Neural computation as a transformation of similarity

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Imaging neural activity *in vivo*

50μm

Raw data: Yuste lab

CalMAn: Giovannucci, Friedrich et al, Chklovskii, Pnevmatikakis

Stimulus vector: $\mathbf{x}_t$

Neural activity vector: $\mathbf{y}_t$
What does neural activity represent?

How is neural representation computed?
Reconstruction approach: linear decoding

What does neural activity represent?

$\mathbf{x} \approx \sum_k w_k y_k$

Abbott, Atick, Bialek, Chklovskii, Olshausen & Field, ...

Challenges

• Activity patterns vary across individuals
• Nonlocal synaptic learning rules
What does neural activity represent?

Similarity of neural activity patterns in IT

Human Cortex

Kriegeskorte et al., 2008
Kiani et al., 2007
What does neural activity represent?

**Similarity of neural activity patterns in IT**

Monkey Cortex

Human Cortex

Kriegeskorte et. al., 2008

Kiani et. al., 2007
Invariance of similarity across individuals may account for the invariance of concepts

Neural representation is representation of similarities

*Shimon Edelman (1998)*
What does neural activity represent?

Similarity along the visual pathway

Kiani et. al., 2007
Kriegeskorte et. al., 2008
What does neural activity represent?

Similarity along the visual pathway

Kiani lab

Kriegeskorte et. al., 2008
How is neural representation computed?

Similarity alignment: Similar input activity patterns evoke similar output activity patterns

Hu, Pehlevan, Chklovskii (2014)
Pehlevan, Chklovskii (2014)
Pehlevan, Chklovskii (2015)
Bahroun, Hunsicker, Soltoggio (2017)
Seung & Zung (2017)
How is neural representation computed?

Categorization

pixel intensity space
Part I: Unsupervised clustering

pixel intensity space

\[ x_1, x_2, x_3, x_4 \]
Normative algorithmic approach

1. Computational objective
2. Online algorithm
3. Neural network
4. Clustering

- Real-time processing, limited memory storage
- Dynamics of neural activity and synaptic plasticity, local learning rules
Clustering by similarity alignment

\[
\max_{y_t, y_\tau} \left( x_t^T x_\tau - \alpha \right) y_t^T y_\tau \quad \text{s.t.} \quad \|y_t\| \leq 1, \|y_\tau\| \leq 1
\]
Similarity alignment objective

\[
\max_{y_t \geq 0, y_{\tau} \geq 0} \frac{1}{T} \sum_{t=1}^{T} \sum_{\tau=1}^{T} (x_t^T x_{\tau} - \alpha) y_t^T y_{\tau} \quad \text{s.t.} \quad \|y_t\| \leq 1, \|y_{\tau}\| \leq 1
\]

similarity of pixel intensity
\[
\begin{pmatrix}
  x_1^T x_1 - \alpha & x_1^T x_2 - \alpha & \cdots & x_1^T x_T - \alpha \\
x_2^T x_1 - \alpha & x_2^T x_2 - \alpha & \cdots & x_2^T x_T - \alpha \\
\vdots & \vdots & \ddots & \vdots \\
x_T^T x_1 - \alpha & x_T^T x_2 - \alpha & \cdots & x_T^T x_T - \alpha
\end{pmatrix}
\]

pets plants

similarity of neural activity
\[
\begin{pmatrix}
y_1^T y_1 & y_1^T y_2 & \cdots & y_1^T y_T \\
y_2^T y_1 & y_2^T y_2 & \cdots & y_2^T y_T \\
\vdots & \vdots & \ddots & \vdots \\
y_T^T y_1 & y_T^T y_2 & \cdots & y_T^T y_T
\end{pmatrix}
\]

pets plants

Clustering
Deriving a neural network

\[
\max_{y_t \geq 0} \min_{z_t \geq 0} \frac{1}{T} \sum_{t=1}^{T} \sum_{\tau=1}^{T} \left( x_t^T x_\tau - \alpha \right) y_t^T y_\tau - \left( y_t^T y_\tau - 1 \right) z_t^T z_\tau
\]

\[
\frac{1}{T} \sum_{t=1}^{T} \sum_{\tau=1}^{T} x_t^T x_\tau y_t^T y_\tau = \sum_{t=1}^{T} y_t^T \left( \frac{1}{T} \sum_{\tau=1}^{T} y_\tau x_\tau^T \right) x_t = \sum_{t=1}^{T} y_t^T W_{yx}^T x_t \quad \text{same for } z_t, W_{yz}^T
\]

neural activity:
\[
y_t \leftarrow y_t + \gamma \left( W_{yx}^T x_t - W_{yz}^T z_t - \alpha b_y \right)
\]

synaptic plasticity:
\[
W_{i,j}^{yx} \leftarrow W_{i,j}^{yx} + \eta \left( y_{t,i} x_{t,j} - W_{i,j}^{yx} \right)
\]

Local learning rule!

Pehlevan, Genkin & Chklovskii (2017)
Similarity alignment can (softly) cluster...
Experimental test in fly larva antennal lobe

Berck et al, 2016

Chapochnikov, Pehlevan, Chklovskii et al, unpublished

Samuel lab
Clustering model of insect olfaction

Decorrelation by interneuron: ORN->I synaptic weight vector ~ centroid of neural activity
Clustering model of insect olfaction

- Rectification by KCs
- Single giant interneuron (GI)
- Non-random connectivity (Eichler et al, 2017)
- Sparse over-complete representation = soft clustering

Pehlevan, Genkin & Chklovskii (2017)
Part II: Manifold learning by similarity alignment
Categorization as manifold disentanglement

*DiCarlo & Cox, 2007*
Manifold learning

\[
\max_{y_i \geq 0, y_\tau \geq 0} \frac{1}{T} \sum_{t=1}^{T} \sum_{\tau=1}^{T} (x_t^T x_\tau - \alpha) y_t^T y_\tau \quad \text{s.t.} \quad \|y_t\| \leq 1, \|y_\tau\| \leq 1
\]

\[
\max_{y_i \geq 0, y_\tau \geq 0} \frac{1}{T} \sum_{t=1}^{T} \sum_{\tau=1}^{T} (\alpha - \|x_t - x_\tau\|_2^2) y_t^T y_\tau \quad \text{s.t.} \quad \|y_t\| \leq 1, \|y_\tau\| \leq 1
\]
Manifold learning

Similarity alignment learns manifolds

Pehlevan et al, unpublished
Similarity alignment network learns V1 features from natural images

Pehlevan & Chklovskii (2014)
Manifold learning

Similarity along the visual pathway

Kiani lab

Kriegeskorte et. al., 2008
Unsupervised manifold disentangling and clustering

Manifold learning

Tepper, Sengupta & Chklovskii (2017)
Pehlevan, Genkin, Chklovskii (2017) and unpublished
# Family of similarity alignment networks

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<th><strong>BIOLOGICAL FEATURE</strong></th>
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The similarity alignment approach yields biologically plausible networks for nontrivial computations.
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