# Neutrino Flavor Puzzle

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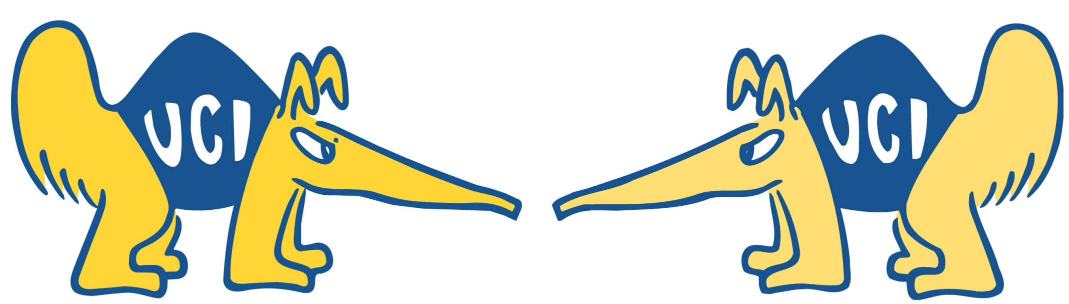
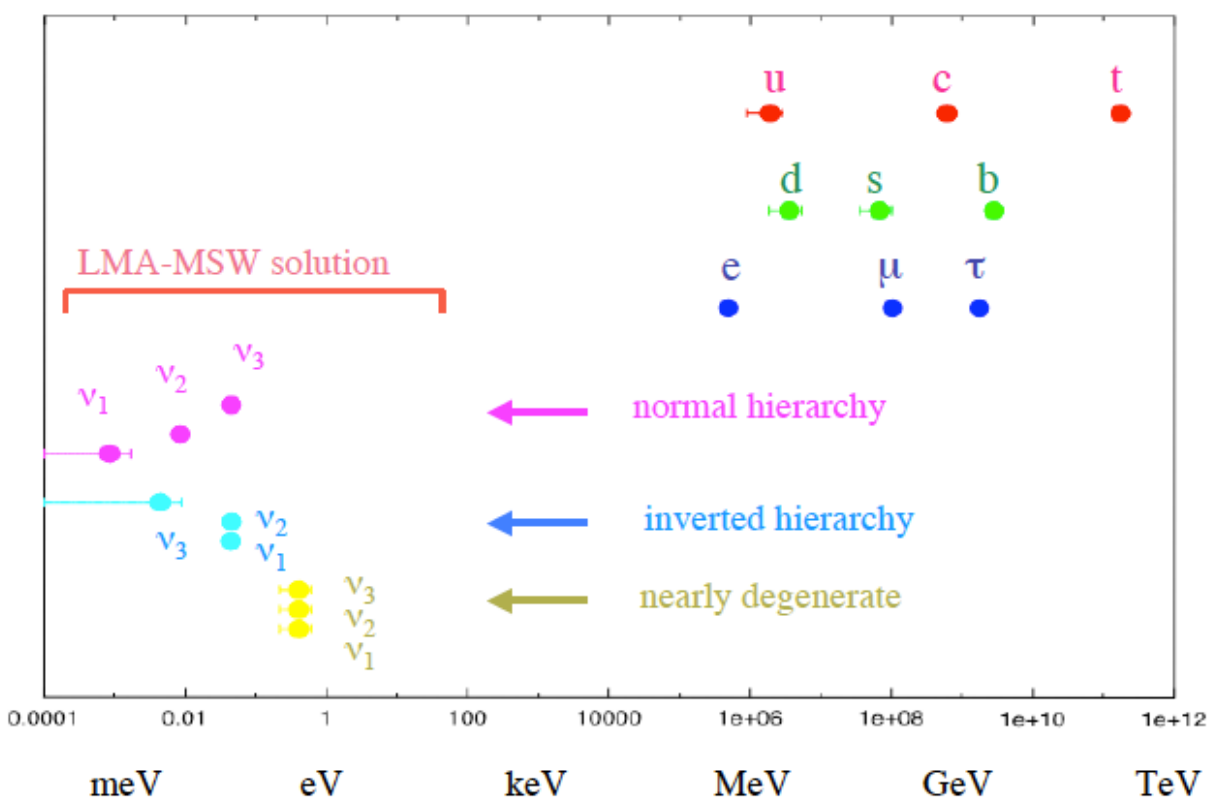



Snowmass Theory Frontier Conference, KITP, UCSB, February 25, 2022

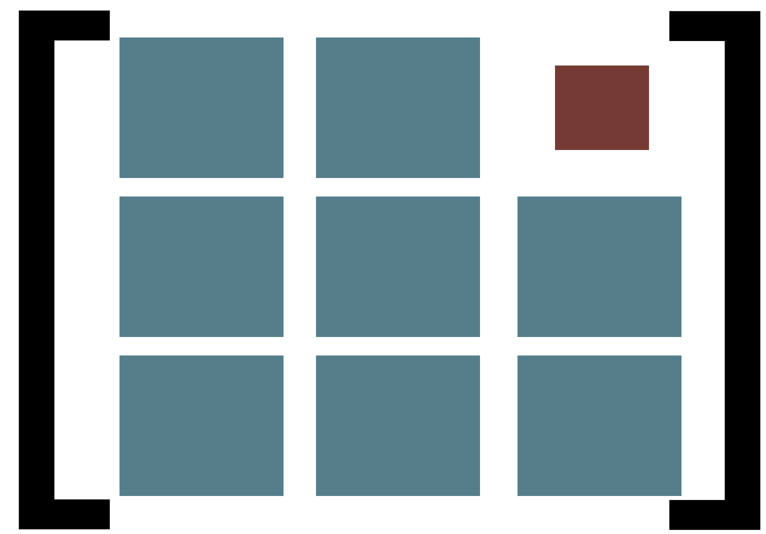
# Open Questions - Theoretical

☞ **Smallness of neutrino mass:**

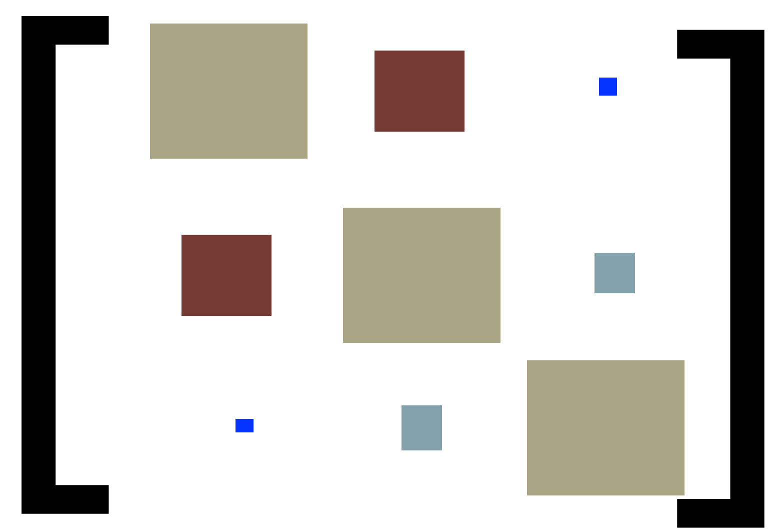
$m_\nu \ll m_{e,\,u,\,d}$




☞ **Flavor structure:**



leptonic mixing



quark mixing

# Why Should We Care?

- Understanding a wealth of data [Talks by Michael Wagman, Pedro Machado]
- SM flavor sector: no understanding of significant fraction (~20/25) of SM parameters; (c.f. SM gauge sector)
- Neutrinos as window into BSM physics [Talk by Zahra Tabrizi ]
  - neutrino mass generation unknown (suppression mechanism, scale)
  - Uniqueness of neutrino masses ➜ connections w/ NP frameworks
- Neutrinos affords opportunities for new explorations
  - New Tools (examples of formal theories' pheno relevance)
  - May address other puzzles in particle physics
    - Window into early Universe [Talk by Alex Friedland ]
  - UV connection - String Theories

# Small Masses - Majorana Neutrinos

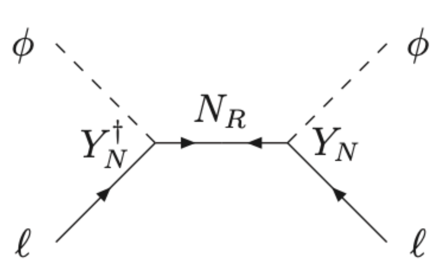
- SM: effective low energy theory $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{\mathcal{O}_{5D}}{M} + \frac{\mathcal{O}_{6D}}{M^2} + \ldots$

$$\frac{\lambda_{ij}}{M} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

Weinberg, 1979

- **only one dim-5 operator: most sensitive to high scale physics**

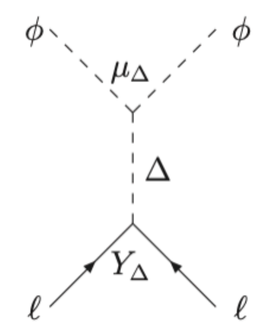
Type-I seesaw




$N_R$: $SU(3)_c \times SU(2)_w \times U(1)_Y \sim (1,1,0)$

Minkowski, 1977; Yanagida, 1979; Glashow, 1979; Gell-mann, Ramond, Slansky,1979; Mohapatra, Senjanovic, 1979;

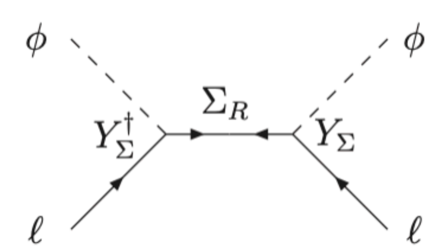
Type-II seesaw




$\Delta$: $SU(3)_c \times SU(2)_w \times U(1)_Y \sim (1,3,2)$

Lazarides, 1980; Mohapatra, Senjanovic, 1980

Type-III seesaw



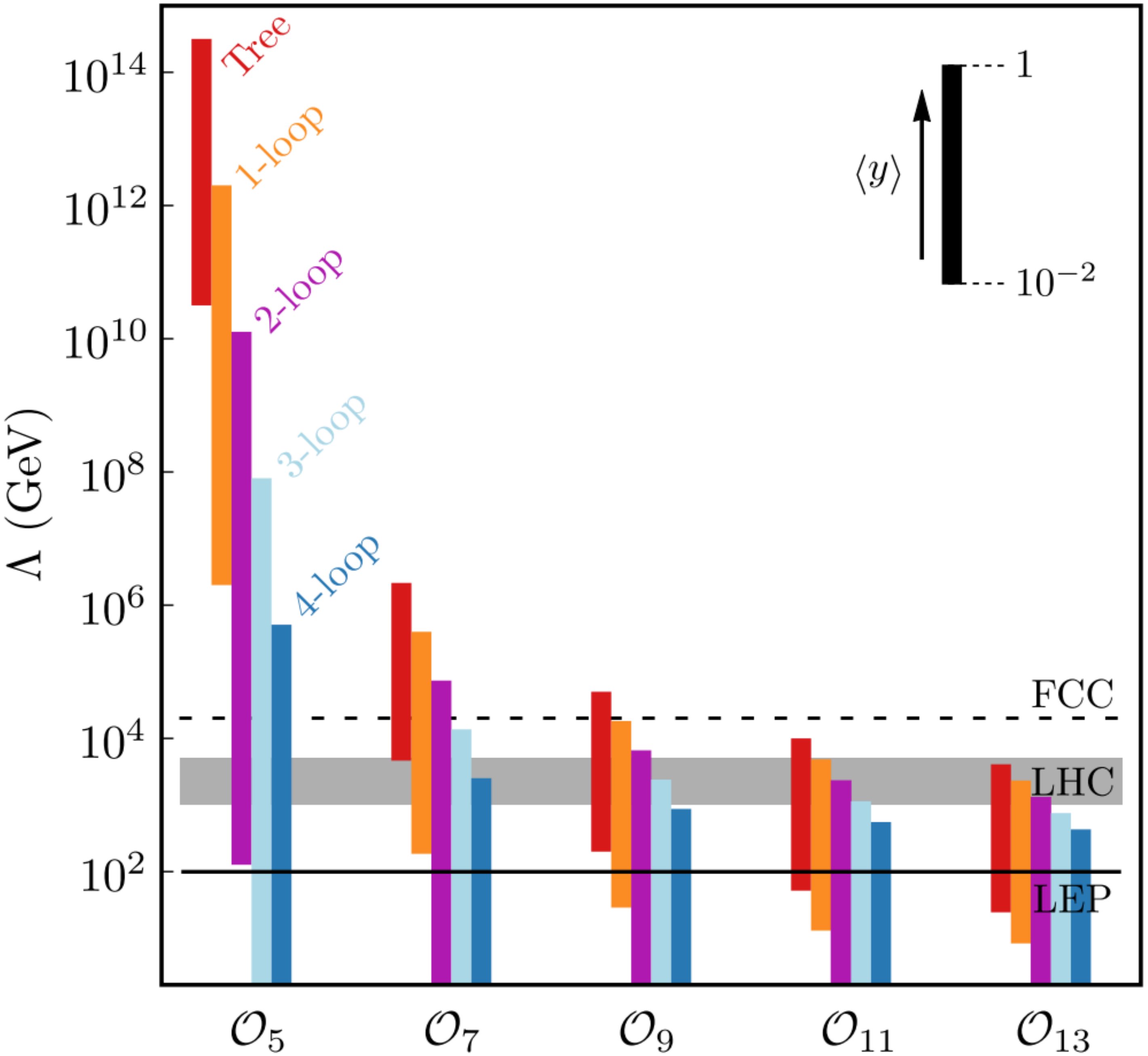

$$\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)$$

$\Sigma_R$: $SU(3)_c \times SU(2)_w \times U(1)_Y \sim (1,3,0)$

Foot, Lew, He, Joshi, 1989; Ma, 1998

- Naturally embedded in **Grand Unified Theories** (Type I)
- **Low Seesaw scale possible**: Type II, Type III, inverse seesaw, radiative mass generation, LR symmetric model, RPV, gauged U(1)' , .....

# Higher Dimensional Neutrino Masses



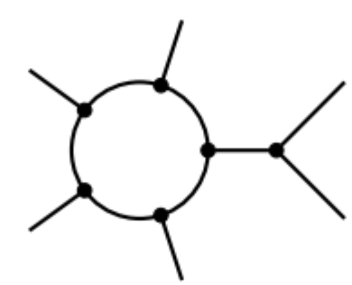

Anamiati, Castillo-Felisola, Fonseca, Helo, Hirsch (2019)

$$m_\nu \propto \epsilon \cdot \left(\frac{1}{16\pi^2}\right)^n \cdot \left(\frac{v}{\Lambda}\right)^{d-5} \cdot \frac{v^2}{\Lambda}.$$

Babu, Leung (2001); de Gouvea, Jenkins (2007); …..

e.g. at dim-7, 1-loop

$$O_1' = LLHH(H^\dagger H)$$

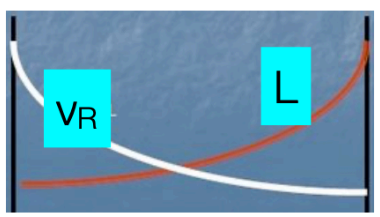


For an excellent review on Radiative Neutrino Mass Generation: Cai, Herrero-García, Schmidt, Vincente, Volkas, 1706.08524

# Small Masses - Dirac Neutrinos

## Randall-Sundrum warped extra dimensions

$$\psi_{(0)} \sim e^{(1/2-c)ky}$$




Grossman, Neubert (2000); Huber, Shafi (2001)

## Clockwork Seesaw Mechanism

S.C. Park, C.S. Shin (2017); Hong, Kurup, Perelstein (2019); Babu, Saad (2020) …

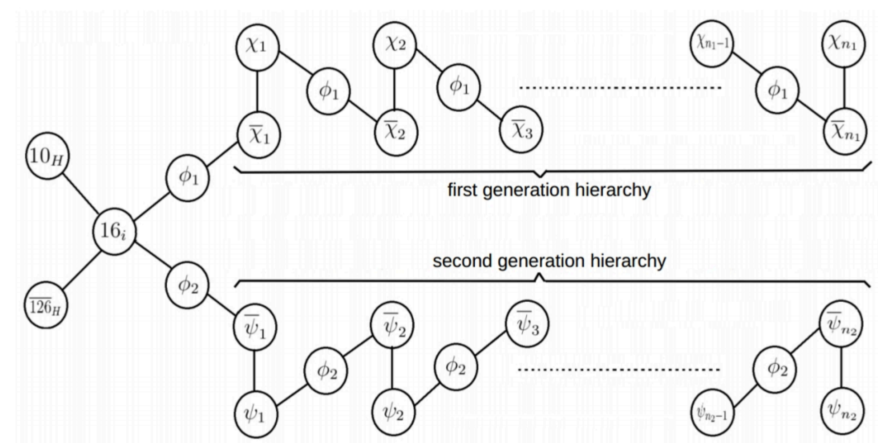
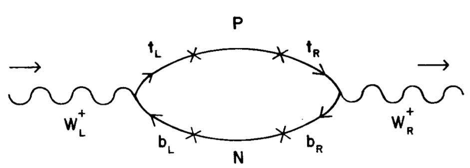



Figure from Babu, Saad (2020)

## Radiative Mass Generation



Cheng, Li (1978); …..

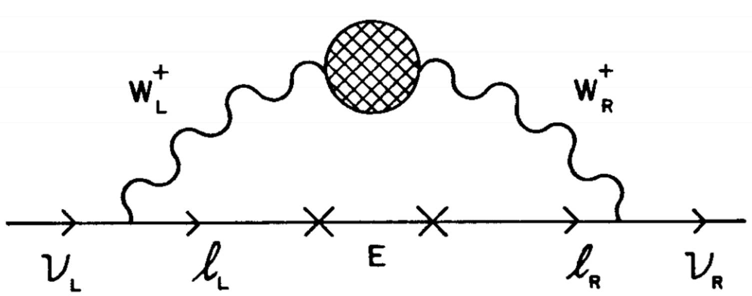
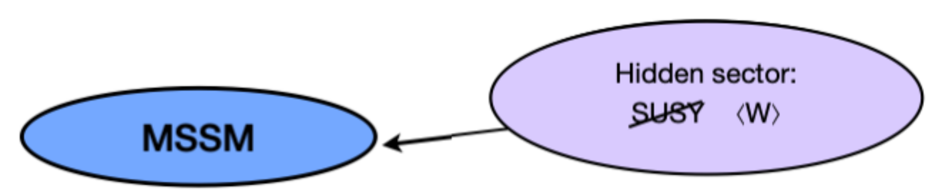



Figure from Babu, He (1988);

For clarifications of radiative Dirac neutrino mass generation: see e.g. Farzan, Pascoli, Schmidt (2012)

## SUSY Breaking

Arkani-Hamed, Hall, Murayama, Tucker-Smith, Weiner (2001)



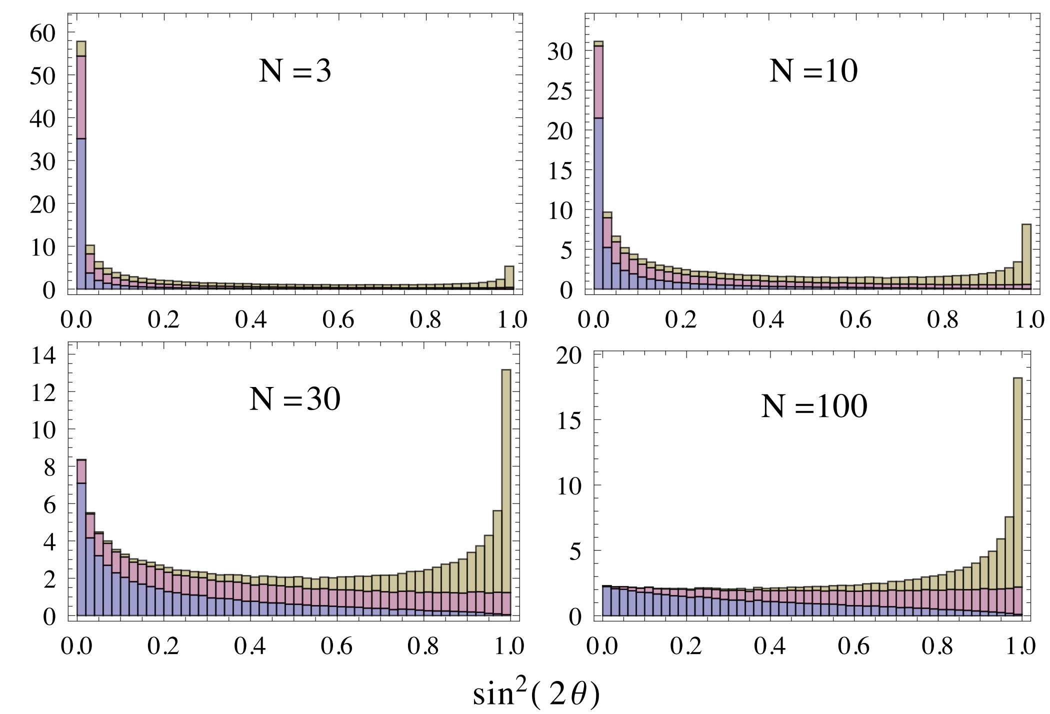

$$Y_\nu \sim \frac{m_{3/2}}{M_P} \sim \frac{\mu}{M_P}$$

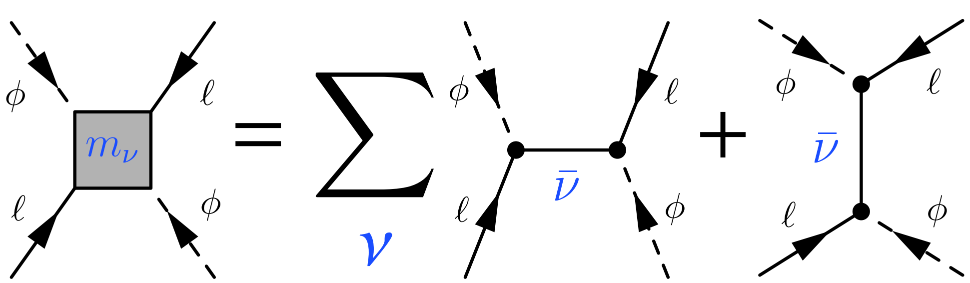
# Flavor Structure - Anarchy

- there are no parametrically small numbers
  Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003);
- large mixing angle, near mass degeneracy statistically preferred
- UV theory prediction can resemble anarchy
  - warped extra dimensions
  - heterotic string models: O(100) RH neutrinos
    Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez, Ratz (2007)

- statistical expectations with large N ( = # of RH neutrinos)

Feldstein, Klemm (2012)



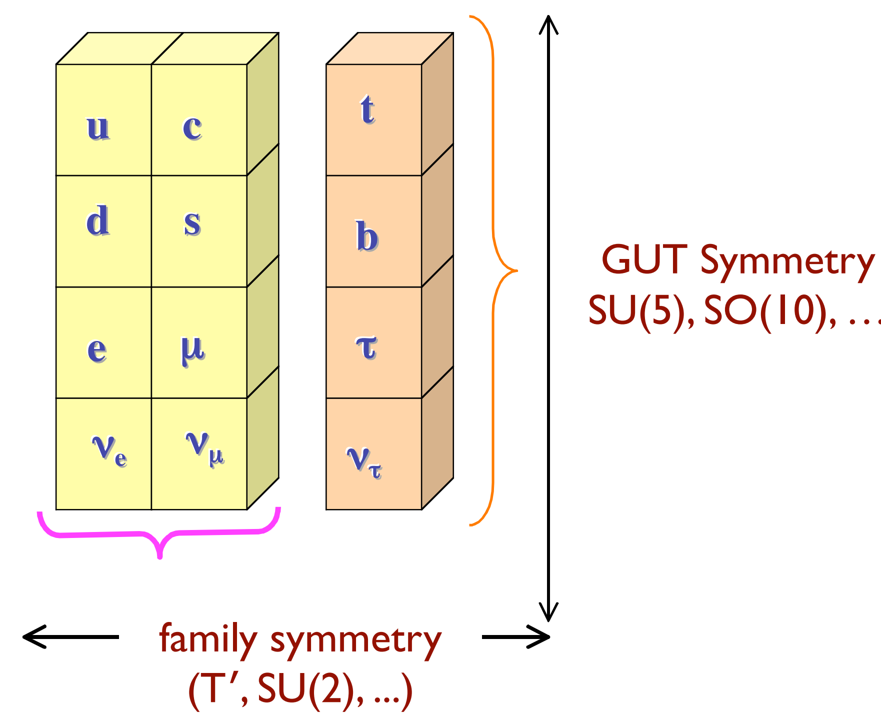



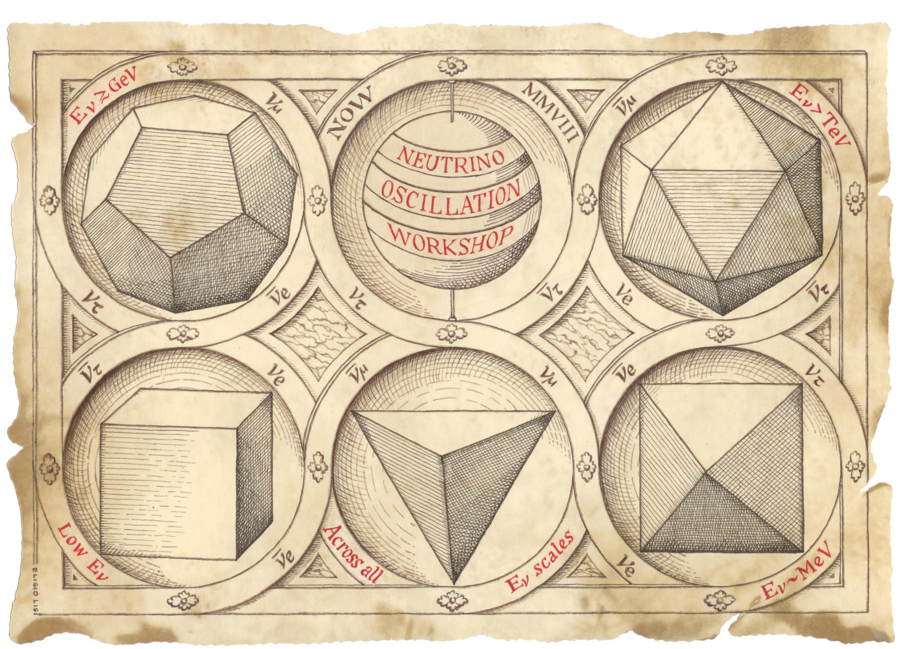

$$m_\nu \sim \frac{v^2}{M_*}$$

$$M_* \sim \frac{M_{\mathrm{GUT}}}{10 \ldots 100}$$

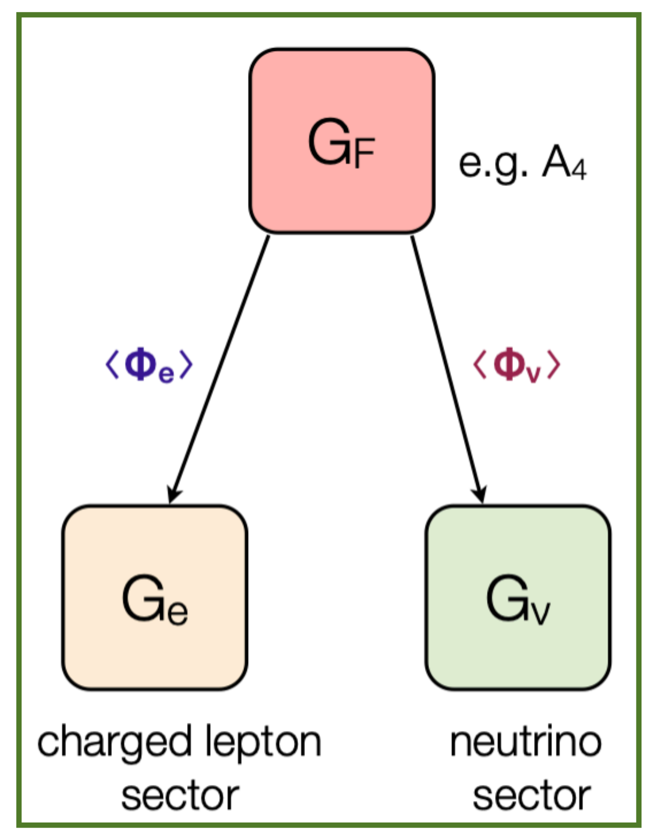
# Non-Abelian Discrete Flavor Symmetries

- models based on discrete family symmetry groups have been constructed
  - $A_4$ (tetrahedron)
  - T´ (double tetrahedron)
  - $S_3$ (equilateral triangle)
  - $S_4$ (octahedron, cube)
  - $A_5$ (icosahedron, dodecahedron)
  - $\Delta_{27}$
  - $Q_6$

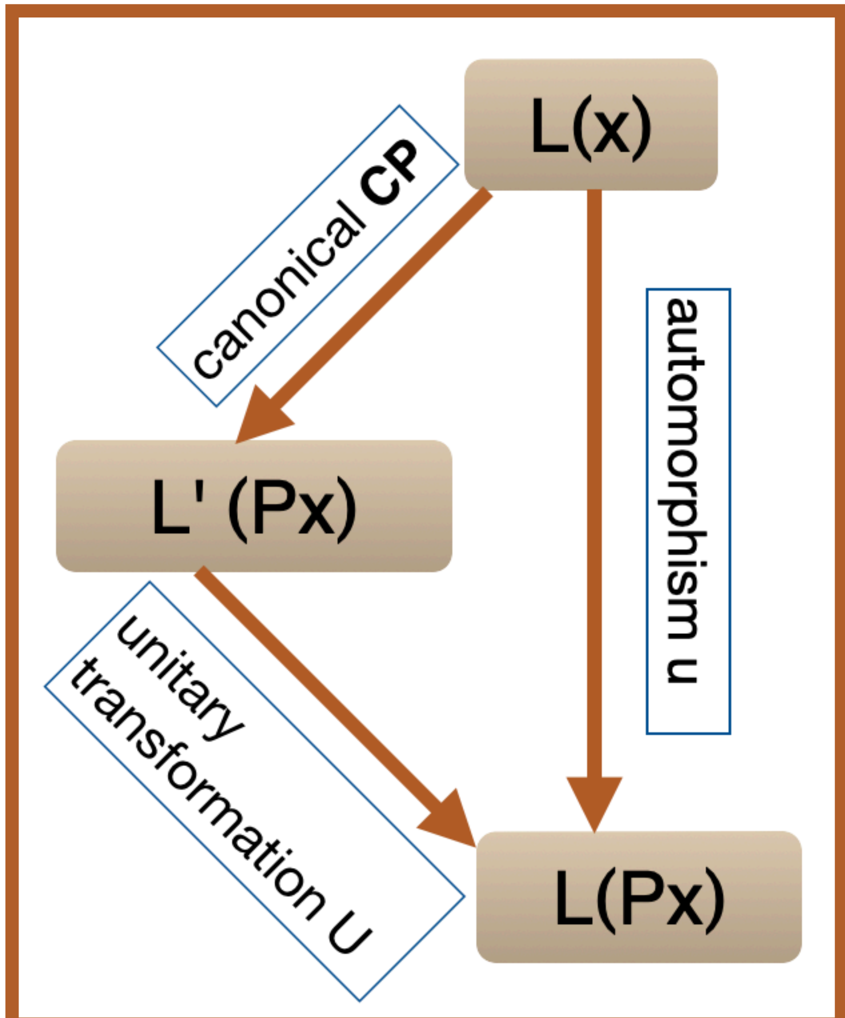



[Eligio Lisi for NOW2008 ]

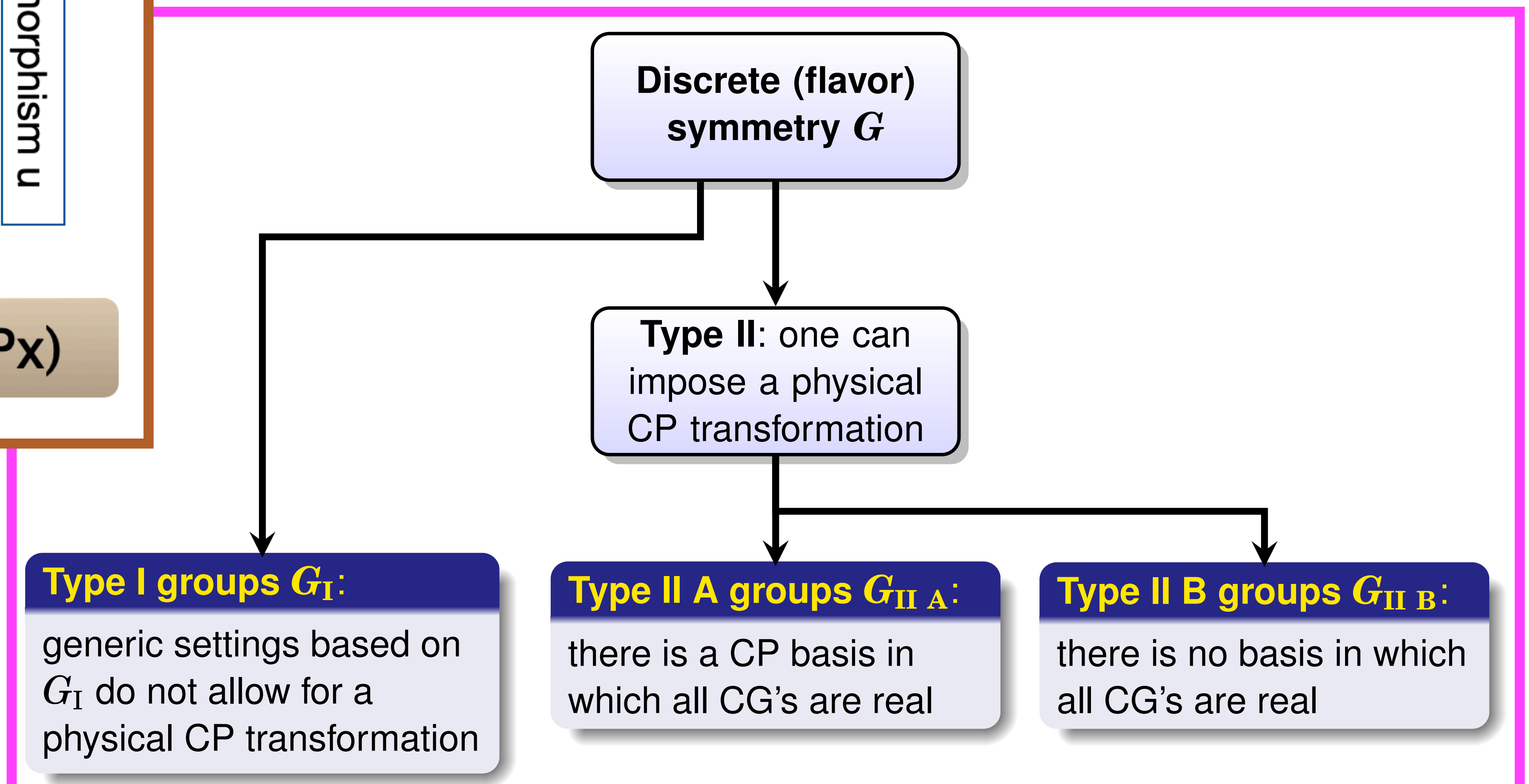


# Group Theoretical Origin of CP Violation



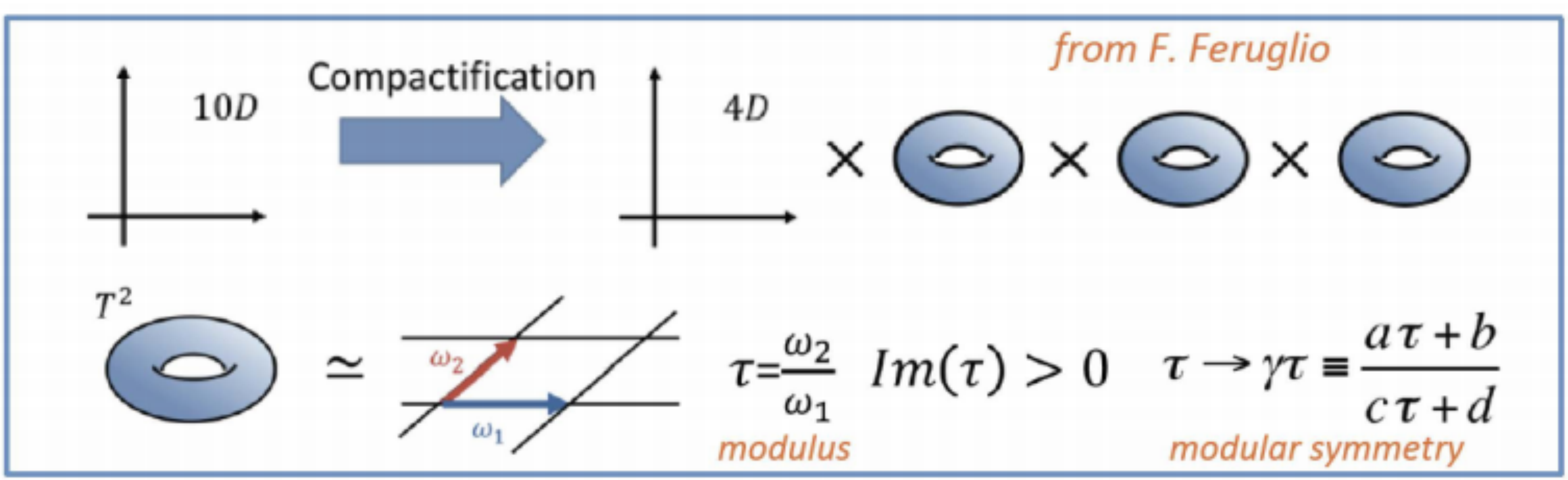

**complex CGs ⇨ G and physical CP transformations do not commute**

M-CC, Mahanthappa (2009); M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)



# Modular Flavor Symmetries

- Extra dimensional origin of non-Abelian discrete symmetries
- Modular symmetry    Altarelli, Feruglio (2005); Feruglio (2017), .....
  - Inspired by string theories
  - Imposing modular invariance $Y = Y(\tau)$
  - Highly predictive models (relatively)



# A Toy Modular $A_4$ Model

Feruglio (2017)

- Weinberg Operator $\mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\ Y\ (H_u \cdot L)]_1$
- Traditional A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Flavon VEVs ($A_4$ triplet, 6 real parameters)

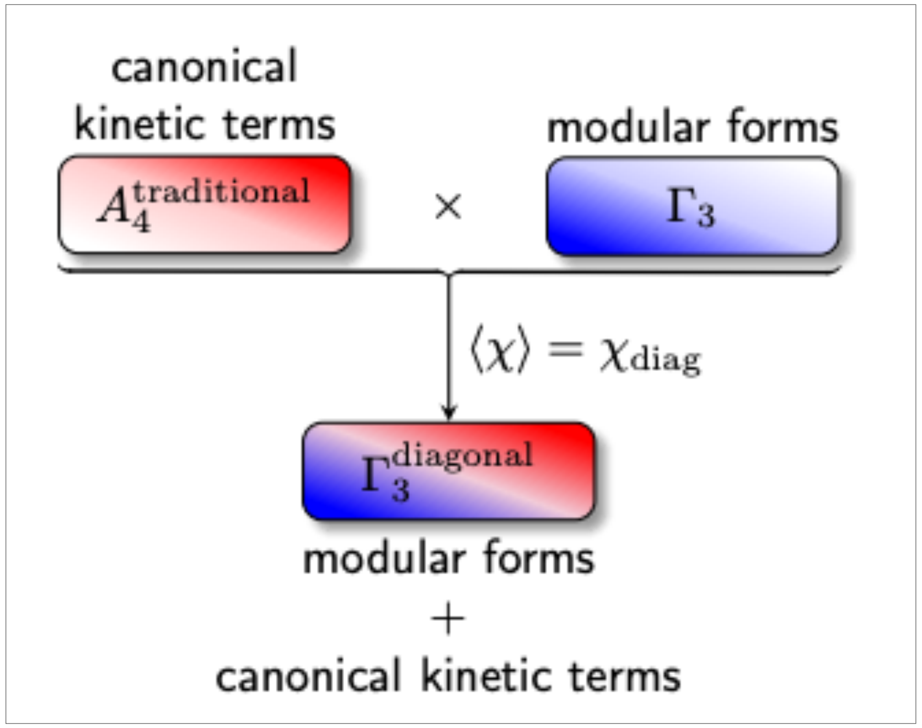
$$Y \rightarrow \langle \phi \rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Longrightarrow m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix}$$

- Modular A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$Y \rightarrow \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \Longrightarrow m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

# Modular Symmetry: Bottom-Up Meet Top-Down

- Bottom-Up:
  - reducing the number of parameters: in extreme case, entire neutrino mass matrix controlled by $\tau$ Feruglio (2017)
  - Traditional flavor symmetries: corrections to kinetic terms generally sizable Leurer, Nir, Seiberg ('93); Dudas, Pokorski, Savoy ('95); M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
  - corrections to kinetic terms can be under control MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)




- Top-Down:
  - Modular flavor symmetries from strings e.g. Baur, Nilles, Trautner, Vaudrevange
  - Modular Symmetries from magnetized tori e.g. Almumin, MCC, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

# Modular Invariance Beyond Neutrino Flavor

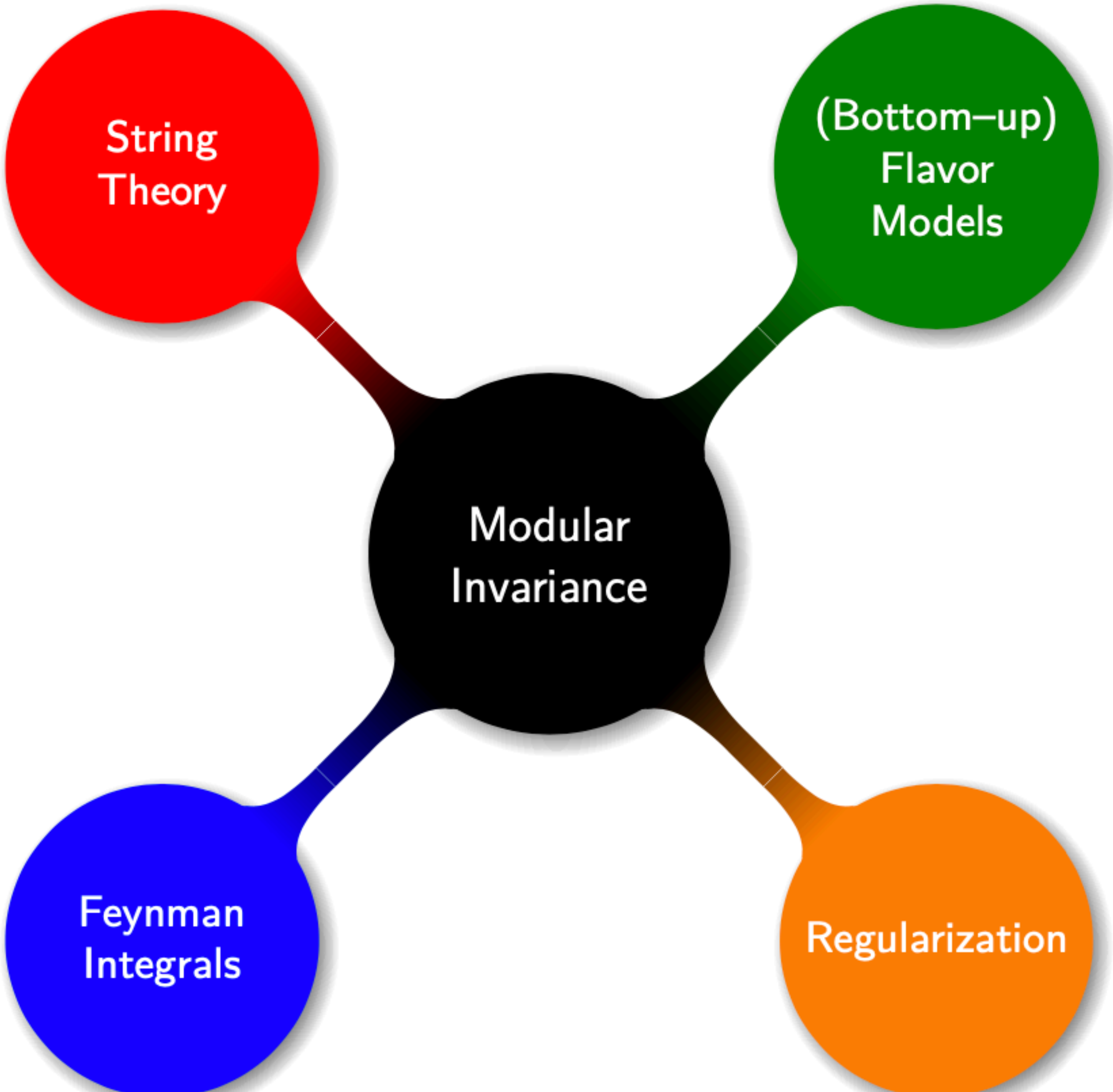
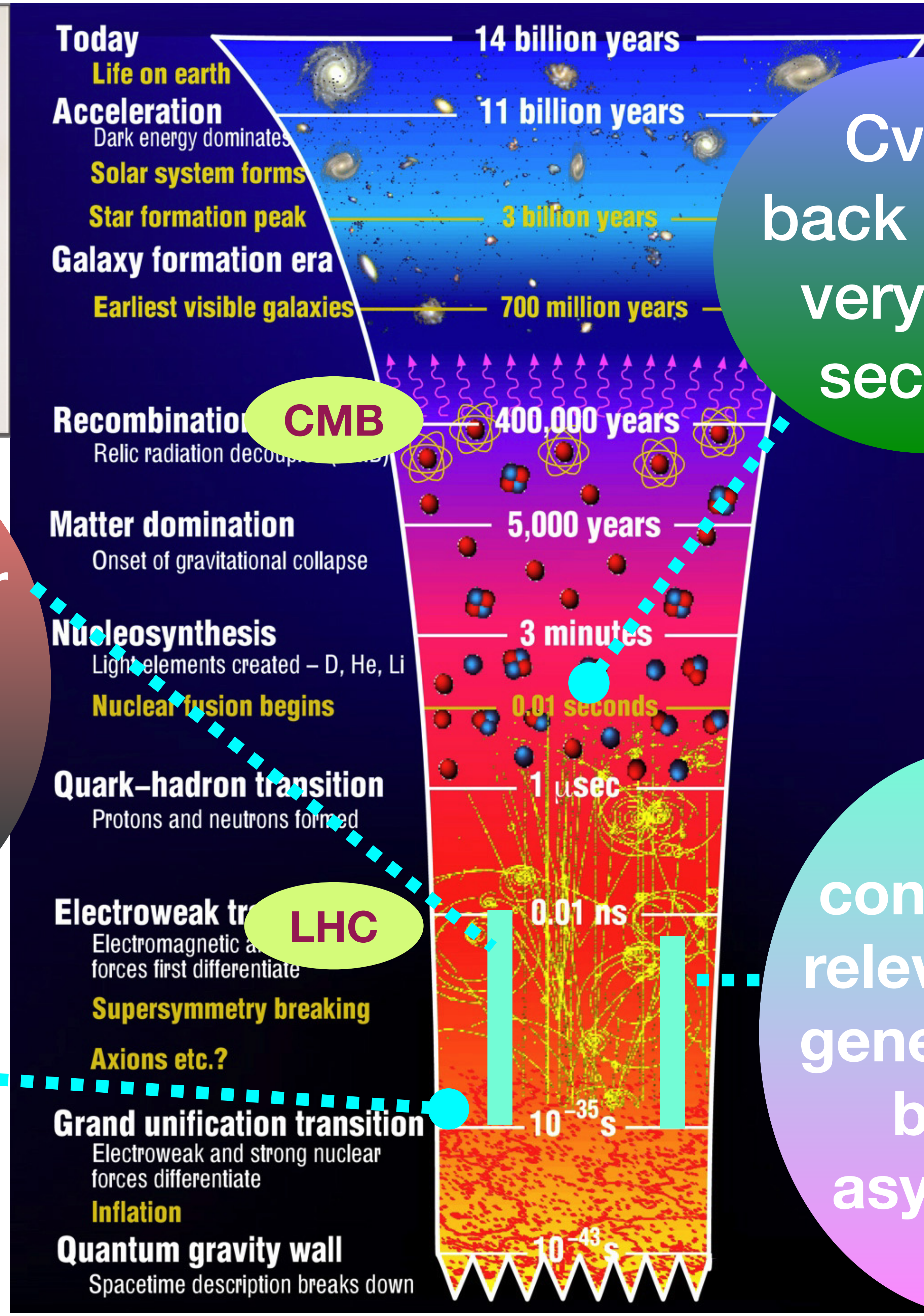



Figure credit: Michael Ratz




CvB - back to the very first second

operator for ν mass generation unknown

unique window into GUT scale physics

conceivable relevance for generation of baryon asymmetry

# Outlook

- Fundamental origin of fermion mass & mixing patterns still unknown
  - It took decades to understand the gauge sector of SM
- Uniqueness of Neutrino masses offers exciting opportunities to explore BSM Physics
  - Many NP frameworks; addressing other puzzles
  - Early Universe (leptogenesis, non-thermal relic neutrinos)
- New Tools/insights: examples of pheno relevance of formal theories
  - Non-Abelian Discrete Flavor Symmetries
    - Deep connection between outer automorphisms and CP
  - Modular Flavor Symmetries
    - Enhanced productivity of flavor models
    - Possible connection to string theories
- **Diverse perspectives/approaches drive intellectual excellence**