Biomimetic 4D Printing

Sabetta Matsumoto
KITP 22 January 2016

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Shape-morphing Systems

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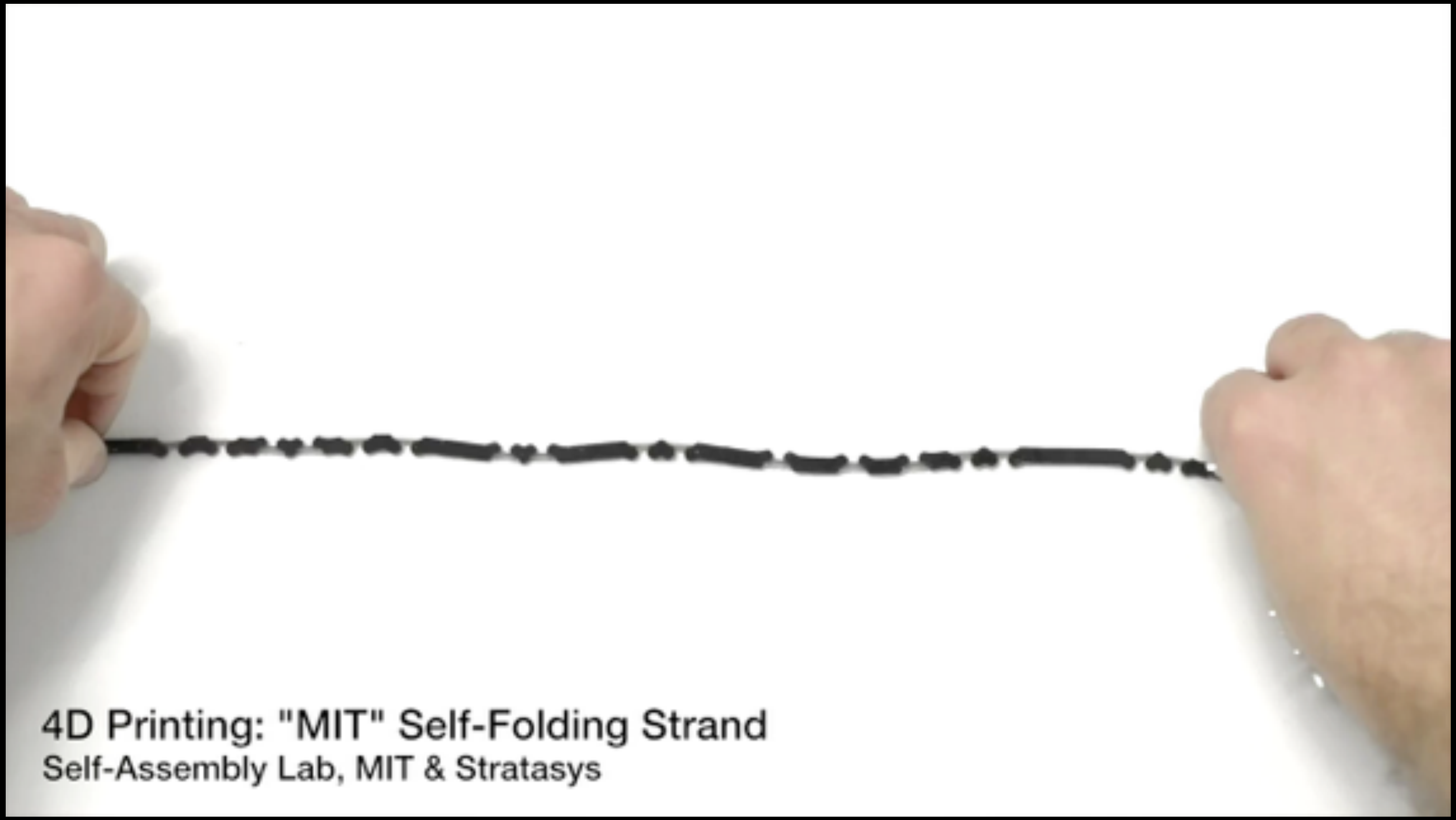
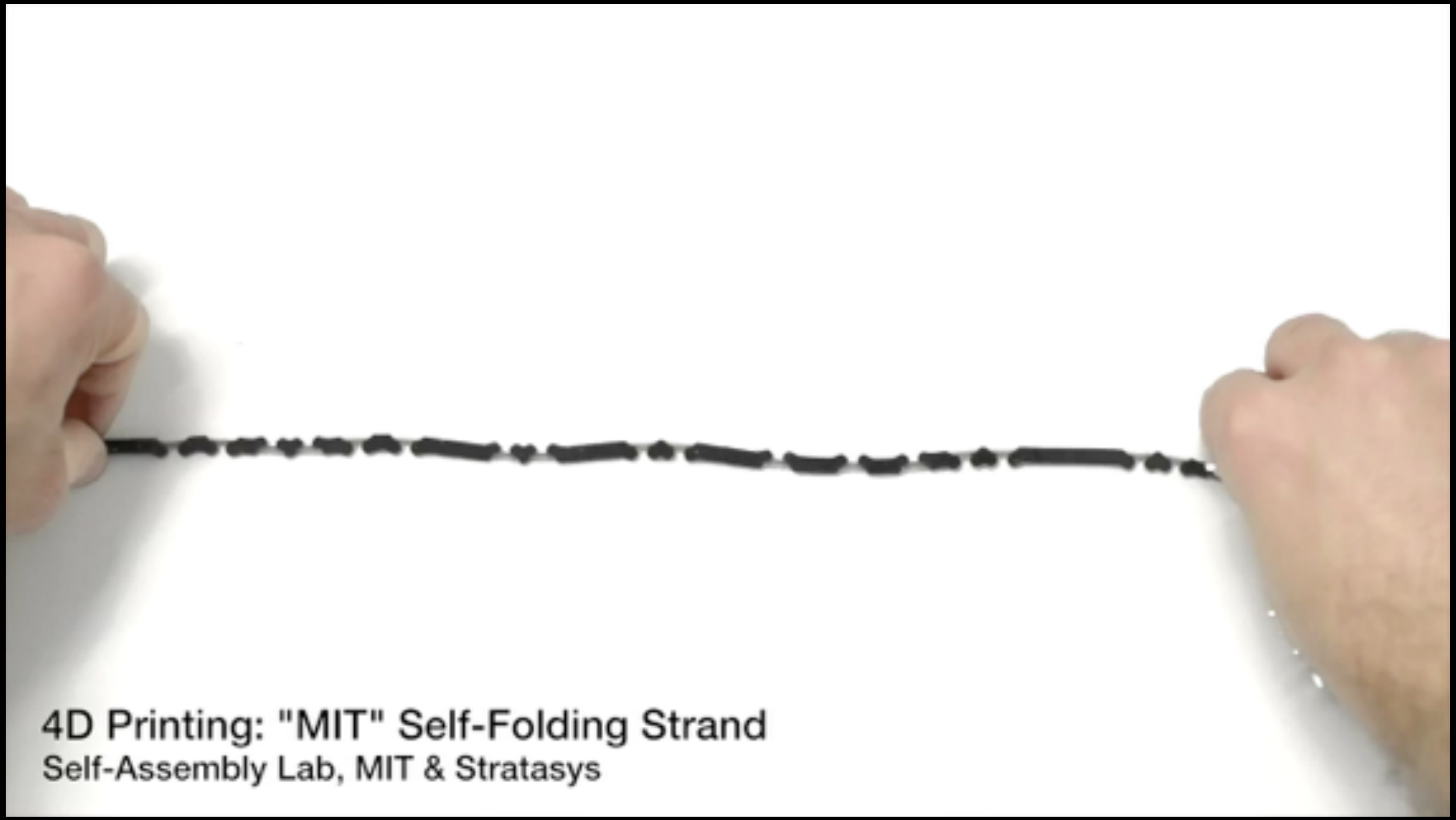
4D Printing


4D Printing: "MIT" Self-Folding Strand
Self-Assembly Lab, MIT & Stratasys

4D Printing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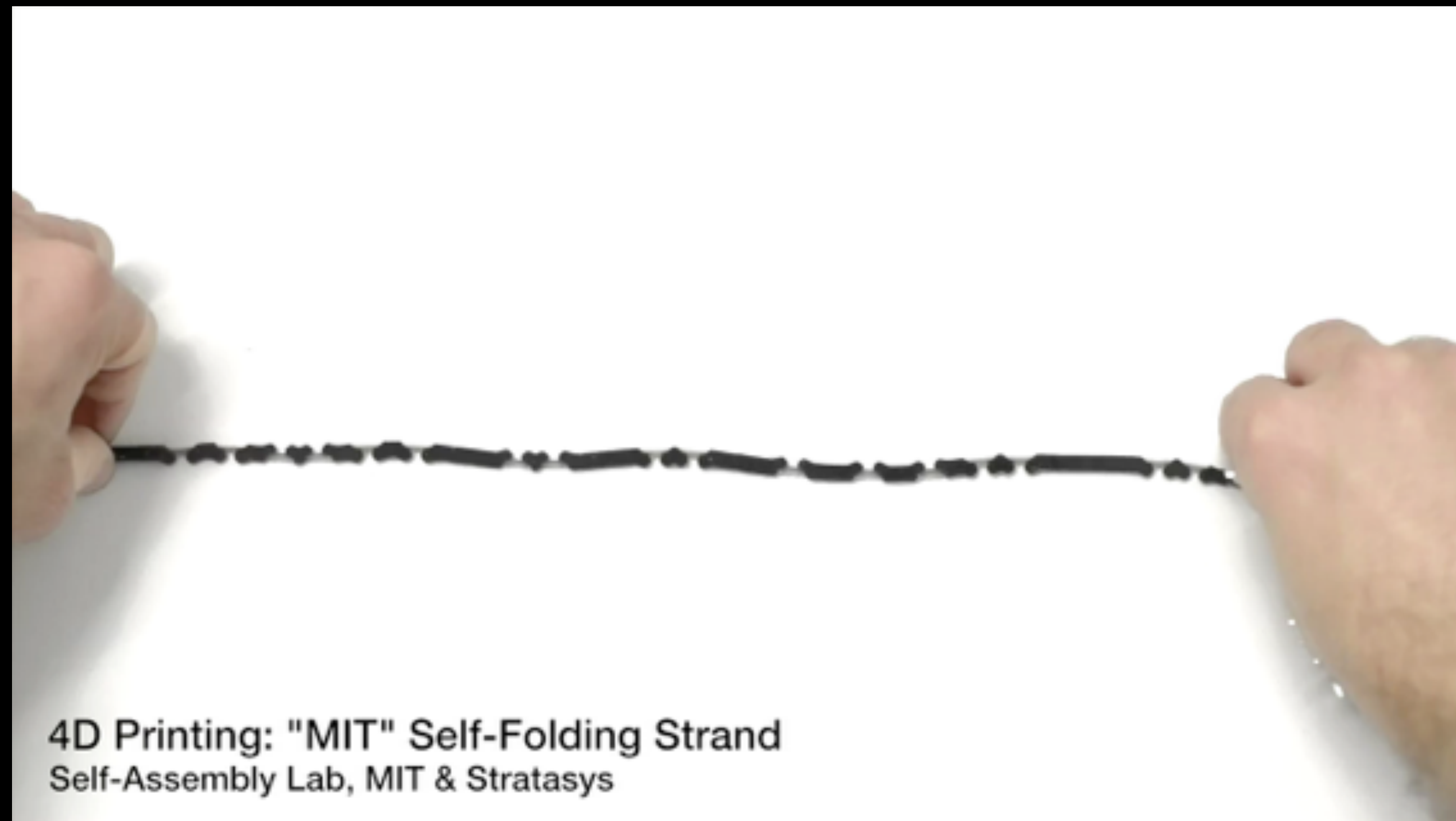
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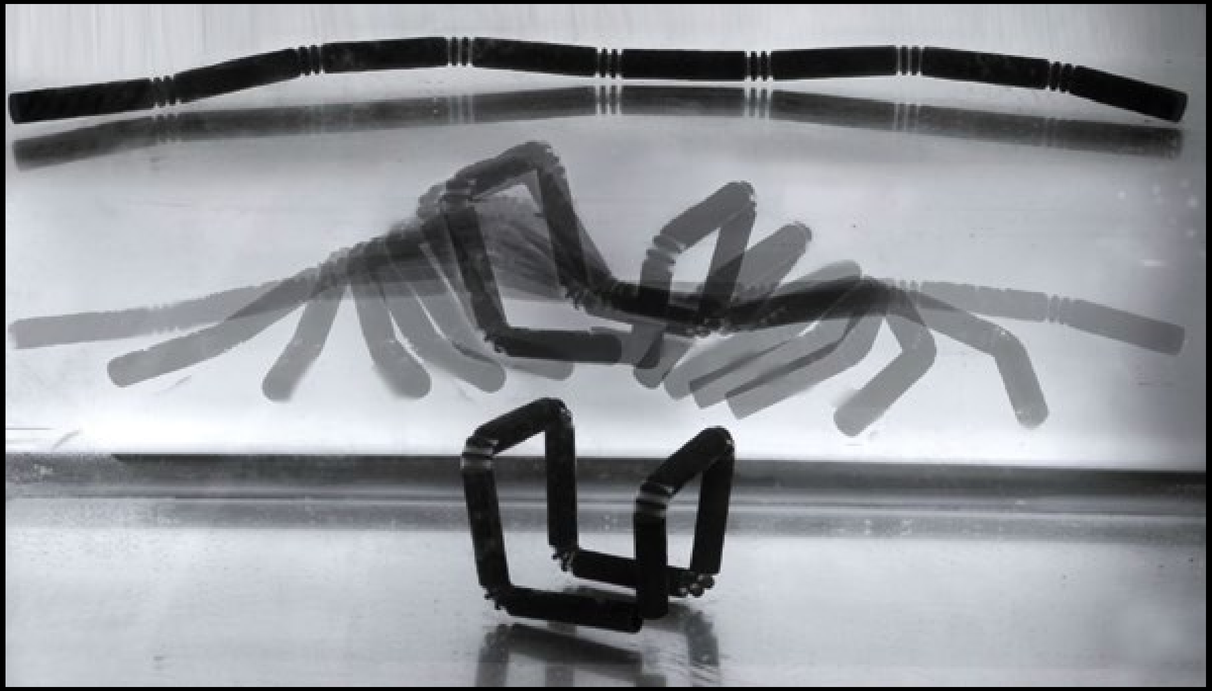
Shape-morphing Systems

Mechanical Hinges



Tibbits, *Arch. Design* **84** 116 (2014)

Shape-morphing Systems

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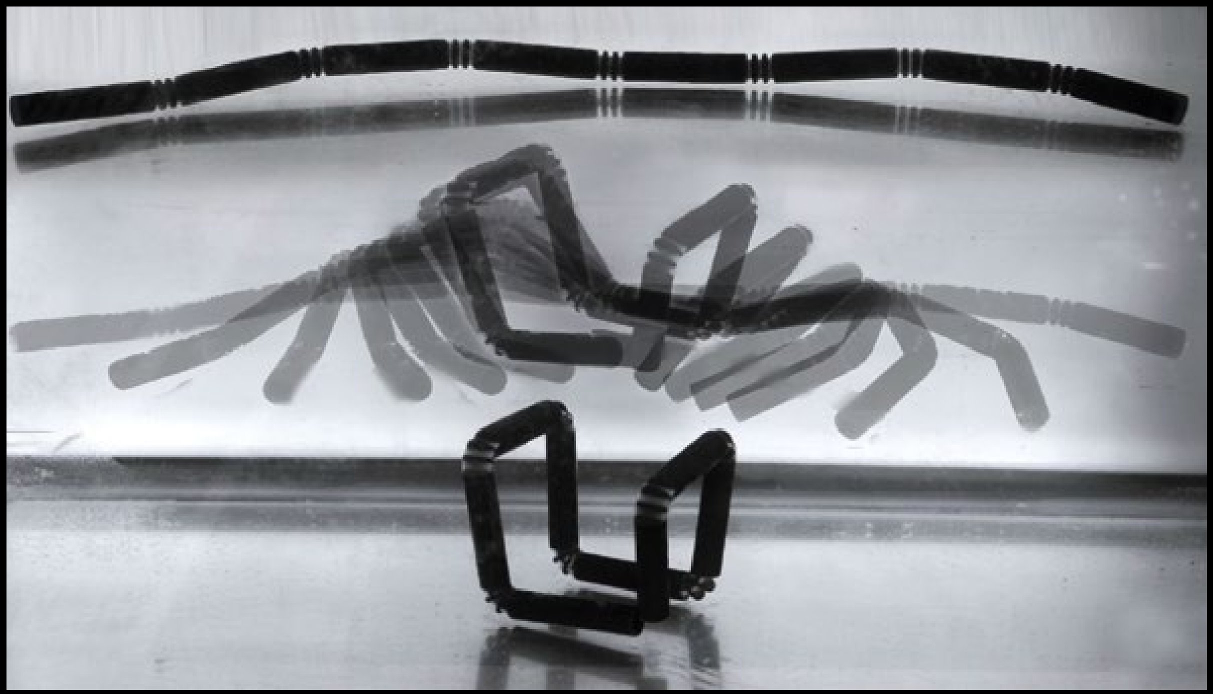
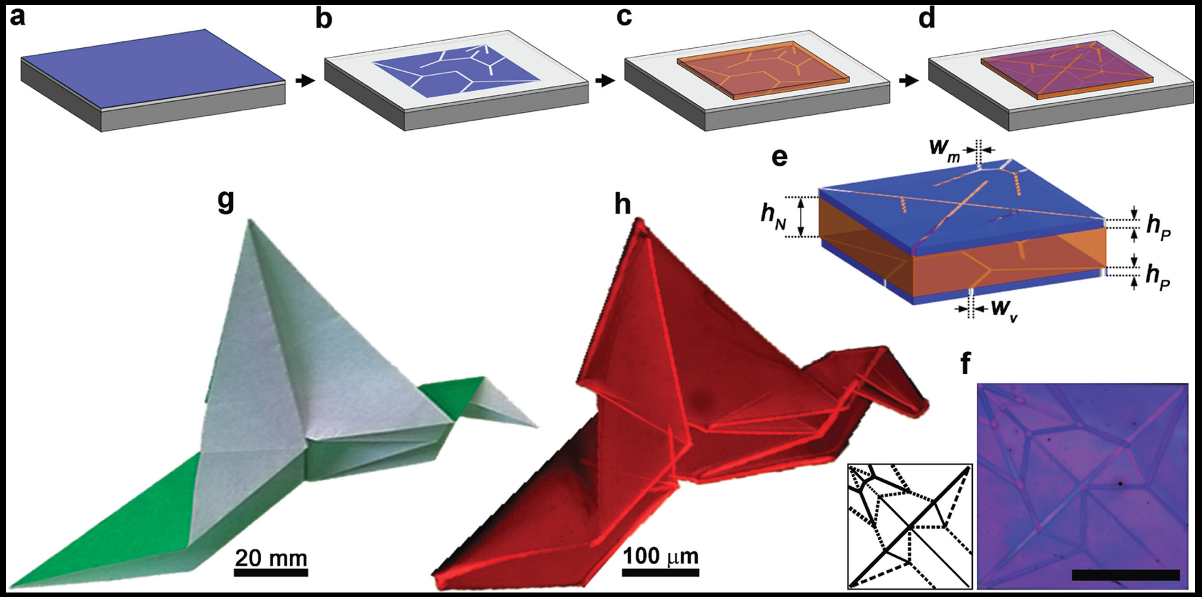


Tibbits, *Arch. Design* **84** 116 (2014)

Origami



Na, *et al. Adv. Mat.* **27** 79 (2015)

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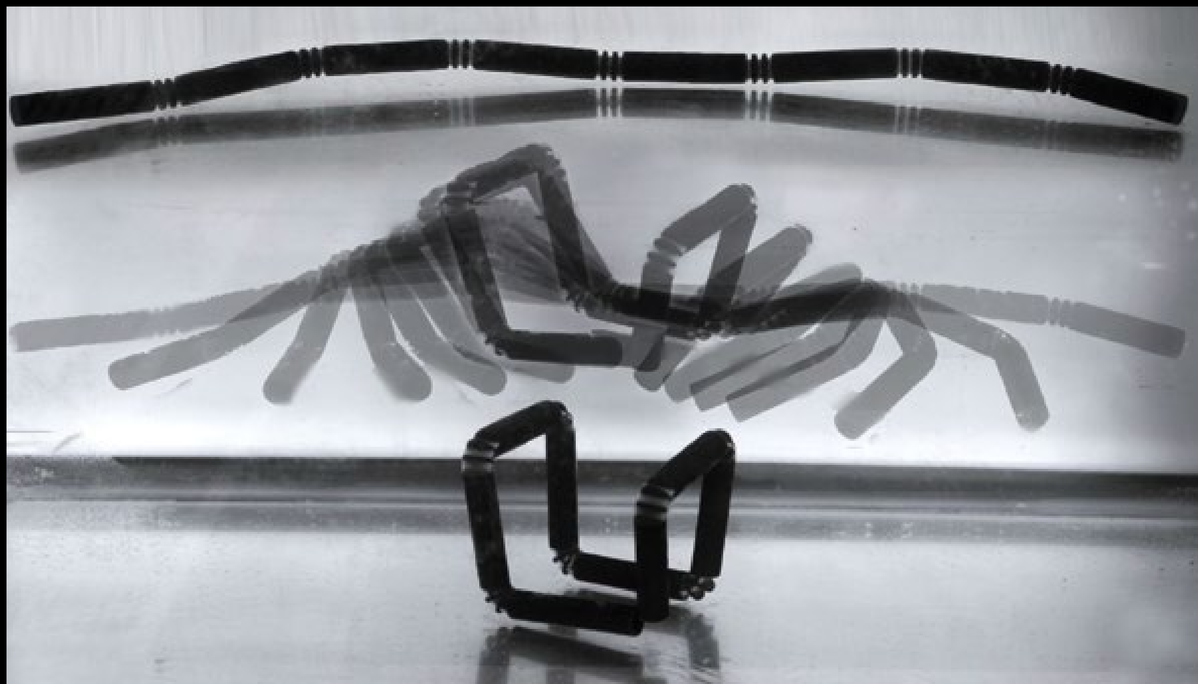

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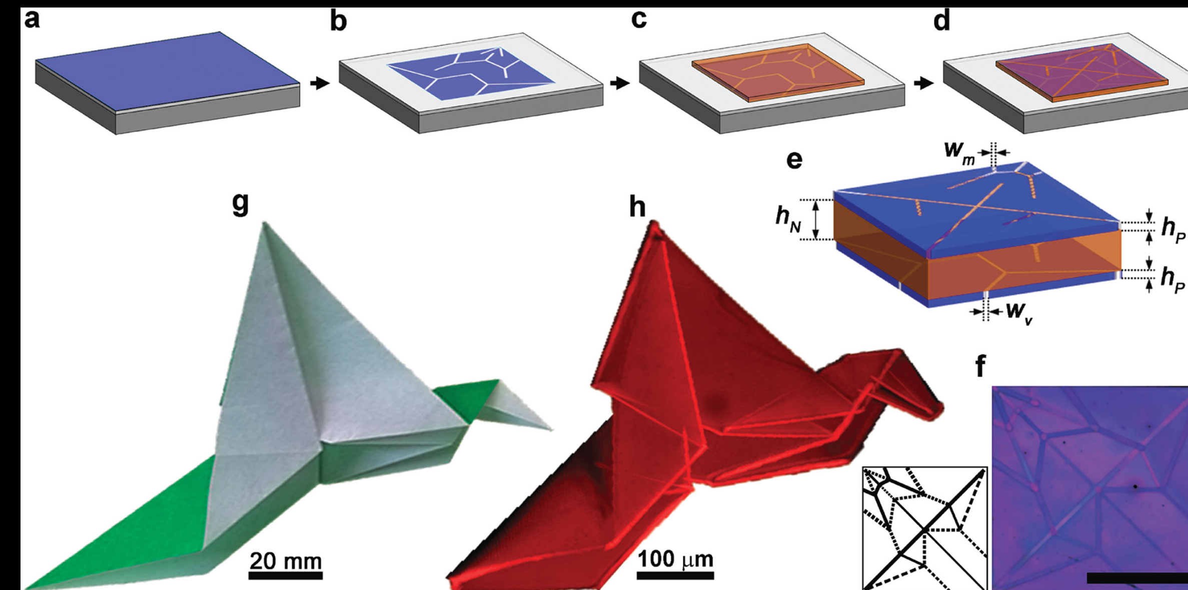
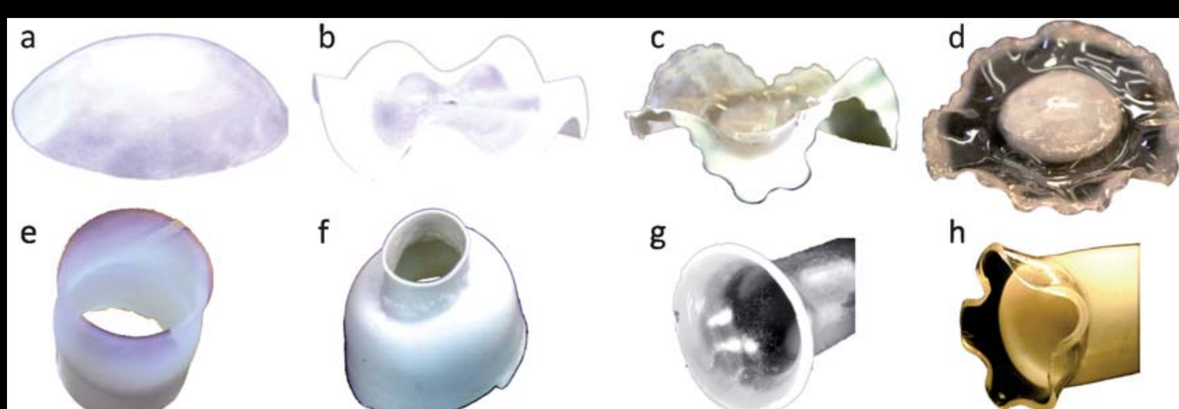


Na, *et al.* *Adv. Mat.* **27** 79 (2015)

Swelling Hydrogels



Sharon & Efrati, *Soft Matter* **6** 5693 (2010)

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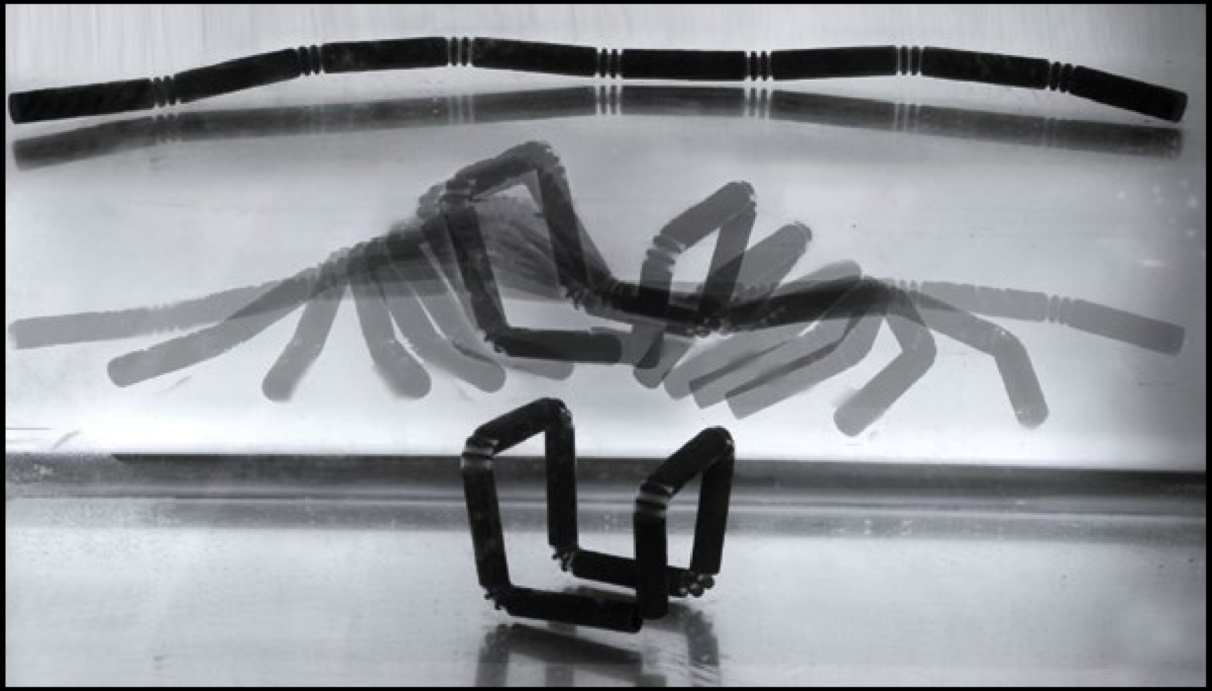

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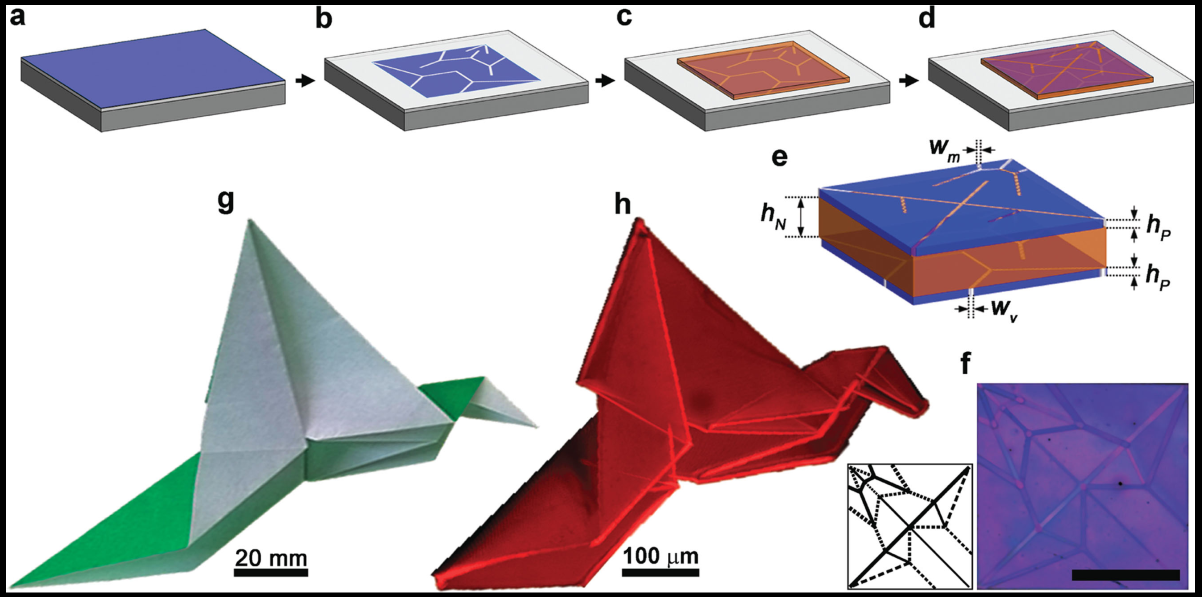

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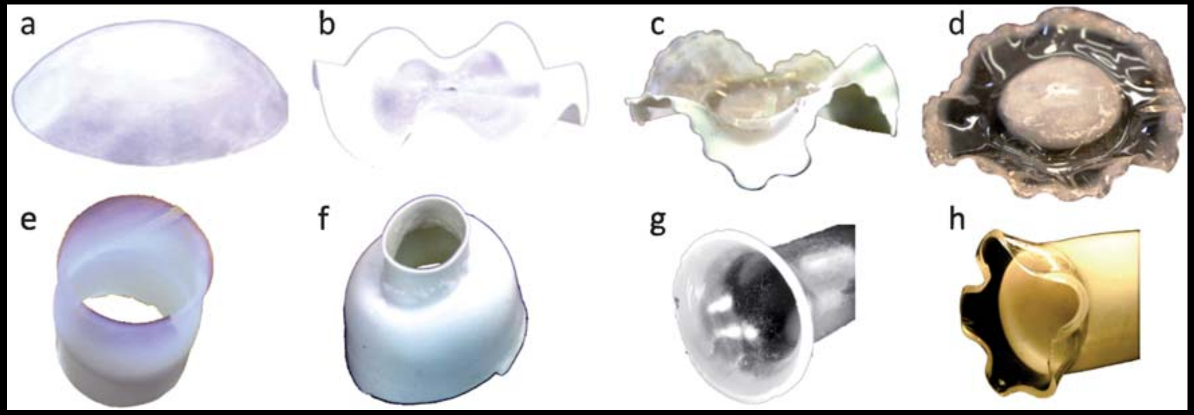
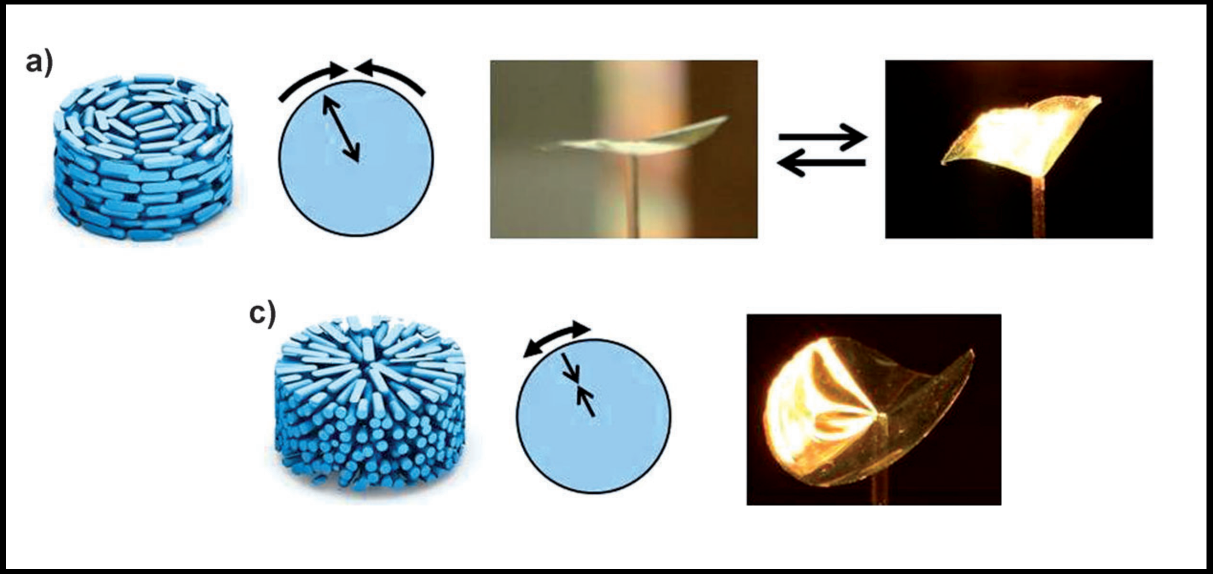


Sharon & Efrati, *Soft Matter* **6** 5693 (2010)

Liquid Crystal Elastomers



de Haan, *et al. Angew. Chem.* **51** 12469 (2012)

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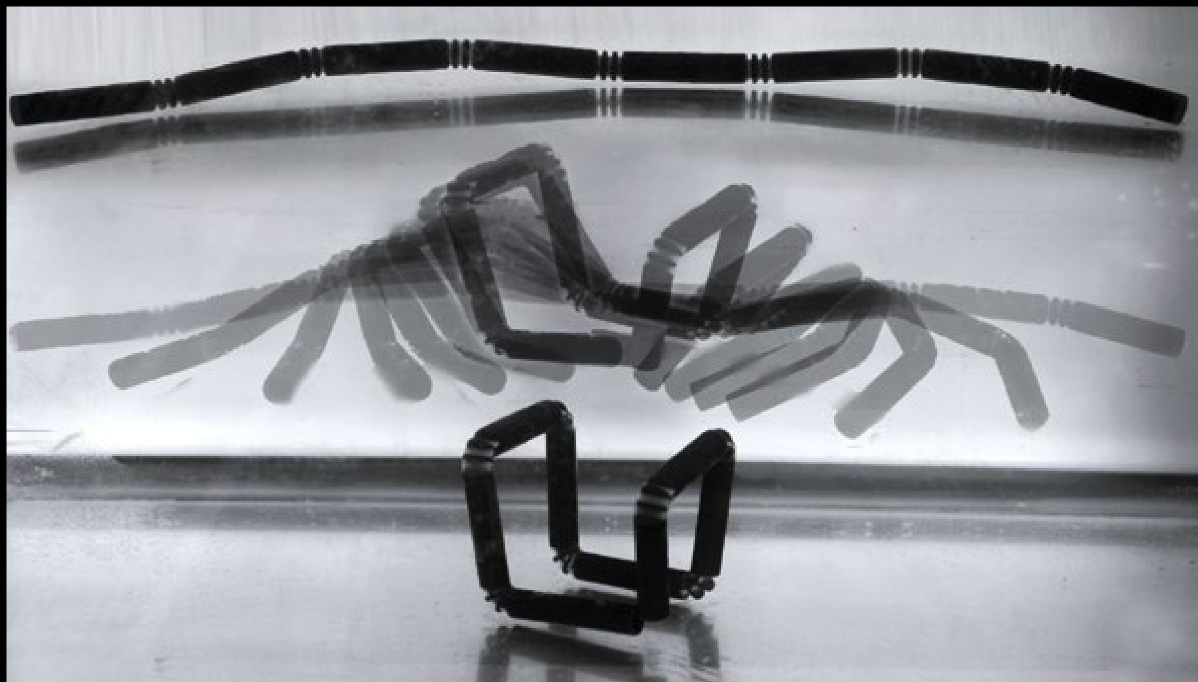

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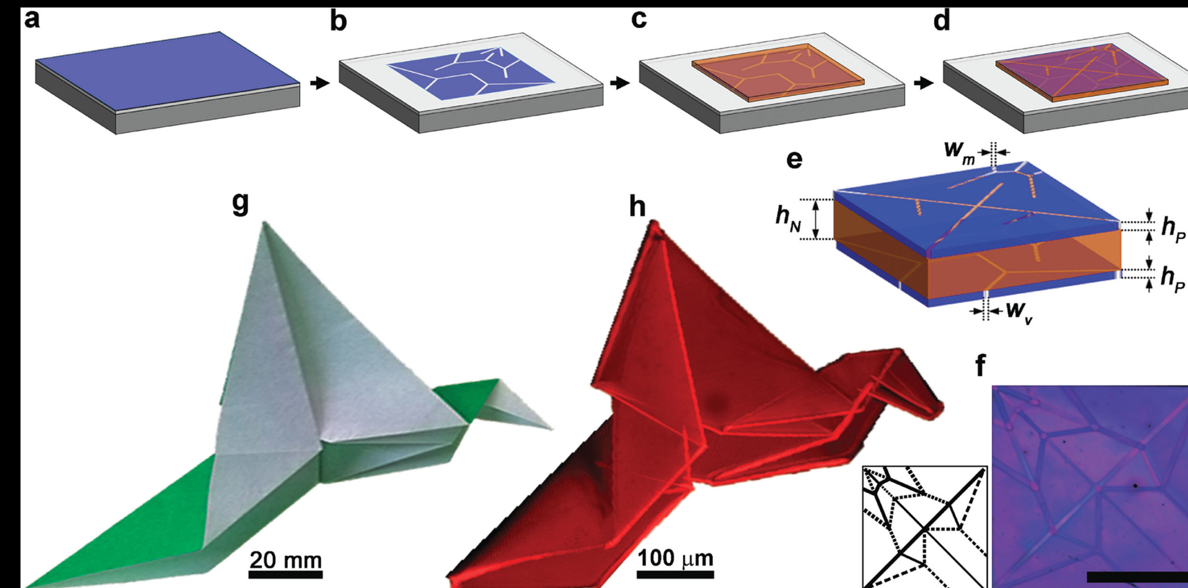

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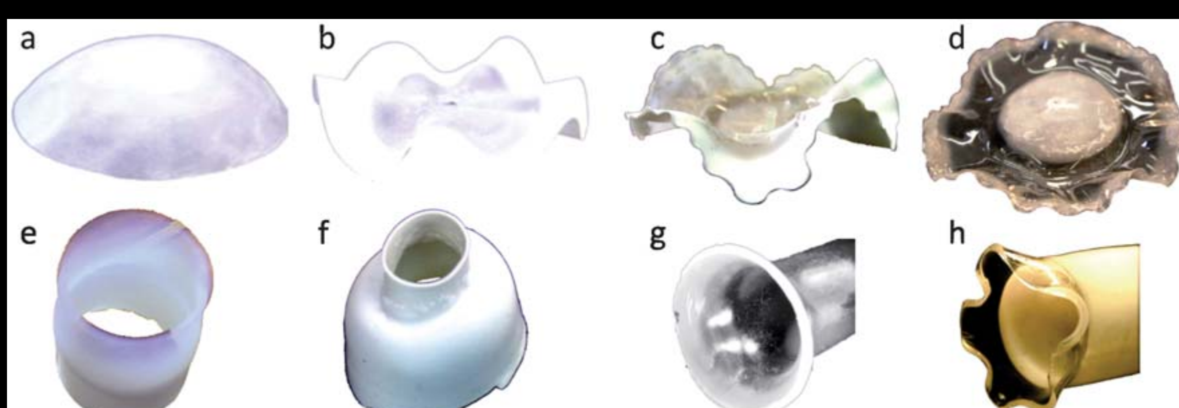

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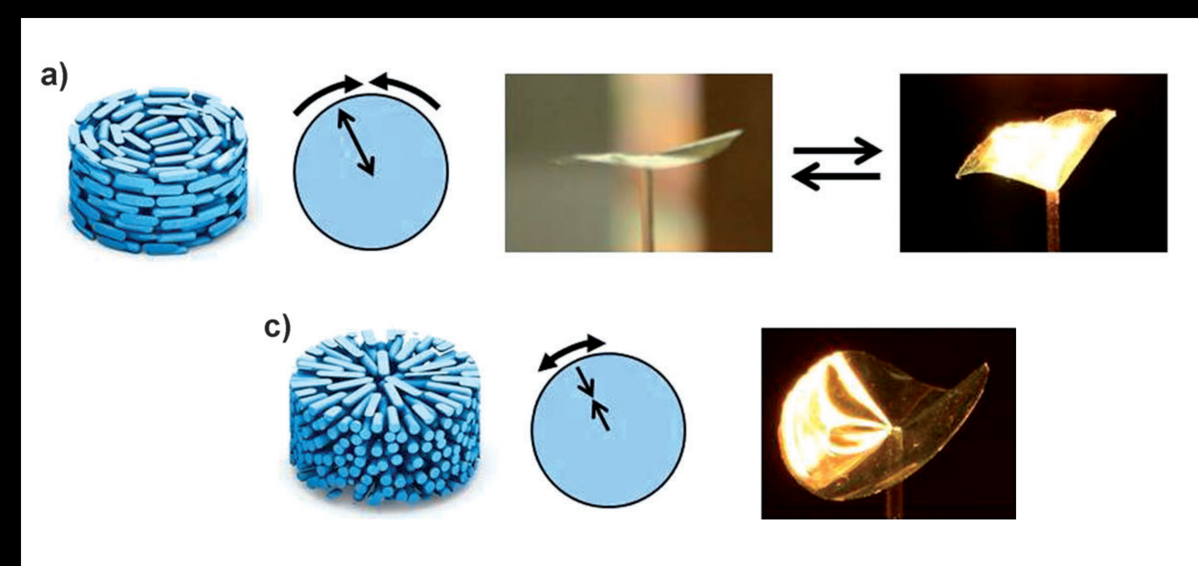

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Hygroscopic Motion

Pine Cone

Erodium Awn

Hygroscopic Motion

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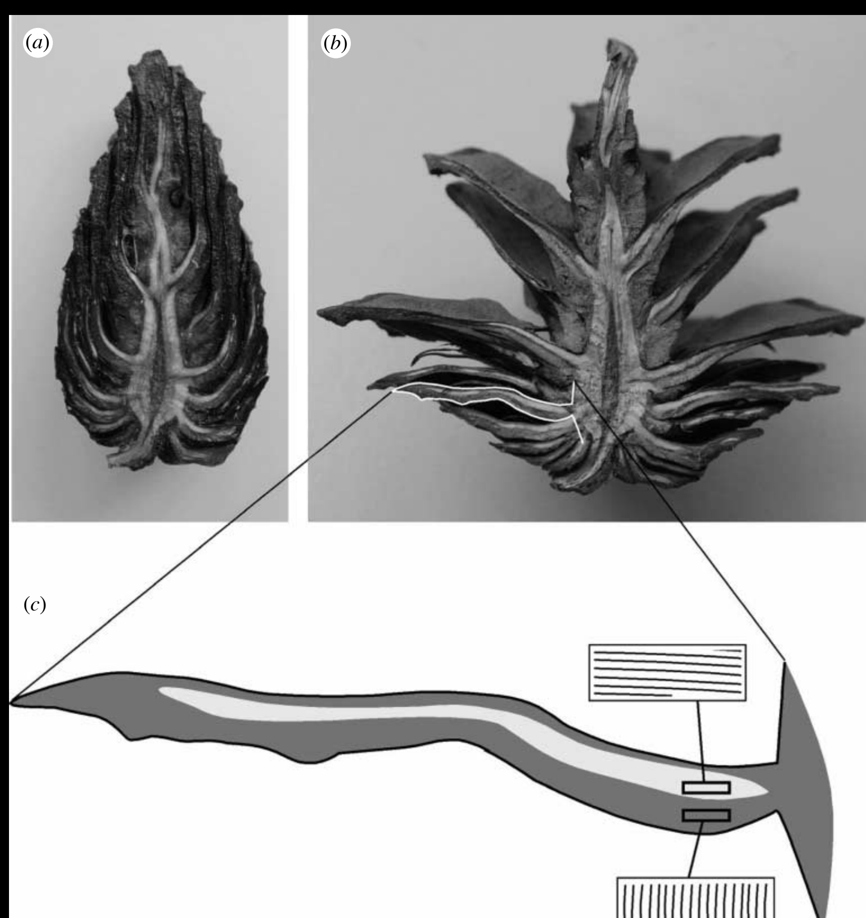

Burgert & Fratzl, *Phil. Trans. R. Soc. A* **367** 1541 (2009)

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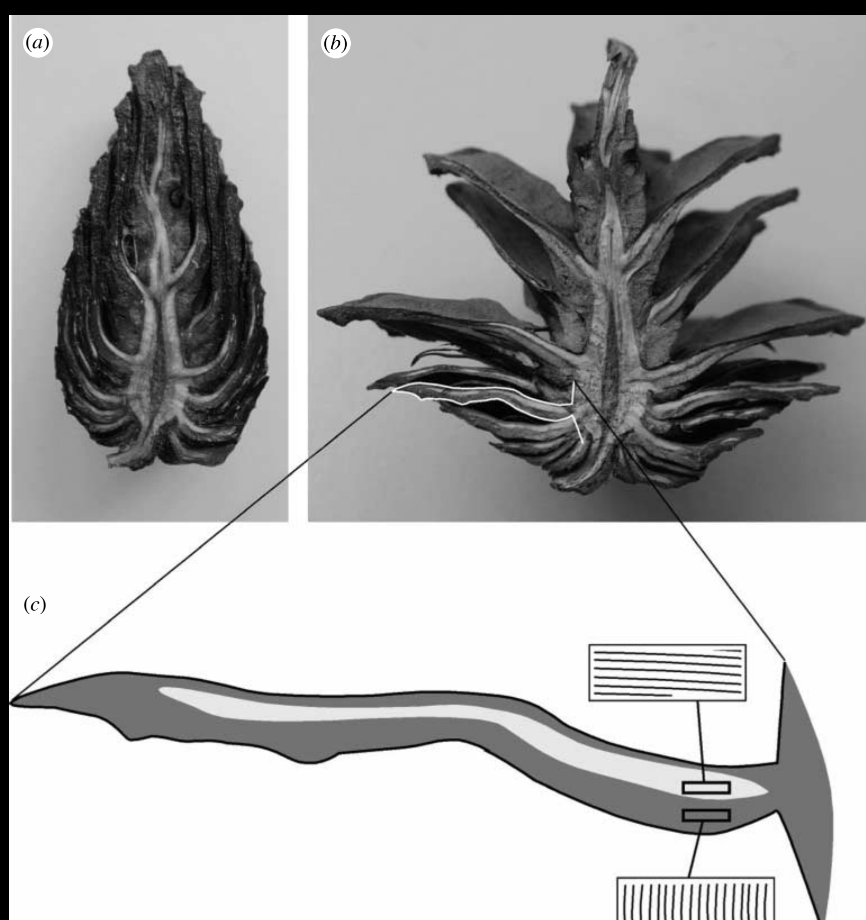

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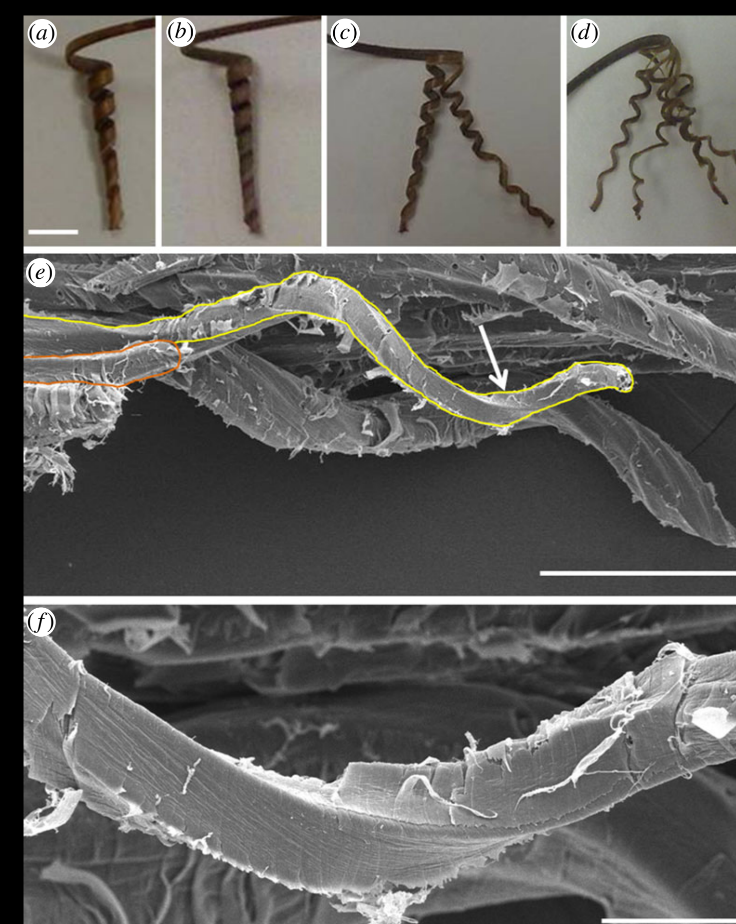
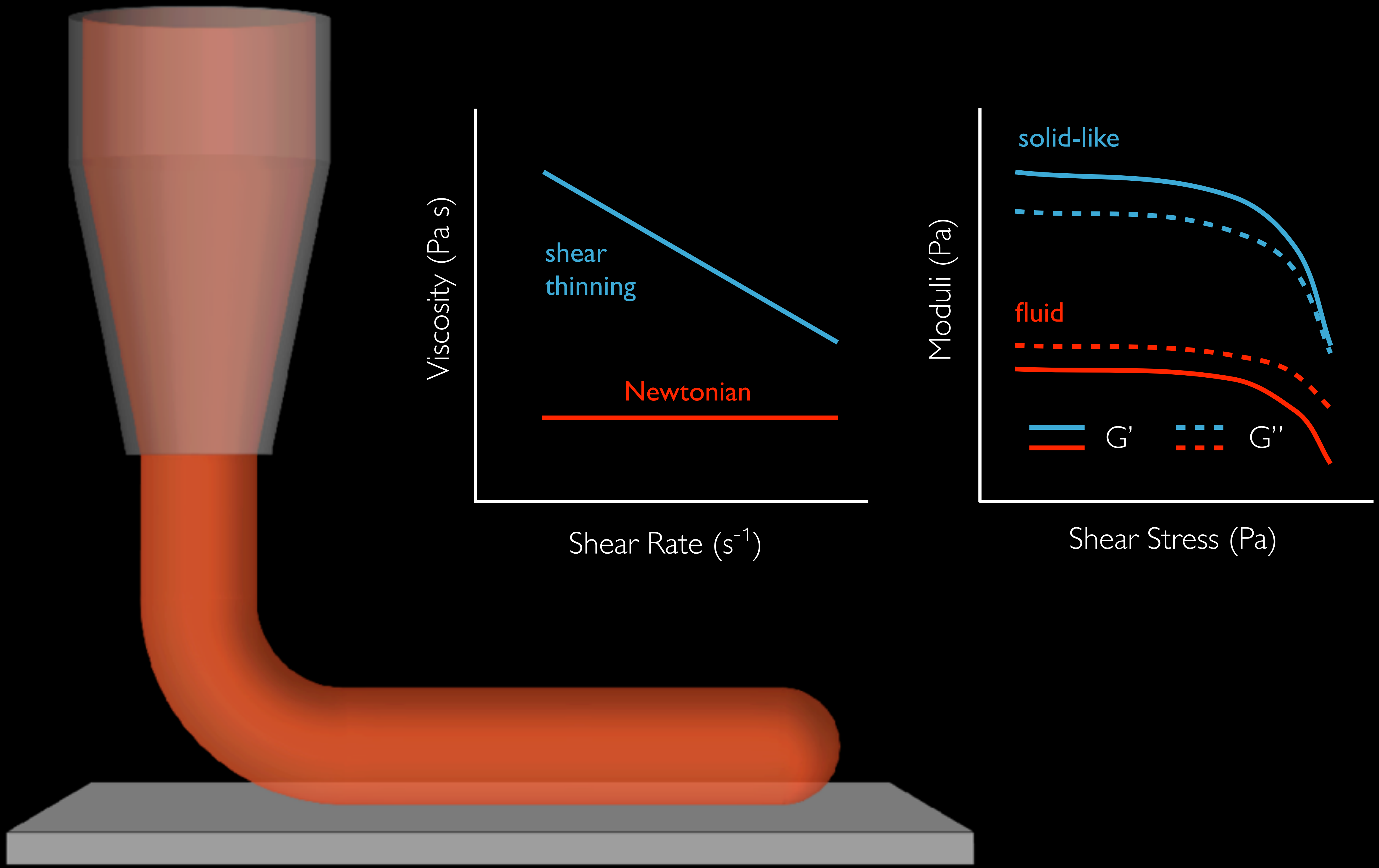


Abraham, *et al. J. R. Soc. Interface* **9** 640 (2012)

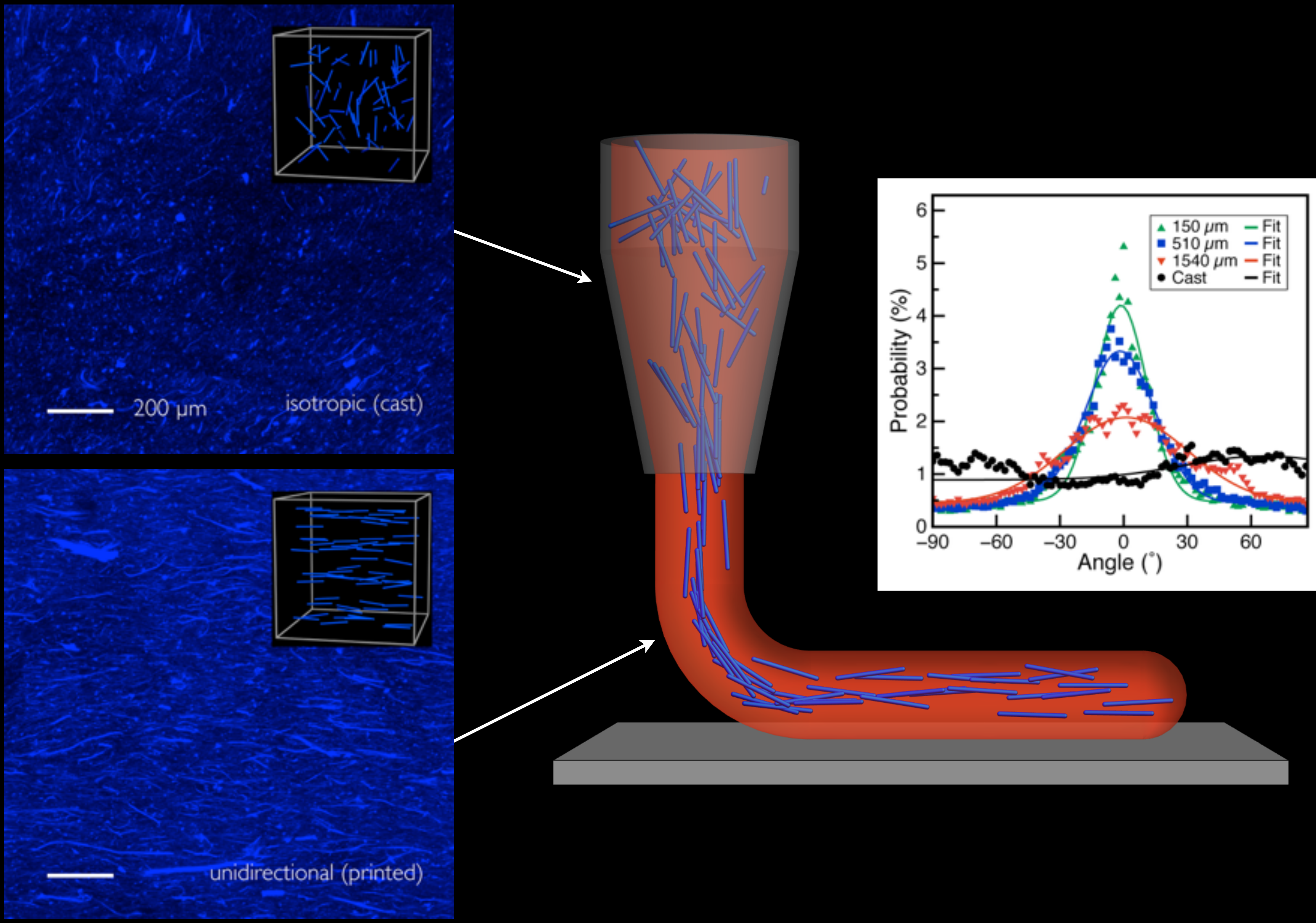
3D Printing



A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

Our Ink

Cellulose Nanofibrils + Acrylamide Monomers + Clay = Composite Ink



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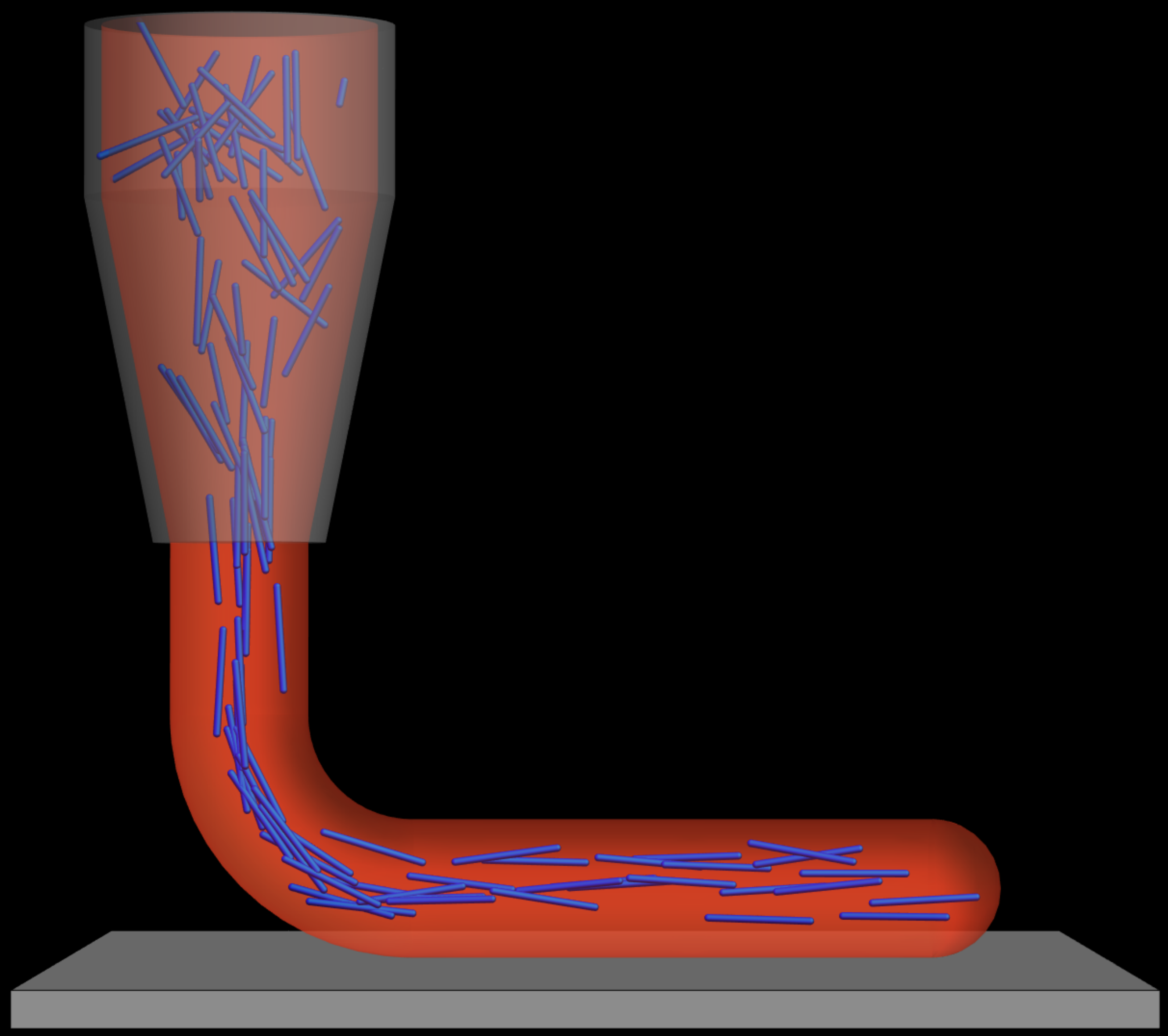

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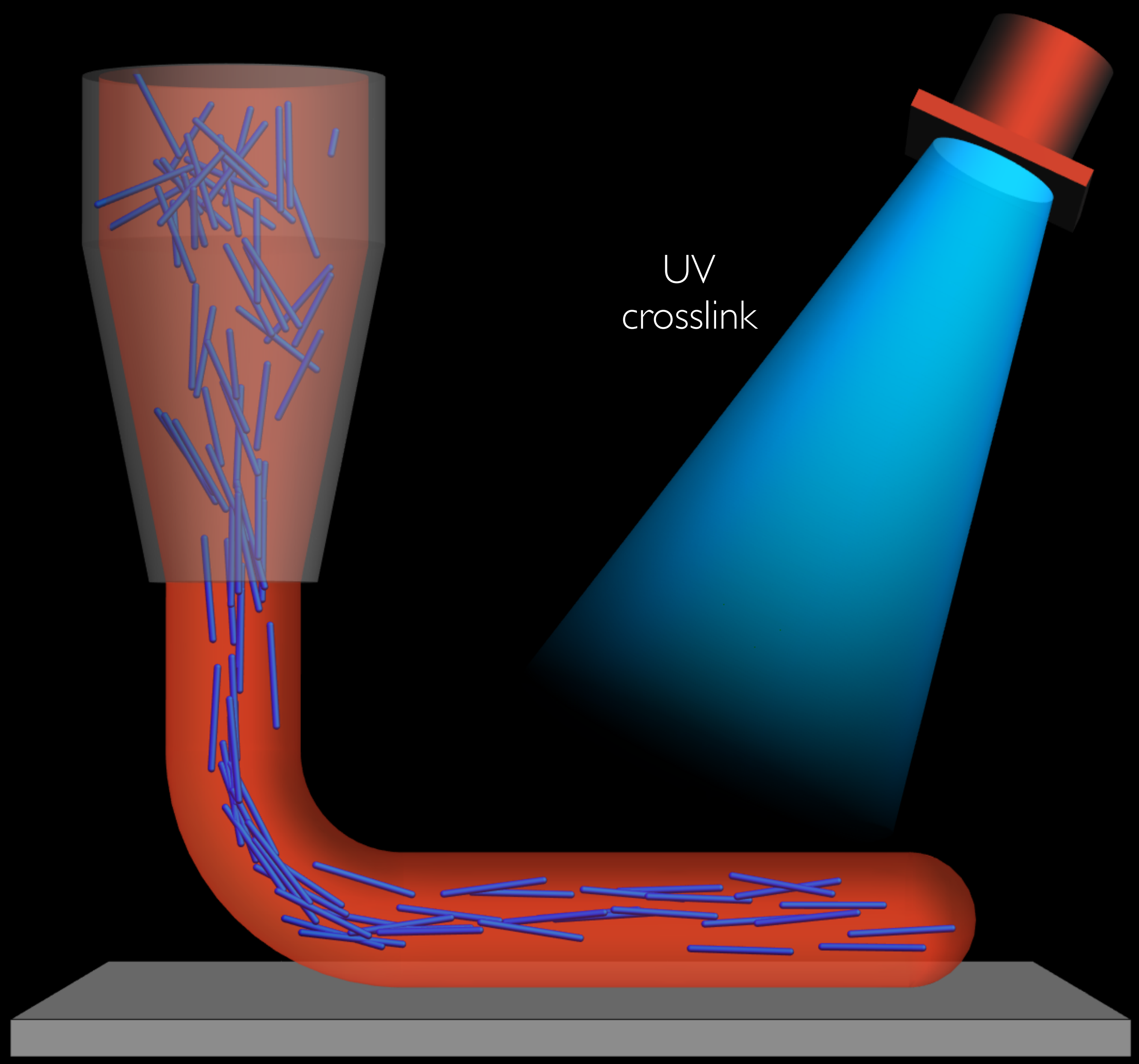

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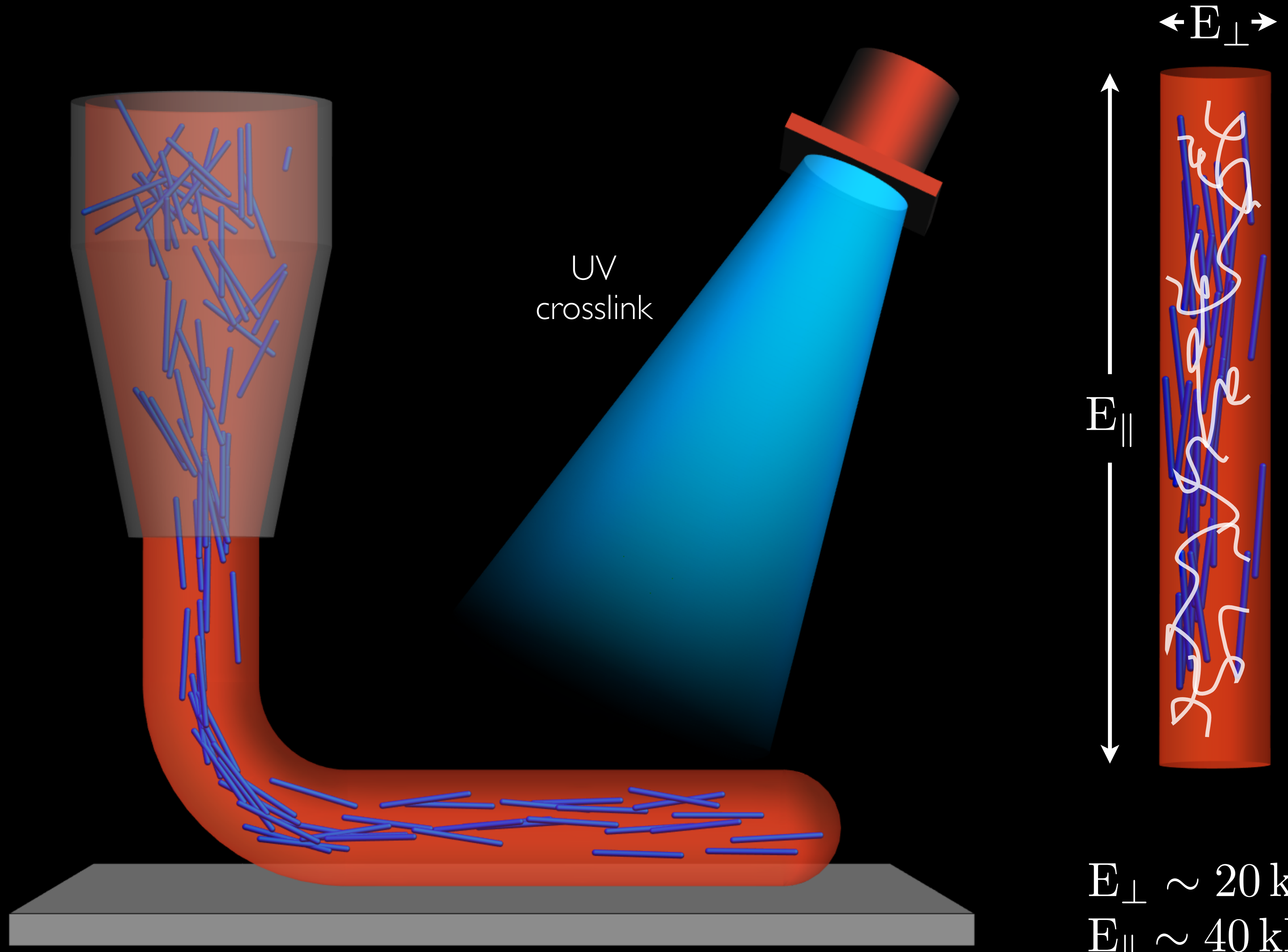

$E_\perp \sim 20\,\mathrm{kPa}$
$E_\parallel \sim 40\,\mathrm{kPa}$

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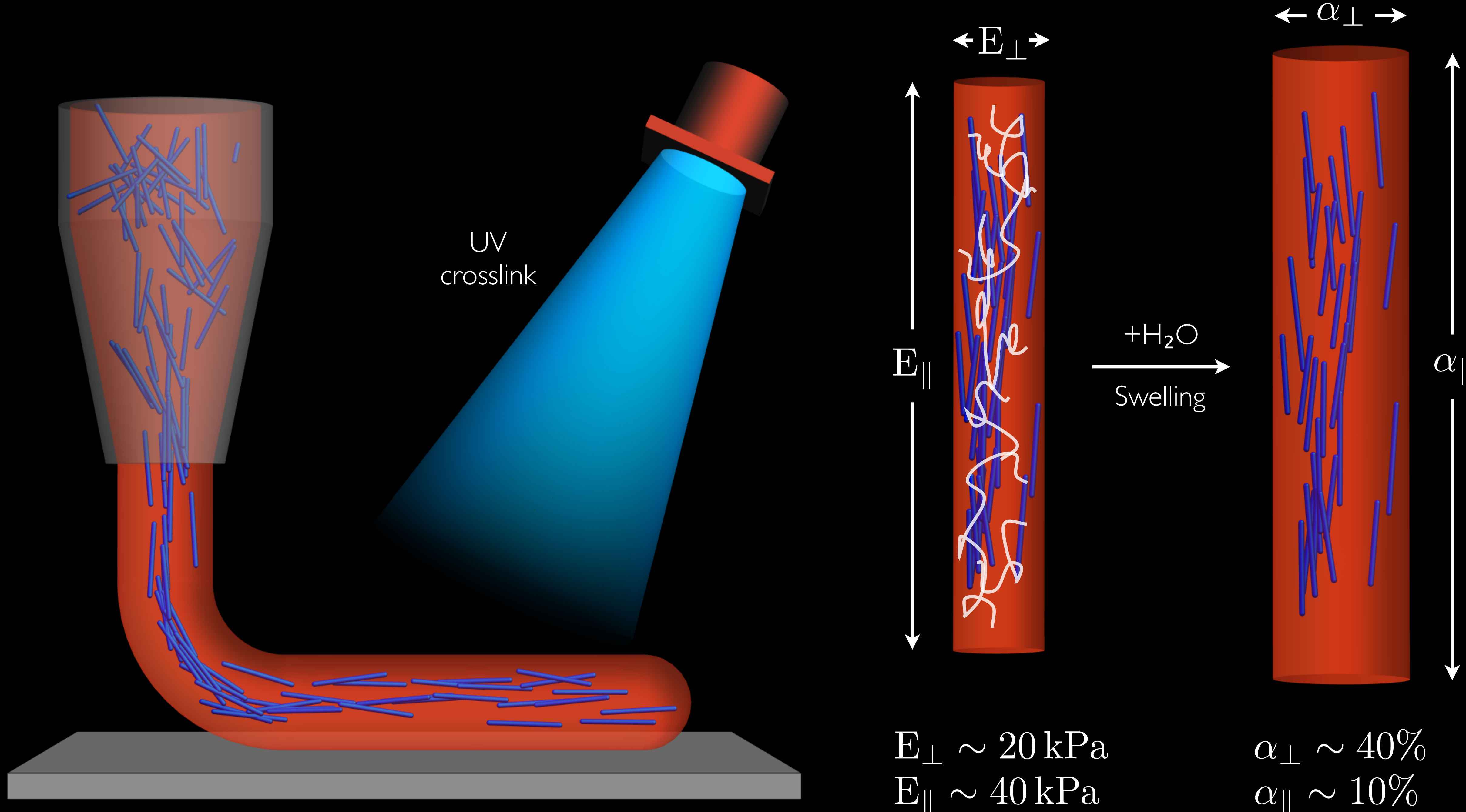
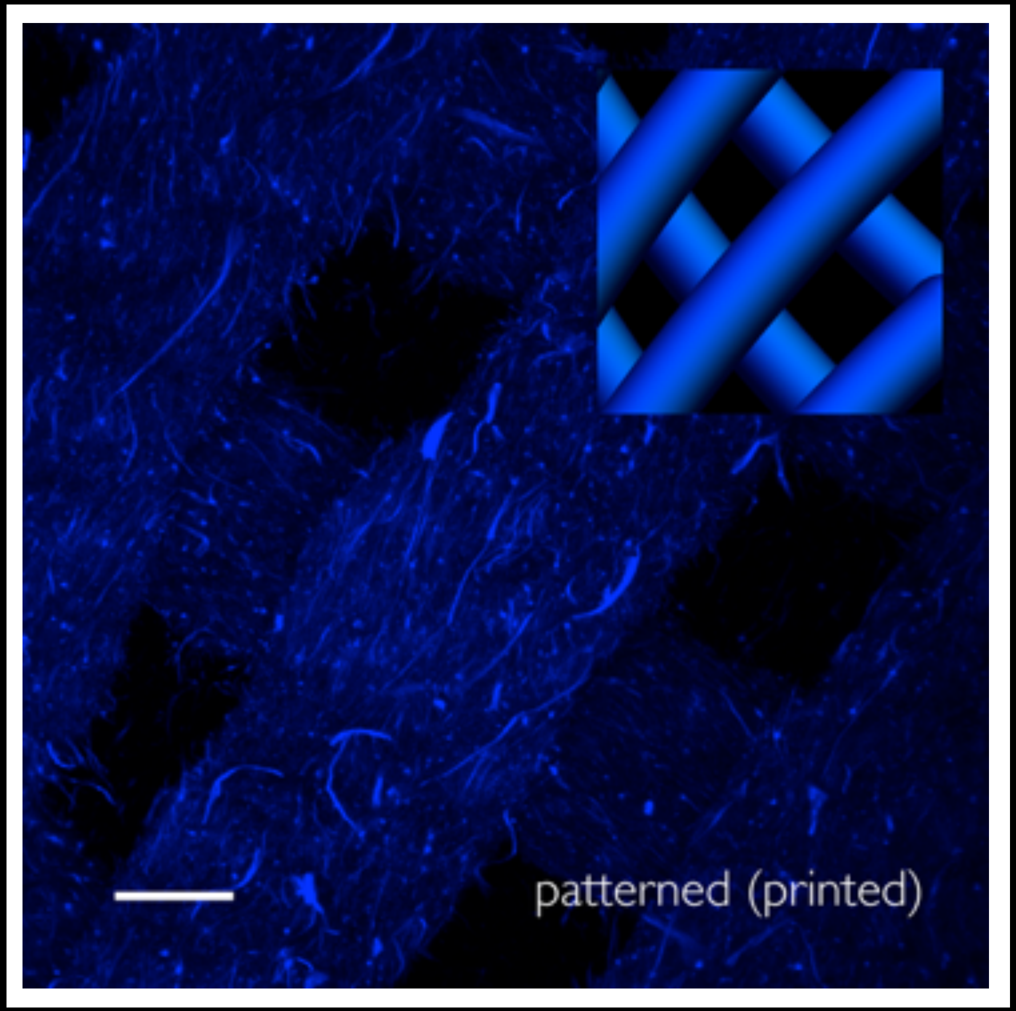


Elastic Anisotropy leads to swelling anisotropy

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

Encoding Local Anisotropy



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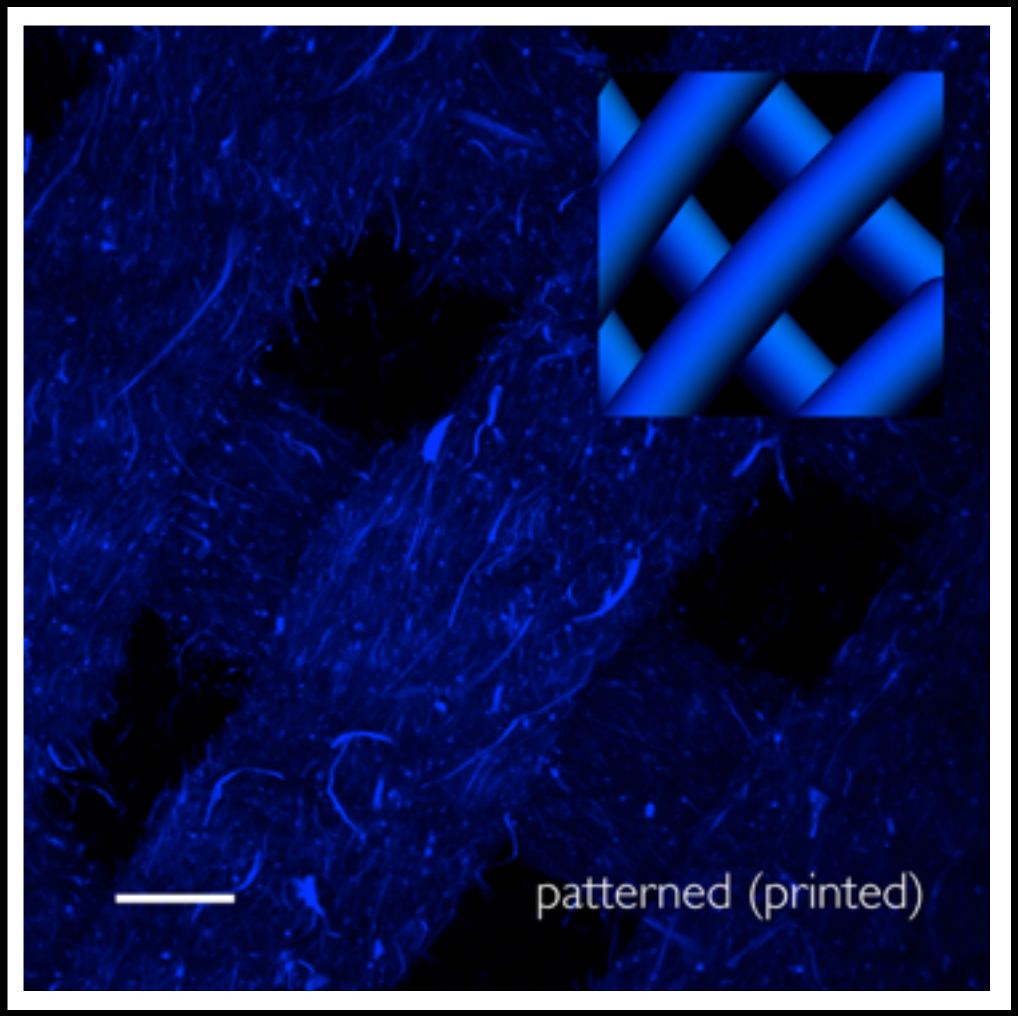

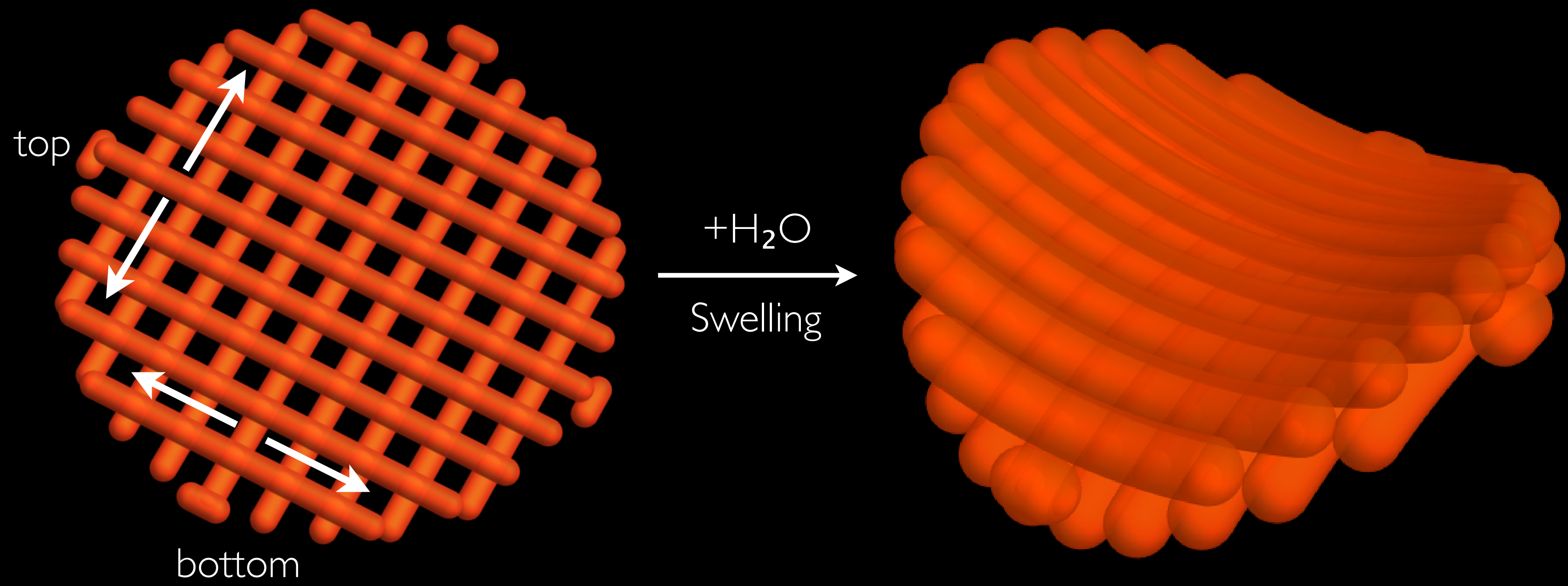

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

Bi-Metallic Strips

1. Equilibrium Condition

"Due to the fact that there are not external forces acting on the strip, all forces acting over any cross-section of the strip must be in equilibrium"

Stress from fictitious force

$$\sigma = \frac{\mathrm{P}^{\mathrm{eff}}}{h}$$

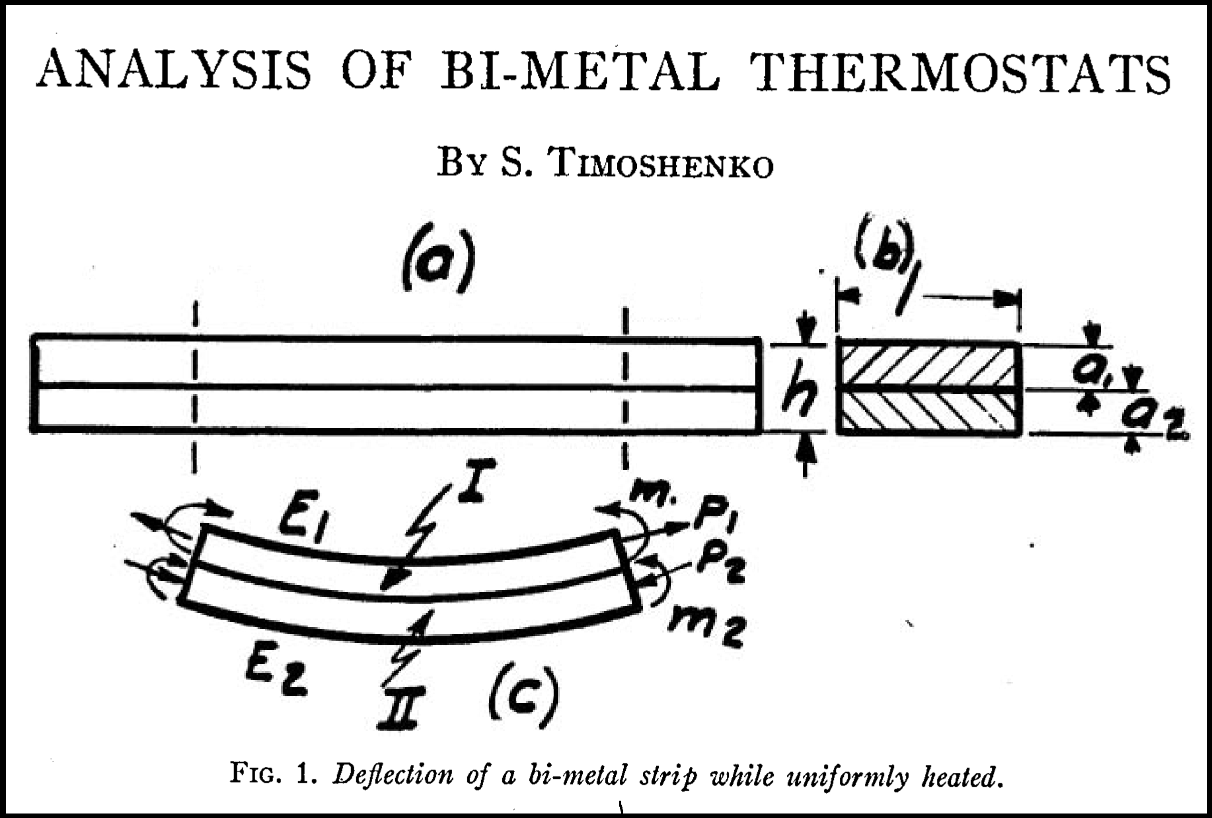

FIG. 1. *Deflection of a bi-metal strip while uniformly heated.*

Timoshenko, *J. Opt. Soc. Am.* **11** 233 (1925)

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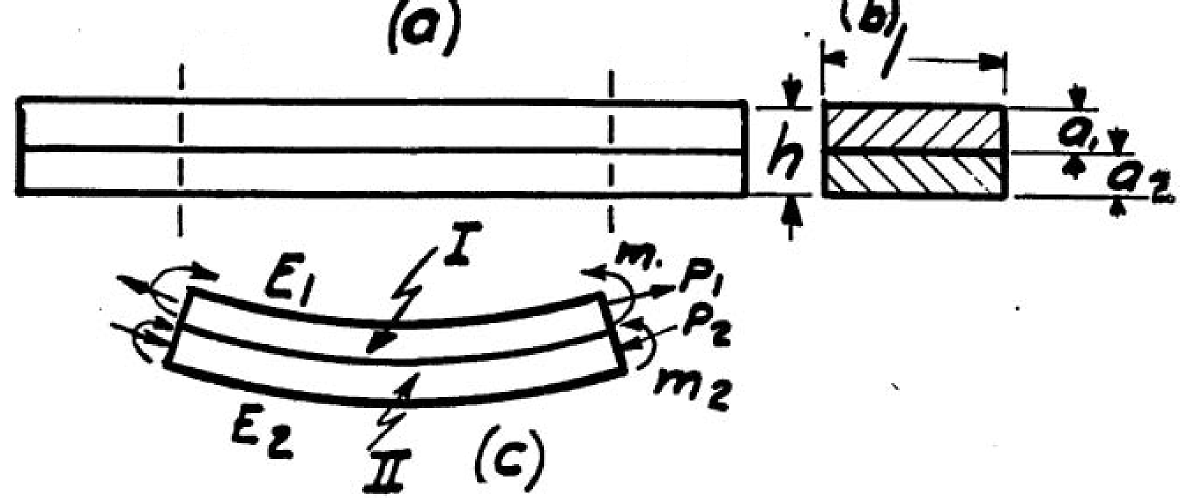
$$M^{\mathrm{tot}} = \int z\sigma dz = \frac{\mathrm{P}^{\mathrm{eff}} h}{2}$$

ANALYSIS OF BI-METAL THERMOSTATS

BY S. TIMOSHENKO



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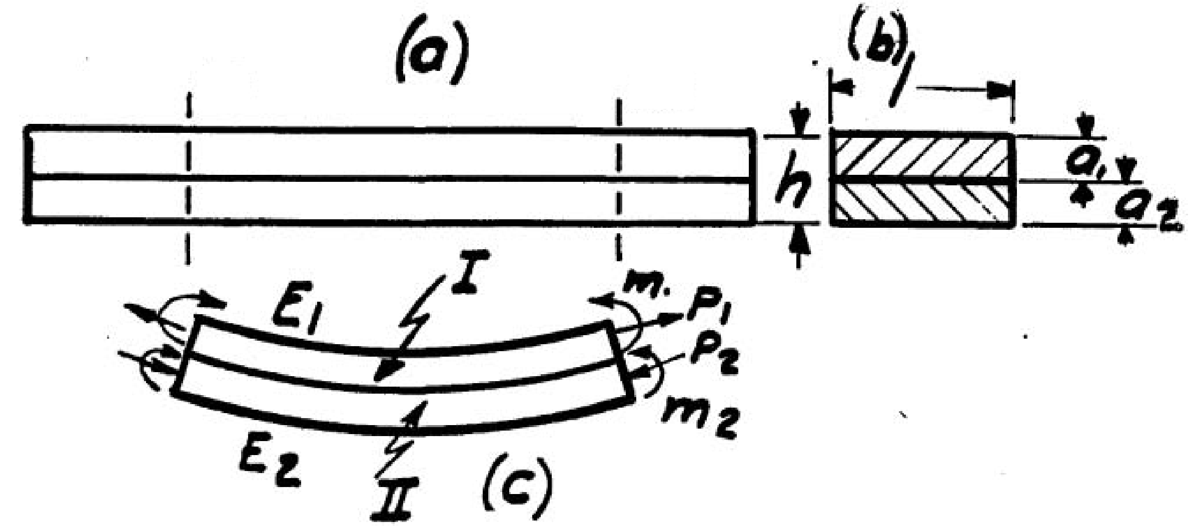
ANALYSIS OF BI-METAL THERMOSTATS

BY S. TIMOSHENKO



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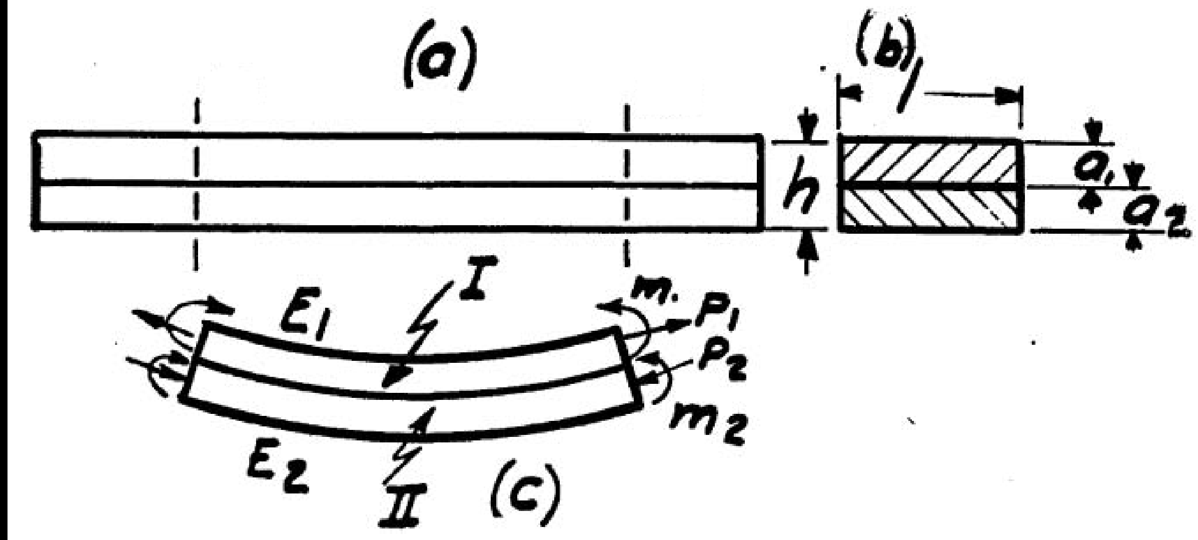

ANALYSIS OF BI-METAL THERMOSTATS

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Moment-stress relationship

$$\sigma^{\mathrm{tot}} = \frac{M^{\mathrm{tot}}}{2h}$$

Timoshenko, *J. Opt. Soc. Am.* **11** 233 (1925)

Bi-Metallic Strips

2. Compatibility Condition

"On the bearing surface of both metals the unit elongation occurring in the longitudinal fibres of metals (1) and (2) must be equal."

$$\varepsilon^{(1)} = \varepsilon^{(2)}$$

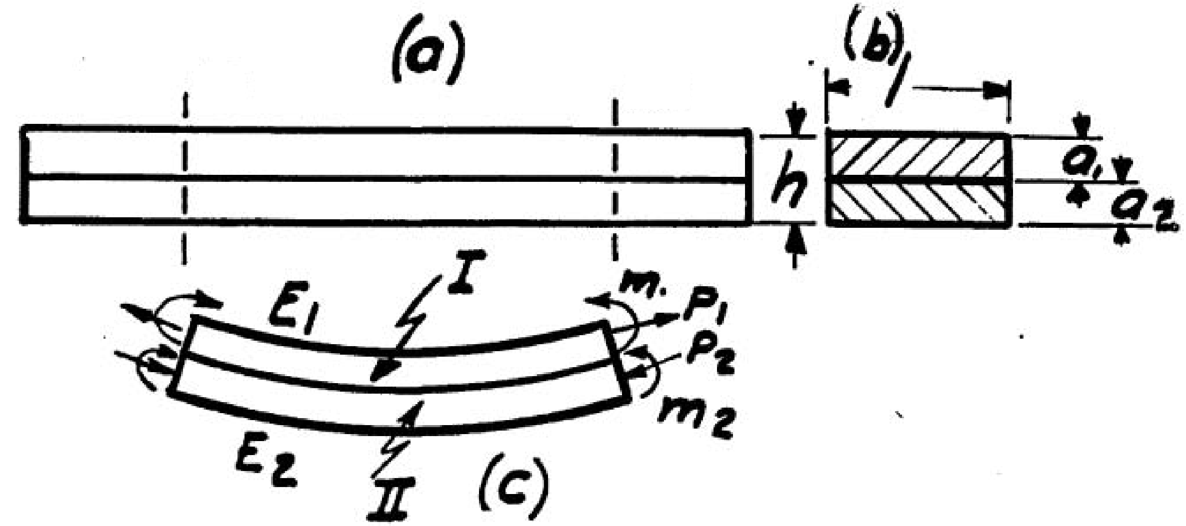
Strain from swelling $\varepsilon^{\mathrm{s}} = \alpha$

Strain from curvature $\varepsilon^{\mathrm{s}} = z\kappa$

Strain from stress $\varepsilon = \mathrm{E}^{-1}\sigma^{\mathrm{tot}} = \dfrac{1}{\mathrm{E}}\dfrac{M^{\mathrm{tot}}}{2h}$

ANALYSIS OF BI-METAL THERMOSTATS

BY S. TIMOSHENKO



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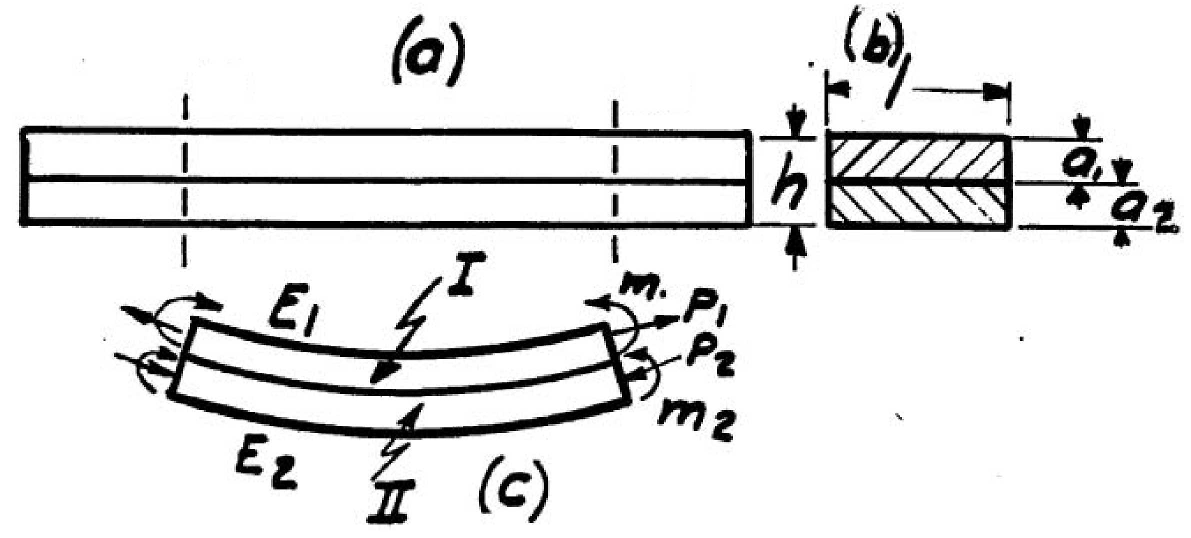
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$$\frac{1}{a_1}\int_0^{a_1}\left(\varepsilon^{\mathrm{s}(1)} + \varepsilon^{\mathrm{e}(1)}(z) + \frac{\sigma^{\mathrm{tot}}}{\mathrm{E}_1}\right)dz = \frac{1}{a_2}\int_{-a_2}^{0}\left(\varepsilon^{\mathrm{s}(2)} + \varepsilon^{\mathrm{e}(2)}(z) + \frac{\sigma^{\mathrm{tot}}}{\mathrm{E}_2}\right)dz$$

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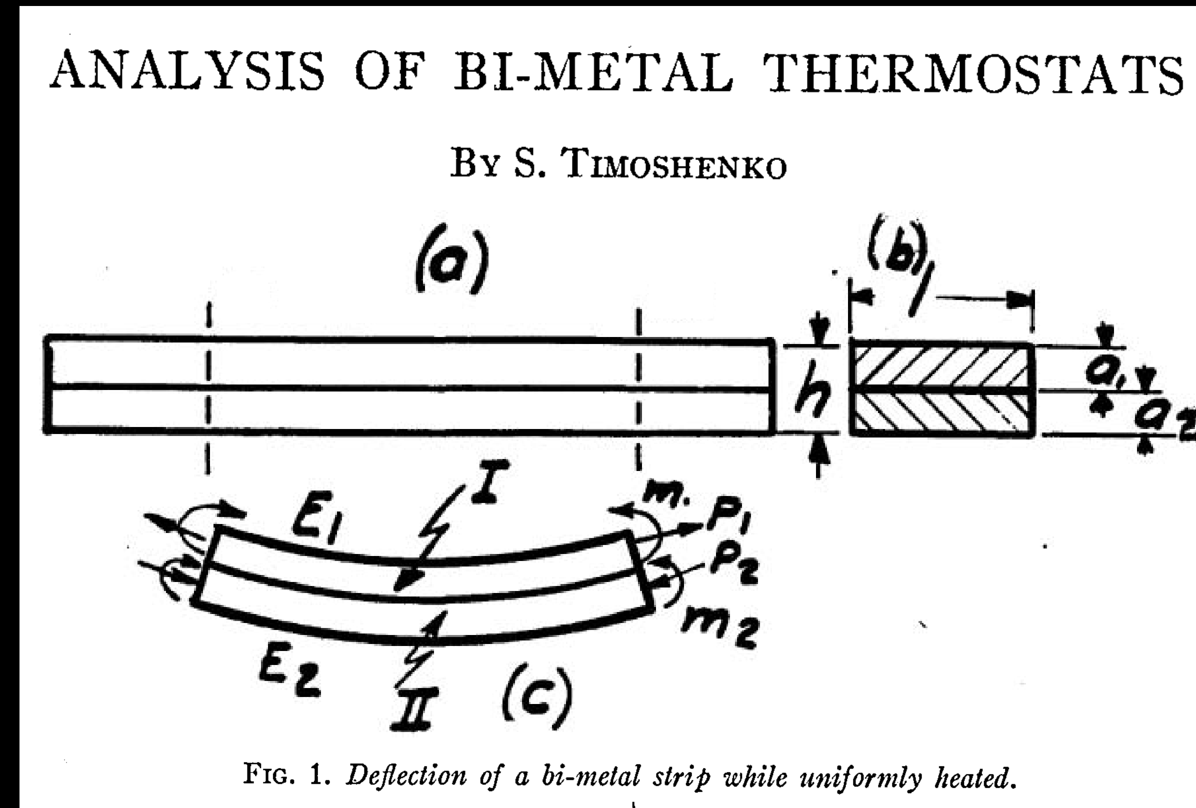
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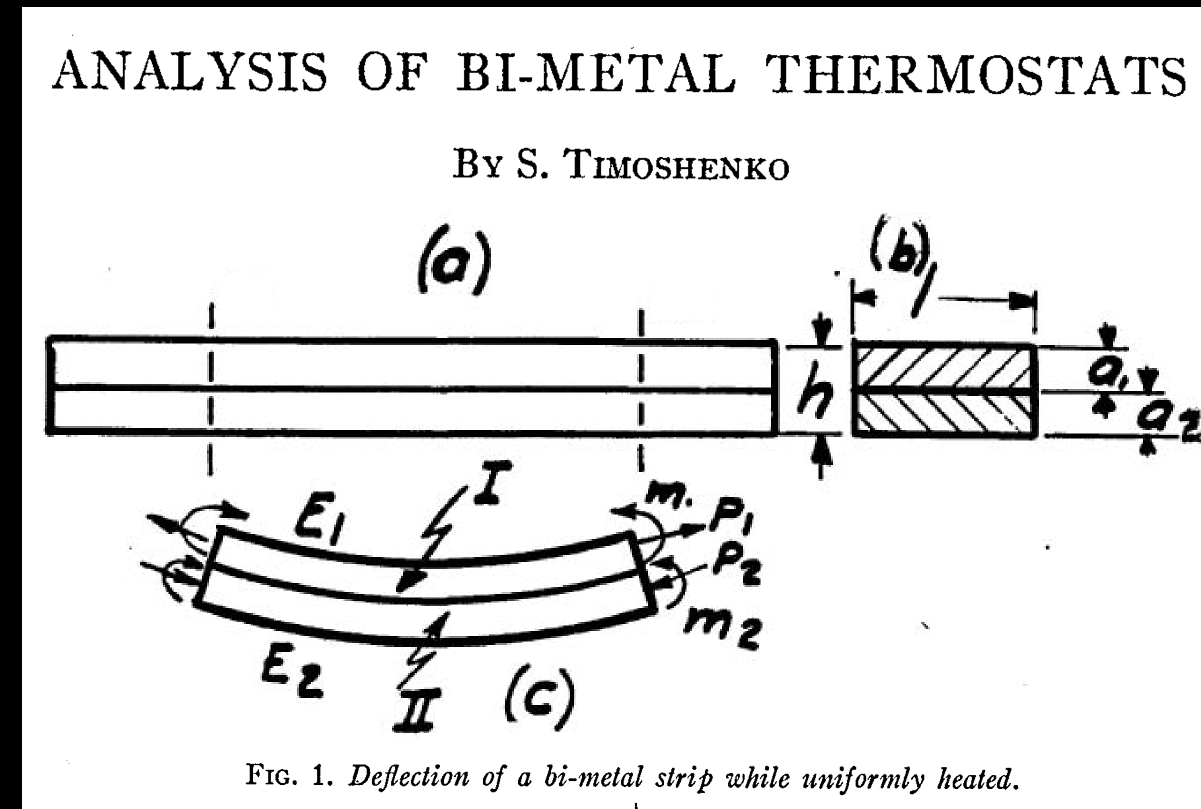
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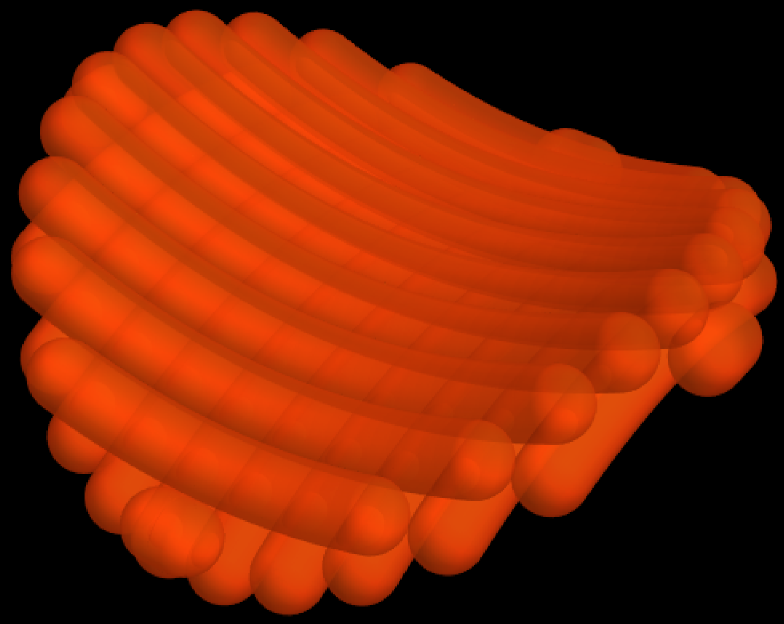
$$\boxed{\kappa = \frac{6(\alpha_2-\alpha_1)(1+m)^2}{h\left(3(1+m)^2 + (1+mn)\left(m^2+\frac{1}{mn}\right)\right)}, \qquad m = \frac{a_1}{a_2},\ n = \frac{\mathrm{E}_1}{\mathrm{E}_2}}$$

Timoshenko, *J. Opt. Soc. Am.* **11** 233 (1925)

A Brief Primer on Curvature



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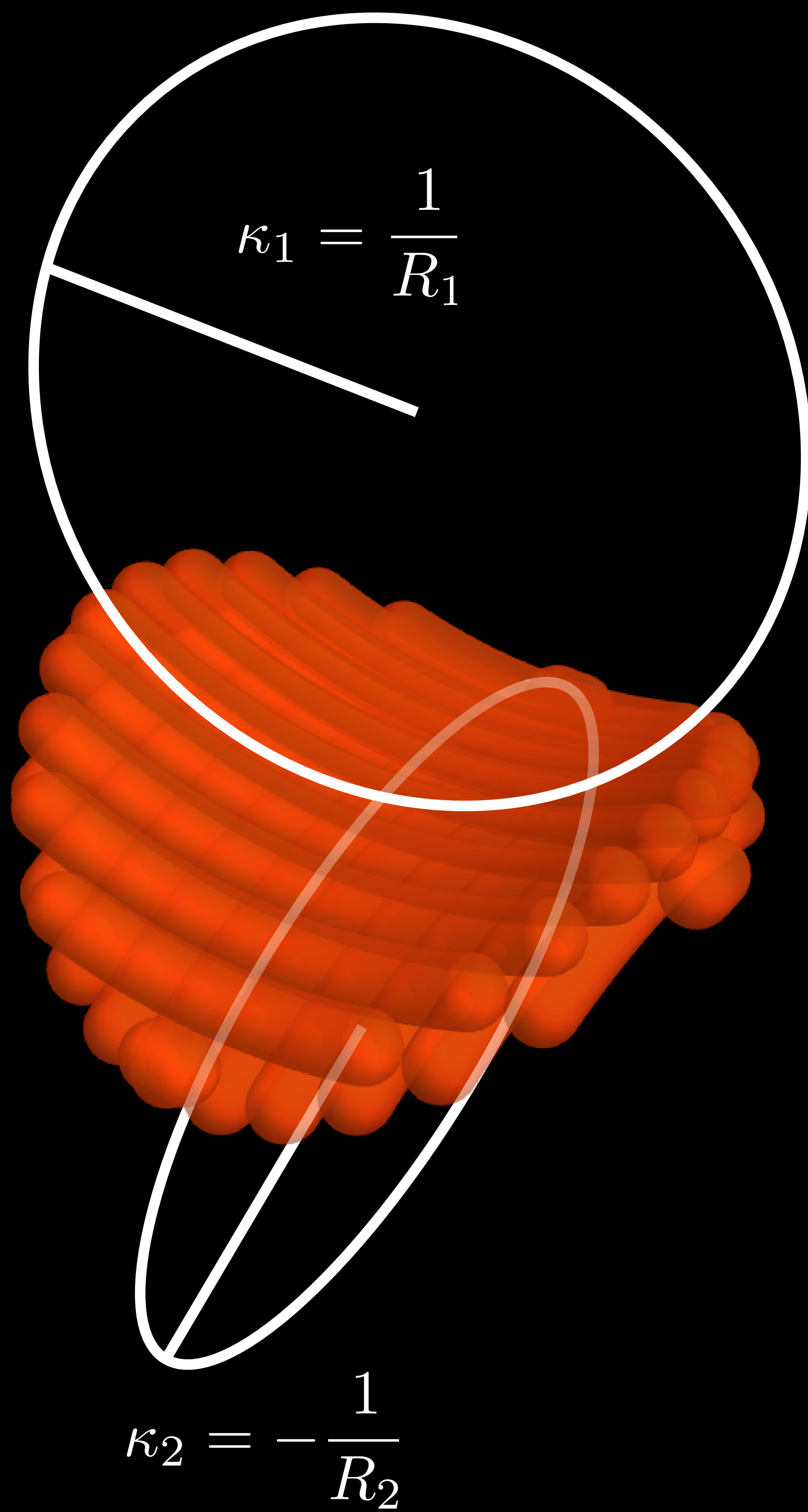

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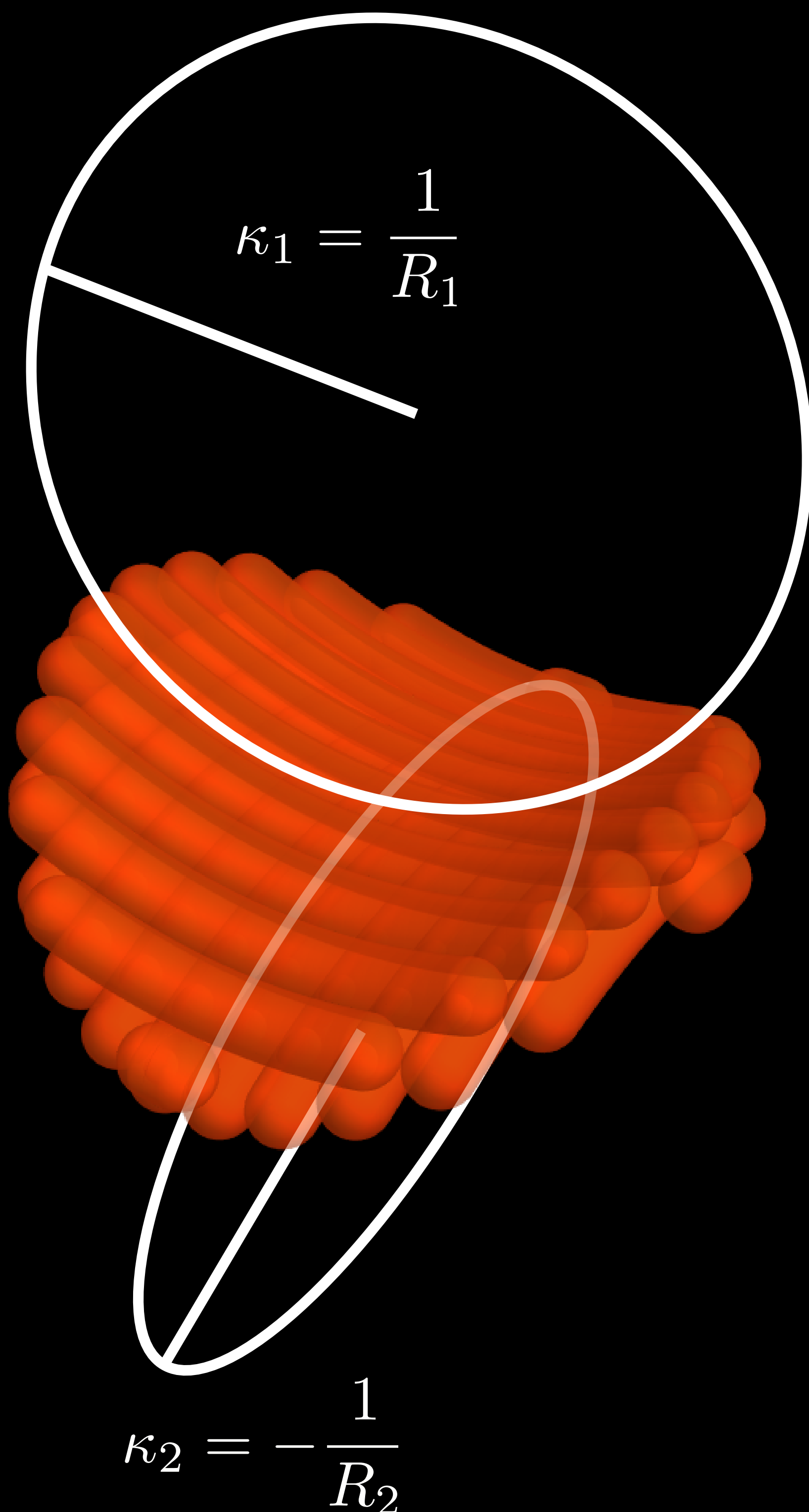
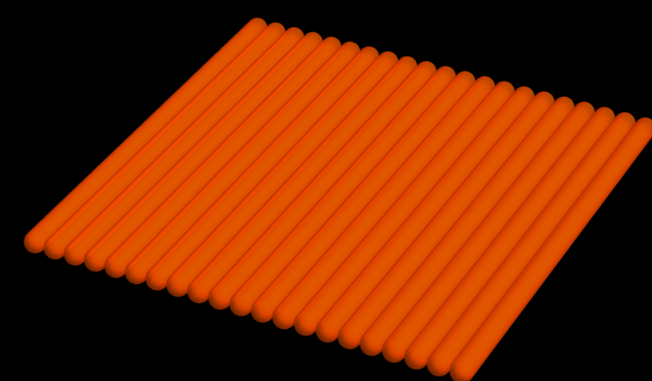


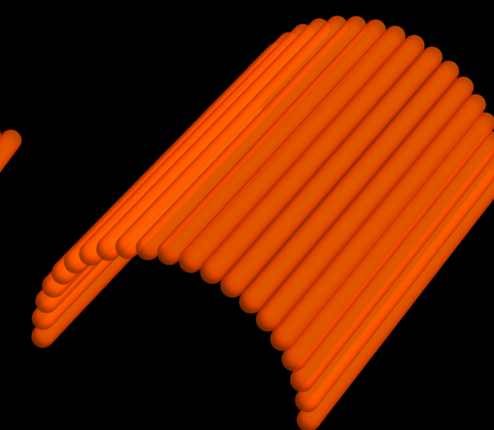
Mean (Extrinsic) Curvature: $H = \frac{1}{2}(\kappa_1 + \kappa_2)$

Bending energy

H=0



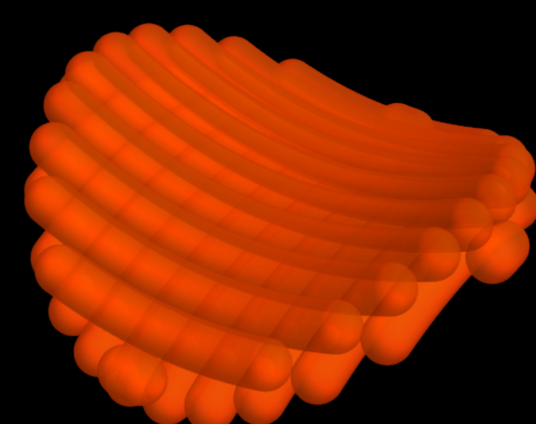
H<0



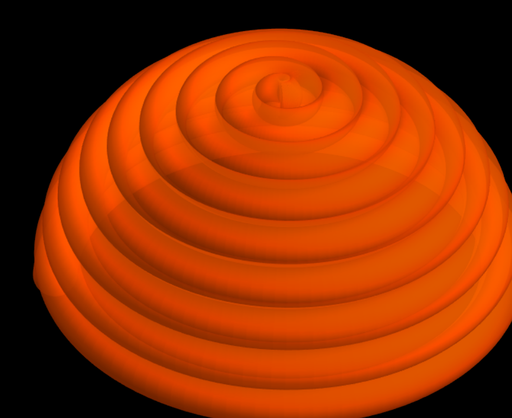
Gaussian (Intrinsic) Curvature: $K = \kappa_1 \kappa_2$

Stretching energy

K<0



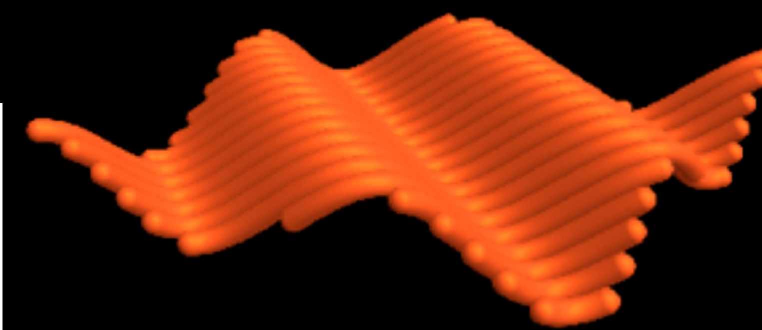
K>0



A Geometric Model

$$g(x,y) = R[\theta(x,y)] \begin{bmatrix} \alpha_\parallel & 0 \\ 0 & \alpha_\perp \end{bmatrix} R^T[\theta(x,y)]$$

Gauss's Theorema Egregium



GENERAL INVESTIGATIONS

OF

CURVED SURFACES

BY

KARL FRIEDRICH GAUSS

PRESENTED TO THE ROYAL SOCIETY, OCTOBER 8, 1827

Thus the formula of the preceding article leads of itself to the remarkable

THEOREM. *If a curved surface is developed upon any other surface whatever, the measure of curvature in each point remains unchanged.*

$$K(x,y) = K\left(g, \partial_x g, \partial_y g, \partial_{xx} g, \partial_{xy} g, \partial_{yy} g\right)$$

Aharoni, Sharon, Kupferman, *PRL* **113** 257801 (2014)

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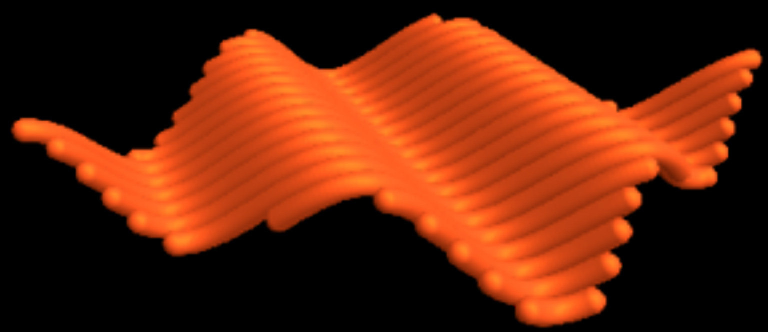

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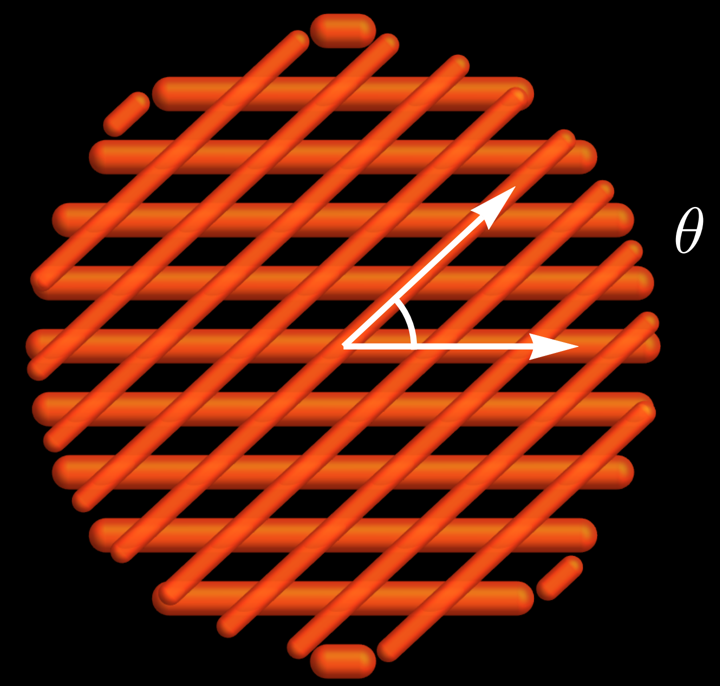
$$K(x,y) = (\alpha_\parallel - \alpha_\perp)\left[\frac{(\phi^2-1)\phi_{xy} - \phi(\phi_{xx} - \phi_{vv})}{(\phi^2+1)^2} + \frac{(3\phi^2-1)(\phi_x^2 - \phi_y^2) - 2\phi(\phi^2-3)\phi_x\phi_y}{(\phi^2+1)^3}\right]$$

Aharoni, Sharon, Kupferman, *PRL* **113** 257801 (2014)

The Model

Theory of Anisotropic Plates and Shells



Curvature in Monge Gauge $\kappa_{ij} = \partial_i \partial_j H(x, y)$

Swelling Strain $\varepsilon^{\mathrm{s}} = \begin{bmatrix} \alpha_\parallel & 0 \\ 0 & \alpha_\perp \end{bmatrix}$

Elastic Strain $\varepsilon^{\mathrm{e}}_{ij} = -z\kappa_{ij}$

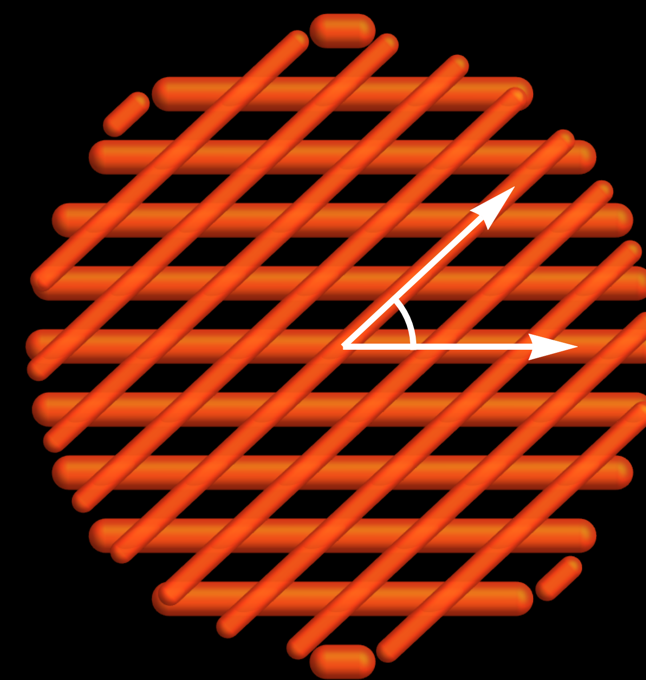
Strain Tensor $\varepsilon = \varepsilon^{\mathrm{s}} + \varepsilon^{\mathrm{e}}$ $\varepsilon_{ij}(\theta) = R_{im}(\theta)\varepsilon_{mn}R^T_{jn}(\theta)$

Elastic Modulus Tensor $\mathrm{E}_{ijkl}(\theta) = R_{im}(\theta)R_{kp}(\theta)\mathrm{E}_{mnpq}R^T_{jn}(\theta)R^T_{lq}(\theta)$

Stress-Strain Relation $\sigma_{ij} = \mathrm{E}_{ijkl}\varepsilon^{\mathrm{e}}_{kl}$

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

The Model



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Bending Moments

$$M_{ij} = \int_{-a_2}^{a_1} z\sigma_{ij}dz = -\int_{-a_2}^{a_1} z^2 \mathrm{E}_{ijkl}\kappa_{kl}dz$$

$$= -\int_{0}^{a_1} \mathrm{E}_{ijkl}(0)\kappa_{kl}z^2 dz - \int_{-a_2}^{0} \mathrm{E}_{ijkl}(\theta)\kappa_{kl}z^2 dz$$

1. Equilibrium Condition

"Due to the fact that there are not external forces acting on the strip, all forces acting over any cross-section of the strip must be in equilibrium"

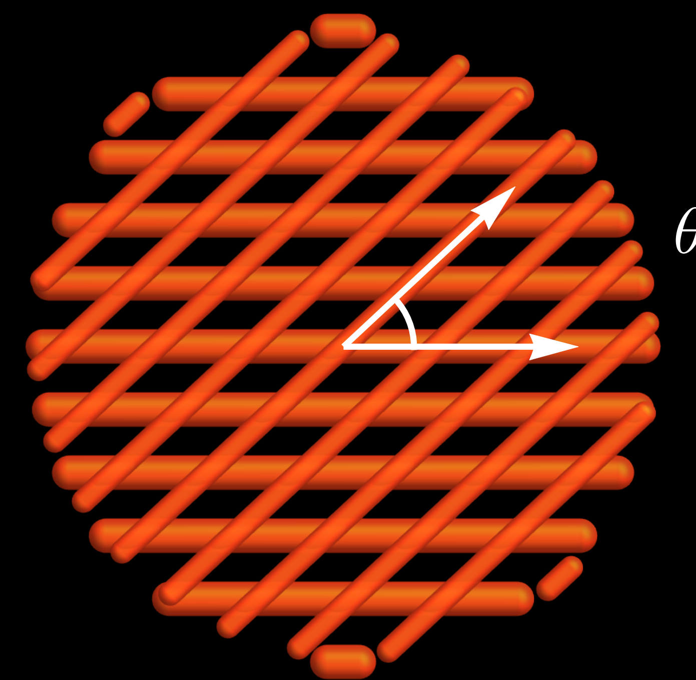
2. Compatibility Condition

"On the bearing surface of both metals the unit elongation occurring in the longitudinal fibres of metals (1) and (2) must be equal."

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

The Model

Theory of Anisotropic Plates and Shells



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$$\frac{1}{\alpha_1}\int_0^{a_1}\left(\varepsilon^{(1)}_{ij} + \frac{\mathrm{E}^{-1}_{ijkl}}{a_1}M_{kl}(\theta)\right)dz = \frac{1}{\alpha_2}\int_{-a_2}^{0}\left(\varepsilon^{(2)}_{ij}(\theta) + \frac{\mathrm{E}^{-1}_{ijkl}(\theta)}{a_2}M_{kl}(\theta)\right)dz$$

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

The Model

$$\frac{1}{\alpha_1}\int_0^{a_1}\left(\varepsilon_{ij}^{(1)}+\frac{\mathrm{E}_{ijkl}^{-1}}{a_1}M_{kl}(\theta)\right)dz=\frac{1}{\alpha_2}\int_{-a_2}^{0}\left(\varepsilon_{ij}^{(2)}(\theta)+\frac{\mathrm{E}_{ijkl}^{-1}(\theta)}{a_2}M_{kl}(\theta)\right)dz$$

Given: $\alpha_\|,\ \alpha_\perp,\ \mathrm{E}_{ijkl},\ a_1,\ a_2, \theta$ Solve for: κ_{ij}

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$$H=\frac{\alpha_{\perp}-\alpha_{\|}}{h}\frac{c_1\sin^2(\theta)}{c_2-c_3\cos(2\theta)+m^4\cos(4\theta)}$$

$$K=-\frac{(\alpha_{\perp}-\alpha_{\|})^2}{h^2}\frac{c_4\sin^2(\theta)}{c_5-c_6\cos(2\theta)+m^4\cos(4\theta)}$$

$$,\quad c_i=c_i(\mathrm{E}^{(1)},\mathrm{E}^{(2)},\mathrm{m}=\mathrm{a}_1/\mathrm{a}_2)$$

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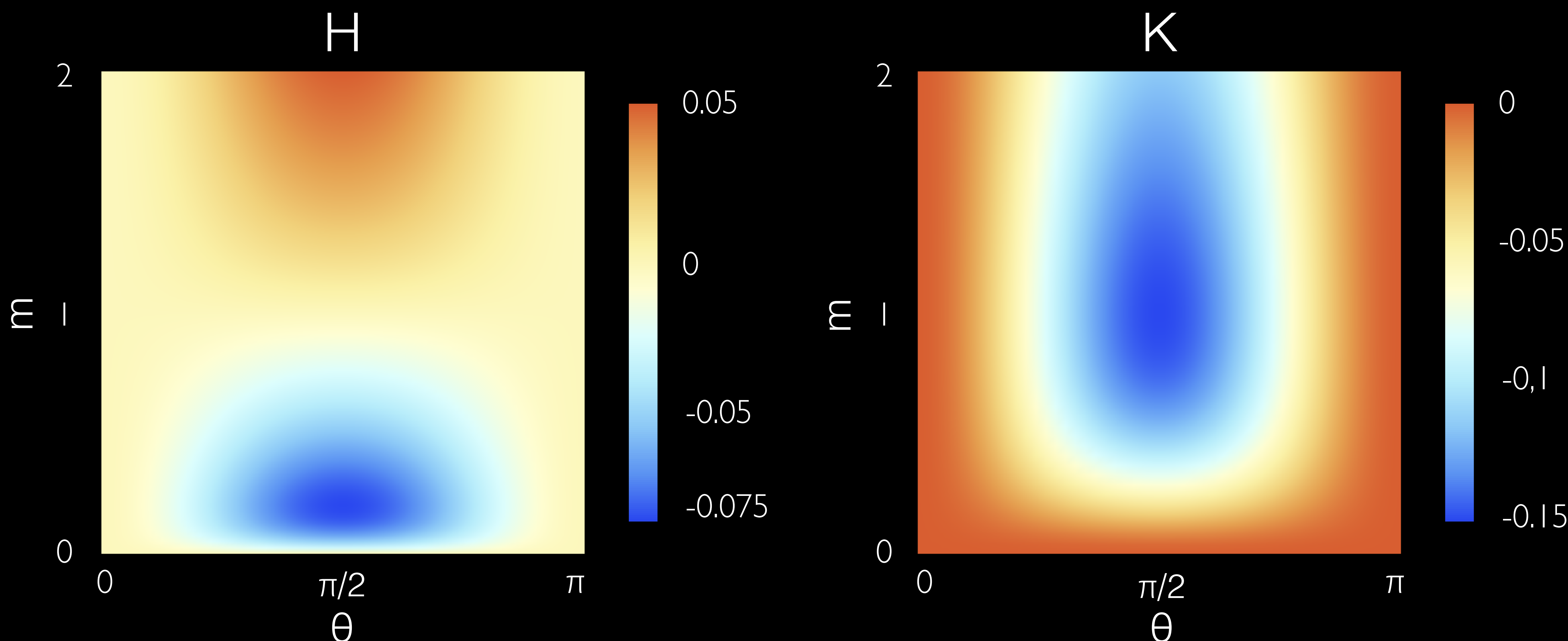
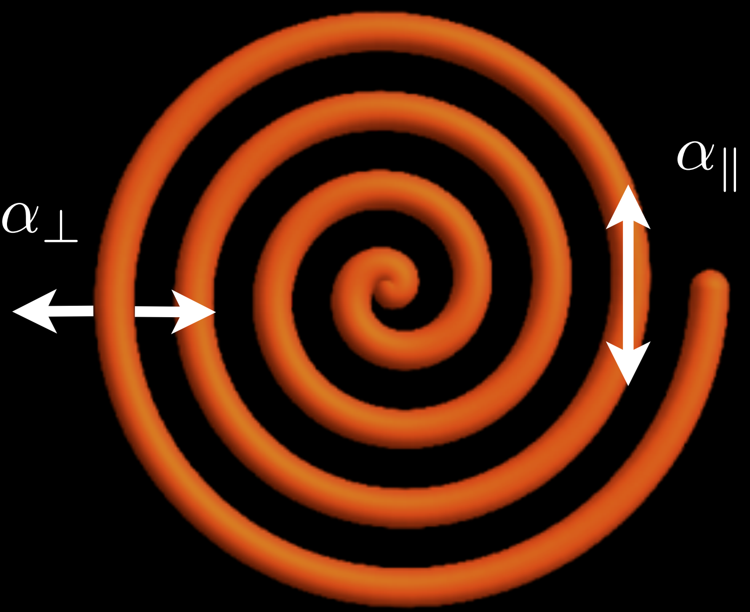


A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

Controlling Gaussian Curvature

K>0



$\alpha_{\perp} > \alpha_{\parallel}$

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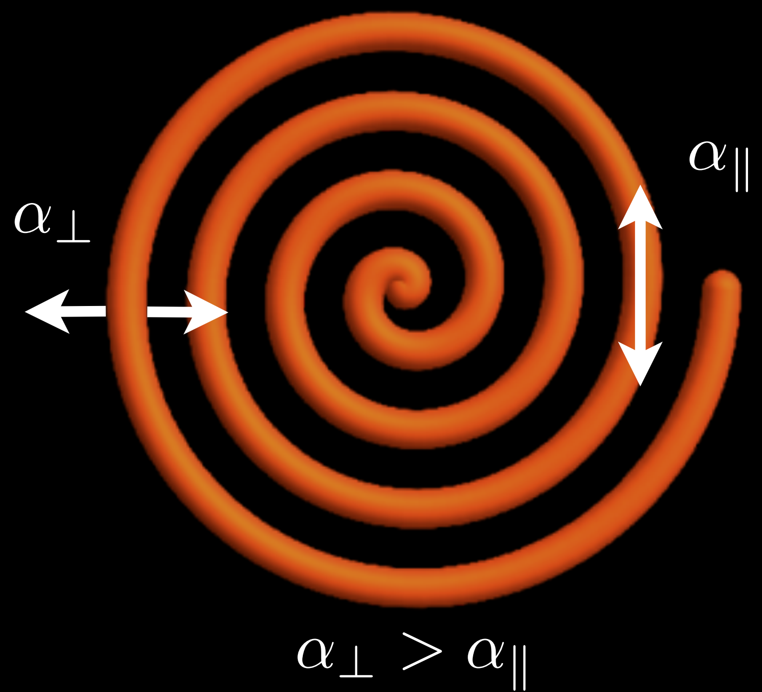

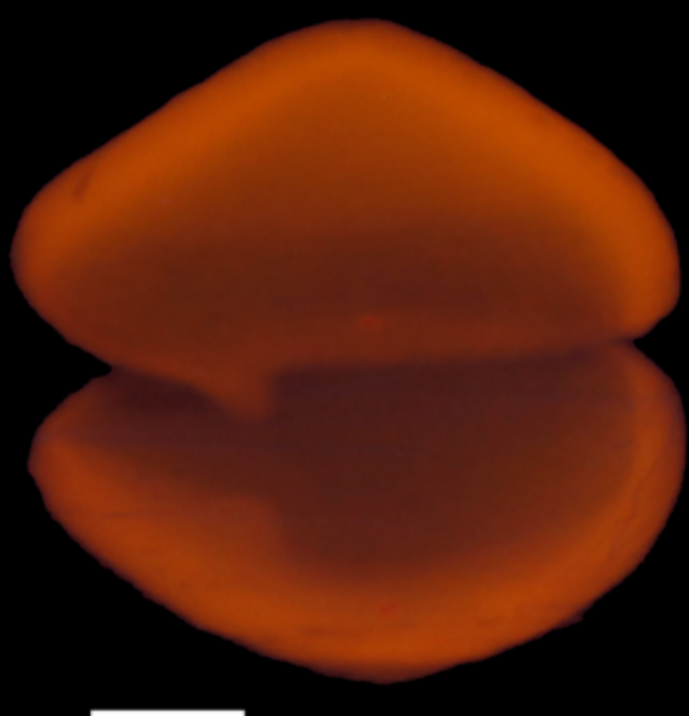

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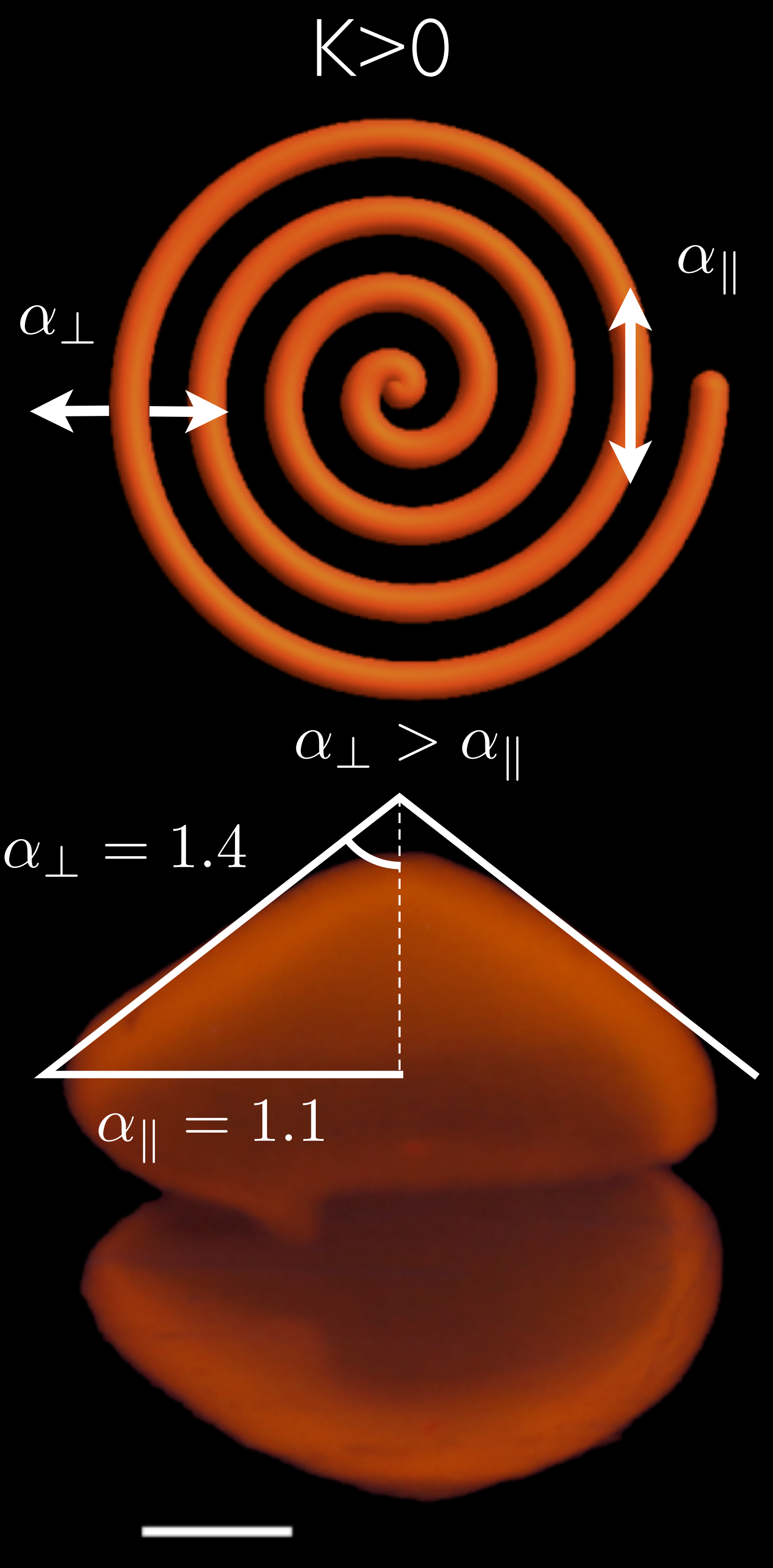

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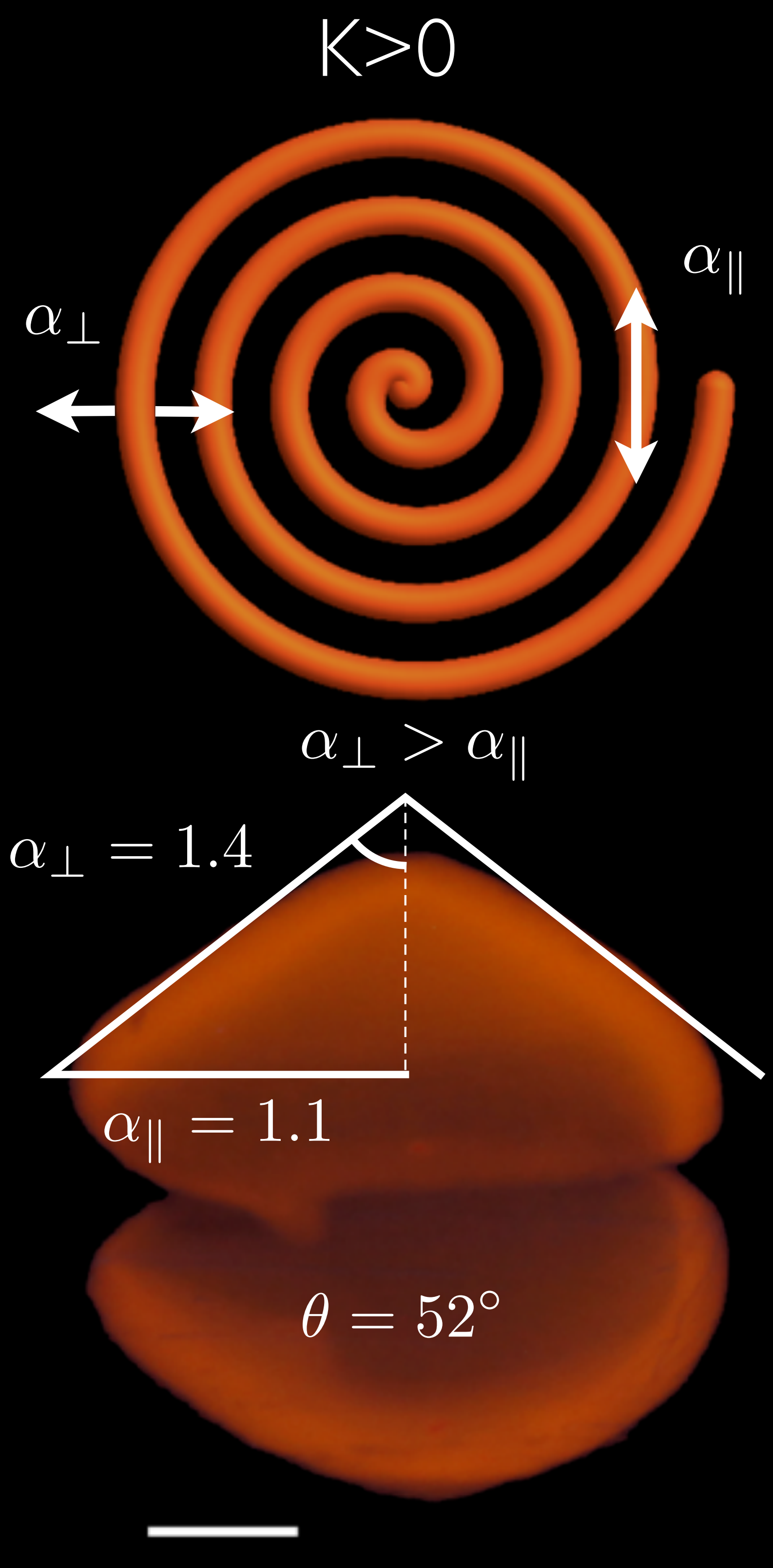

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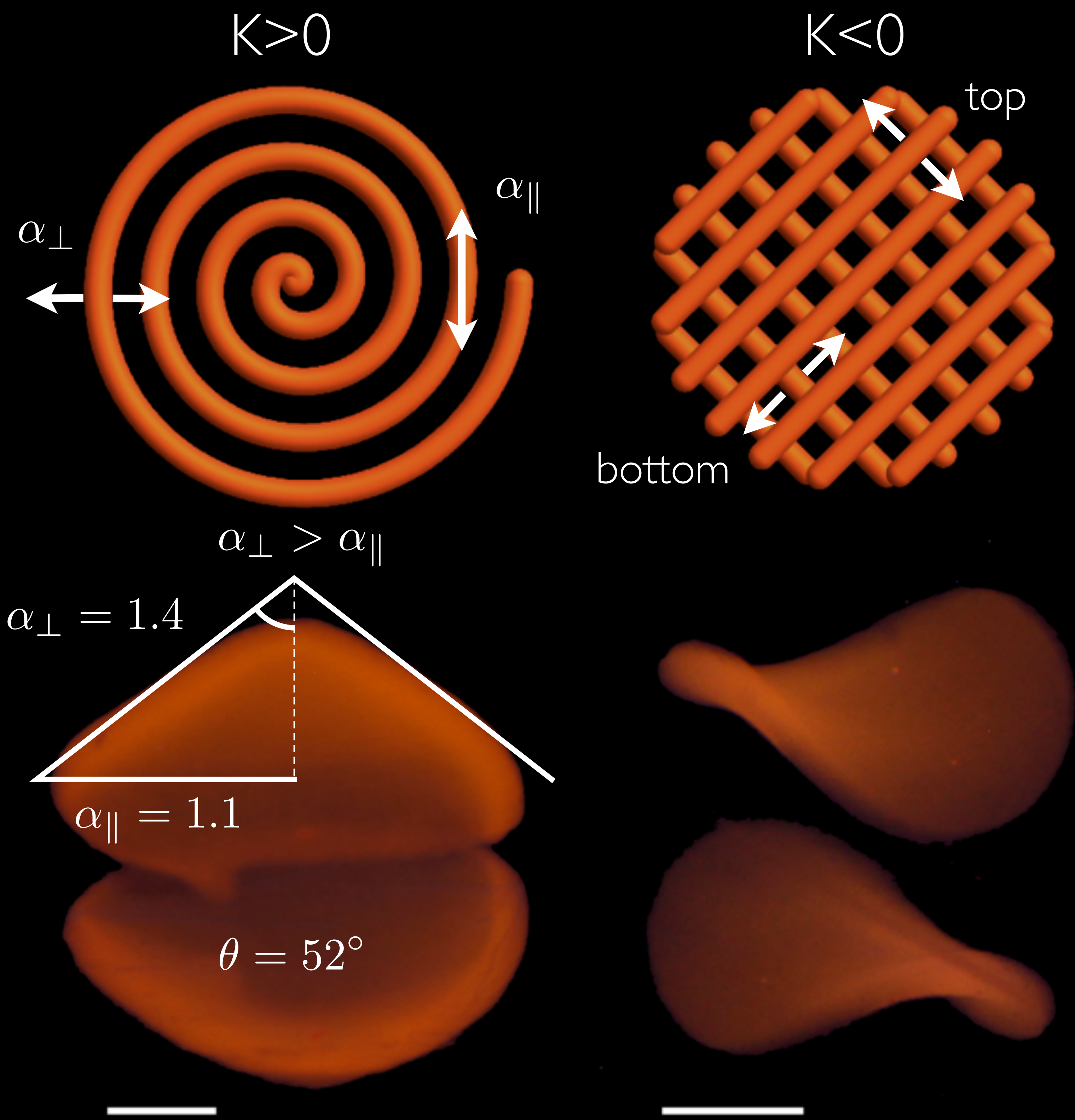

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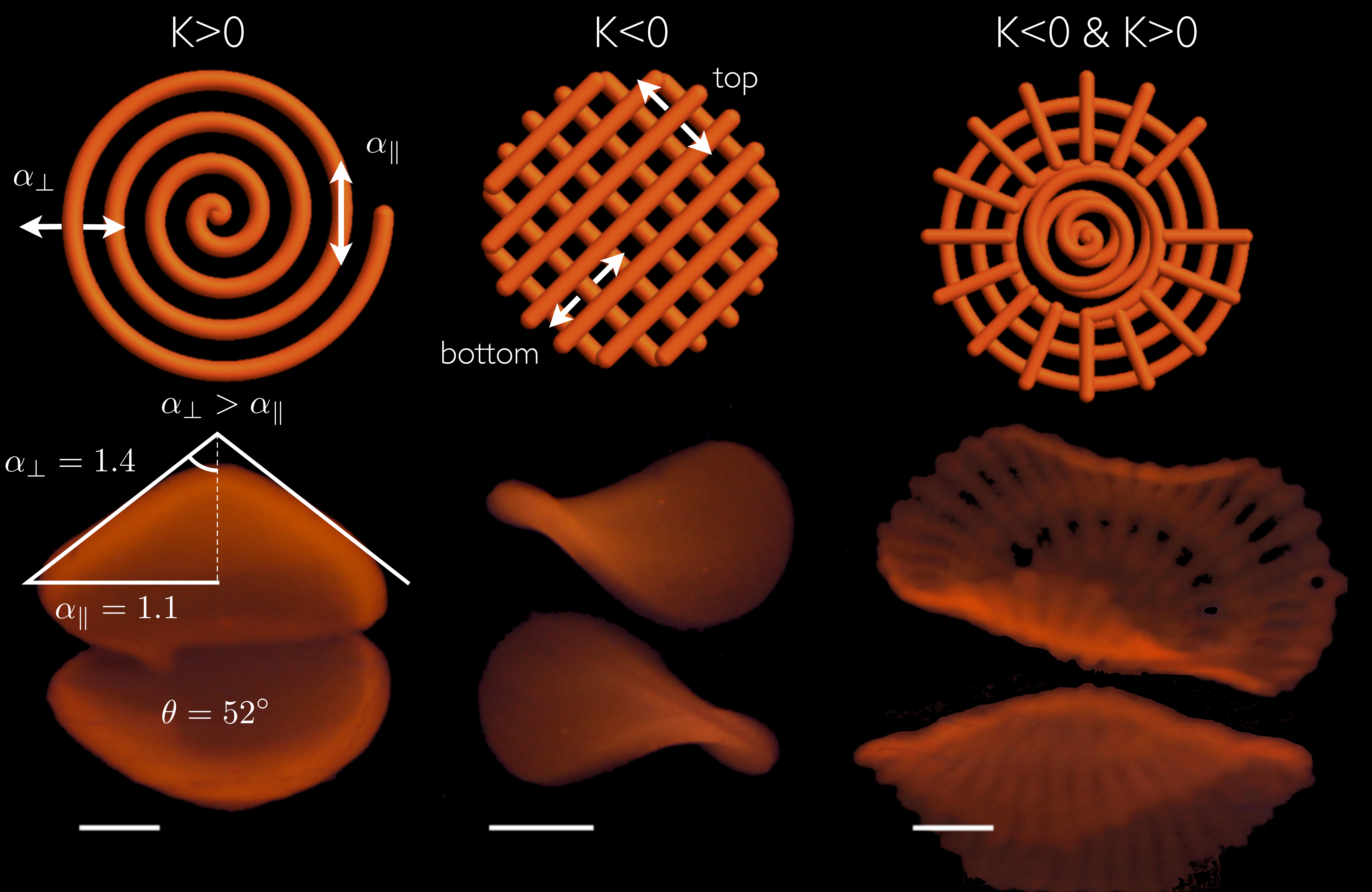
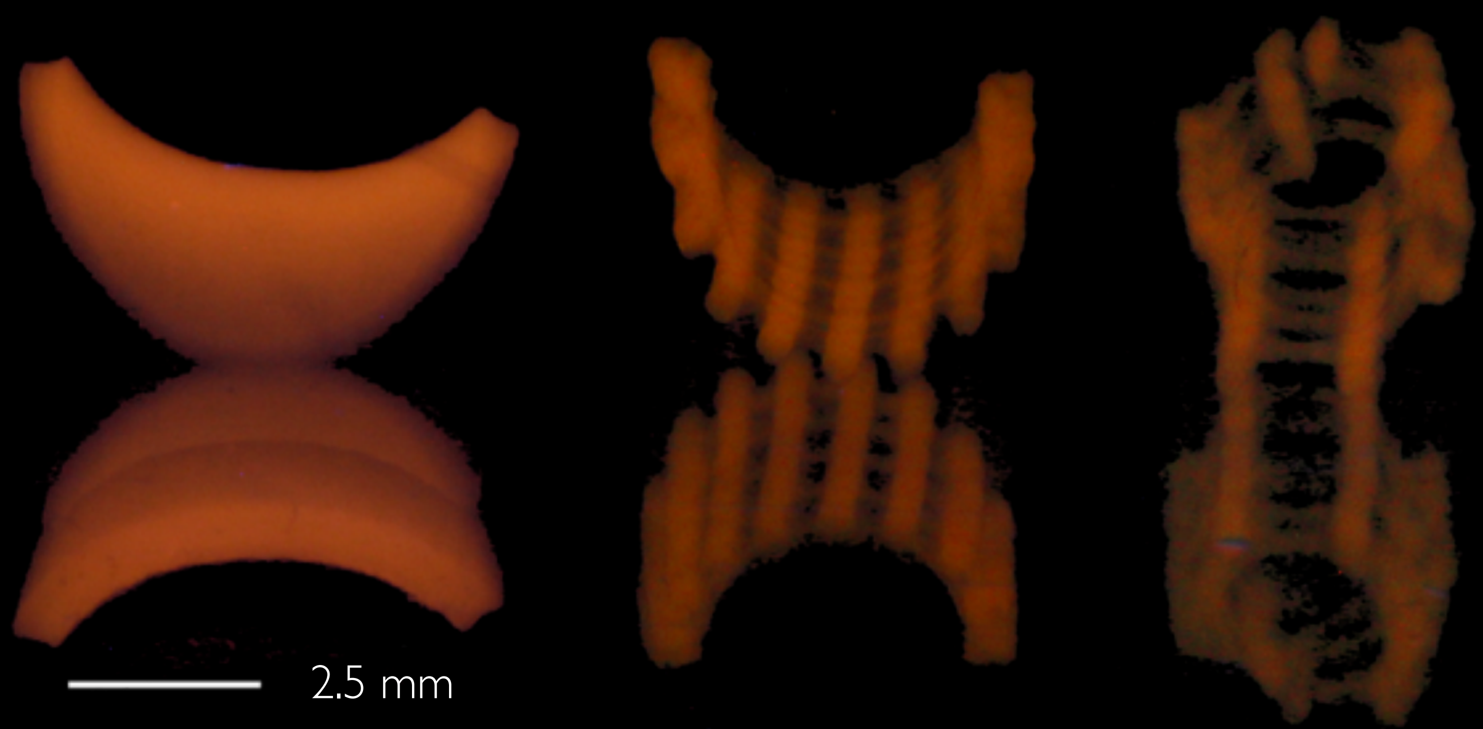


A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

Controlling Mean Curvature

$\kappa = 3/2(\alpha_1-\alpha_2)/h = 0.45/h\ \mathrm{mm}^{-1}$

h = 1.25 mm   h = 0.75 mm   h = 0.5 mm



predicted

$\kappa = 0.36\ \mathrm{mm}^{-1}$ $\kappa = 0.6\ \mathrm{mm}^{-1}$ $\kappa = 0.9\ \mathrm{mm}^{-1}$

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

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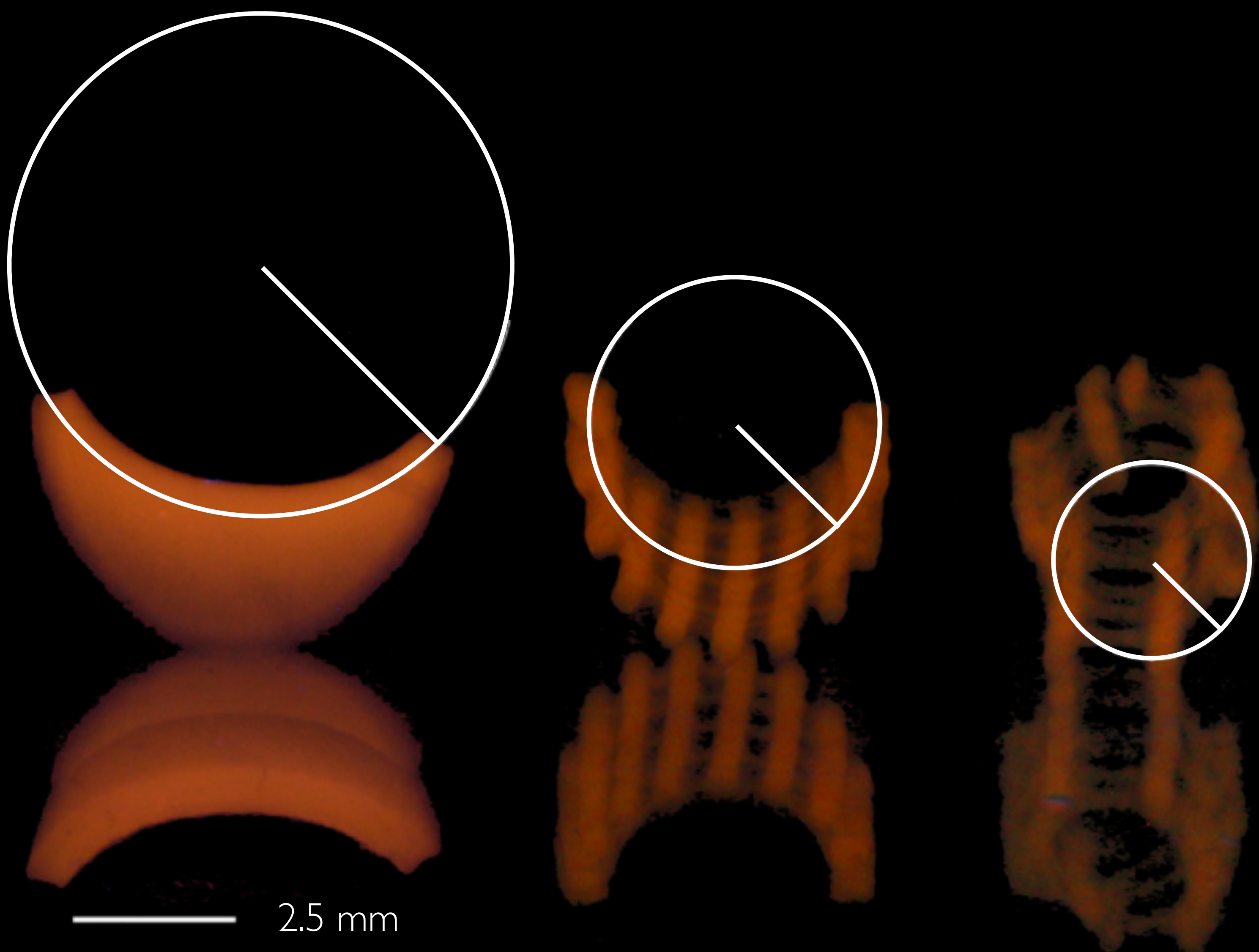

predicted

$\kappa = 0.36\ mm^{-1}$ | $\kappa = 0.6\ mm^{-1}$ | $\kappa = 0.9\ mm^{-1}$

measured

$\kappa = 0.34\ mm^{-1}$ | $\kappa = 0.61\ mm^{-1}$ | $\kappa = 0.85\ mm^{-1}$

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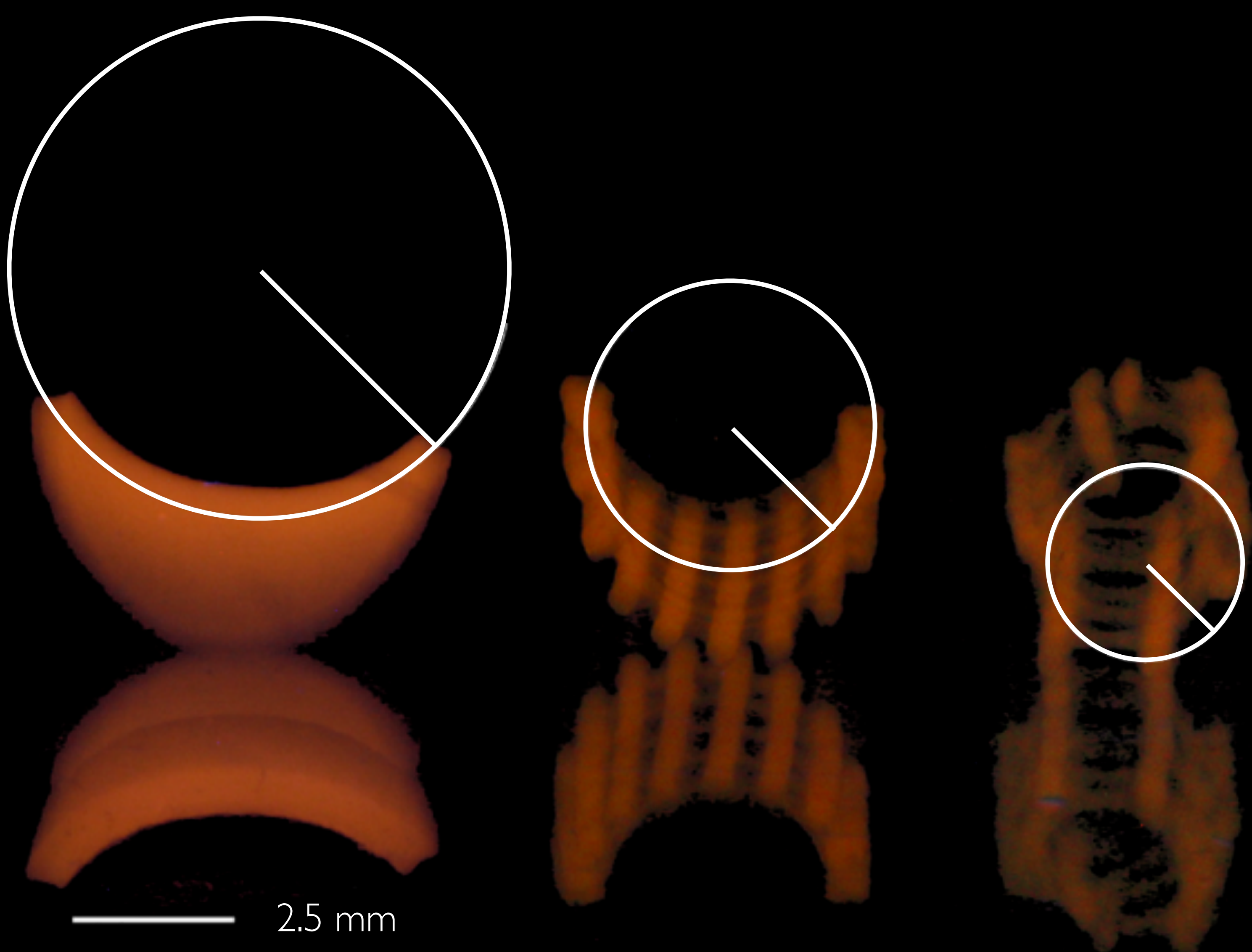

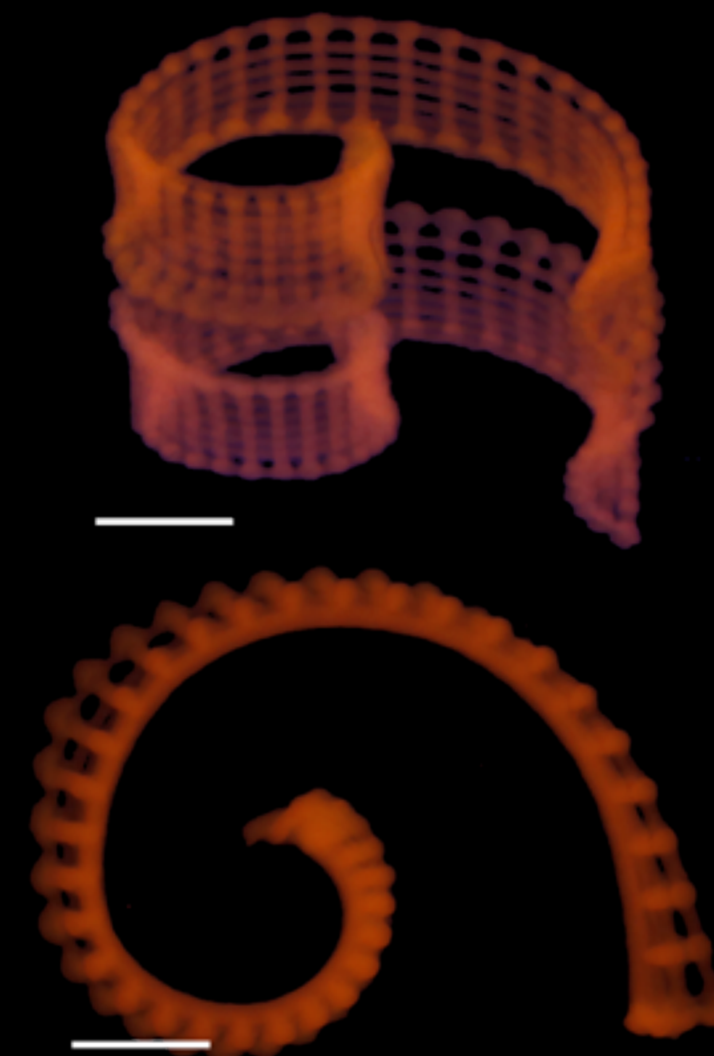

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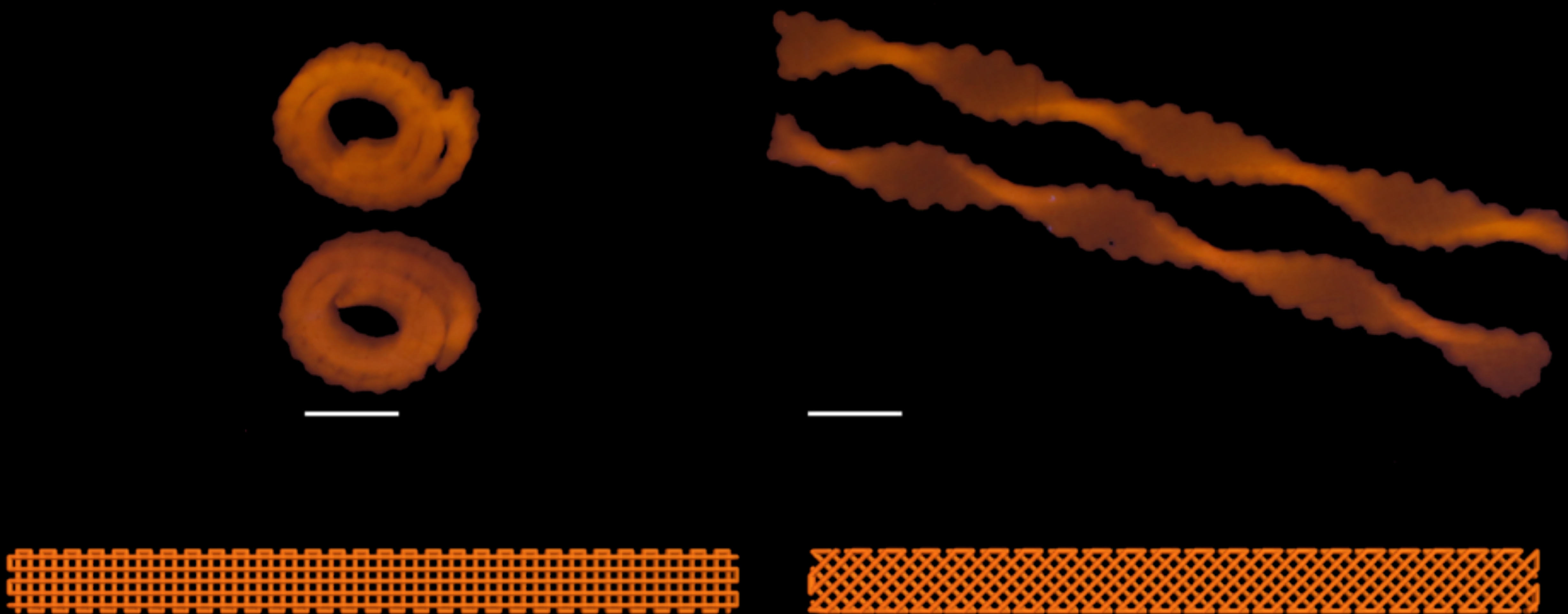
To Twist or Not To Twist, That is the Question



Bottom Layer: 0°
Top Layer: 90°

Bottom Layer: -45°
Top Layer: 45°

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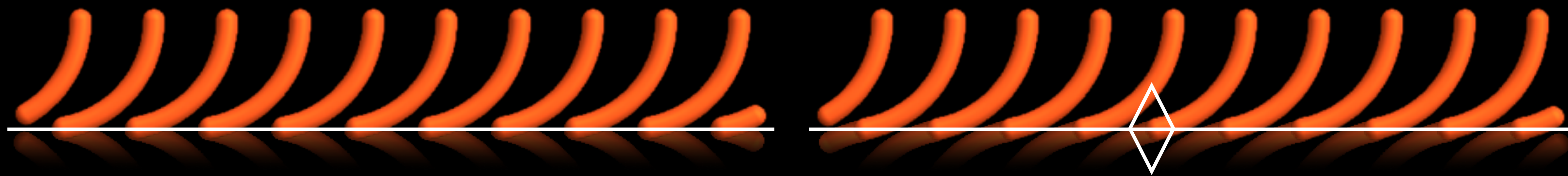

mirror rotation by 180°

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

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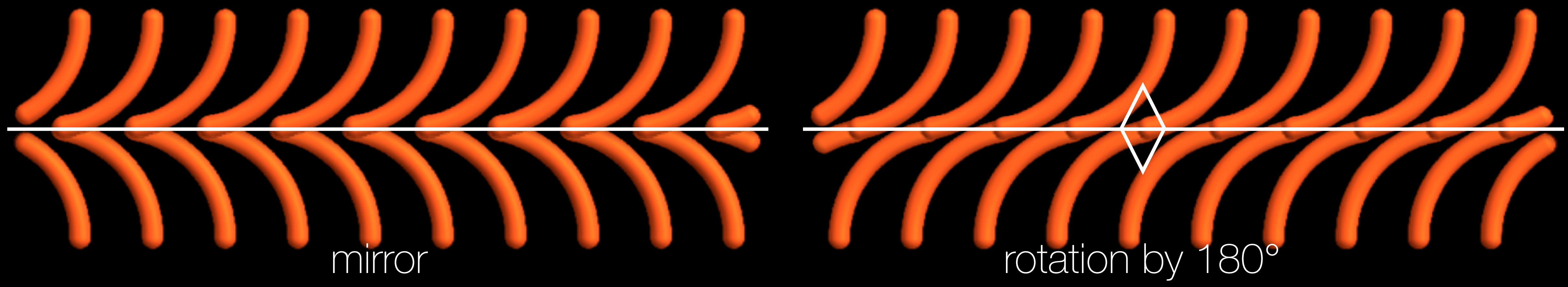

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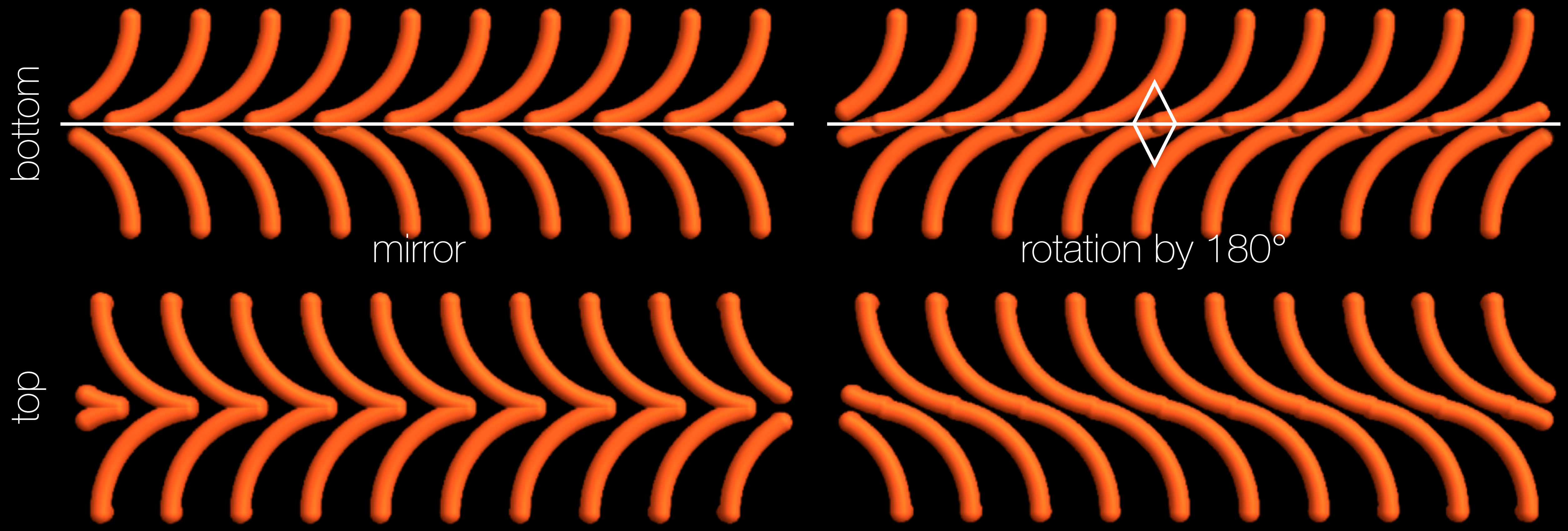

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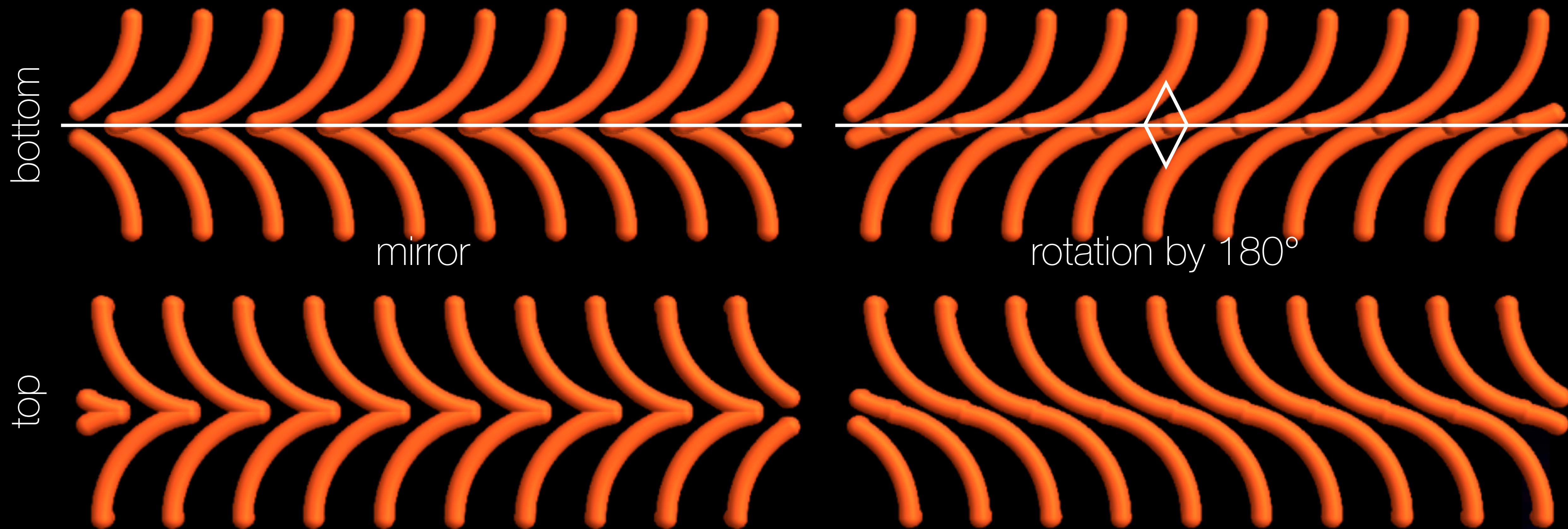

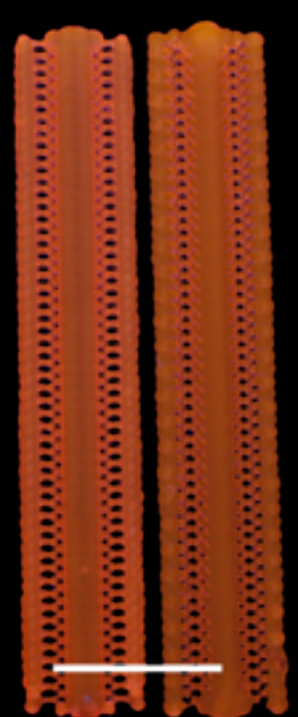

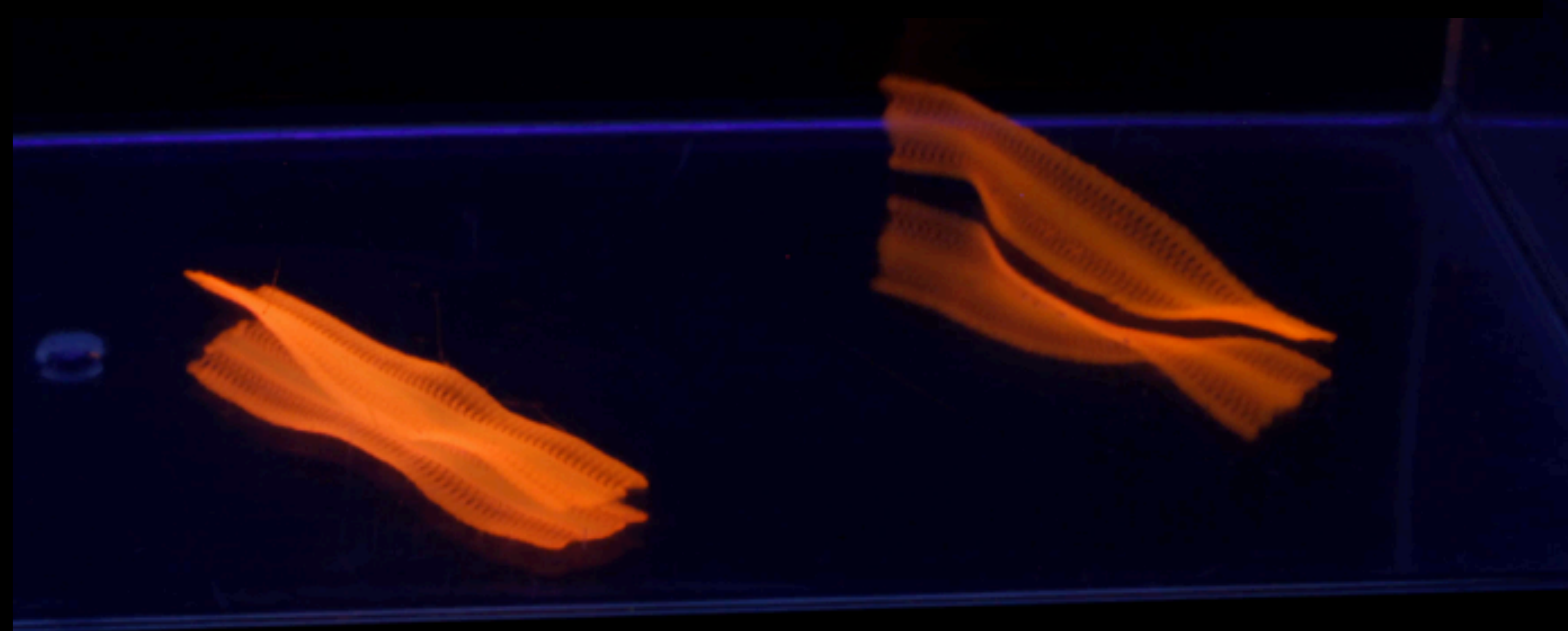

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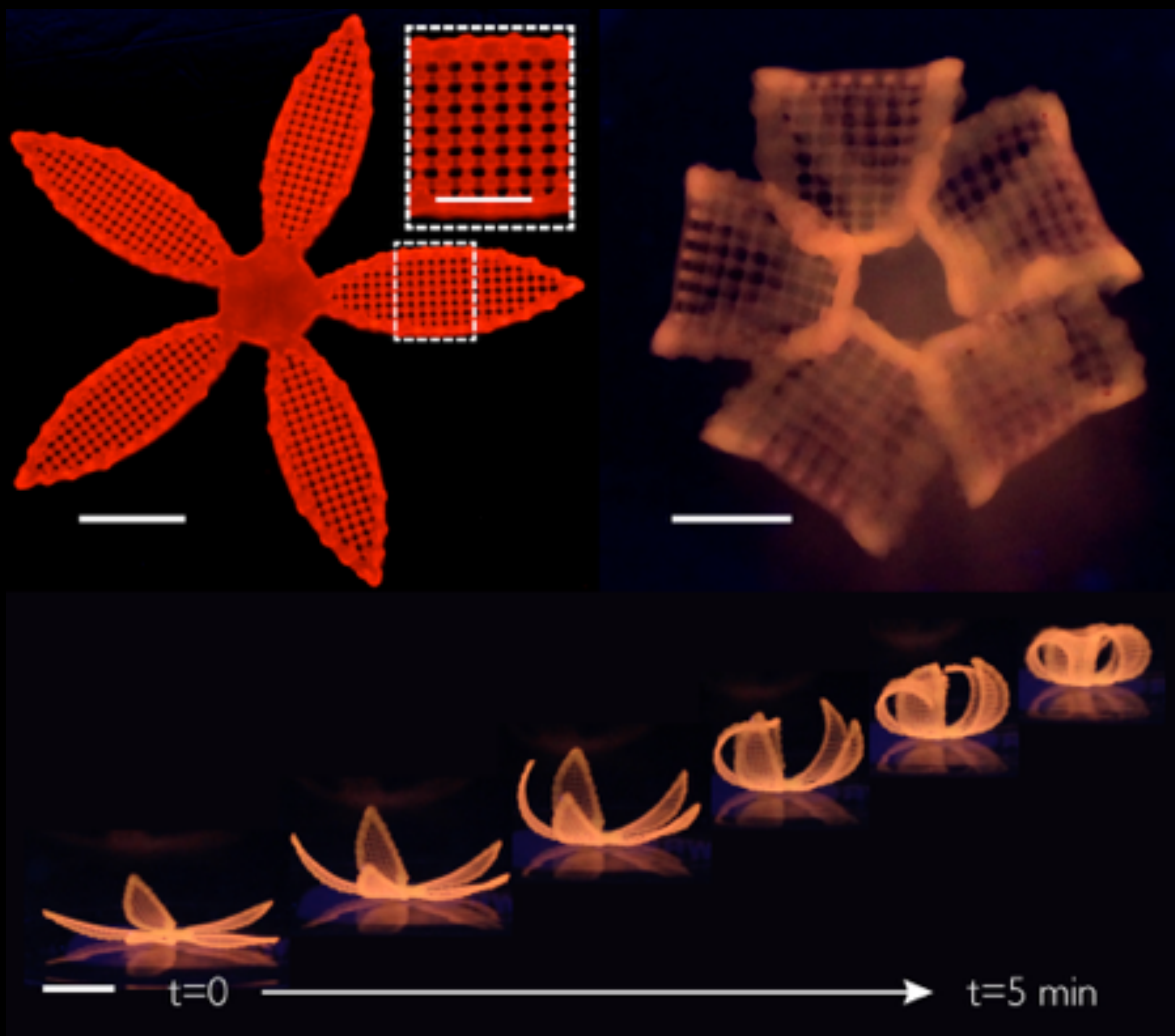
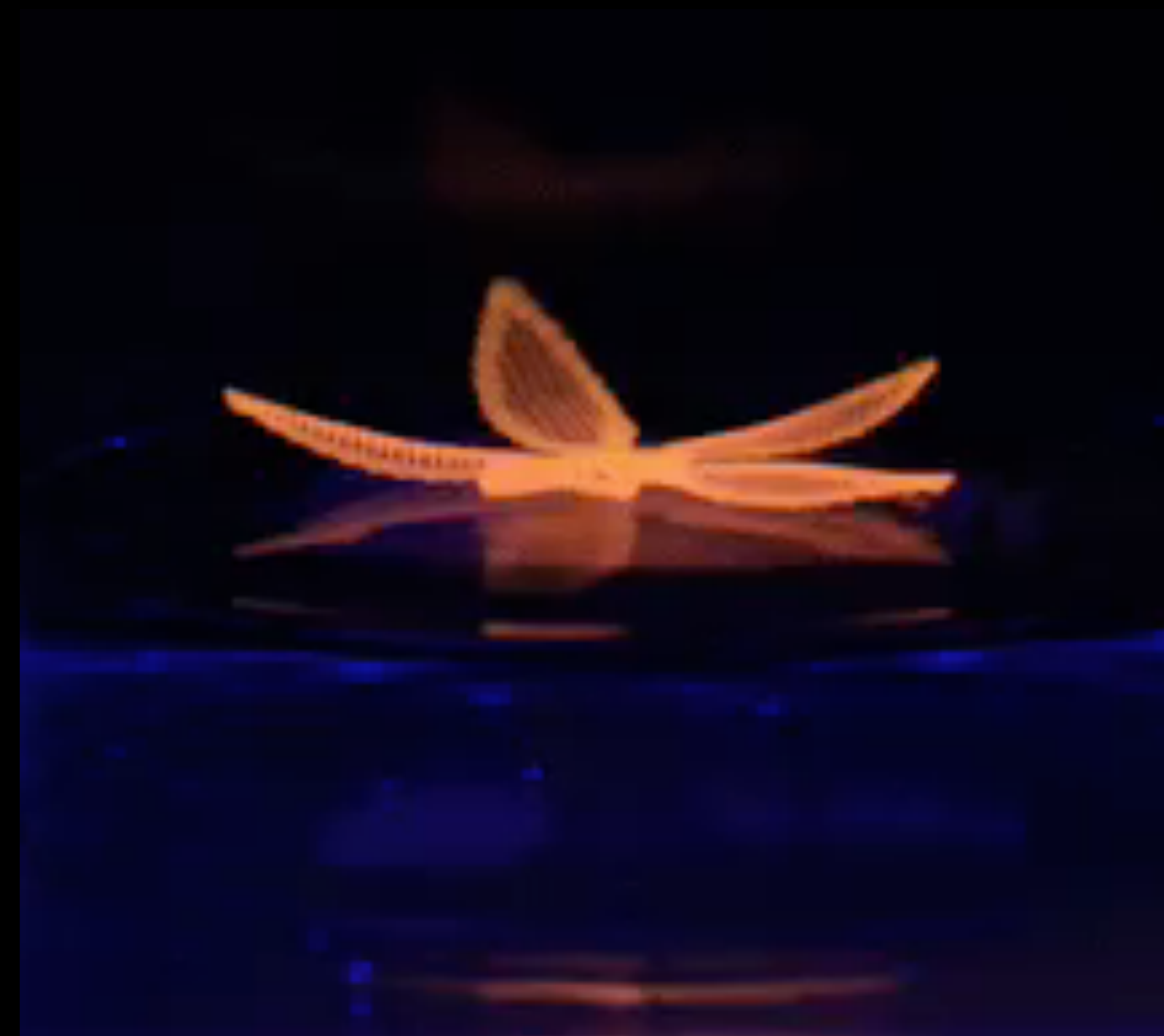
Forty 4D Folding Flowers





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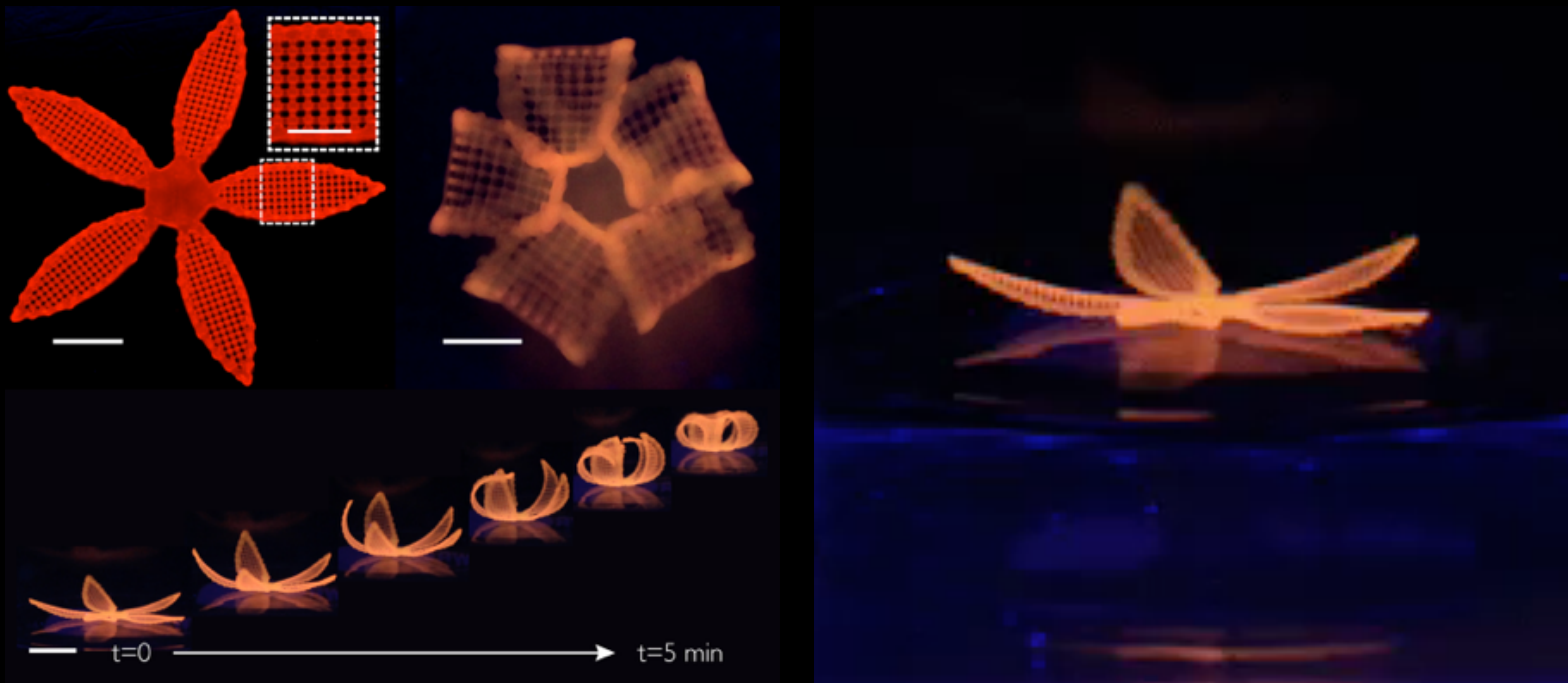

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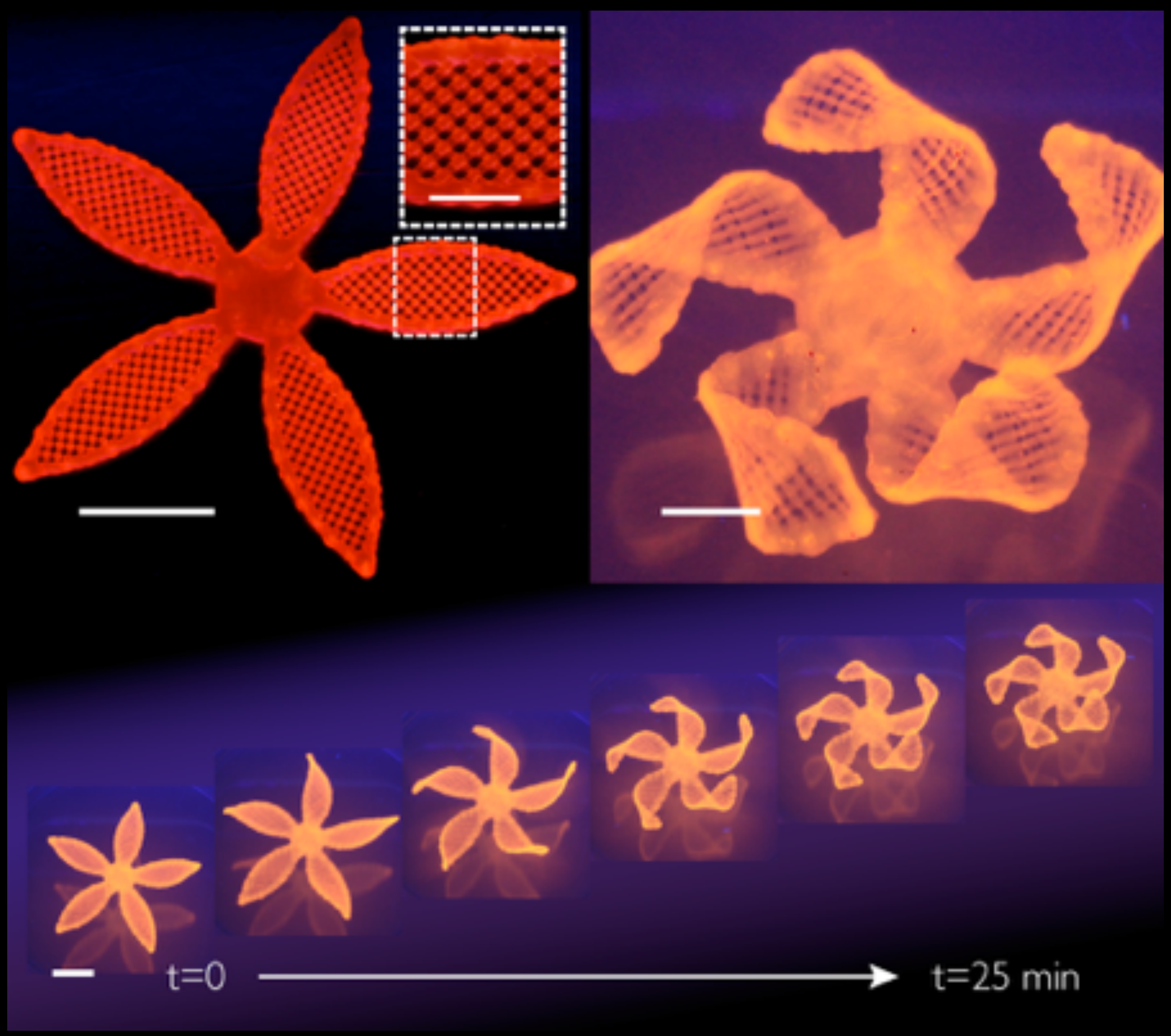

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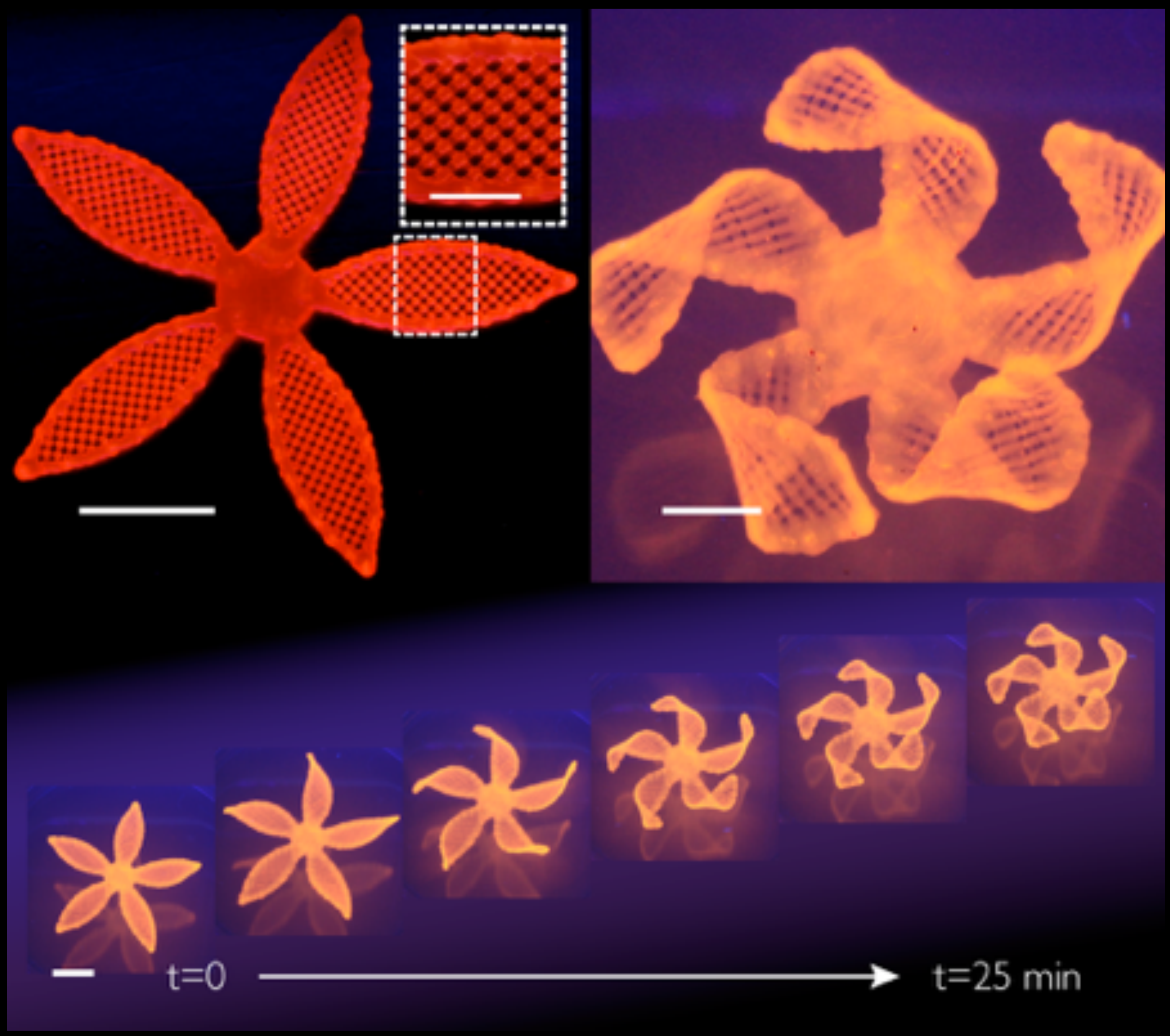

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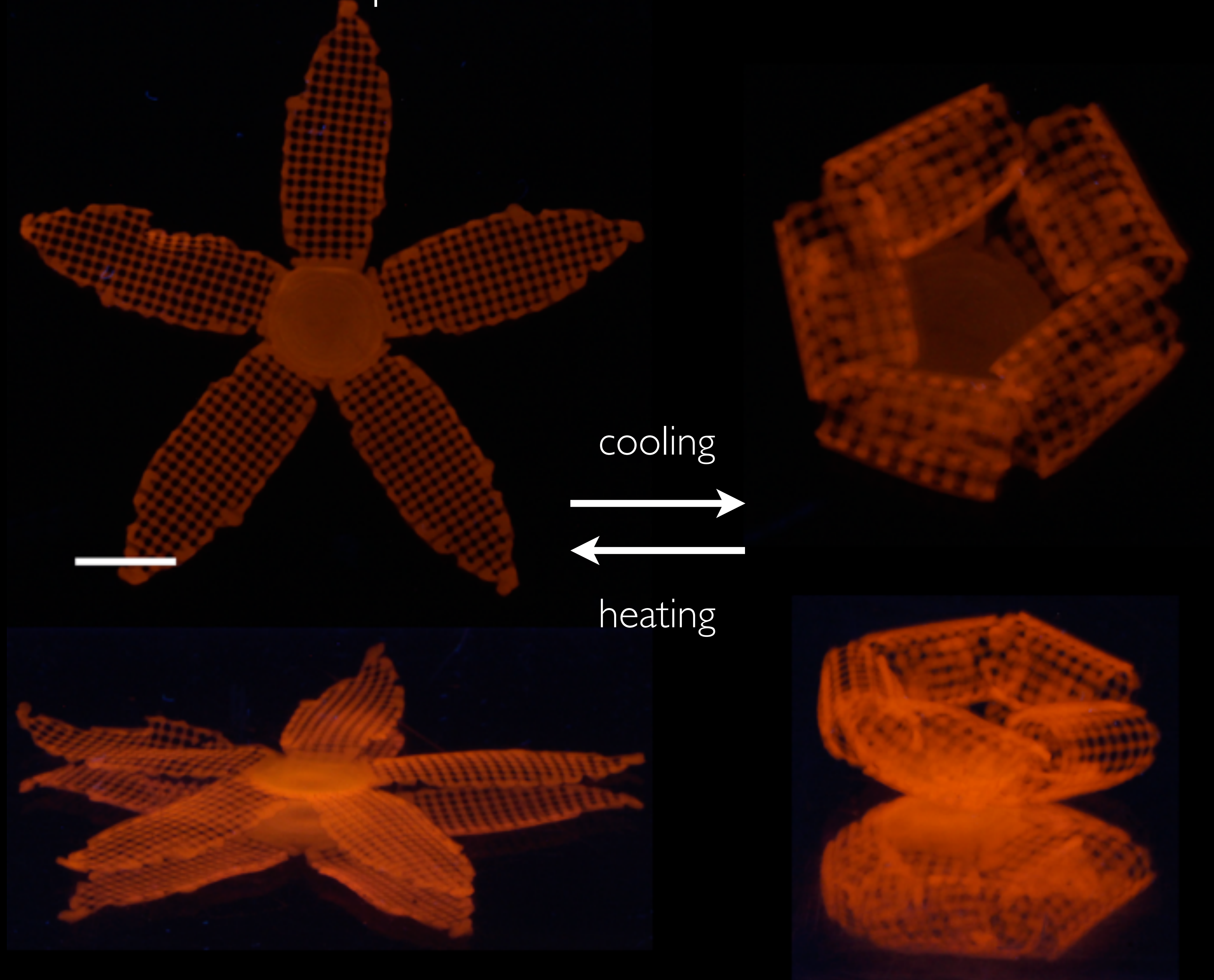
Reversibility

Replace PDMA with PNIPA



PNIPA undergoes a hydrophilic-hydrophobic transition at 40°C.

A. S. Gladman, **EAM**, R. Nuzzo, L. Mahadevan, and J. Lewis, *Nature Materials* (advanced online publication) 2016.

Forty 4D Folding Flowers

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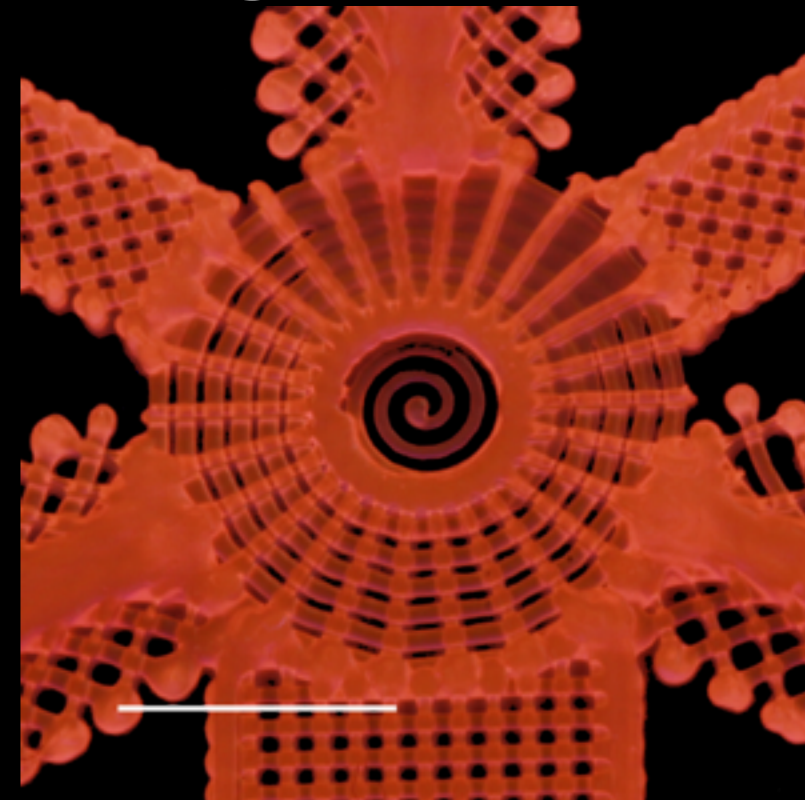

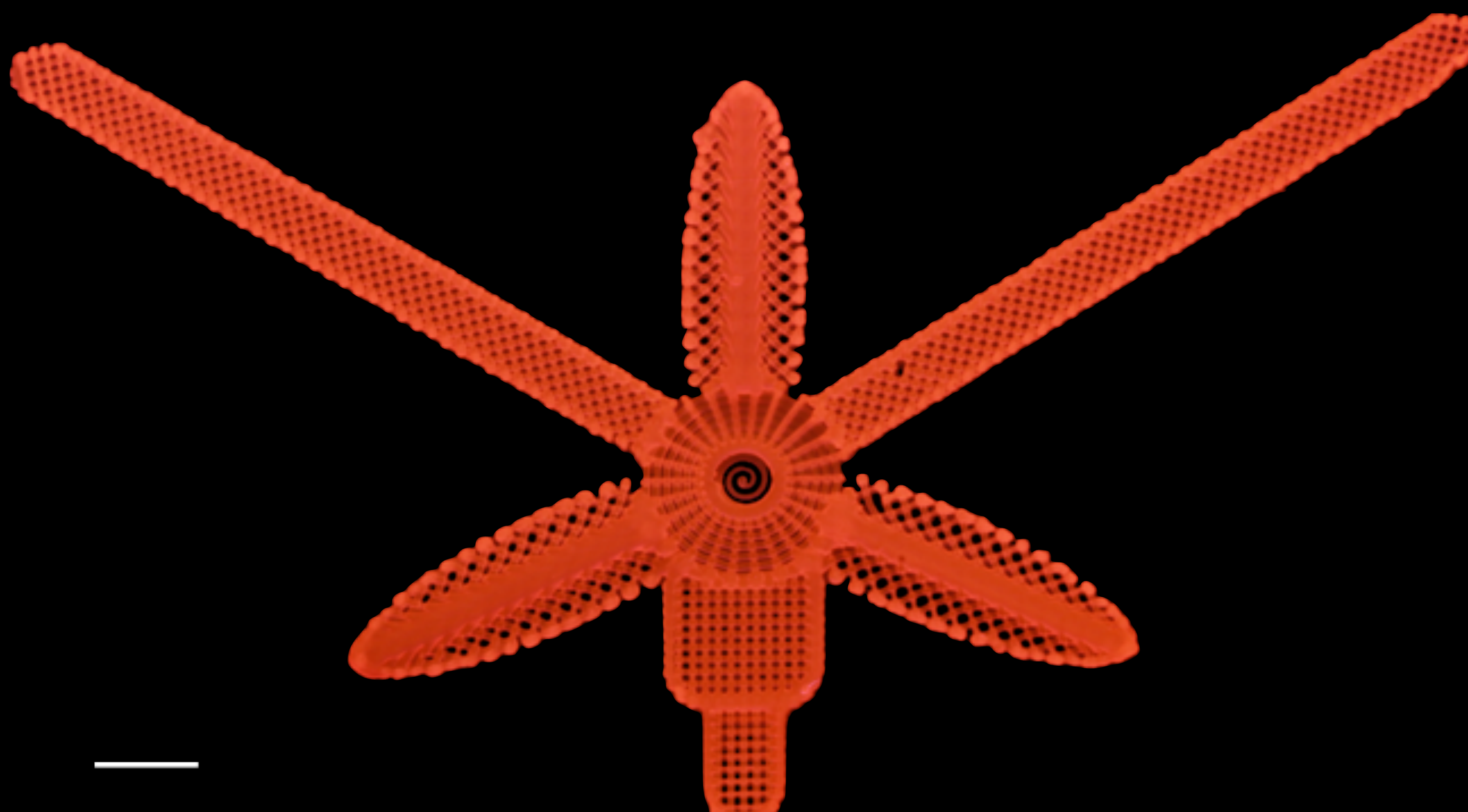

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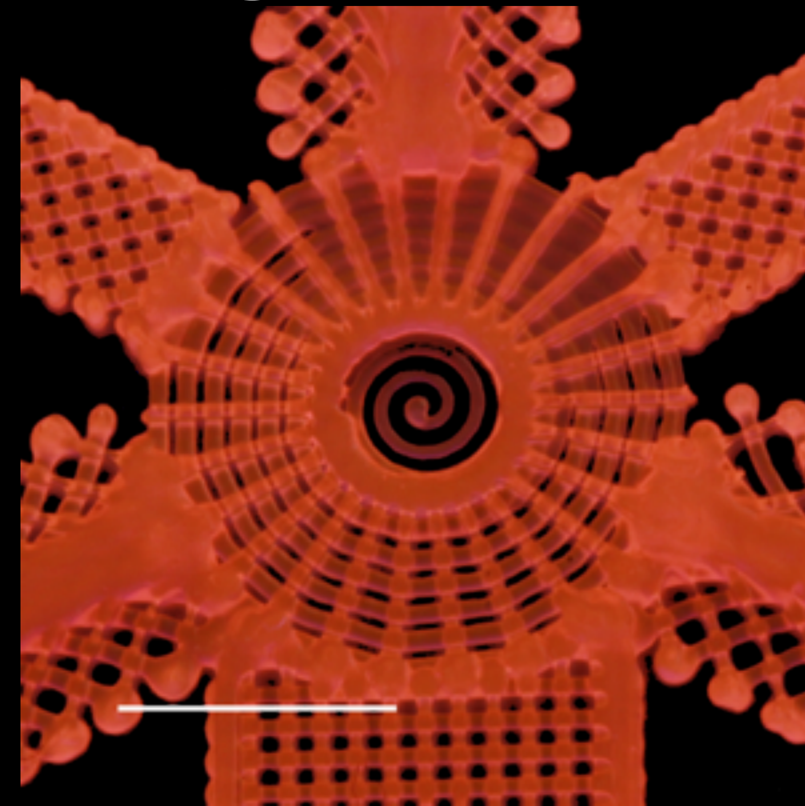
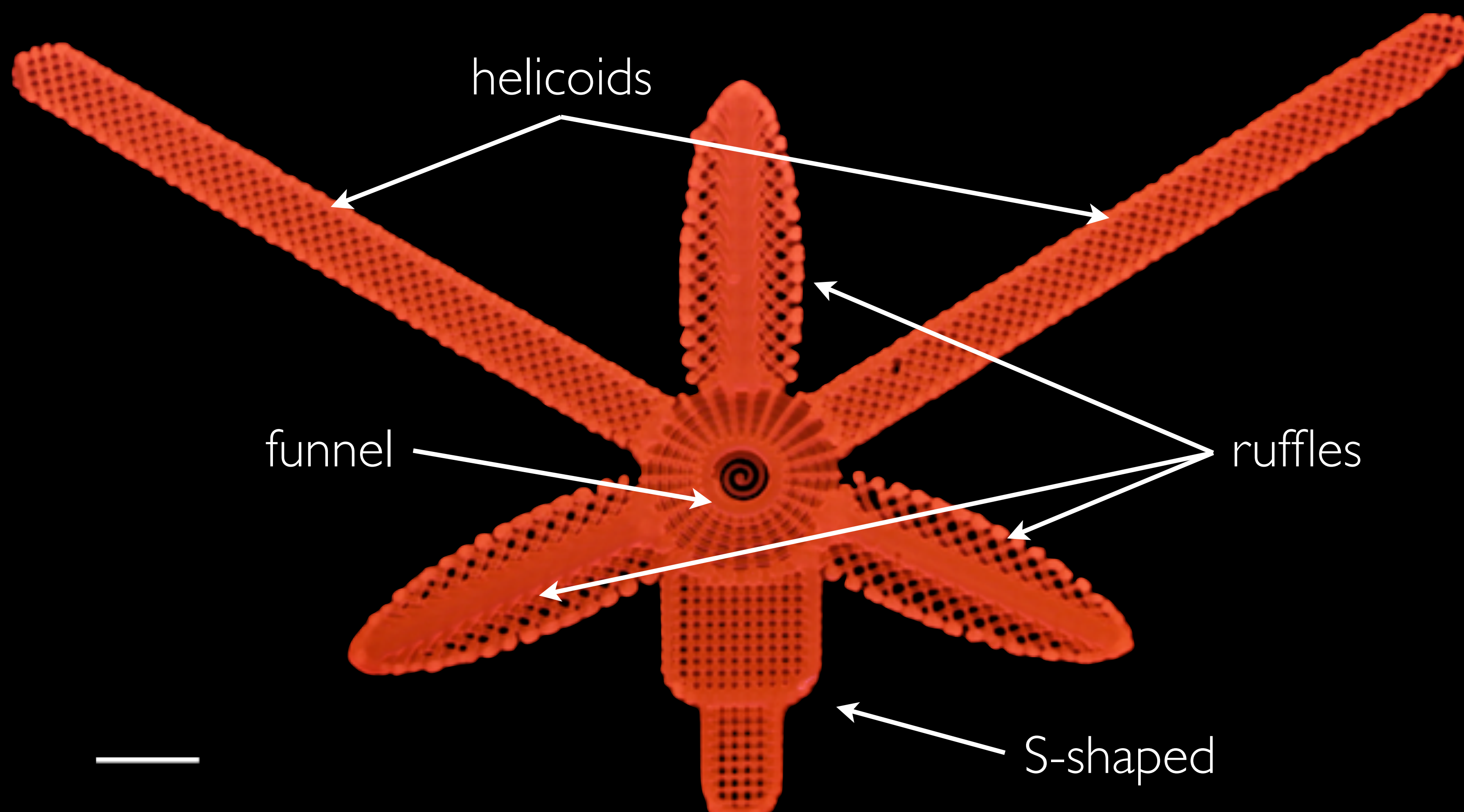
Forty 4D Folding Flowers





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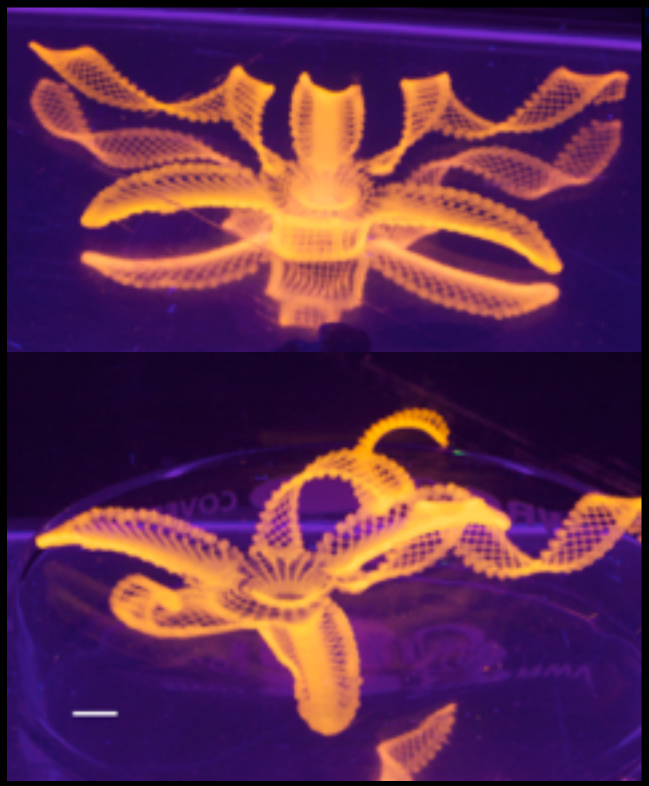

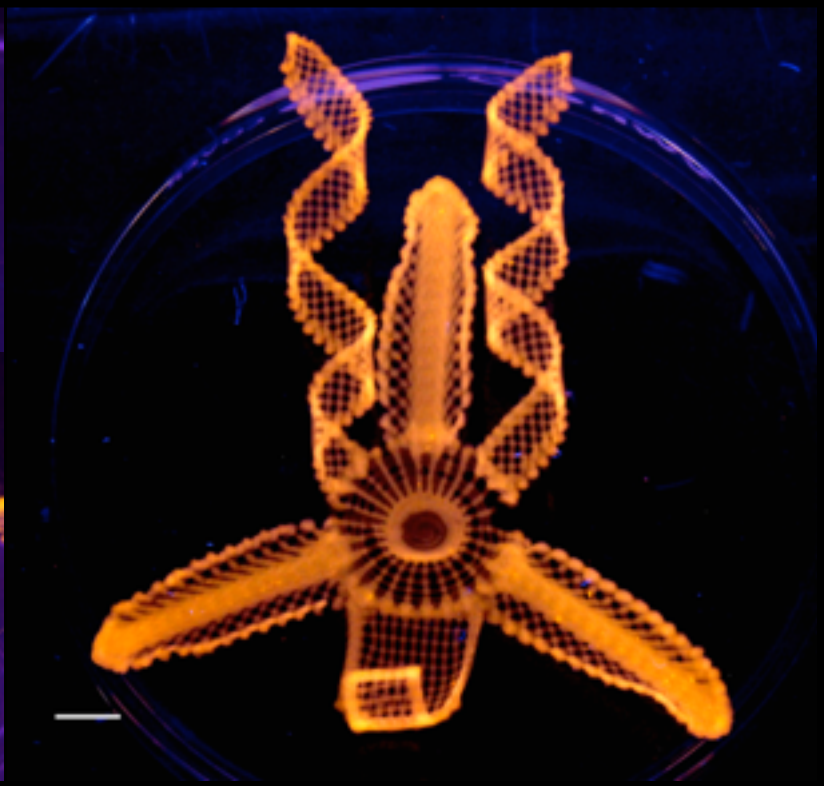

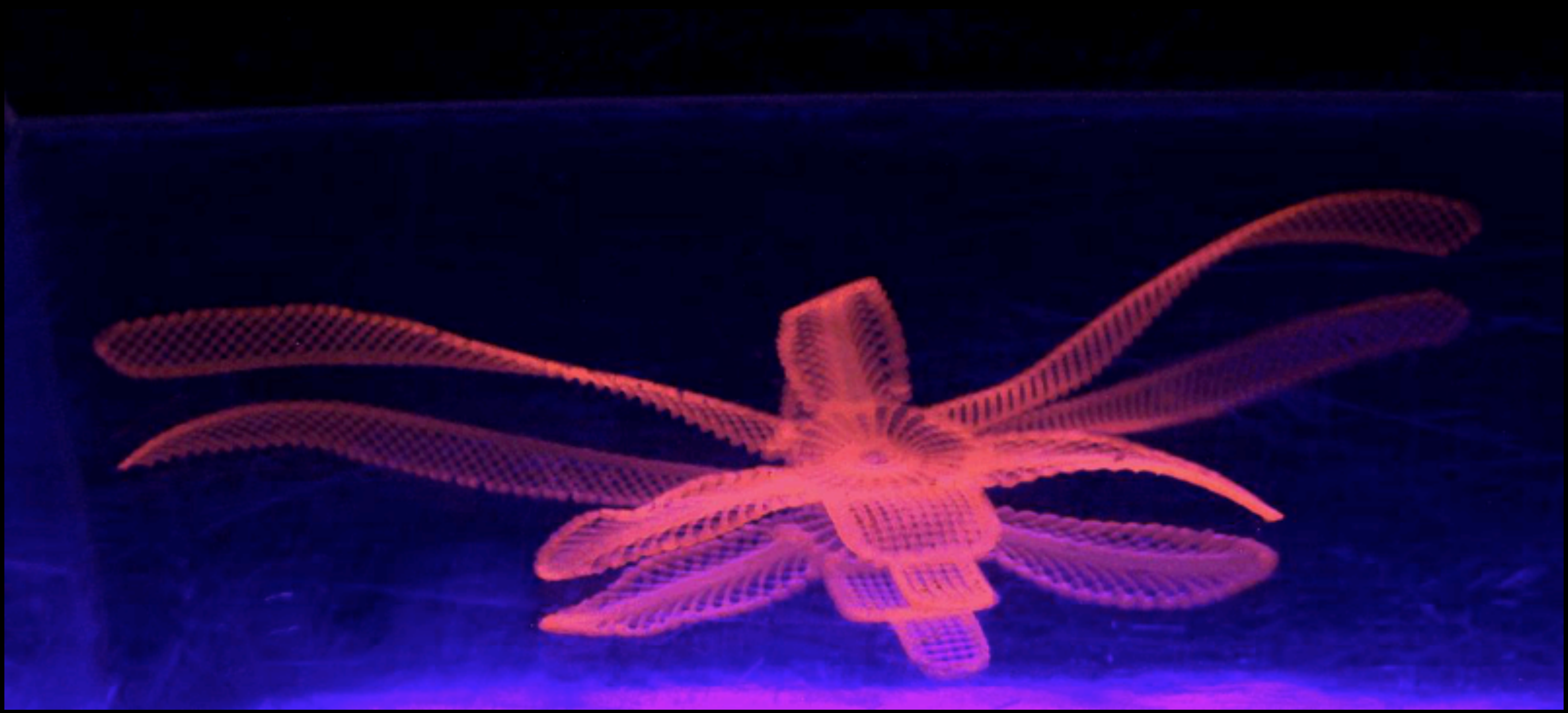

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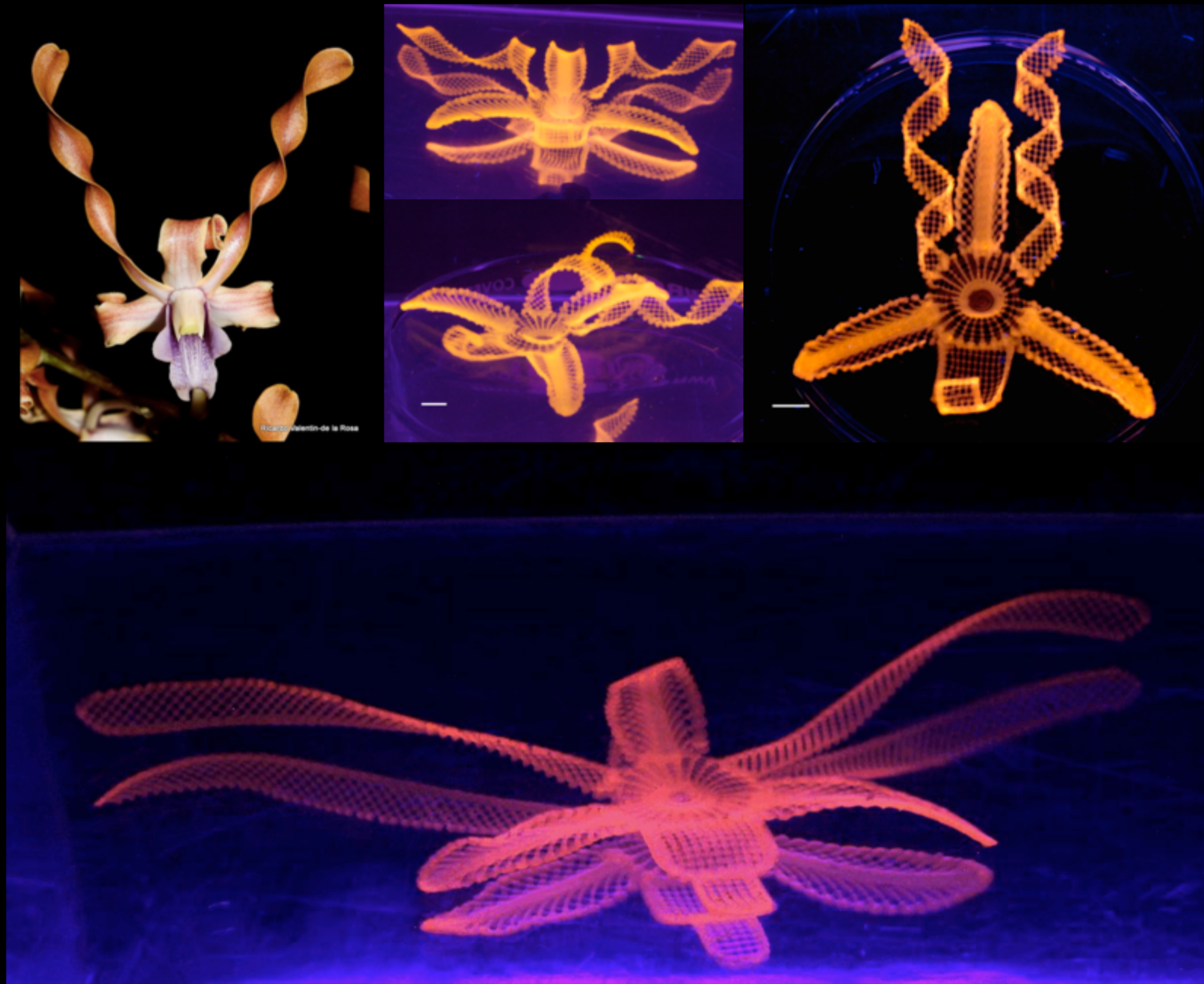

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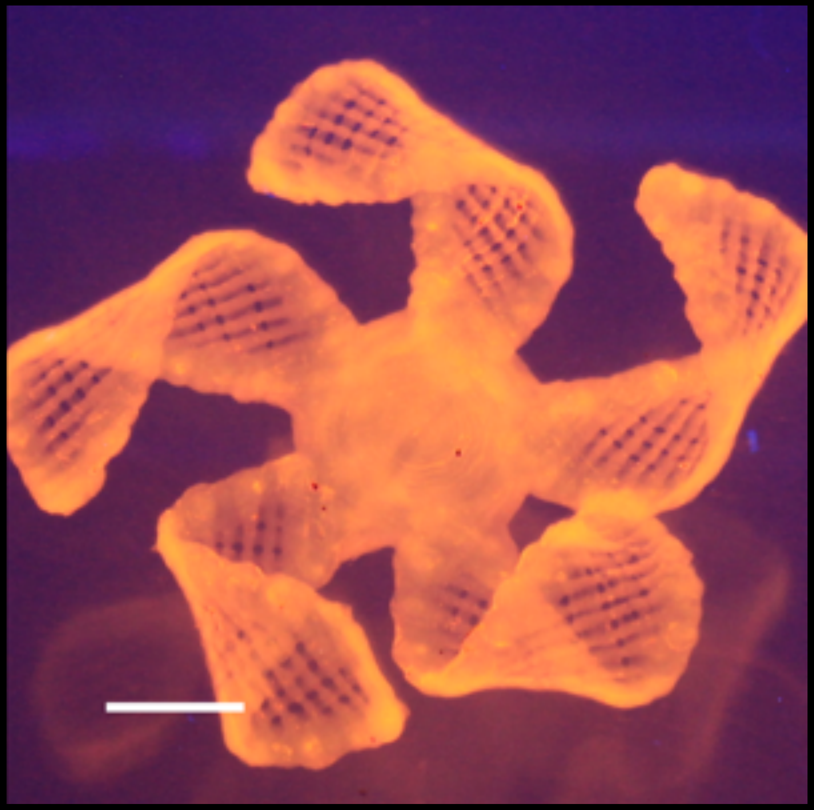
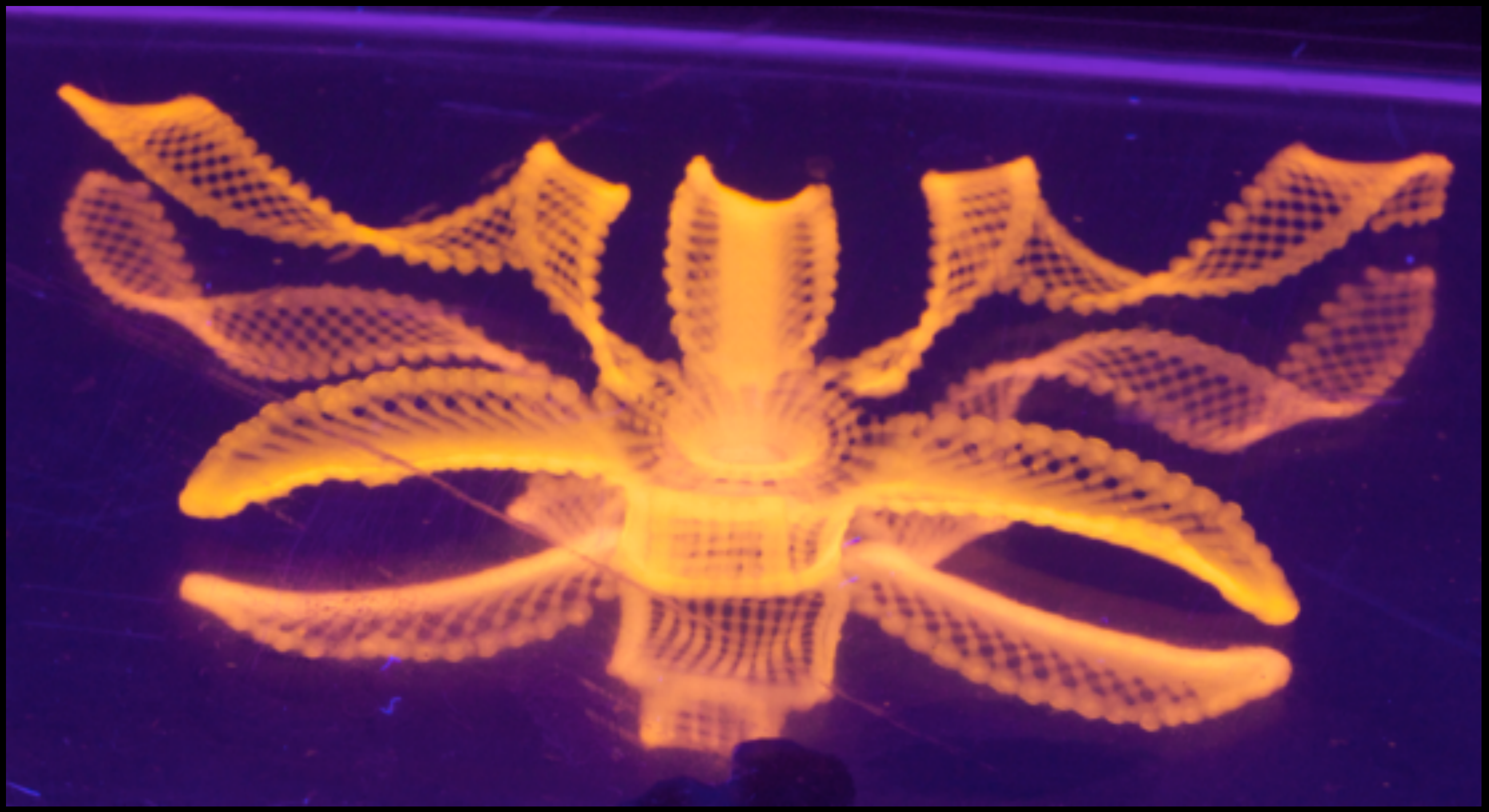
Left-handed or Right-handed?





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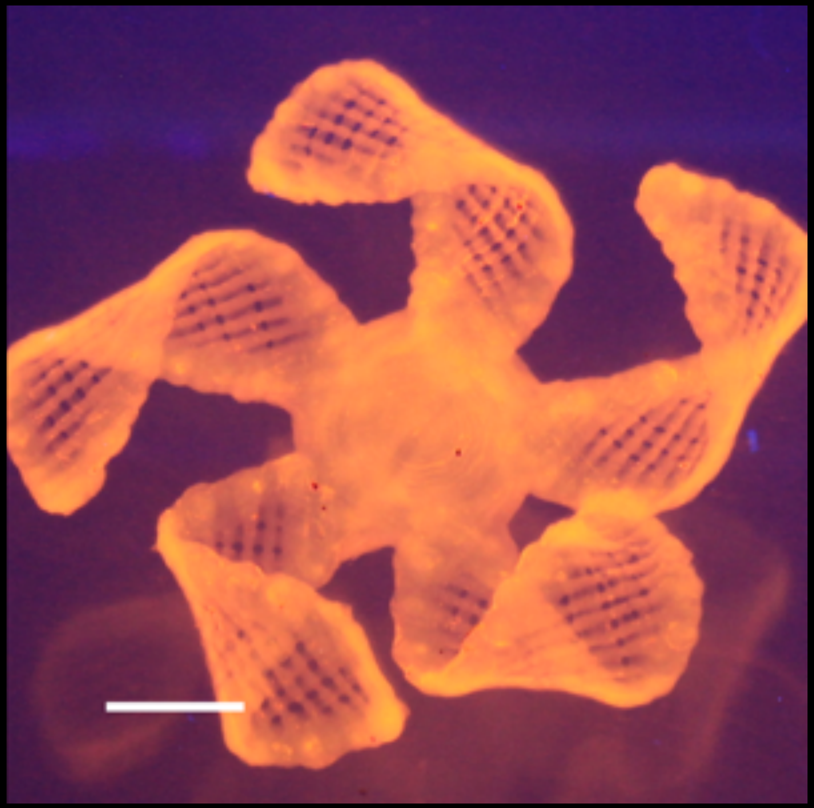

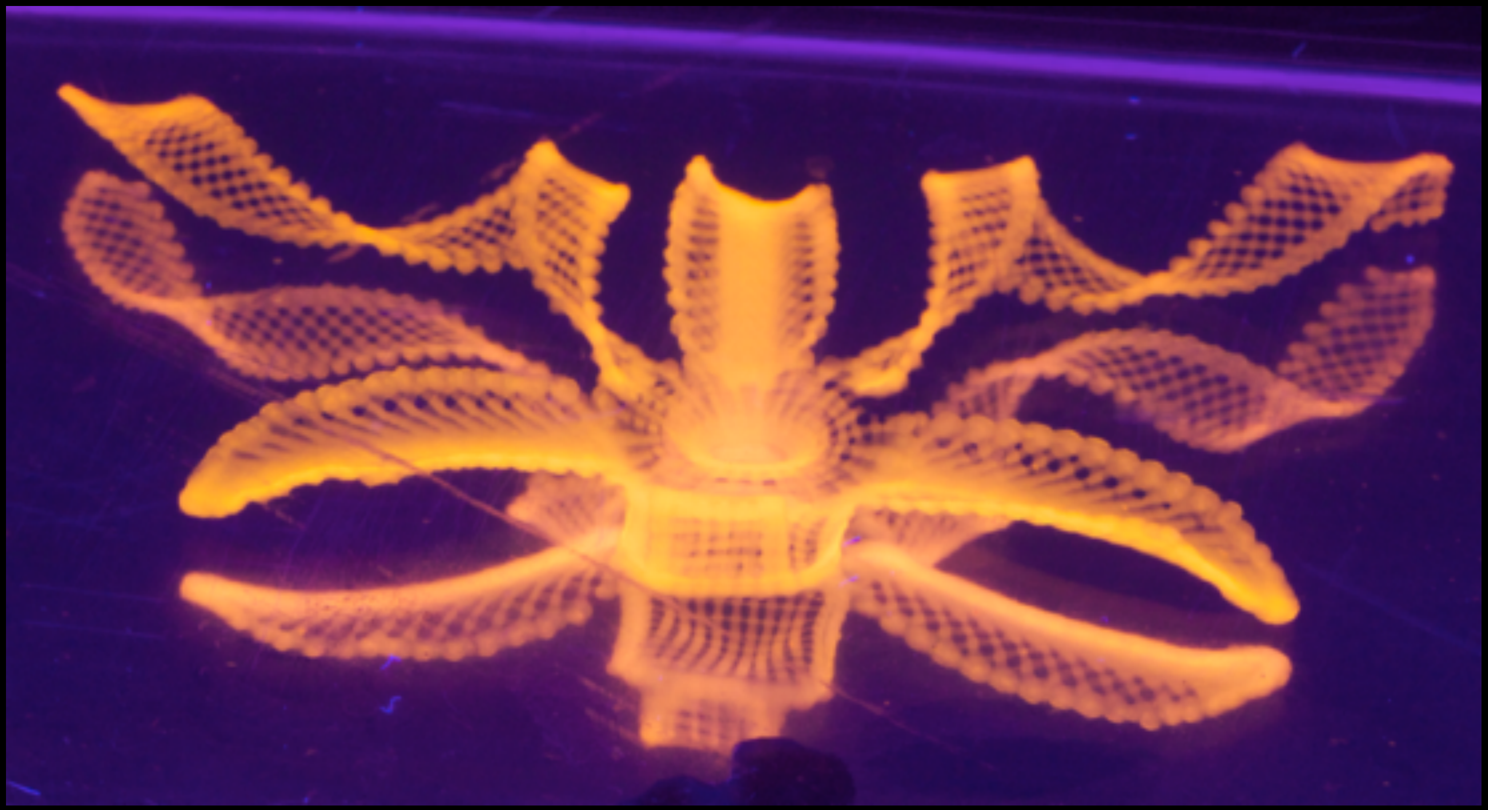

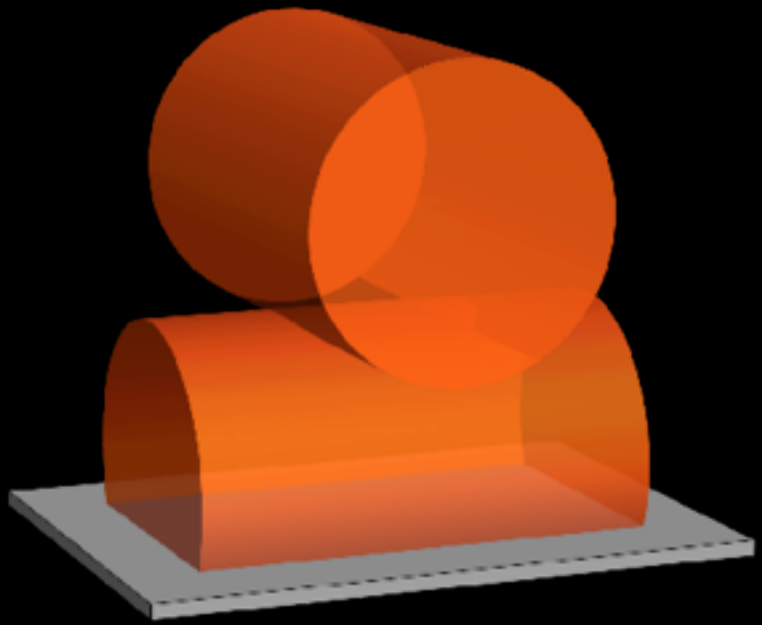

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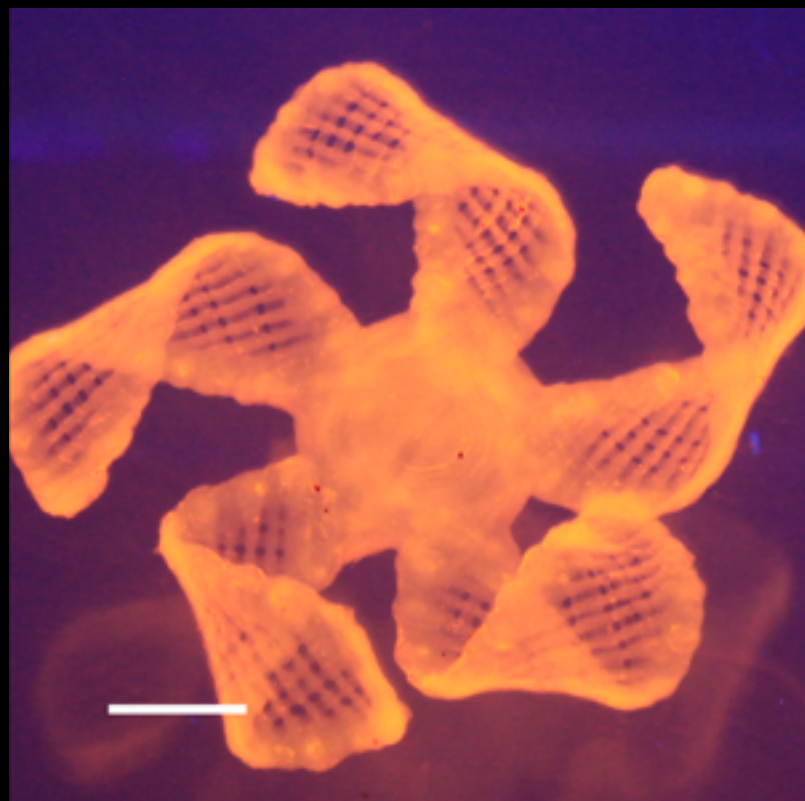

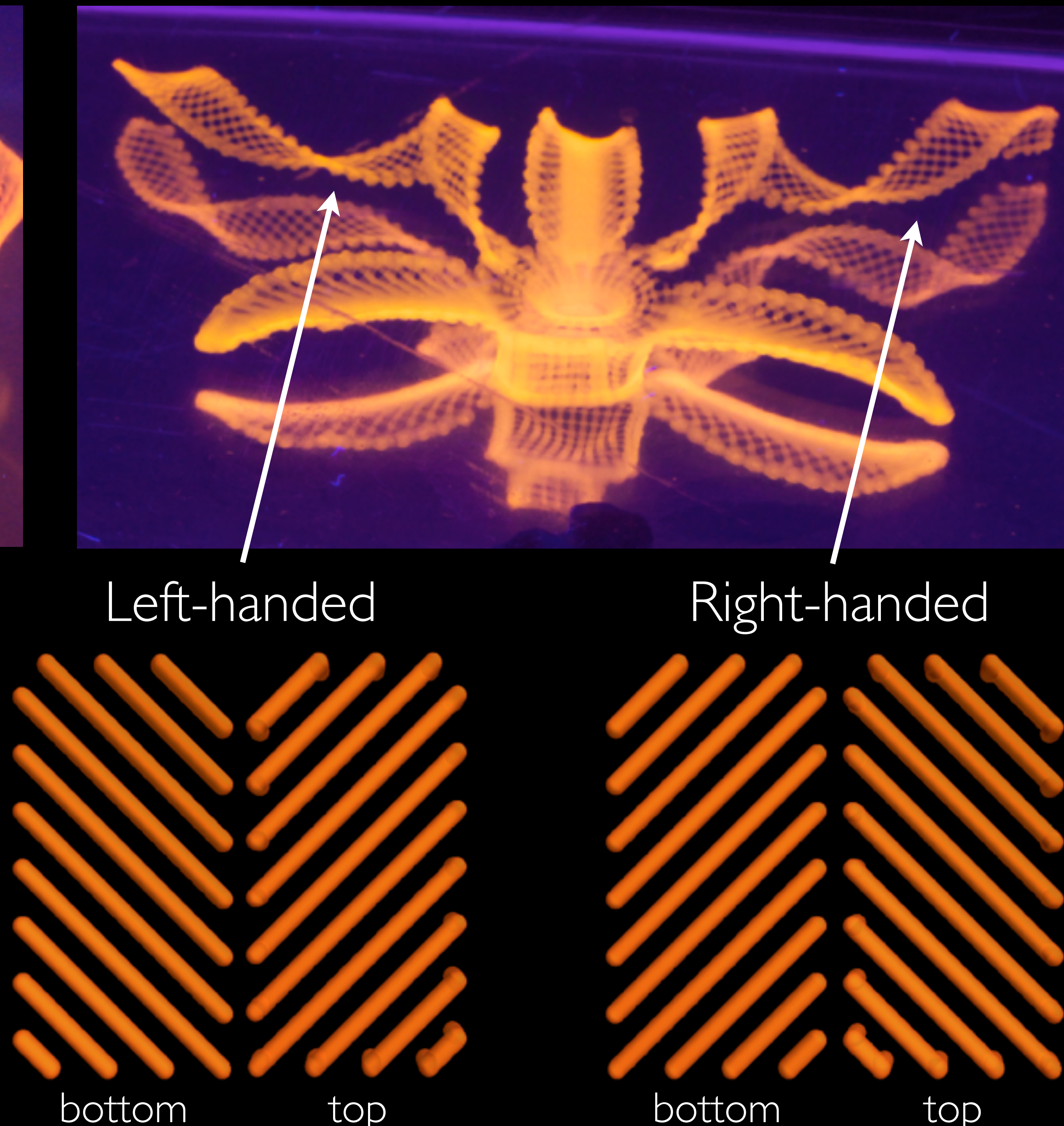

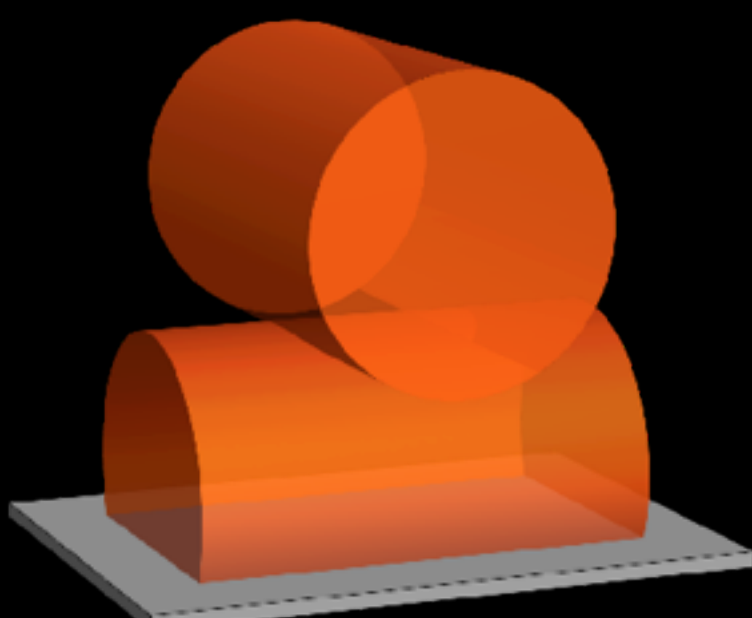

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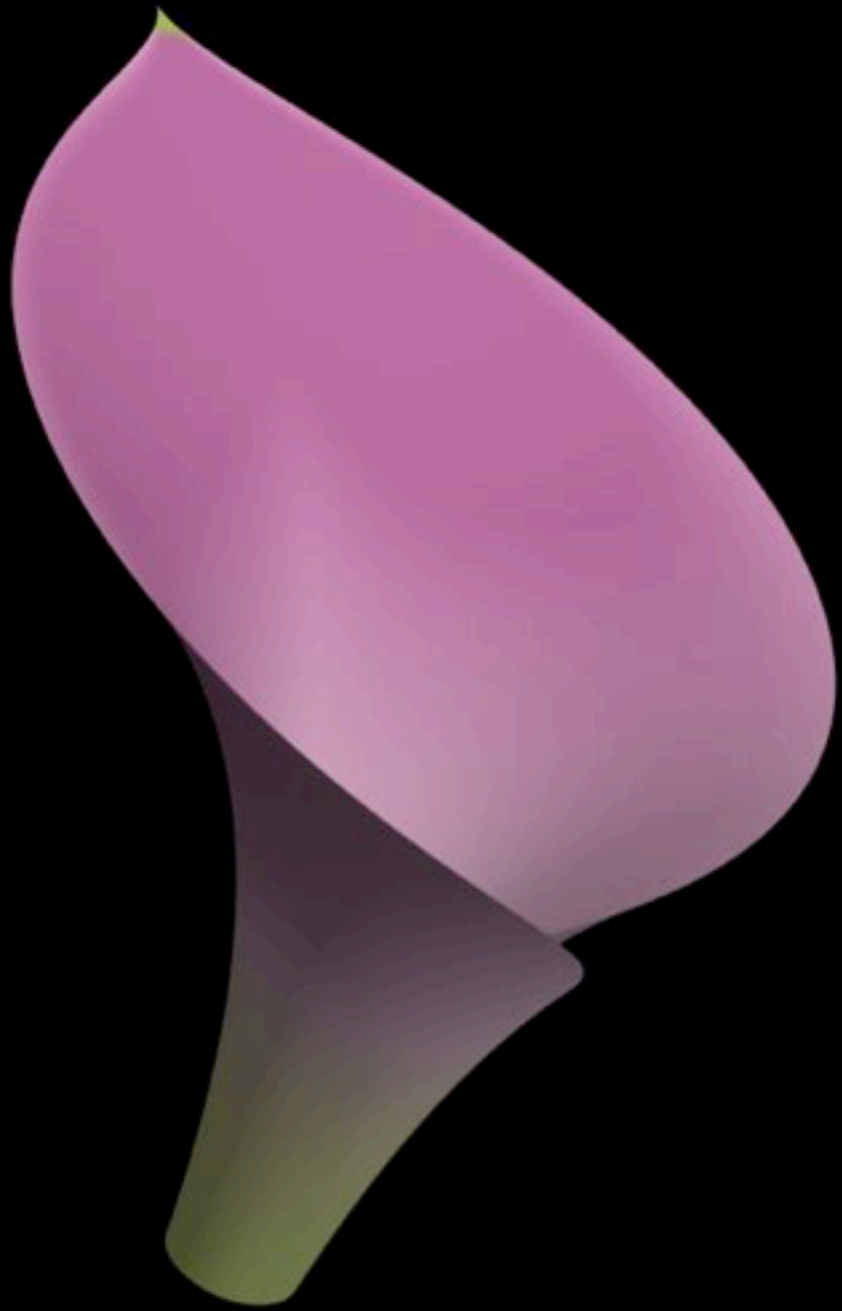
The Inverse Problem



$$\Phi(u,v) = \left\{ \frac{e^{\frac{4u}{8+u}}}{2}\cos(v), \frac{e^{\frac{4u}{8+u}}}{2}\sin(v), \frac{u}{1+u/8} \right\}$$

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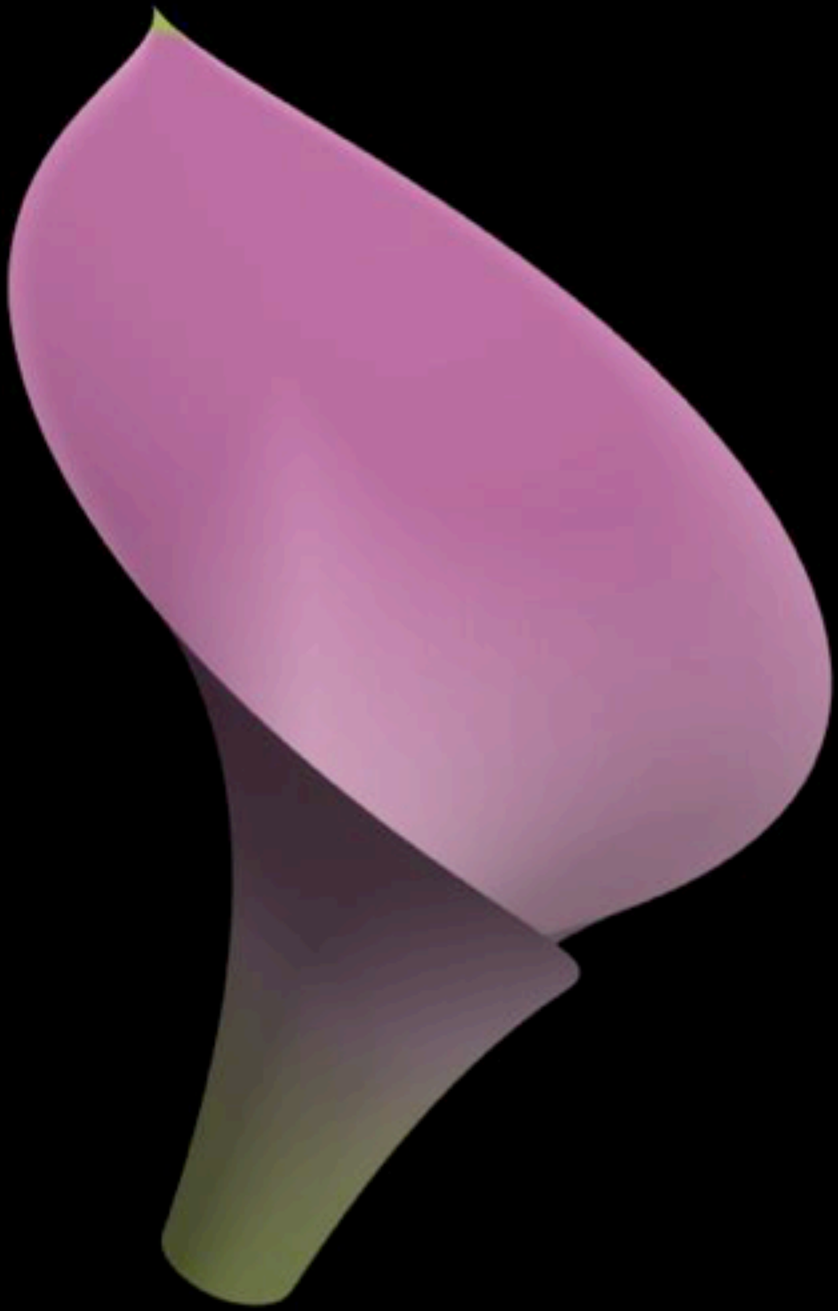
The Inverse Problem



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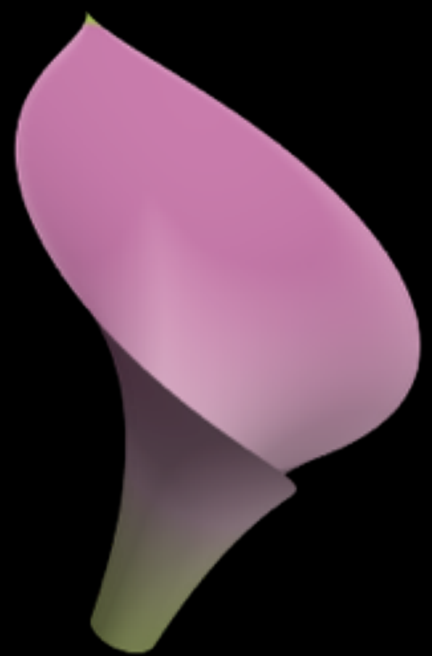
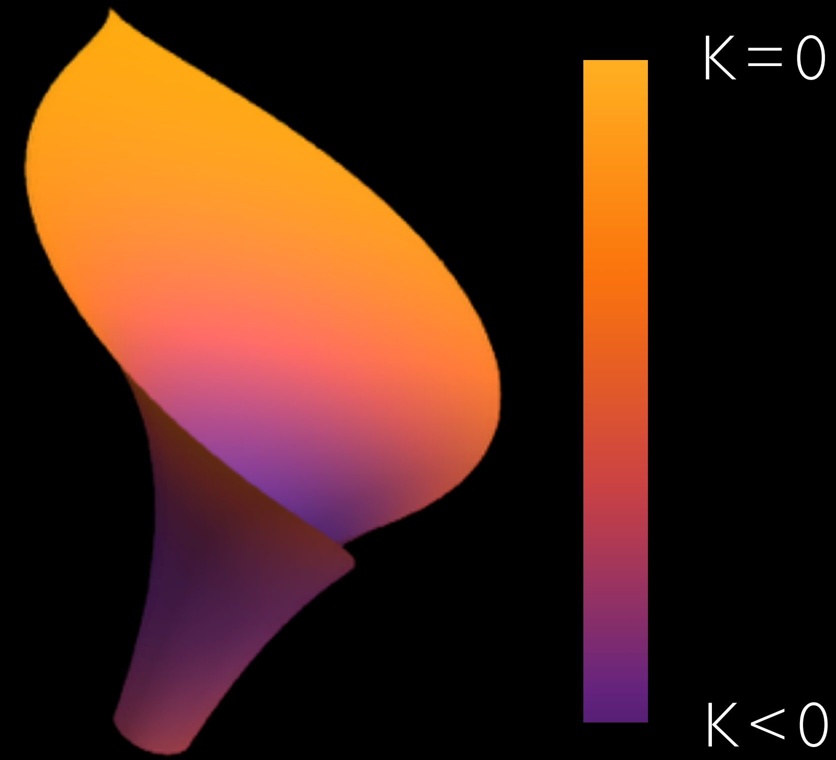
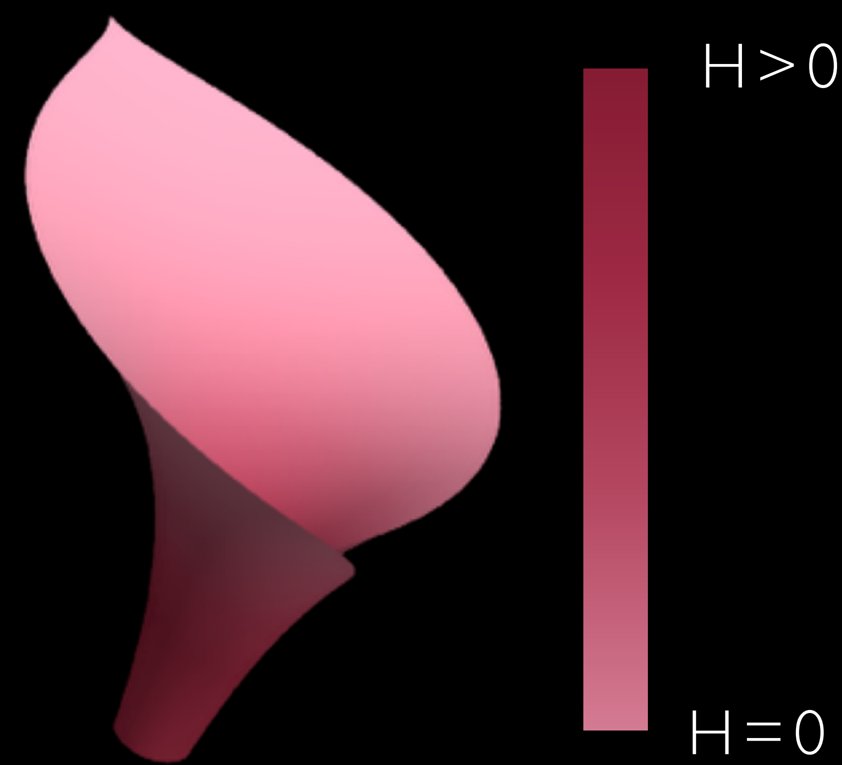




$$K = -\frac{64}{\left(e^{\frac{8u}{u+8}}+16\right)^2}$$

$$\begin{aligned} X(u,v) &= \frac{e^{\frac{4u}{8+u}}}{2}\cos(v) \\ Y(u,v) &= \frac{e^{\frac{4u}{8+u}}}{2}\sin(v) \\ Z(u,v) &= \frac{u}{1+u/8} \end{aligned}$$



$$H = \frac{128e^{-\frac{4u}{u+8}}}{\left(e^{\frac{8u}{u+8}}+16\right)^{3/2}}$$

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Programming Local Curvatures

$$H = \frac{\alpha_\perp - \alpha_\parallel}{h} \frac{c_1 \sin^2(\theta)}{c_2 - c_3 \cos(2\theta) + m^4 \cos(4\theta)}, \quad K = -\frac{(\alpha_\perp - \alpha_\parallel)^2}{h^2} \frac{c_4 \sin^2(\theta)}{c_5 - c_6 \cos(2\theta) + m^4 \cos(4\theta)}$$

Given: $H,\ K,\ \alpha_\parallel,\ \alpha_\perp,\ \mathrm{E}^{(1)},\ \mathrm{E}^{(2)}$ Solve for: $\theta,\ m = a_1/a_2$

bottom

top

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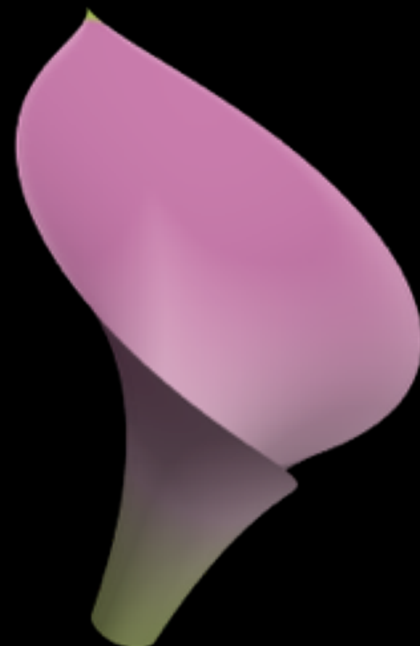

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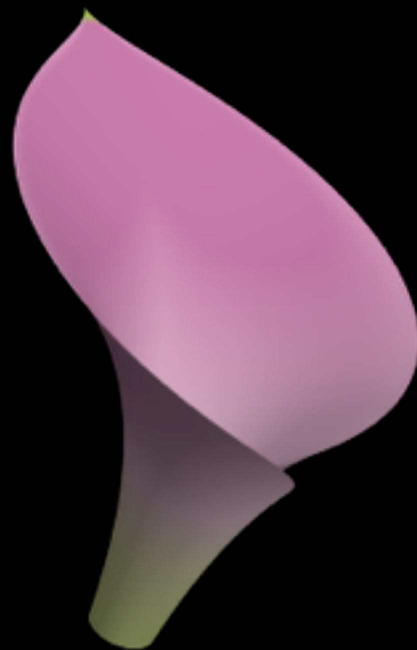

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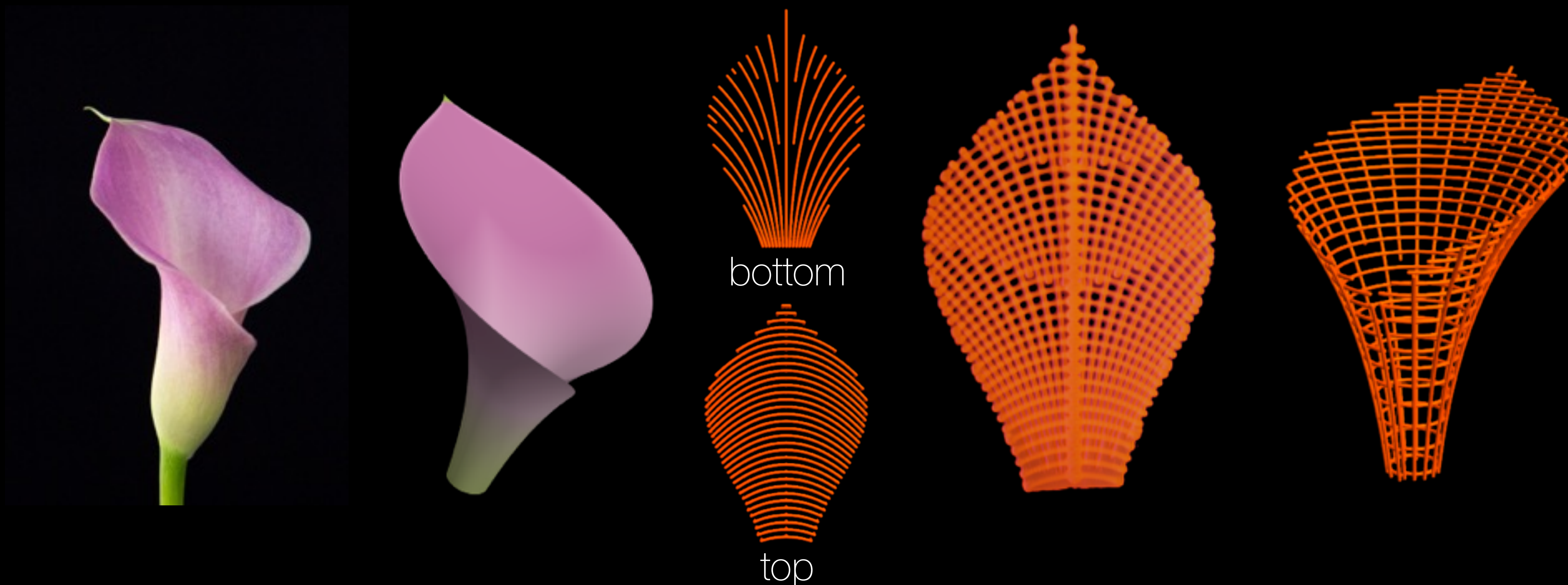

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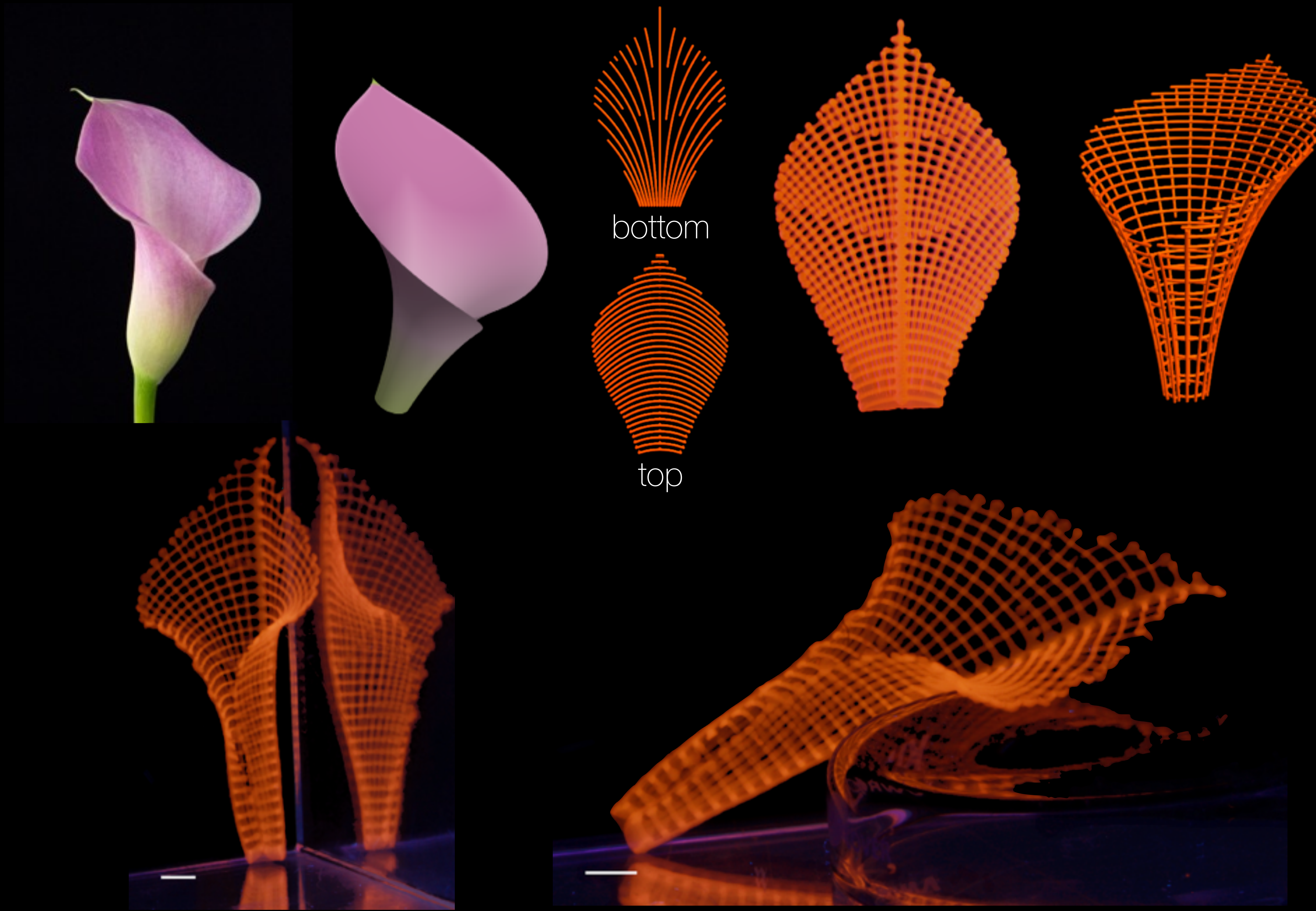

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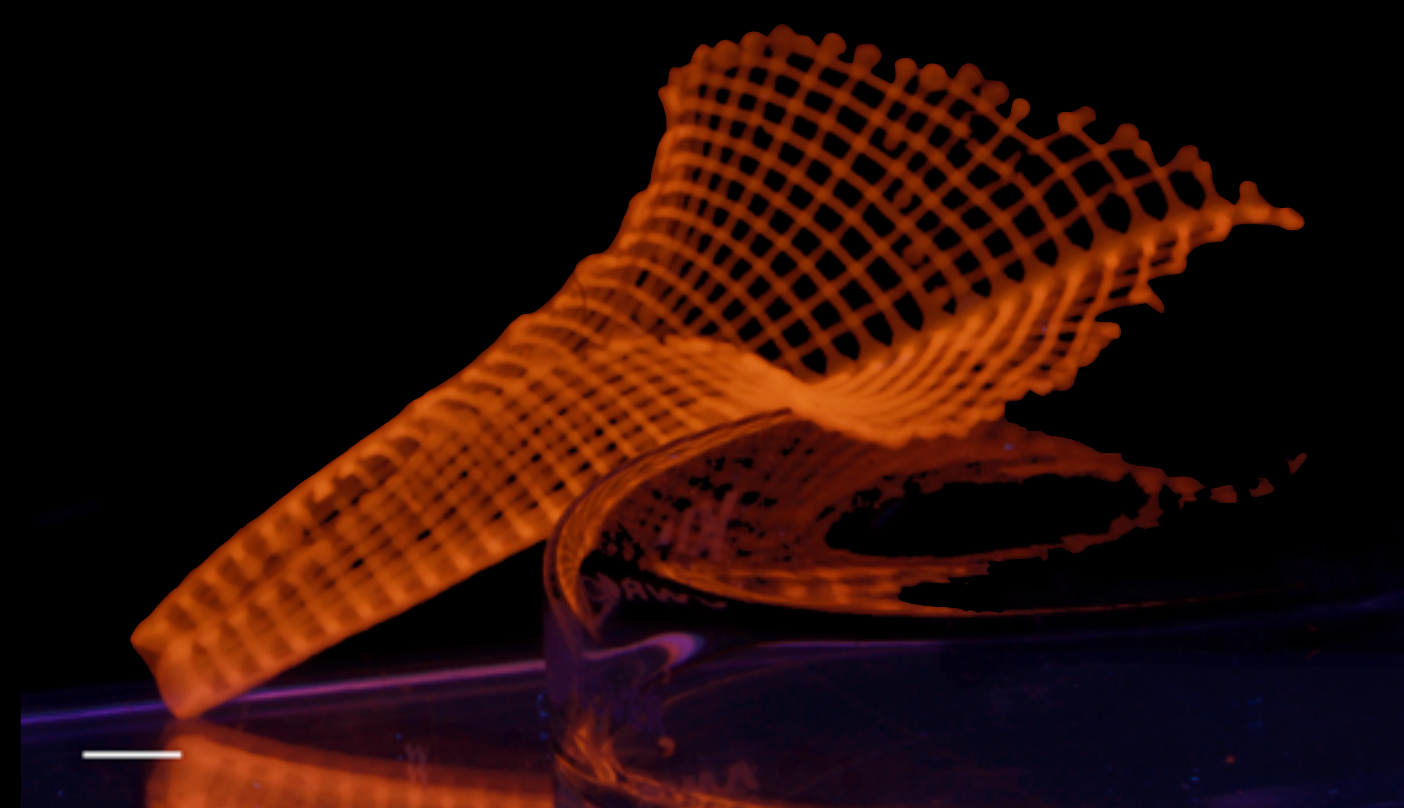
Conclusions and Future Directions

- 3D printing hydrogel ink + cellulose nanofibrils simultaneously encodes anisotropy in swelling and elastic modulus. Complexity is free with additive manufacturing techniques.
- Local swelling anisotropy in a bilayer system generates curvature.
- Elasticity theory of anisotropic plates and shells allows us to predict mean and Gaussian curvatures.
- The inverse problem: How may we design print paths associated with specific target surfaces?
- Platform technology can be used with multi-stimuli responsive inks: light, temperature, electric field, hydration.



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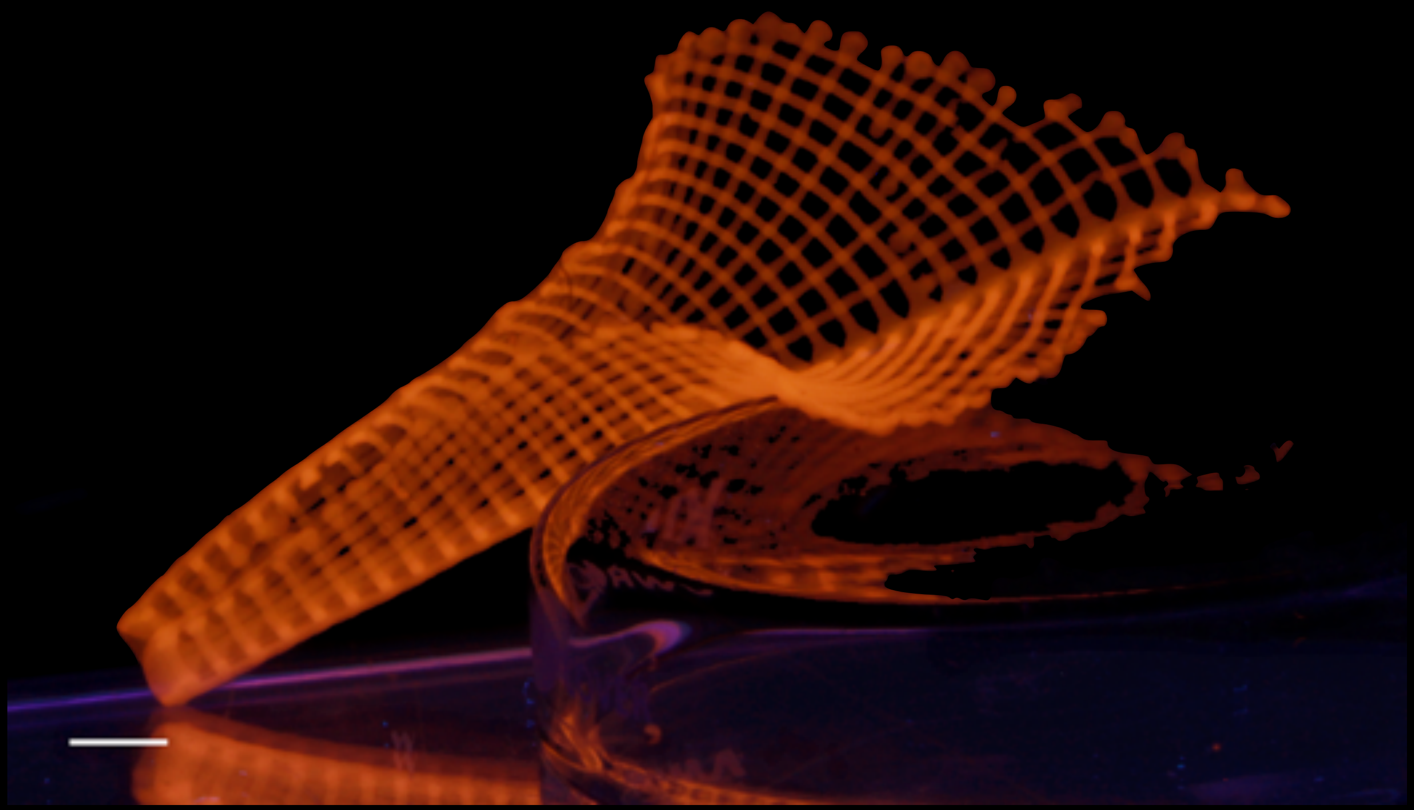

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Thank you!