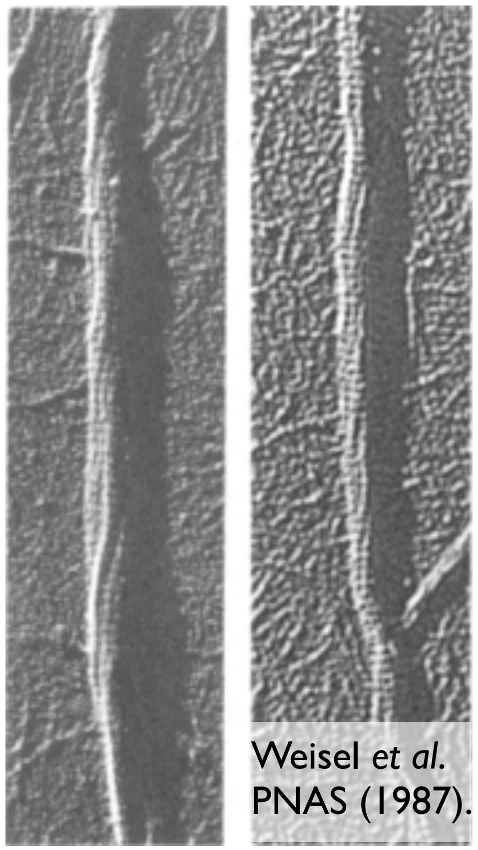
Surveying the metric geometry of filamentous matter:
Defects and instabilities of incompatible bundles

collagen fibrils

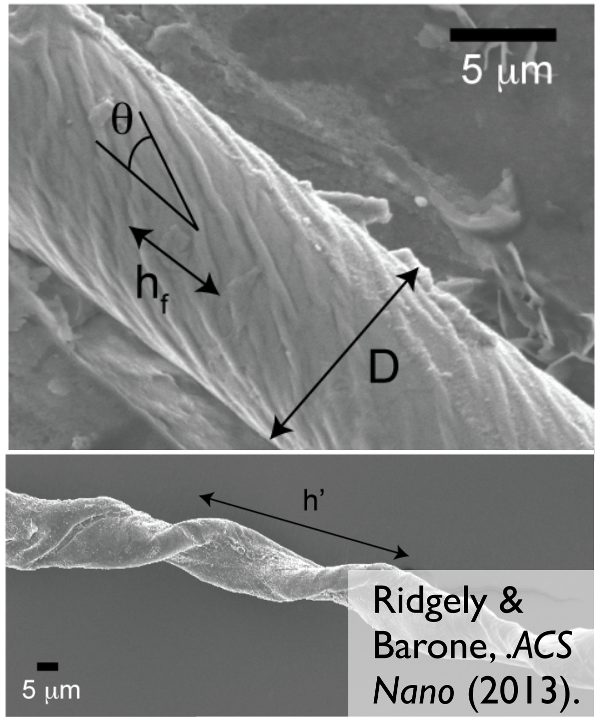
Ottani *et al.* Micron. (2002).

fibrin bundles



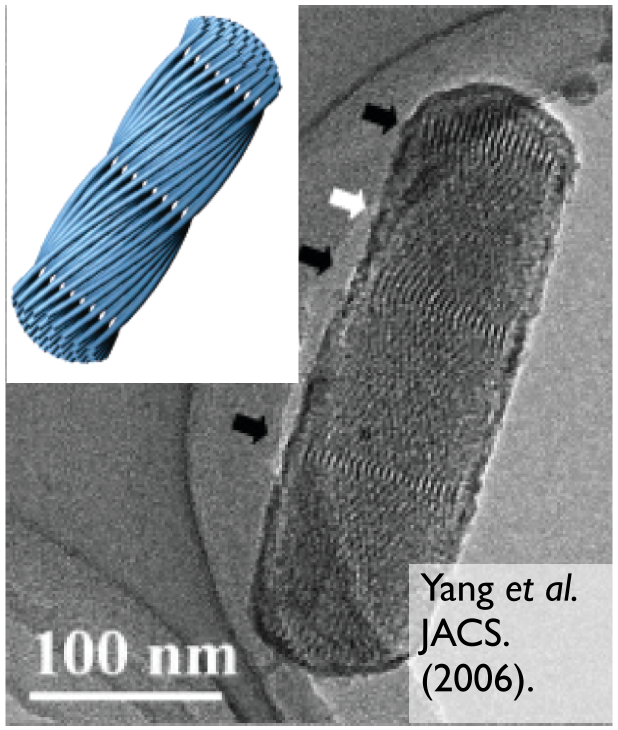
Weisel *et al.* PNAS (1987).

amyloid fibrils



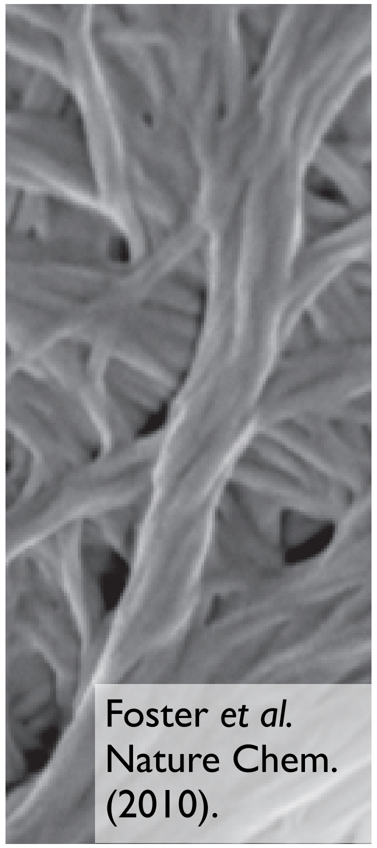
Ridgely & Barone, *ACS Nano* (2013).

mesoporous silica (columnar micelles)



Yang *et al.* JACS. (2006).

organogel fibers



Foster *et al.* Nature Chem. (2010).

Isaac Bruss, Amir Azadi, Doug Hall & Gregory M. Grason
Department of Polymer Science & Engineering
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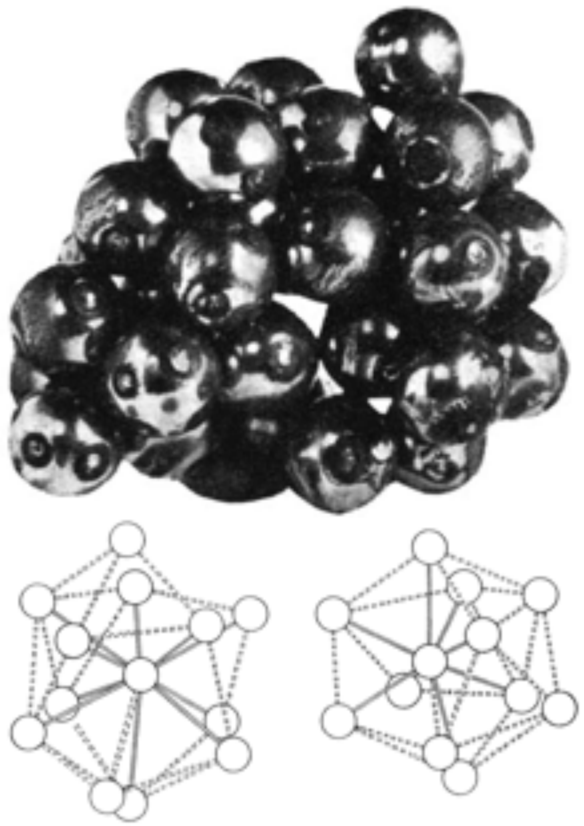
http://www.pse.umass.edu/ggrason

NSF CAREER DMR 09-55760; Alfred P. Sloan Foundation;
UMass Center for Hierarchical Manufacturing (NSF NSEC)

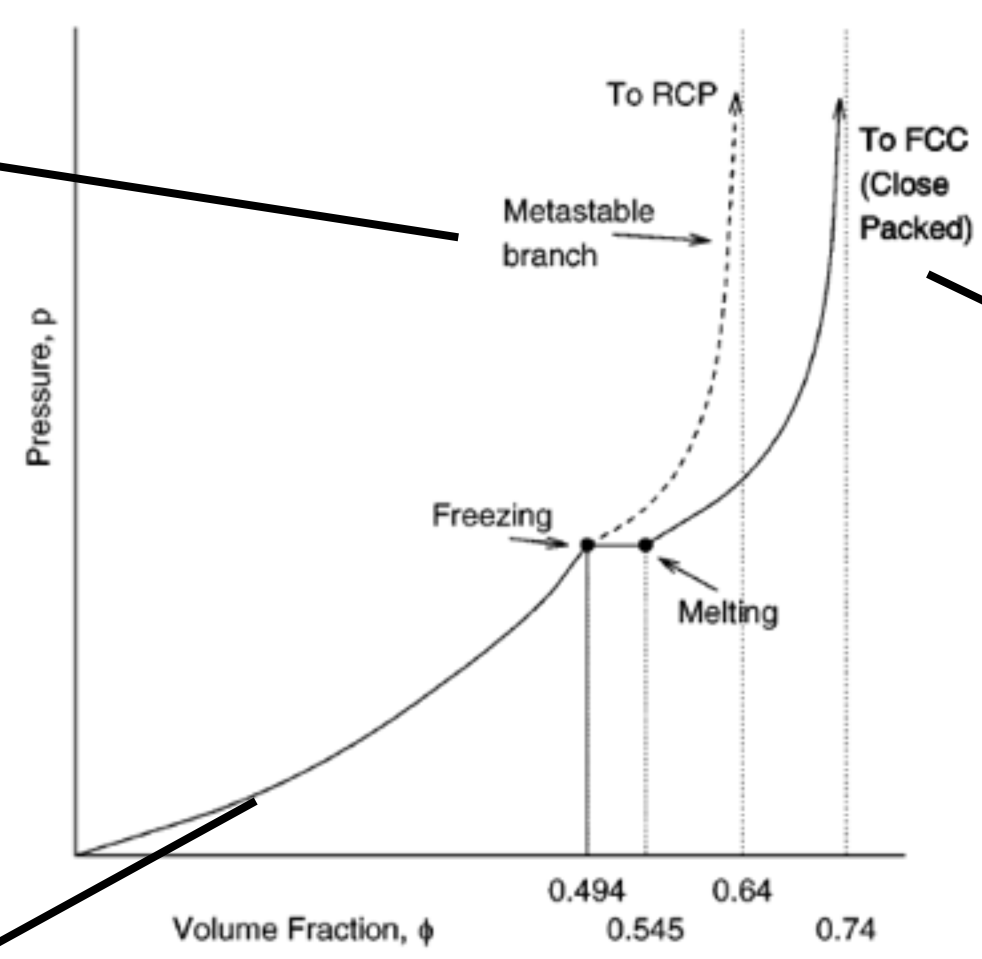
Models of matter: Spheres from Kepler to colloids

random-close packing:
from dense liquids to glassiness



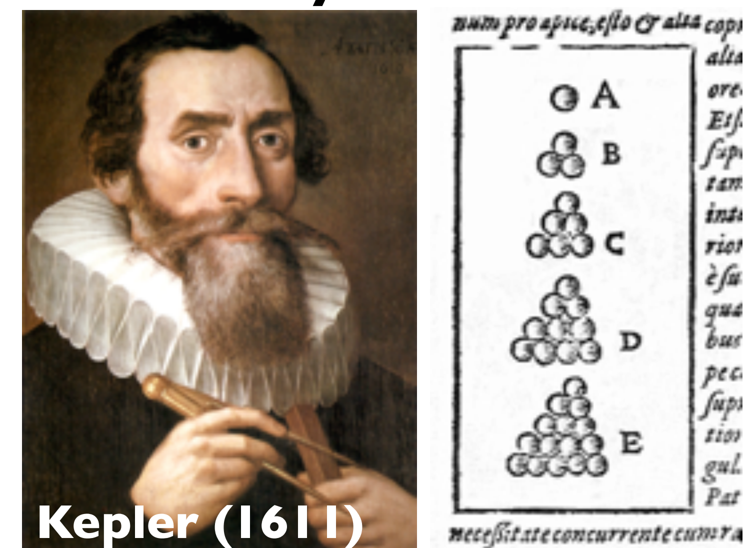
Bernal, *Nature* (1960).

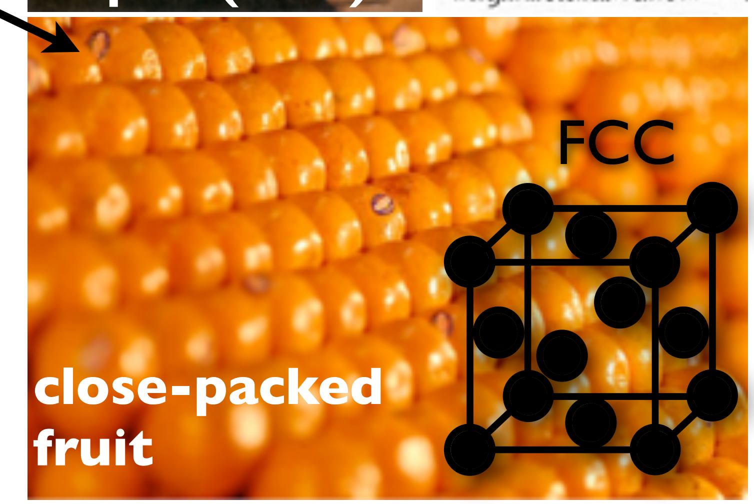
equation of state hard spheres

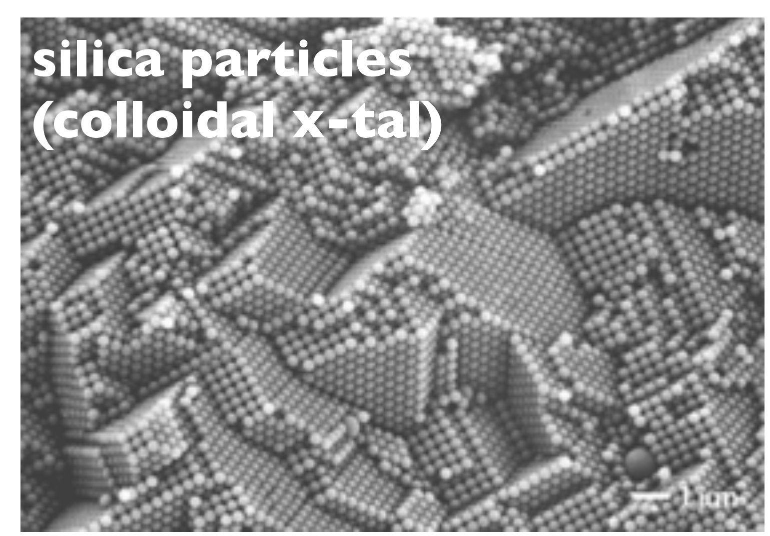


Rintoul & Torquato, *PRL* (1996).

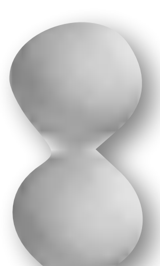
sphere packing, optimal lattices
& crystallization



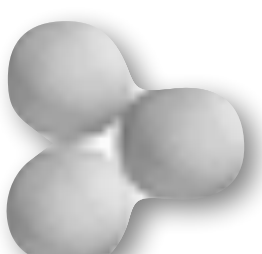




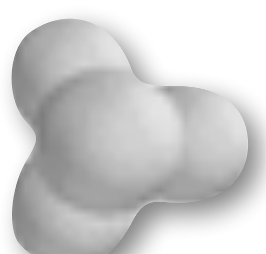
***N*-body cluster geometry**
& viral expansion



2-sphere



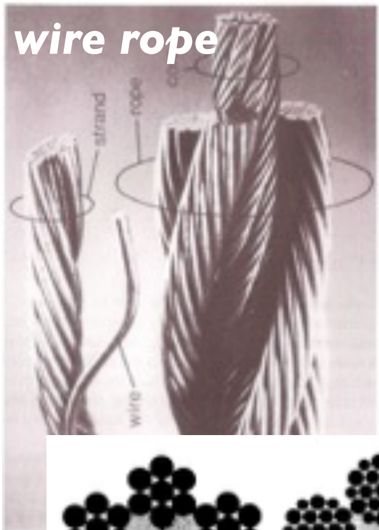
3-sphere

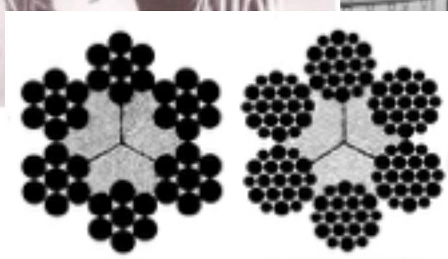


4-sphere

Models of matter: Filamentous matter from Galileo to nanomaterials

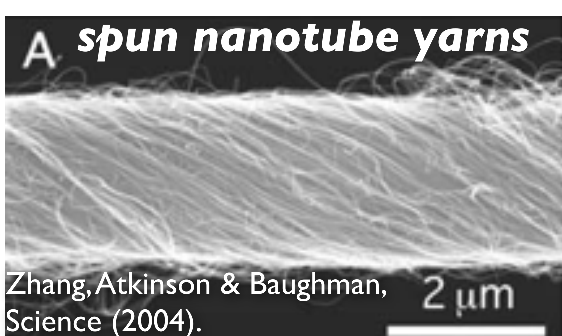
macroscale, manufactured materials





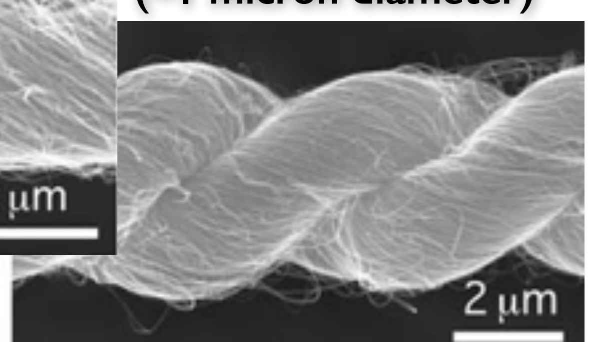
(~0.1-1m diameter)

(~0.1-1cm diameter)



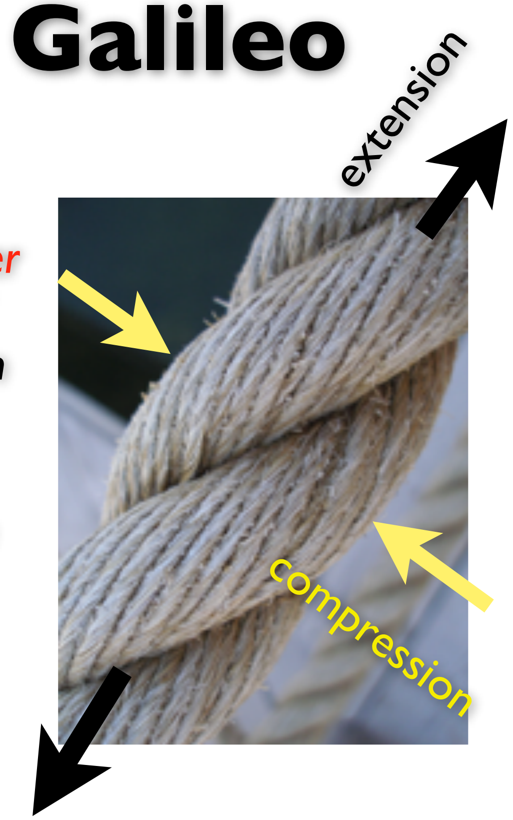
Zhang, Atkinson & Baughman, Science (2004).

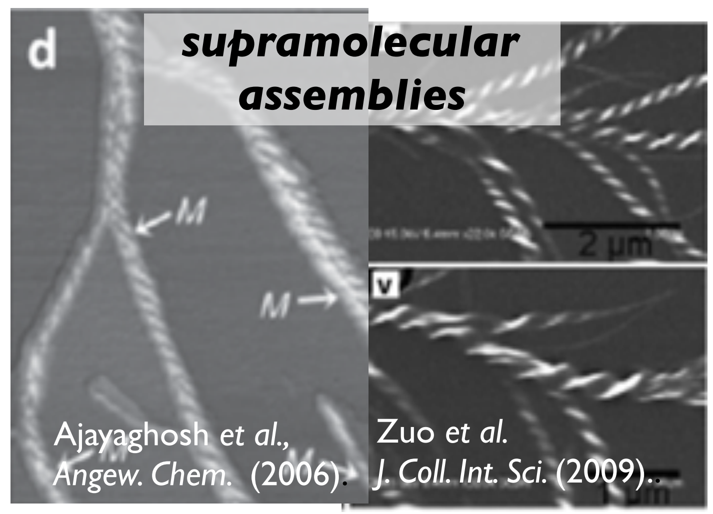
(~1 micron diameter)



"But in the case of the rope, the very act of twisting causes the threads to bind one another in such a way that when the rope is stretched with a great force, the fibers break rather than separate from each other."

Galileo, *Strength of Materials* (1638)



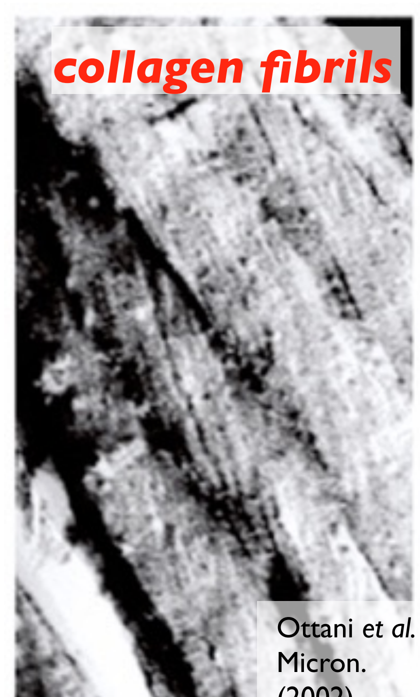


Ajayaghosh *et al.*, *Angew. Chem.* (2006).

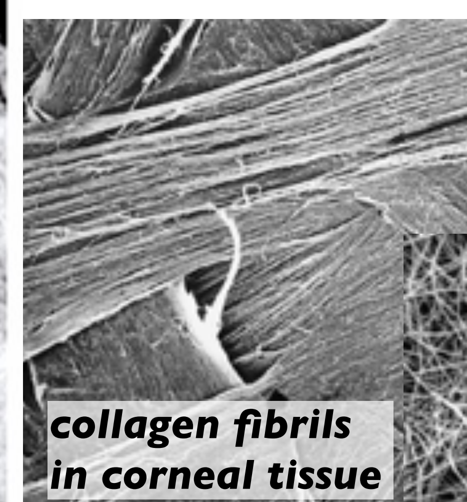
Zuo *et al.* *J. Coll. Int. Sci.* (2009).

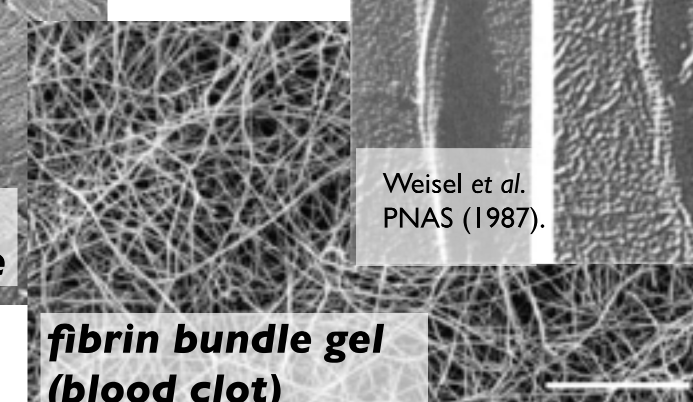
nanoscale self-assembled materials

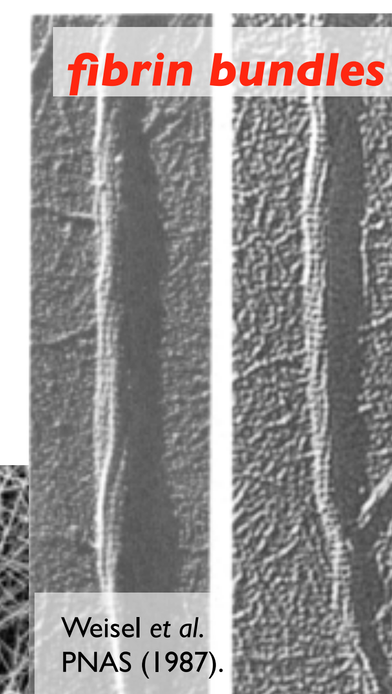
(~10-1000 nm "rope" diameter; ~1-10 nm "filament" diameter)



Ottani *et al.* Micron. (2002).







Weisel *et al.* PNAS (1987).

Generic challenges to understanding cohesive filament assembly

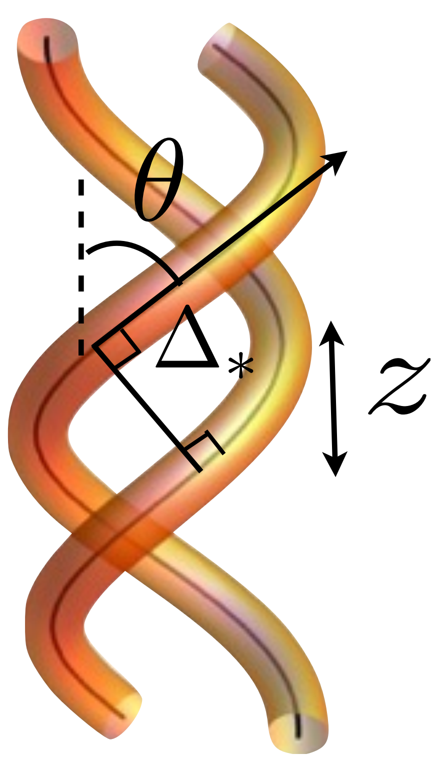
1) Extreme aspect ratio (*L/d* >> 1) ➙ *extreme* *flexibility* relative to *inter-filament cohesion*

$$E_{mech} \approx \frac{B}{2} L\kappa^2 \quad (B \sim d^4)$$

bending stiffness

$$E_{coh} \approx -\epsilon L$$

2) "Distance" and contact are *non-local* ➙ coupling between *orientation* & *spacing*



Distance of closest approach:

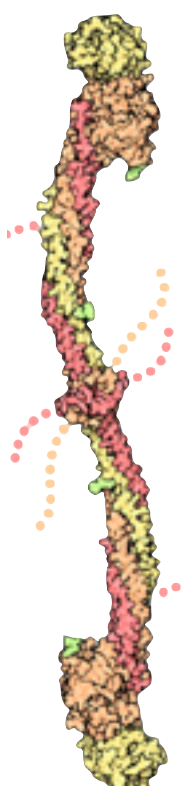
$$\Delta_* = \min_z [\Delta(z)]$$

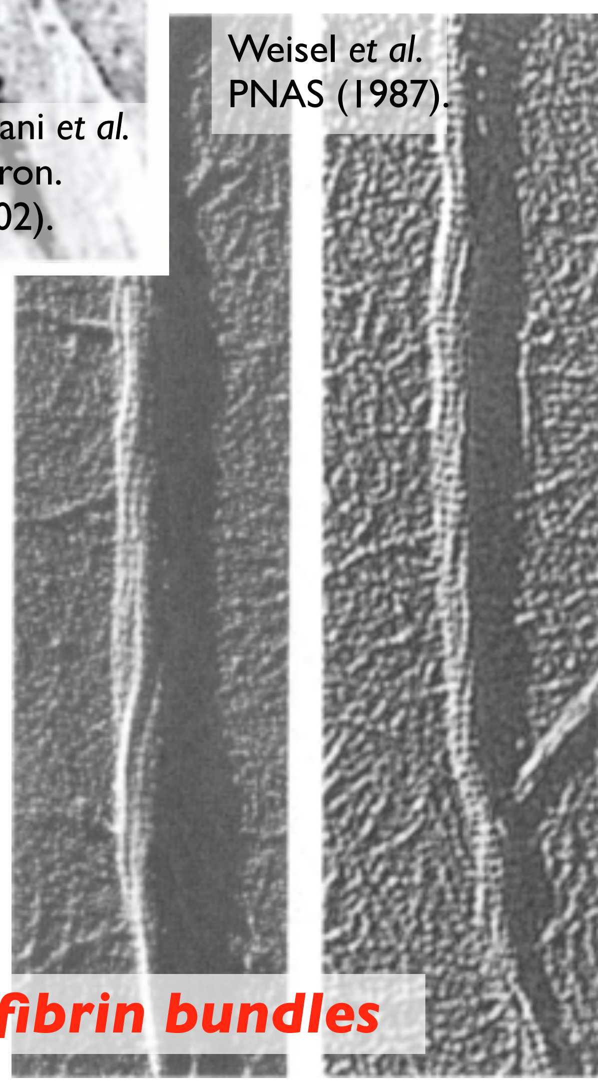
Broad Questions:

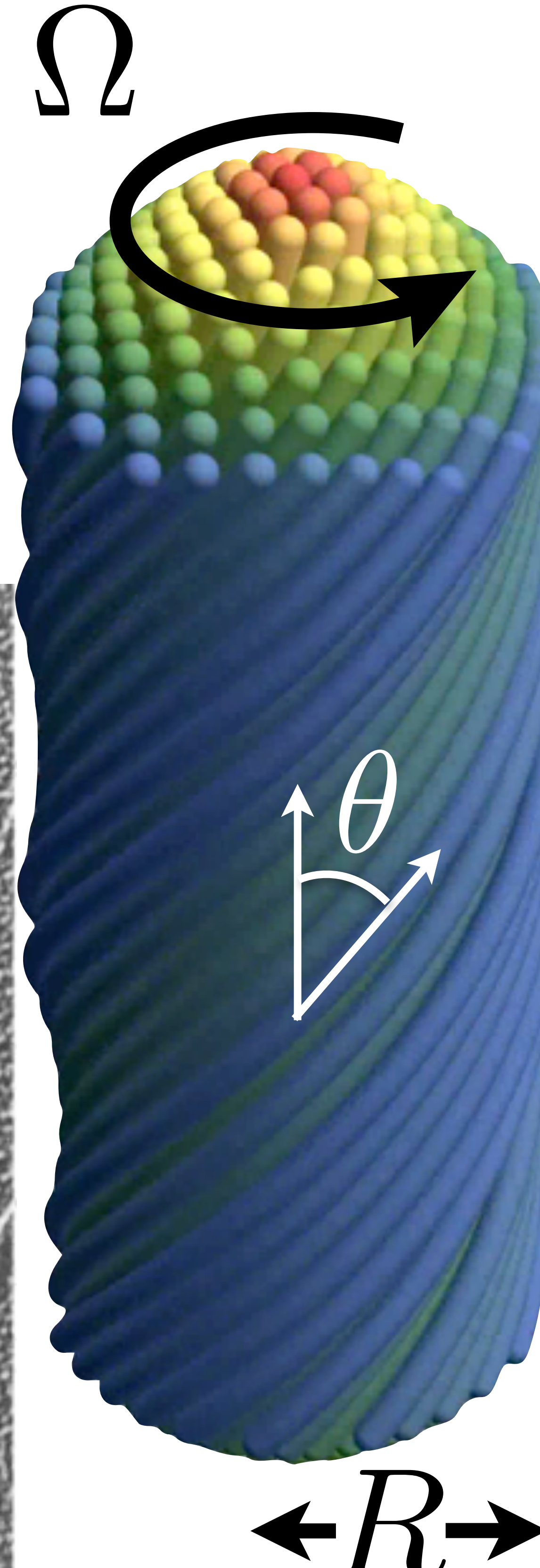
How does 3D shape (e.g. **twist, bend**) of a multi-filament assembly (e.g. bundle) influence structure & energetics of lateral order?

What are optimal packings for non-trivial bundle geometry? What determines these?

Structure & assembly of twisted, cohesive bundles







Motivations:

1) "Self-twisting" *chiral filament* bundle: common structural motif of biofilament assemblies

2) Twist: Simplest, non-trivial example of coupling between filament tilt and spacing

R - radius

Ω - helical rotation rate (2π/pitch)

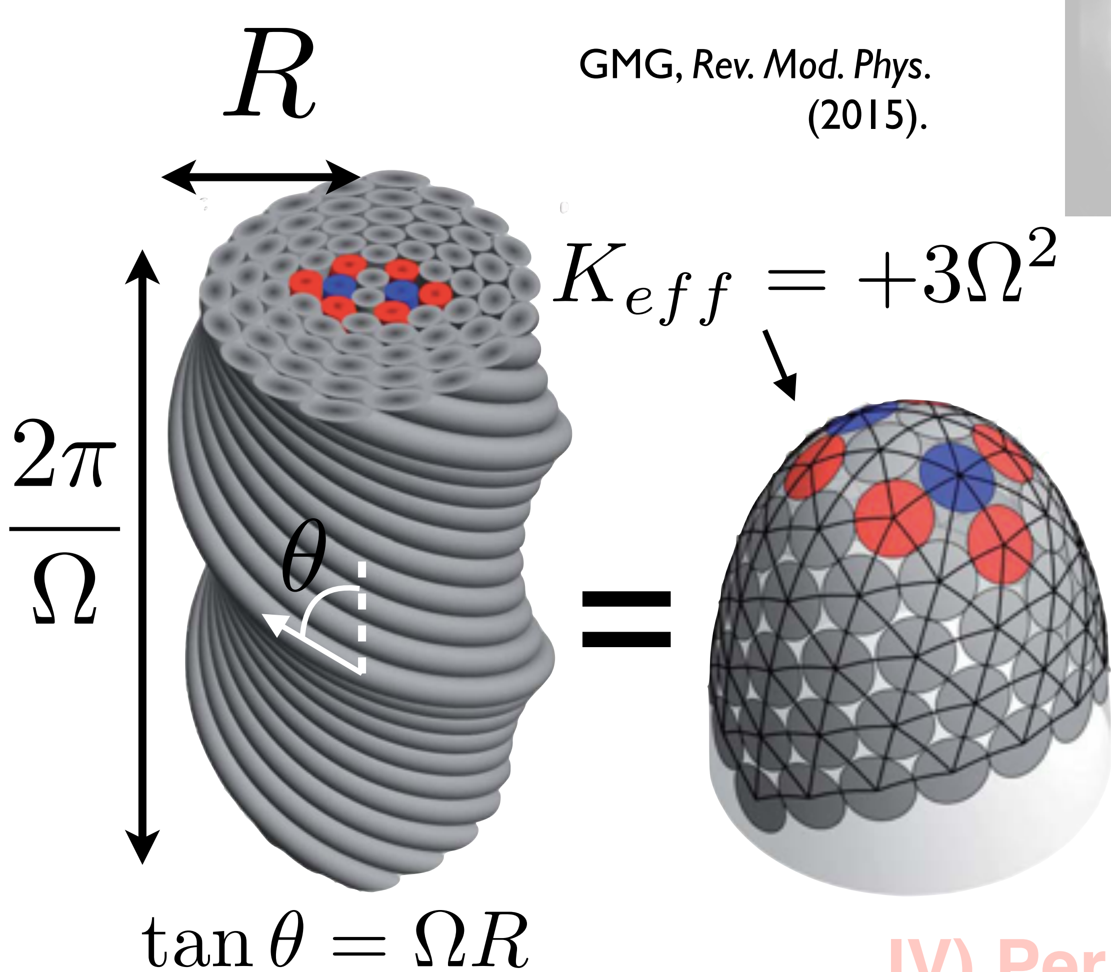
$$\tan\theta = \Omega R$$

θ - helical tilt angle

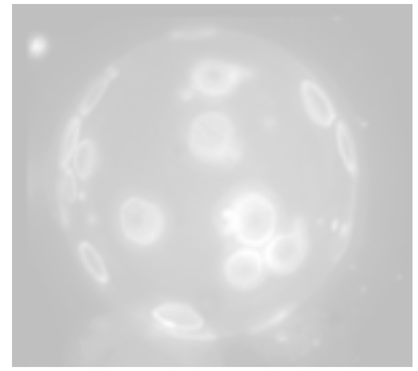
Twisted bundles: non-Euclidean geometry & anomalous assembly

I) Non-euclidean "metric" geometry of bundles

GMG, *Rev. Mod. Phys.* (2015).

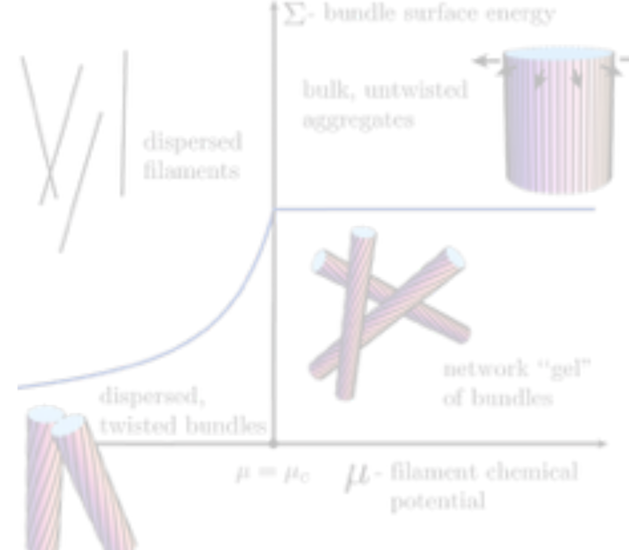


solid domains on lipid vesicles



Bandekar & Sofou, Langmuir (2012)

chiral filament assembly

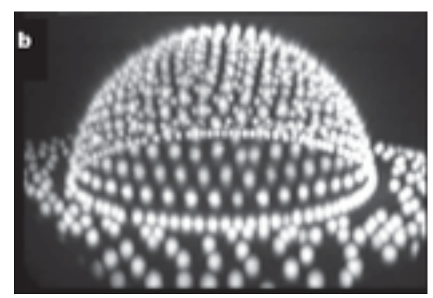


II) Self-limiting Assembly

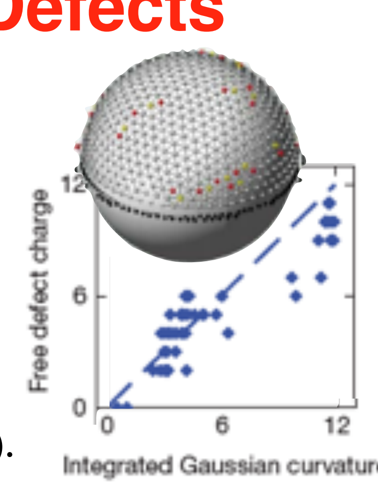
GMG & Bruinsma, PRL (2007); GMG, PRE (2009)

III) Topological Defects

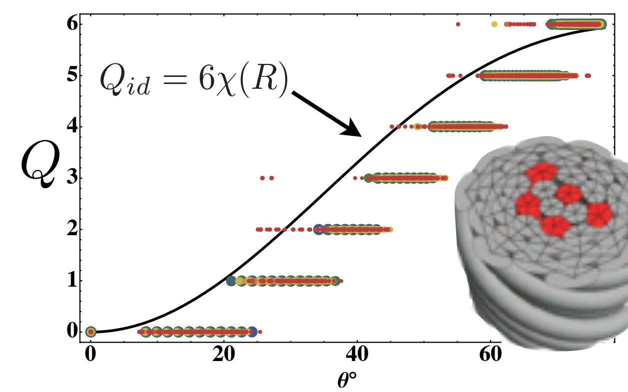
defects in curved crystals



Irvine *et al.*, *Nature* (2010).



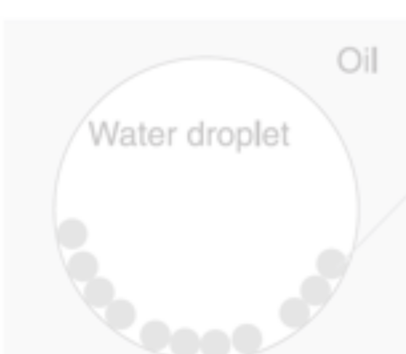
defects in twisted bundles



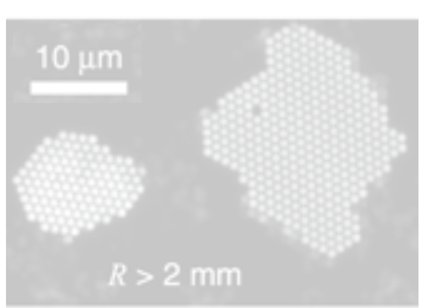
Bruss & GMG, *PNAS* (2012); *Soft Matter* (2013).

IV) Perimeter Instability & Anisotropic Domains

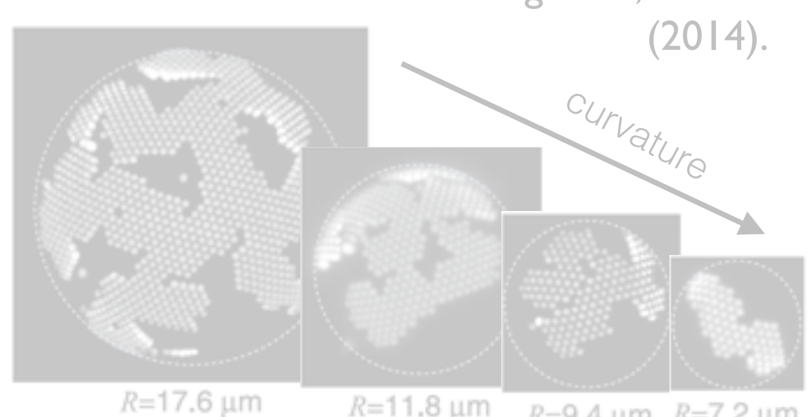
colloidal crystals on spherical droplets



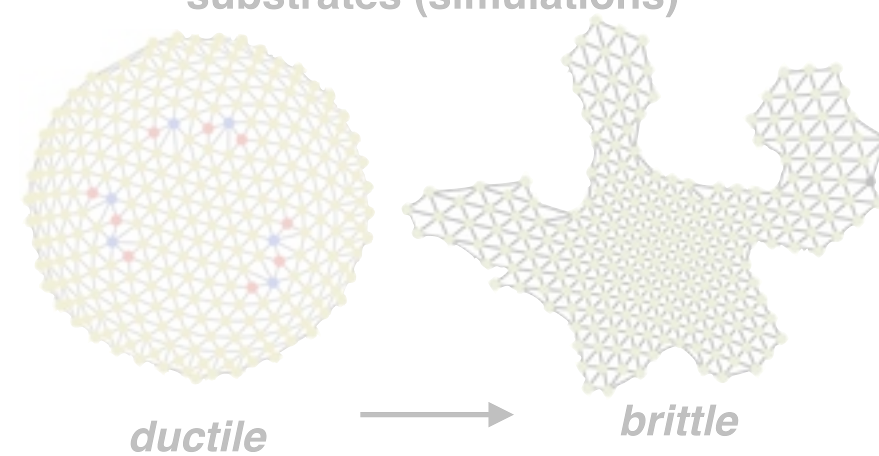
large droplets



Meng *et al.*, *Science* (2014).



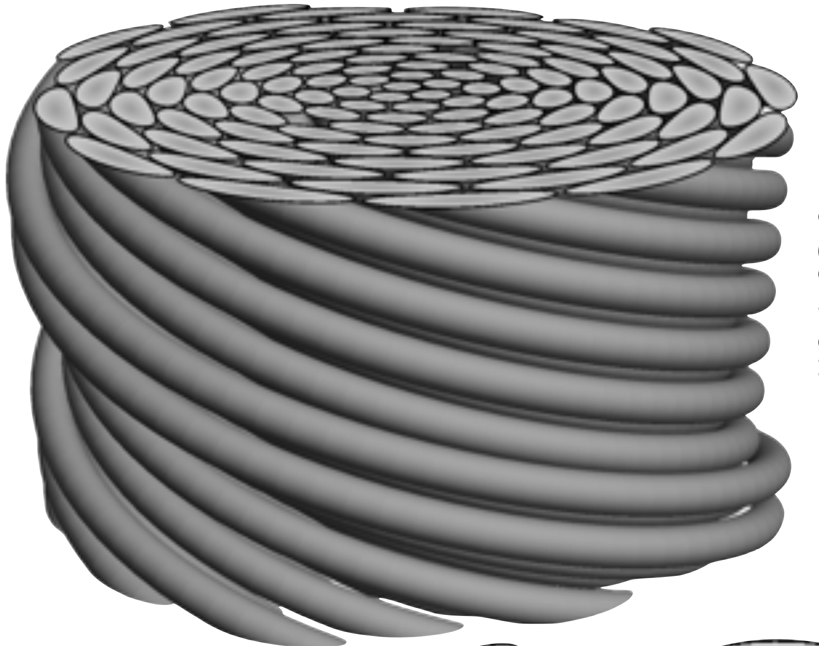
cohesive membranes on spherical substrates (simulations)



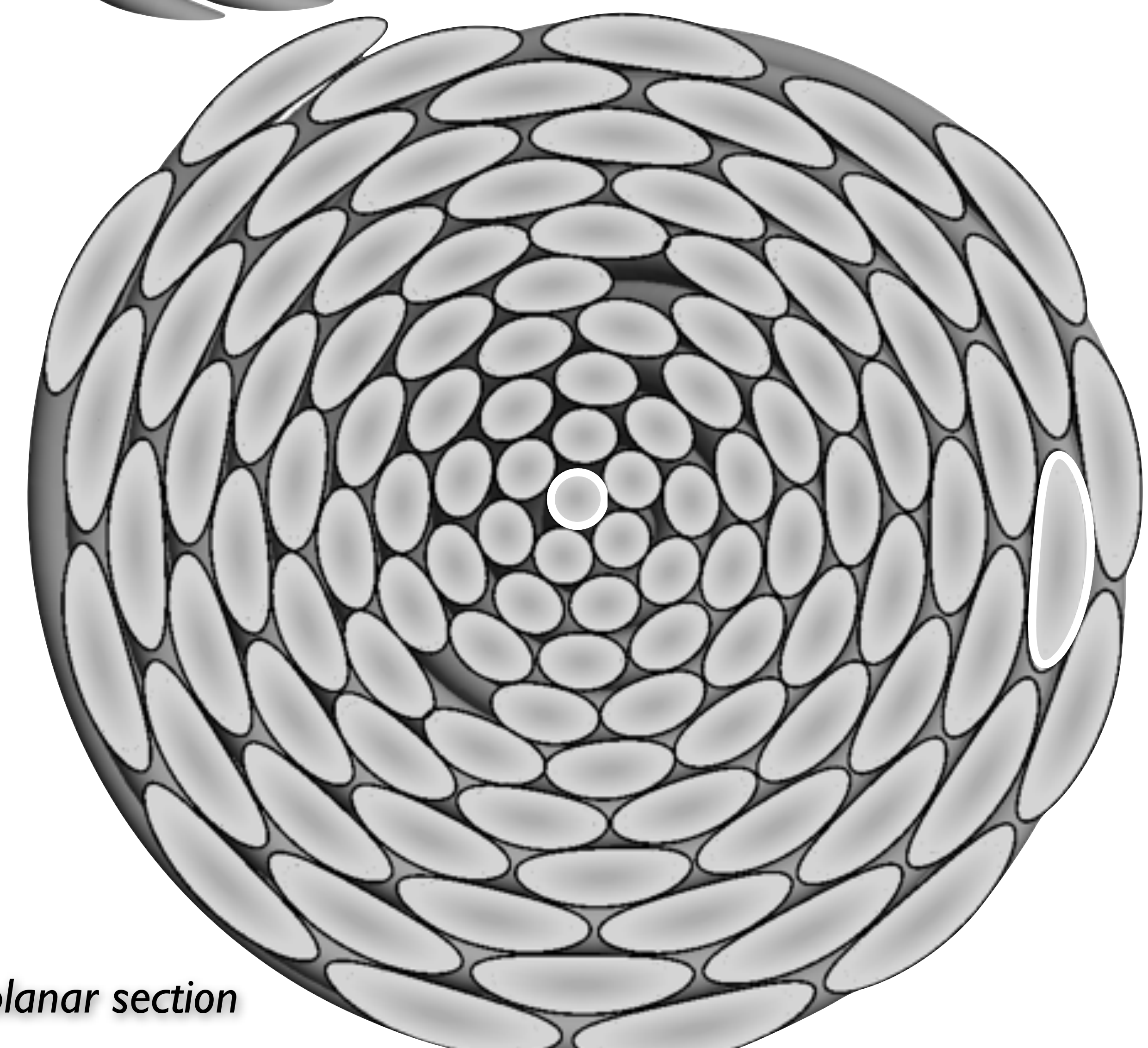
Amir Azadi, *to be published*.

Mapping frustration in filament packing

cross-section of twisted bundle



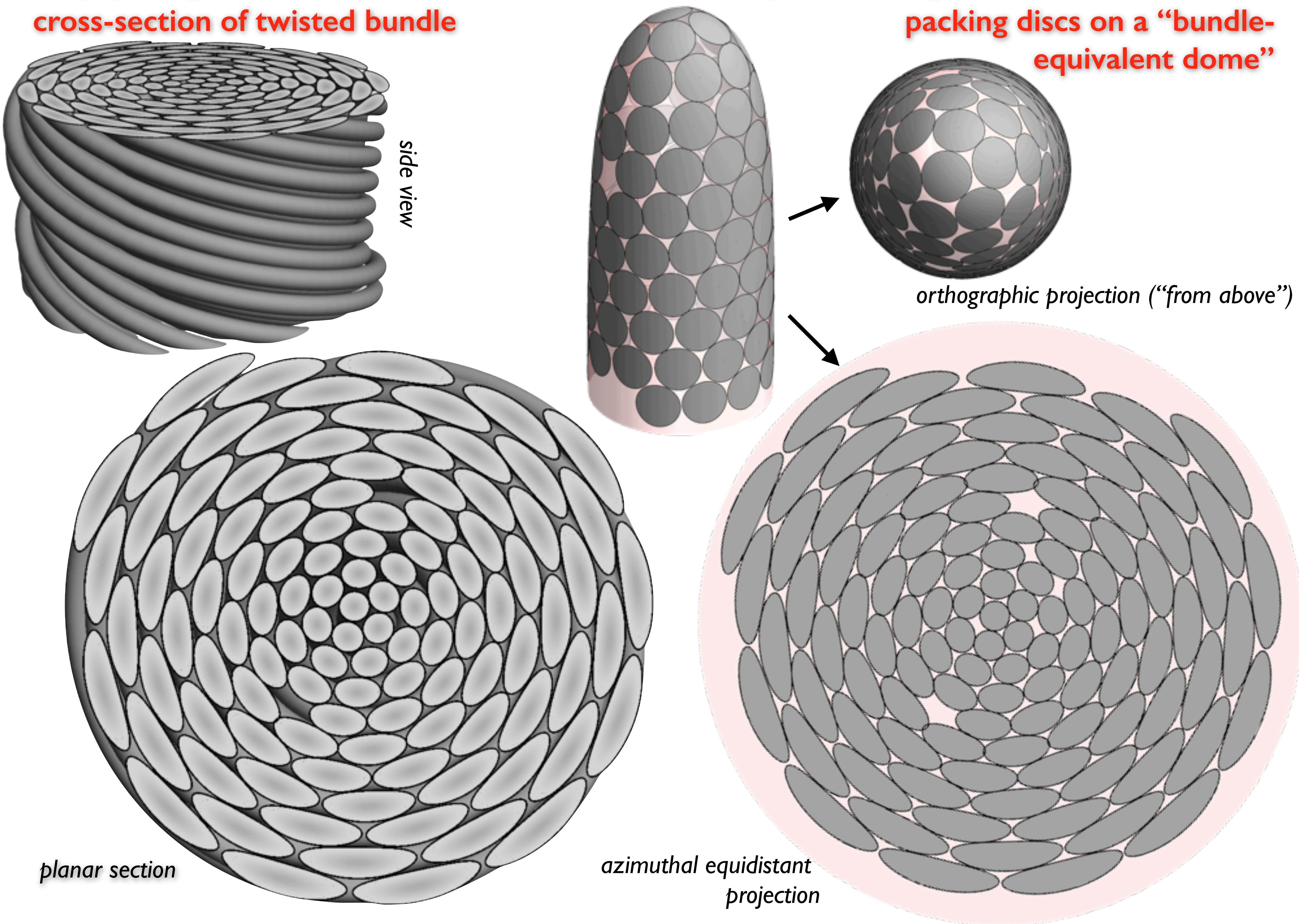
side view



planar section

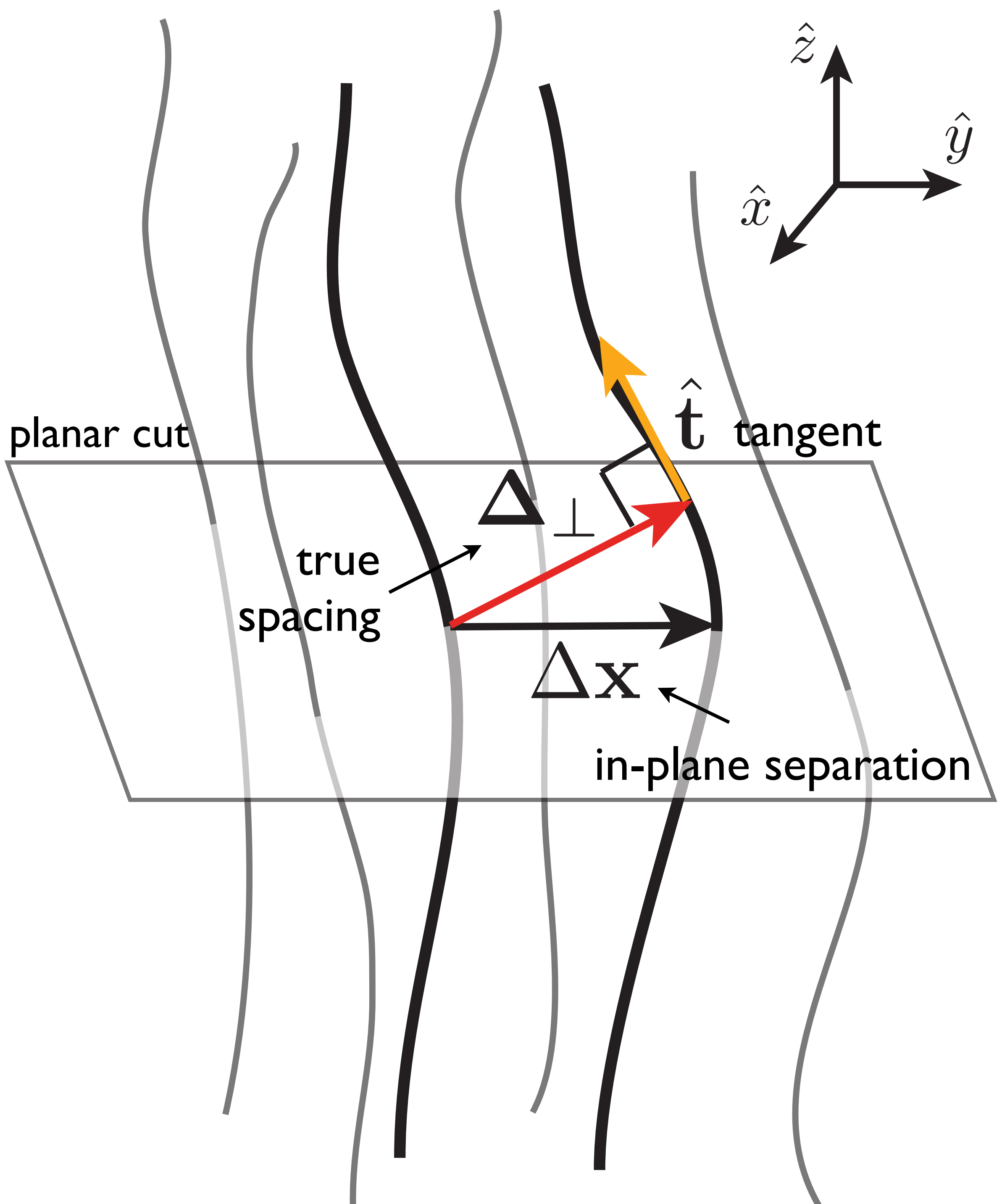
packing discs on sphere
(orthographic projection)

Mapping frustration in filament packing



Geometric connection between filament arrays & 2D sheets

GMG, *Rev. Mod. Phys.* (2015).



infinitesimal filament spacing

in-place coords.

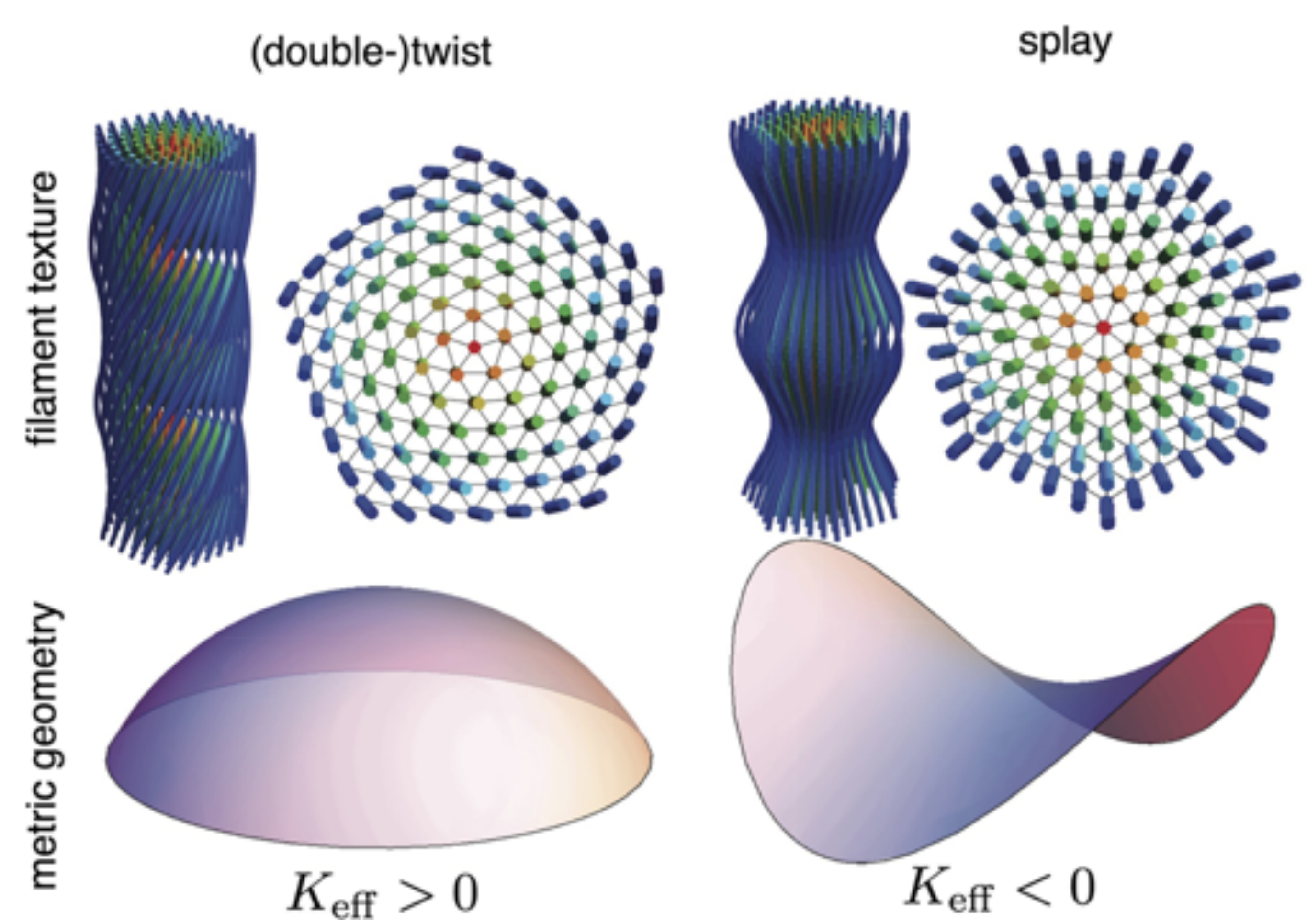
$$|d\Delta_\perp|^2 = g_{ij}(\mathbf{x})dx_i dx_j$$

inter-filament (2D) metric:

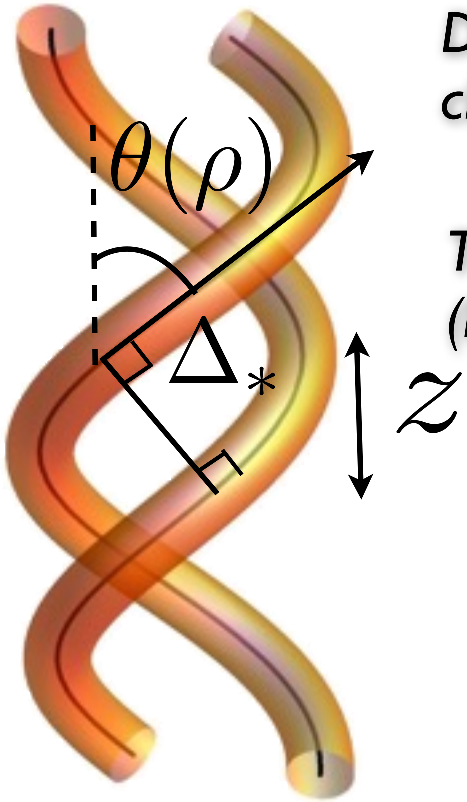
$$g_{ij}(\mathbf{x}) = \delta_{ij} - t_i(\mathbf{x})t_j(\mathbf{x})$$

Gaussian curvature (filament metric):

$$K_{\rm eff} \simeq \frac{1}{2}(\partial_x^2 t_y^2 + \partial_y^2 t_x^2 - 2\partial_x\partial_y t_x t_y)$$



Mapping frustration in filament packing



Distance of closest approach: $\Delta_* = \min_z \left[\Delta(z)\right]$

Tilt angle (local): $\sin\theta(\rho) = \dfrac{\Omega\rho}{\sqrt{1+(\Omega\rho)^2}}$

Distance between helical curves:

$$\Delta^2(z) = \rho_1^2 + \rho_2^2 - 2\rho_1\rho_2\cos\left(\Omega z + \phi_1 - \phi_2\right) + z^2$$

Metric geometry of bundles:

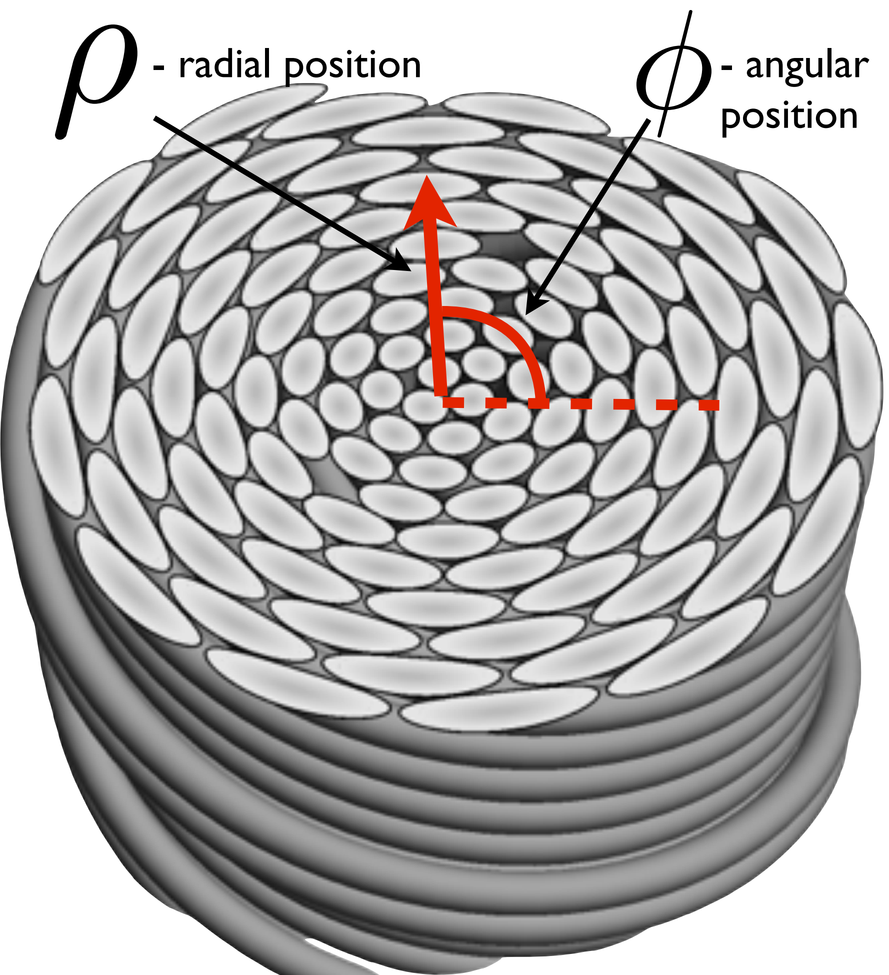
$$\lim_{\delta\phi\to 0,\delta\rho\to 0} \Delta_*^2 \equiv ds^2 = (\delta\rho)^2 + \Omega^{-2}\sin^2\theta(\rho)(\delta\phi)^2$$

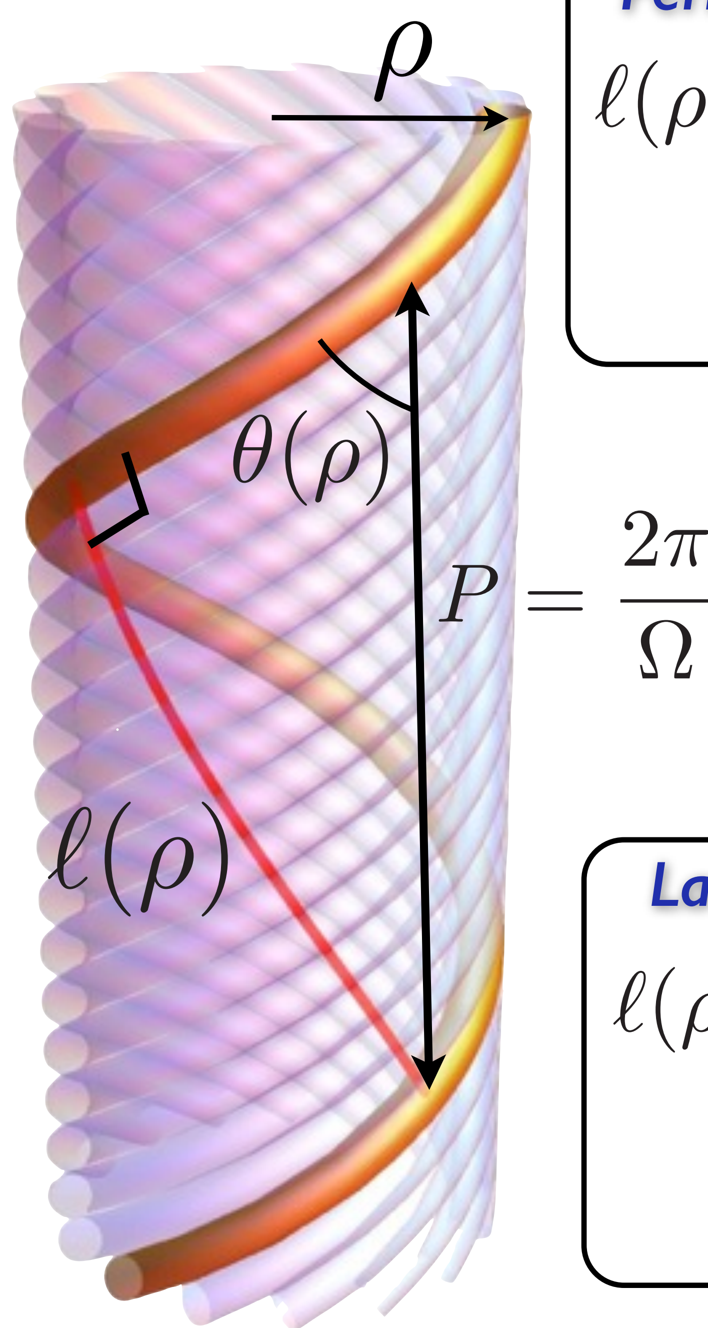
Perimeter: space available in bundle @ ρ

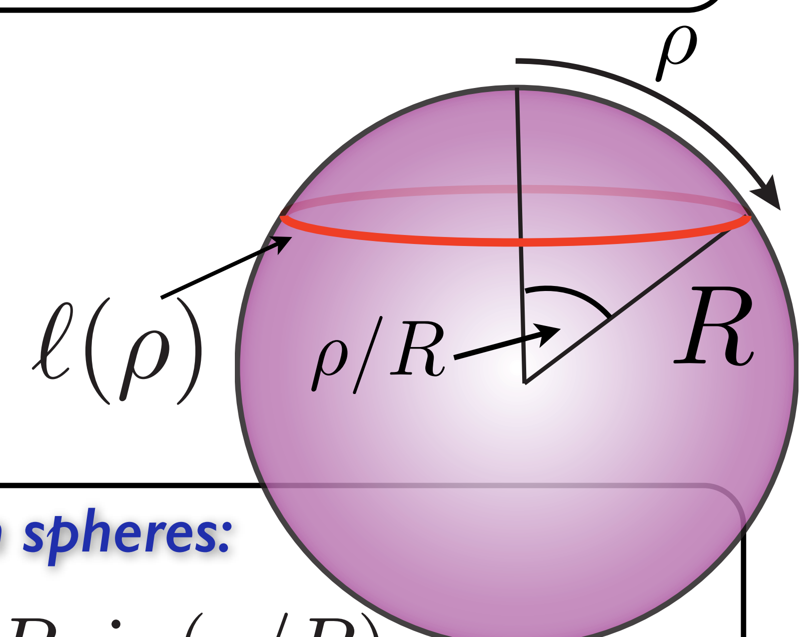
$$\ell(\rho) = P\sin\theta(\rho)$$

$$\simeq 2\pi\rho\Big(1 - \frac{\Omega^2}{2}\rho^2\Big)$$

less than planar packing!







Latitudes on spheres:

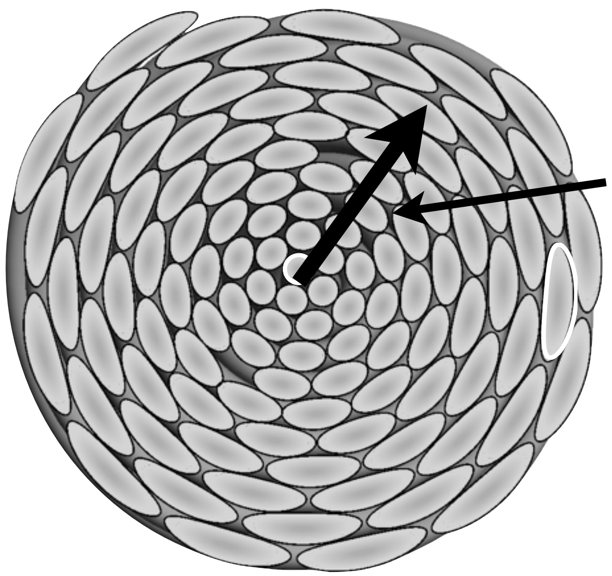
$$\ell(\rho) = 2\pi R\sin(\rho/R)$$

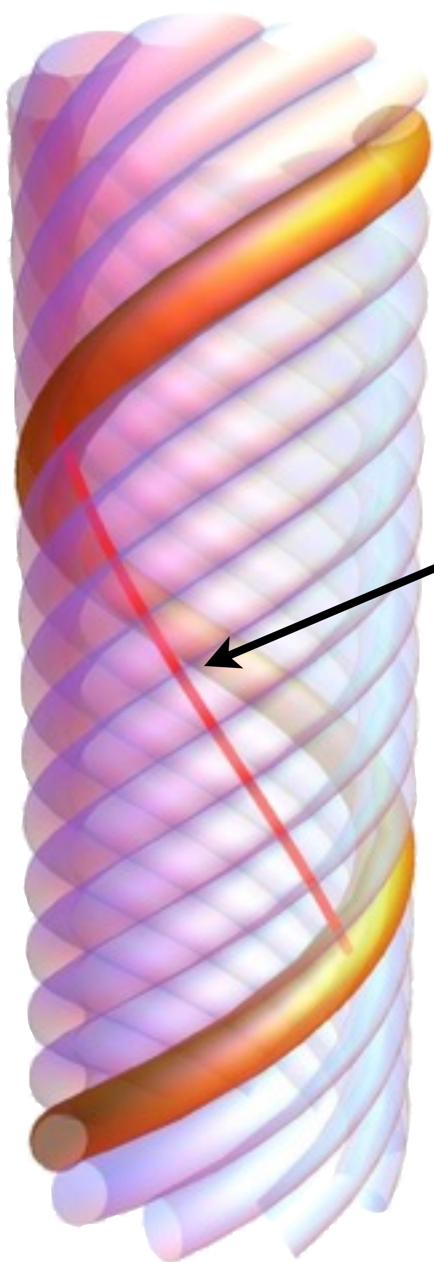
$$\simeq 2\pi\rho\Big(1 - \frac{1}{6R^2}\rho^2\Big)$$

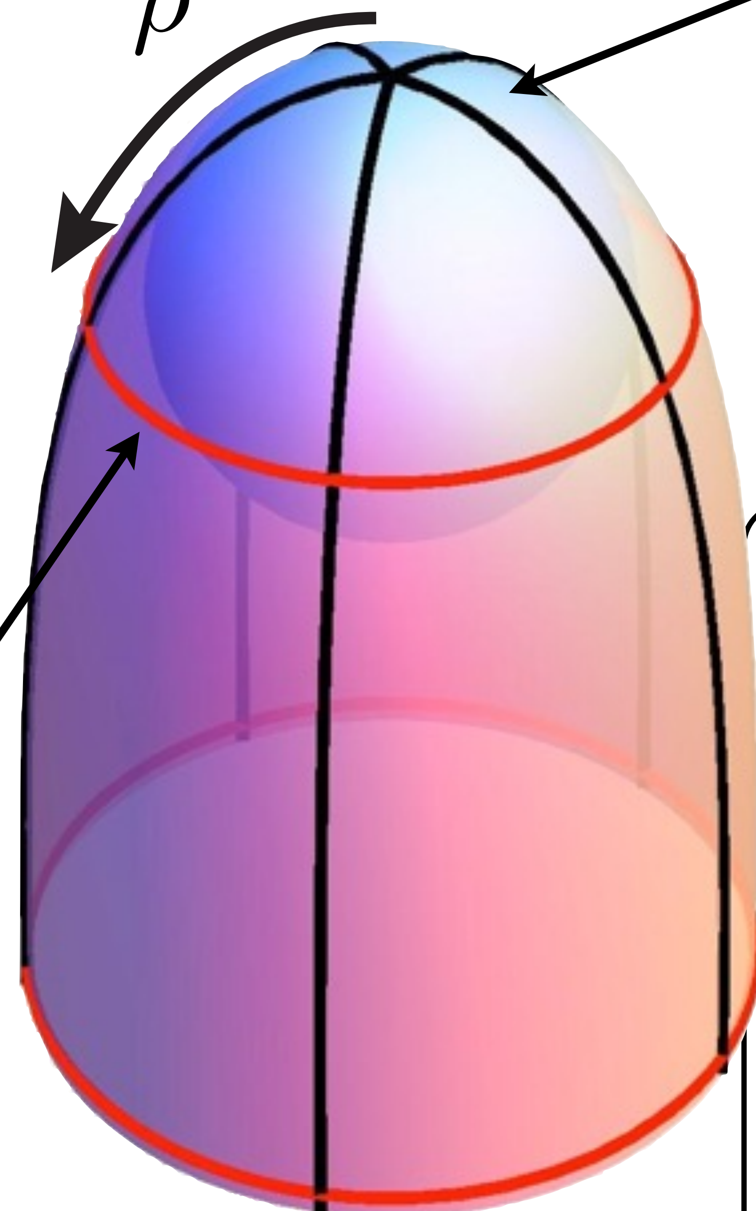
Bruss and GMG, *PNAS* (2012)

Frustration in filament packing: hidden geometry

Twisted bundle packing is equivalent to packing on curved surface!







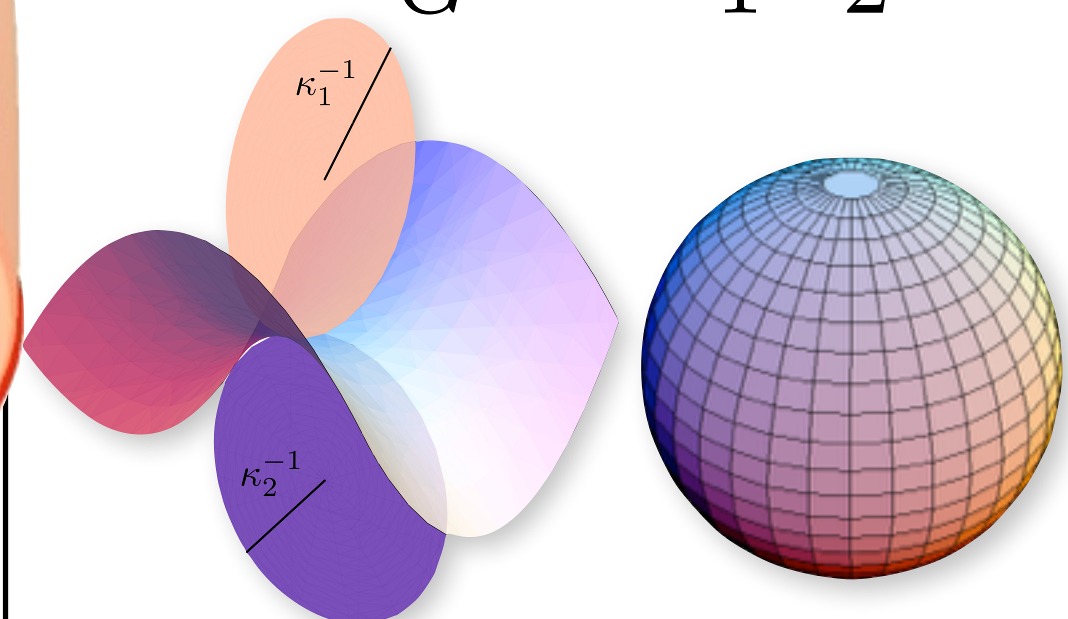
spherical radius $= \frac{\Omega^{-1}}{\sqrt{3}}$

$$K_G = \frac{3\Omega^2}{\left[1 + (\Omega\rho)^2\right]^2}$$

Gaussian curvature of "dual surface" to twisted filament bundle

cylindrical radius $= \Omega^{-1}$

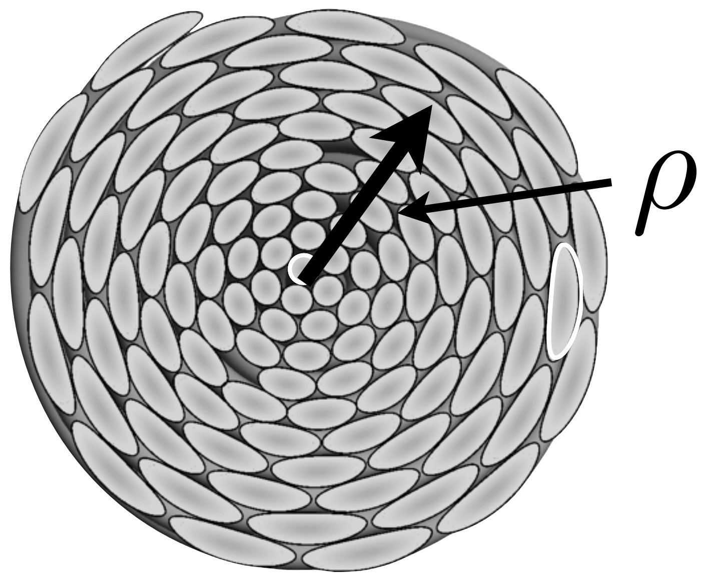
$$K_G = \kappa_1 \kappa_2$$

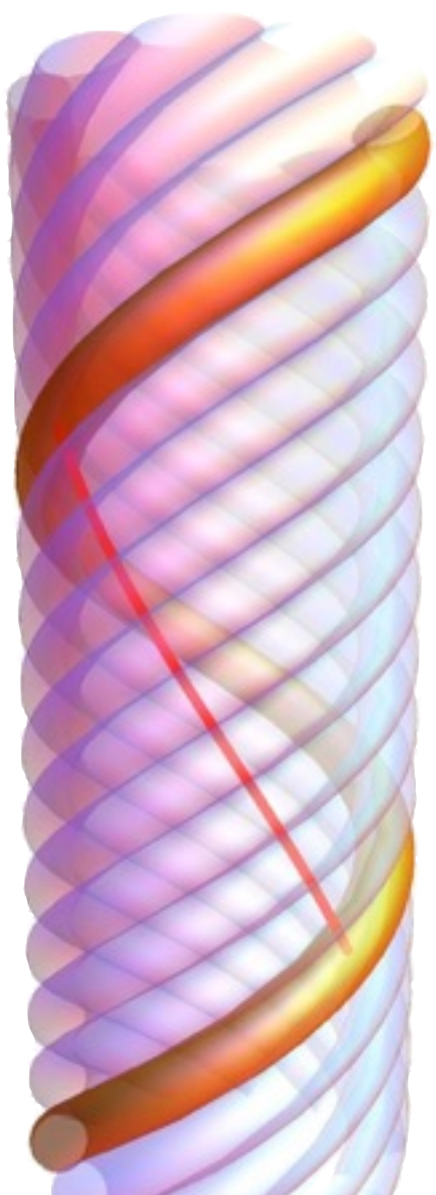


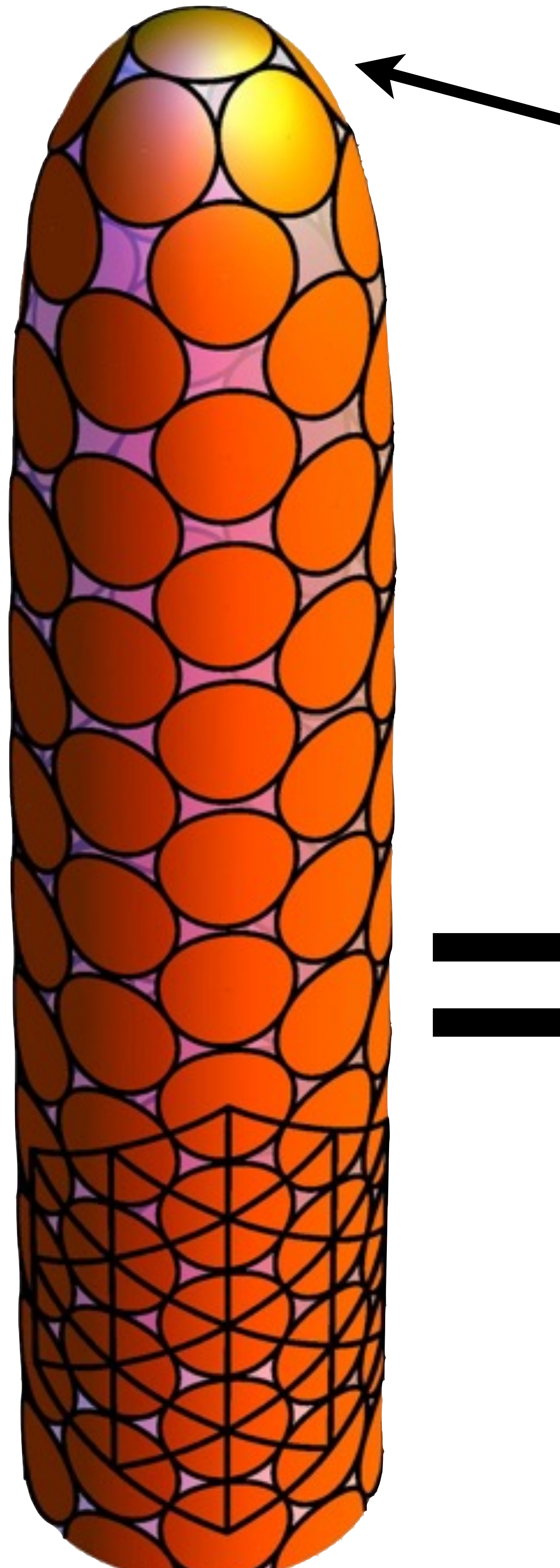
$K_G < 0$ $\quad$ $K_G > 0$

Bruss and GMG, *PNAS* (2012)

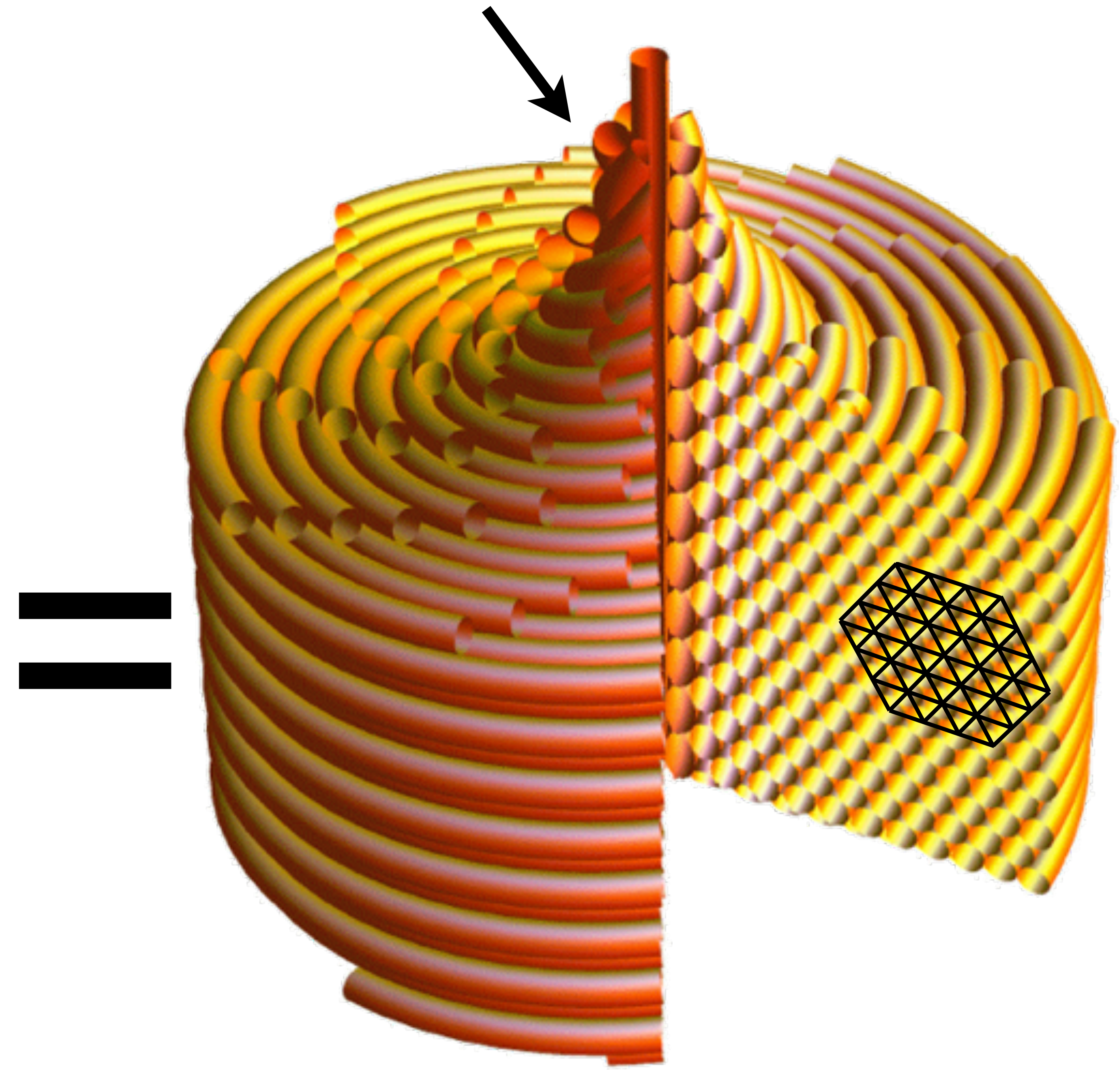
Frustration in filament packing: hidden geometry







Perfect, regular packing is frustrated (disrupted) at the *central core* of twisted bundles



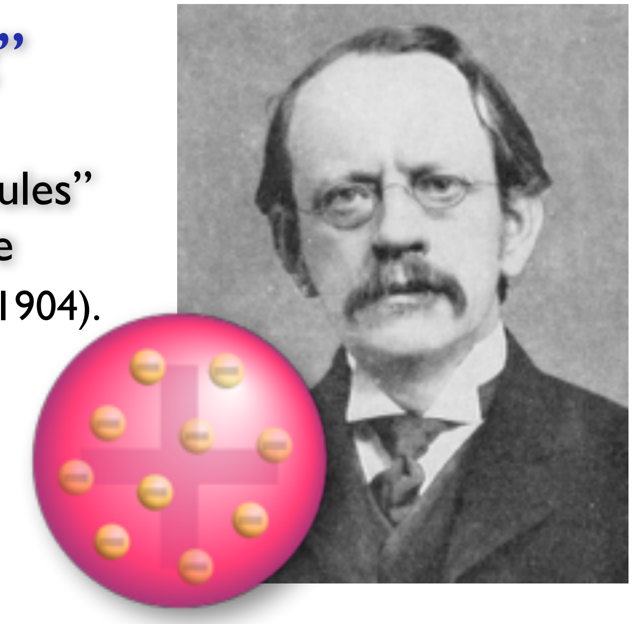
Bruss and GMG, *PNAS* (2012)

Spherical Crystallography: Generalized Thomson Problem

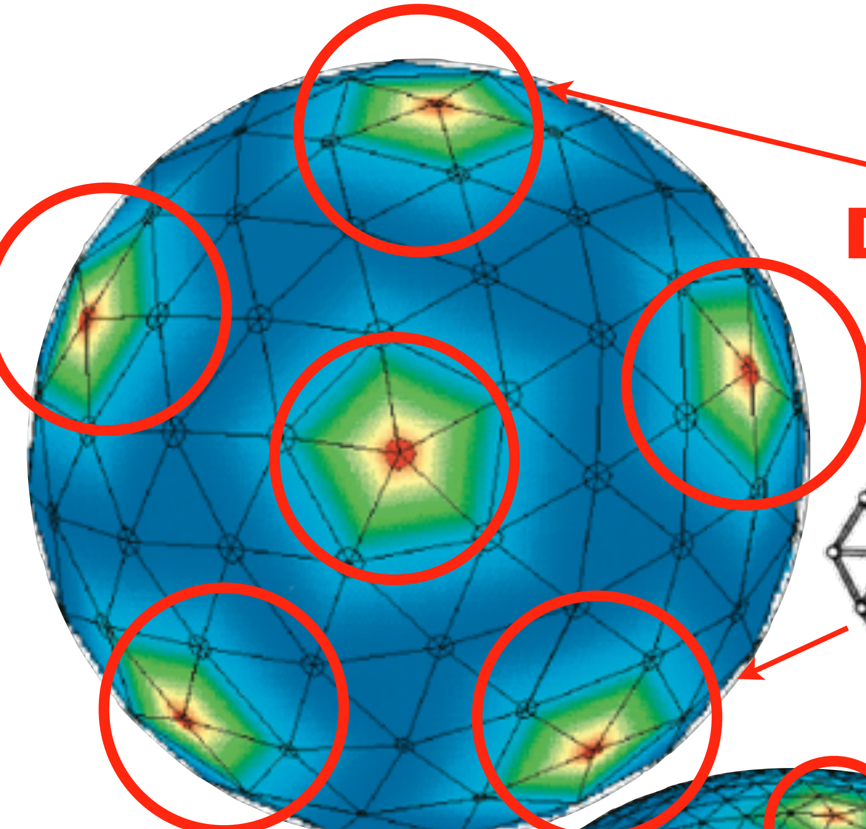
"plum pudding" model:

electronic "corpuscules" on a charged sphere

Thomson, *Phil. Mag.* (1904).

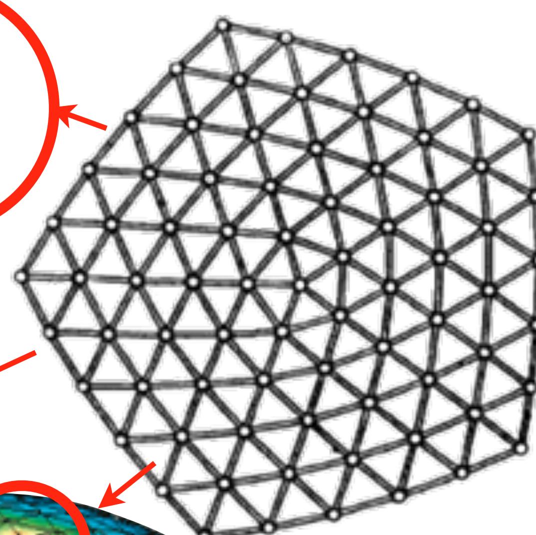


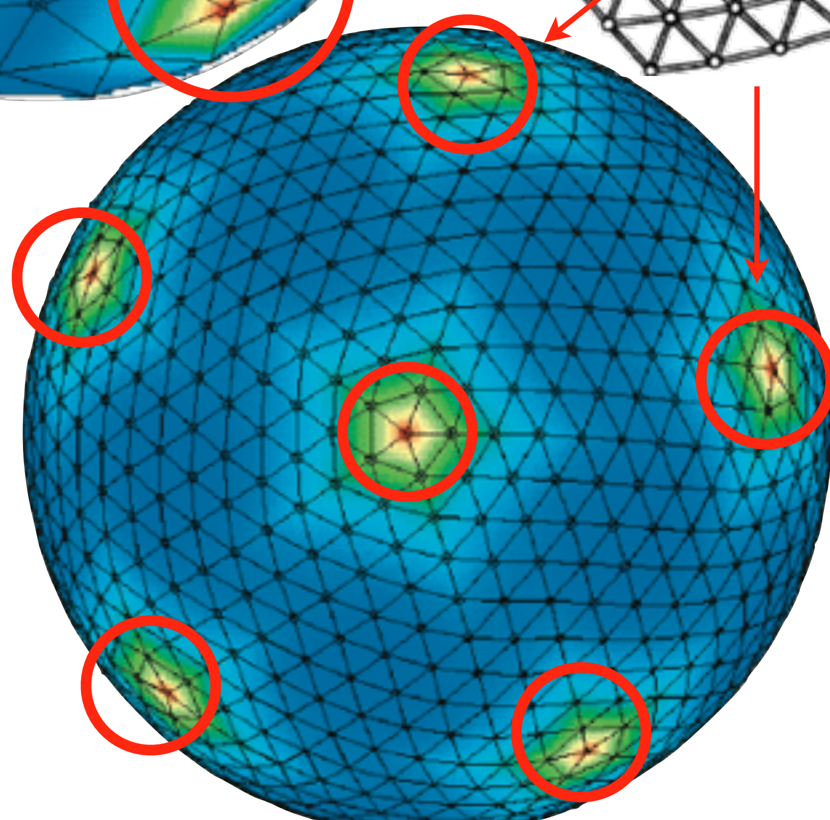
Repulsive (electrostatic) particles on spheres



N = 132

(5-fold) Disclinations!

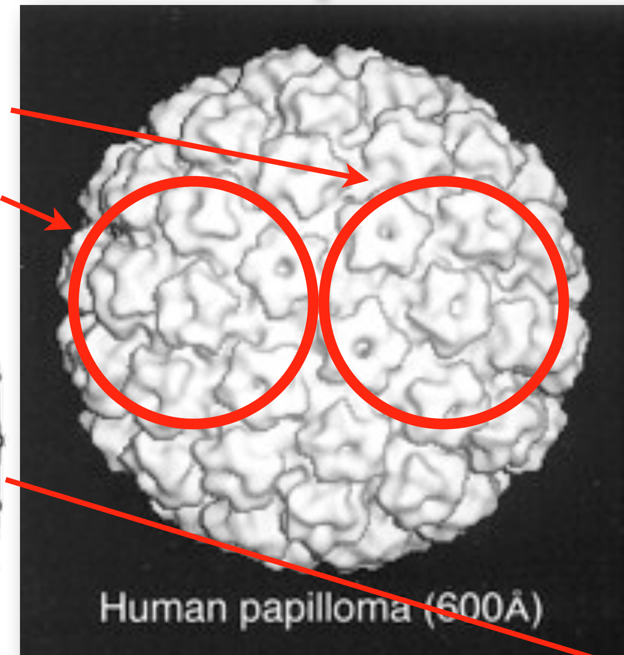




N = 1032

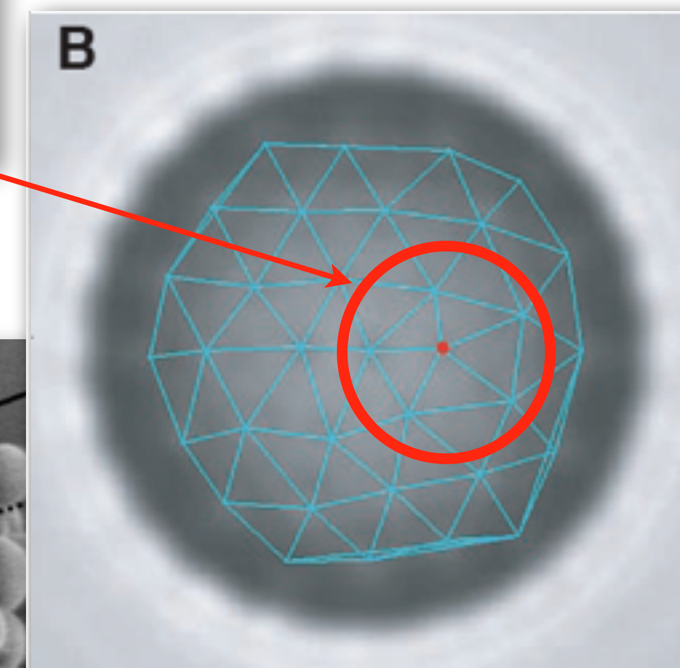
Altschuler, Williams, Ratner, Tipton, Stong, Dowla & Wooten, *Phys Rev Lett* (1999)

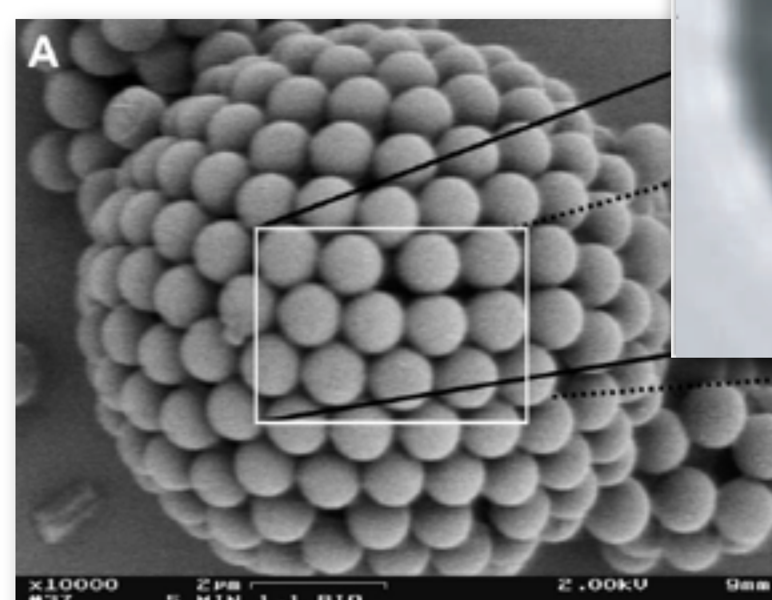
Viral capsids



Baker, Oslen & Fuller, *Micro. & Mol. Biol. Rev.* (1994).

Colloidsomes





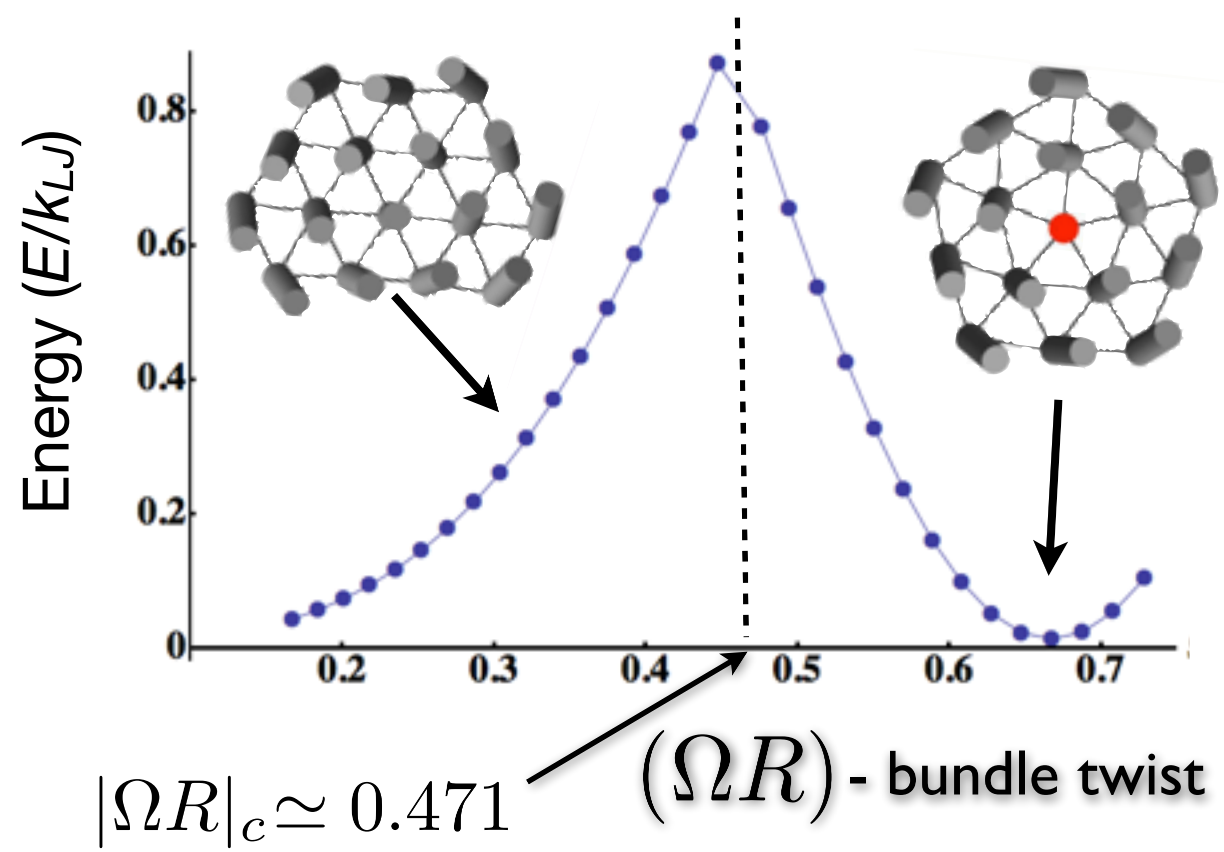
Dinsmore, Hsu, Nikolaides, Marquez & Weitz, *Science* (2002)

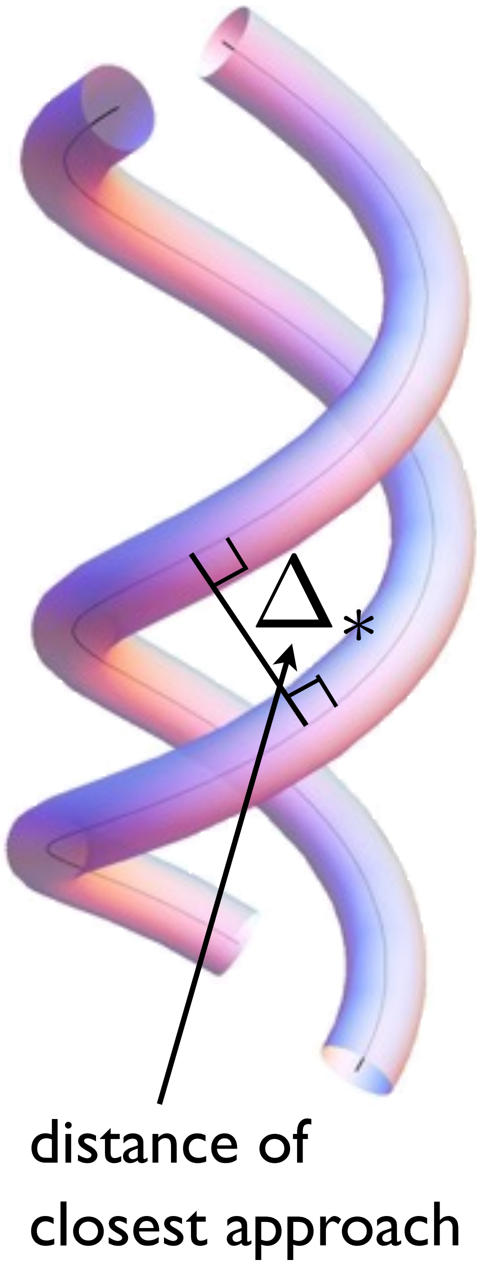
Bausch, Bowick, Cacciuto, Dinsmore, Hsu, Nelson, Nikolaides, Travesset & Weitz, *Science* (2003)

Simulations of adhesive filament, twisted assemblies

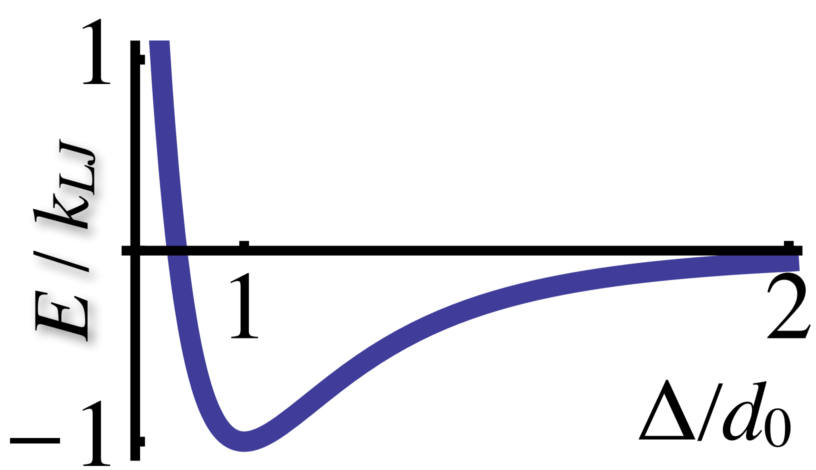
Method: *(numerically) minimize 2D cross-section of N filament bundles of fixed twist interacting via attractive, pair-wise forces*

16 filament bundle



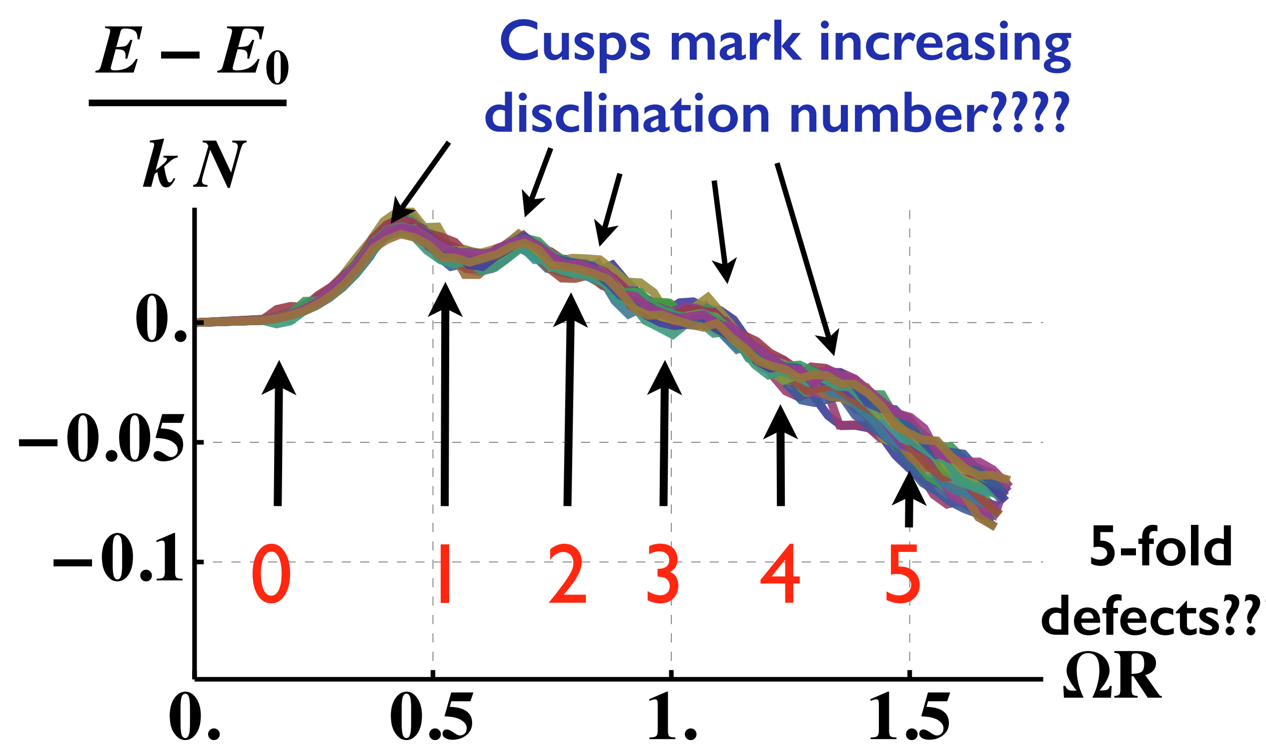


$$E_{LJ} = \frac{k_{LJ}}{2}\left(\frac{5d_o^{12}}{6\Delta^{11}} - \frac{11d_o^{\ 6}}{6\Delta^5}\right)$$



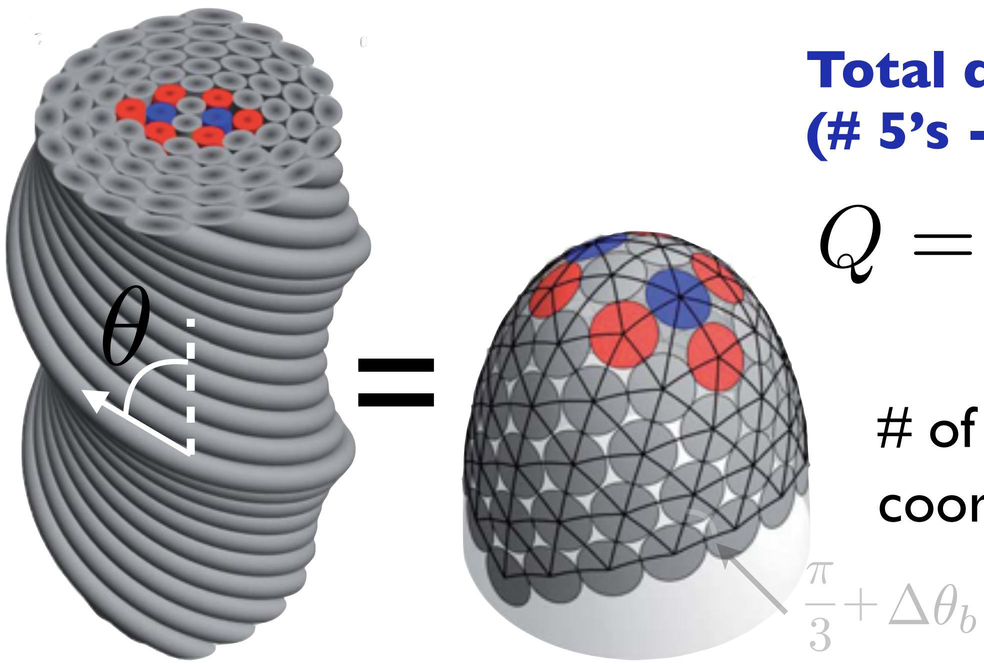
Pair-wise adhesive energy ("Curva-Lennard Jones")

***N* = 85 - 154 filaments**



Bruss & GMG, *PNAS* (2012); Bruss & GMG, *Soft Matter* (2013).

Defects in Ground States of Small-N Bundles



↖ "equilateral excess"

Total disclination "charge" (# 5's - #7's):

$$Q = \sum_n (n-6) V_n$$

of *n*-fold coordinated filaments

Generalized Euler-Poincare formula (triangulation w/open boundary):

$$6 \int dA \; K_G = 2\pi Q + \sum_b \Delta\theta_b$$

distortion @ boundary

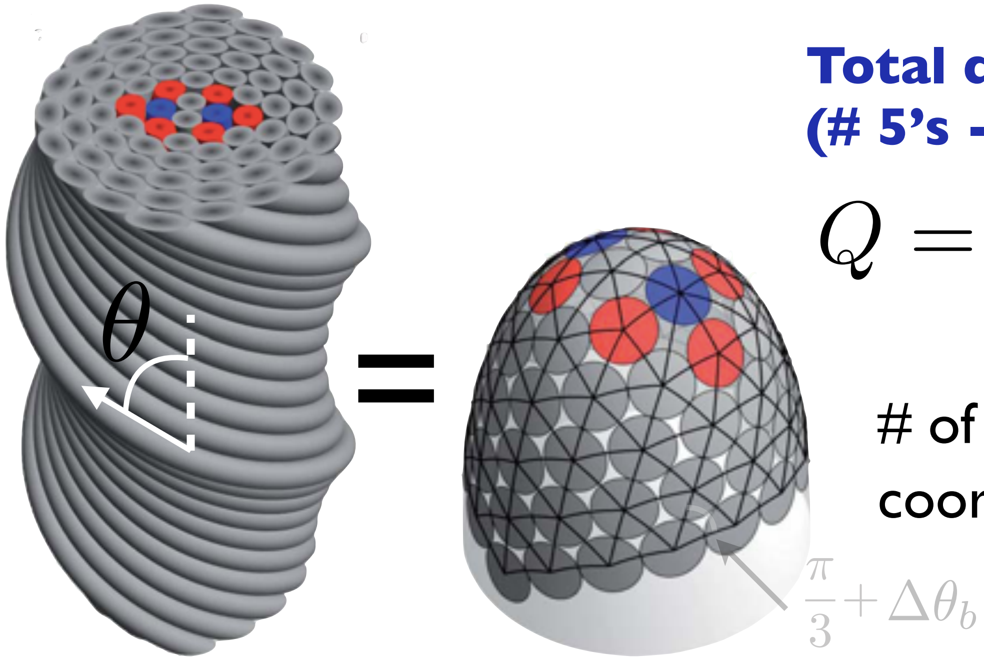
"Net neutral" disclination charge:

$$Q_{id} = \frac{3}{\pi} \int dA \; K_{\rm eff} = 6(1 - \cos^3 \theta)$$

bundle twist angle

Bruss & GMG, *PNAS* (2012); Bruss & GMG, *Soft Matter* (2013).

Defects in Ground States of Small-N Bundles



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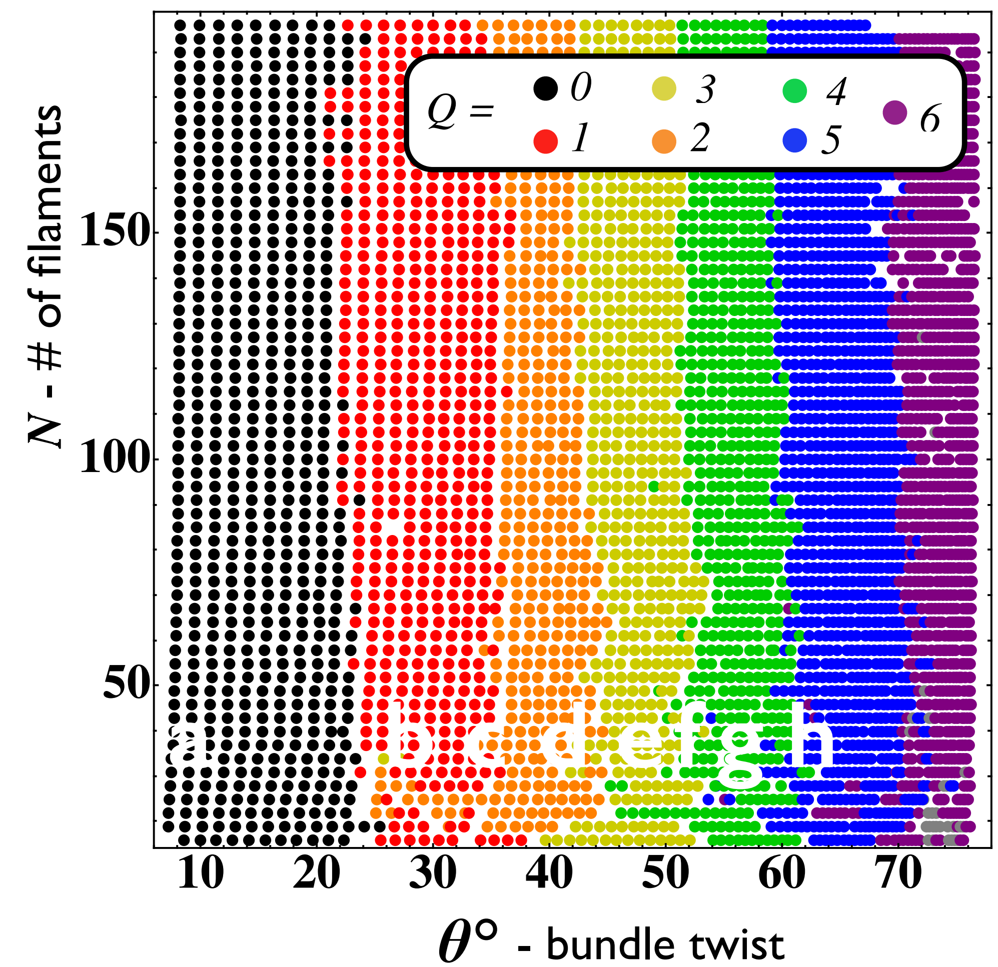
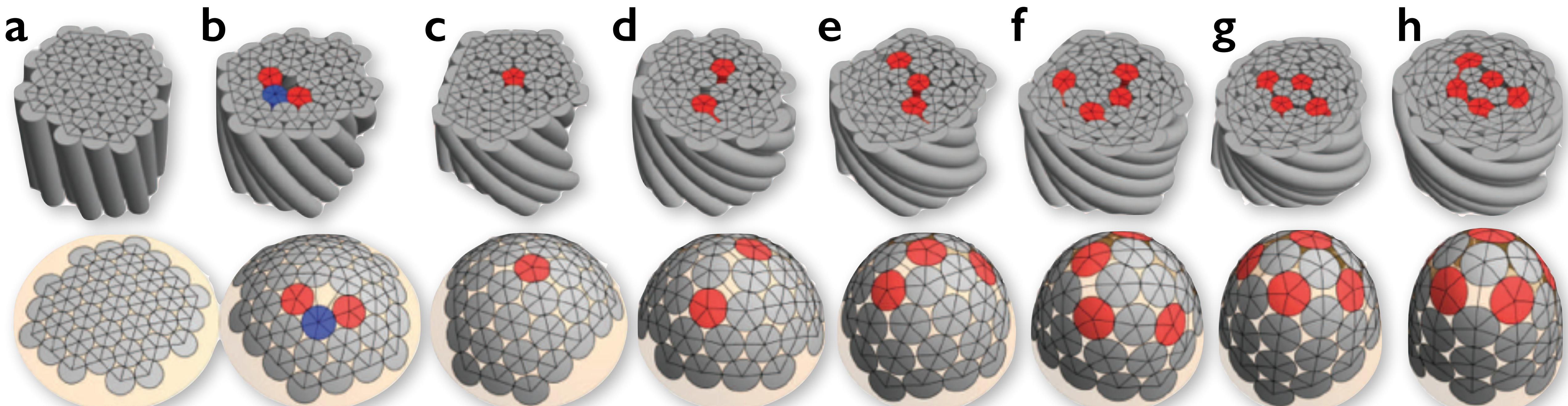
$$Q_{id} = \frac{3}{\pi} \int dA \; K_{\rm eff} = 6(1 - \cos^3 \theta)$$

bundle twist angle

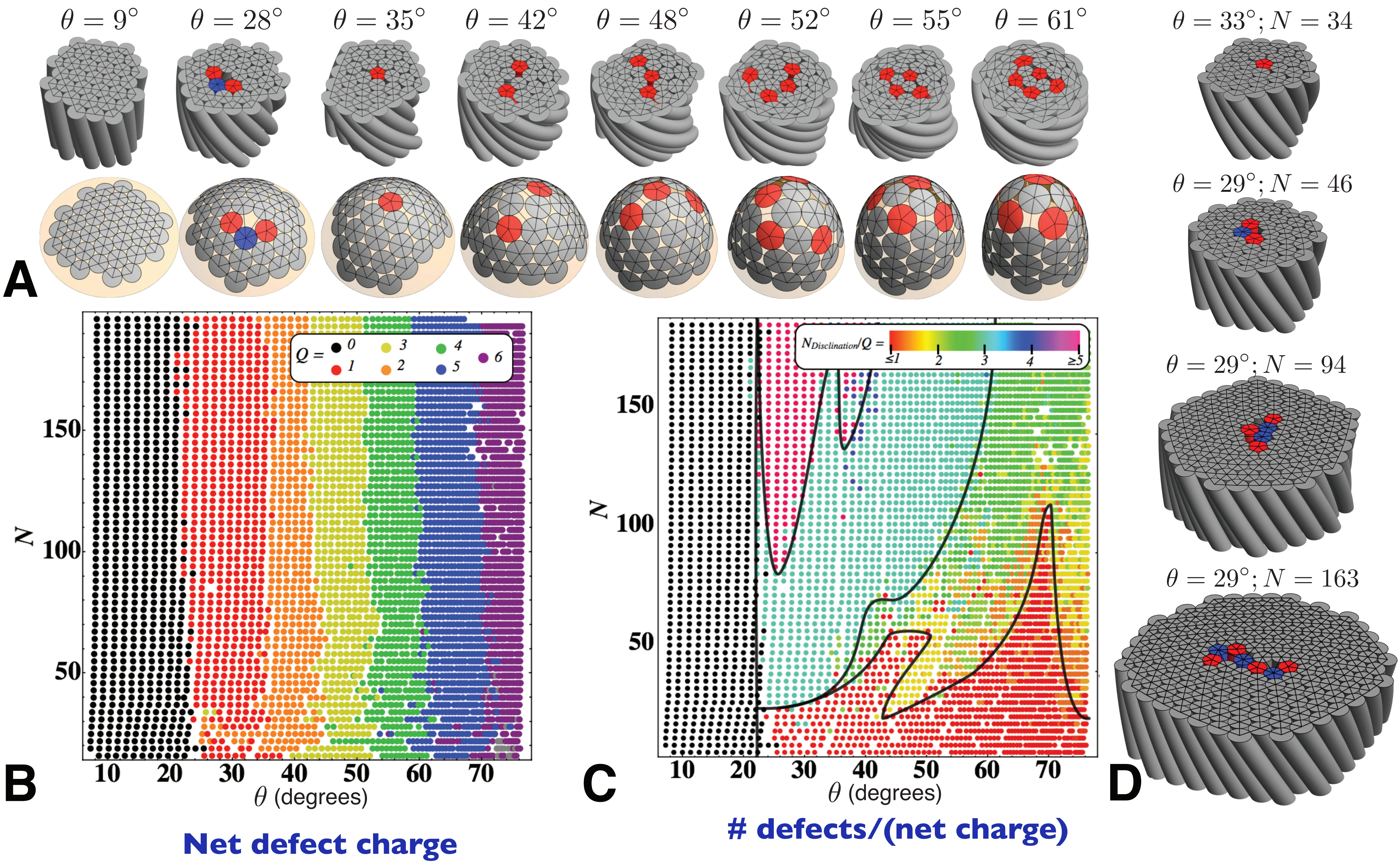
ground state defect charge, Q





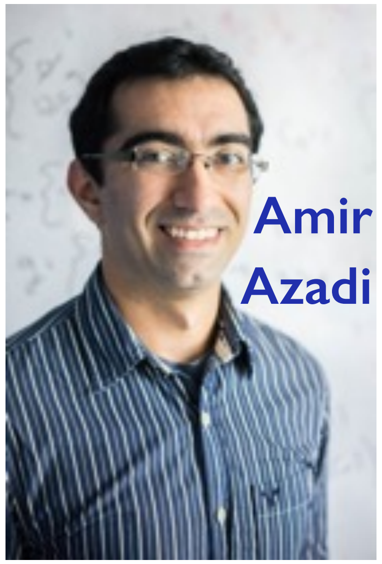
Bruss & GMG, *PNAS* (2012); Bruss & GMG, *Soft Matter* (2013).

Numerical "ground states" of small-*N* bundles:



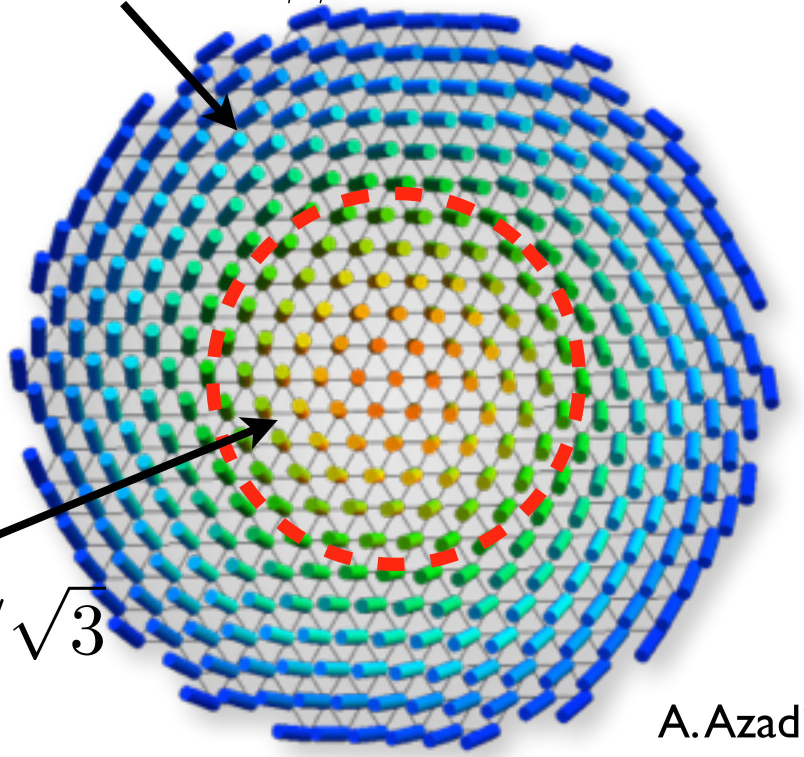
Bruss & GMG, *PNAS* (2012); Bruss & GMG, *Soft Matter* (2013).

Large-*N* Bundles: Multi-dislocation ground states



twist-induced stresses:

compression: $\sigma_{\phi\phi} < 0;\ \rho > R/\sqrt{3}$

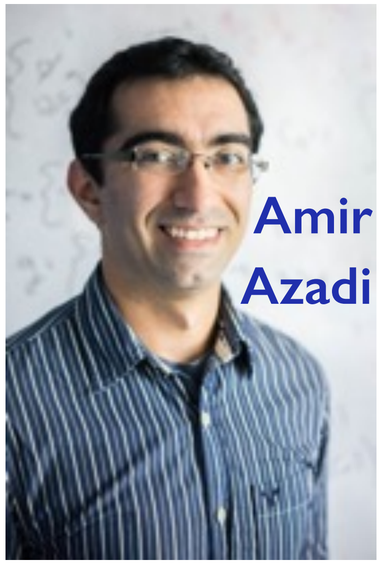


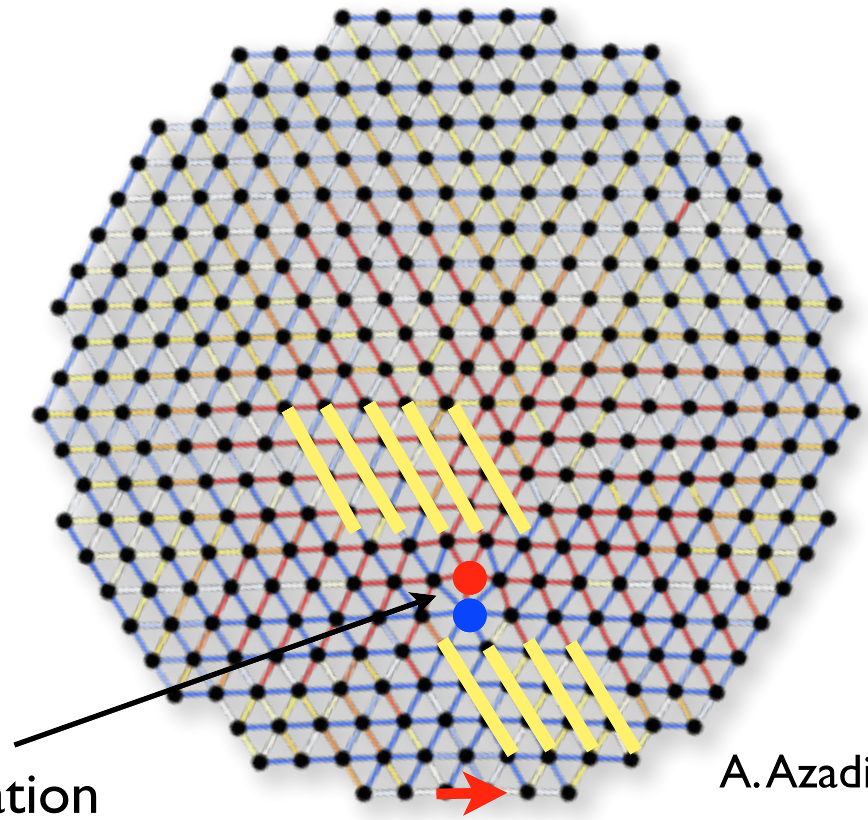
tension:

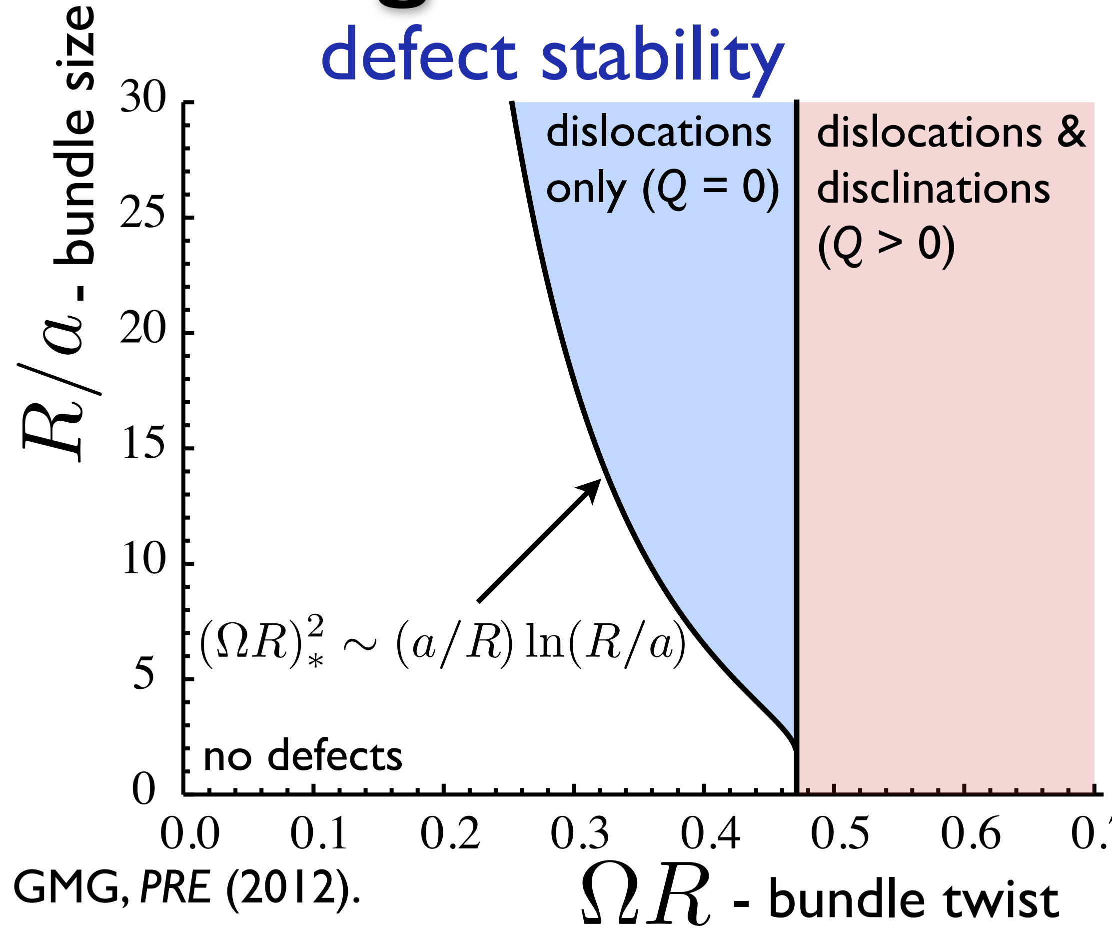
$\sigma_{\phi\phi} > 0;\ \rho < R/\sqrt{3}$

A. Azadi & GMG, *PRE* (2012).

Large-*N* Bundles: Multi-dislocation ground states

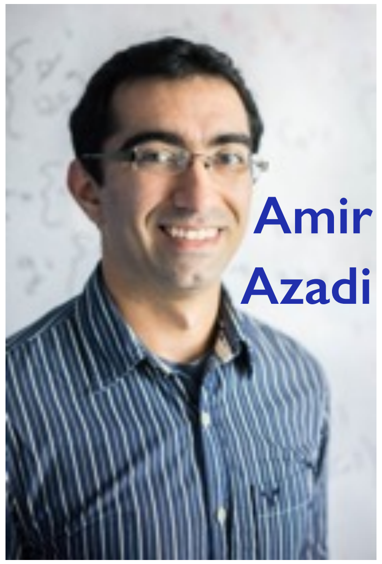


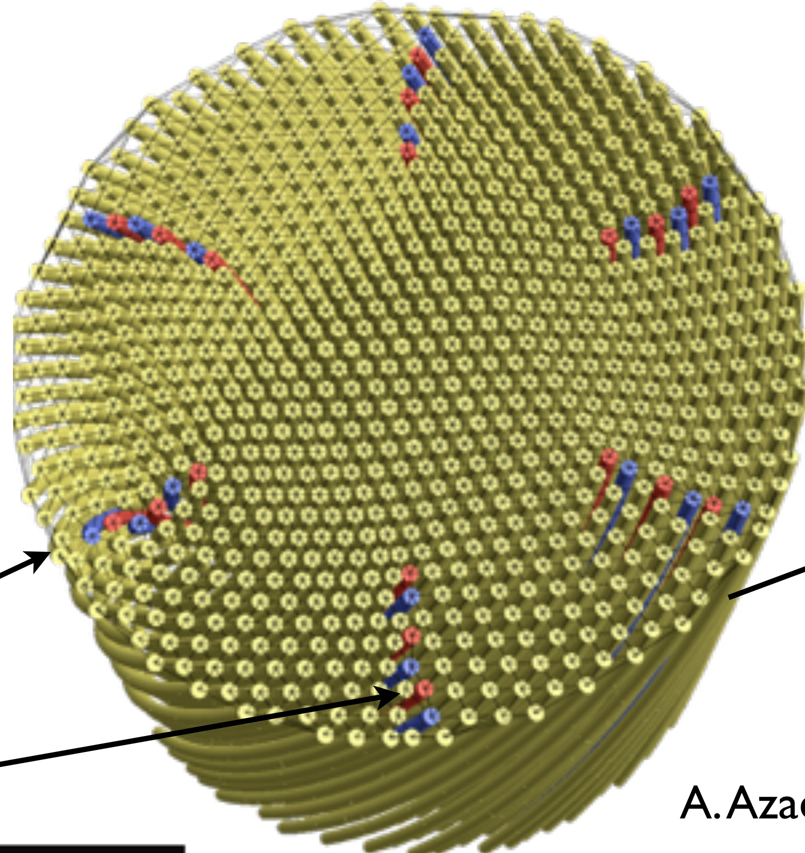




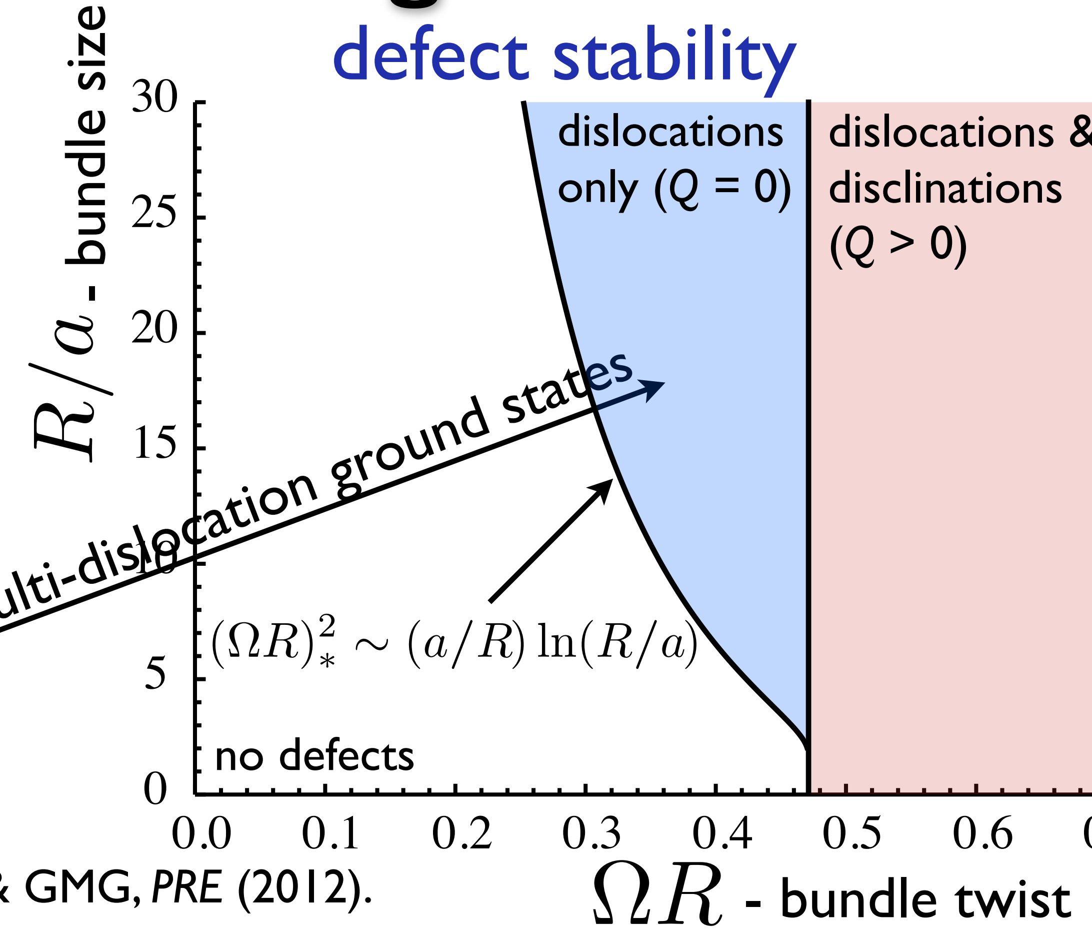
A. Azadi & GMG, *PRE* (2012).

Large-*N* Bundles: Multi-dislocation ground states

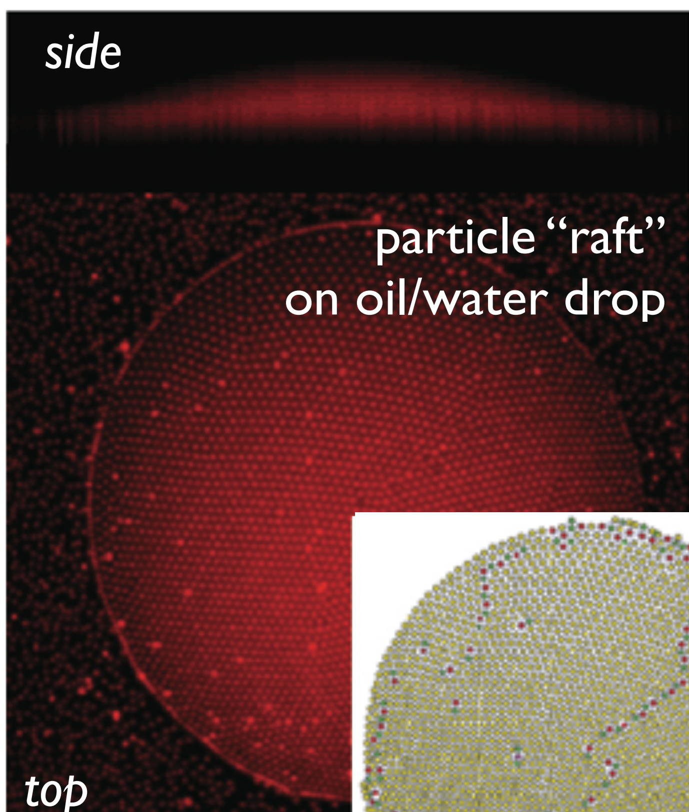




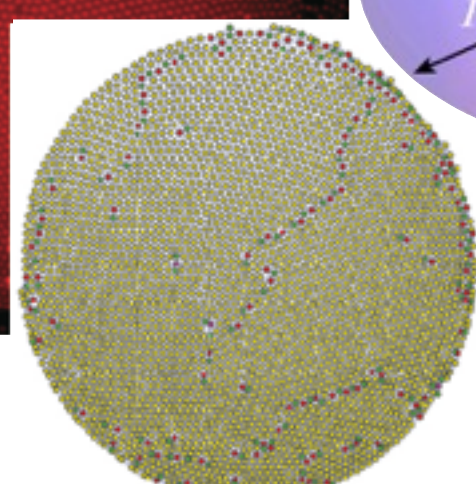
low-angle grain boundary "scars"



A. Azadi & GMG, *PRE* (2012).

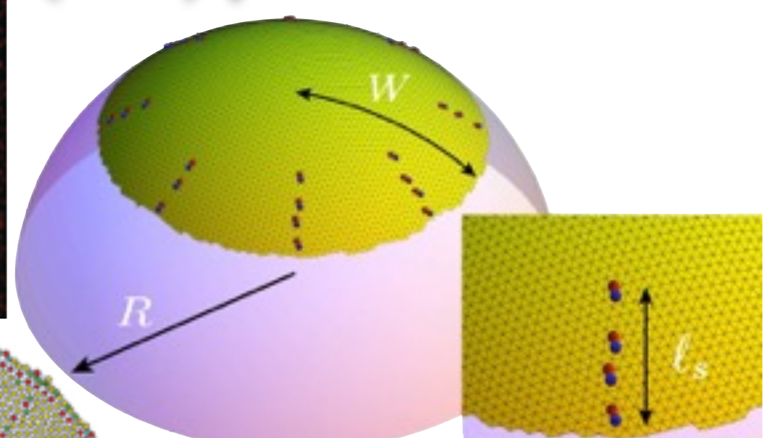


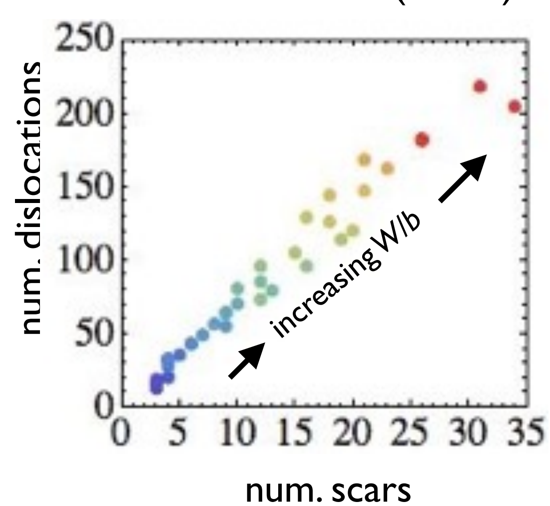
Irvine
@ U Chicago

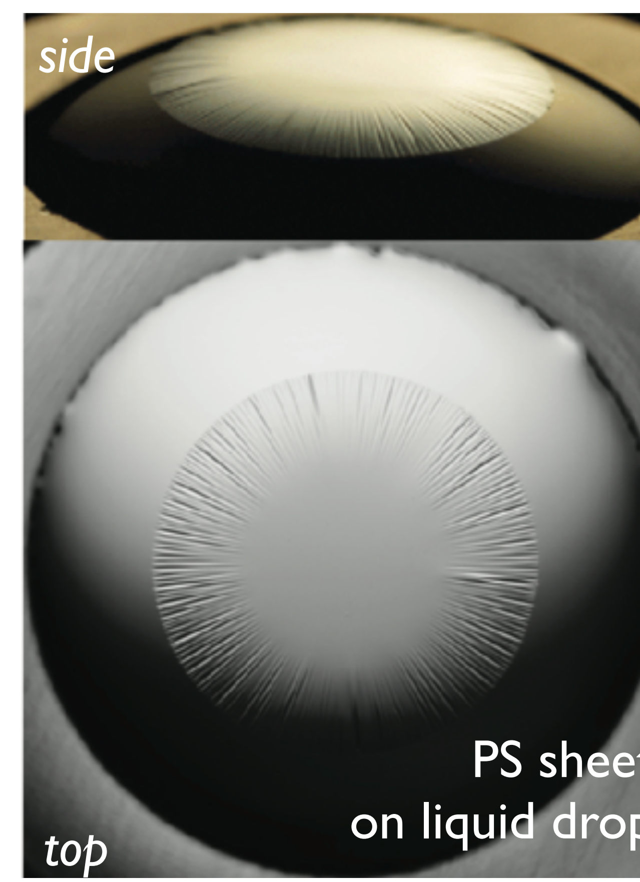


Optimal symmetry of multi-dislocation (scar) patterns?

A. Azadi & GMG, *PRL* (2014).







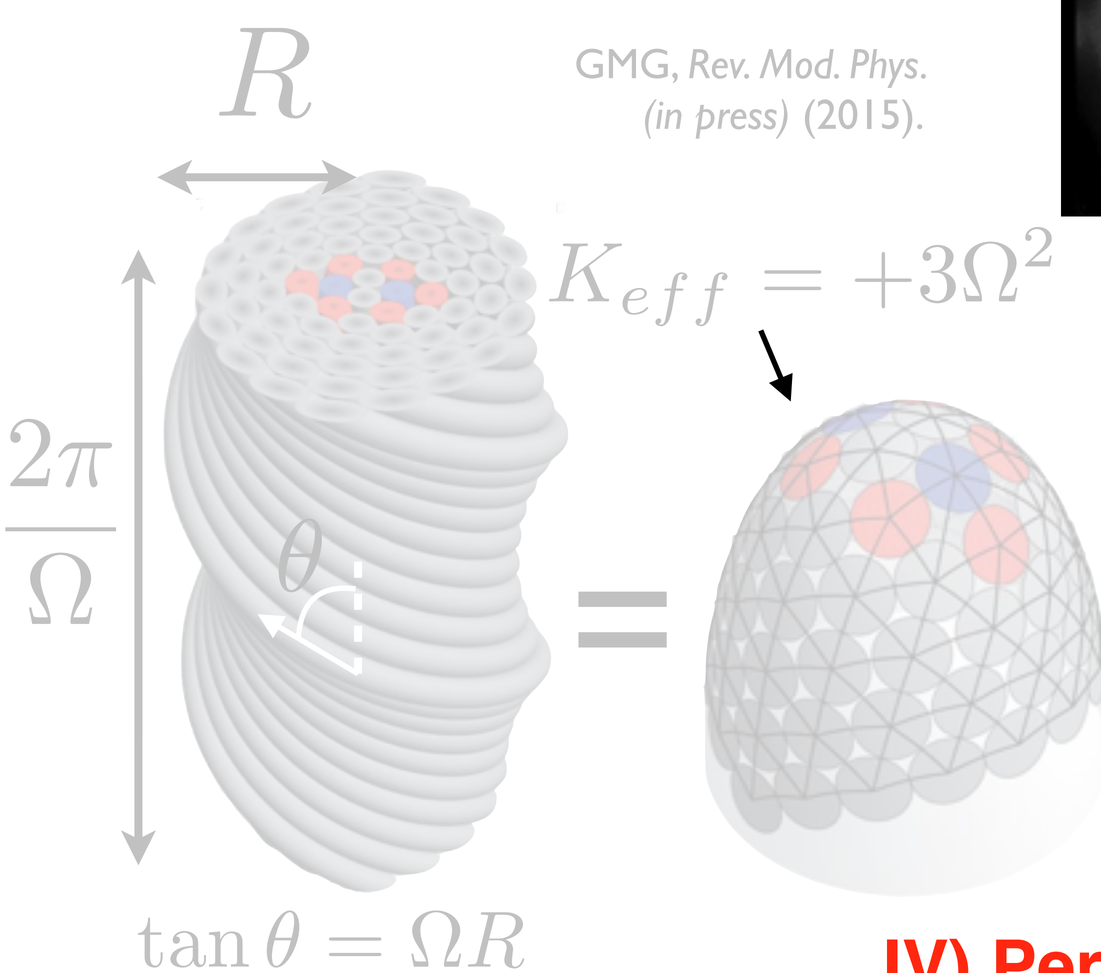
Menon @ UMass

Ground states of surface confined assemblies: *"elastic" wrinkle vs. "plastic" defect patterns?*

GMG & Davidovitch, *PNAS* (2013).

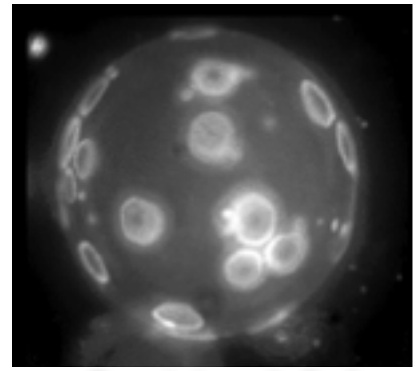
Twisted bundles: non-Euclidean geometry & anomalous assembly

I) Non-euclidean metric geometry of bundles



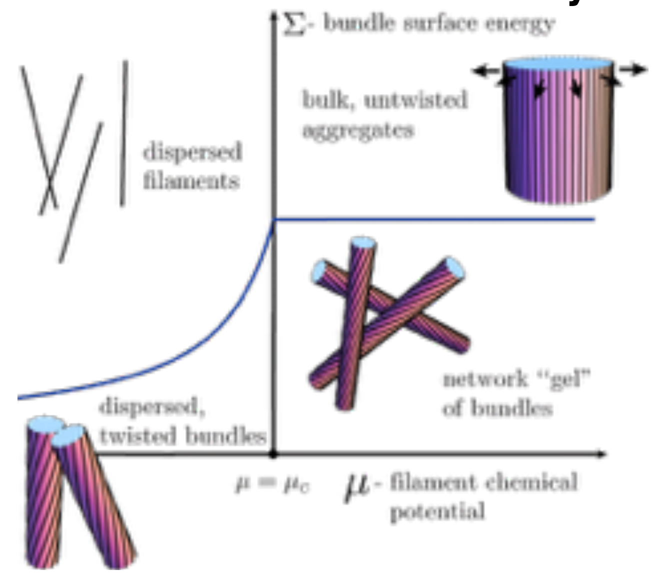
GMG, *Rev. Mod. Phys.* (*in press*) (2015).

solid domains on lipid vesicles



Bandekar & Sofou, Langmuir (2012)

chiral filament assembly



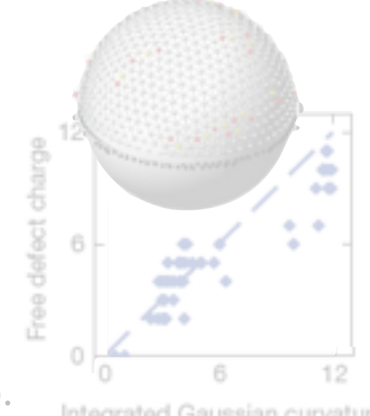
II) Self-limiting Assembly

GMG & Bruinsma, PRL (2007); GMG, PRE (2009)

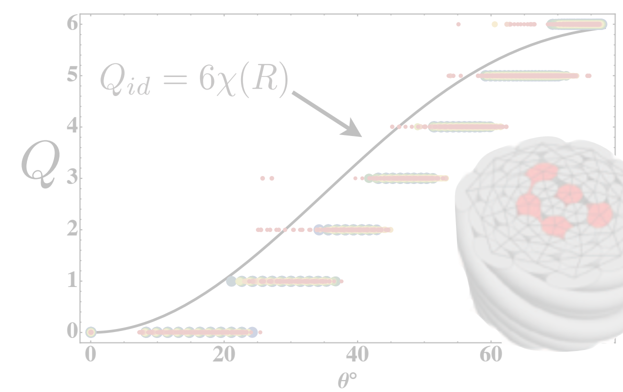
III) Topological Defects

defects in curved crystals

Irvine *et al.*, *Nature* (2010).



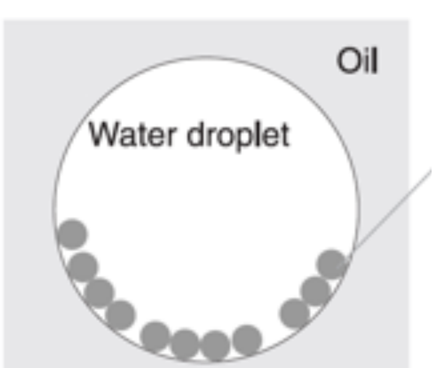
defects in twisted bundles



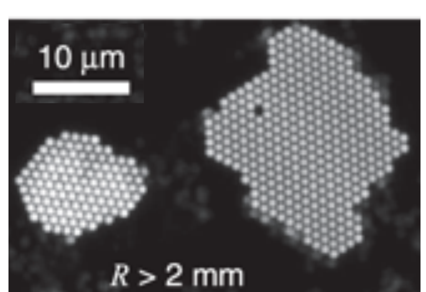
Bruss & GMG, *PNAS* (2012); *Soft Matter* (2013).

IV) Perimeter Instability & Anisotropic Domains

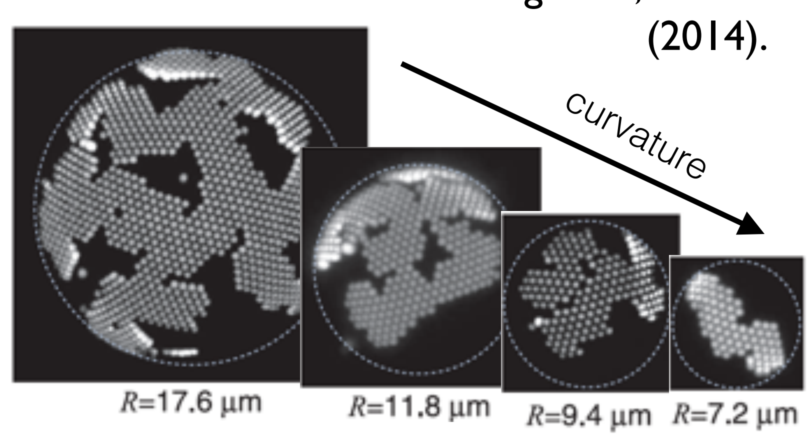
colloidal crystals on spherical droplets



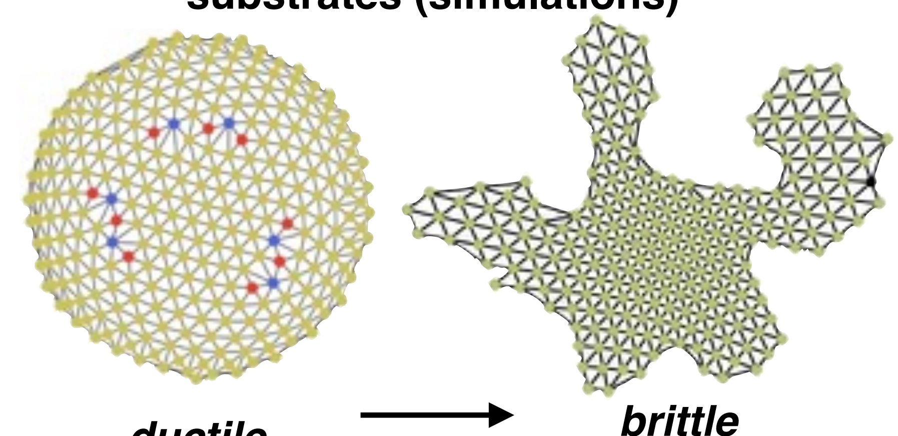
large droplets



Meng *et al.*, *Science* (2014).



cohesive membranes on spherical substrates (simulations)



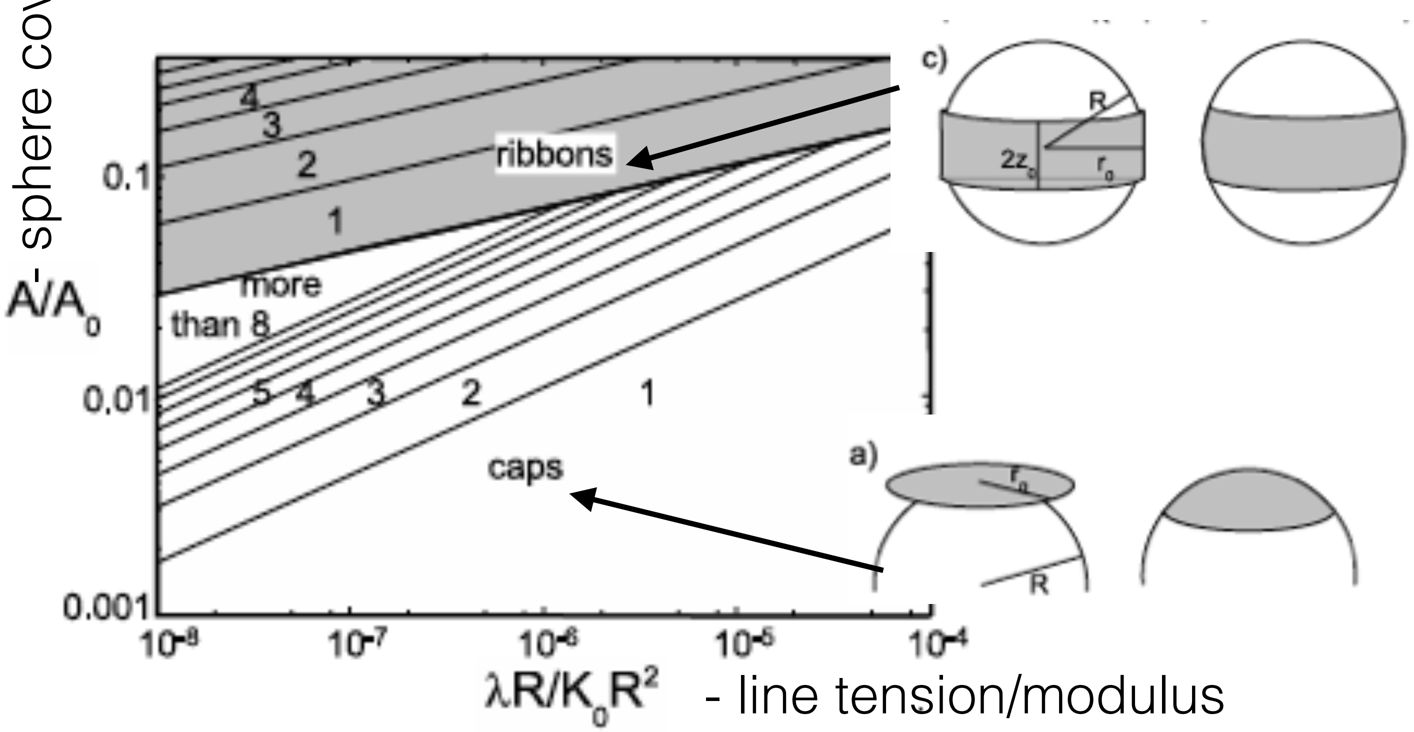
Amir Azadi, *to be published.*

Elastic Perimeter Instability of Curved Crystals

EUROPHYSICS LETTERS 1 April 2005
Europhys. Lett., **70** (1), pp. 136–142 (2005)
DOI: 10.1209/epl/i2004-10464-2

Shapes of crystalline domains on spherical fluid vesicles

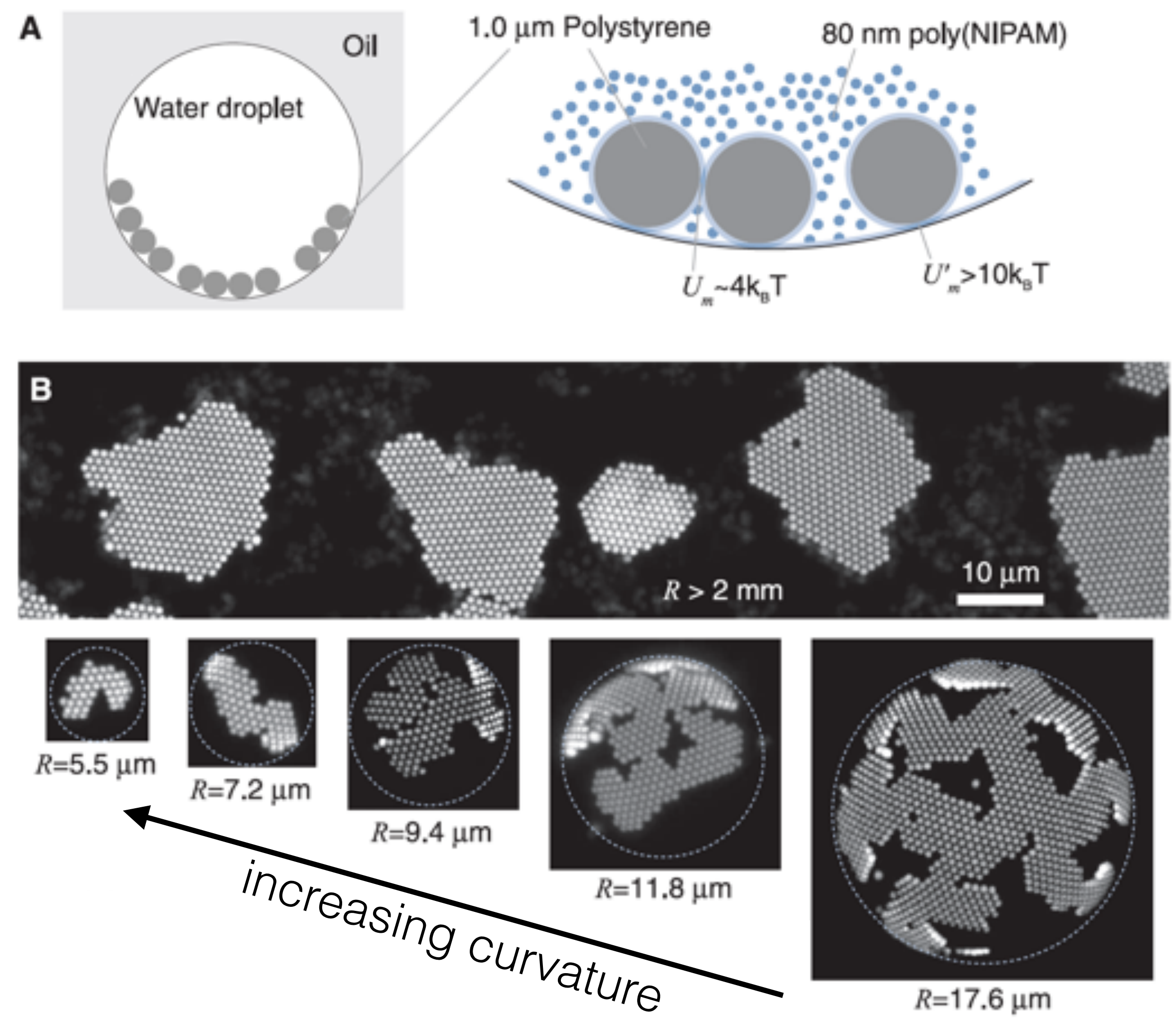
S. Schneider and G. Gompper
Institut für Festkörperforschung - Forschungszentrum Jülich, D-52425 Jülich, Germany



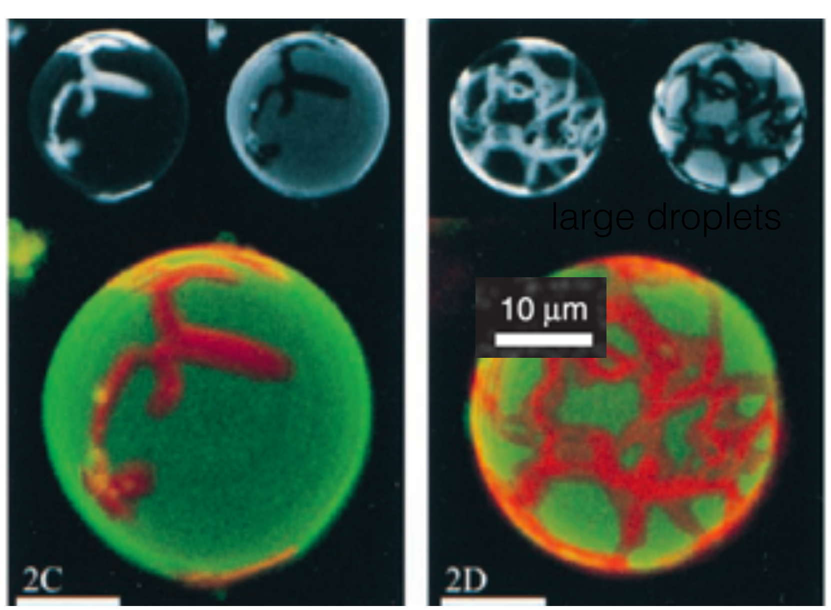
Elastic Instability of a Crystal Growing on a Curved Surface

Guangnan Meng,[1] Jayson Paulose,[2] David R. Nelson,[1,2] Vinothan N. Manoharan[2,1]*

7 FEBRUARY 2014 VOL 343 SCIENCE

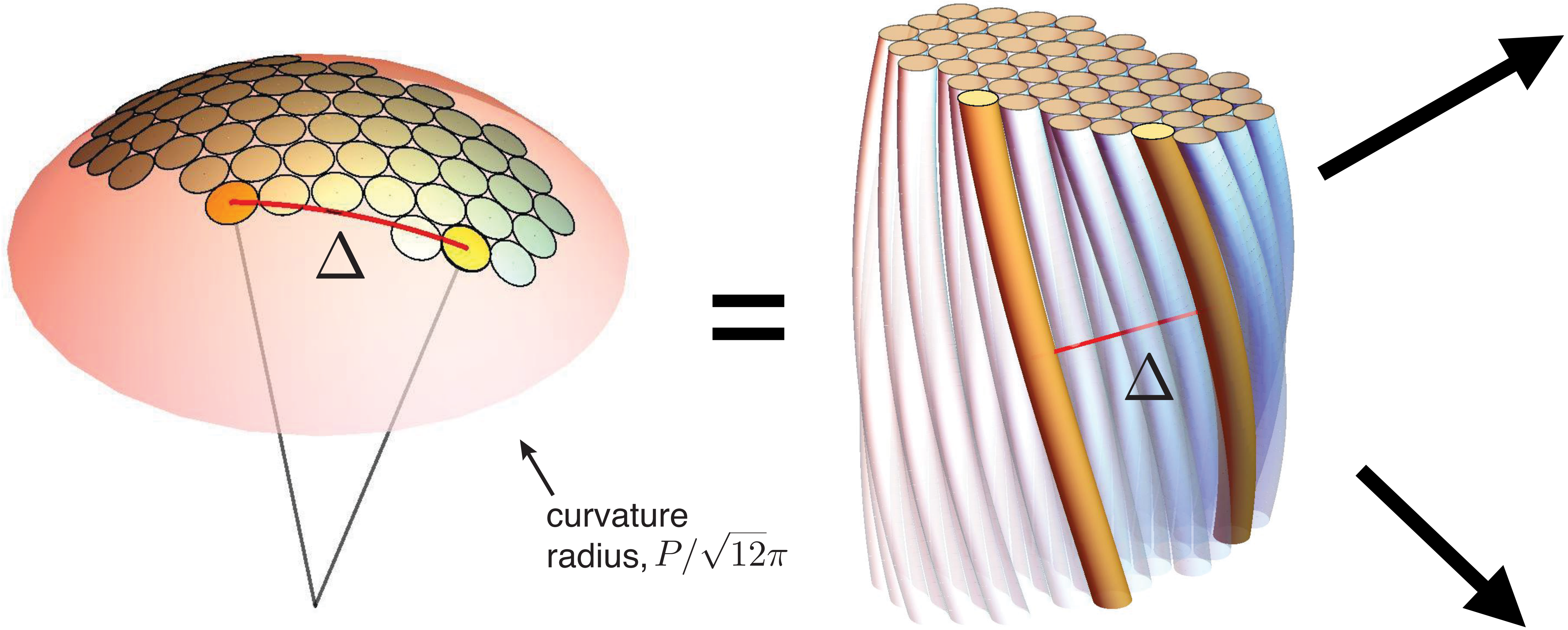


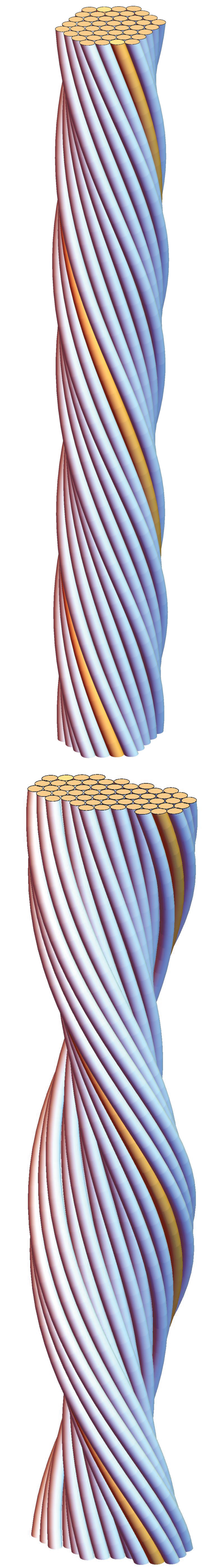
lipid solid-fluid coexistence on GUVS



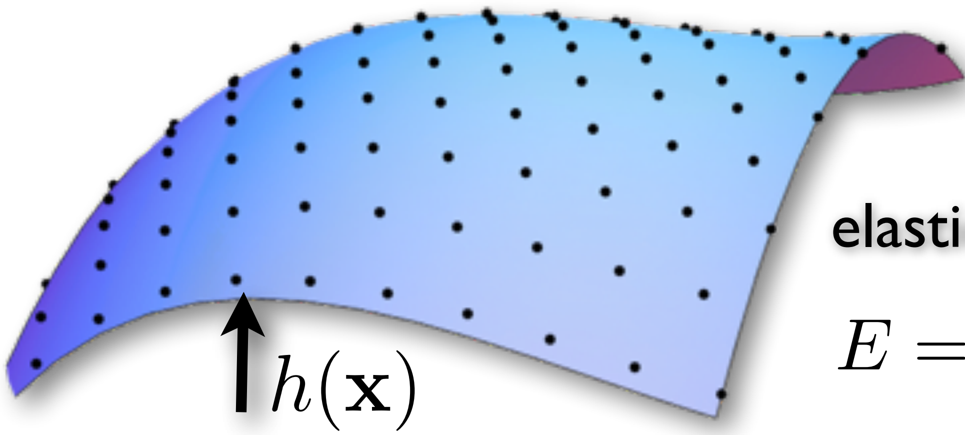
Webb *et al.*
PNAS (1999).

How does metric frustration and boundary instability select optimal “morphology” of twisted bundles of chiral filaments?





Curved Membranes vs. Twisted Bundles: Elasticity



membrane displacement

in-plane stress:

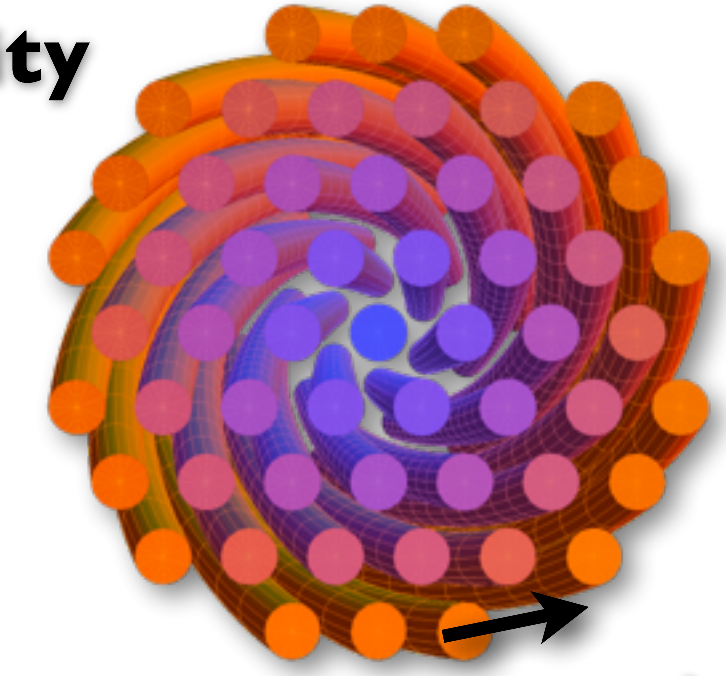
$$\sigma_{ij} = \lambda \delta_{ij} u_{kk} + 2\mu\, u_{ij}$$

elastic energy:

$$E = \frac{1}{2}\int dV\ \sigma_{ij} u_{ij}\ ;$$

Young's modulus:

$$Y = \frac{4\mu(\lambda+\mu)}{\lambda + 2\mu}$$



$$\mathbf{t}_\perp \simeq (\Omega \rho)\hat{\phi}$$

filament tangent

non-linear strain

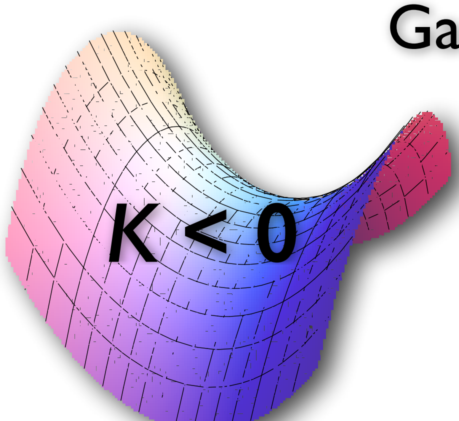
$$u_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h\right) \qquad u^{\perp}_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i - t_i t_j\right)$$

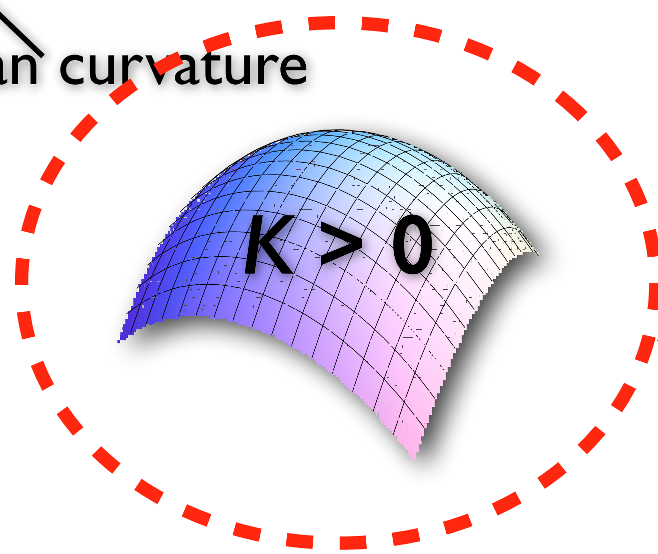
Compatibility relation

$$\frac{\nabla^2_\perp \sigma_{ii}}{Y} = -(\partial_x^2 h)(\partial_y^2 h) + (\partial_x \partial_y h)^2 = -K$$

$$\frac{\nabla^2_\perp \sigma_{ii}}{Y} = -\frac{1}{2}\Big[(\partial_x t_y)^2 + (\partial_y t_x)^2 - 2\partial_x \partial_y (t_x t_y)\Big] \equiv -K_{eff} = -3\Omega^2$$

Gaussian curvature





Bundle twist generates interfilament stress identical to *positive* Gaussian curvature in 2D membranes

GMG, *PRL* (2010).
GMG, *PRE* (2012).

Isotropic, Twisted Cylindrical Bundles: Thermodynamics

intra-filament (bending) elasticity

inter-filament (packing) elasticity

surface energy (cohesion)

R - bundle radius

Ω - twist (2π/pitch)

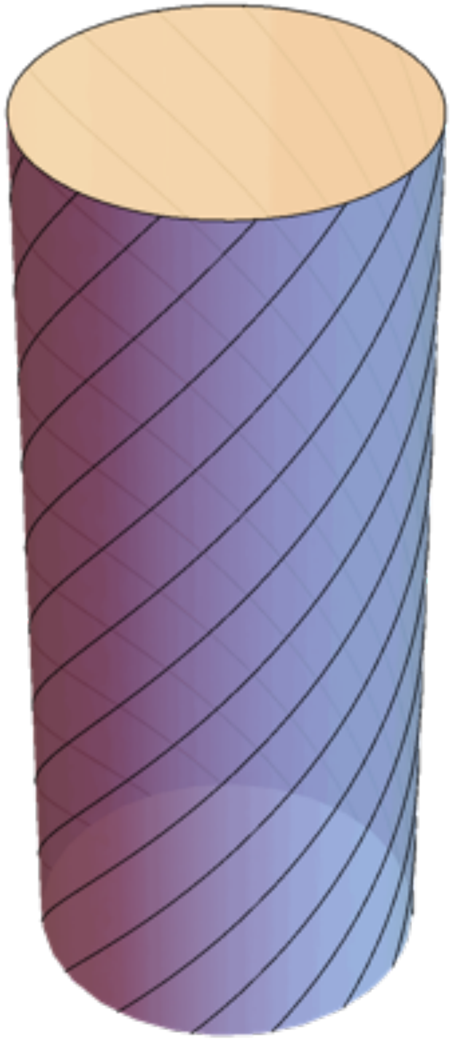
$$F = \frac{B\rho_0}{2} \int dV \ \kappa^2(r) + \frac{1}{2} \int dV \ \sigma_{ij} u_{ij} + \Sigma(2\pi R L)$$

$\kappa(r) \simeq \Omega^2 r$

filament curv.

$\sigma_{ii} \approx -Y(\Omega R)^2$

geometric strain



Isotropic, Twisted Cylindrical Bundles: Thermodynamics

intra-filament (bending) elasticity — *inter*-filament (packing) elasticity — surface energy (cohesion)

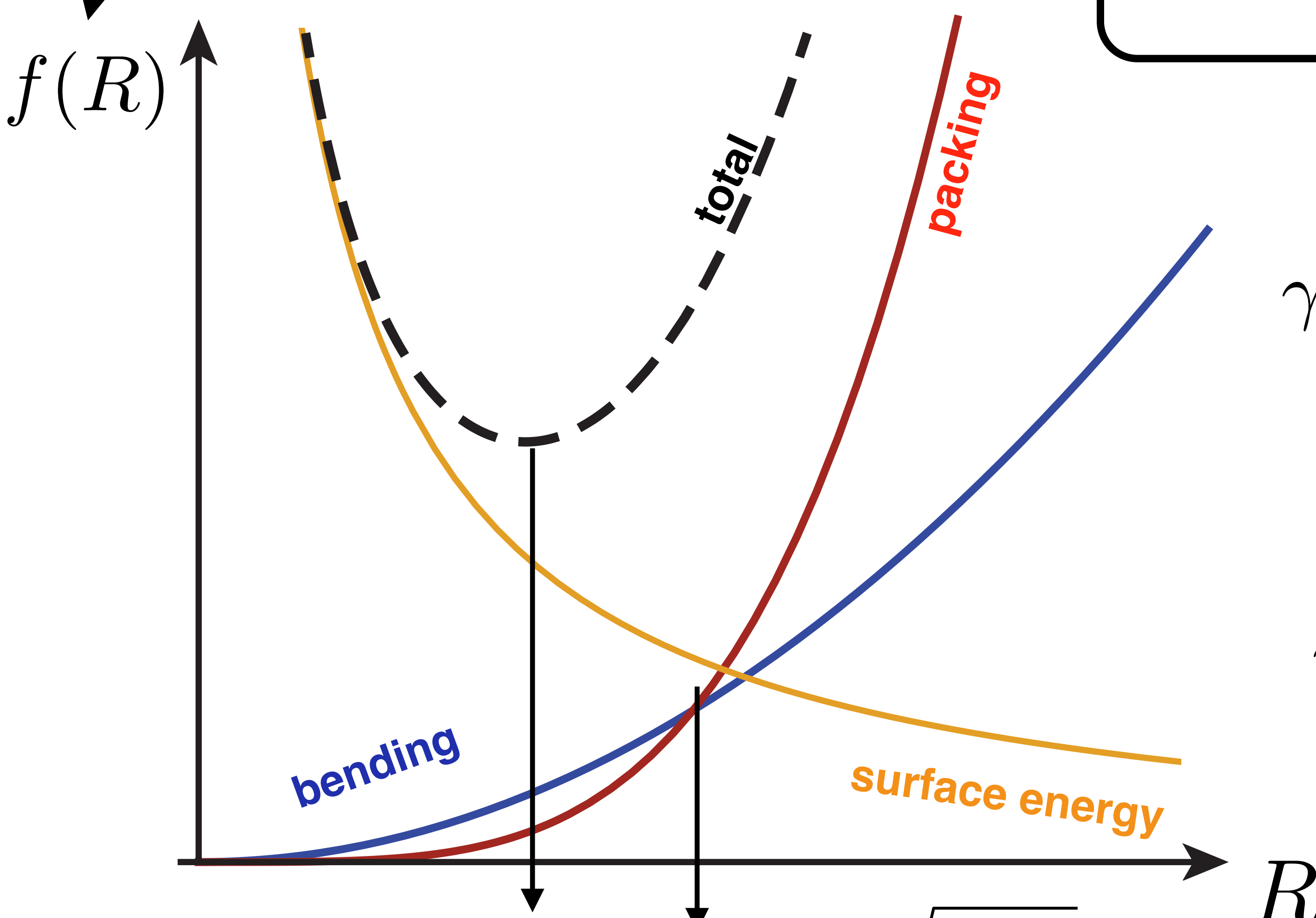
R - bundle radius

Ω - twist (2π/pitch)

$$f(R) = \frac{B\rho_0}{4}\Omega^4 R^2 + \frac{3Y}{128}(\Omega R)^4 + \frac{2\Sigma}{R}$$

free-energy density

$$\gamma \equiv (R_0/\lambda_B)^2 = \frac{\text{“packing” cost}}{\text{bending cost}}$$



R_0 equilibrium radius

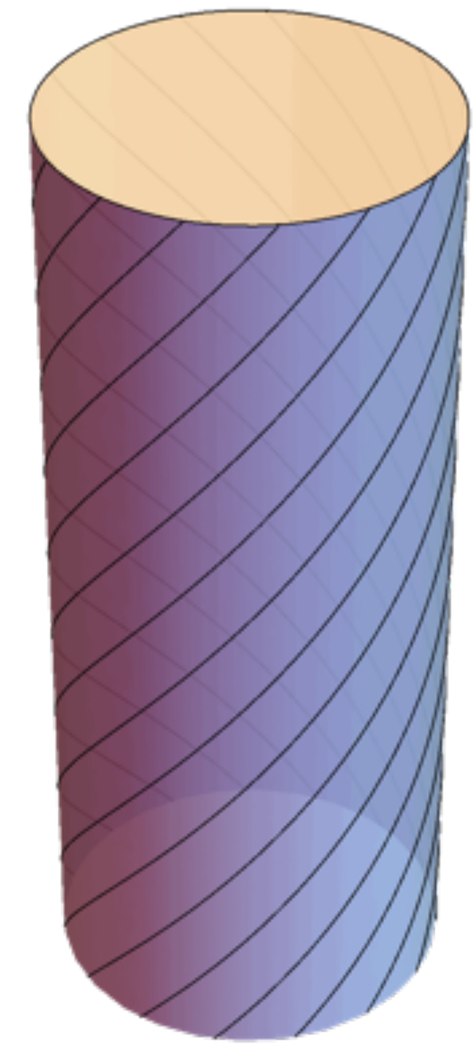
$$\lambda_B \equiv \sqrt{\frac{B\rho_0}{Y}}$$

$\gamma \ll 1$: **bending** limited

$$R_0 \sim (\Sigma/B\Omega^4)^{1/3}$$

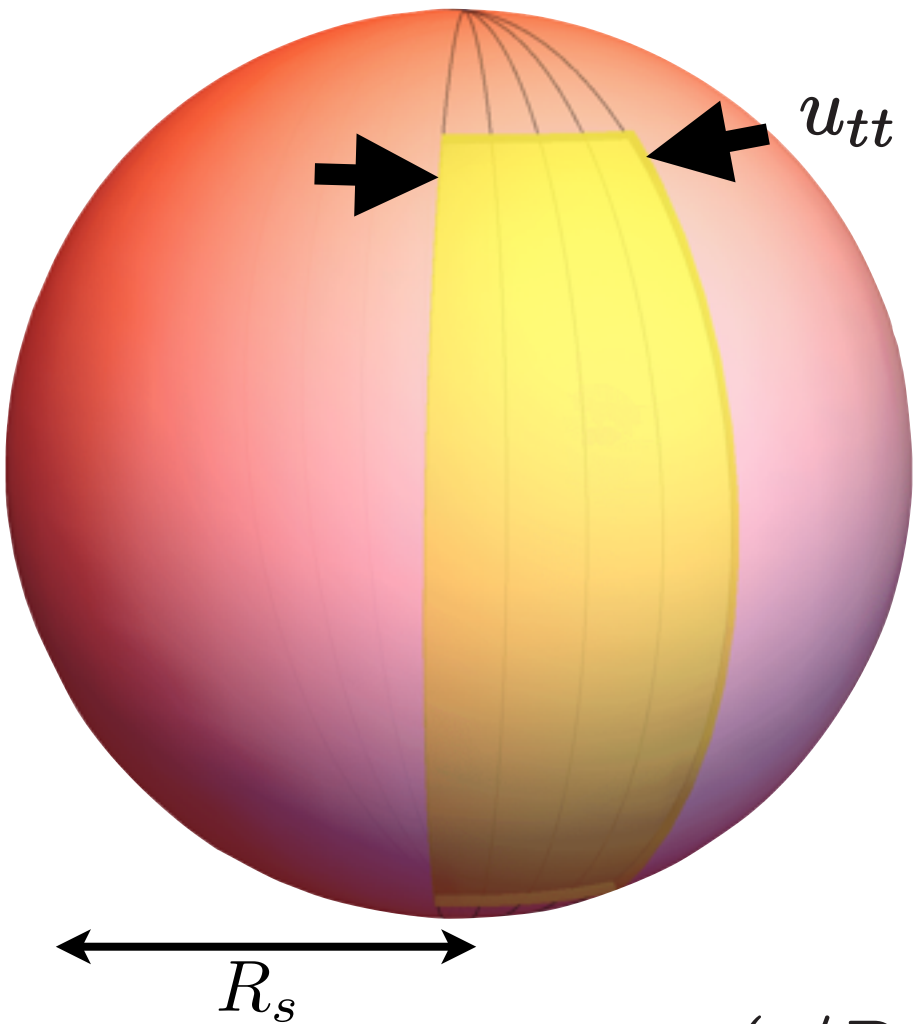
$\gamma \gg 1$: **packing** limited

$$R_0 \sim (\Sigma/Y\Omega^4)^{1/5}$$



Anisotropic, Helical "Tapes": Inter- vs. Intra-filament Strain

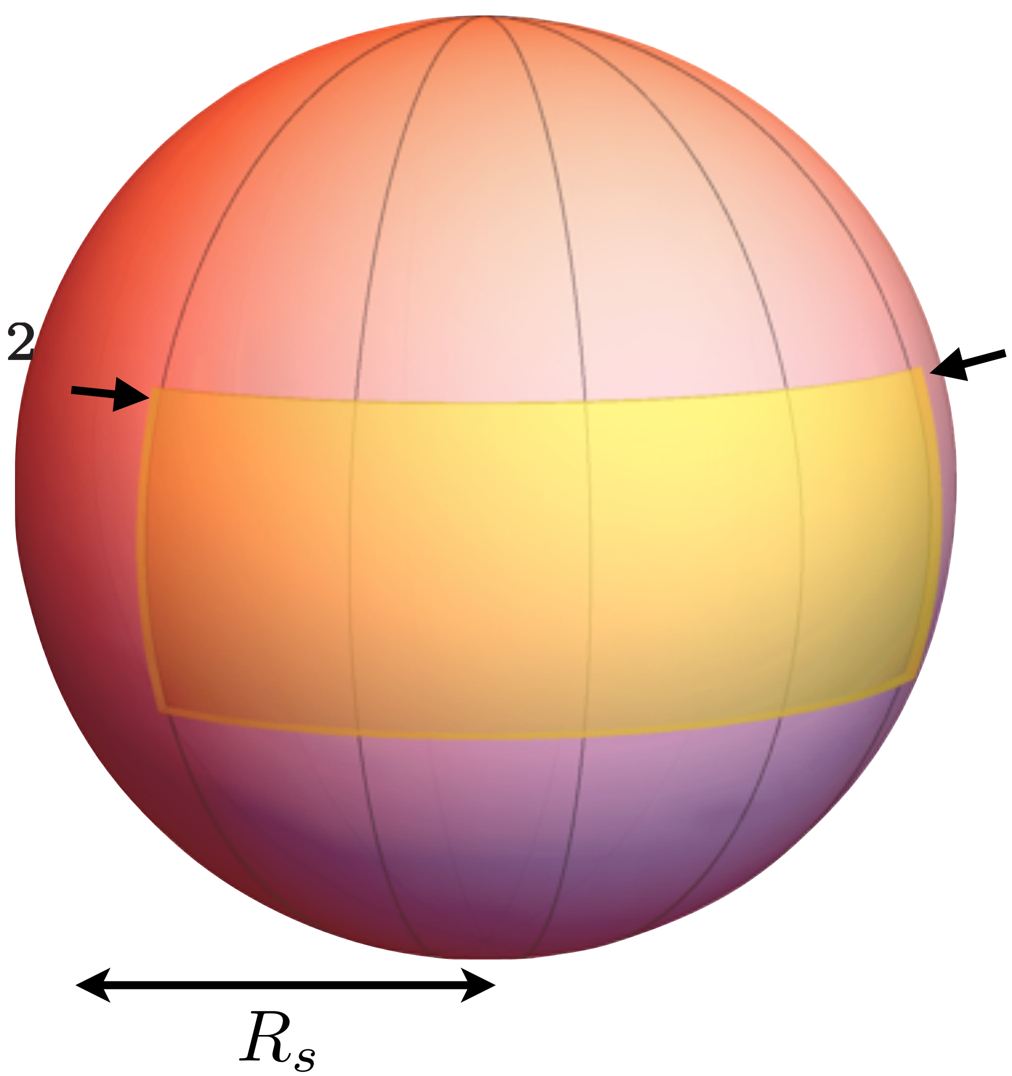
width preserving map



$u_{tt} \approx -(w/R_s)^2$

strain grows with *wide* dimension ⇒ ***large elastic energy***

thickness preserving map

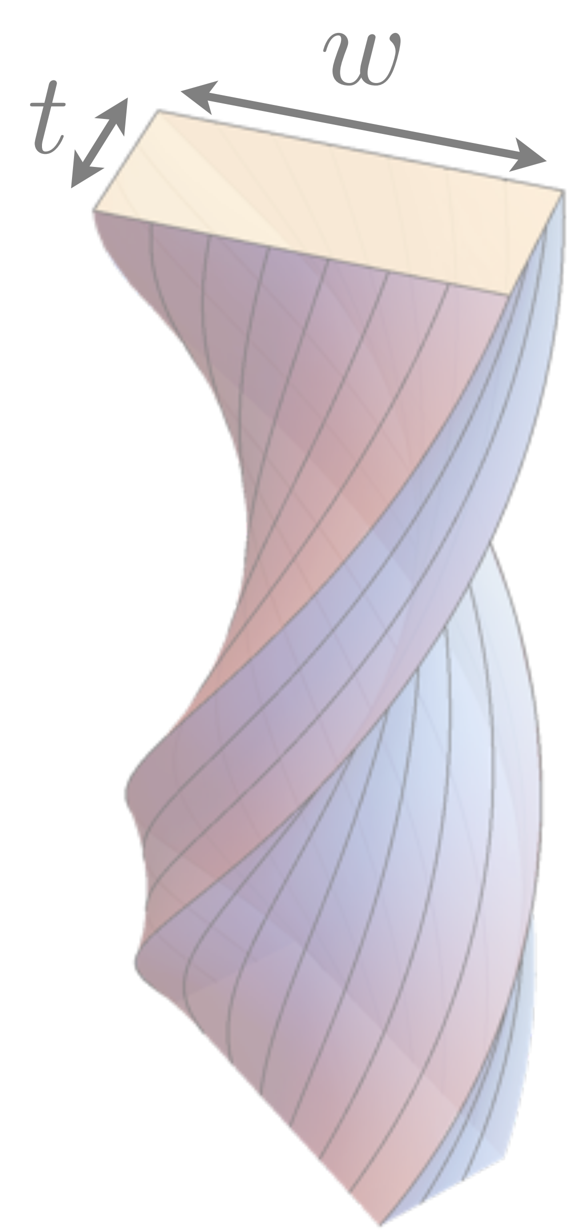


$u_{ww} \approx -(t/R_s)^2$

strain grows with *thin* dimension

⇓

low elastic energy



Anisotropic, Helical "Tapes": Inter- vs. Intra-filament Strain

intra-filament (bending) elasticity — *inter*-filament (packing) elasticity — surface energy (cohesion)

$$f(w \gg t) \simeq \frac{B\rho_0}{24}\Omega^4 w^2 + \frac{Y}{160}(\Omega t)^4 + 2\Sigma\Big(\frac{1}{w} + \frac{1}{t}\Big)$$

$\kappa \approx \Omega^2 w$ — $u_{ww} \approx -(t/R_s)^2$

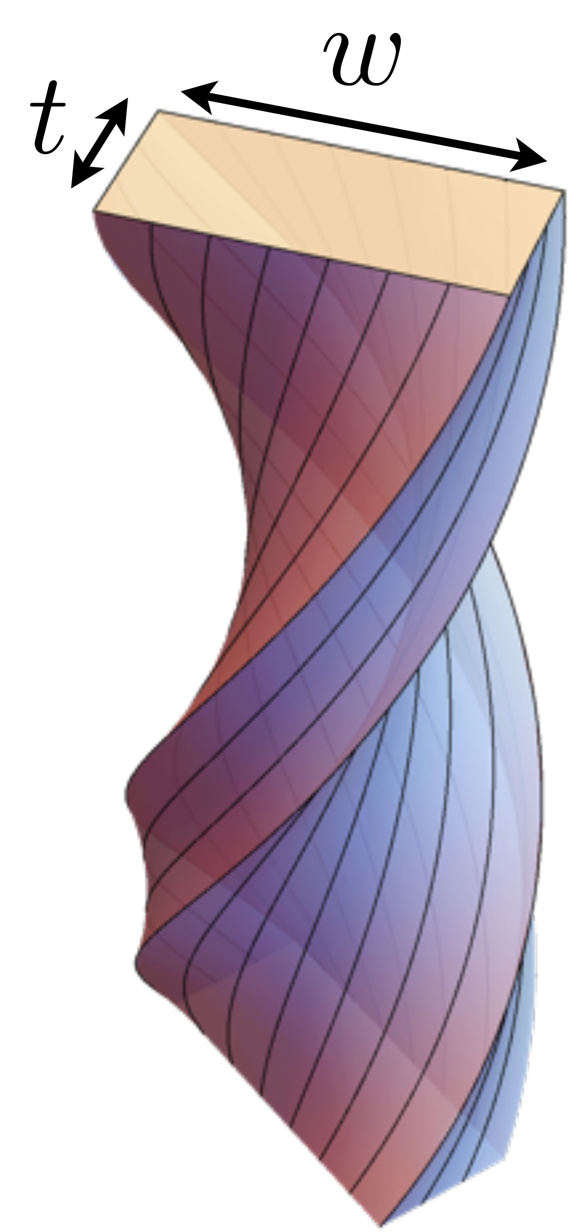
equilibrium dimensions

tape width: **bending** limited

$$w_0 = 24^{1/3}\Big(\frac{\Sigma}{\Omega^4 B\rho_0}\Big)^{1/3}$$

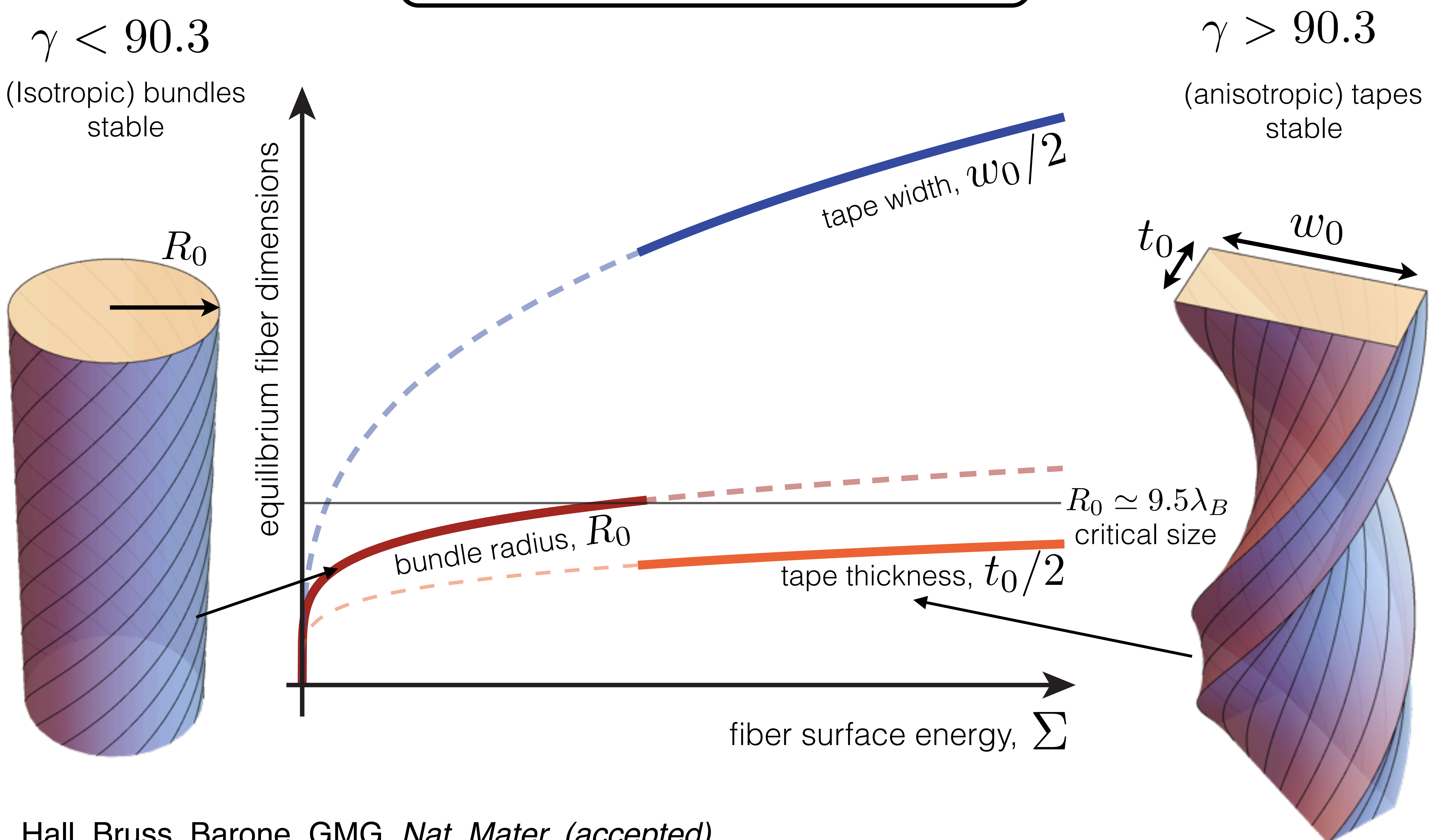
tape thickness: **packing** limited

$$t_0 = 80^{1/5}\Big(\frac{\Sigma}{\Omega^4 Y}\Big)^{1/5}$$



From Bundles to Tapes: Critical Bundle Size

$$\gamma \equiv (R_0/\lambda_B)^2 = \frac{\text{“packing” cost}}{\text{bending cost}}$$

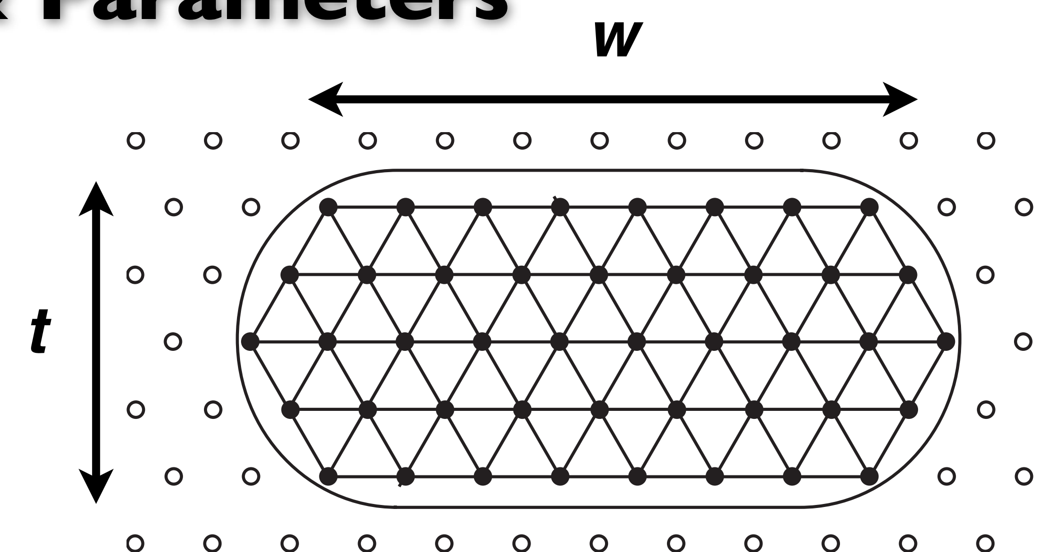


Hall, Bruss, Barone, GMG, *Nat. Mater. (accepted)*.

Discrete filament simulations: Model & Parameters

Doug Hall (UMass)

Method: *(numerically) optimize 2D cross-section and N of filament bundles of fixed twist interacting via attractive, pair-wise forces*



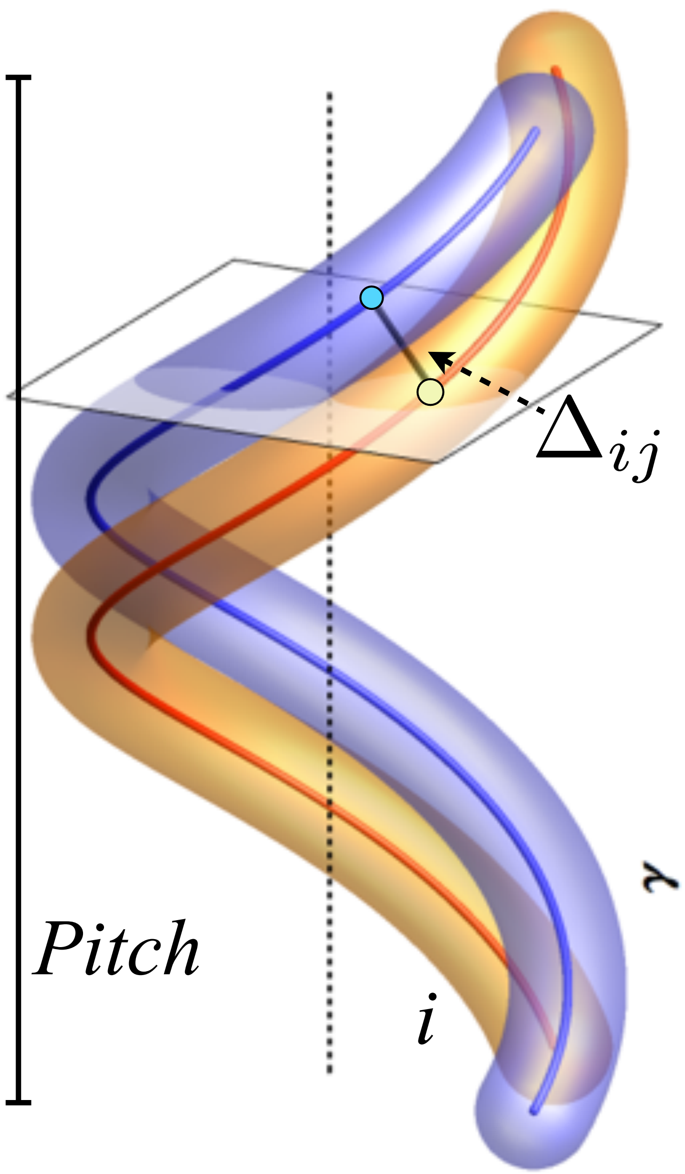
anisotropic trial packing

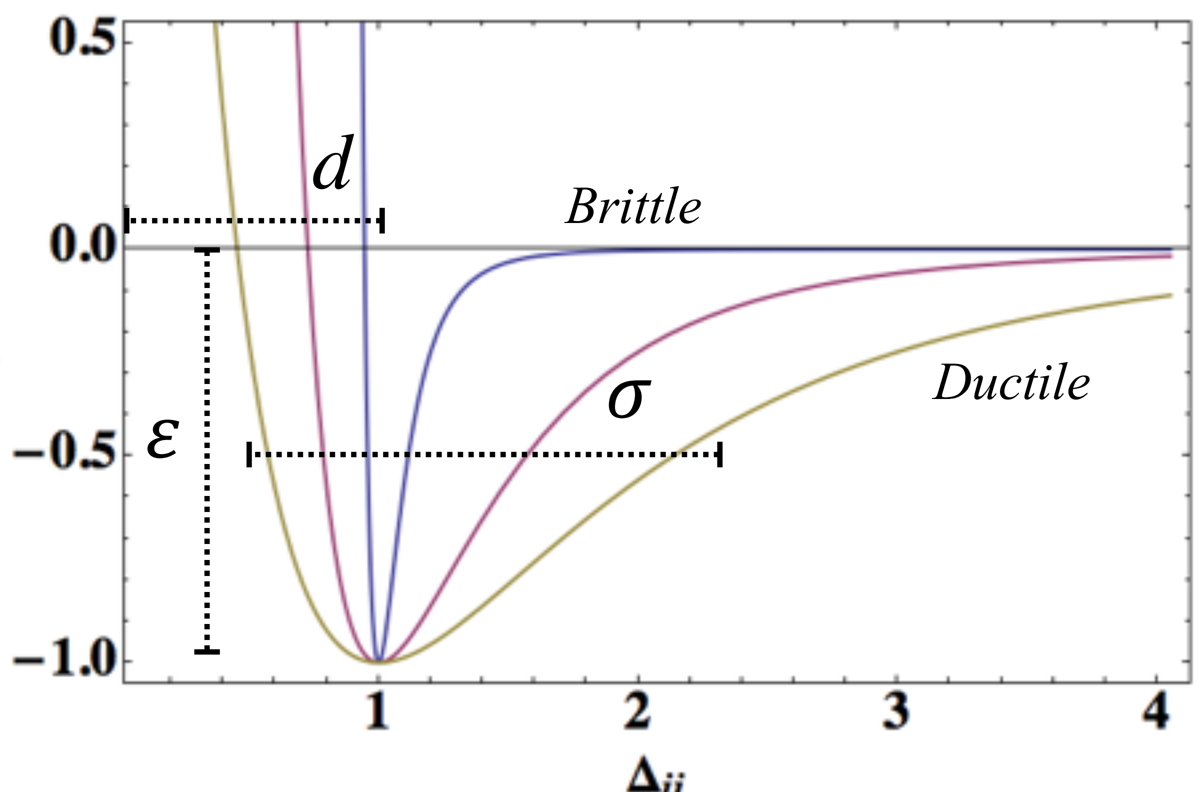
$$E_{Bend} = \frac{B}{2} L \sum_i \kappa_i^2$$

Curvature

$$E_{co} = \sum_{ij} L_{ij} \gamma(\Delta_{ij})$$

Length — *Energy per unit length*





$$\lambda_B \equiv \sqrt{\frac{B\rho_0}{Y}}$$

$$\propto (B/\epsilon)^{1/2}(d/\sigma)$$

$$\lambda_S \equiv \Sigma/Y$$

$$\propto d(\sigma/d)^2$$

Discrete filament simulations: Energy Landscapes & Optimal Morphologies

scaled parameters:

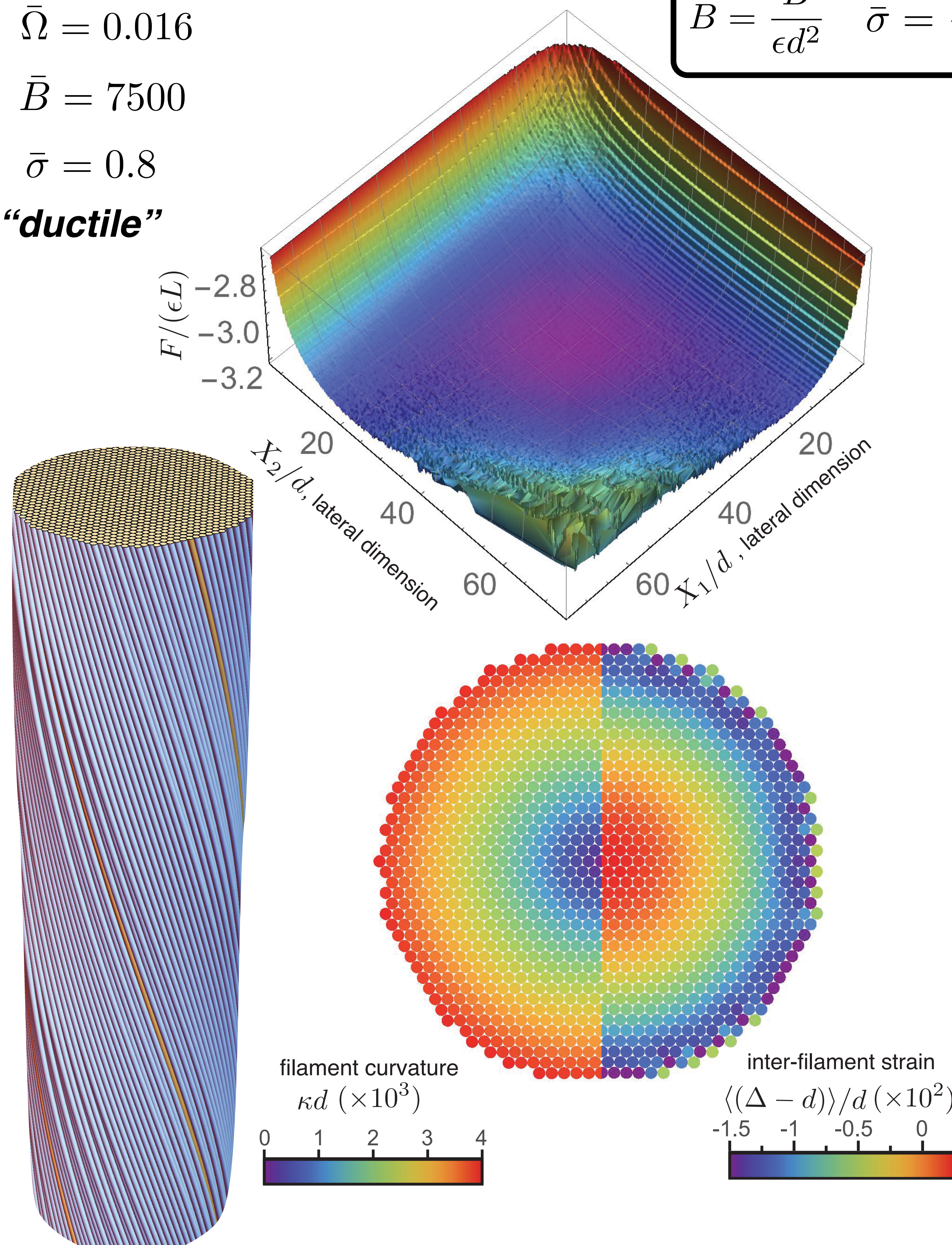
$$\bar{B} = \frac{B}{\epsilon d^2} \quad \bar{\sigma} = \frac{\sigma}{d} \quad \bar{\Omega} = 2\pi d/Pitch$$

$\bar{\Omega} = 0.016$

$\bar{B} = 7500$

$\bar{\sigma} = 0.8$

"ductile"

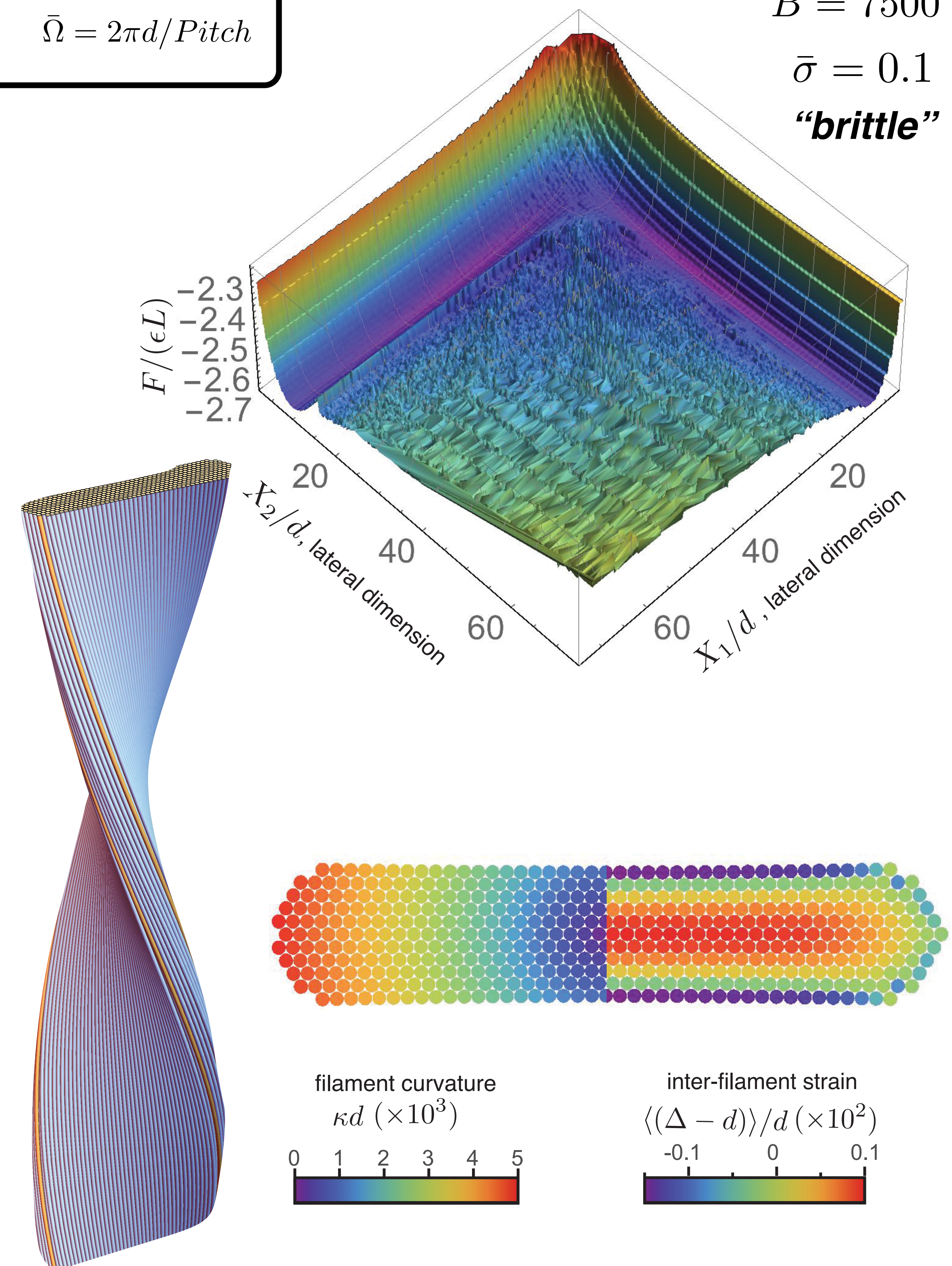


$\bar{\Omega} = 0.016$

$\bar{B} = 7500$

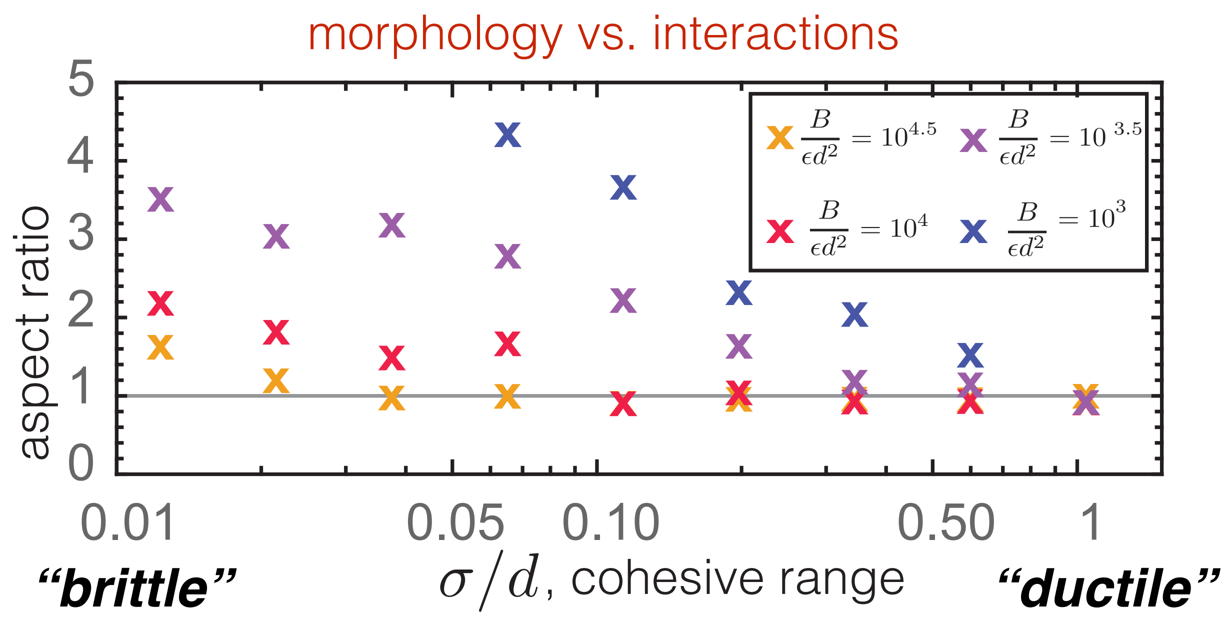
$\bar{\sigma} = 0.1$

"brittle"

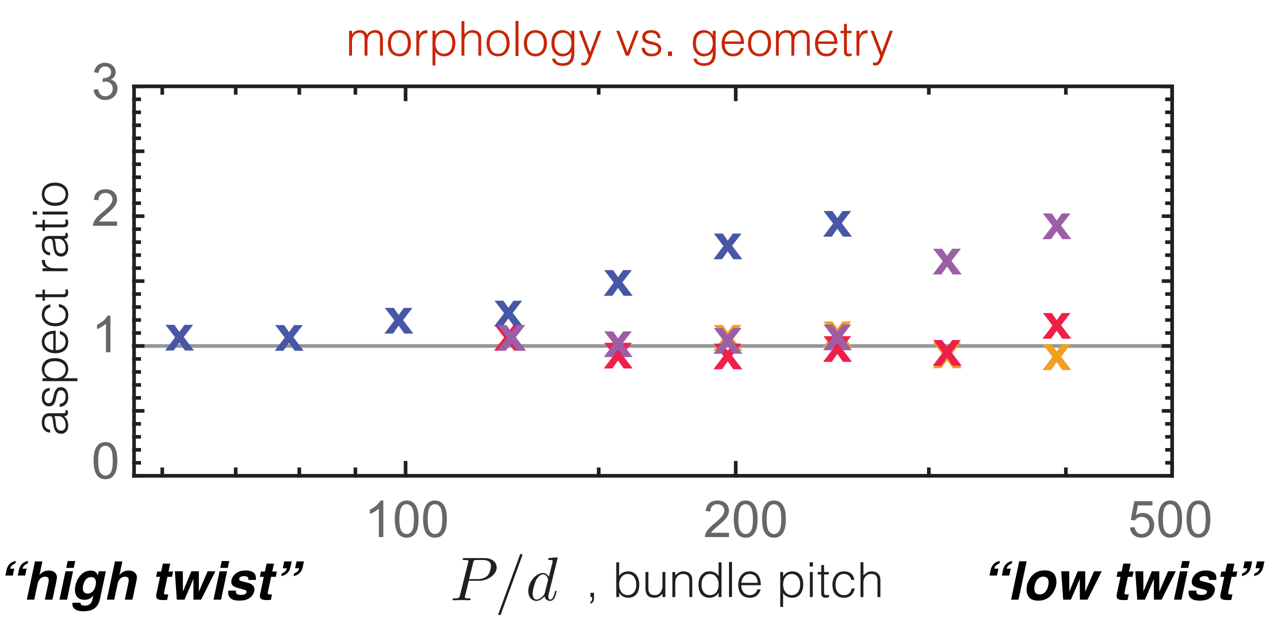


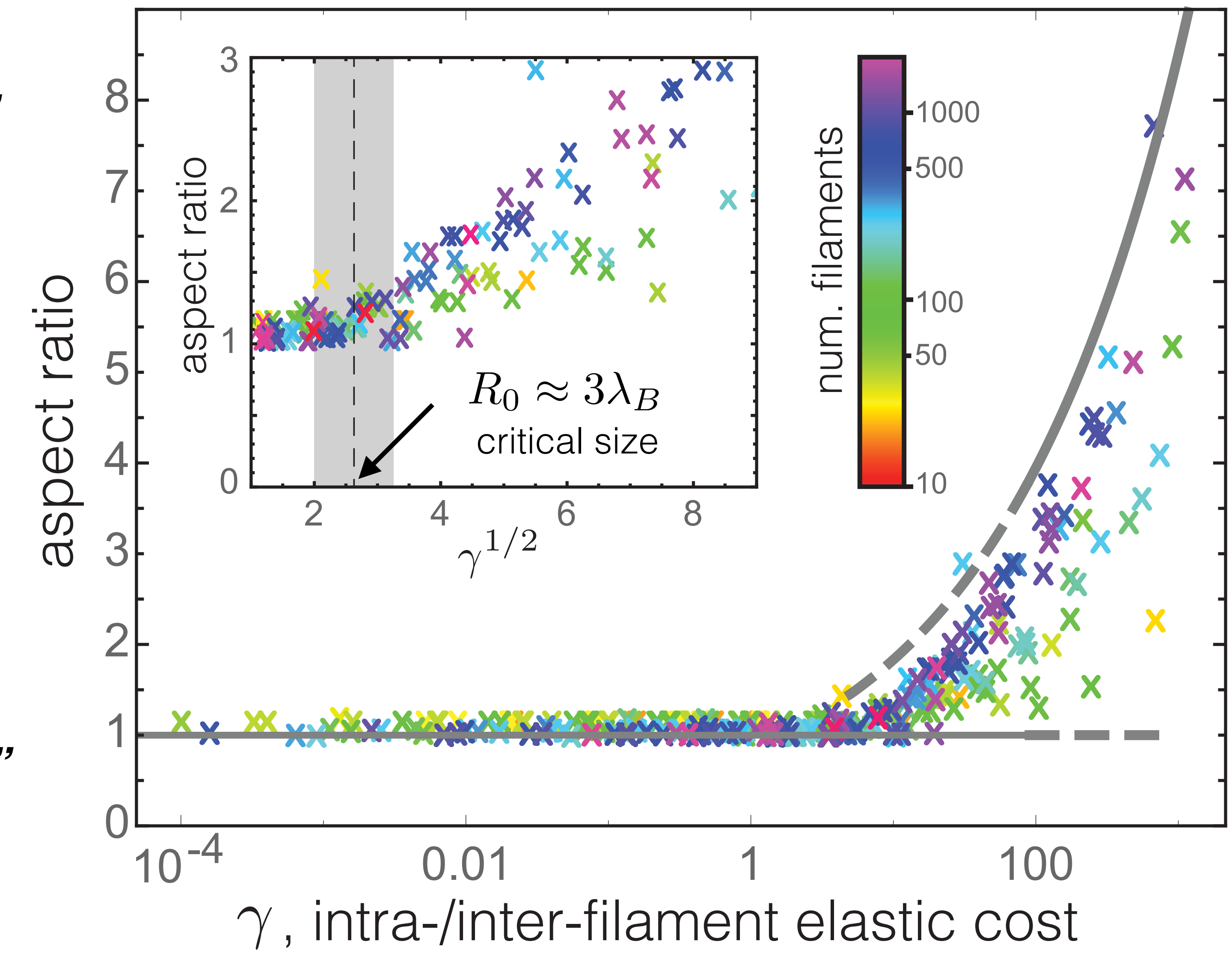
Hall, Bruss, Barone, GMG, *Nat. Mater. (accepted)*.

Discrete filament simulations: Equilibrium aspect ratio



$$\gamma \equiv (R_0/\lambda_B)^2 = \frac{\text{“packing” cost}}{\text{bending cost}}$$





Hall, Bruss, Barone, GMG, *Nat. Mater. (accepted)*.

Twisted Amyloid Tapes: From Mesoscale Dimensions to "Molecular" Scale Parameters

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Evolution of the Amyloid Fiber over Multiple Length Scales

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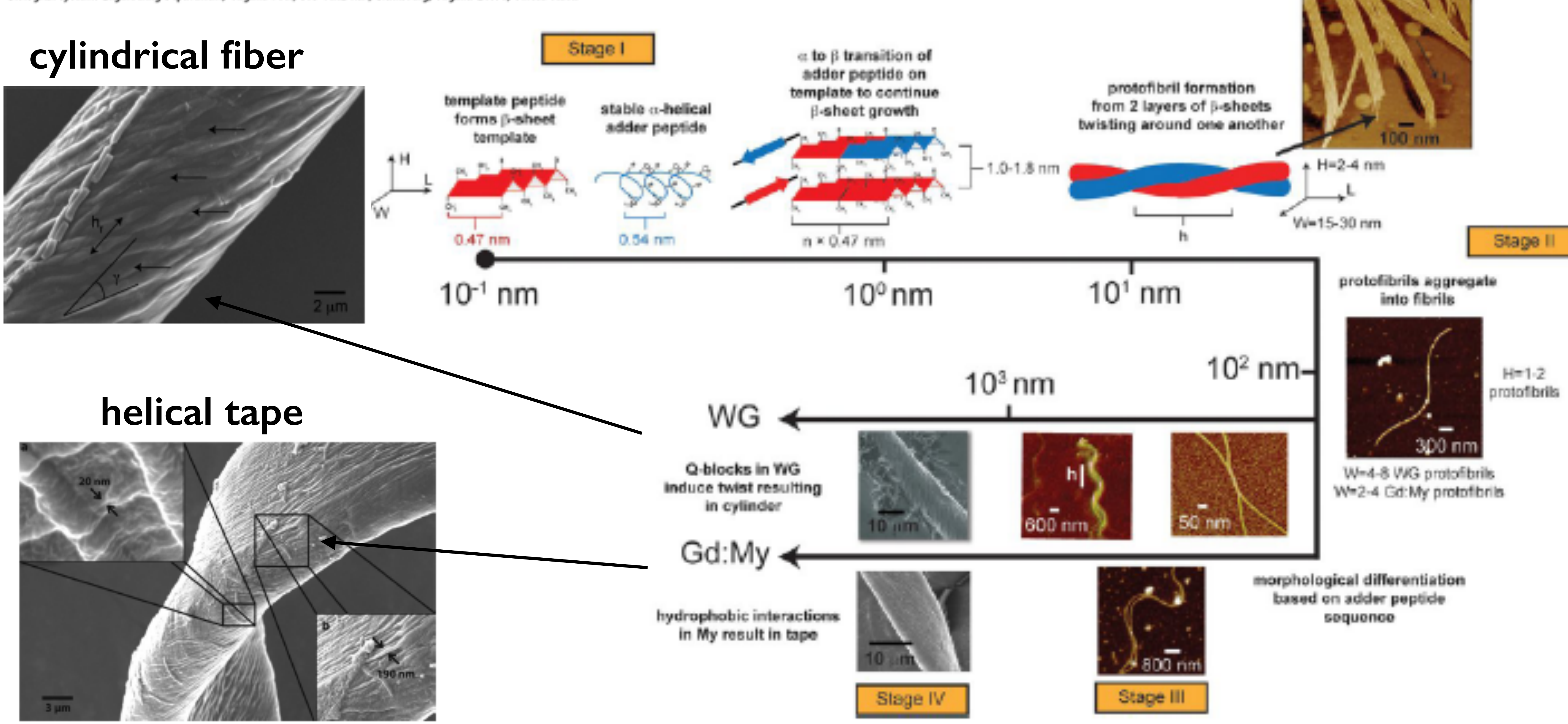
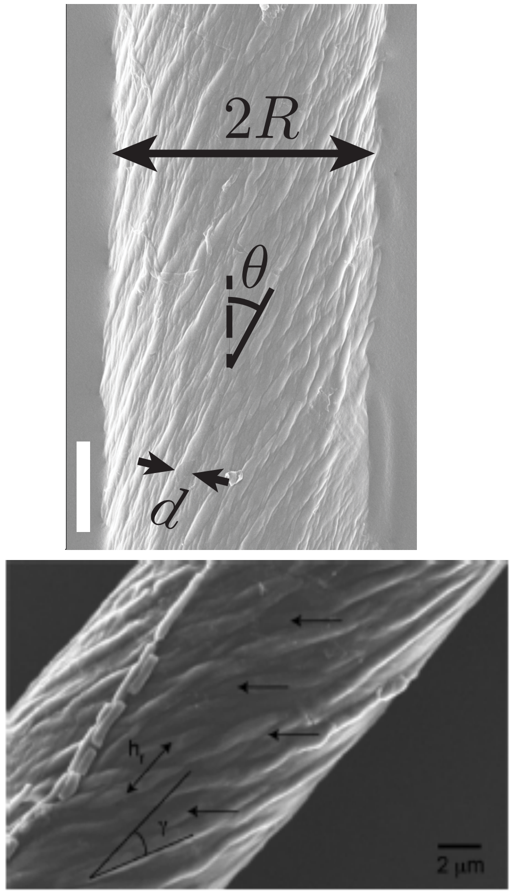
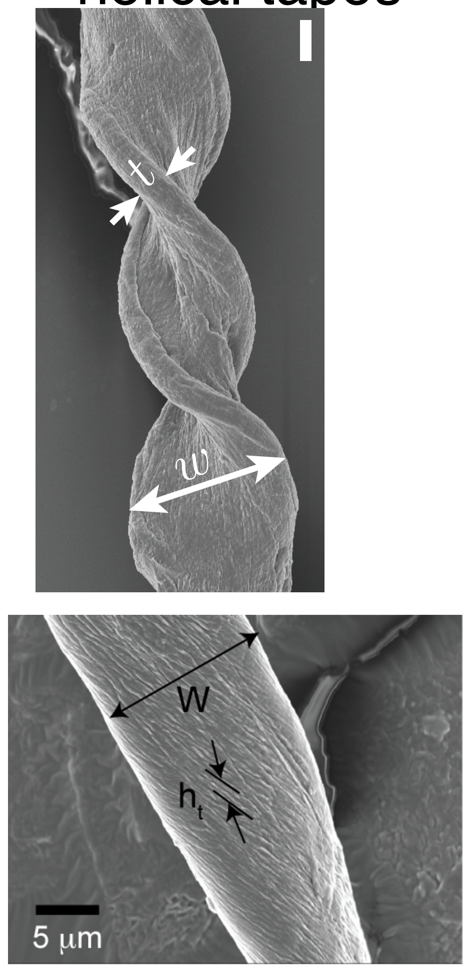


Twisted Amyloid Tapes: From Mesoscale Dimensions to "Molecular" Scale Parameters

cylindrical bundles



helical tapes

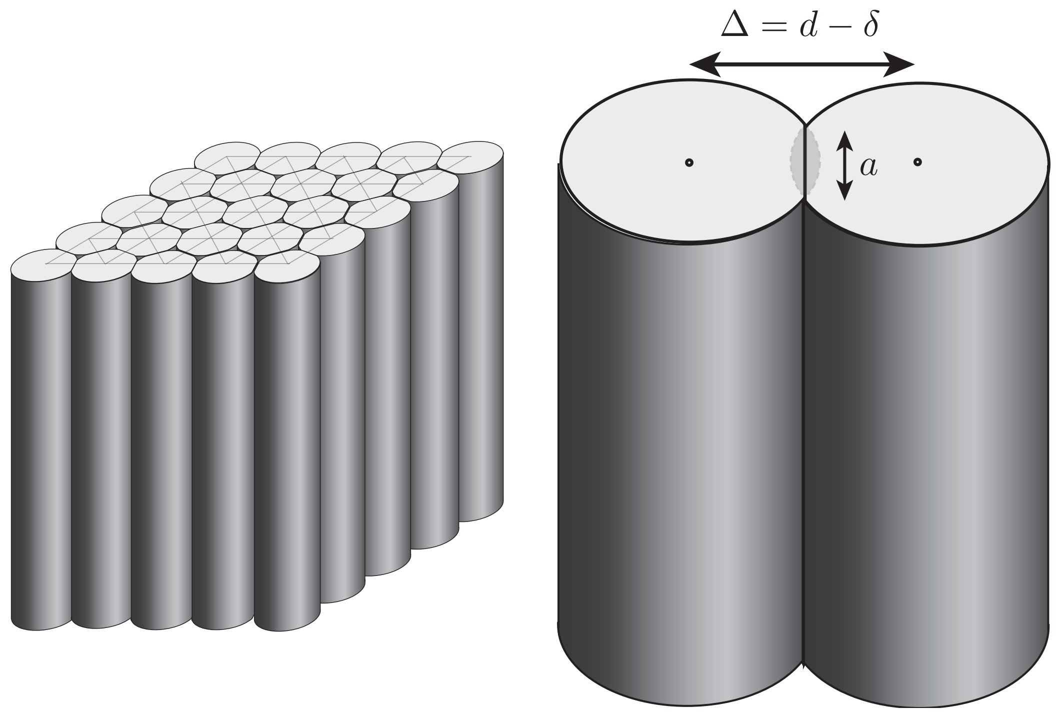


T		pH 4	6	8	10	100 mM NaCl
22 °C	WG			▬		
	Gd:My			●/▬		
37 °C	WG	●	●/▬	●	●/▬	▬
	Gd:My	●/▬	▬	●/▬	▬	▬
60 °C	WG			▬		
	Gd:My			●/▬		
80 °C	WG			▬		
	Gd:My			●/▬		

Ridgely & Barone, *Soft Matter* (2012).

Twisted Amyloid Tapes: From Mesoscale Dimensions to "Molecular" Scale Parameters

cohesive elastic cylinder array



predicted tape dimensions:

$$w_0 = 24^{1/3}\left(\frac{\Sigma}{\Omega^4 B \rho_0}\right)^{1/3} \qquad t_0 = 80^{1/5}\left(\frac{\Sigma}{\Omega^4 Y}\right)^{1/5}$$

characteristic lengths (material dependent):

$$\lambda_B \equiv \sqrt{\frac{B\rho_0}{Y}} = \sqrt{\frac{3t_0^5}{10w_0^3}} \qquad \lambda_S \equiv \Sigma/Y = \frac{\Omega^4 t_0^5}{80}$$

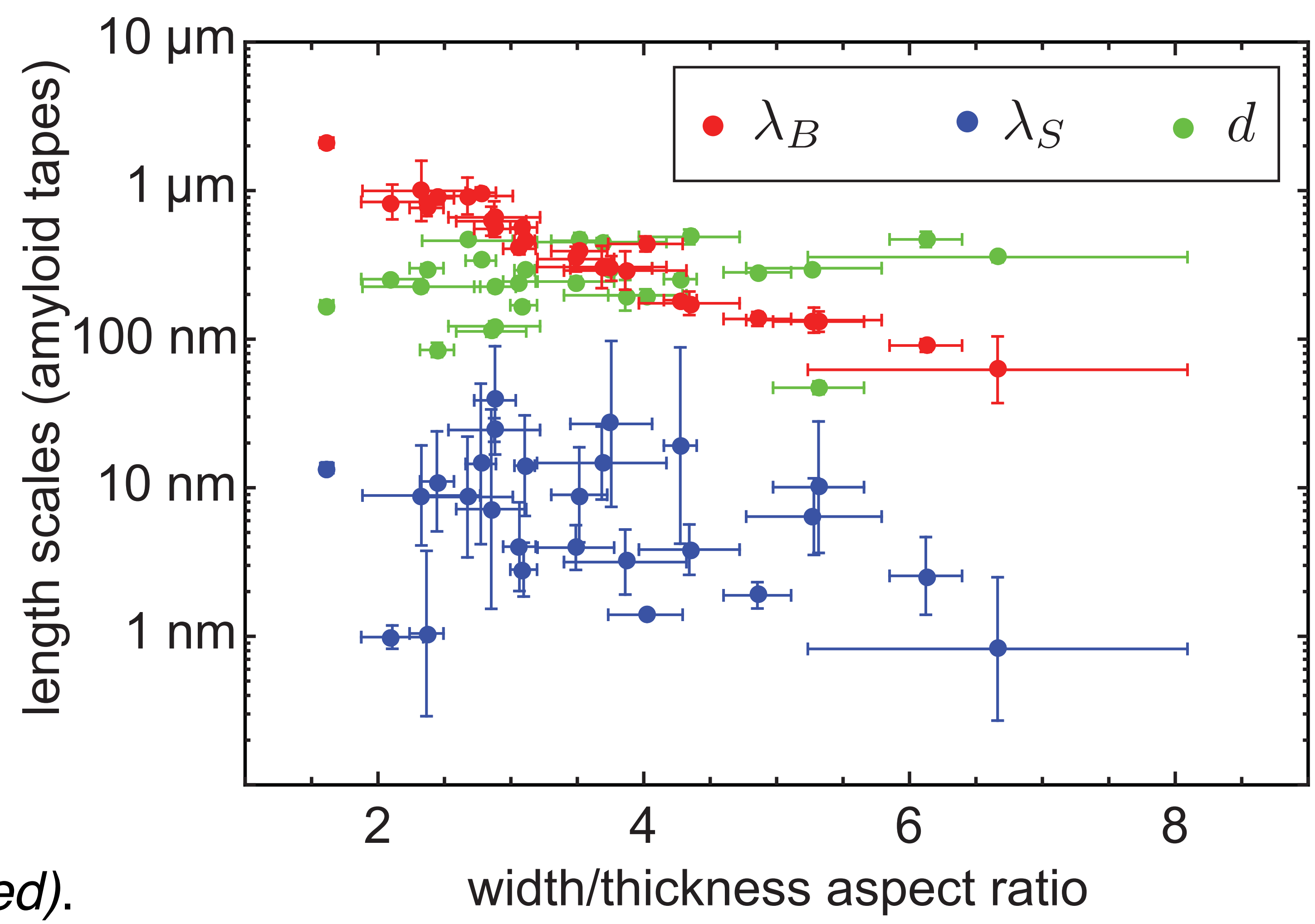
array modulus   filament modulus

$$Y \approx E \Rightarrow \lambda_B \sim d$$

$$\rho_0 B \approx E d^2$$

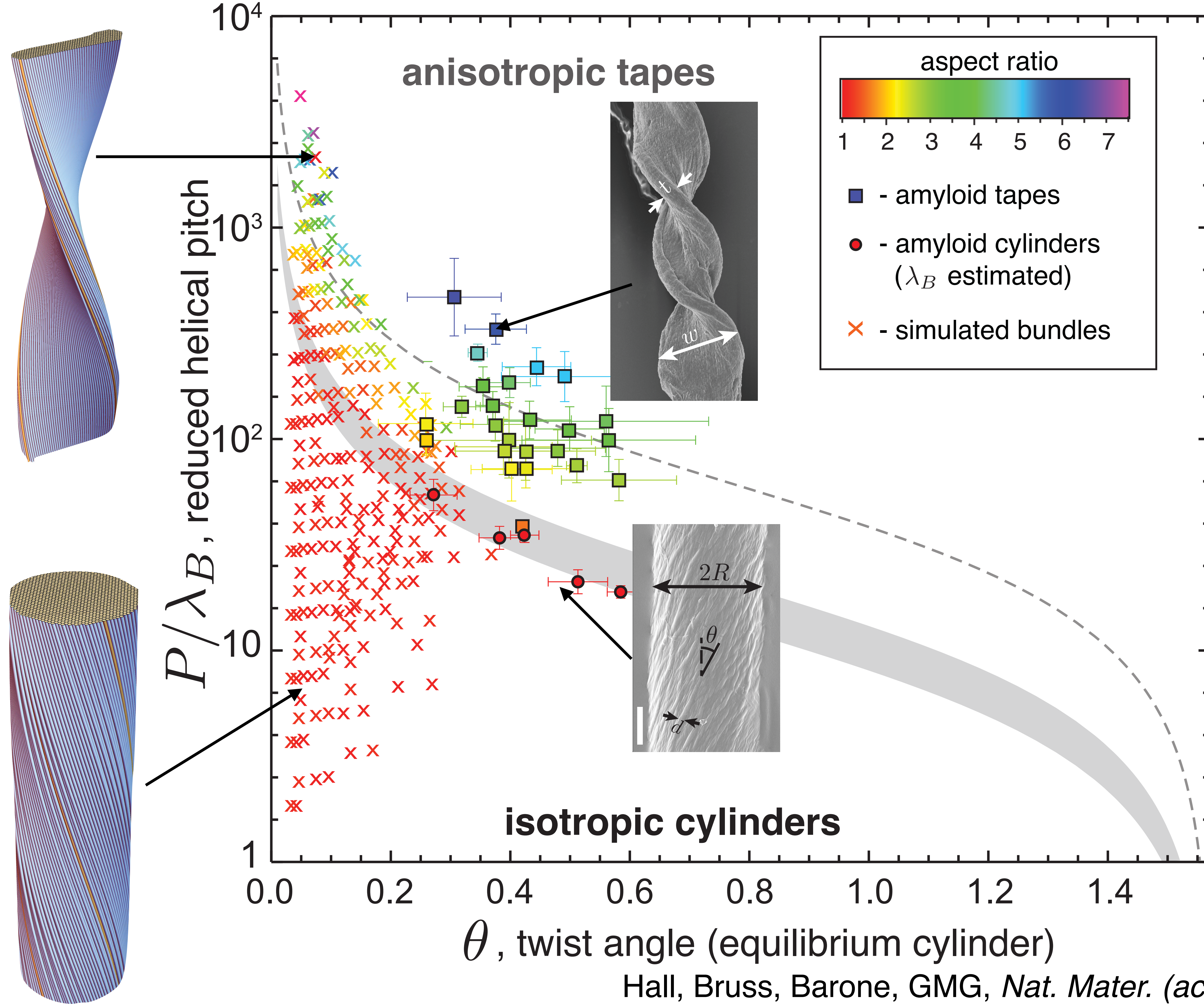
$$\sigma \sim \delta \Rightarrow \lambda_S \sim \delta^2/d \ll d$$

cohesive range   contact "depth"



Hall, Bruss, Barone, GMG, *Nat. Mater. (accepted)*.

Bundle morphology: a universal phase diagram



Hall, Bruss, Barone, GMG, *Nat. Mater. (accepted)*.

Summary (Frustration & Morphology Selection):

1) Geometric strains drive anisotropy in fiber sections above critical size

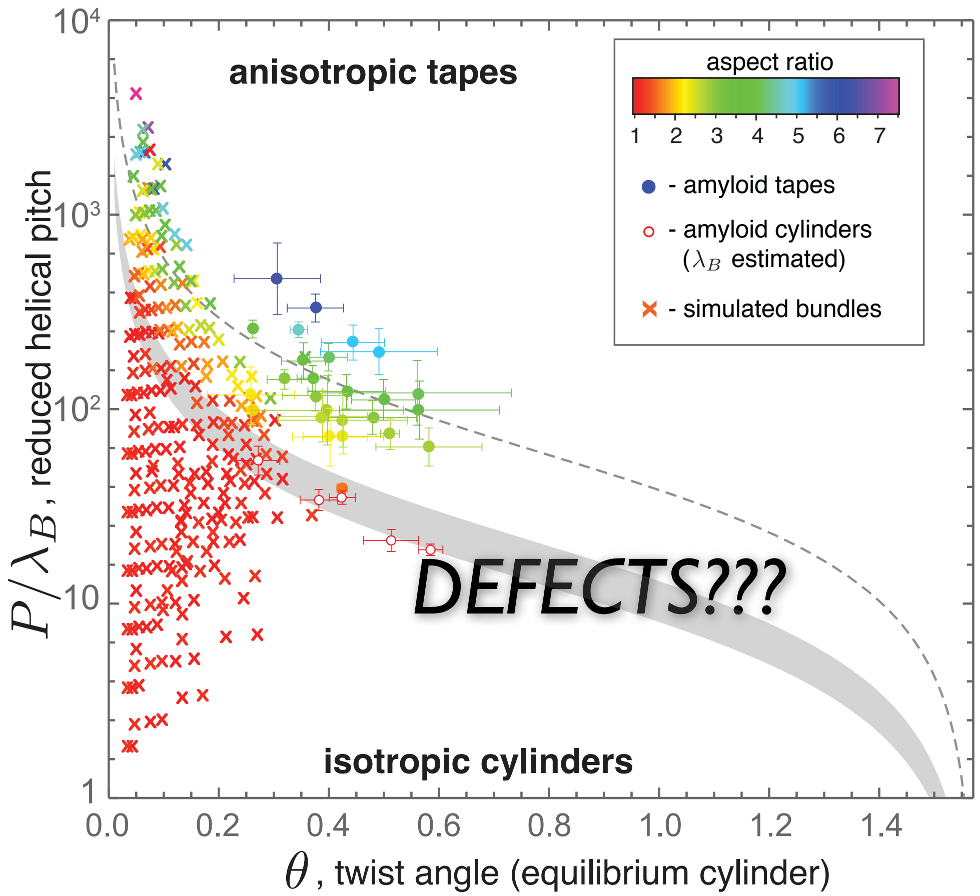
2) Tape width/thickness dimensions are selected by intra-filament (bending)/ intra-filament (packing) elasticity

3) Mesoscale dimensions directly quantify microscopic parameters (inter-filament elasticity; intra-filament elasticity; inter-fil. cohesion)

Open questions:

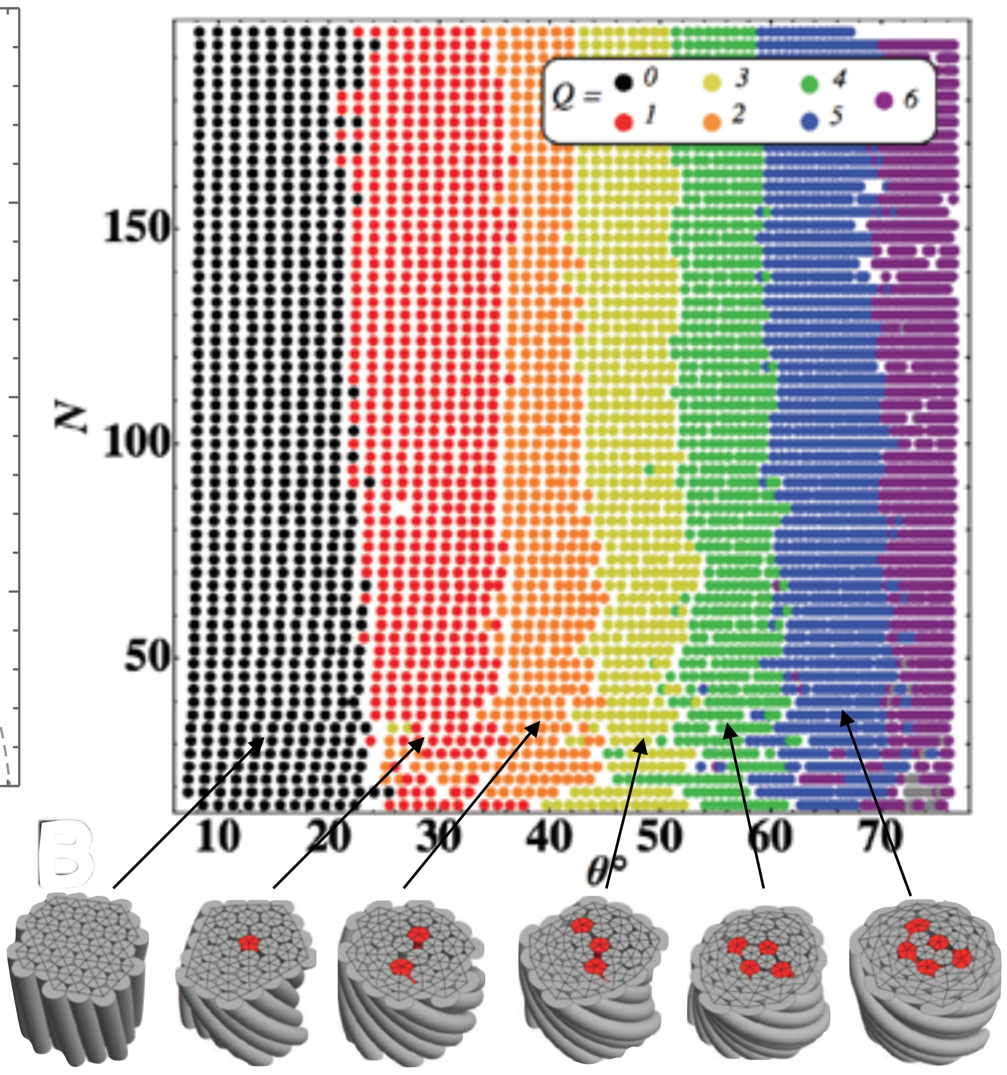
Stiff filaments at large twist?

How do defects mitigate bundle/tape transition & alter self-limitation?

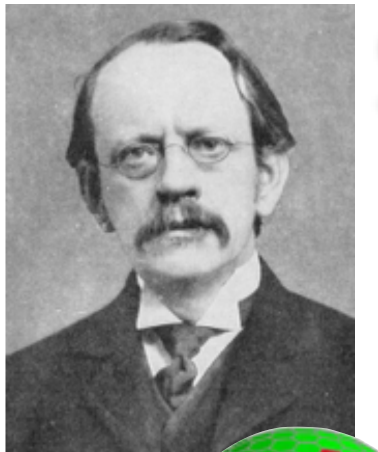


Disclinations in twisted bundles

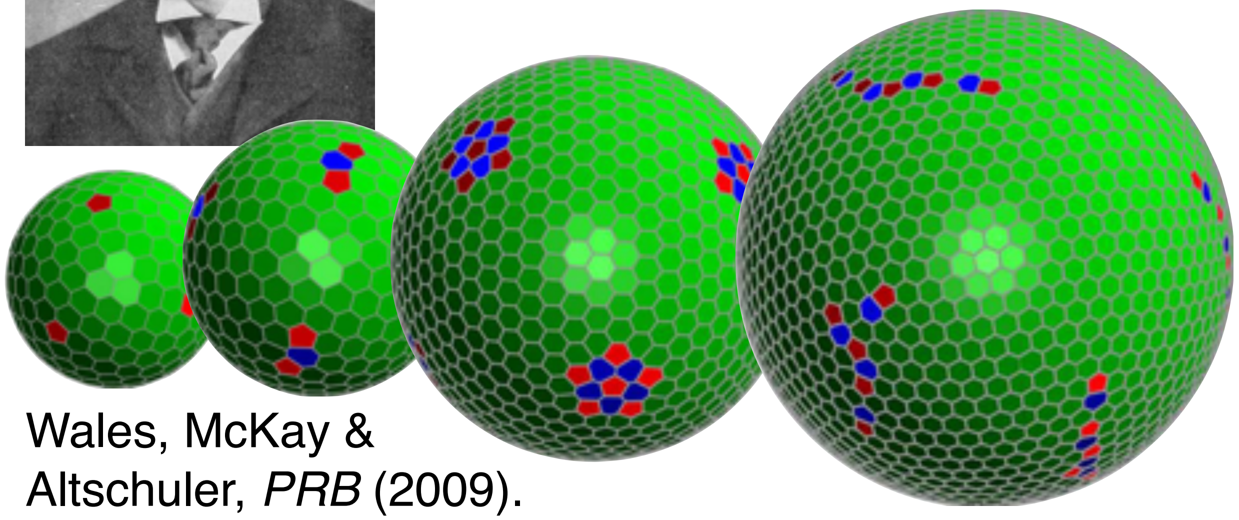
(Bruss & GMG PNAS, 2012; Soft Matter 2013)



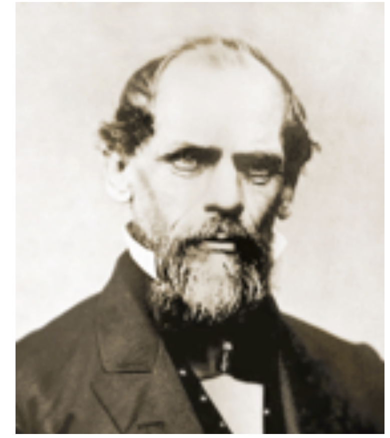
Geometry of Filamentous Matter:



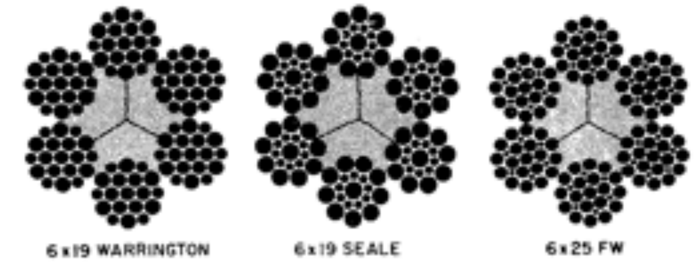
Optimal packing on spheres:
Thomson Problem (1904)

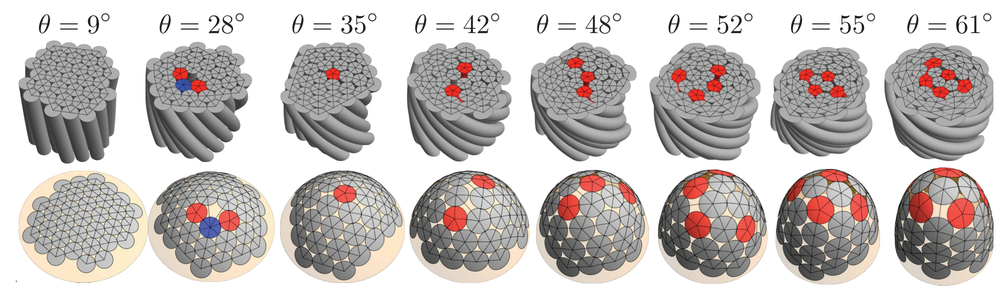


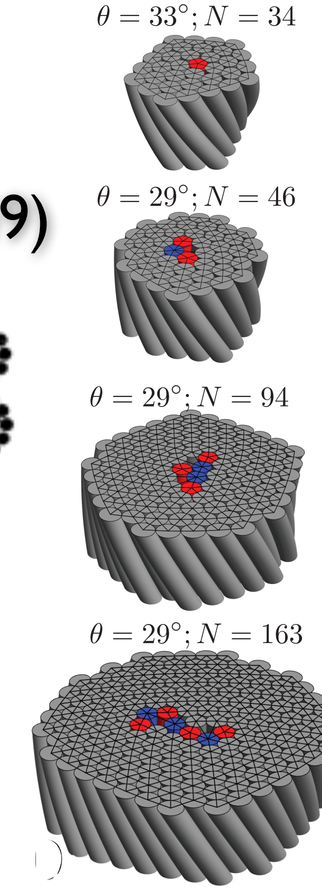
Wales, McKay & Altschuler, *PRB* (2009).



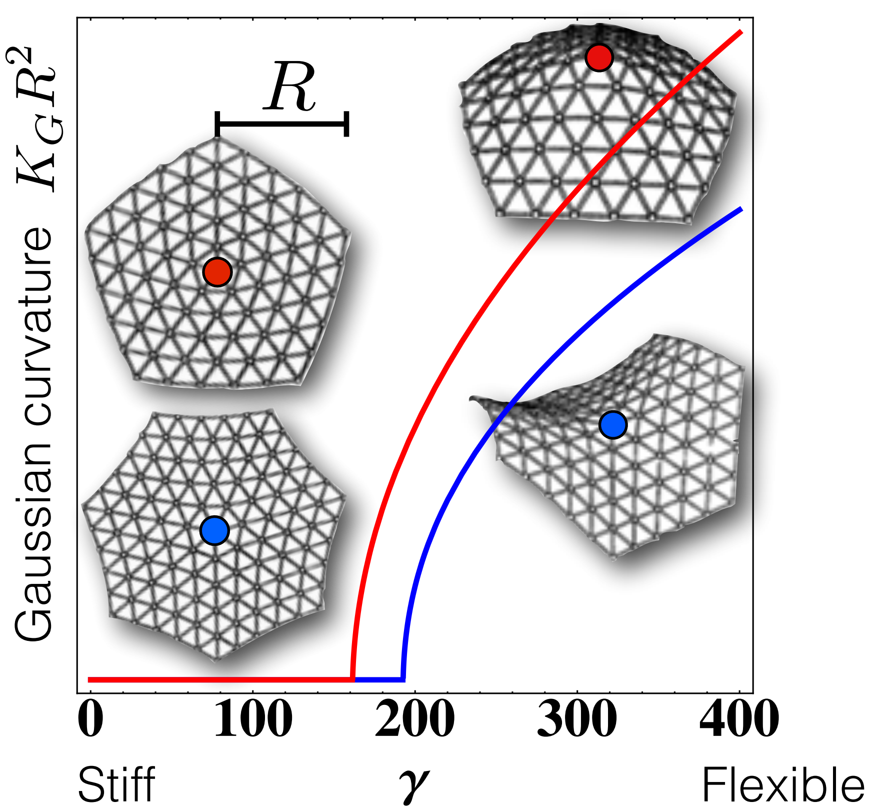
Optimal packing in ropes:
"Roebling Problem" (~1849)



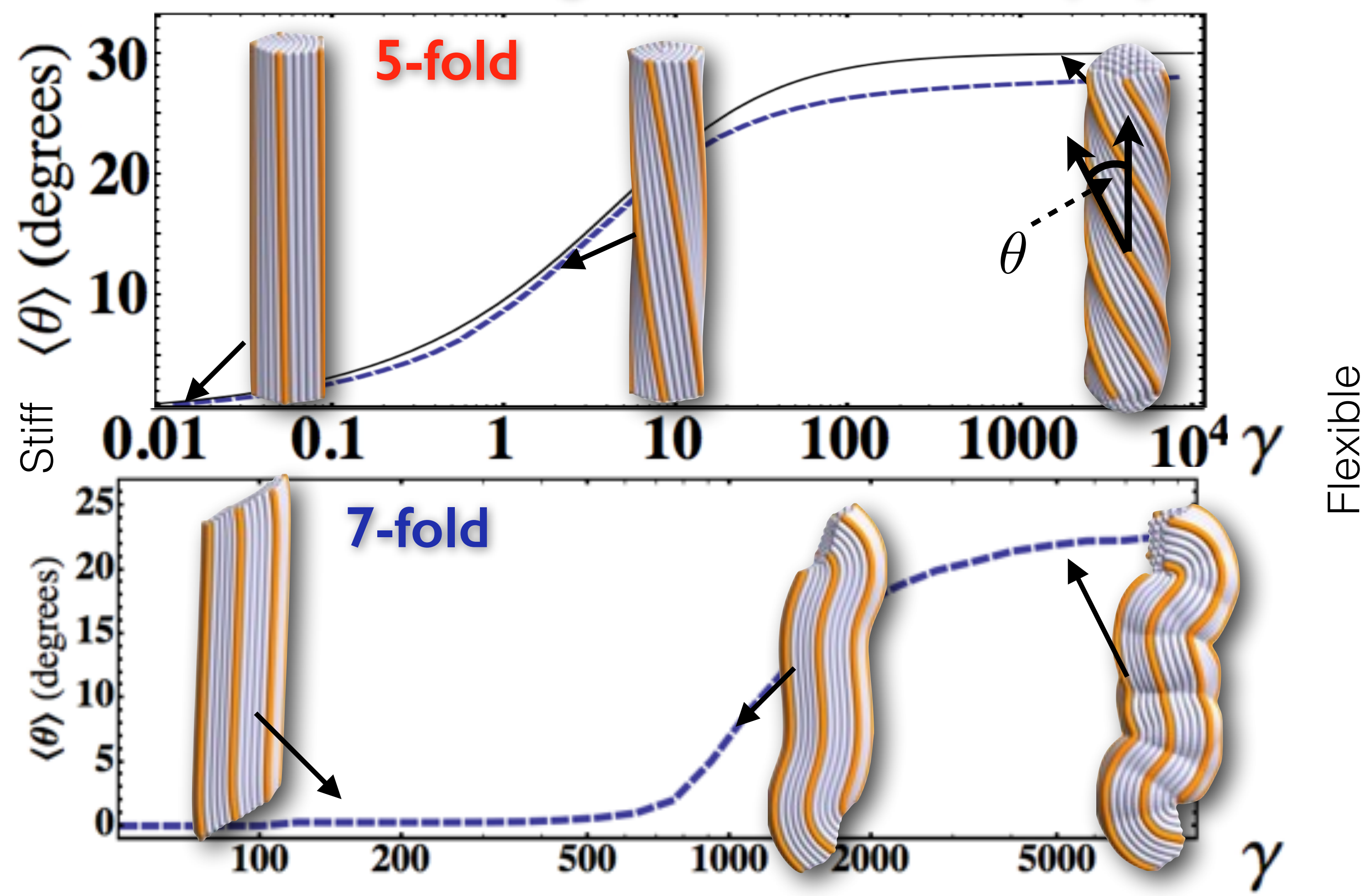




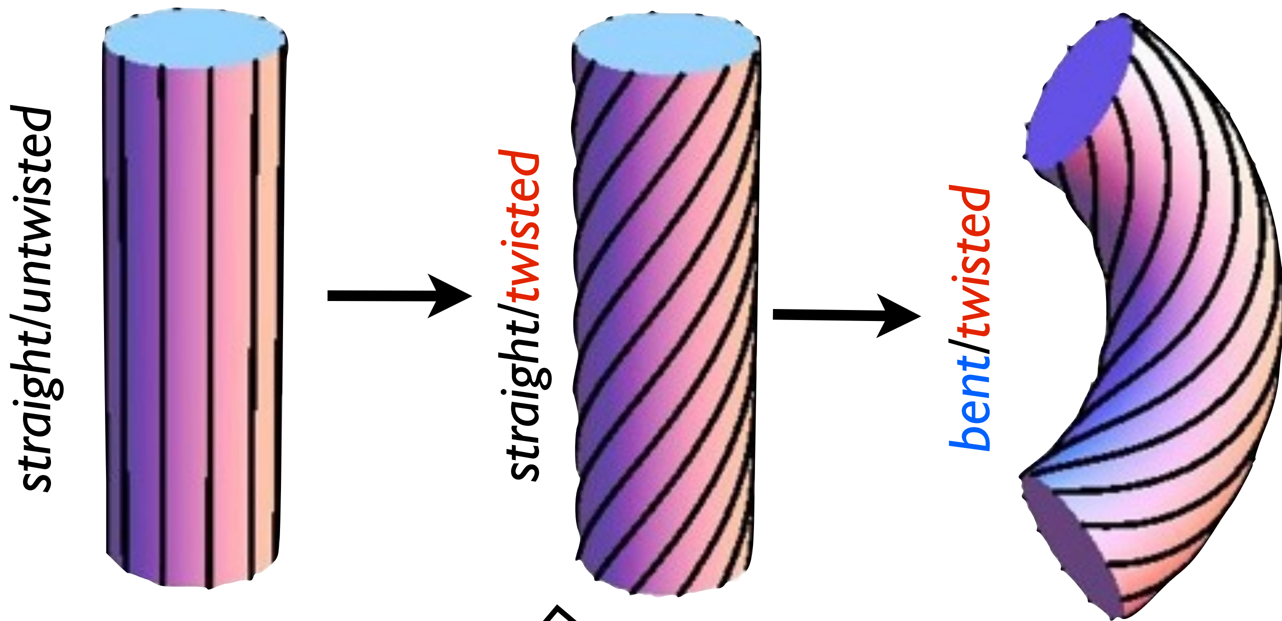
Defect-induced buckling of 2D x-tals: Seung & Nelson (1988)



Defect-induced buckling of bundles: Bruss *in preparation*

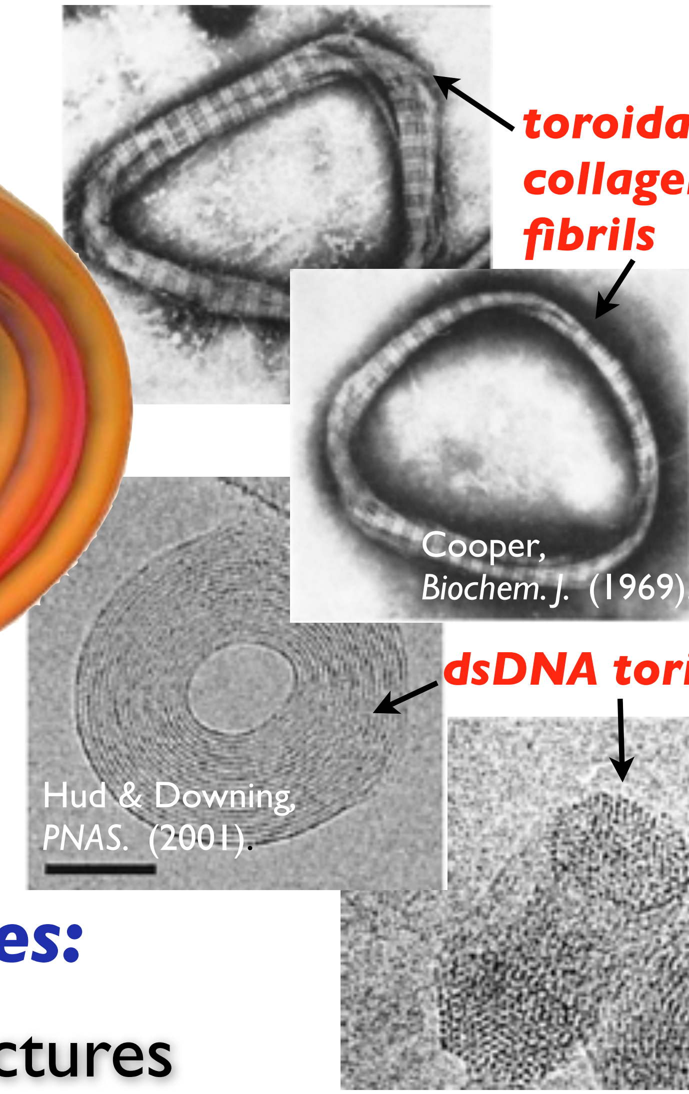


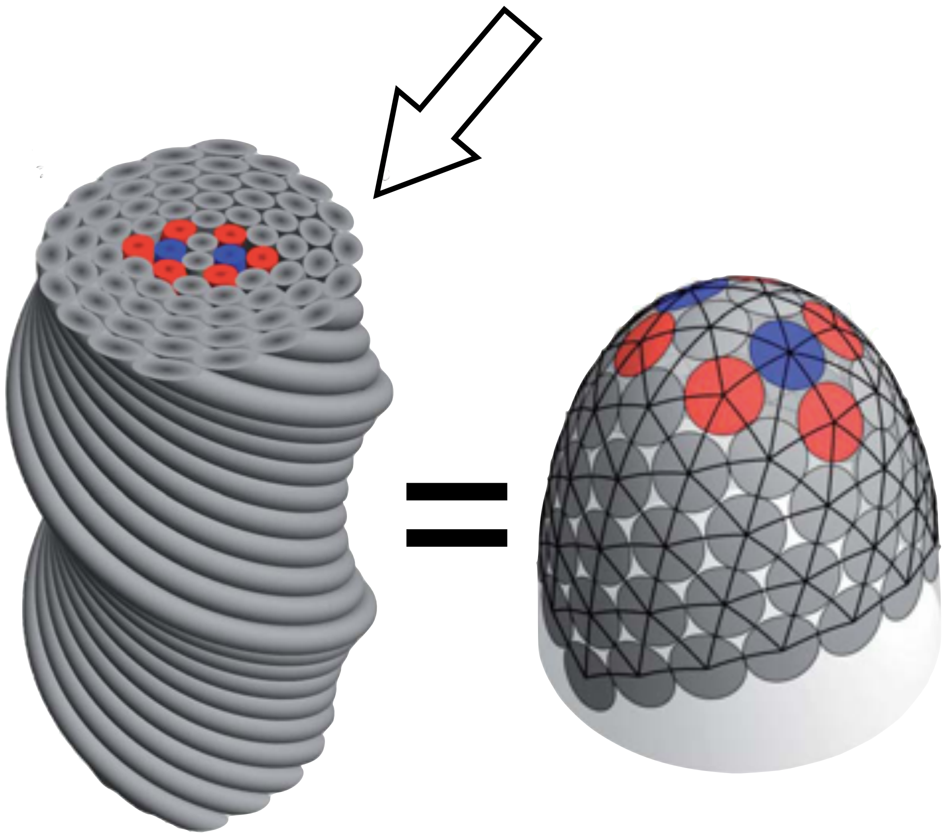
Packing filament beyond pure twist:



Prototypes of *bent* & *twisted* bundles:

twisted toroidal bundles





screw symmetry (straight/twisted) ⇨ *equidistance filament contact encoded in single 2D surface*

"Cartography" of twisted toroidal bundles:

What are appropriate structures (embeddings) for "charting"/packing intrinsic inter-filament geometry?

How does metric geometry (or geometries) depend on bundle *curvature*?

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- Justin Barone (V.Tech)
- Benny Davidovitch (UMass)