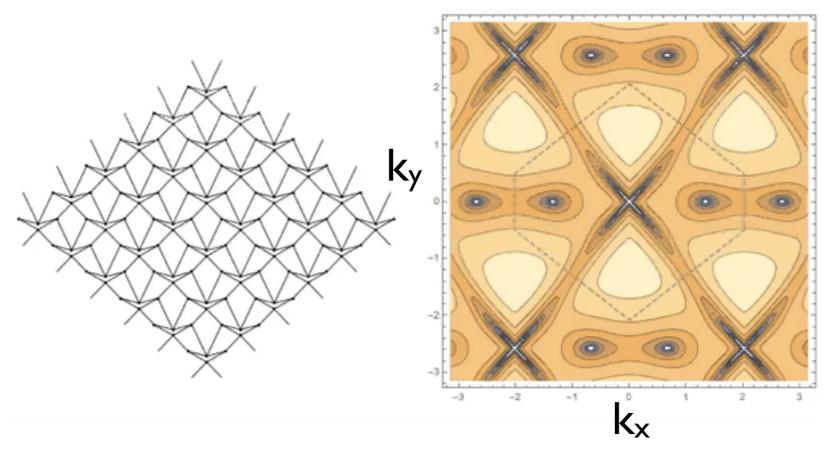




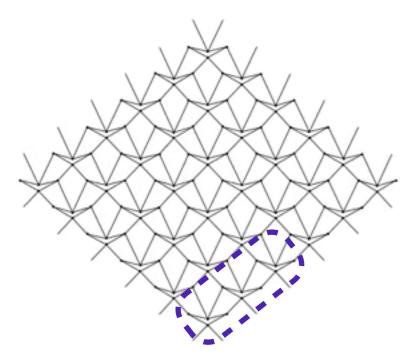




BGC, Rocklin, Falk, Vitelli, Lubensky. In preparation.

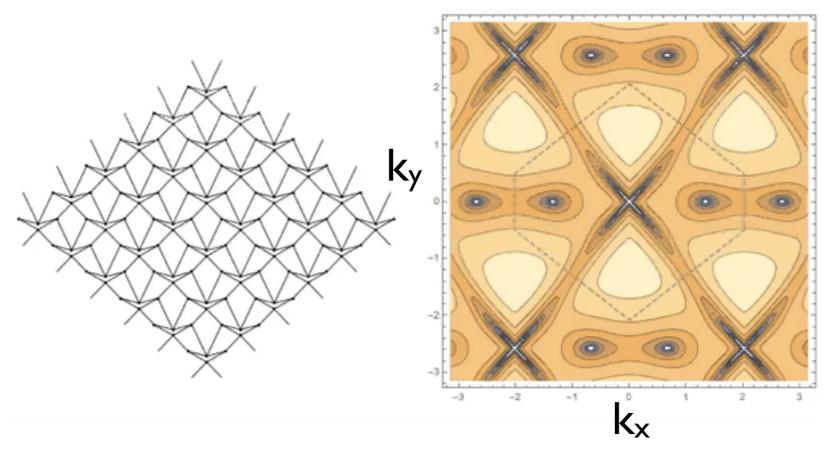




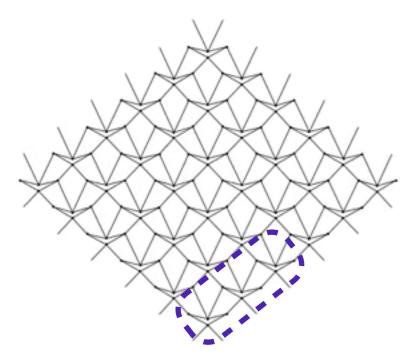


Rocklin, Zhou, Sun, Mao. arXiv:1510.06389



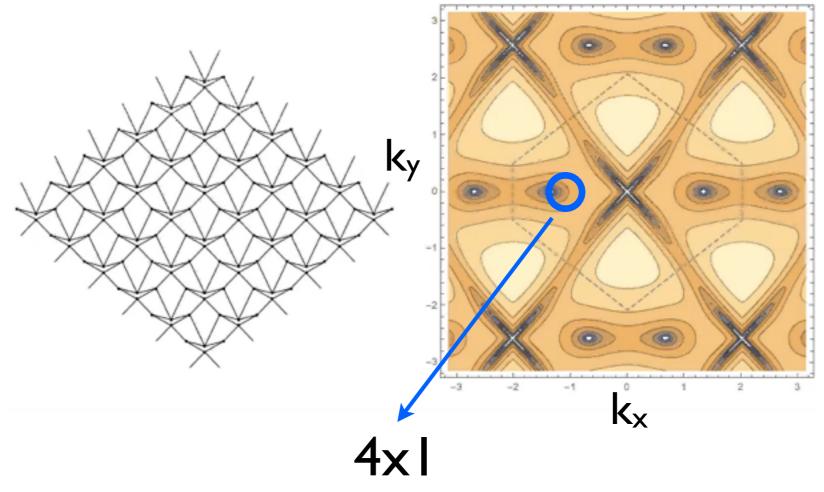


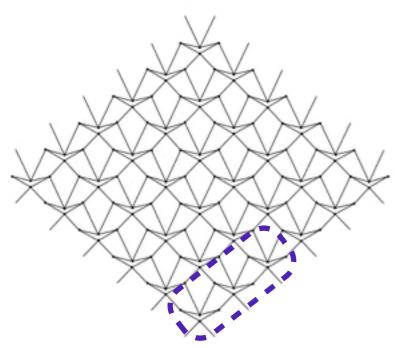




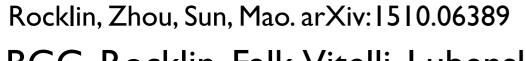
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389



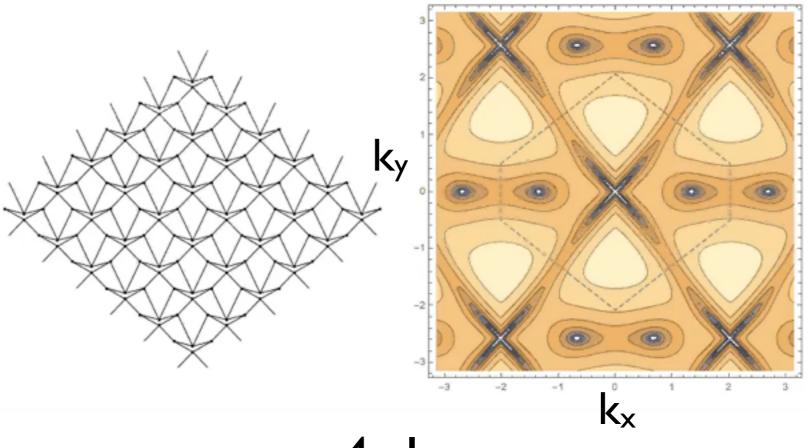




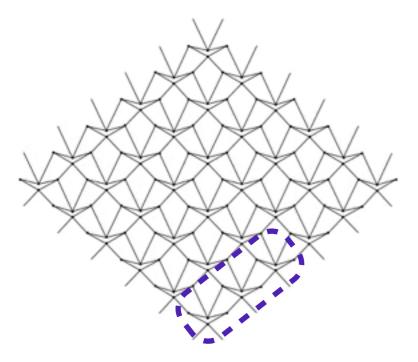
3xI





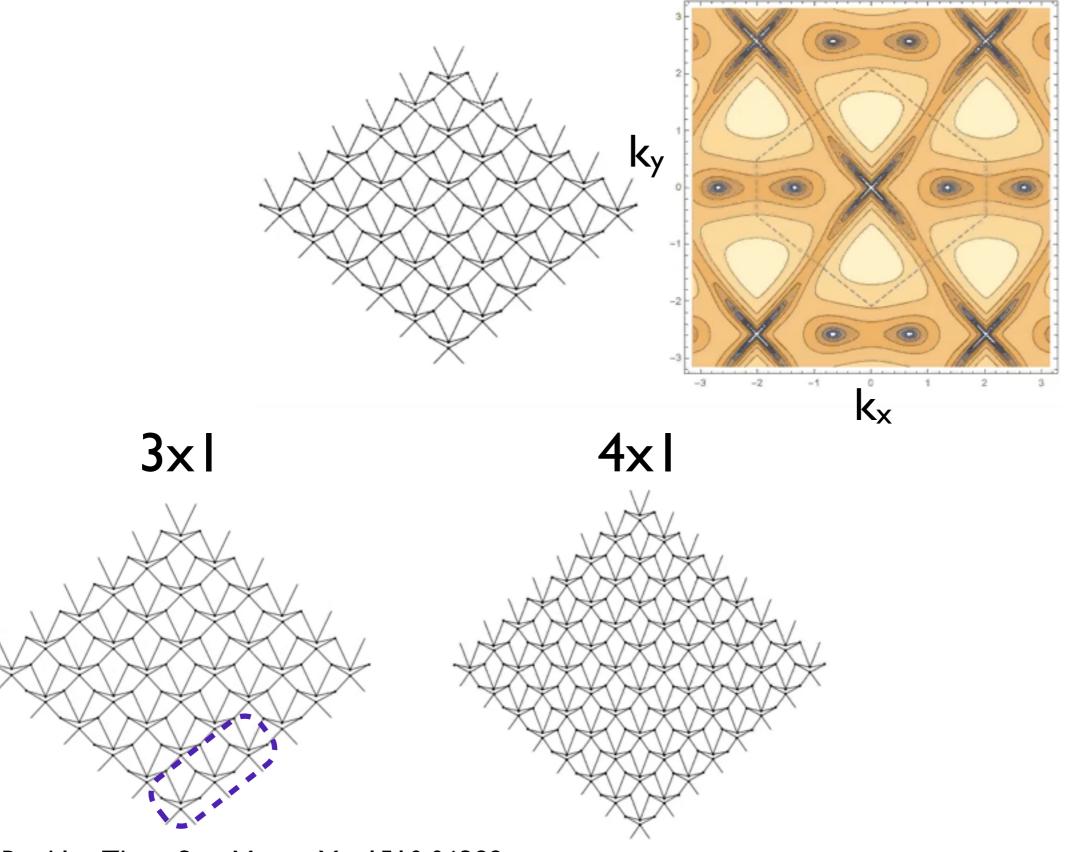


3xI 4xI



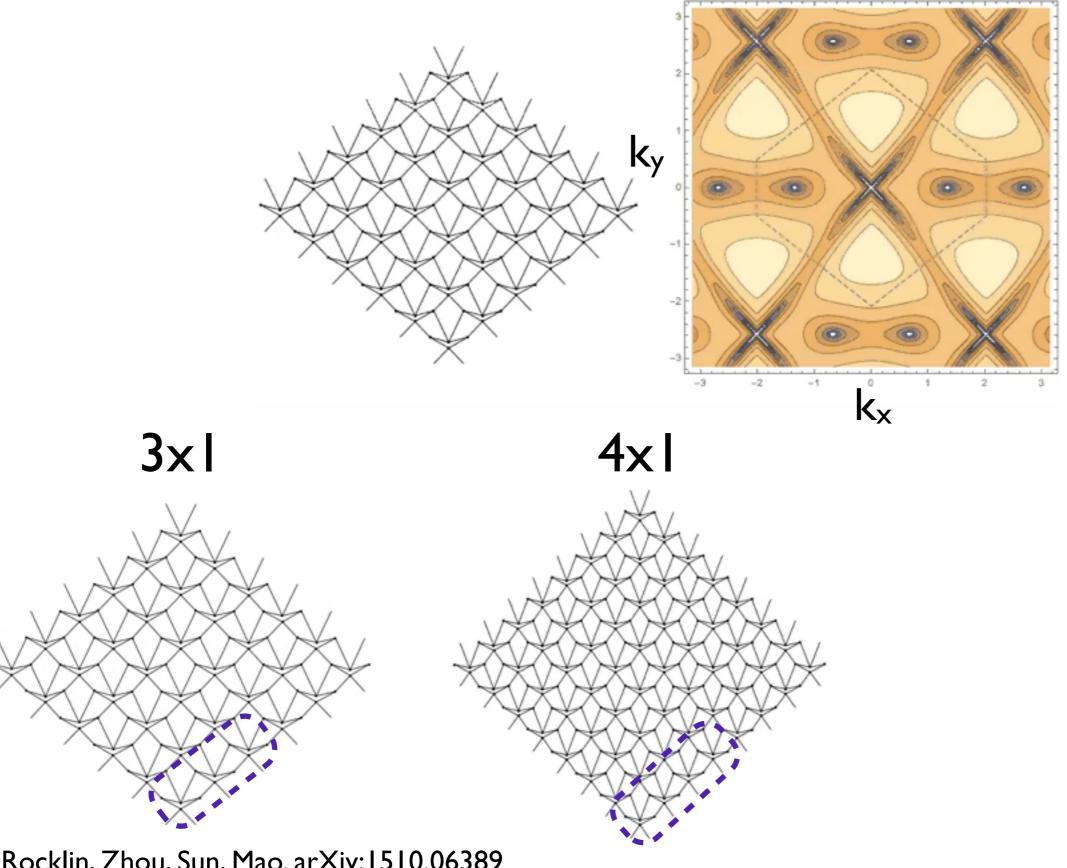
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389





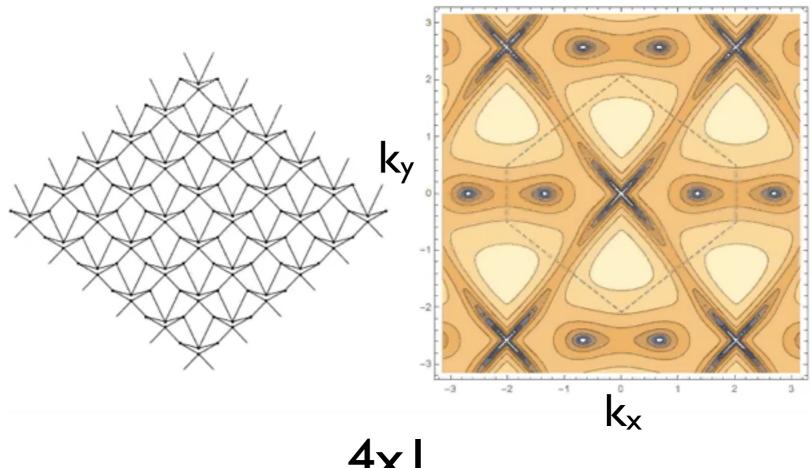
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389



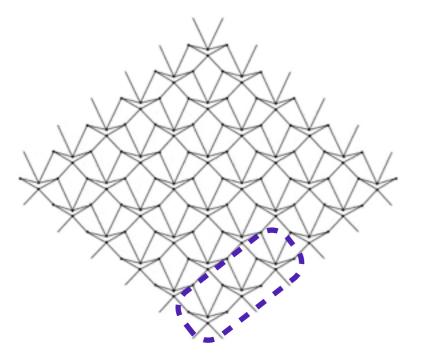


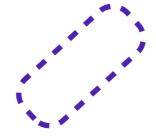
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389





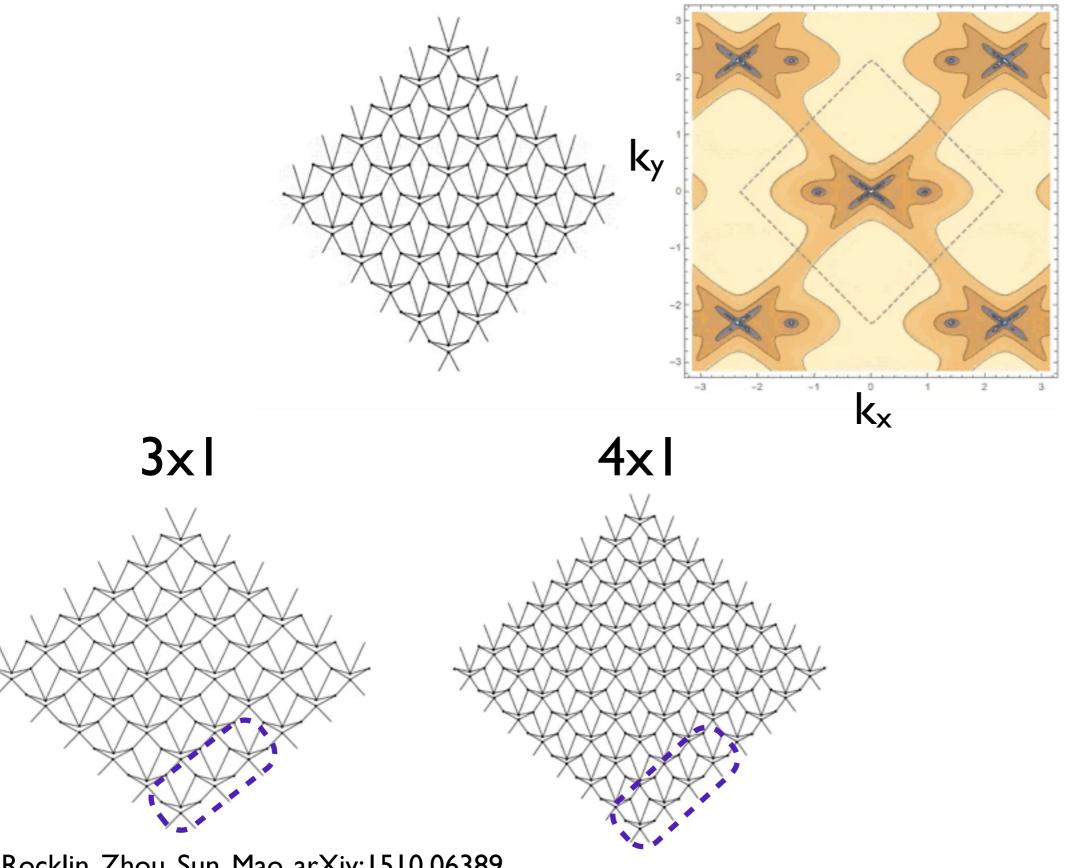
3xI 4xI





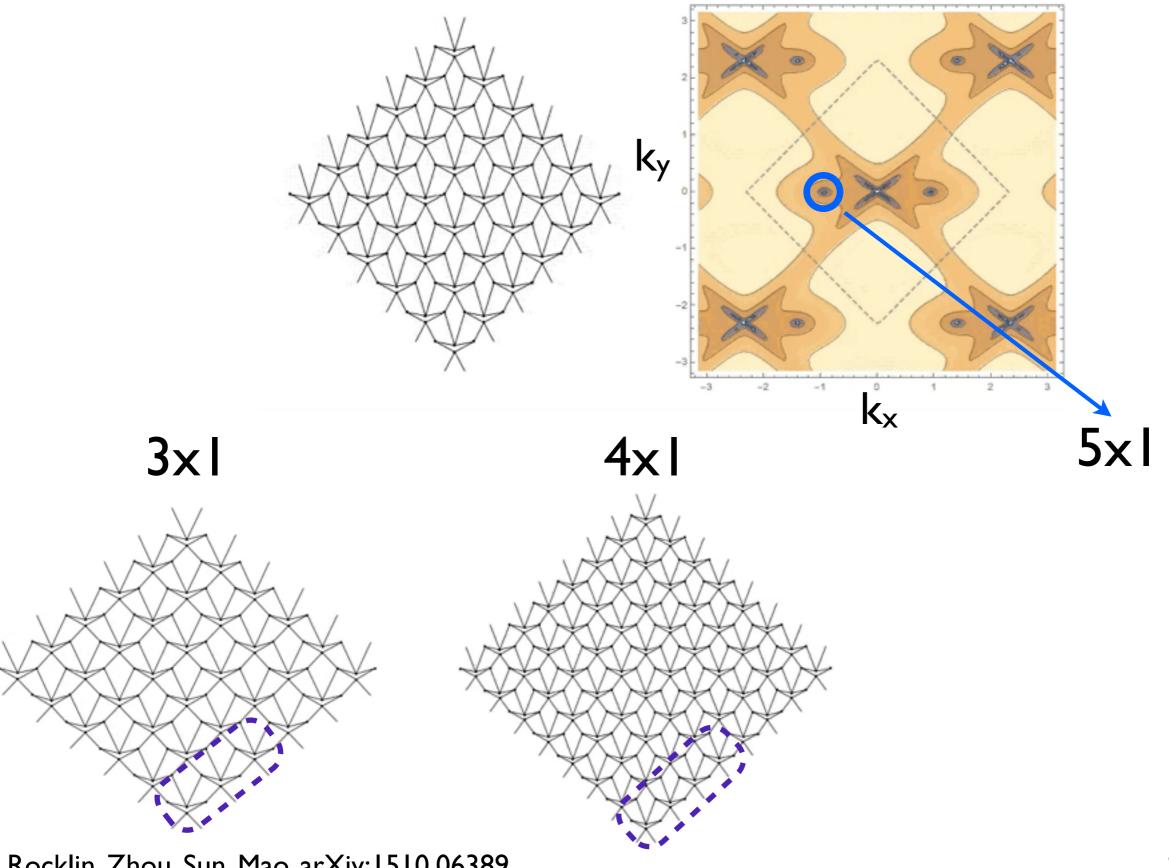
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389





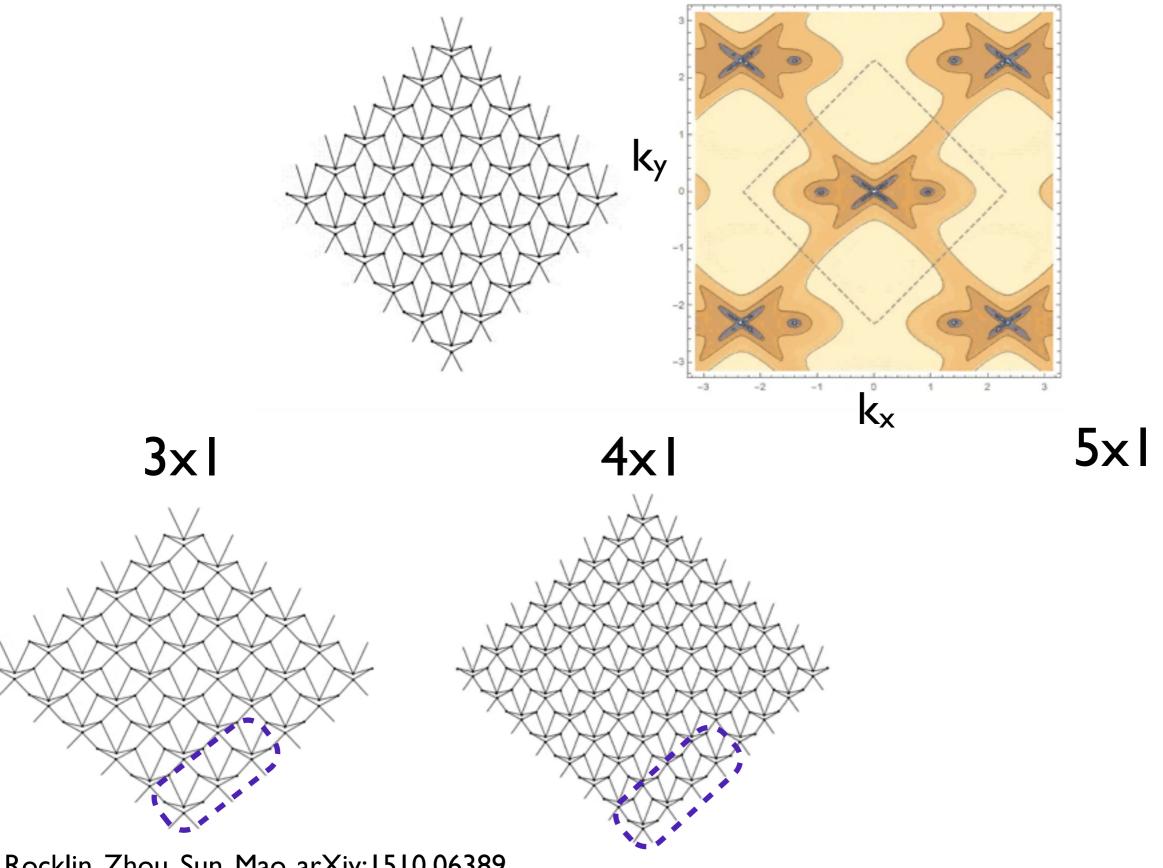
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389





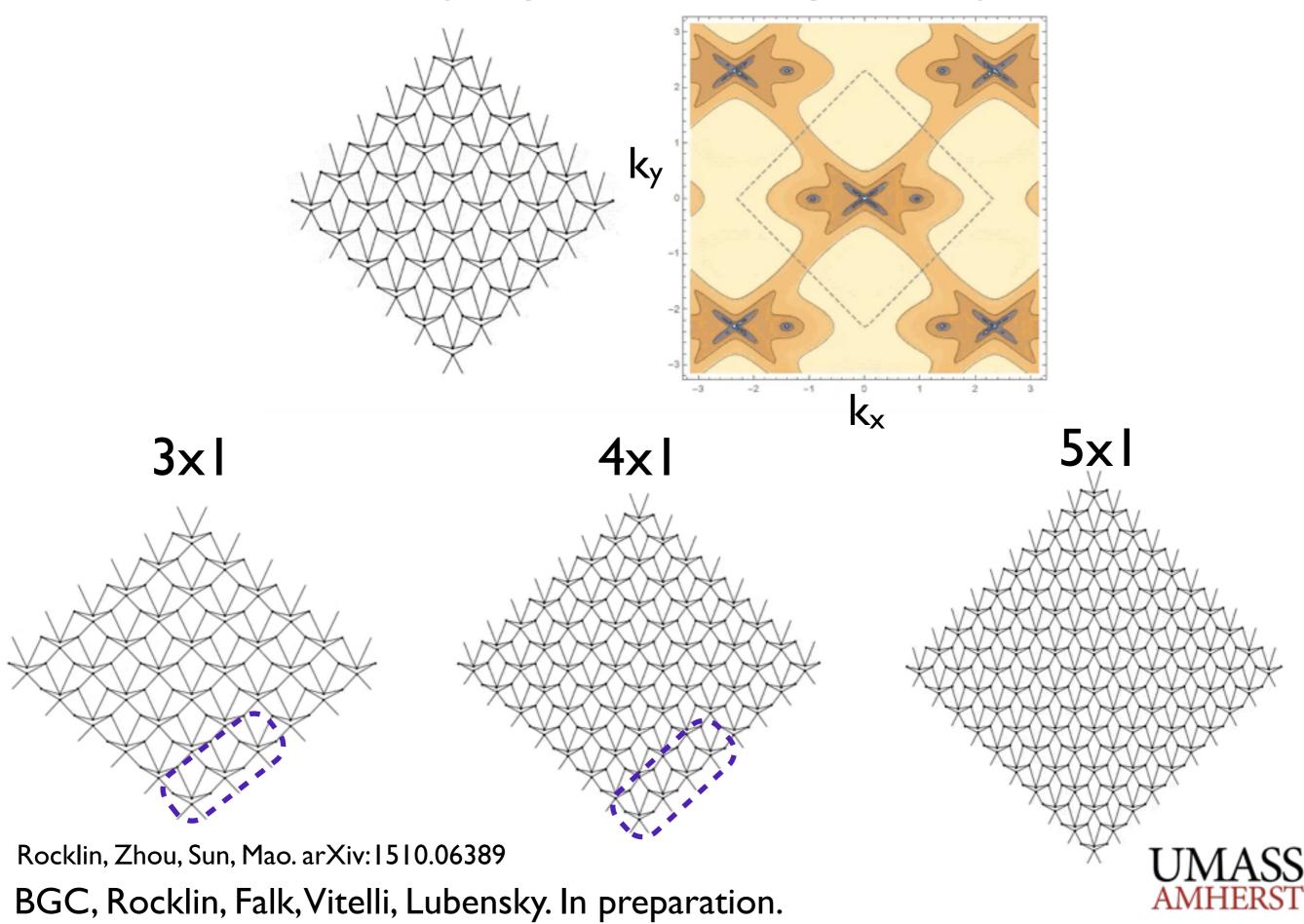
Rocklin, Zhou, Sun, Mao. arXiv:1510.06389

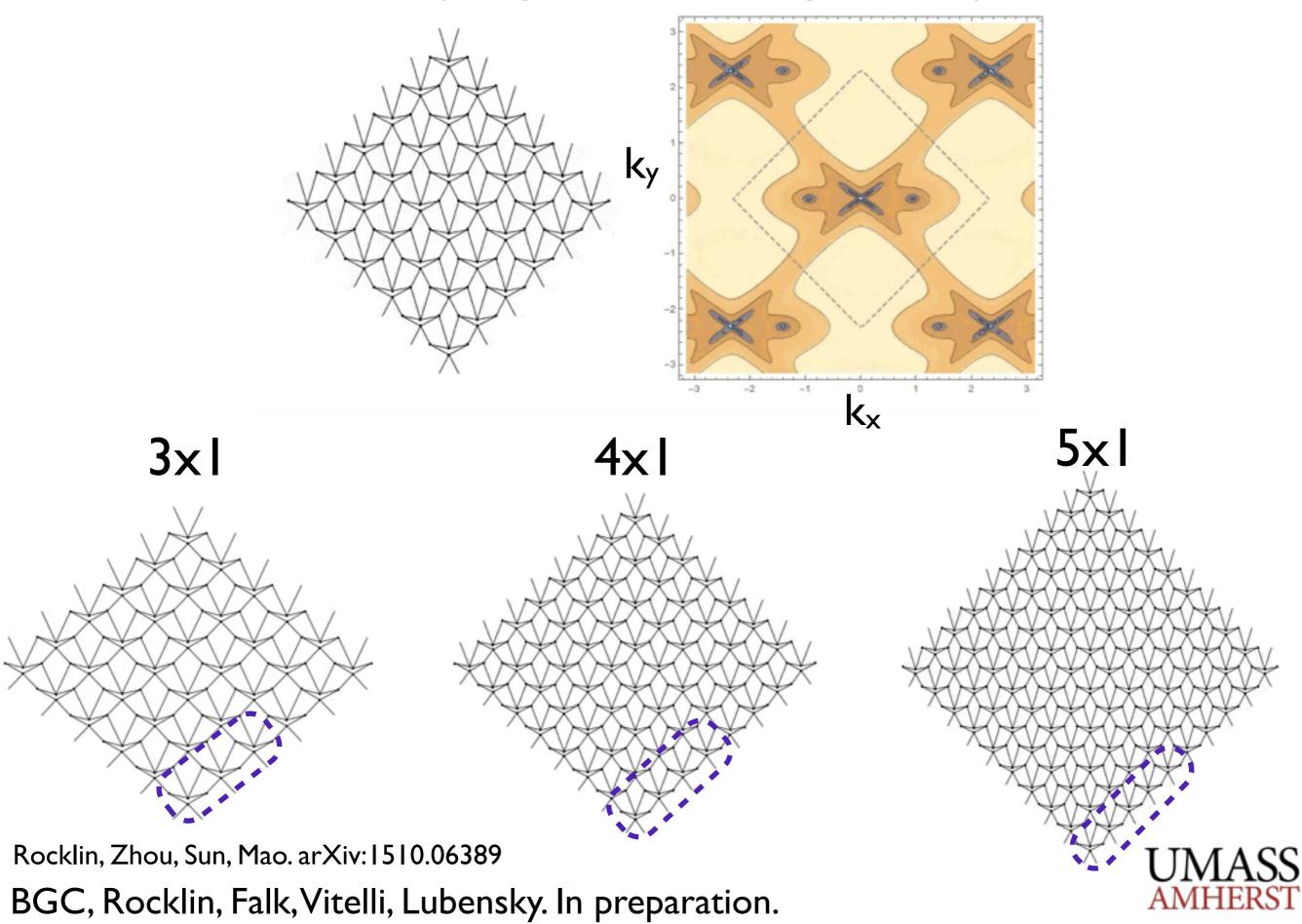


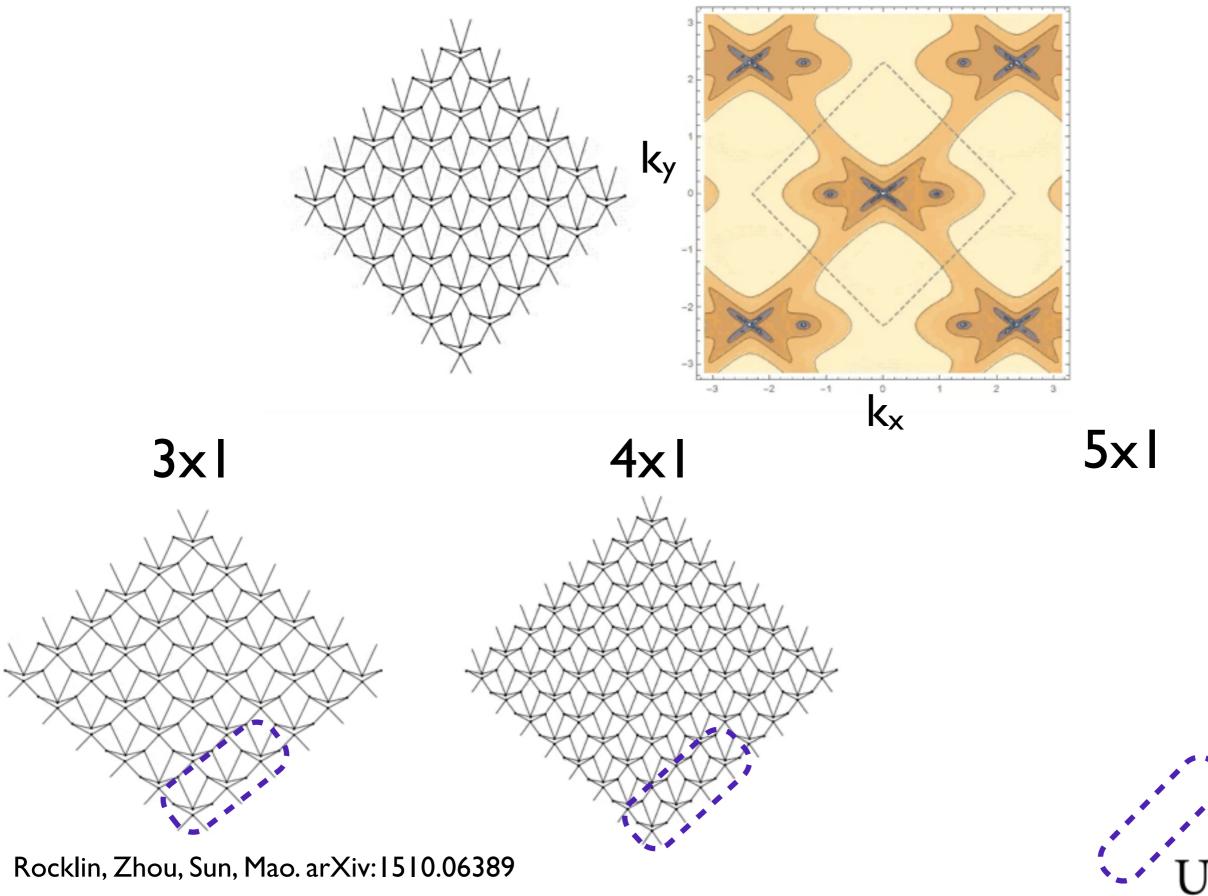


Rocklin, Zhou, Sun, Mao. arXiv:1510.06389





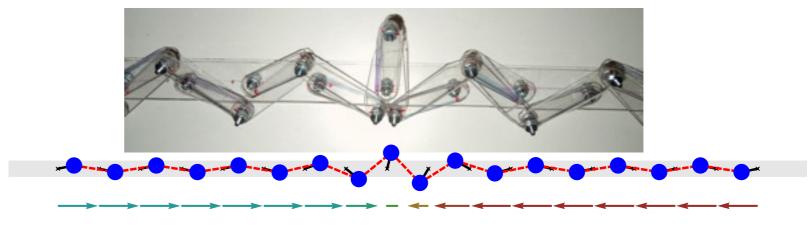




In a **topological mechanical insulator**, energy may be transported by a **soliton**:

Thanks!

Tien-syh Chen
Jayson Paulose
Jeffrey Teo
Ari Turner
Yujie Zhou



There are **topologically protected** bulk zero modes in generic isostatic lattices:

Chen, Upadhyaya, Vitelli, PNAS 111, 13004 (2014)

Rocklin, Chen, Falk, Vitelli, Lubensky, arXiv:1510.04970

"Vitelli Lab" youtube







