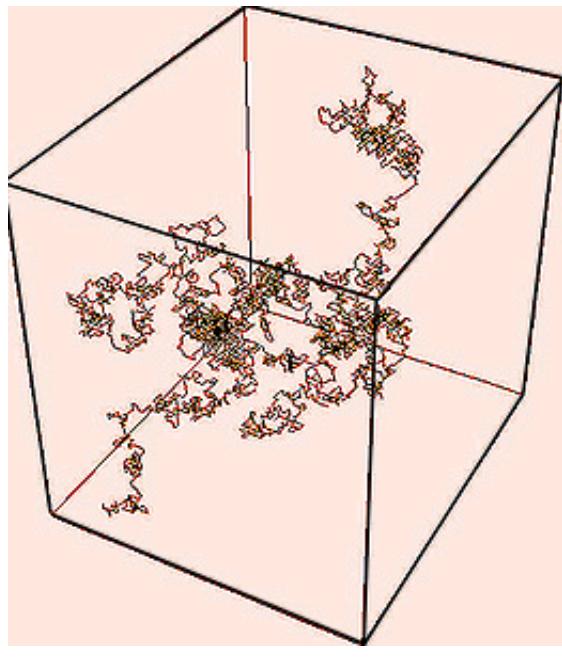
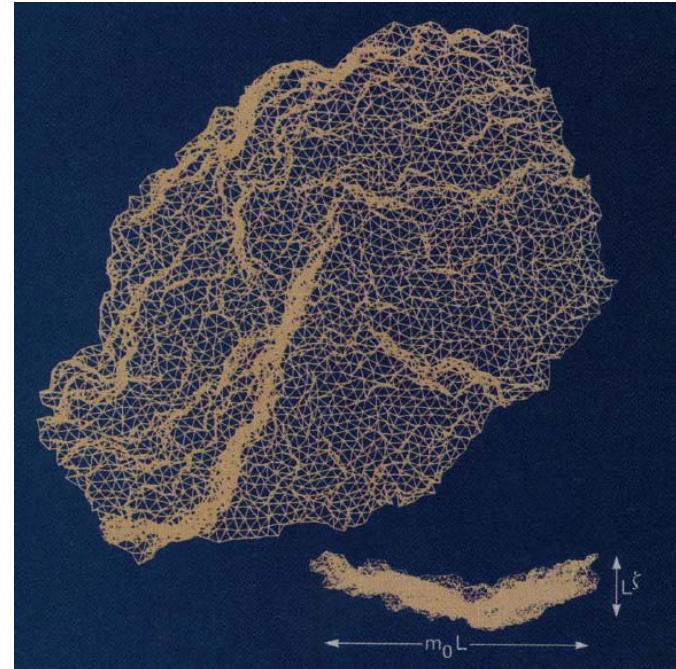
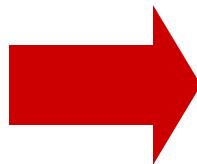


Ancient History: from linear polymers to tethered surfaces

✿ By the early 1990's, theories of linear polymer chains in a good solvent were generalized to treat the statistical mechanics of flexible sheet polymers



linear polymer



sheet polymer

F. Abraham and drn, Science 249, 393 (1990)

- Remarkably, “tethered surfaces” with a shear modulus are able to resist thermal crumpling and exhibit a low temperature “wrinkled”, flat phase...
- A continuous broken symmetry -- long range order in the surface normals -- arises in two dimensions (the Mermin-Wagner-Hohenberg theorem is evaded!)
- Experiments on the spectrin by Christoph Schmidt & Cyrus Safinya

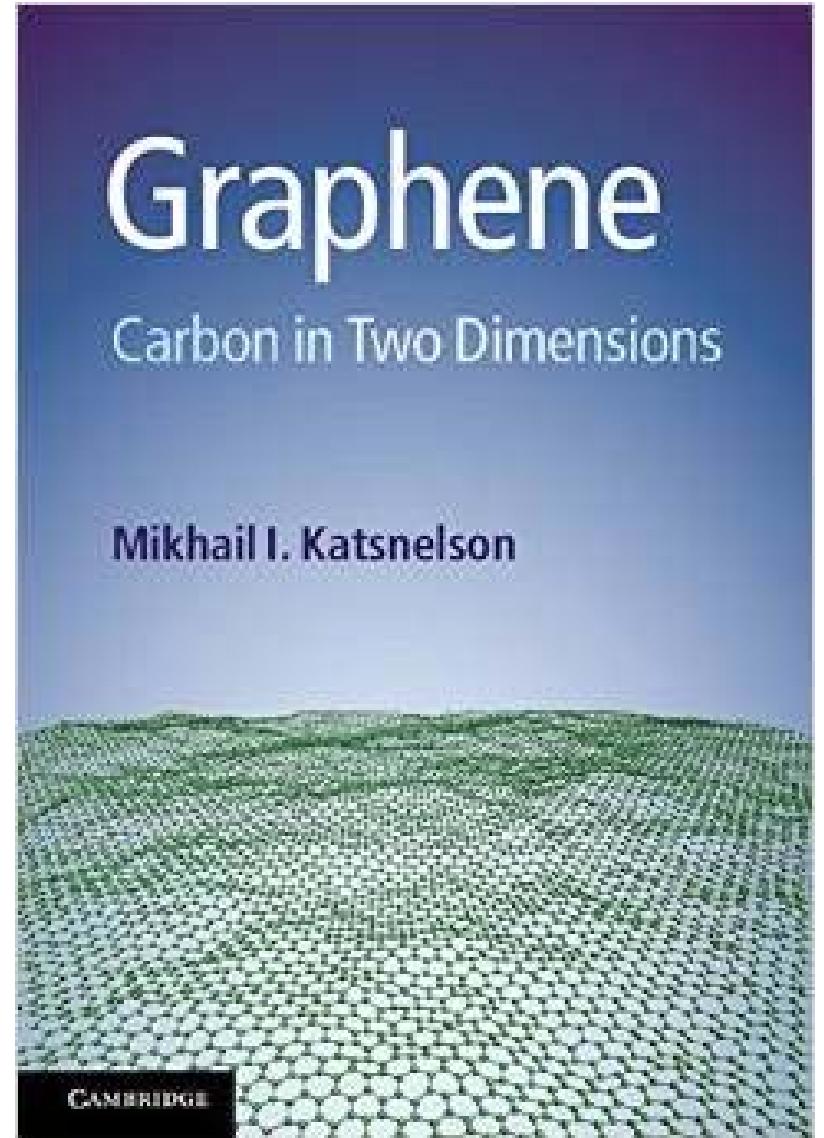
Lots of recent interest in the flat phase among graphene theorists, but (until recently) not many experiments....

Influence of out-of-plane phonons on electronic properties:

1. E. Mariani and F. von Oppen, Phys. Rev. Lett. **100**, 076801 (2008).
2. K. S. Tkhonov, W. L. Z. Zhao and A. M. Finkel'stein, Phys. Rev. Lett. **113**, 076601 (2014).

Quantum effects at low temperatures:

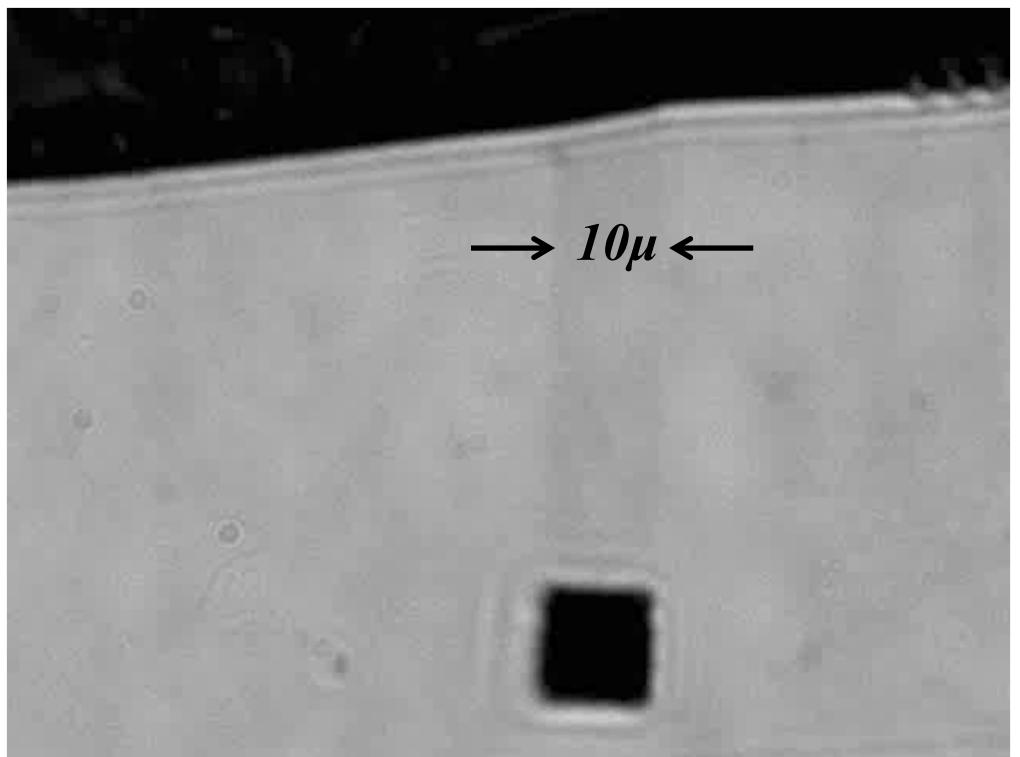
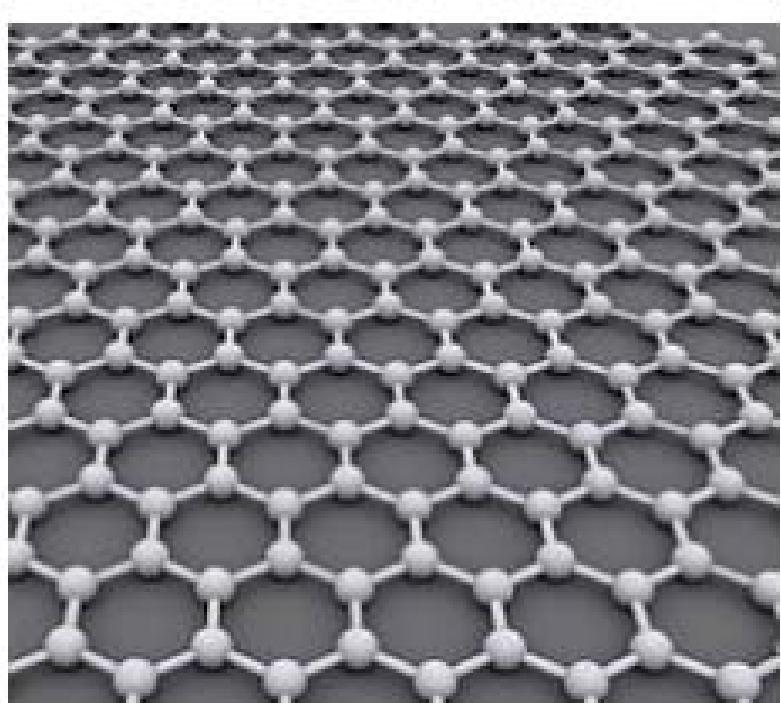
1. E. I. Kats and V. V. Lebedev, Phys. Rev. B **89**, 125433 (2014).
2. B. Amorim, R. Roldan, E. Cappelluti, F. Guinea, A. Fasolino and M. I. Katsnelson, Phys. Rev. B **89**, 224307 (2014)



*Experiments of the McEuen group
at Cornell: “Single molecule
polymer physics” for graphene*

M. Blees et al., Nature 524, 204 (2015)

graphene



Thermalized sheets and shells: topology matters

Equations of thin plate theory

- nonlinear bending and stretching energies*
- $vK = \text{Föppl-von Karman number} = YR^2/\kappa \gg 1$

Recent experiments:
Paul McEuen group
(Cornell)

Physics of thermalized sheets

- “self-organized criticality” of the flat phase
- strongly scale-dependent elastic parameters
- hard condensed matter applications : graphene, MoS_2 , BN , WS_2 ,

G. Gompper
G. Vliegenthart



Jayson
Paulose

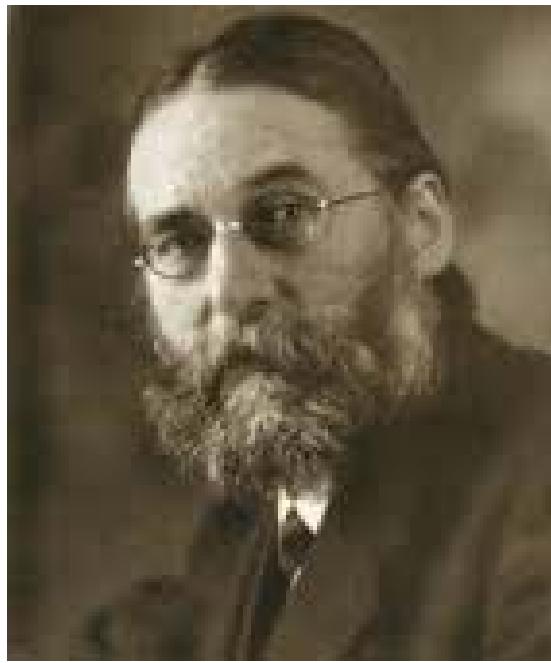
Physics of thermalized shells

- spherical shells accessible via soft matter experiments on diblock copolymers.
- shells with zero pressure difference are crushed by thermal fluctuations for $R > R_c(T)$!

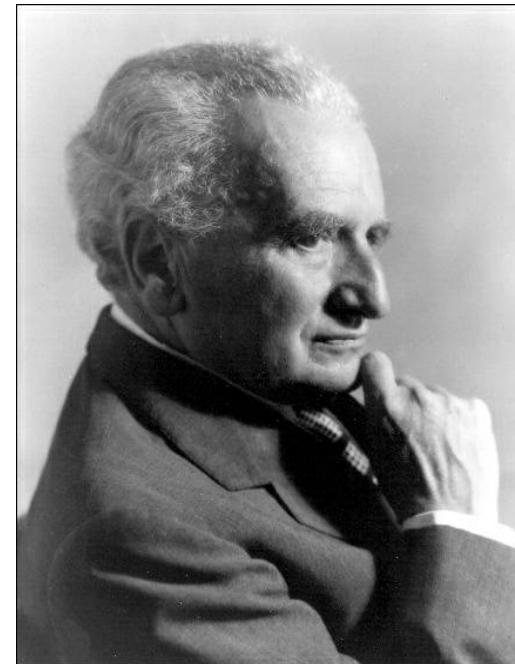


Andrej
Kosmrlj

Elastic membranes: in 1904, Föppl & von Kármán studied large deflections of elastic plates

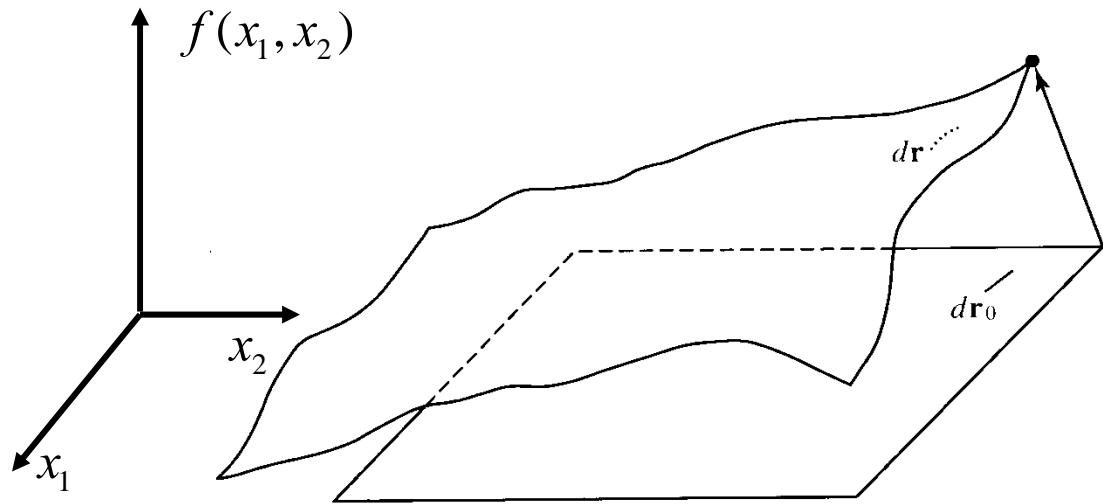


August Föppl
(1854-1924)
Pioneer of
elasticity theory



Theodore von Kármán
(1881-1963) Hungarian-American physicist & aeronautical engineer

To study deformed surfaces, expand about a flat reference state...



$$\vec{r}(x_1, x_2) = \vec{r}_0 + \begin{pmatrix} f(x_1, x_2) \\ u_1(x_1, x_2) \\ u_2(x_1, x_2) \end{pmatrix}$$

$$dr^2 = dr_0^2 + 2u_{ij}dx_i dx_j$$

$$u_{ij}(\vec{x}) = \frac{1}{2} \left[\frac{\partial u_i(\vec{x})}{\partial x_j} + \frac{\partial u_j(\vec{x})}{\partial x_i} + \frac{\partial f(\vec{x})}{\partial x_i} \frac{\partial f(\vec{x})}{\partial x_j} \right]$$

$$E = \frac{1}{2} \int d^2x \left[\underbrace{\kappa (\nabla^2 f(\vec{x}))^2}_{bending energy} + \underbrace{2\mu u_{ij}^2(\vec{x}) + \lambda u_{kk}^2(\vec{x})}_{stretching energy} \right]$$

κ = bending rigidity

μ = shear modulus

$\mu + \lambda$ = bulk modulus

Flexural phonons can escape softly into the 3rd dimension...

Nonlinear Föppl -von Karman Equations (1905)

$$\partial_i \sigma_{ij} = 0 \Rightarrow \sigma_{ij}(\vec{x}) = 2\mu u_{ij}(\vec{x}) + \lambda \delta_{ij} u_{kk}(\vec{x}) \equiv \varepsilon_{im} \varepsilon_{jn} \partial_m \partial_n \chi(\vec{x})$$

$\chi(\vec{x})$ = Airy stress function

Bending modes $f(\vec{x})$ coupled to stretching modes $\vec{u}(\vec{x})$;

Minimize energy over $f(\vec{x})$ and $\chi(\vec{x})$...

$$\kappa \nabla^4 f = \frac{\partial^2 \chi}{\partial y^2} \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 \chi}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - 2 \frac{\partial^2 \chi}{\partial x \partial y} \frac{\partial^2 f}{\partial x \partial y}$$

$$Y = \frac{4\mu(\mu + \lambda)}{2\mu + \lambda} = \text{Young's modulus}$$

$$\frac{1}{Y} \nabla^4 \chi = -\frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} + \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 = \text{Gaussian curvature}$$

κ = bending rigidity

⇒ resembles a simplified form of general relativity (developed 10 years later....)

⇒ exact solutions available only in very special cases

⇒ dimensionless "Föppl-von Karman number" $vK = YL^2 / \kappa \gg 1$ (linear size = L)

[compare Reynolds number in fluid mechanics, $Re = uL / v$]

How big is the Föppl von-Karman number?

$$\kappa \nabla^4 f = \frac{\partial^2 \chi}{\partial y^2} \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 \chi}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - 2 \frac{\partial^2 \chi}{\partial x \partial y} \frac{\partial^2 f}{\partial x \partial y}; \quad \frac{1}{Y} \nabla^4 \chi = - \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} + \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 \quad \text{let } f = L \tilde{f}, \chi = Y L^2 \tilde{\chi} \dots$$

$$\frac{1}{vK} \nabla^4 \tilde{f} = \frac{\partial^2 \tilde{\chi}}{\partial y^2} \frac{\partial^2 \tilde{f}}{\partial x^2} + \frac{\partial^2 \tilde{\chi}}{\partial x^2} \frac{\partial^2 \tilde{f}}{\partial y^2} - 2 \frac{\partial^2 \tilde{\chi}}{\partial x \partial y} \frac{\partial^2 \tilde{f}}{\partial x \partial y}$$

$$\nabla^4 \tilde{\chi} = - \frac{\partial^2 \tilde{f}}{\partial x^2} \frac{\partial^2 \tilde{f}}{\partial y^2} + \left(\frac{\partial^2 \tilde{f}}{\partial x \partial y} \right)^2 \quad vK = YL^2 / \kappa$$

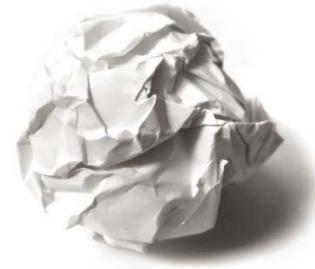
$1/vK \ll 1$ multiplies highest derivative in first equation

\Rightarrow crumpled foil or paper of thickness h

$$\kappa = E_3 h^3 / [12(1 - \nu^2)]; \quad Y = E_3 h$$

E_3 = 3d isotropic Young's modulus

$$vK = YL^2 / \kappa = 12(L/h)^2(1 - \nu^2) = 4 \times 10^7$$



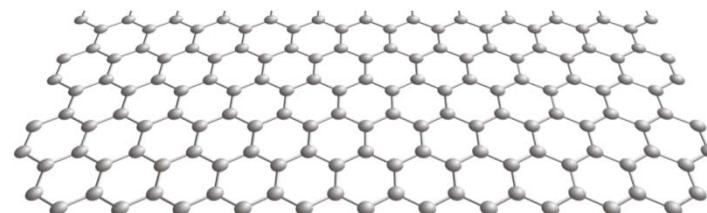
$$L = 8.5 \text{ in}, \quad h = 0.1 \text{ mm}$$

$$\nu = \text{Poisson ratio} \approx 0.25$$

$\Rightarrow L = 10 \mu$ square of graphene

$$Y = 30 \text{ eV} / \text{Å}^2; \quad \kappa = 1.2 \text{ eV}$$

$$vK = YL^2 / \kappa \approx 10^{12} !!$$



Applications: thin solid shells and structures

macroscopic (1cm - 100m)



plant
leaves



egg
shell

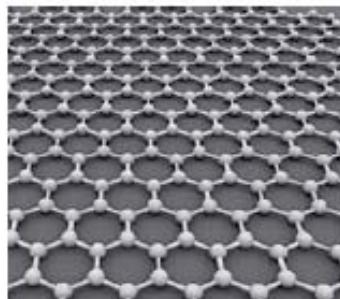


aluminum foil

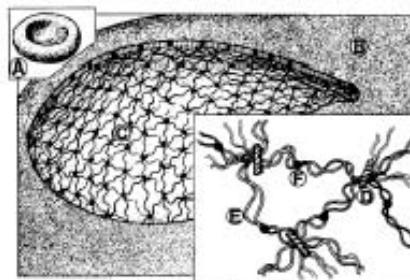


pipes

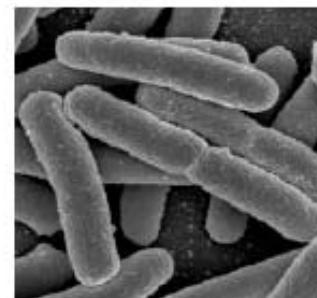
& microscopic (0.1nm - 1μm) -- is Brownian motion important ??



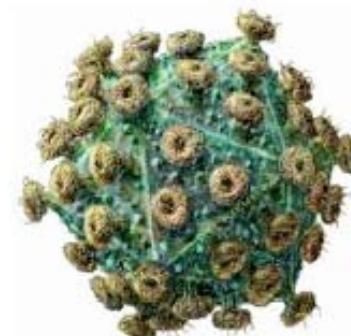
graphene



cell membrane
with cytoskeleton



bacterial
cell wall



viral
capsid

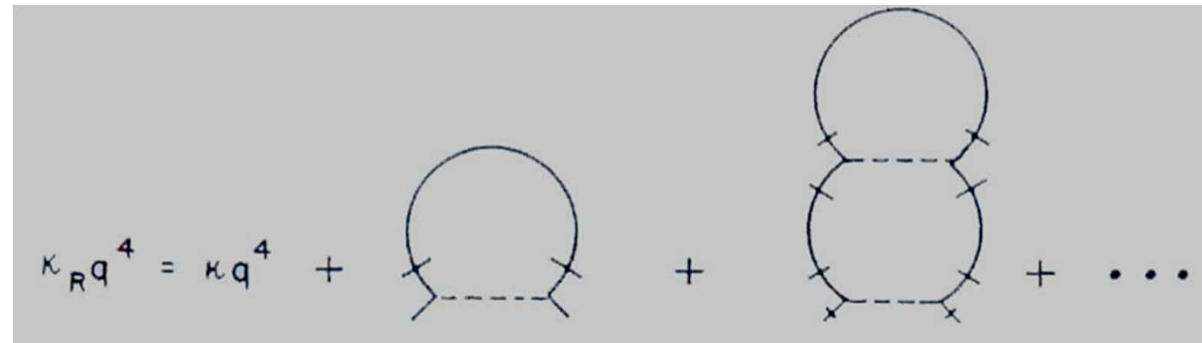
What About Thermally Excited Membranes? (L. Peliti & drn)

✿ Tracing out in-plane phonon degrees of freedom yields a massless nonlinear field theory

$$F_{\text{eff}} = -k_B T \ln \left(\int D\{u_x(x, y)\} \int D\{u_y(x, y)\} e^{-E/k_B T} \right) \quad Y = \frac{4\mu(\mu + \lambda)}{2\mu + \lambda} = \text{Young's modulus}$$

$$F_{\text{eff}} = \frac{1}{2} \kappa \int d^2x \left[(\nabla^2 f)^2 \right] + \frac{1}{4} Y \int d^2x \left[P_{ij}^T (\partial_i f \partial_j f) \right]^2 \equiv F_0 + F_1; \quad P_{ij}^T = \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}$$

◆ Assume $k_B T / \kappa \ll 1$, and do low temperature perturbation theory



$$\begin{aligned} \kappa_R(q) &= \kappa + k_B T Y \int \frac{d^2k}{(2\pi)^2} \frac{\hat{q}_i P_{ij}^T(\vec{k}) \hat{q}_j}{\kappa |\vec{q} + \vec{k}|^4} + \dots \\ &\approx \kappa [1 + (vK) k_B T / (4\pi^3 \kappa) + \dots] \end{aligned}$$

$$vK = YL^2 / \kappa \approx (L/h)^2 \gg 1$$

L = membrane size

h = membrane thickness

Self-consistent bending rigidity, $\kappa_R(q) \sim 1/q$ & diverges as $q \rightarrow 0!$

Renormalization Group for Thermally Excited Sheets

$$E = \frac{1}{2} \int d^2x [\kappa (\nabla^2 f(\vec{x}))^2 + 2\mu u_{ij}^2(\vec{x}) + \lambda u_{kk}^2(\vec{x})]$$

$$u_{ij}(\vec{x}) = \frac{1}{2} \left[\frac{\partial u_i(\vec{x})}{\partial x_j} + \frac{\partial u_j(\vec{x})}{\partial x_i} + \frac{\partial f(\vec{x})}{\partial x_i} \frac{\partial f(\vec{x})}{\partial x_j} \right]$$

$$Z = \int \mathcal{D}\bar{u}(x_1, x_2) \int \mathcal{D}f(x_1, x_2) \exp(-E / k_B T)$$

$$\kappa_R(l) \approx \kappa(l / l_{th})^\eta$$

$$Y_R(l) \approx Y(l_{th} / l)^{\eta_u}$$

$$\eta \approx 0.82, \quad \eta_u \approx 0.36$$

Thermal fluctuations

dominate whenever $L > l_{th}$

$$l_{th} \approx \sqrt{\kappa^2 / (k_B T Y)}$$

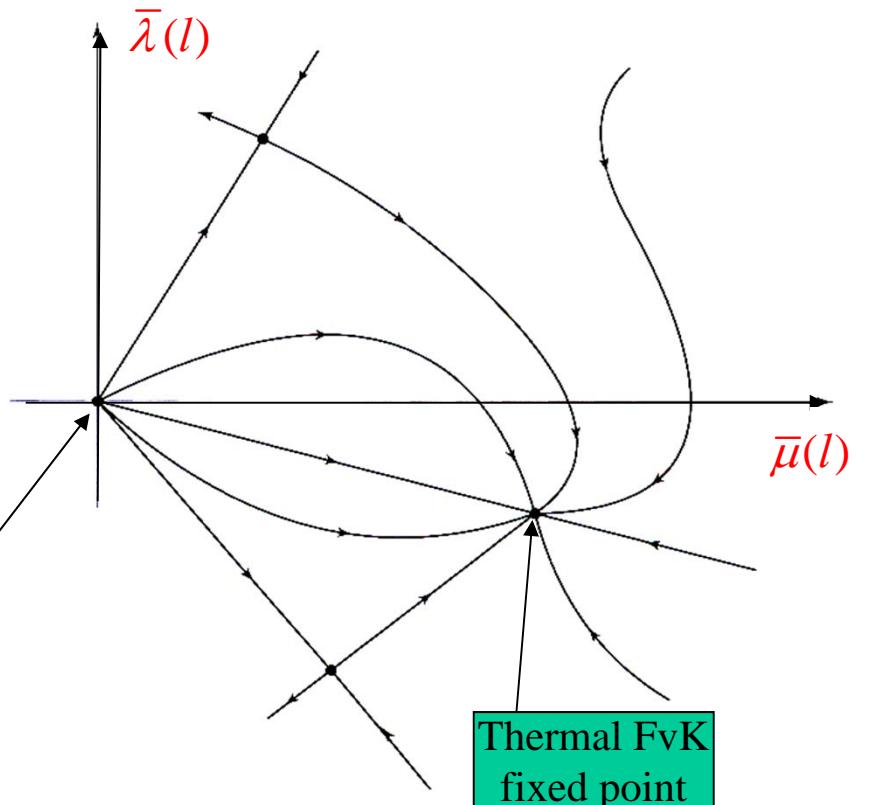
*L. Peliti & drn (~1987)
J. Aronovitz and T. Lubensky
P. Le Doussal and L. Radzihovsky*

define running coupling constants....

$$\bar{\mu}(l) = k_B T \mu a_0^2 / \kappa^2; \quad \bar{\lambda}(l) = k_B T \lambda a_0^2 / \kappa^2$$

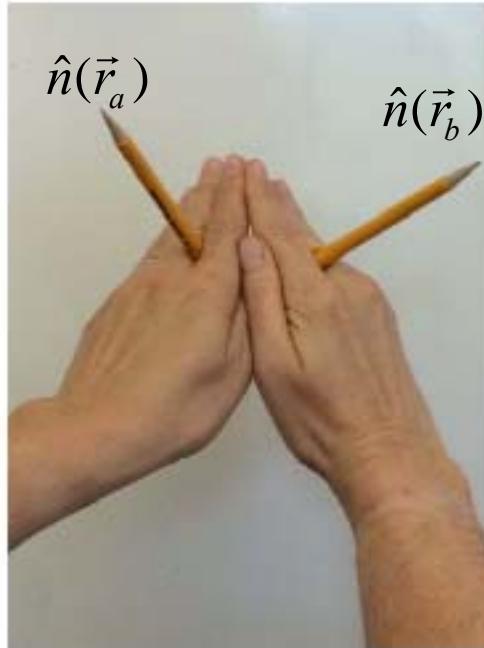
scale dependent Young's modulus

$$Y(l) = \frac{4\mu(l)[\mu(l) + \lambda(l)]}{2\mu(l) + \lambda(l)}$$

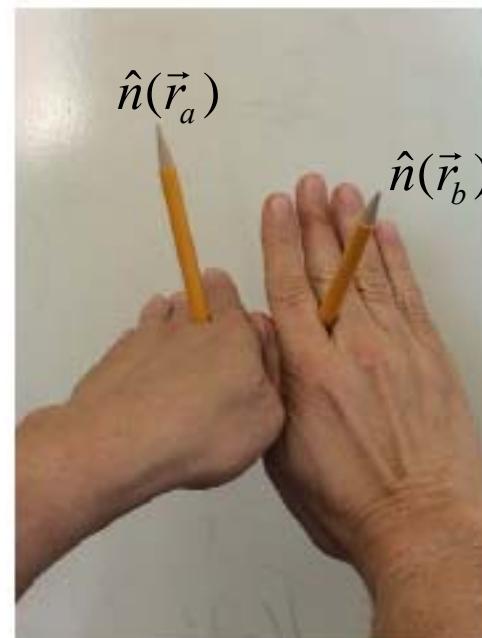


Why long range order? Longitudinal vs. transverse spin waves

$$F_{\text{eff}} = \frac{1}{2} \kappa \int d^2x \left[(\nabla^2 f)^2 \right] + \frac{1}{4} Y \int d^2x \left[P_{ij}^T (\partial_i f \partial_j f) \right]^2 \equiv F_0 + F_1; \quad P_{ij}^T = \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}$$



A longitudinal spin wave fluctuation only requires low energy bending



A transverse spin wave fluctuation requires energetically costly stretching, unless this happens along a cut

$$\langle \hat{n}(\vec{r}_a) \cdot \hat{n}(\vec{r}_b) \rangle = 1 - \frac{k_B T}{2\pi\kappa} \left[\eta^{-1} + \ln(l_{th}/a_0) \right] + C \frac{k_B T}{\kappa} \left(\frac{l_{th}}{|\vec{r}_a - \vec{r}_b|} \right)^\eta$$

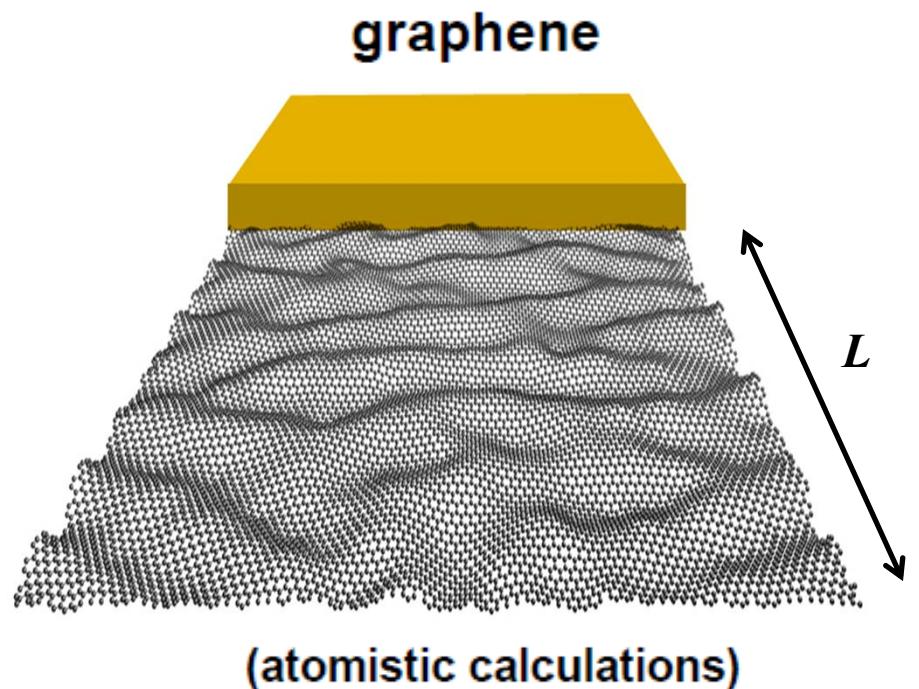
Freely supported ($\sigma_{ij} = 0$) graphene tests the theory

Graphene is the ultimate 2D crystalline membrane:

- One atom thick
- Very stiff in-plane (Young's modulus $Y = 20\text{eV}/\text{A}^2$)
- $L = 10\mu$
- $\kappa = 1.2\text{eV}$

With graphene, we have reached the “Moore’s Law” limit of large Foppl-von Karman numbers

$$\nu K = YL^2/\kappa \sim 10^{12}$$



$$\kappa_0 \approx 1.2\text{eV} \approx 2 \times 10^{-19} \text{ J}$$

Extremely flexible!

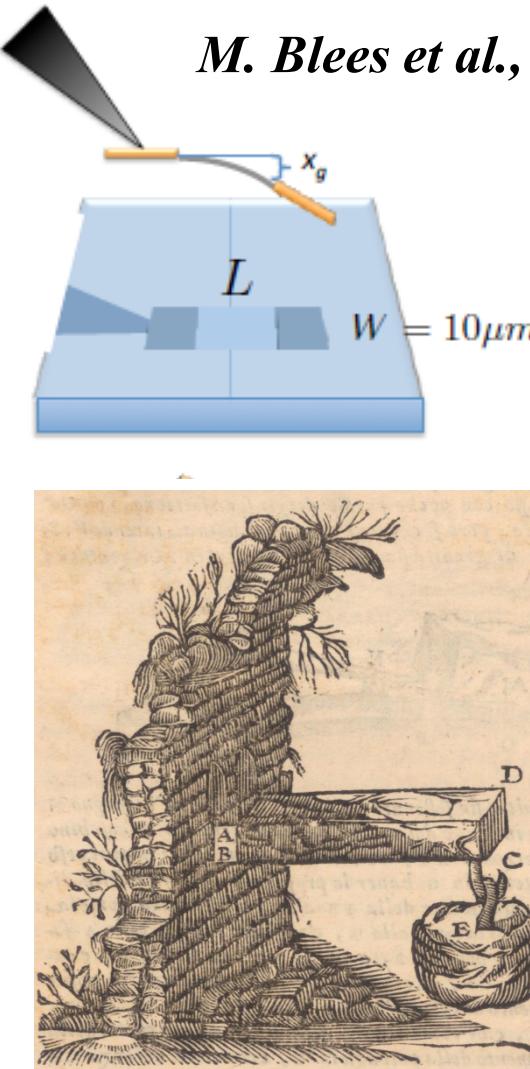
R. Nicklow, N. Wakabayashi and H. G. Smith,
PRB 5, 4951 (1972)

fluctuations dominate for $L > l_{th}$

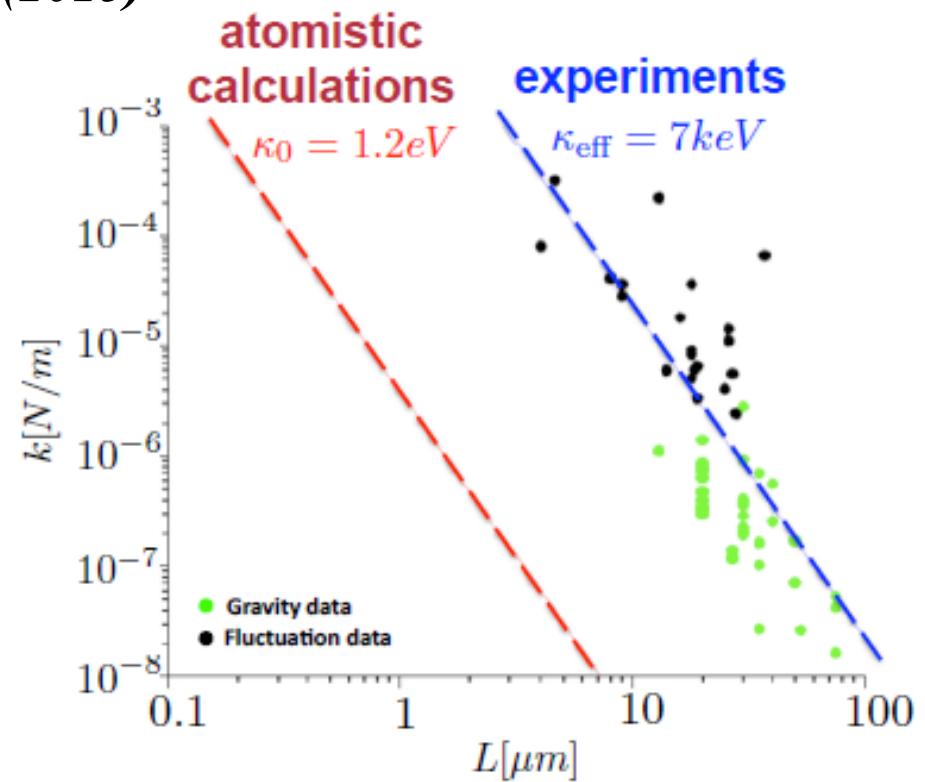
$$l_{th} \approx \sqrt{\kappa^2 / (k_B T Y)} \approx 0.2\text{nm}!$$

Bending rigidity of graphene membranes

M. Blees et al., Nature 524, 204 (2015)



Galileo Galilei (1638)
Cantilever experiment



Bending rigidity enhanced ~5000 fold
Agrees with $\kappa_R(l) \approx \kappa(W/l_{th})^{0.8}$
($l_{th} \sim 0.2 \text{ nm}$ for graphene)

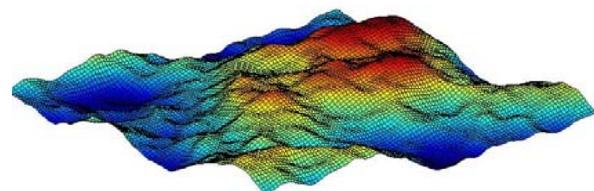


Thermal fluctuations of warped membranes

Adapt self-consistent screening approx. of Le Doussal and Radzihovsky, Phys. Rev. B **48**, 3548 (1993).



$$\langle |h(\mathbf{q})|^2 \rangle \sim q^{-4}$$



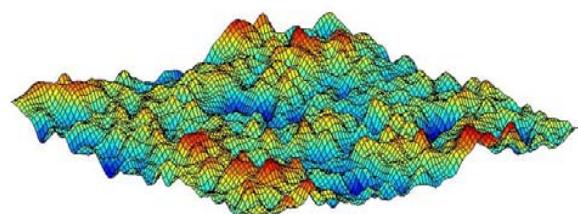
$$\langle \kappa_{\text{eff}} \rangle \sim L^{+\eta_f} \quad \langle Y_{\text{eff}} \rangle, \langle \mu_{\text{eff}} \rangle \sim L^{-\eta_u}$$

$$\eta_f = 1$$

$$\eta_u = 1$$

**geometry
disorder
dominates**

$$\langle |h(\mathbf{q})|^2 \rangle \sim q^{-2}$$

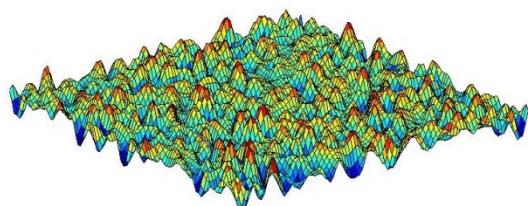


$$\eta_f \approx 0.82$$

$$\eta_u \approx 0.36$$

**thermal
fluctuations
dominate**

$$\langle |h(\mathbf{q})|^2 \rangle \sim 1$$



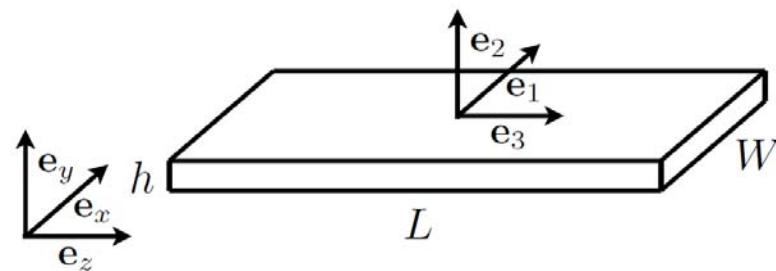
$$\eta_f \approx 0.82$$

$$\eta_u \approx 0.36$$

**thermal
fluctuations
dominate**

Theory of thermalized cantilever ribbons (see Andrej's talk, late february)

$$\frac{d\mathbf{e}_i}{ds} = \boldsymbol{\Omega} \times \mathbf{e}_i$$



$$E = \int \frac{ds}{2} \left[A_1 \Omega_1^2 + A_2 \Omega_2^2 + C \Omega_3^2 \right] - F_Z$$

$$A_1 = \kappa W(1-\nu^2)/12; \quad C = 2\kappa W(1-\nu)$$

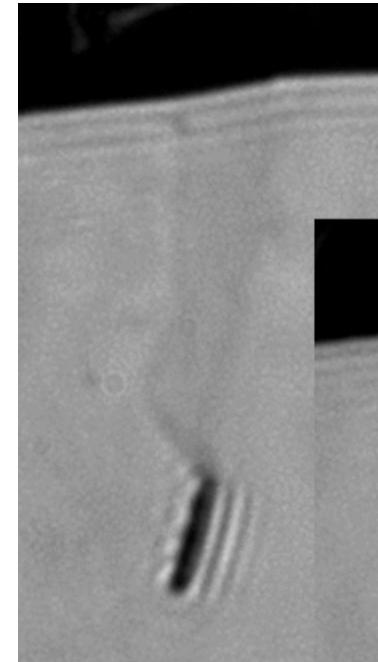
$$A_2 = YW^3/12 \gg A_1, C$$

Y = 2d *Young's modulus*

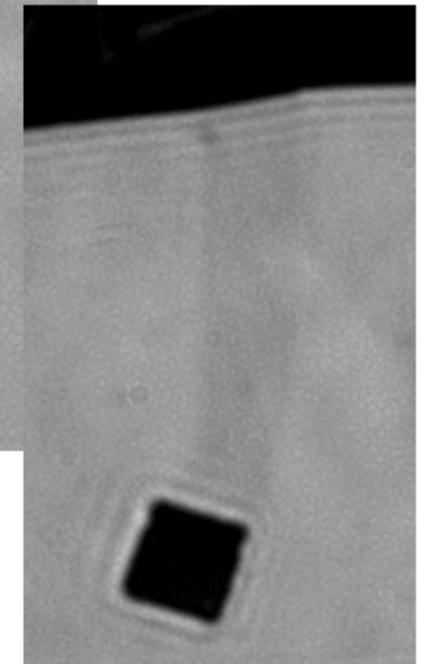
κ = 2d *bending rigidity*

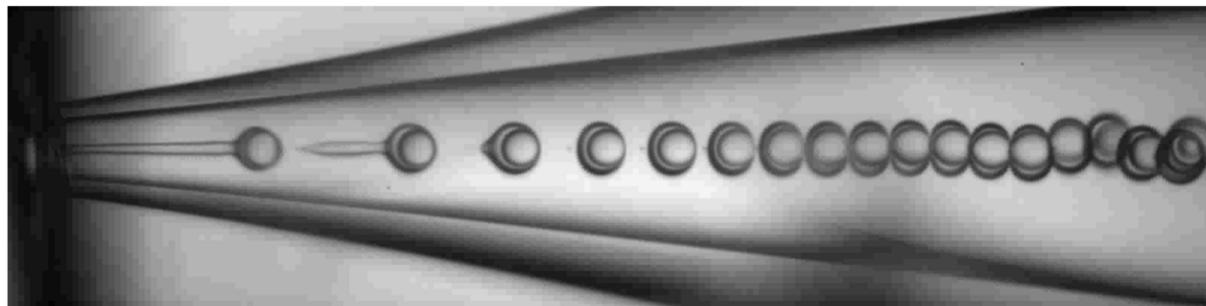
ν = 2d *Poisson ratio*

Map 1d path integral statistical mechanics onto the quantum mechanics of a rigid rotor in an external gravitational field



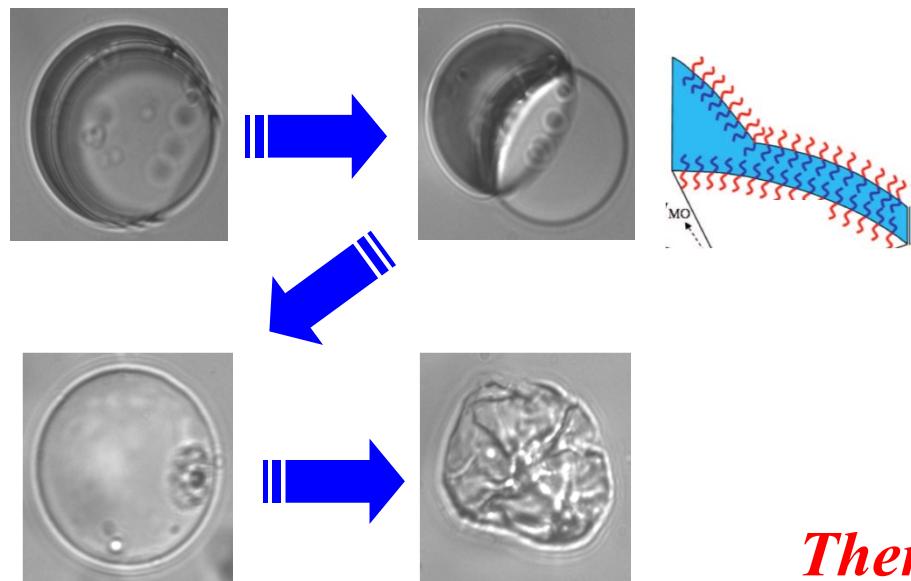
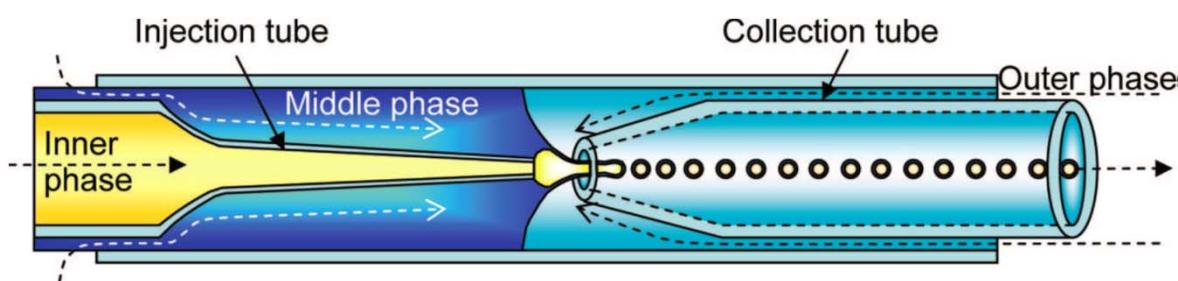
McEuen
group





Microfluidic fabrication of polymersomes

Shum et al., JACS 2008, 130, 9543



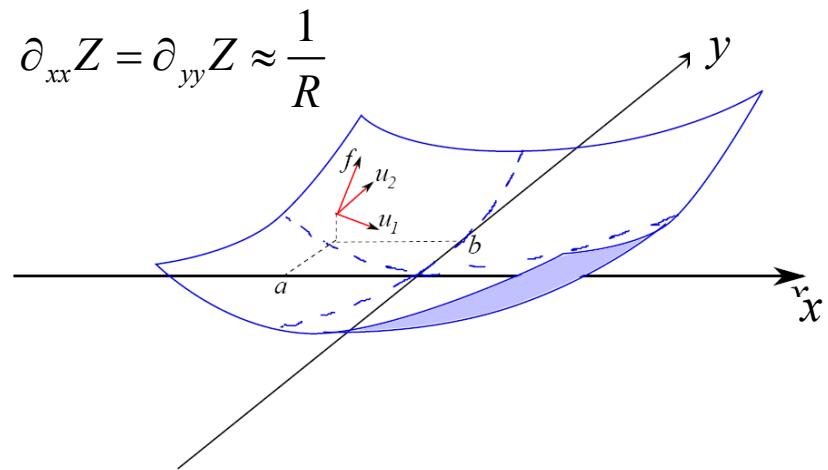
- Start with “double emulsion” of amphiphilic diblock copolymers (PEG-b-PLA).
- Tune wetting properties to eject thin *crystalline* bilayer shells.
- Result is a delivery vehicle for drugs, flavors, colorings and fragrances that can be crushed by osmotic pressure

*Polymersome Radius, $R = 30 \mu\text{m}$
Thickness, $h = 10 \text{ nm}$*

$$\begin{aligned}\gamma &= \text{Foppl-von Karman number} \\ &\approx (R / h)^2 = 10^8\end{aligned}$$

Thermal fluctuations again matter...

Initial shape:
$$\begin{cases} z = Z(x, y) \\ z = \sqrt{R^2 - x^2 - y^2} \end{cases}$$



To study thermal deformations of spherical shells, we use shallow shell theory....

$$\partial_{xx}Z = \partial_{yy}Z \approx \frac{1}{R}$$

$$\begin{pmatrix} x \\ y \\ Z(x, y) \end{pmatrix} \rightarrow \begin{pmatrix} x + u_x(x, y) - f(x, y)\partial_x Z(x, y) \\ y + u_y(x, y) - f(x, y)\partial_y Z(x, y) \\ Z(x, y) + f(x, y) \end{pmatrix}$$

$$E = \frac{1}{2} \int d^2x [\kappa (\nabla^2 f(\vec{x}))^2 + 2\mu u_{ij}^2(\vec{x}) + \lambda u_{kk}^2(\vec{x})]$$

$$ds'^2 = ds^2 + 2u_{ij}dx_i dx_j$$

$$u_{ij}(\vec{x}) = \frac{1}{2} \left[\frac{\partial u_i(\vec{x})}{\partial x_j} + \frac{\partial u_j(\vec{x})}{\partial x_i} + \frac{\partial f(\vec{x})}{\partial x_i} \frac{\partial f(\vec{x})}{\partial x_j} \right]$$



$$u_{ij}(\vec{x}) = \frac{1}{2} \left[\frac{\partial u_i(\vec{x})}{\partial x_j} + \frac{\partial u_j(\vec{x})}{\partial x_i} + \frac{\partial f(\vec{x})}{\partial x_i} \frac{\partial f(\vec{x})}{\partial x_j} - \delta_{ij} \frac{f}{R} \right]$$

Nonlinear Field Theory for Thermally Excited Shells...

✿ Trace out in-plane phonons and radial shrinkage $f_0 \dots$

$$F_{\text{eff}} = -k_B T \ln \left(\int D\{u_x(x, y)\} \int D\{u_y(x, y)\} \int df_0 e^{-F/k_B T} \right)$$

$$F_{\text{eff}} = \frac{\kappa}{2} \int d^2x (\nabla^2 f)^2 + \frac{Y}{2} \int d^2x \left(\frac{1}{2} P_{ij}^T \partial_i f \partial_j f - \left(\frac{f}{R} \right)^2 - \frac{pR}{4} \int d^2x |\nabla f|^2 \right)$$

New for curved membranes!

$$F_{\text{eff}} = F_0 + F_1$$

$$F_0 = \frac{1}{2} \int d^2x \left[\kappa (\nabla^2 f)^2 - \frac{pR}{2} (\vec{\nabla} f)^2 + \frac{Y}{R^2} f^2 \right]$$

$$F_1 = \int d^2x \left[\frac{1}{4} P_{ij}^T (\partial_i f \partial_j f)^2 - \frac{f}{R} P_{ij}^T \partial_i f \partial_j f \right]$$

$\ell^* = \text{curvature length scale}$
 $= R / vK^{1/4} \sim \sqrt{Rh} \ll R$

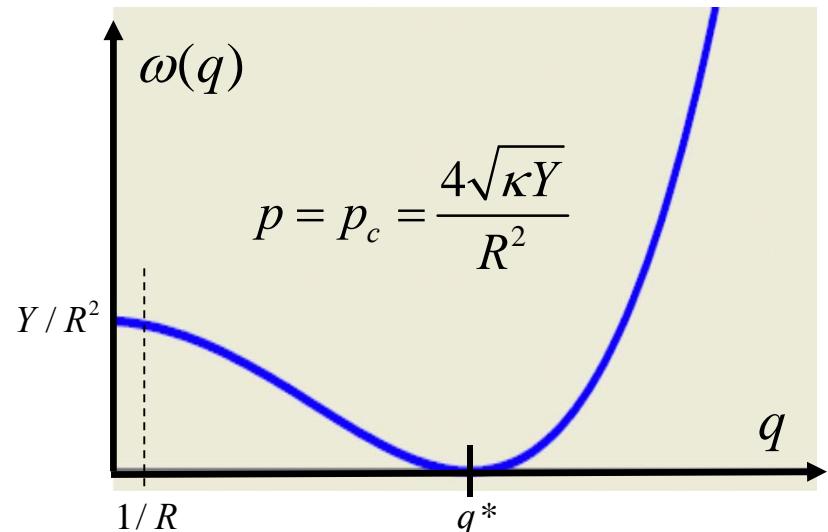
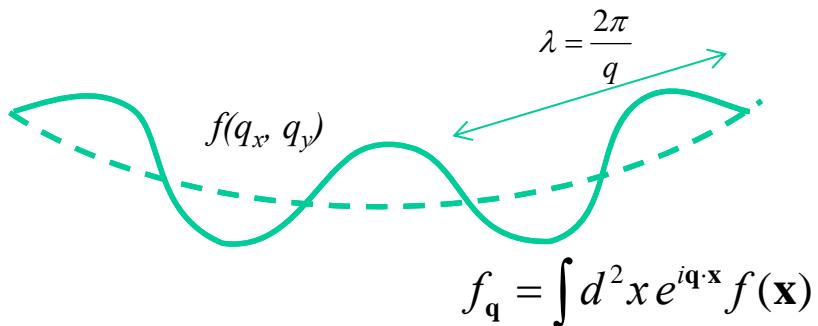
Gaussian Fluctuation Spectrum in Fourier Space

$$F_0 = \frac{1}{2} \int d^2x \left[\kappa (\nabla^2 f)^2 - \frac{pR}{2} (\vec{\nabla} f)^2 + \frac{Y}{R^2} f^2 \right]$$

$$= \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} \left(\kappa q^4 - \frac{pR}{2} q^2 + \frac{Y}{R^2} \right) |f_{\mathbf{q}}|^2$$

$$F_0 \equiv \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} \omega(q) |f_{\mathbf{q}}|^2, \quad \omega(q) = \frac{Y}{R^2} - \frac{pR}{2} q^2 + \kappa q^4$$

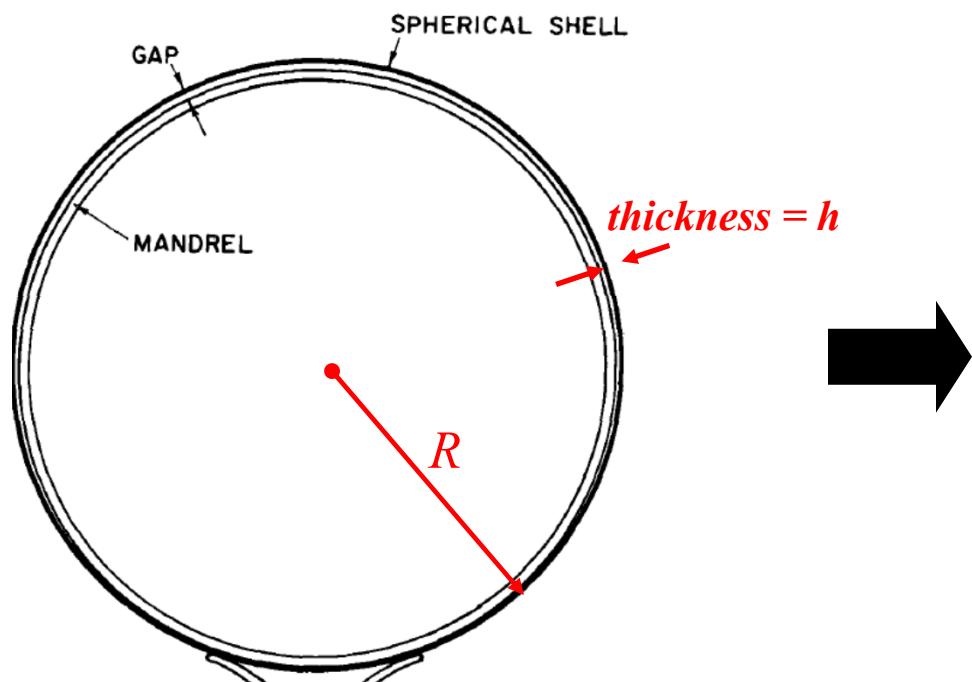
✿ There is a soft phonon mode above a critical pressure!



$$q^* = vK^{1/4} / R = 1/l^* \quad (vK = YR^2 / \kappa)$$

Macroscopic Buckling Instability Arrested by a Wax Mandrel...

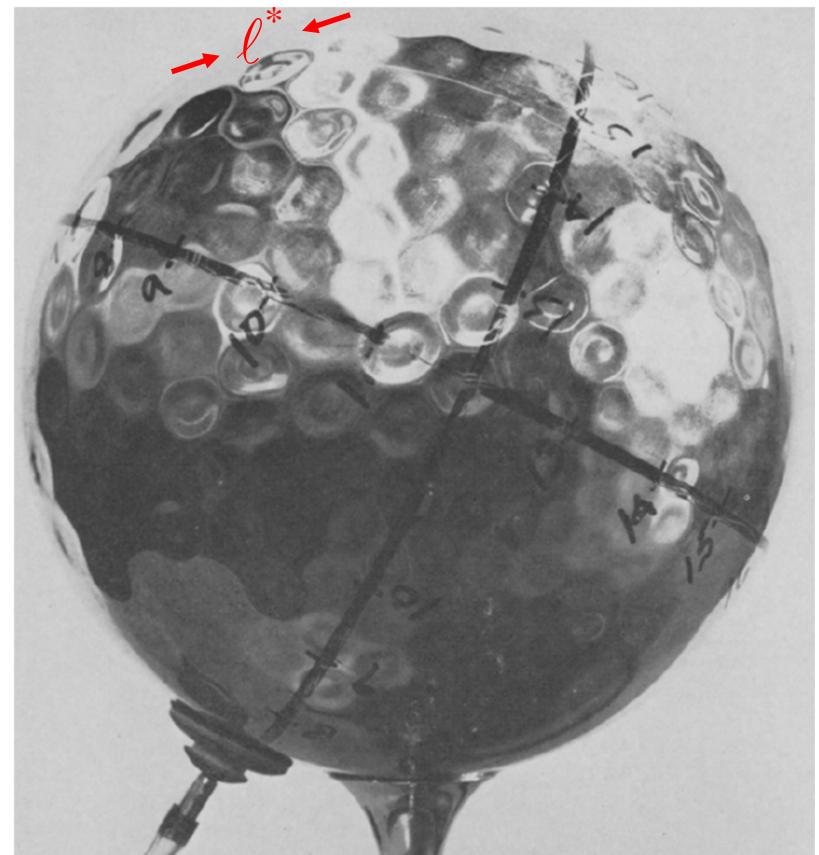
R. L. Carlson et al.,
Exp. Mech. 7, 281 (1962)



Classical
shell
theory

Koiter, 1963 (1945)
Hutchinson, 1967

$$\ell^* = R / \nu K^{1/4} \sim \sqrt{Rh} \ll R$$



$R = 4.25$ in., $R/h \sim 2000$

Evaluate effect of nonlinearities with perturbation theory...

J. Paulose, G. Vliegenthart, Gompper, drn, PNAS **109**, 19551 (2012)

$$\kappa_R \approx \kappa \left[1 + \frac{61}{4096} \frac{k_B T}{\kappa} \sqrt{vK} \right]; \quad Y_R \approx Y \left[1 - \frac{3}{256} \frac{k_B T}{\kappa} \sqrt{vK} \right]$$

$$p_R \approx \frac{1}{24\pi} \frac{k_B T}{\kappa} p_c \sqrt{vK}; \quad vK = YR^2 / \kappa; \quad p_c = 4\sqrt{\kappa Y} / R^2$$

Assume bare pressure = 0 (e.g., hemisphere!) and use renormalized elastic parameters $\kappa_R(\ell^*)$ and $Y_R(\ell^*)$, $\ell^* = (\kappa R^2 / Y)^{1/4}$, to evaluate the pressure generated by thermal

fluctuations: $p_R(\ell^*) = \frac{1}{24\pi} \frac{kT}{\kappa_R(\ell^*)} p_c(\ell^*) \sqrt{vK(\ell^*)}$; Thermal fluctuations will crush

spheres or hemispheres whenever $p_R(\ell^*) > p_c(\ell^*)$

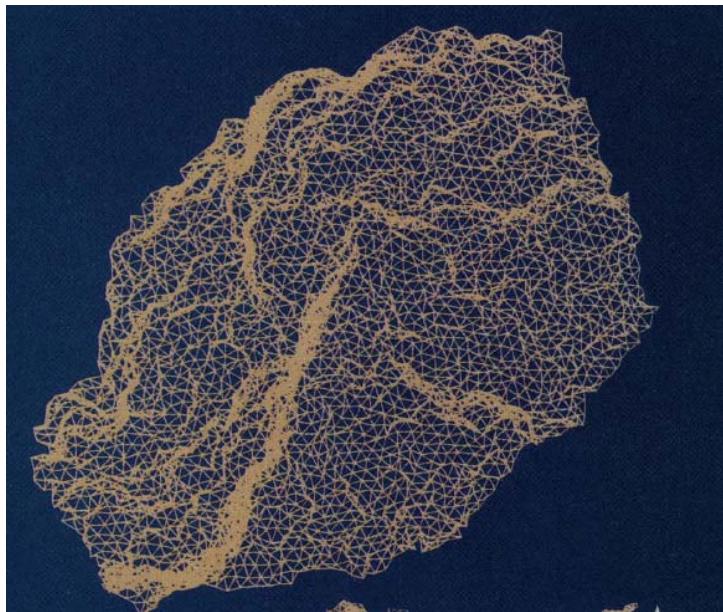
Criterion for self-crushing is $R > R_c \approx 54 \left[\frac{\kappa^3}{Y(k_B T)^2} \right]^{1/2}$; $R_c \approx \text{const.} \frac{\kappa}{k_B T} h$

$R_c \approx 60 \text{ nm}$ for graphene....

(see talk of Andrej Kosmrlj in late February)

Thermalized sheets and shells: topology matters

F. Abraham

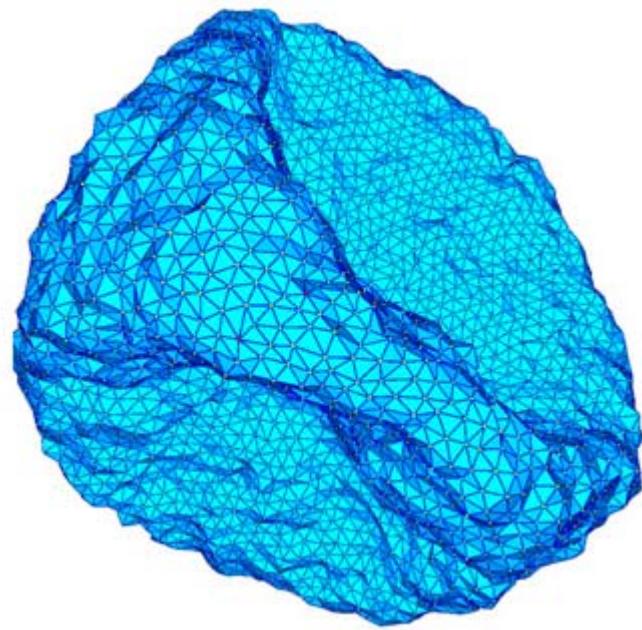


Scale-dependent bending rigidity κ
~5000 fold enhancement for graphene
at room temperature



Jayson
Paulose

vs.



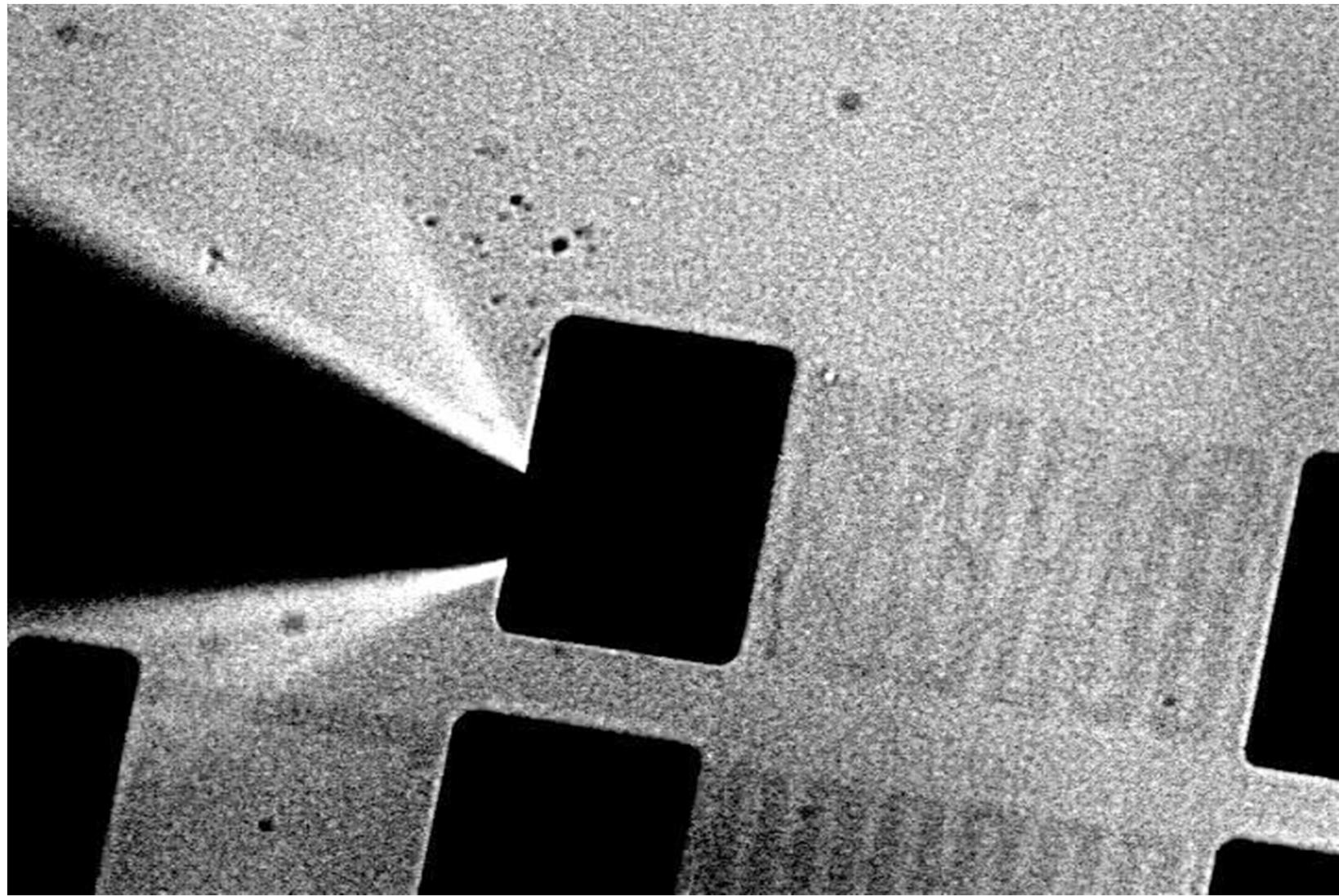
G. Gompper & G. Vliegenthart

Enhancement of bending rigidity κ only ~70 fold at room temperature; but spheres (and hemispheres) crush themselves for $R > R_c = 54[\kappa^3 / Y(k_B T)^2]^{1/2}$

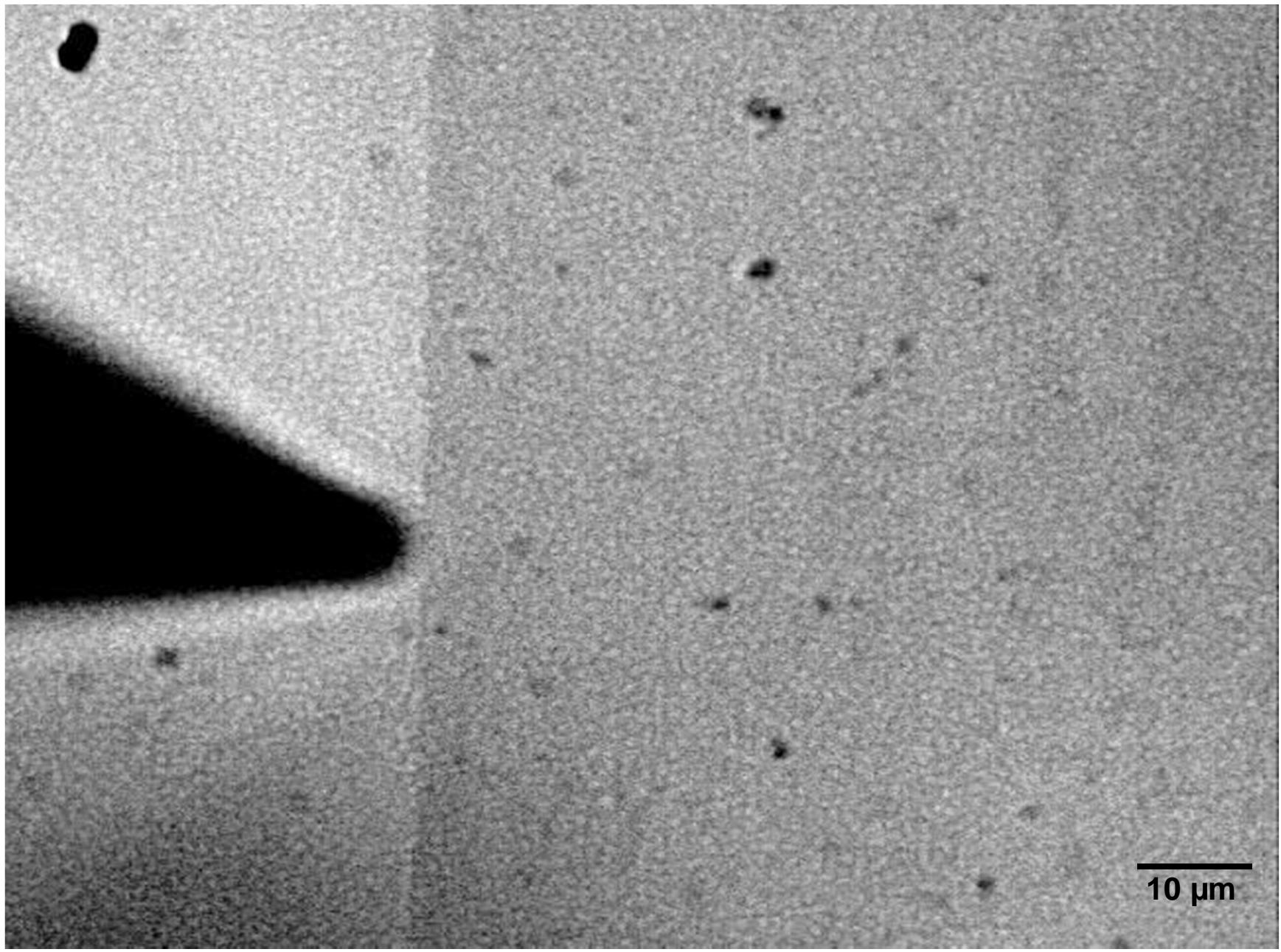


Andrej
Kosmrlj

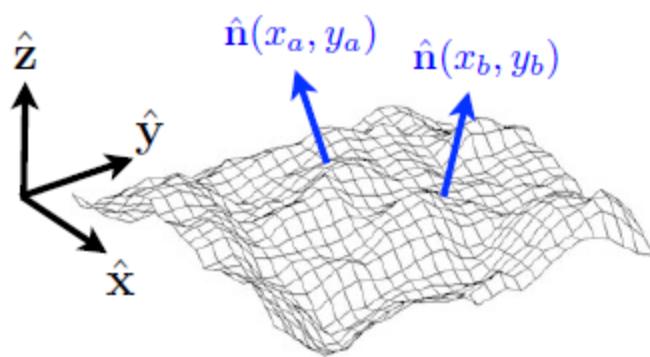
Future directions: new, atomically thin springs



10 μm



Future directions: flat vs. crumpled phases



$$\hat{\mathbf{n}}(x, y) = \frac{1}{\sqrt{1 + (\nabla f)^2}} \begin{pmatrix} -\partial_x f \\ -\partial_y f \\ 1 \end{pmatrix}$$

$$\kappa_R(q) \sim \kappa / (q \ell_{\text{th}})^\eta \quad \ell_{\text{th}} \sim \kappa / \sqrt{k_B T Y} \quad \eta \approx 0.82$$

$$\langle \hat{\mathbf{n}}(\mathbf{r}_a) \cdot \hat{\mathbf{n}}(\mathbf{r}_b) \rangle = 1 - \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{k_B T}{\kappa_R(q) q^2} \left[1 - e^{i \mathbf{q} \cdot (\mathbf{r}_b - \mathbf{r}_a)} \right]$$

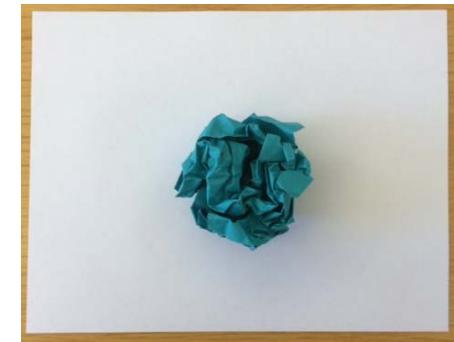
$$\langle \hat{\mathbf{n}}(\mathbf{r}_a) \cdot \hat{\mathbf{n}}(\mathbf{r}_b) \rangle \approx 1 - \frac{k_B T}{2\pi\kappa} \left[\eta^{-1} + \ln \left(\frac{\ell_{\text{th}}}{a_0} \right) \right] + C \frac{k_B T}{\kappa} \left(\frac{\ell_{\text{th}}}{|\mathbf{r}_b - \mathbf{r}_a|} \right)^\eta$$

**Normal-normal correlations approach
constant value at large separation.**

*Projected
Area $\sim L^2$
low T*



OR



*Projected
area $\sim L^{8/5}$
high T*



Crumpled paper experiment

Thank you!!

Figure: Melina Blees, Cornell

Thermalized sheets and shells: topology matters

Equations of thin plate theory

- nonlinear bending and stretching energies*
- $vK = \text{Föppl-von Karman number} = YR^2/\kappa \gg 1$

Recent experiments:
Paul McEuen group
(Cornell)

Physics of thermalized sheets

- “self-organized criticality” of the flat phase
- strongly scale-dependent elastic parameters
- hard condensed matter physics : graphene , MoS_2 , BN , WS_2 ,....

G. Gompper
G. Vliegenthart



Jayson
Paulose

Physics of thermalized shells

- spherical shells accessible via soft matter experiments on diblock copolymers.
- shells with zero pressure difference are crushed by thermal fluctuations for $R > R_c(T)$!



Andrej
Kosmrlj