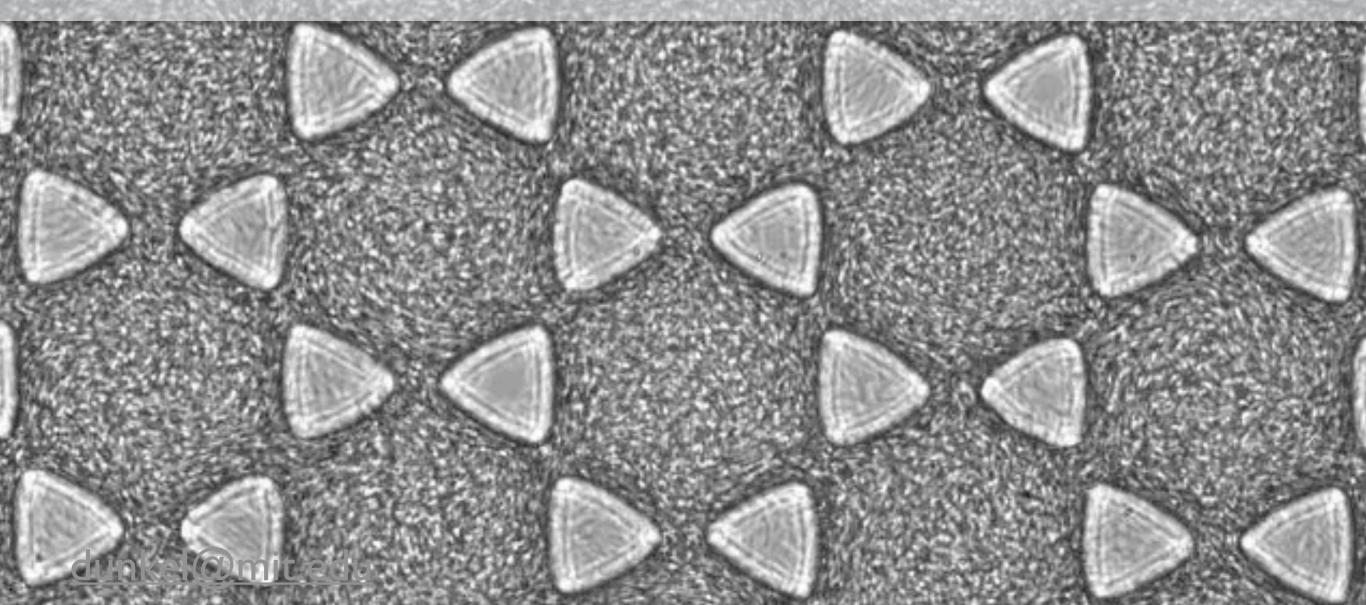
Discussion session: Bacterial "Sheets"

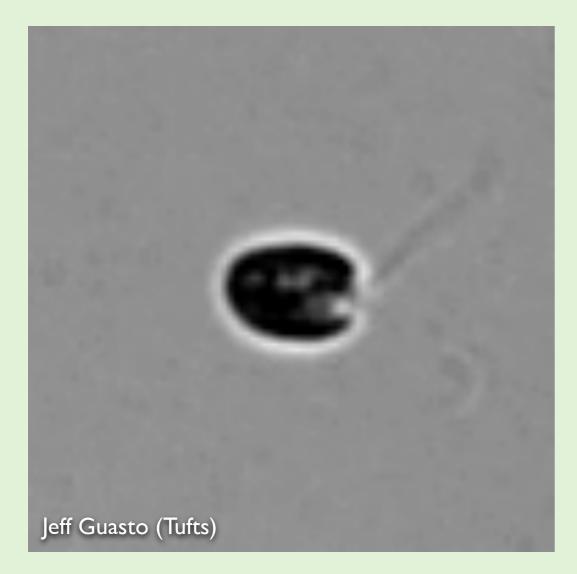
Jörn Dunkel



- effects of spatial dimensionality on individual microbial swimming (2D vs. 3D)
- intrinsic vortex scale selection in bacterial suspensions
- confinement & collective dynamics of quasi-2D suspensions (edge currents, magnetic order, quasi-"superfluidity", etc.)
- defect dynamics and long-range order in 2D planar/curved active nematics

Model organisms

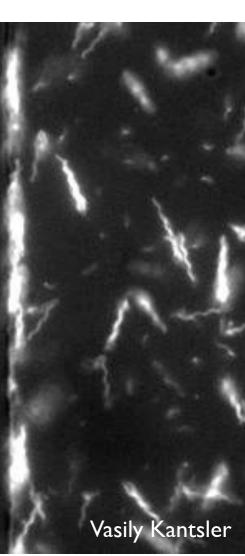
Chlamydomonas alga



Drescher et al (2010) PRL Guasto et al (2010) PRL Human sperm cell



Kantsler et al (2014) eLife Friedrich et al (2010) JEB Bacteria



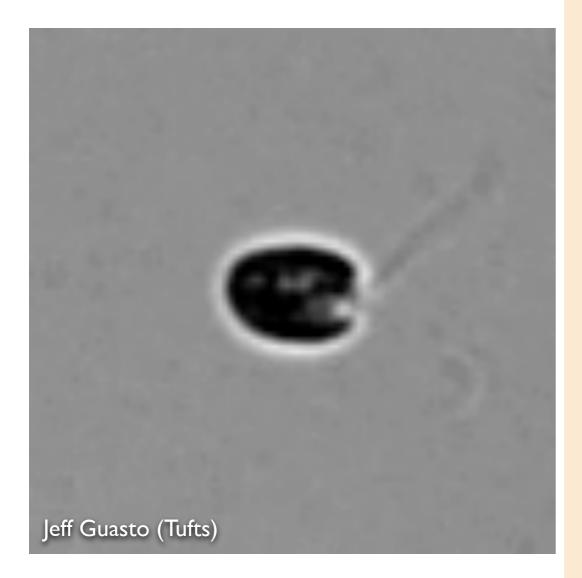
Drescher et al (2011) PNAS

Eukaryotes

Prokaryote

Swimming strategies

Chlamydomonas alga



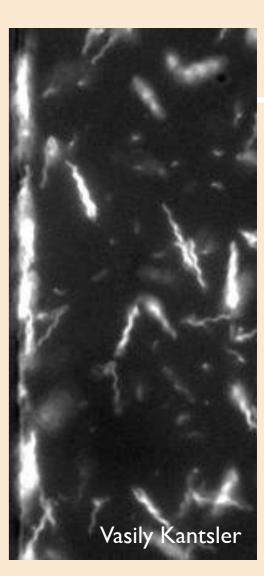
Drescher et al (2010) PRL Guasto et al (2010) PRL

Puller

Human sperm cell

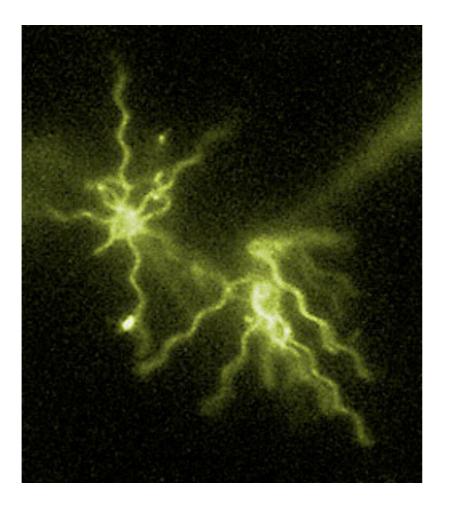


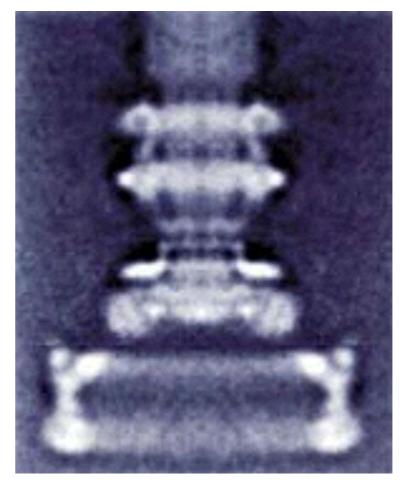
Kantsler et al (2014) eLife Friedrich et al (2010) JEB Bacteria



Drescher et al (2011) PNAS

Pusher





Berg (1999) Physics Today

Bacterial Hydrodynamics*

Eric Lauga[†]

Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom

Swimming at low Reynolds number

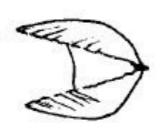


Navier - Stokes:

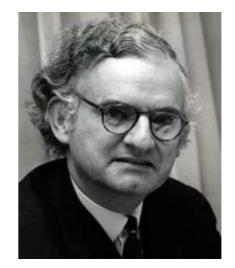
$$\mathscr{H} \qquad \mathcal{R} \sim U L \rho / \eta \ll 1$$

Time doesn't matter. The pattern of motion is the same, whether slow or fast, whether forward or backward in time.

The Scallop Theorem







James Lighthill



Edward Purcell

$$0 = \mu \nabla^2 \boldsymbol{u} - \nabla p + \boldsymbol{f},$$

$$0 = \nabla \cdot \boldsymbol{u}.$$

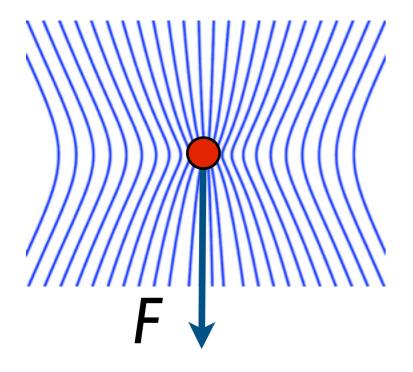
+ time-dependent BCs

American Journal of Physics, Vol. 45, No. 1, January 1977

Superposition of singularities



3D stokeslet



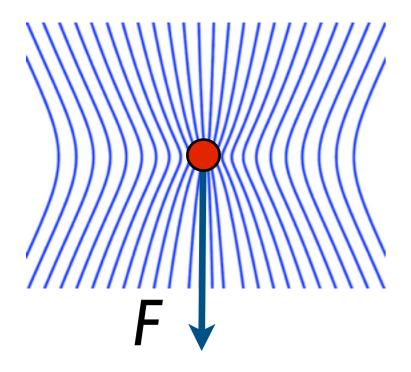
$$p(\mathbf{r}) = \frac{\hat{\mathbf{r}} \cdot \mathbf{F}}{4\pi r^2} + p_0$$
$$v_i(\mathbf{r}) = \frac{(8\pi\mu)^{-1}}{r} [\delta_{ij} + \hat{r}_i \hat{r}_j] F_j$$

flow ~
$$r^{-1}$$

Superposition of singularities



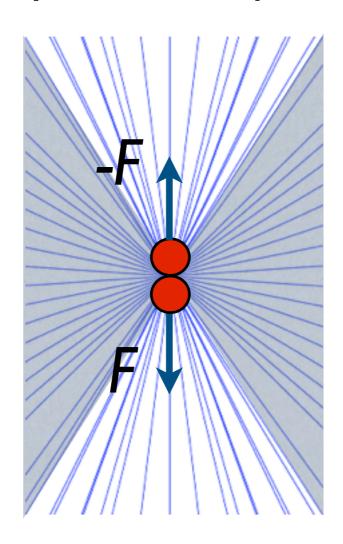
3D stokeslet



$$p(\mathbf{r}) = \frac{\hat{\mathbf{r}} \cdot \mathbf{F}}{4\pi r^2} + p_0$$
$$v_i(\mathbf{r}) = \frac{(8\pi\mu)^{-1}}{r} [\delta_{ij} + \hat{r}_i \hat{r}_j] F_j$$

flow ~ r^{-1}

2 x stokeslet = symmetric dipole



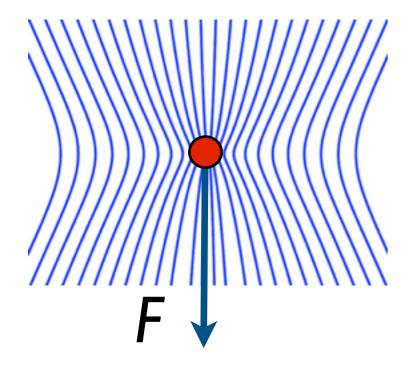
$$r^{-2}$$

'pusher' dipole

Superposition of singularities



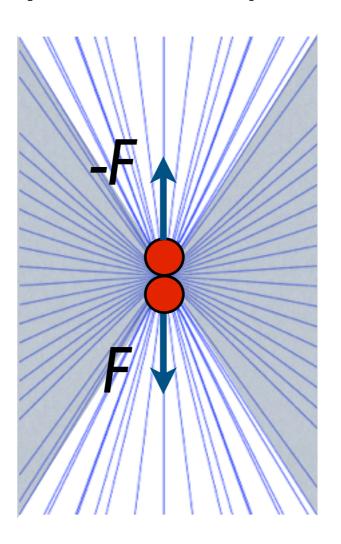
stokeslet



$$p(\mathbf{r}) = \frac{\hat{\mathbf{r}} \cdot \mathbf{F}}{4\pi r^2} + p_0$$
$$v_i(\mathbf{r}) = \frac{(8\pi\mu)^{-1}}{r} [\delta_{ij} + \hat{r}_i \hat{r}_j] F_j$$

flow $\sim r^{-1}$

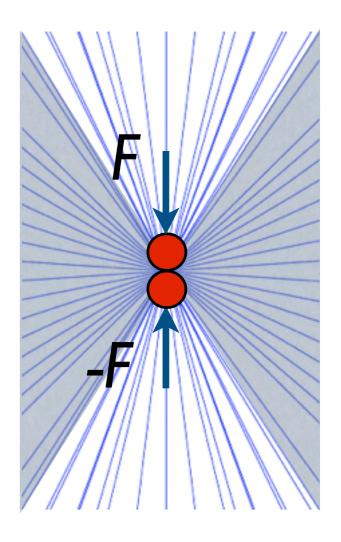
2 x stokeslet = symmetric dipole



 r^{-2}

'pusher' dipole

2 x stokeslet = symmetric dipole

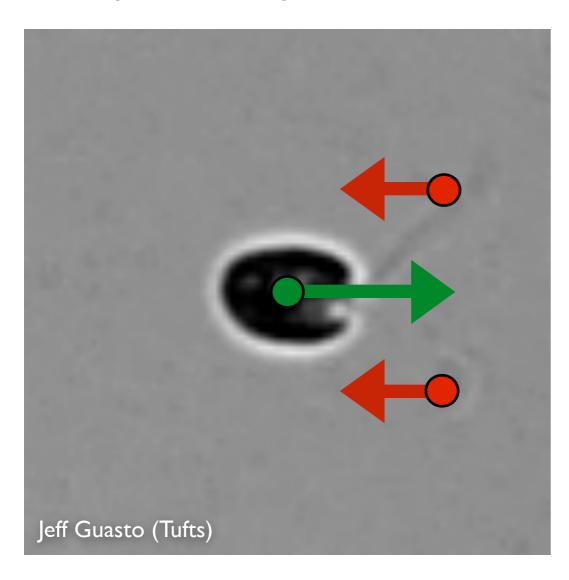


 r^{-2}

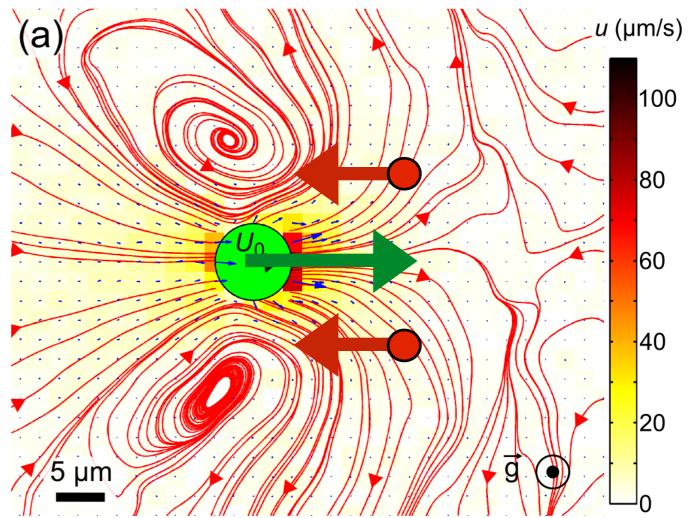
'puller' dipole

Algal flow field: not a dipole

Chlamydomonas alga



Drescher et al (2010) PRL Guasto et al (2010) PRL



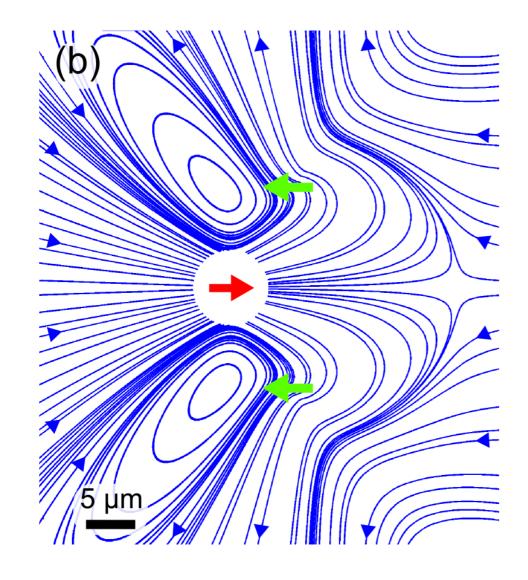
size: 20 µm

speed: 100 µm/s

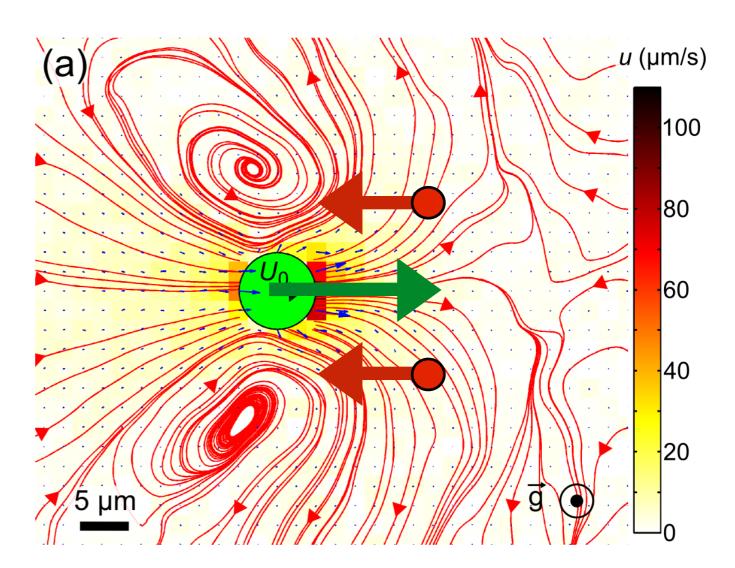
beat frequency: 30 Hz

Algal flow field: not a dipole

Chlamydomonas alga



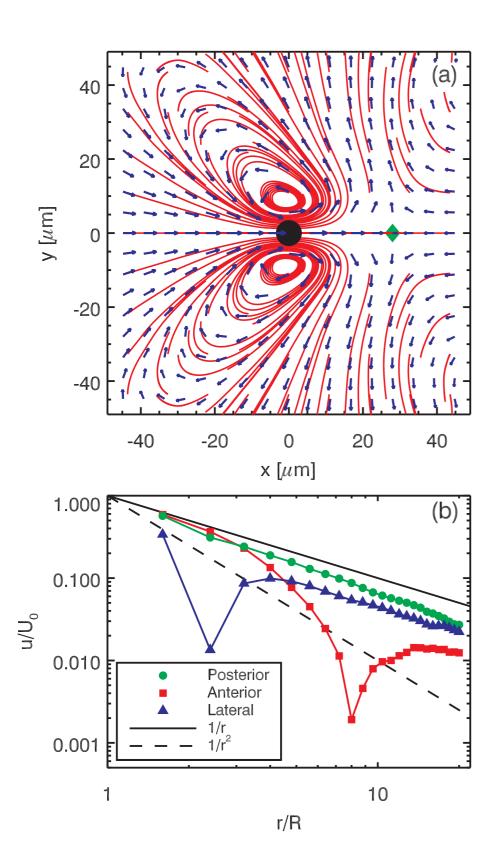
Drescher et al (2010) PRL Guasto et al (2010) PRL



size: 20 µm

speed: 100 µm/s

beat frequency: 30 Hz



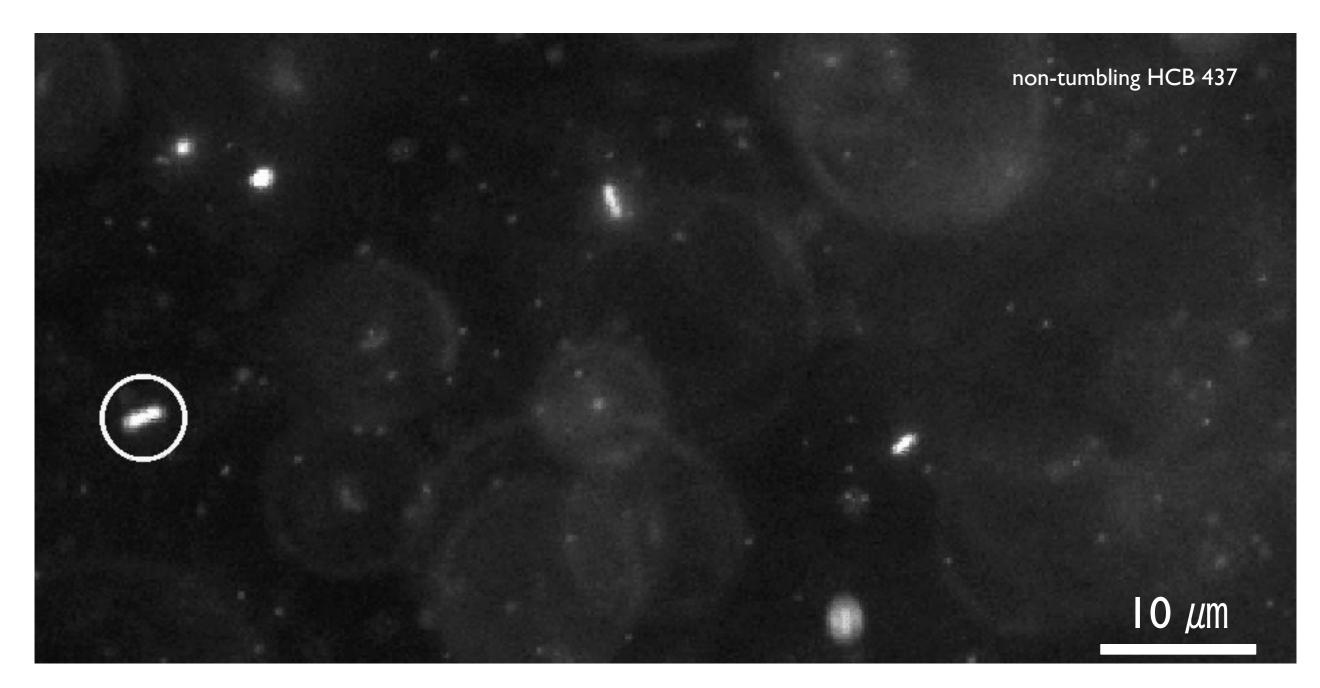
u (µm/s) 100 80 60 40 20 10⁰ (c) 10 u/U_0 10⁻² vortex' 10⁻³ stagnation point 10 r/R

Guasto et al (2010) PRL

Drescher et al (2010) PRL

E. coli (non-tumbling)

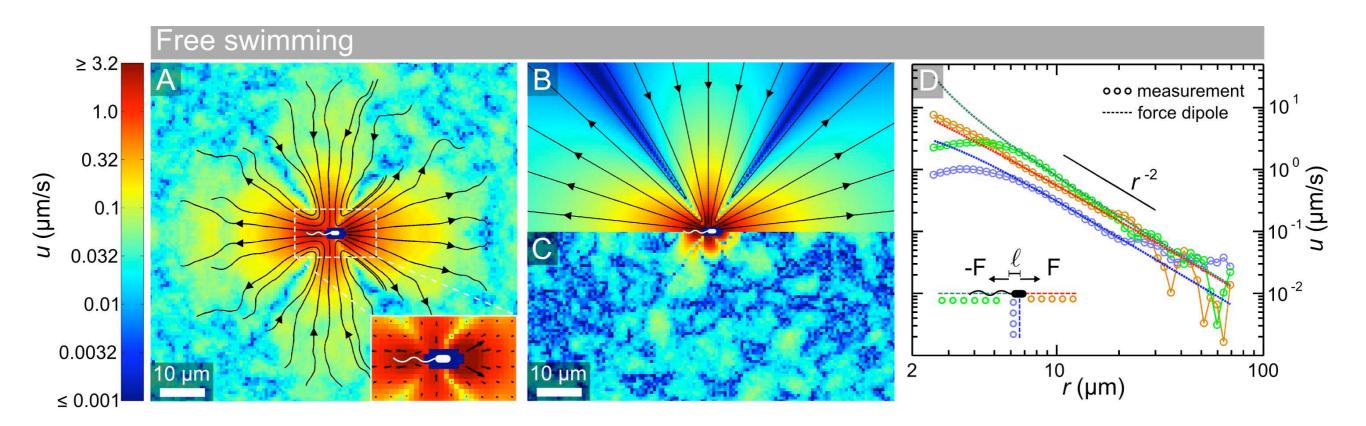




Drescher et al (2011) PNAS

E.coli (non-tumbling HCB 437)





$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2} \left[3(\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{d}})^2 - 1 \right] \hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi \eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

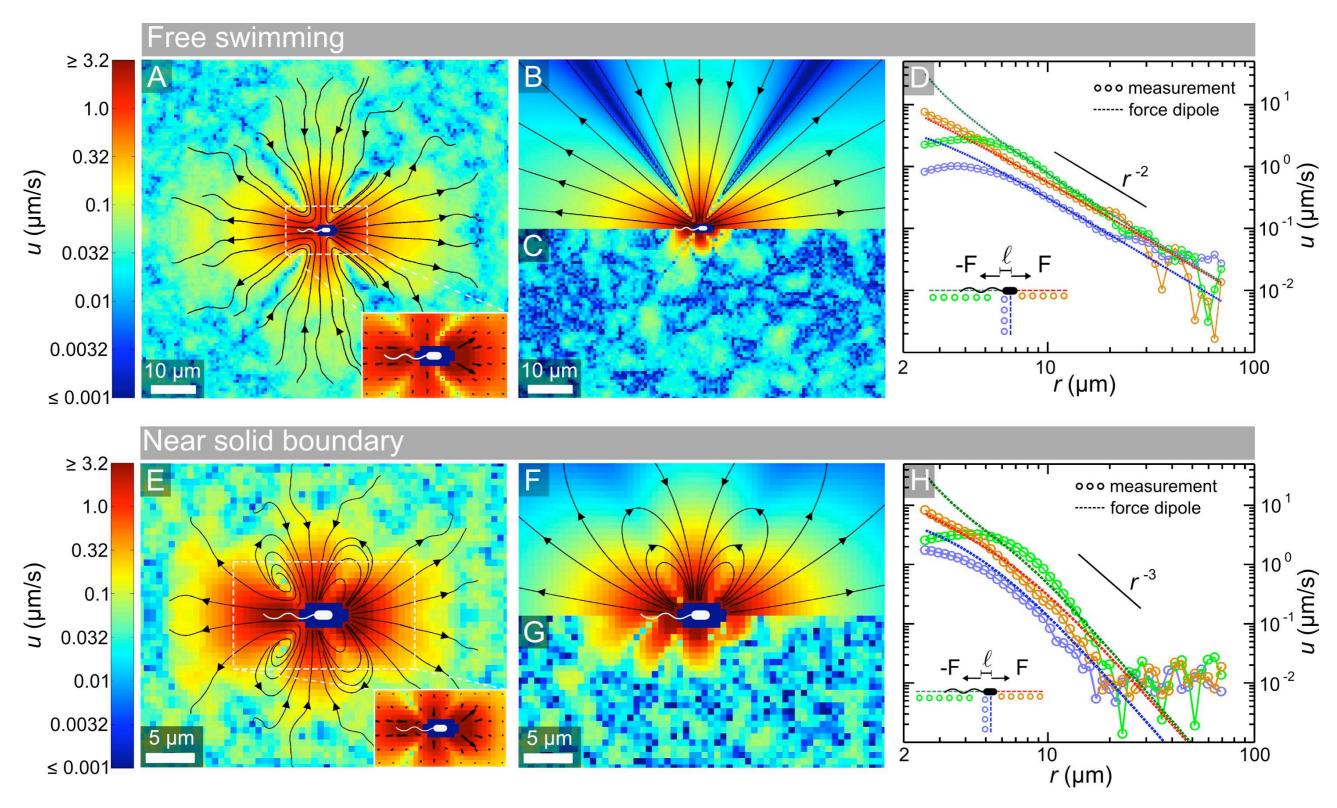
$$\begin{aligned} & V_0 = 22 \pm 5 \ \mu \text{M} \\ & \ell = 1.9 \ \mu \text{M} \\ & F = 0.42 \ \text{pN} \end{aligned}$$

$$V_0 = 22 \pm 5 \ \mu \text{m/s}$$
 $\ell = 1.9 \ \mu \text{m}$
 $F = 0.42 \ \text{pN}$

'pusher' dipole

E.coli (non-tumbling HCB 437)





Drescher et al (2011) PNAS

Hydrodynamic scattering

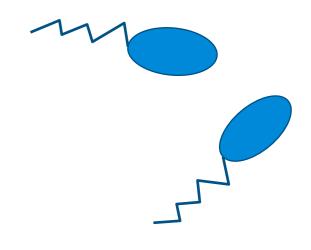


dipole flow

$$oldsymbol{v} \sim rac{A}{r^2}$$

vorticity

$$\omega = \nabla \times \boldsymbol{v} \sim \frac{A}{r^3}$$

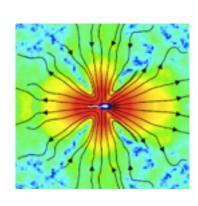


encounter time

$$au \sim \ell/V$$

HD rotation

$$\langle |\Delta \phi|^2 \rangle \sim (\omega \tau)^2 \sim \left(\frac{A\tau}{r^3}\right)^2$$



rotational diffusion

$$\langle |\Delta \phi|^2 \rangle \sim D_r \tau$$

$$D_r = 0.057 \text{ rad}^2/\text{s}$$

balance

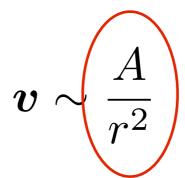
$$r_H \sim \left(\frac{A^2 \tau}{D_r}\right)^{1/6}$$

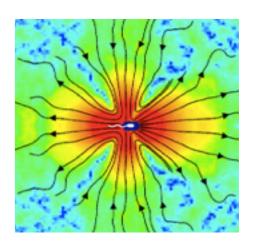
 $3.3 \ \mu \text{m} \text{ for } E. \ coli$

Implications for collective swimming



√ hydrodynamic advection





- √ steric alignment
- hydrodynamic alignment less important

$$\omega =
abla imes oldsymbol{v} \left\langle rac{A}{r^3}
ight
angle$$

✓ intrinsic rotational noise much larger than thermal noise

$$D_r = 0.057 \text{ rad}^2/\text{s}$$

- effects of spatial dimensionality on individual microbial swimming (2D vs. 3D)
- intrinsic vortex scale selection in bacterial suspensions
- confinement & collective dynamics of quasi-2D suspensions (edge currents, magnetic order, quasi-"superfluidity", etc.)
- defect dynamics and long-range order in 2D planar/curved active nematics



Self-Concentration and Large-Scale Coherence in Bacterial Dynamics

Christopher Dombrowski, ¹ Luis Cisneros, ¹ Sunita Chatkaew, ¹ Raymond E. Goldstein, ^{1,2} and John O. Kessler ¹ Department of Physics, University of Arizona, Tucson, Arizona 85721, USA ² Program in Applied Mathematics, University of Arizona, Tucson, Arizona 85721, USA (Received 23 December 2003; published 24 August 2004)

B. subtilis

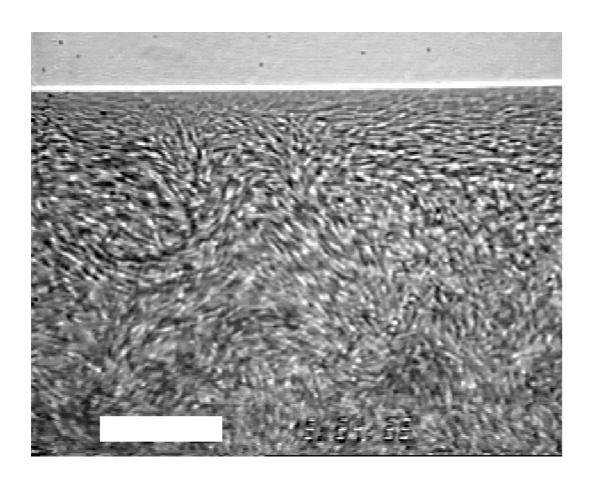
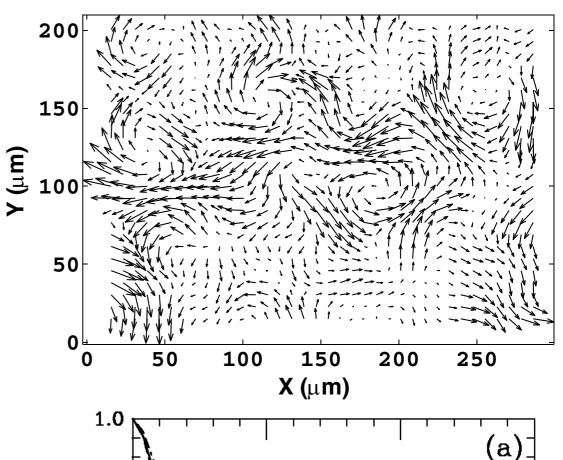
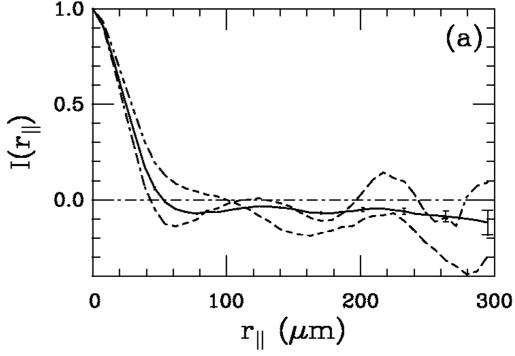


FIG. 3. Bacterial "turbulence" in a sessile drop, viewed from below through the bottom of a petri dish. Gravity is perpendicular to the plane of the picture, and the horizontal white line near the top is the air-water-plastic contact line. The central fuzziness is due to collective motion, not quite captured at the frame rate of 1/30 s. The scale bar is 35 μ m.

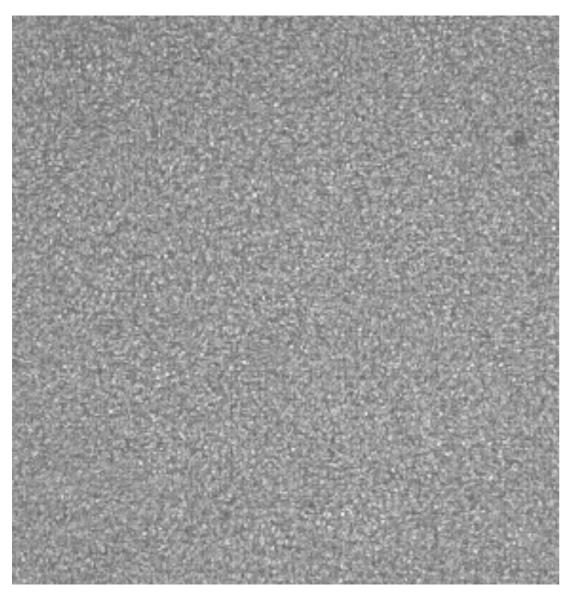




Bacterial 'turbulence'

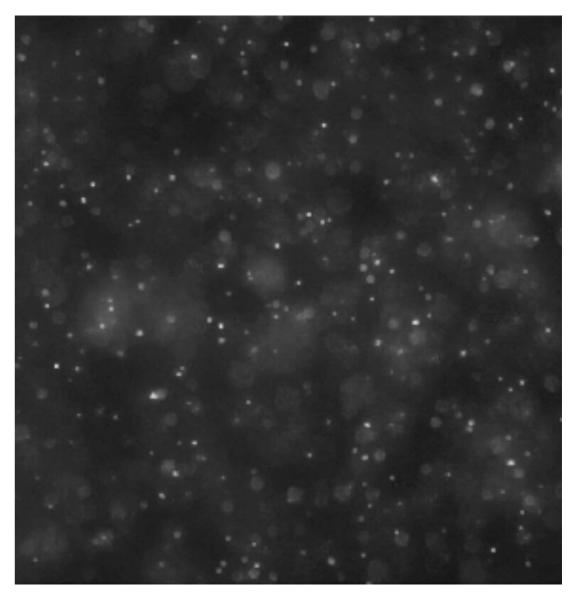


B. subtilis



bright field





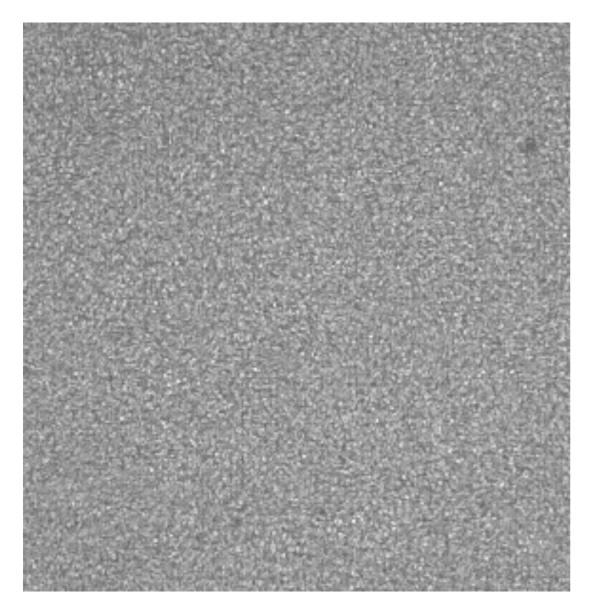
fluorescence

Dunkel et al PRL 2013

Bacterial 'turbulence'

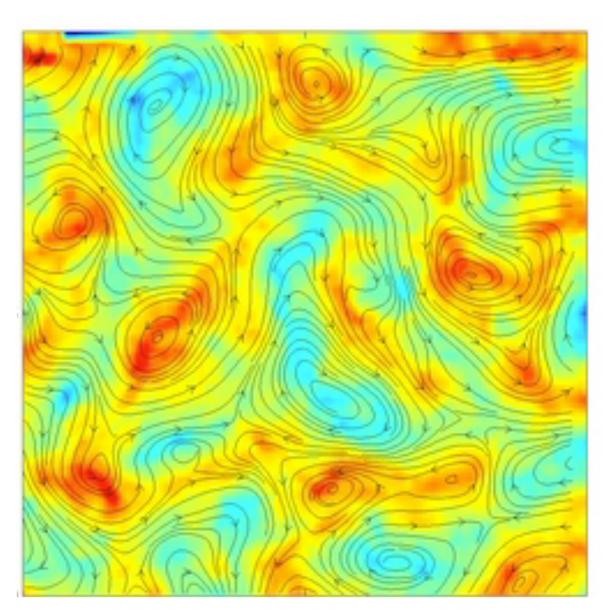


B. subtilis



bright field

Wensink et al PNAS 2012 Dunkel et al (2013) PRL

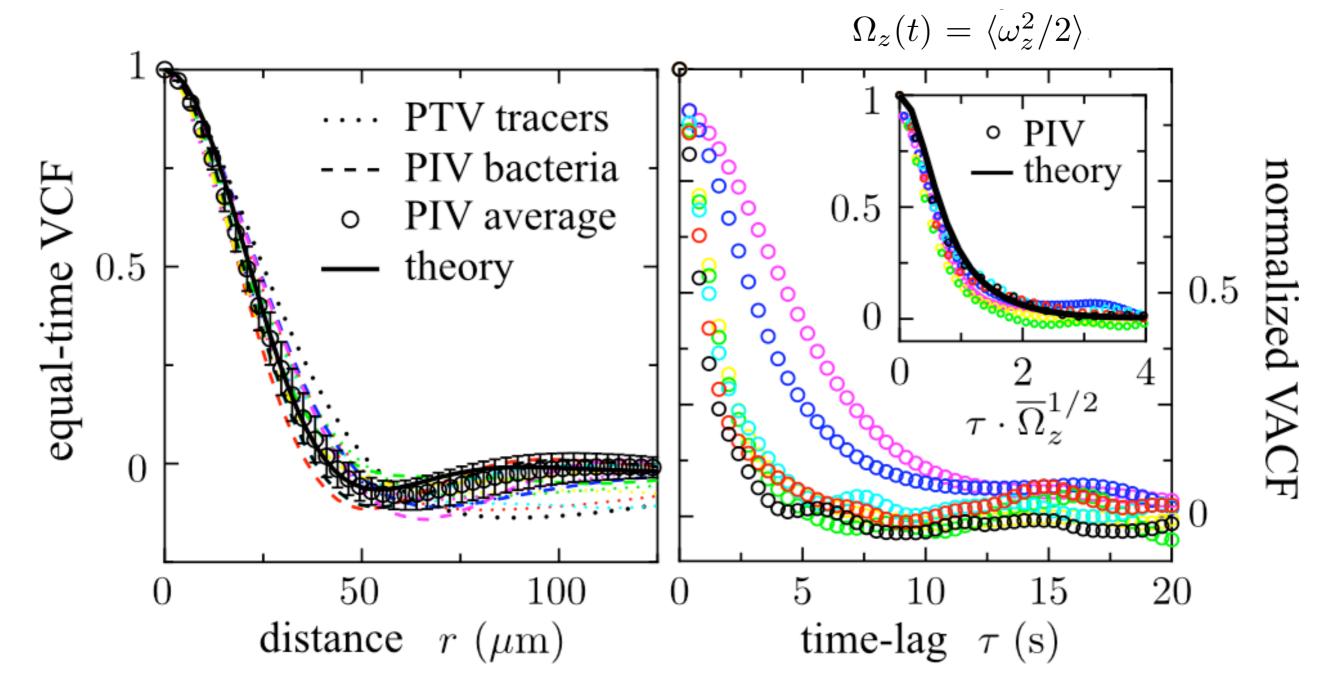


PIV

see also: Sokolov & Aronson (2012) PRL

Velocity correlations





Vortex diameter ~ 70 µm

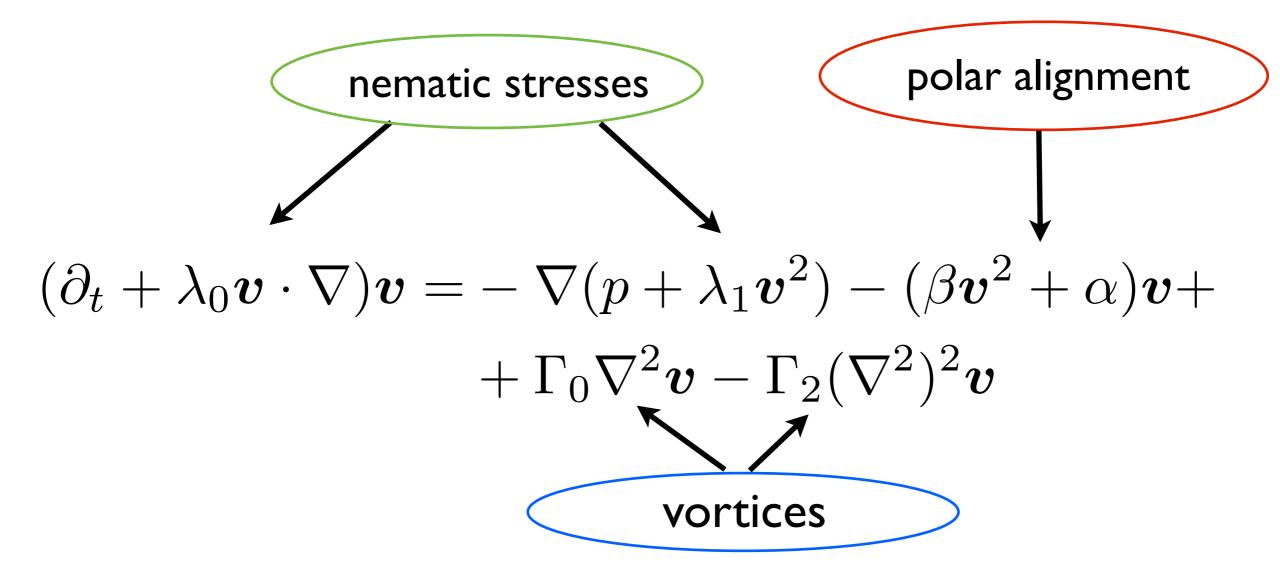
Life time ~ seconds

Minimal continuum theory for bacterial velocity field



incompressibility

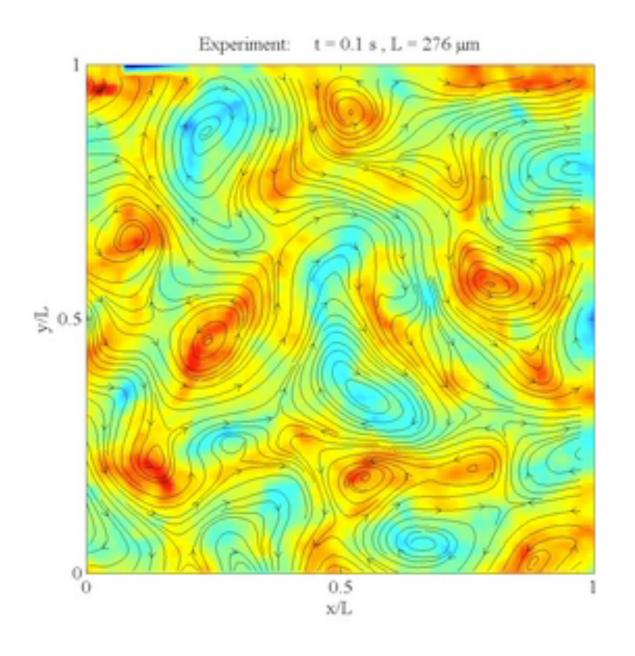
$$\nabla \cdot \boldsymbol{v} = 0$$



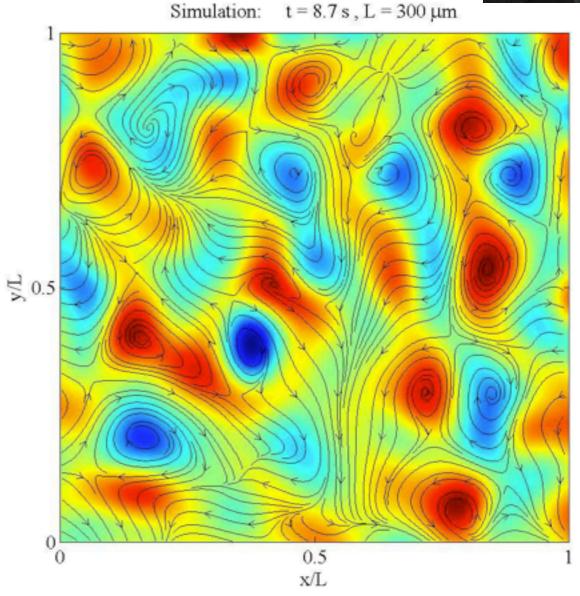
Dunkel et al (2014) PRL Wensink et al (2012) PNAS

experiment vs. theory





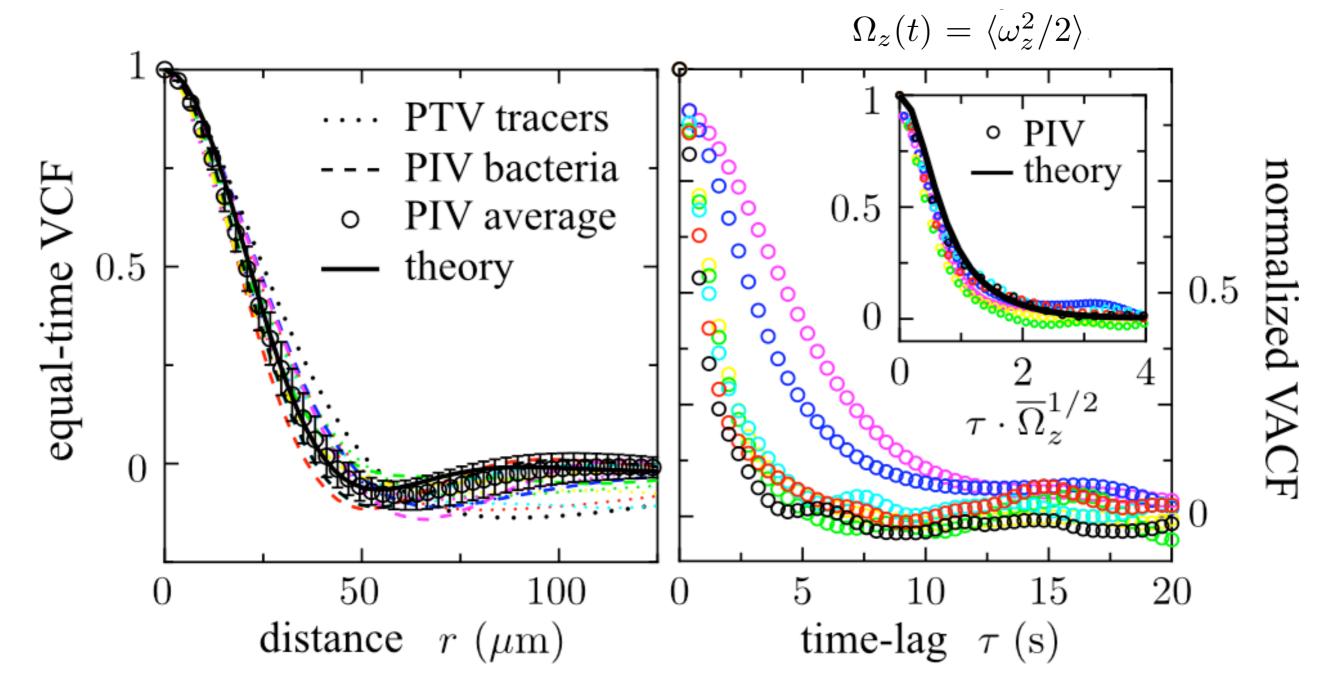
quasi-2D slice



2D slice from 3D simulation

Velocity correlations





Vortex diameter ~ 70 µm

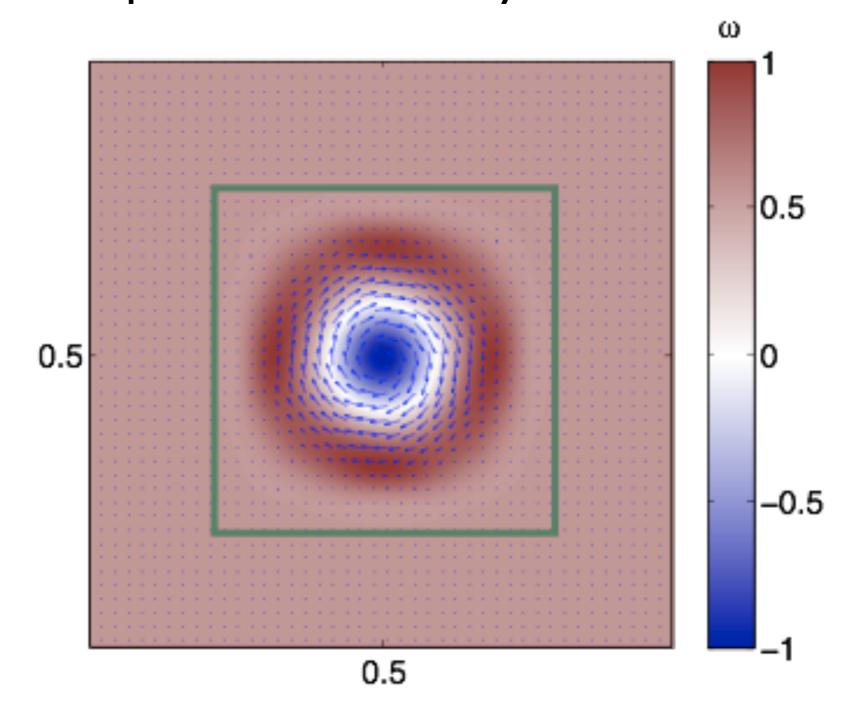
Life time ~ seconds

- effects of spatial dimensionality on individual microbial swimming (2D vs. 3D)
- intrinsic vortex scale selection in bacterial suspensions
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- defect dynamics and long-range order in 2D planar/curved active nematics

Vortex stabilization

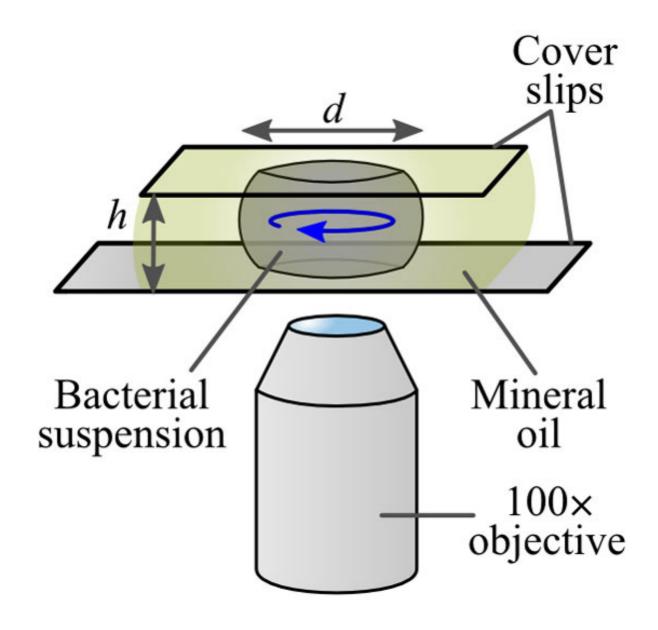


'prediction' of many models



Can we stabilize vortices?





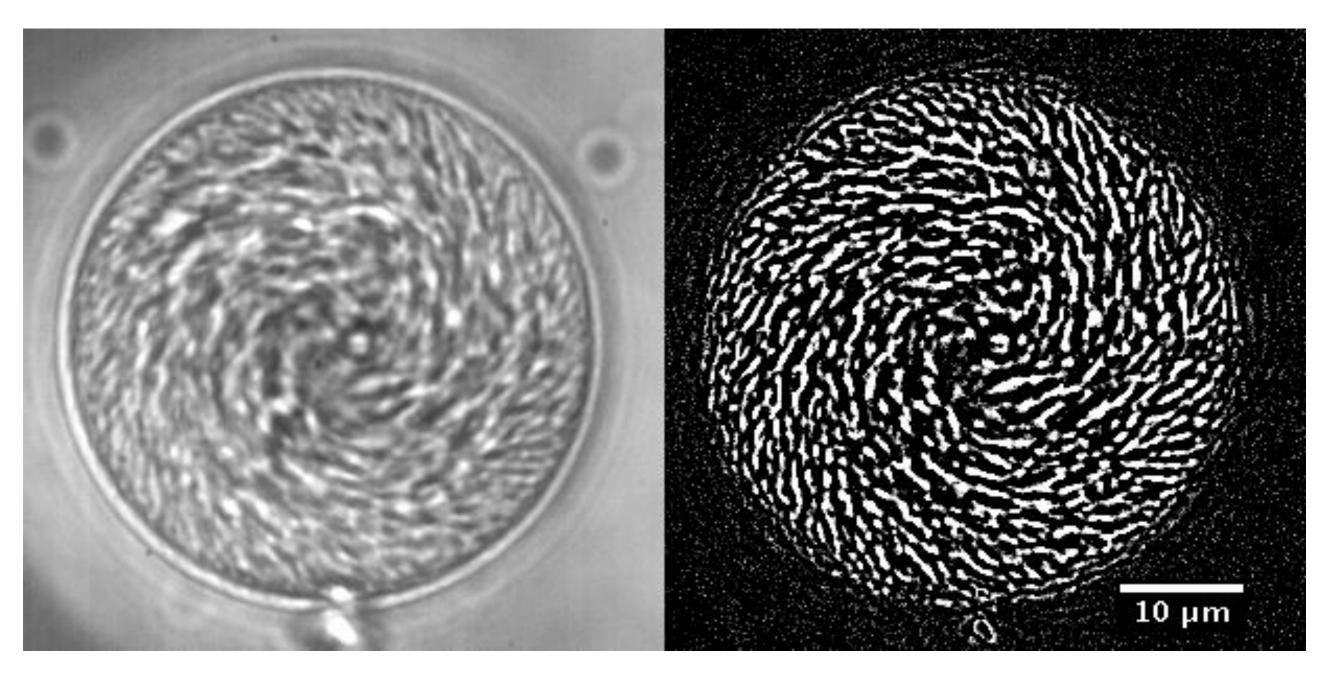


Wioland et al (2013) PRL



Stable bacterial spiral vortex

Vortex life time ~ minutes

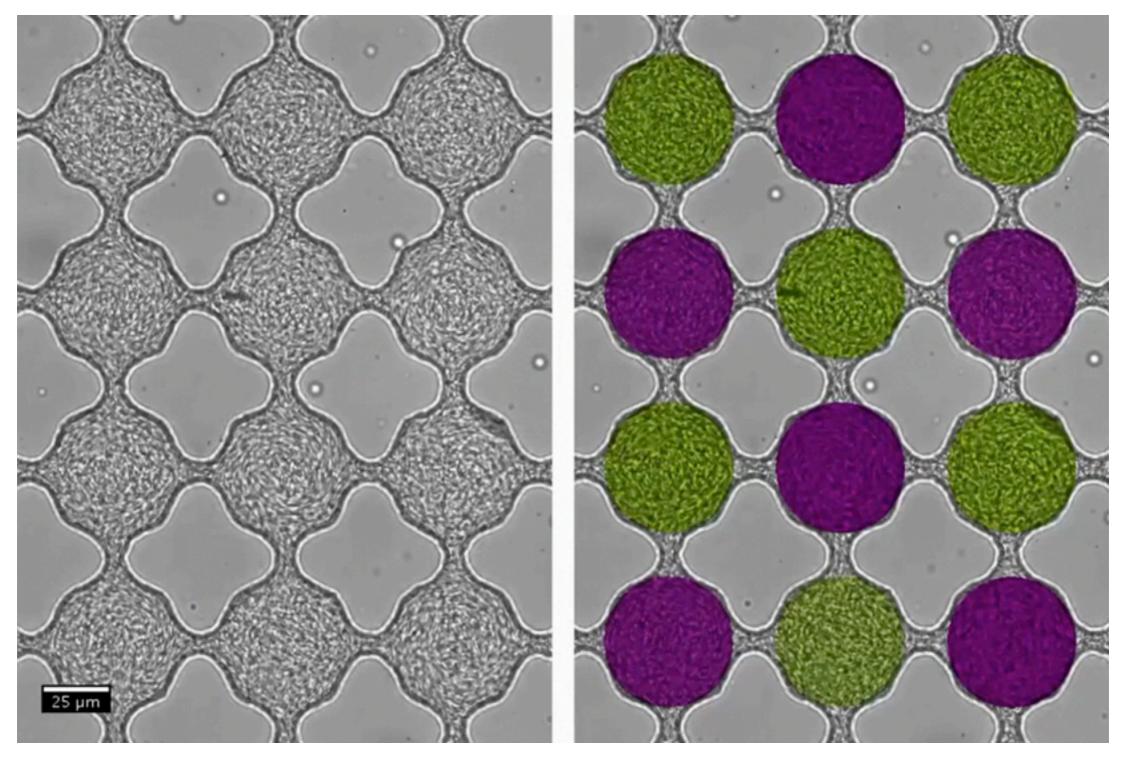


Wioland et al (2013) PRL Lushi et al (2014) PNAS

Edge currents!

Anti-ferromagnetic order

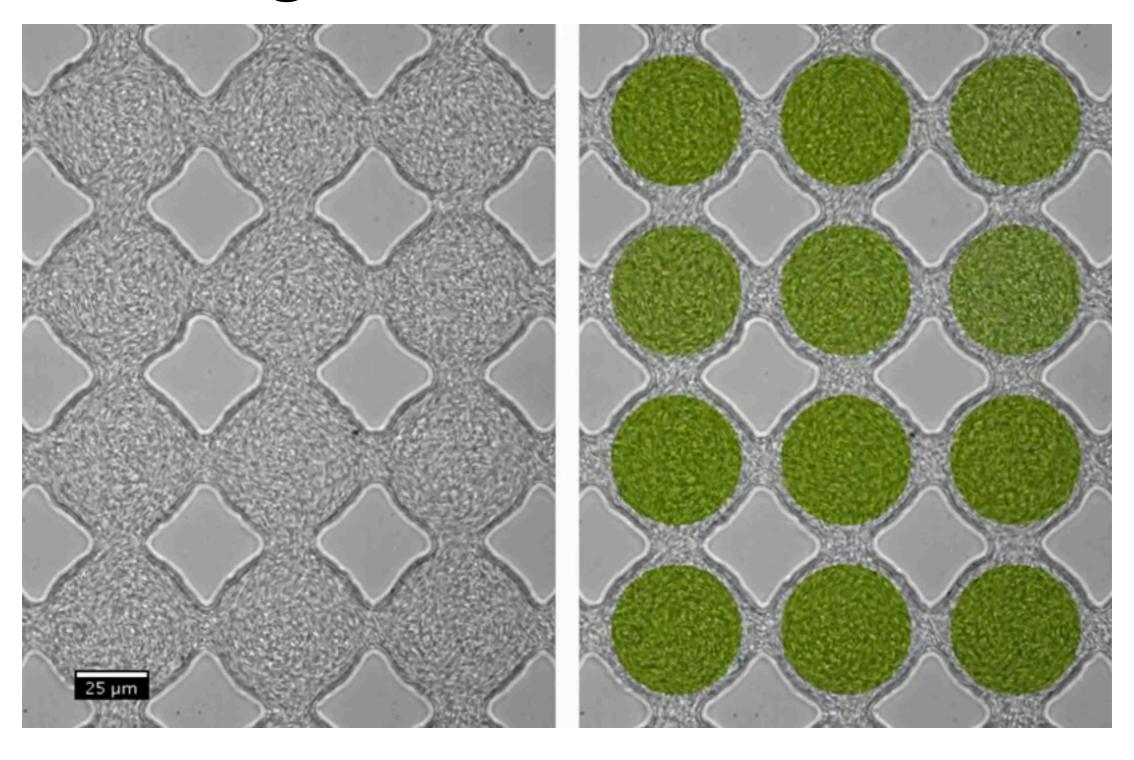




Wioland et but only for weak coupling (small gaps)

Ferromagnetic order





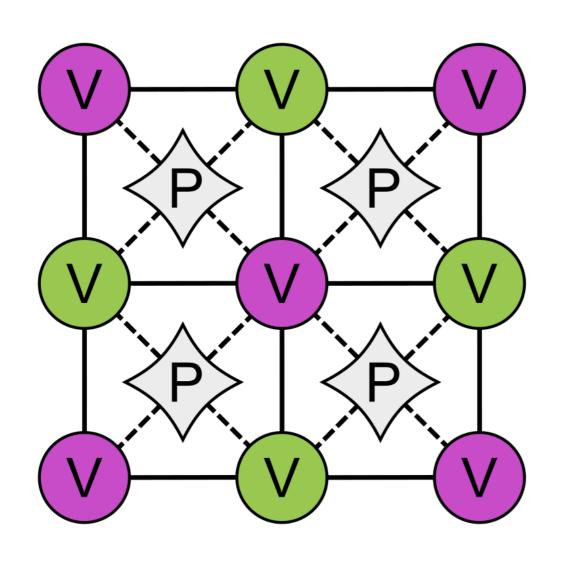
Wioland et al (2016) n by edge currents around pillars Nature Physics

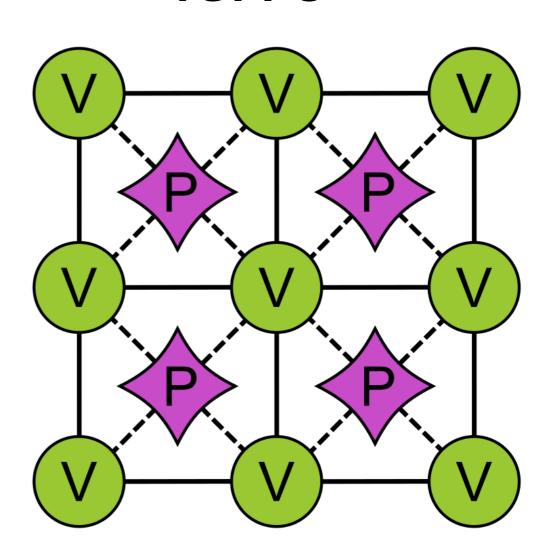


anti-ferro

VS.

ferro

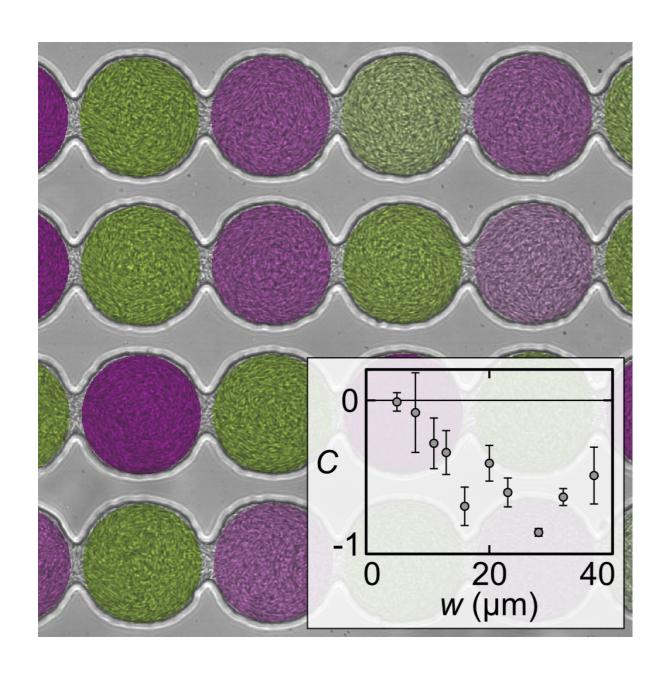


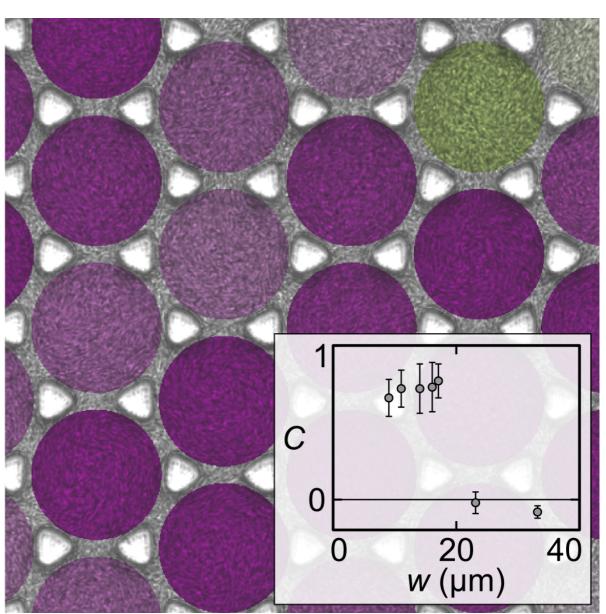


$$H(\mathbf{V}, \mathbf{E}) = -J_v \sum_{V_i \sim V_j} V_i V_j - J_e \sum_{V_i \sim E_j} V_i E_j + \sum_{V_i} \left(\frac{1}{2} a_v V_i^2 + \frac{1}{4} b_v V_i^4\right) + \sum_{E_i} \frac{1}{2} a_p E_i^2$$

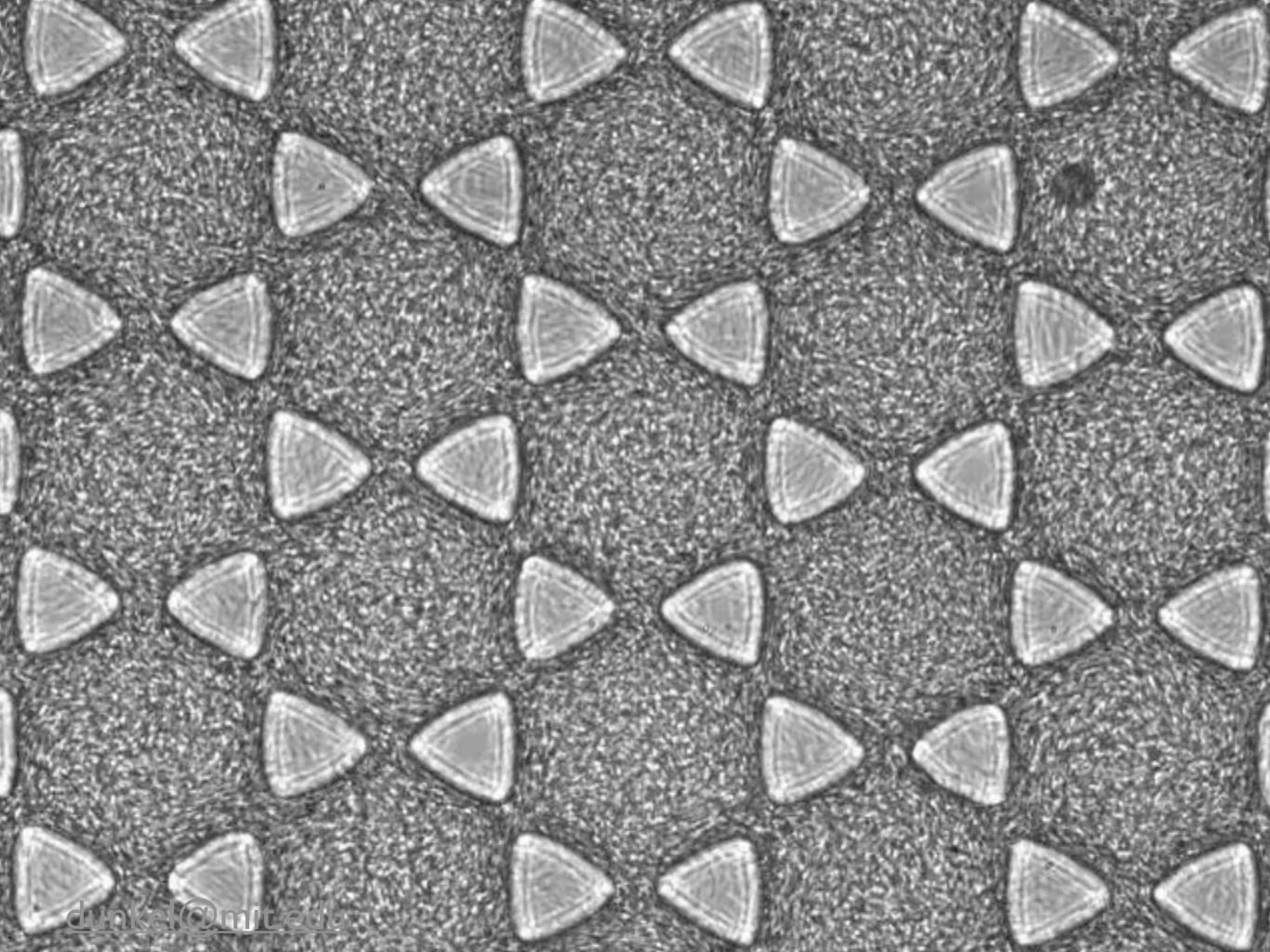
Geometry & frustration







$$C = \overline{\langle V_i V_j \rangle_{i \sim j}}$$



Reduction of Viscosity in Suspension of Swimming Bacteria

Andrey Sokolov^{1,2} and Igor S. Aranson²

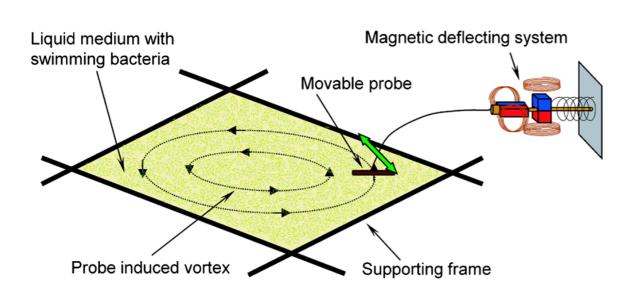


FIG. 1 (color online). Experimental setup 1: a thin liquid film with bacteria spans between four movable fibers. A micromanipulator with a magnetic deflecting system is used to initiate a large vortex through movement of the probe.

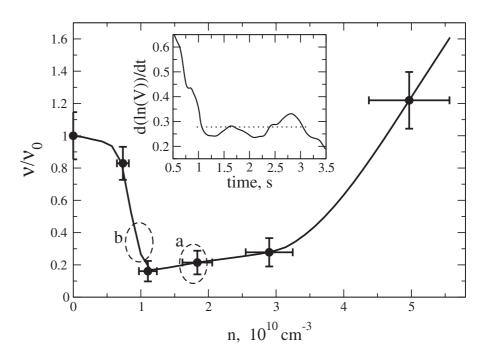


FIG. 3. Viscosity for 6 different concentrations of bacteria. ν_0 is the viscosity of the liquid without bacteria. Inset: instant viscosity vs time during decay of the vortex for density $n = 2.9 \times 10^{10}$. The dashed line is the average value of the viscosity during the slow phase of decay. See movies 1 and 2 in [19].



Non-Newtonian Viscosity of Escherichia coli Suspensions

Jérémie Gachelin, Gastón Miño, Hélène Berthet, Anke Lindner,* Annie Rousselet, and Éric Clément *PMMH-ESPCI, UMR 7636 CNRS-ESPCI-Universities Pierre et Marie Curie and Denis Diderot,* 10 rue Vauquelin, 75005 Paris, France (Received 5 October 2012; published 26 June 2013)

$$\eta_r = \frac{\eta_1}{\eta_0} = \frac{d_1}{d_0}.\tag{1}$$

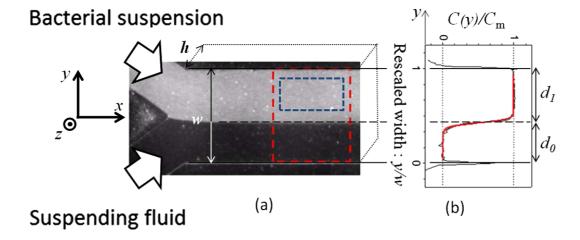
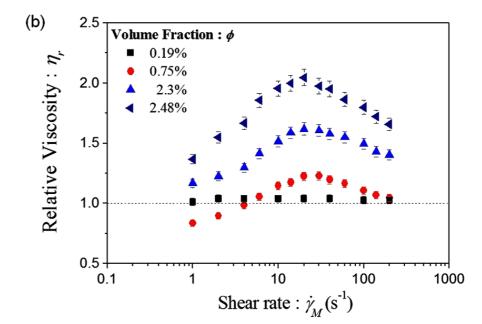
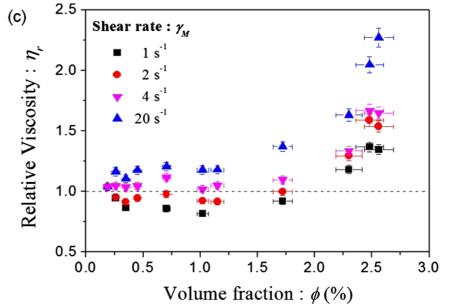


FIG. 1 (color online). Experimental setup. (a) Time-averaged image of the microchannel ($W=600~\mu\mathrm{m}$) for $Q=10~\mathrm{nl/sec}$ in each branch and volume fraction $\phi=0.35\%$. Bacteria are visualized using a white light microscope. The red and blue frames indicate the measurement areas. (b) Concentration profile C(y) normalized by the maximum concentration C_M (black line) and error function fit used to determine the interface position (red line).

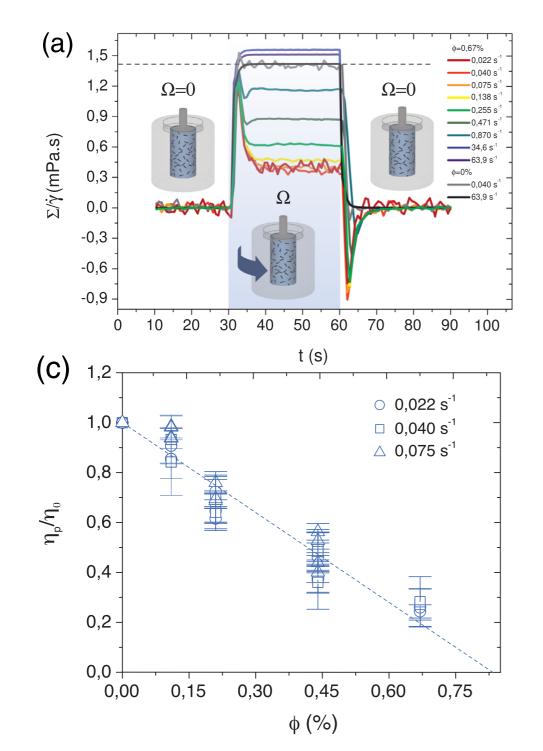


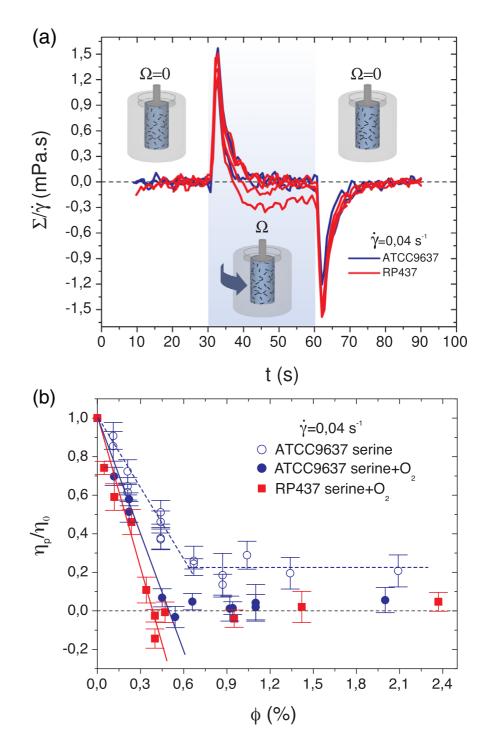




Turning Bacteria Suspensions into Superfluids

Héctor Matías López, ¹ Jérémie Gachelin, ² Carine Douarche, ³ Harold Auradou, ^{1,*} and Eric Clément ² ¹ Université Paris-Sud, CNRS, F-91405, Lab FAST, Bâtiment 502, Campus Univ, Orsay F-91405, France ² Physique et Mécanique des Milieux Hétérogenes (UMR 7636 ESPCI/CNRS/Université P.M. Curie/Université Paris-Diderot), 10 rue Vauquelin, 75005 Paris, France ³ Laboratoire de Physique des Solides, Université Paris-Sud, CNRS UMR 8502, F-91405 Orsay, France (Received 19 March 2015; revised manuscript received 24 May 2015; published 7 July 2015)





Shearing Active Gels Close to the Isotropic-Nematic Transition

M. E. Cates, S. M. Fielding, D. Marenduzzo, E. Orlandini, and J. M. Yeomans

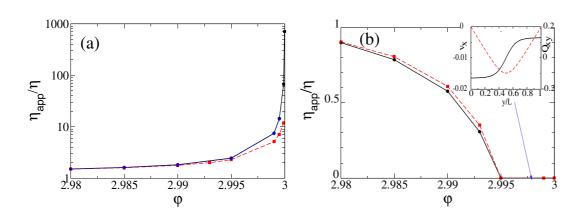


FIG. 2 (color online). (a) Apparent zero shear viscosity for an isotropic contractile gel in a slab under shear. Solid curve(s): free-boundary conditions; HLB data are indistinguishable from those derived from the bulk flow curve [13]. Dashed curve: fixed boundaries (director along flow). (b) Apparent viscosity in a linear regime for an active extensile gel in a slab under shear. Solid and dashed curves correspond to systems with free and fixed boundaries, respectively. The insets show plots of Q_{xy} and u_x (solid and dashed lines) in the $\eta_{app} = 0$ phase.

PHYSICAL REVIEW E 81, 051908 (2010)

Sheared active fluids: Thickening, thinning, and vanishing viscosity

Luca Giomi, 1,2 Tanniemola B. Liverpool, and M. Cristina Marchetti 4

$$\Pi_{\alpha\beta} = 2\xi \left(Q_{\alpha\beta} + \frac{1}{3} \delta_{\alpha\beta} \right) Q_{\gamma\epsilon} H_{\gamma\epsilon} - \xi H_{\alpha\gamma} \left(Q_{\gamma\beta} + \frac{1}{3} \delta_{\gamma\beta} \right) \\
- \xi \left(Q_{\alpha\gamma} + \frac{1}{3} \delta_{\alpha\gamma} \right) H_{\gamma\beta} - \partial_{\alpha} Q_{\gamma\nu} \frac{\delta \mathcal{F}}{\delta \partial_{\beta} Q_{\gamma\nu}} \\
+ Q_{\alpha\gamma} H_{\gamma\beta} - H_{\alpha\gamma} Q_{\gamma\beta} - \zeta Q_{\alpha\beta}. \tag{2}$$

$$\eta_{\rm app}(\dot{\gamma}, L, \ldots) = (\Pi_{xy} + \eta \partial_y u_x)/\dot{\gamma}.$$

New Journal of Physics The open-access journal for physics

The Taylor-Couette motor: spontaneous flows of active polar fluids between two coaxial cylinders

S Fürthauer^{1,2}, M Neef³, S W Grill^{1,2}, K Kruse^{3,4} and F Jülicher¹

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¹ Max-Planck-Institut für Physik komplexer Systeme Nöthnitzer Straße 38, 01187 Dresden, Germany

² Max Planck Institute of Molecular Cell Biology and Genetics, Pfotenhauerstr. 108, 01307 Dresden, Germany

³ Theoretische Physik, Universität des Saarlandes, Postfach 151150, 66041 Saarbrücken, Germany

$$\partial_t c = -\nabla[c(\mathbf{v} + c\boldsymbol{\beta}_1 \mathbf{P}) + \Gamma' \mathbf{h} + \Gamma'' \mathbf{f}], \qquad (1a)$$

$$[\partial_t + (\mathbf{v} + c\beta_2 \mathbf{P})\nabla]P_i + \omega_{ij}P_j = \lambda u_{ij}P_j + \Gamma h_i + \Gamma' f_i, \quad (1b)$$

with $\omega_{ij} = (\partial_i v_j - \partial_j v_i)/2$ the vorticity tensor, $\mathbf{h} = -\delta F/\delta \mathbf{P}$ the molecular field, and $\mathbf{f} = -\nabla(\delta F/\delta c)$. The flow velocity satisfies the Navier-Stokes equation [27]

$$\rho(\partial_t + \mathbf{v} \cdot \nabla) v_i = \partial_j \sigma_{ij}, \tag{2}$$

$$F = \int_{\mathbf{r}} \left\{ \frac{C}{2} \left(\frac{\delta c}{c_0} \right)^2 + \frac{a_2}{2} |\mathbf{P}|^2 + \frac{a_4}{4} |\mathbf{P}|^4 + \frac{K_1}{2} (\nabla \cdot \mathbf{P})^2 \right.$$
$$\left. + \frac{K_3}{2} (\nabla \times \mathbf{P})^2 + B_1 \frac{\delta c}{c_0} \nabla \cdot \mathbf{P} + B_2 |\mathbf{P}|^2 \nabla \cdot \mathbf{P} \right.$$
$$\left. + \frac{B_3}{c_0} |\mathbf{P}|^2 \mathbf{P} \cdot \nabla c \right\},$$

$$\sigma_{ij} = 2 \eta u_{ij} + \sigma_{ij}^r + \sigma_{ij}^\alpha + \sigma_{ij}^\beta, \text{ with}$$

$$\sigma_{ij}^\alpha = \frac{\alpha c^2}{\Gamma} (P_i P_j + \delta_{ij}),$$

$$\sigma_{ij}^\beta = \frac{\beta_3 c^2}{\Gamma} [\partial_i P_j + \partial_j P_i + \delta_{ij} \nabla \cdot \mathbf{P}]$$

$$\sigma_{ij}^r = -\delta_{ij} \Pi + \lambda p_i p_j p_k \left[\frac{w}{c_0 \Gamma} \partial_k c + K \nabla^2 p_k \right]$$

$$-\frac{\lambda}{2} \left[\frac{w}{c_0 \Gamma} (p_i \partial_j c + p_j \partial_i c) + K (p_i \nabla^2 p_j + p_j \nabla^2 p_i) \right]$$

$$+ \frac{1}{2} \left[\frac{w}{c_0 \Gamma} (p_i \partial_j c - p_j \partial_i c) + K (p_i \nabla^2 p_j - p_j \nabla^2 p_i) \right]$$

$$-\lambda \Gamma' \xi p_i p_j (D p_k \partial_k c + w p_k \partial_k \partial_l p_l) + \frac{\lambda^2}{\Gamma} p_i p_j u_{kl} p_k p_l.$$

Minimal momentum-conserving model for solvent flow



Jonasz Slomka

Flow equations

$$0 = \nabla \cdot \boldsymbol{v}$$
$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla) \boldsymbol{v} = -\nabla p + \nabla \cdot \boldsymbol{\sigma}$$

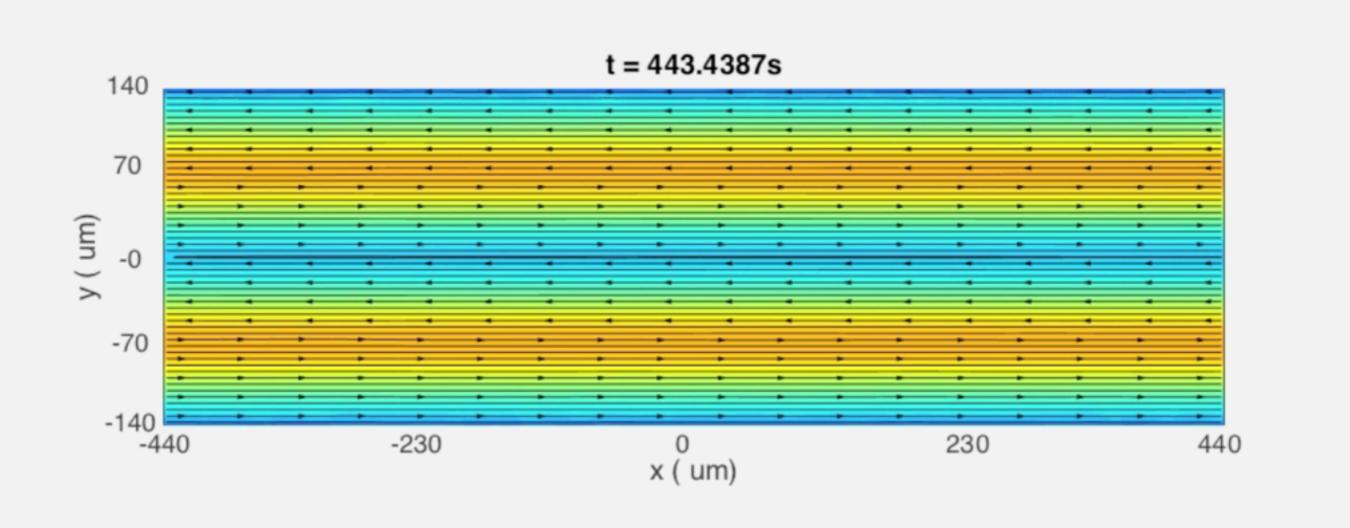
with stress tensor

$$\boldsymbol{\sigma} = [\Gamma_0 - \Gamma_2(\nabla^2) + \Gamma_4(\nabla^2)^2](\nabla^\top \boldsymbol{v} + \nabla \boldsymbol{v}^\top)$$

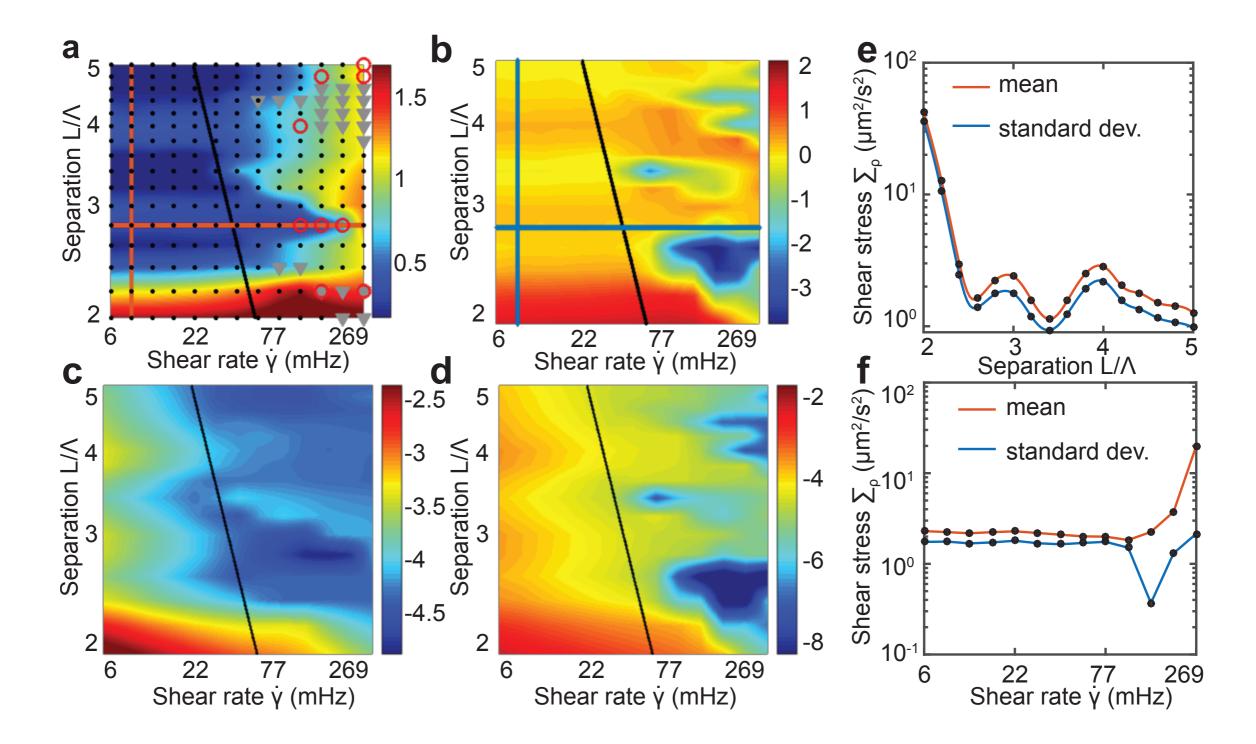
6th order PDE + no-slip + different types of higher order BC

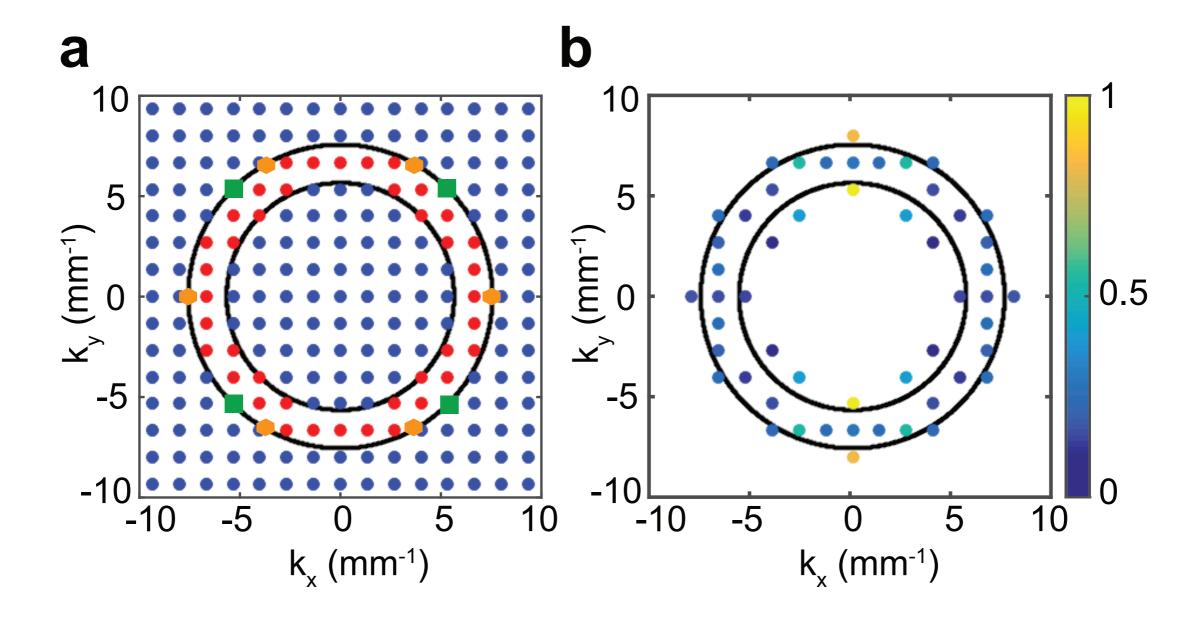


Preliminary numerical results







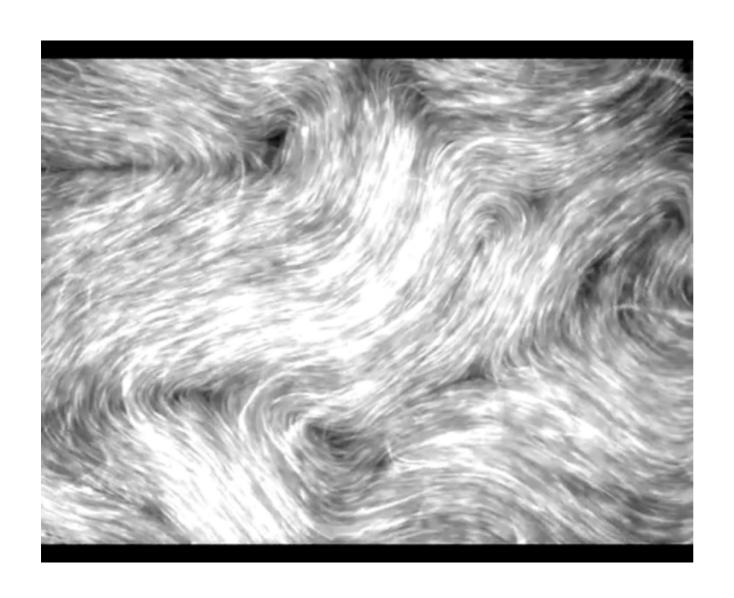


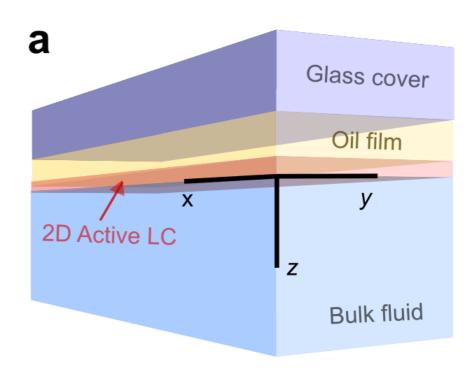
- effects of spatial dimensionality on individual microbial swimming (2D vs. 3D)
- intrinsic vortex scale selection in bacterial suspensions
- confinement & collective dynamics of quasi-2D suspensions (edge currents, magnetic order, quasi-"superfluidity", etc.)
- defect dynamics and long-range order in 2D planar/curved active nematics



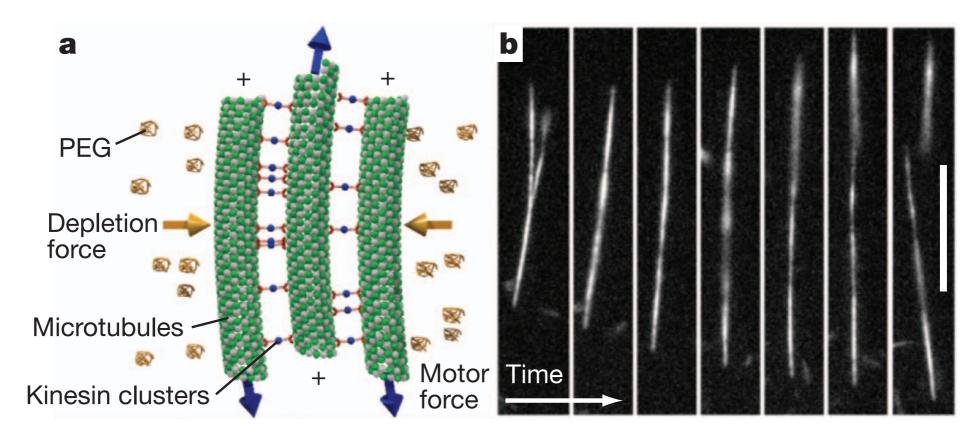
Spontaneous motion in hierarchically assembled active matter

Tim Sanchez¹*, Daniel T. N. Chen¹*, Stephen J. DeCamp¹*, Michael Heymann^{1,2} & Zvonimir Dogic¹





Active nematics



Dogic lab (Brandeis) Nature 2012

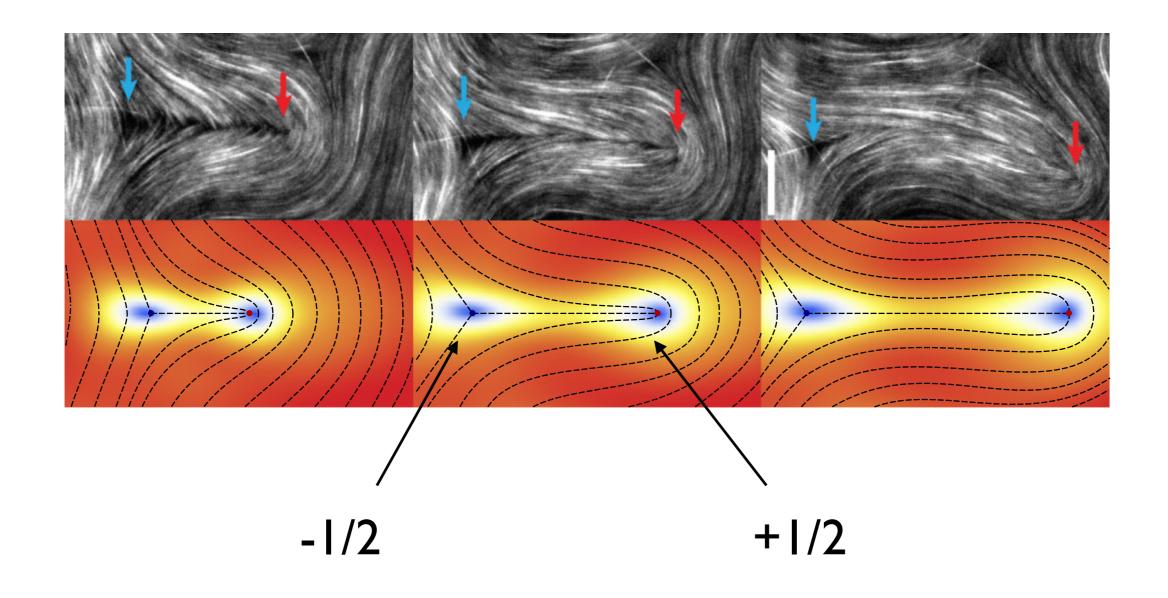
no head or tail \Rightarrow Q-tensor order-parameter

$$Q_{ij} = Q_{ji}$$
, $\operatorname{Tr} Q = 0$ $Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix}$.

$$\Delta = \sqrt{\lambda^2 + \mu^2}, \qquad \Lambda^{\pm} = \pm \Delta$$



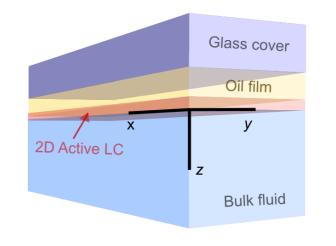
Active nematics



Giomi et al PRL 2012



Active LCs





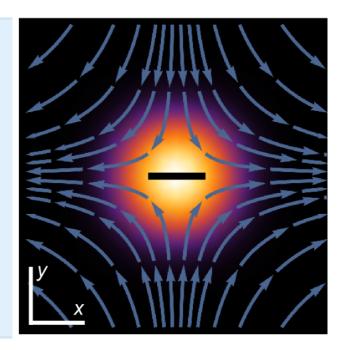
$$\partial_t Q + \nabla \cdot (\boldsymbol{v}Q) = -\frac{\delta \mathcal{F}}{\delta Q}$$

$$F = \text{Tr}\left\{-\frac{a}{2}Q^2 + \frac{b}{4}Q^4 - \frac{\gamma_2}{2}(\nabla Q)^2 + \frac{\gamma_4}{4}(\nabla \nabla Q)^2\right\}$$

Non-incompressible overdamped HD

$$-\eta \nabla^2 \boldsymbol{v} + \nu \boldsymbol{v} = -\zeta \nabla \cdot Q$$

$$\boldsymbol{v} = -D\nabla \cdot Q \qquad \qquad D = \zeta/\nu$$



arXiv:1507.01055

Complex representation

Generalization of analogy between smectic LCs and Abrikosov vortex (De Gennes 1972, Renn & Lubensky 1988, Pindak and co-workers 1990)

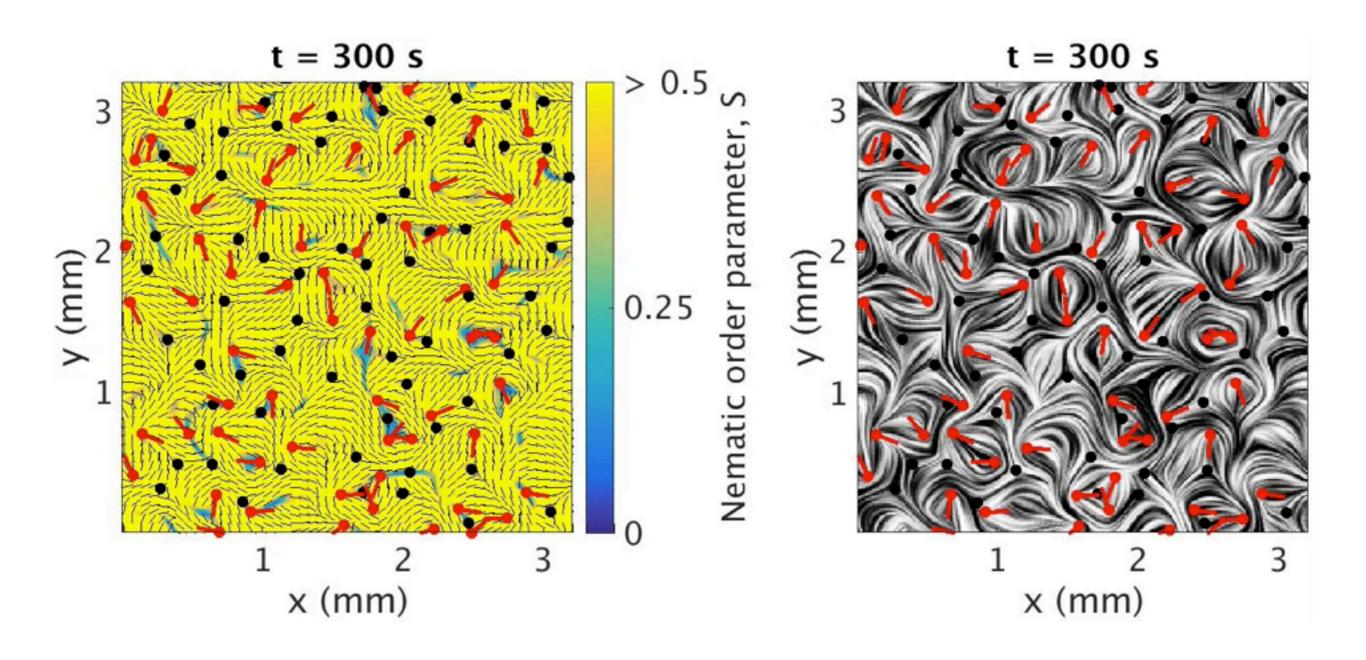
$$\partial_t Q - D\nabla \cdot [(\nabla \cdot Q)Q] = aQ - bQ^3 - \gamma_2 \nabla^2 Q - \gamma_4 (\nabla^2)^2 Q$$

$$Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix} \qquad \psi(t, z) = \lambda + i\mu \qquad \qquad \partial = \frac{1}{2}(\partial_x - i\partial_y)$$

$$\partial_t \psi - 4D \Re\{(\partial^2 \psi) + (\partial \psi)\partial\} \psi = \left(\frac{1}{4} - |\psi|^2\right) \psi - \gamma_2 (4\bar{\partial}\partial)\psi - (4\bar{\partial}\partial)^2 \psi$$

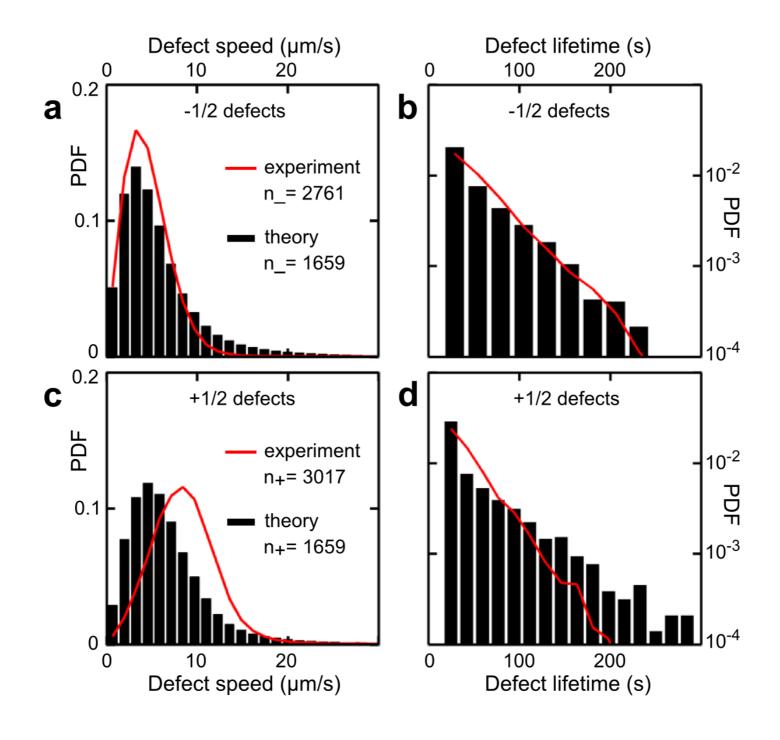
Generalized Gross-Pitaevski equation with double-well dispersion

Chaotic phase



arXiv:1507.01055

Experiment vs. theory



Experimental data kindly provided by Zvonimir Dogic and Steve DeCamp

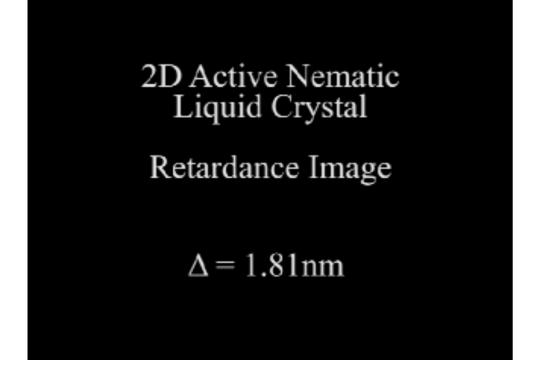
Open problem: longe-range ordering

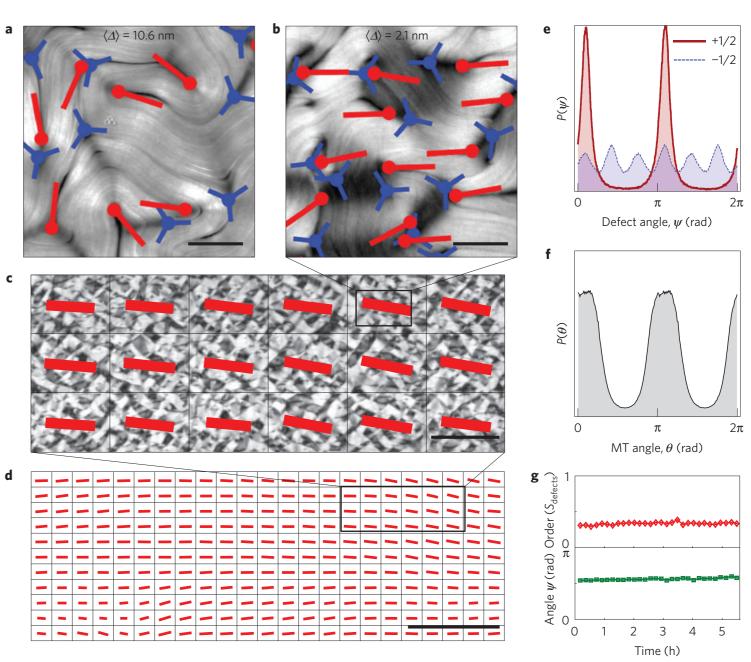


nature materials

Orientational order of motile defects in active nematics

Stephen J. DeCamp[†], Gabriel S. Redner[†], Aparna Baskaran, Michael F. Hagan* and Zvonimir Dogic*



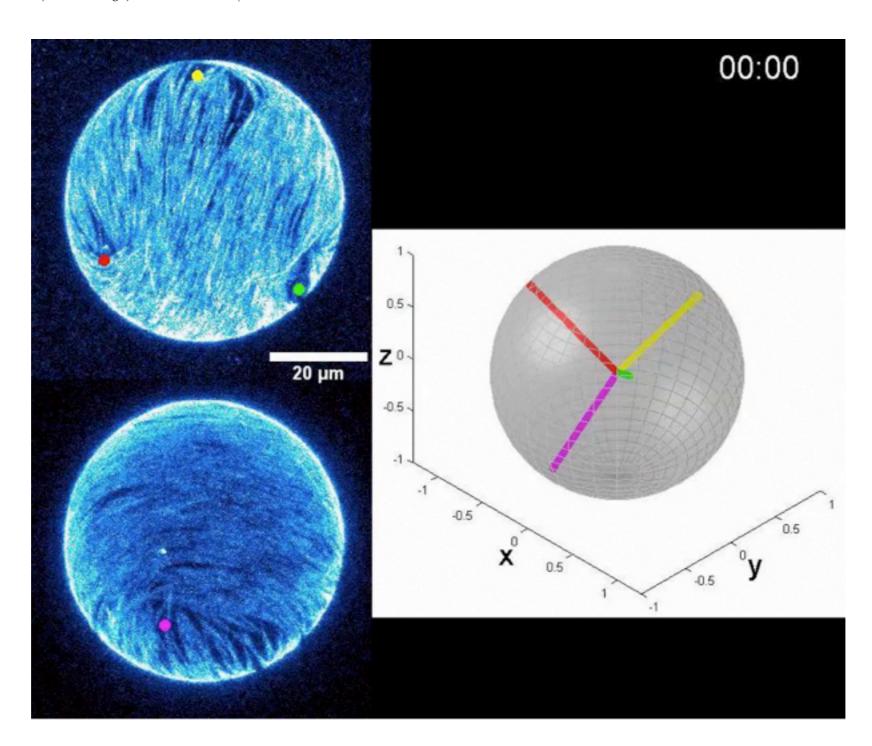


RESEARCH ARTICLES

DYNAMIC ORDERING

Topology and dynamics of active nematic vesicles

Felix C. Keber, 1,2* Etienne Loiseau, 1* Tim Sanchez, 3* Stephen J. DeCamp, 3 Luca Giomi, 4,5 Mark J. Bowick, 6 M. Cristina Marchetti, 6 Zvonimir Dogic, 2,3 Andreas R. Bausch 1†

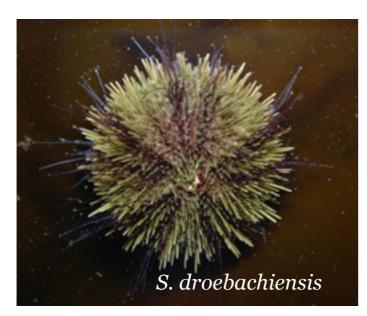


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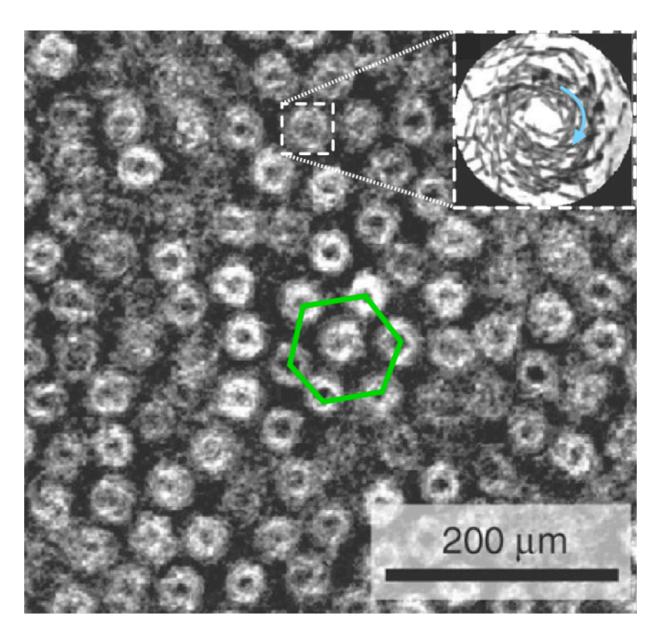
Surface interactions



Sea urchin sperm







Riedel et al (2005) Science