Quantum Physics Beyond Classical Singularities: Cosmological Models

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AA, T. Pawlowski, P. Singh & K. Vandersloot (gr-qc/0612104);
Szulc, Kaminski & Lewandowski (gr-qc/0612101)
(AA, conference review gr-qc/0605011)
(M. Bojowald, Liv. Rev. Rel. (2005))

Related recent work by:

Eloisa Bentevigna, Martin Bojowald, Dah Wei Chiou, W. Kaminski,

Jerzy Lewandowski, Lukasz Szulc, Victor Taveras, Thomas Thiemann,

Kevin Vandersloot, G. V. Vereschagin, Josh Willis.

In general relativity, gravitational field encoded in the very geometry of space-time $\Rightarrow$ space-time itself ends at singularities. General expectation: theory is pushed beyond its domain of applicability. Must incorporate Quantum Physics.

In LQG, physics does not stop at these singularities. Quantum Geometry extends its life. Resolution of space-like singularities has been analyzed at three levels:

i) Quantum Hamiltonian constraint does not break down. (Cosmological and black hole interior models, some midi-superspaces) (Bojowald);

ii) + construction of the Physical Hilbert space, Dirac observables, emergent time (homogeneous models);

iii) + Detailed numerical solutions, effective equations and comparison between the two (homogeneous, isotropic models).

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iii) + Detailed numerical solutions, effective equations and comparison between the two (homogeneous, isotropic models).

Goal of this talk: Provide a bird's eye view. Focus on iii) and comment on i) and ii). Will enable me to bypass more technical issues of LQG and emphasize the recent qualitative change in the degree of completeness & precision.

Organization:
1. Conceptual Setting
2. Loop Quantum Cosmology
3. Detailed Models.

1. Conceptual Setting

Some Long-Standing Questions expected to be answered by Quantum Gravity Theories from first principles:

⋆ How close to the big-bang does a smooth space-time of GR make sense? (Onset of inflation?)

⋆ Is the Big-Bang singularity naturally resolved by quantum gravity?
(answer is ‘No’ in the Wheeler-DeWitt theory)

⋆ Is a new principle/ boundary condition at the Big Bang essential?
(e.g. The Hartle-Hawking ‘no-boundary proposal’.)

⋆ Is the quantum evolution across the ‘singularity’ deterministic?
(answer ‘No’ e.g. in the Pre-Big-Bang and Ekpyrotic scenarios)

⋆ What is on the ‘other side’? A quantum foam? Another large, classical universe? ...

Long Standing Questions (contd)

⋆ How does one extract physics from solutions to the Hamiltonian constraint (e.g. WDW equation)? dynamics from the frozen formalism? Dirac observables? Emergent time?

(Scale factor —natural candidate in the Misner parametrization— not single-valued in closed models.)

⋆ Can one have a deterministic evolution across the singularity and agreement with GR at low curvatures, e.g., recollpase in the closed models?

(Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second (Green and Unruh))

All these issues resolved in LQC.

Emerging Scenario: Physical sector of the theory can be constructed in detail. Continuum a good approximation till curvature attains Planck scale. In simplest models: Vast classical regions bridged deterministically by quantum geometry. No new principle needed.

Older Quantum Cosmology (DeWitt, Misner, Wheeler ... 70's)

- Since only finite number of DOF $a(t), \phi(t)$, field theoretical difficulties bypassed; analysis reduced to standard quantum mechanics.

- Quantum States: $\Psi(a,\phi)$; $\hat{a}\Psi(a,\phi) = a\Psi(a,\phi)$ etc.

Quantum evolution governed by the Wheeler-DeWitt differential equation

k=1, FRW: $$-(a\frac{\partial}{\partial a})^2\Psi(a,\phi) + C_1\ a^4\Psi(a,\phi) = C_2\, a^3\ \hat{H}_\phi\ \Psi(a,\phi)$$

Without additional assumptions, singularity is not resolved.

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Without additional assumptions, singularity is not resolved.

- In LQC, situation is very different due to the Quantum Riemannian Geometry. How is this possible? In QM we have von Neumann's uniqueness theorem!

If one follows the procedure used in LQG, one of the assumptions of the von Neumann theorem violated $\Rightarrow$ uniqueness result bypassed. Inequivalent representations even for mini-superspaces. New quantum mechanics! (AA, Bojowald, Lewandowski) Novel features precisely in the deep Planck regime.

Connection Dynamics: Holonomies and Triads

• Configuration variable: A spin-connection A_a^i which parallel transports chiral spinors. Conjugate momentum: non-Abelian electric fields E_i^a. Dual interpretation: *orthonormal triads*. Gauge group: $\mathrm{SU}(2)$: Rotations of triads (or spin-dyads).

• Surprising uniqueness result: The quantum algebra of holonomies and triad-fluxes admits a unique *diff invariant* state (i.e., unique background independent representation). (Lewandowski, Okolow, Sahlmann,Thiemann; Fleischhack,) .

• This representation was first constructed explicitly in the early nineties. High mathematical precision. Provides a Quantum Geometry which replaces the Riemannian geometry used in classical gravity theories. (AA, Baez, Lewandowski, Rovelli, Smolin, Thiemann, ...)

• No operator corresponding to the connection itself. Mathematical Analogy: $\hat{U}(\lambda) := \widehat{\exp i\lambda x}$ well-defined but not $\hat{x}$. This is why von-Neumann's result is bypassed in quantum cosmology.

2. Loop Quantum Cosmology

- In LQG the canonically conjugate variables are:

A_a^i, SU(2) connections and, E_i^a, orthonormal triads.

Spatial homogeneity and isotropy implies

$$\star \qquad A_a = c \underbrace{\mathring{\omega}_a^i \sigma_i}_{\text{fixed}}, \quad E^a = p \underbrace{\mathring{e}_i^a \sigma^i}_{\text{fixed}}$$

–$c \sim \dot{a}$

–holonomy: $h_e(c) = \cos \mu c \; \mathbf{1} + \sin \mu c \; \dot{e}^a \mathring{\omega}_a^i \sigma_i$

(Almost periodic in c)

– $|p| = a^2$.

– $p \to -p$ changes only the orientation of the triad.

Large gauge transformation; leaves physics invariant.

$\star$ Canonically conjugate pairs:

c, p for gravity $\qquad \phi, p_\phi$ for matter

- Loop quantum cosmology:

Key strategy:

Do not naively set $\mathcal{H} = L^2(R, \mathrm{d}c)$ and $\hat{c}\Psi(c) = c\Psi(c)$; $\hat{p}\Psi(c) = -i\hbar \frac{\mathrm{d}\Psi}{\mathrm{d}c}$.

Rather, Follow full theory.

New Quantum Mechanics

Gravitational Sector:

- States: $\Psi(p) = \sum_i \Psi_i \, |p_i\rangle \qquad ||\Psi||^2 = \sum_i |\Psi_i|^2$

Note: $< p_i|p_j >= \delta_{ij}$ (Kronecker delta, not Dirac!)

- Operators: $\hat{p}\Psi(p) = p\Psi(p)$ (self-adjoint);

  $\widehat{\exp i\mu c}\,\Psi(p) = \Psi(p+\mu)$ (unitary)

But no connection operator $\hat{c}$!

Representation is inequivalent to Schrödinger's
New Quantum Mechanics possible.
This kinematic structure mimics that of the full LQG.

- Similarly for the Hamiltonian constraint:

$$C_{\rm grav} \sim \underbrace{(\epsilon^{ij}{}_k E^a_i E^b_j/\sqrt{q})}_{\rm Thiemann} \quad \underbrace{F^k_{ab}}_{\rm holonomy}$$

Quantum Dynamics

- Quantum Einstein's Equation: Use a representation in which geometry (i.e. $\hat{V} \sim \hat{a}^3$) and matter field (i.e., $\hat{\phi}$) are diagonal : $\Psi(v,\phi)$

Then the Wheeler DeWitt equation is replaced by a difference equation:

$$C^+(v)\,\Psi(v+4,\,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\Psi(v-4,\,\phi) \;=\; \hat{H}_\phi \Psi(v,\phi)$$

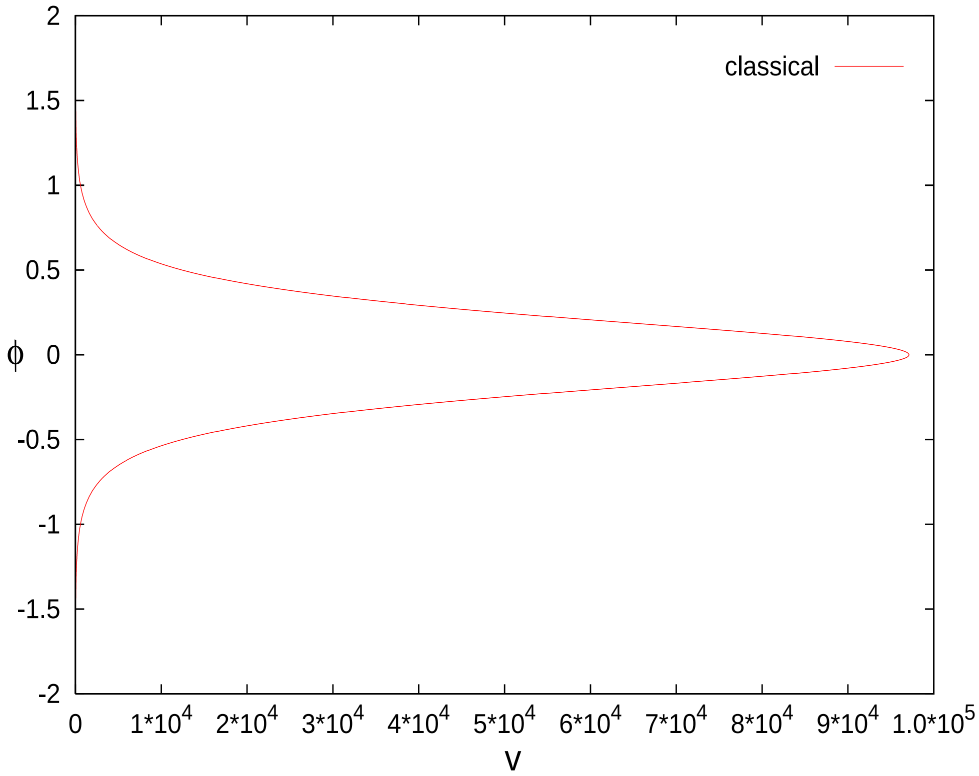
Fundamentally, a constraint equation. Selects physical states. However, this equation also dictates quantum dynamics.

- To extract physics, we need to:

⋆ Isolate 'time' to give meaning to 'evolution'.
⋆ Solutions to the constraint: Physical states. Introduce a physical inner product and suitable observables.
⋆ Construct states which represent the actual universe at late time. 'Evolve back' towards the big bang. Is the classical singularity 'resolved'? In what sense? What is on the 'other side' of the classical big-bang? (Quantum foam?? Another classical universe??)

3. A Detailed Model

A concrete Example: $k = 1$ FRW model with a massless scalar field ϕ. Instructive because every classical solution is singular; scale factor not a good global clock; there is classical re-collapse. Provides a foundation for more complicated models.



Classical trajectories (symmetry under $v \to -v$)

Basic Strategy

- The quantum Hamiltonian constraint takes the form:

$$-\Theta\,\Psi(v,\phi) = \partial_\phi^2 \Psi(v,\phi) \qquad (\star)$$

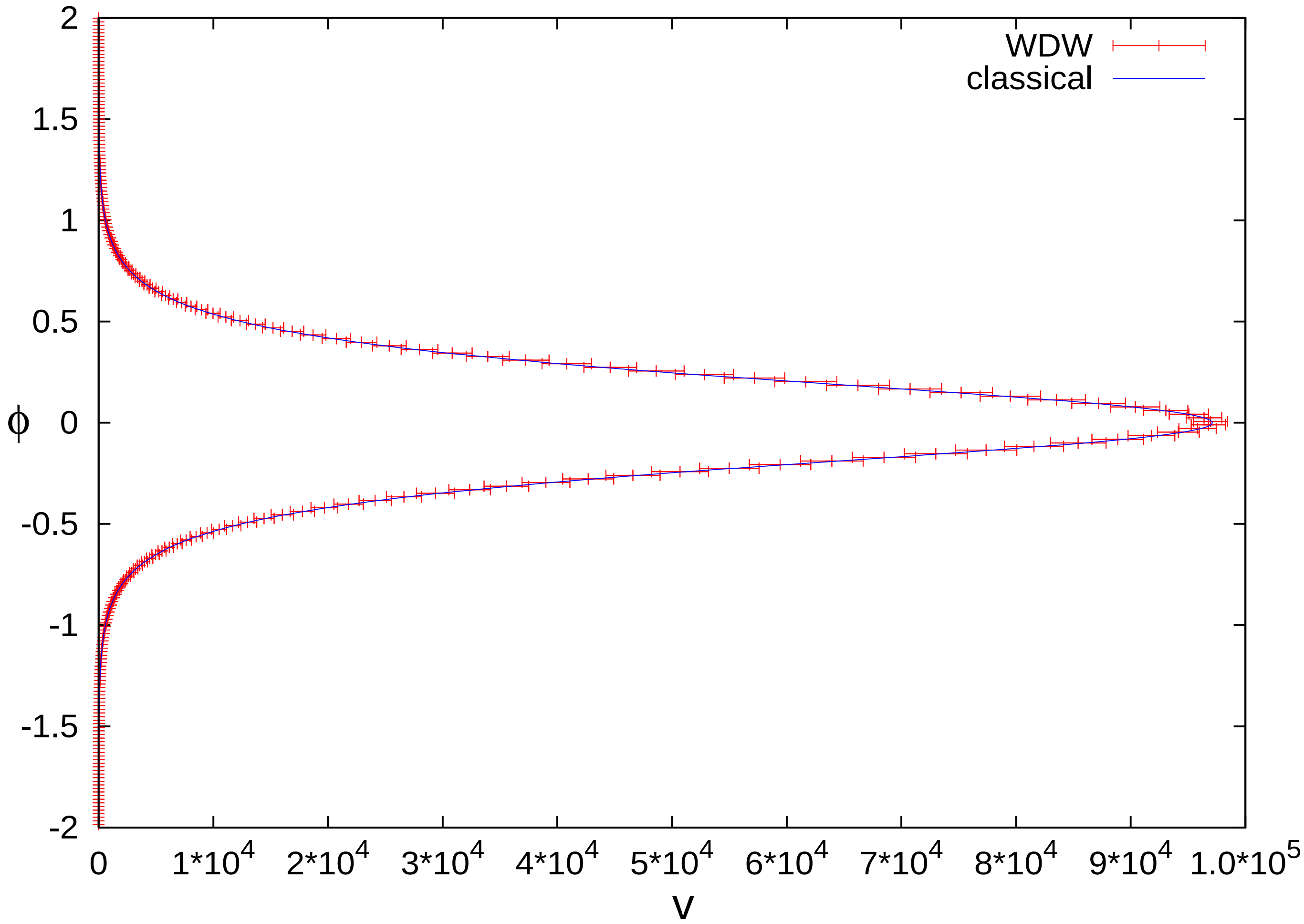
where Θ is a positive, self-adjoint difference operator independent of ϕ

$$\Theta\,\Psi(v,\phi) = C^+(v)\,\Psi(v+4,\,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\,\Psi(v-4,\,\phi).$$

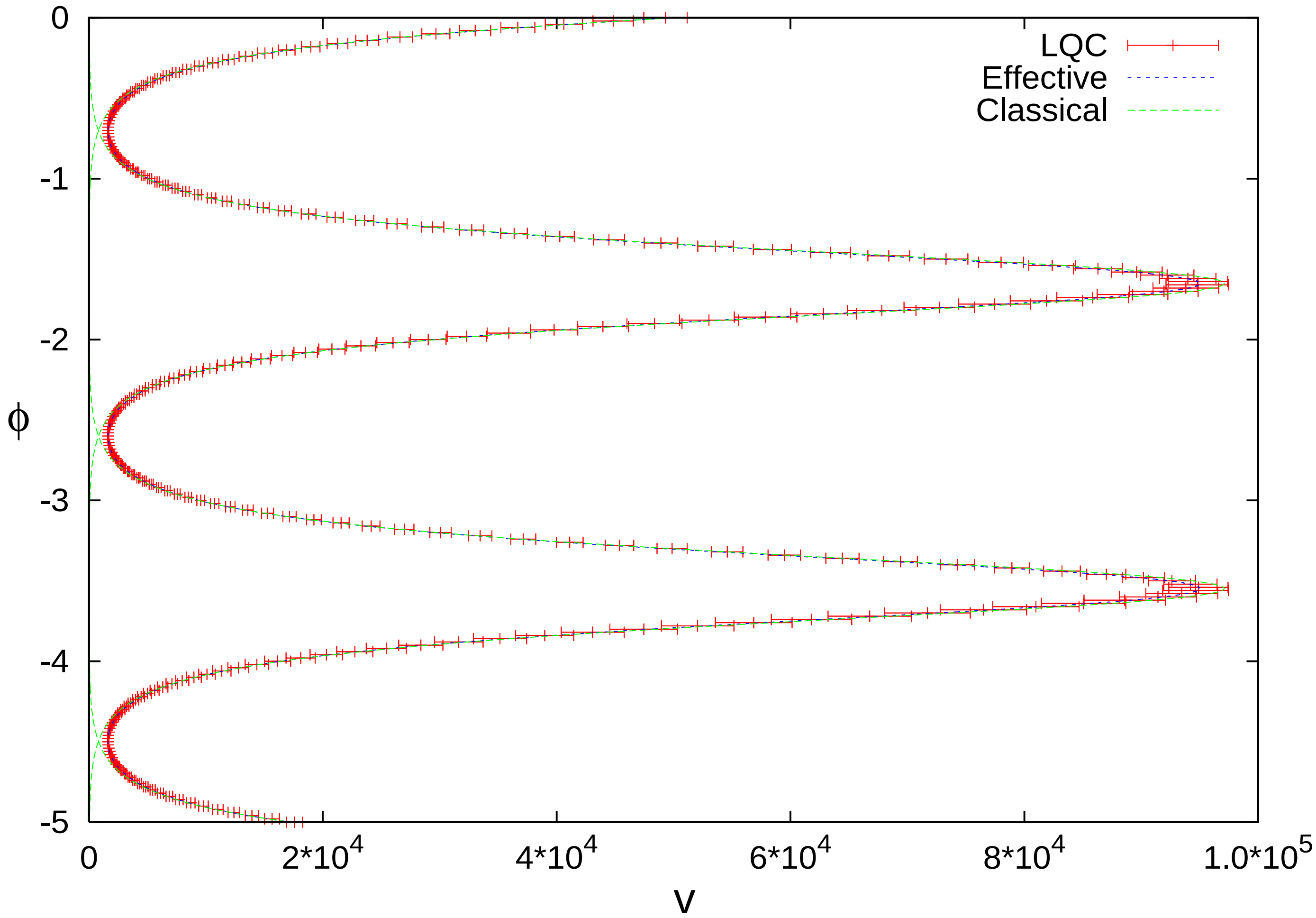
Suggests ϕ could be used as 'emergent time' also in the quantum theory. Relational dynamics.

Physical states: solutions to ($\star$), invariant under $v \to -v$. Observables: $\hat{p_\phi}$ and $\hat{V}|_{\phi=\phi_o}$. Inner product: Makes these self-adjoint or, equivalently, use group averaging. Analogy with KG equation in a static space-time. Semi-classical states: Generalized coherent states.

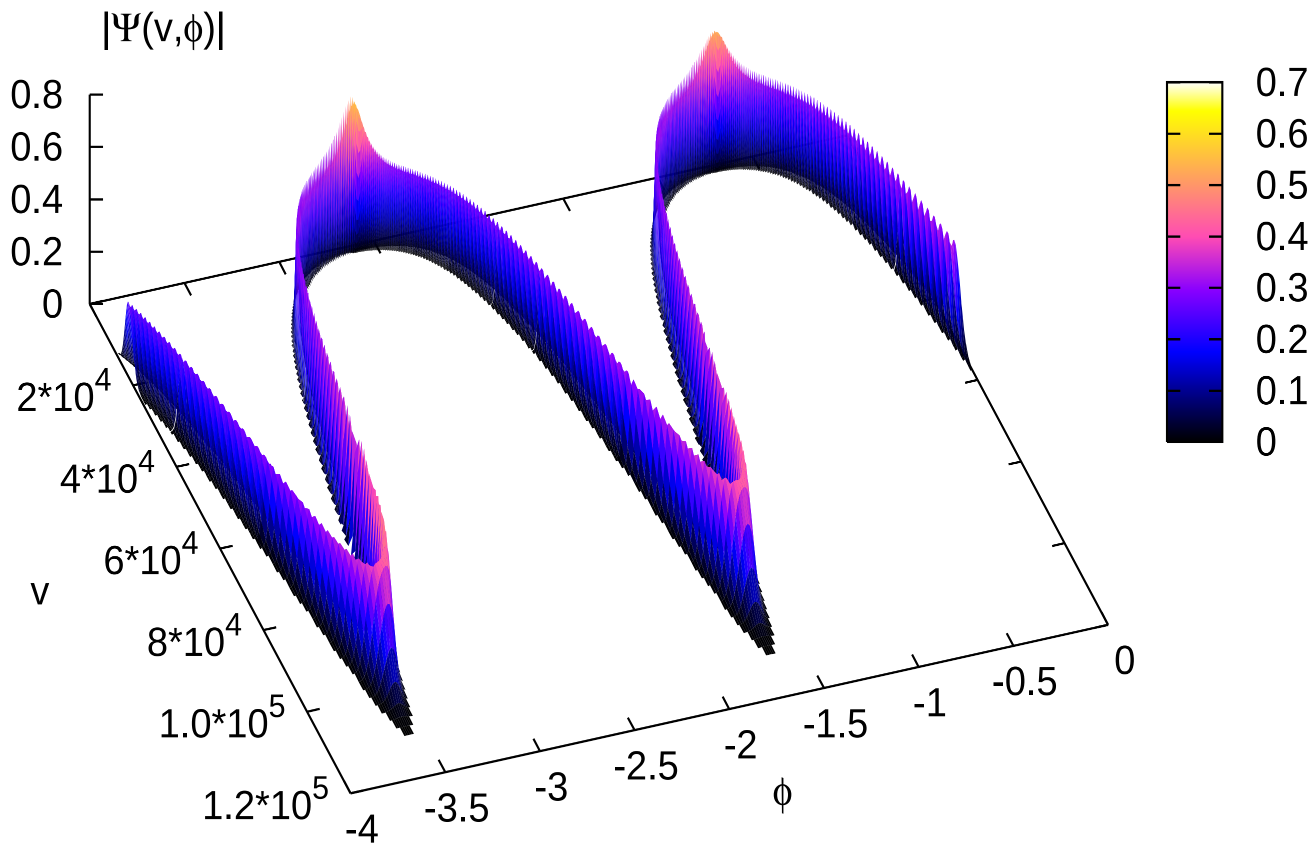
- Use numerical methods to solve the Quantum Constraint. Numerics Essential! Furthermore, not entirely straightforward.



WDW Theory: Expectations values and dispersions of $\hat{V}|_{\phi}$.



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.



Absolute value of the physical state $\Psi(v, \phi)$ $\quad (\Psi(v,\phi) = \Psi(-v,\phi))$

Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then:

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \approx 13\pi/lp^2$ or $\rho \approx 0.82\rho_{\rm Pl}$. $\Rightarrow$ Space-time can be taken to be classical during the inflationary era.

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic. No new principle needed.

• Big bang replaced by a quantum bounce. A new 'repulsive force' due to quantum geometry. Unlike in other approaches with bounces, unambiguous evolution across the 'bridge', provided by the quantum Einstein equation. Hartle-Hawking 'No Boundary Proposal'.

• No unphysical matter. Notion of semi-classicality precise ($\sim$ Coherent States). Unlike in WKB methods, fluctuations under full control. Notion of semi-classicality very weak.

- Effective Friedmann Eq:

$$(\dot{a}/a)^2 = (8\pi G/3)(\rho - \rho_1(v)) \, [\rho_2(v) - \rho/\rho_{\rm crit}]$$

with $\rho_{\rm crit} \approx 0.82 \rho_{\rm Planck}$

- Recollapse: $\rho(v) = \rho_1(v)$; $\rho_{\rm min} = (3/8\pi G a^2_{\rm max}) \left(1 + O(\ell^4_{\rm Pl}/a^4)\right)$

For $p_\phi = 5 \times 10^3 \, \hbar$, $a_{\rm max} \approx 23 \ell_{\rm Pl}$, Agreement with the classical Friedmann formula to one part in 10^5. For macroscopic universes, LQC prediction on recollapse indistinguishable from the classical Friedmann formula.

- Bounces: $\rho(v) = \rho_2(v)$; $\rho_{\rm max} = \rho_{\rm crit} \left(1 + O(\ell^2_{\rm Pl}/a^2_{\rm min})\right)$

For $p_\phi = 5 \times 10^3 \, \hbar$, $a_{\rm min} \approx 5.9 \ell_{\rm Pl}$; $\rho_{\rm max}$ equals $\rho_{\rm crit}$ to within 2%. For large universes, the two are indistinguishable.

- For a universe which attains $a_{\rm max} \approx 1$ Mpc, $a_{\rm min} \approx 6 \times 10^{16} {\rm cm}^3 \approx 10^{115} \ell^3_{\rm Pl}$! Quantum geometry corrections to 'inverse volume' play no role at all. Rather, the origin of the bounce lies in the non-local nature of $\hat{F}_{ab}$ operator (holonomy).

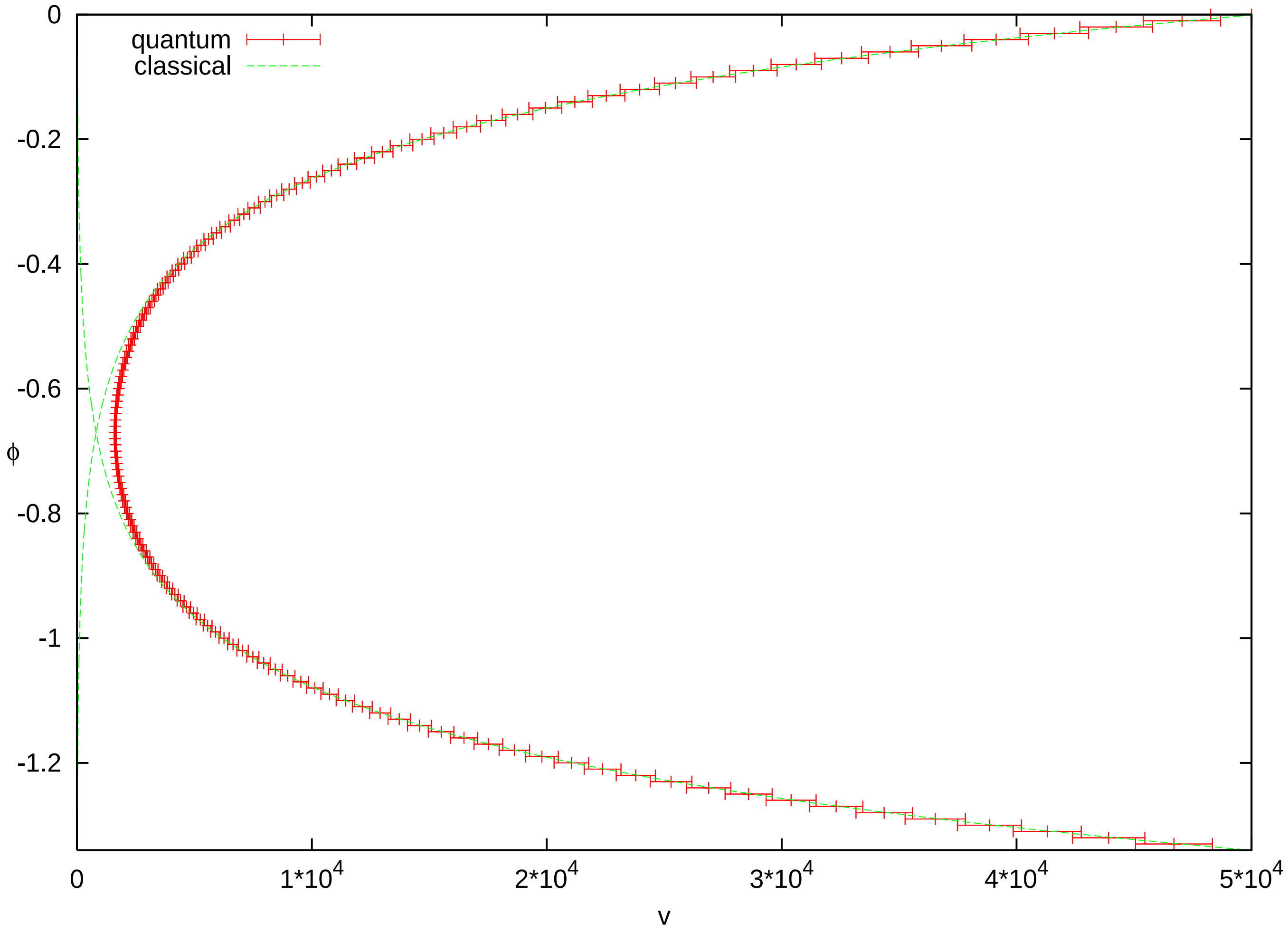
Summary

• In LQG, the interplay between geometry and physics is elevated to quantum level. Physics does not end at singularities.

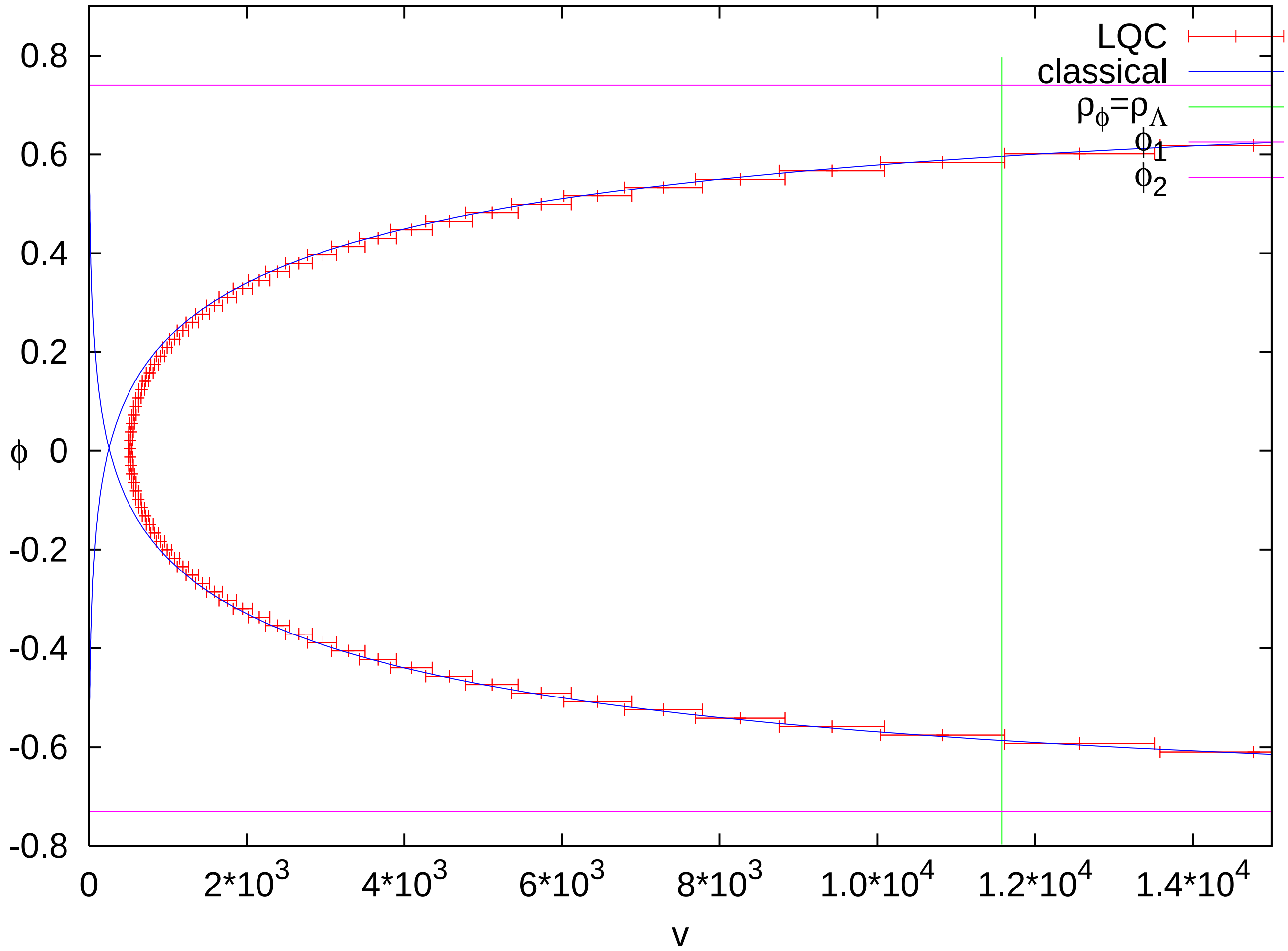
• In $k = 1$ and $k = 0$ FRW models with or without Λ, complete control on the physical sector of the theory. LQC evolution deterministic across the classical big bang and big crunch for all quantum states. For states which are 'semi-classical at late times,' detailed numerical and effective descriptions & comparison between the two. $\rho_{\rm crit} \approx 0.8\rho_{\rm Pl}$ for all these models.

• Universes with $a_{\rm max} \sim 25\ell_{\rm Pl}$ already semi-classical! Repulsive force of quantum geometry arises and dies *very* quickly but makes dramatic changes to classical dynamics.

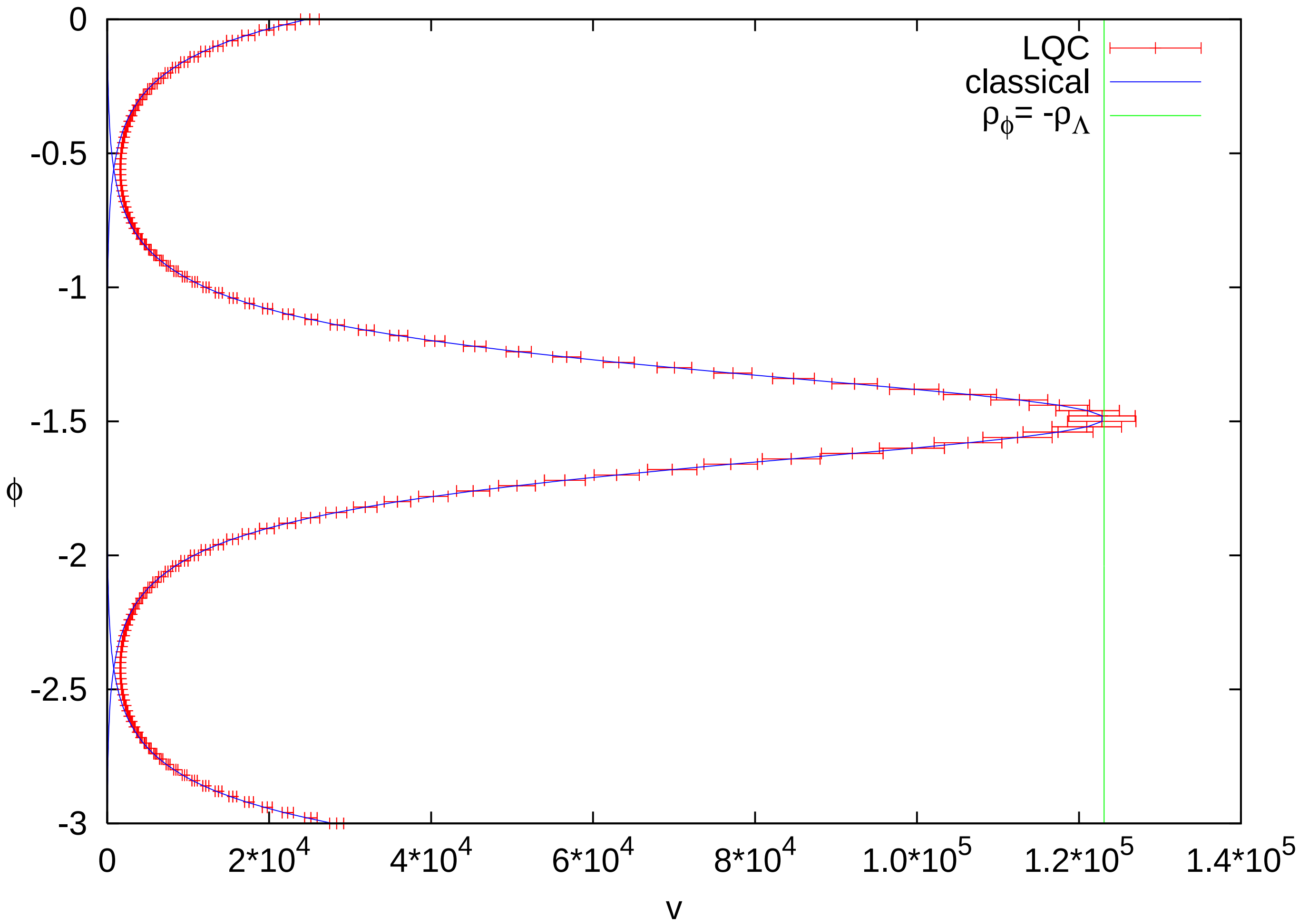
• Challenge to background independent theories: Detailed recovery of classical GR at low curvatures/densities (Green and Unruh). Met in cosmological models.



$k = 0$: Expectations values and dispersions of $\hat{V}|_\phi$.



Positive Cosmological constant: Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories. (Numerics are currently being improved.)



Negative Cosmological constant: Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories. (Numerics are currently being improved.)

- More general models:

Level ii) Anisotropies (AA, Chiou, Vandersloot).
level i) Anisotropies (Bojowald, Date, Vandersloot); black hole interiors (AA, Bojowald, Modesto, Cartin & Khanna); midi-superspaces (Bojowald).
A large body of work on Effective equations and their applications, e.g. Chaos in Bianchi IX; inflation; power suppression at large angular scales ....

A Sample of Current work:

- Midisuperspaces, particularly Gowdy Models (Several groups); Inclusion of Inhomogeneities (Bojowald et al); Implications to string theory inspired models (Singh, Verschagin, Vandersloot); Relation between LQC and LQG (AA, Engle, Lewandowski, Koslowski, Thiemann, ...).

- Effective Equations: The Hamiltonian analog of effective action framework developed to obtain quantum corrections to Einstein's equations and systematically applied to quantum cosmology (AA, Bojowald, Skirzewski, Taveras, Willis, ...).

- General singularity resolution theorems? (Planck scale non-locality of F_{ab} & quantum geometry repulsion.)

Supplemet 1:Uniqueness of the LQG Representation

Question postponed during the seminar (regarding the unique diff invariant state in LQG kinematics). Need to go back a bit and recall a few facts re general mathematical framework of QFT

In the algebraic approach to QFT, one begins with a $\star$-algebra A of basic variables. A state f is a positive linear function (PLF) on this algebra. Given such a state, there is a canonical way to construct a representation. In this representation, the given (abstract) state f determines a (concrete) vector $\Psi_f\rangle$ in the Hilbert space and the original PLF equals the expectation value of operators in this vector: $f(a) = \langle\Psi_f|a|\Psi_f\rangle$

In LQG kinematics, the basic algebra is generated by holonomies and fluxes. The recent theorem of Lewandowski et al is that this algebra admits a unique diff invariant state. The GNS construction gives the kinematical Hilbert space of LQG. Of course in final quantum dynamics there are infinitely many diff invariant states. But they are states on the algebra of diff invariant operators which is distinct from the original holonomy-flux algebra.