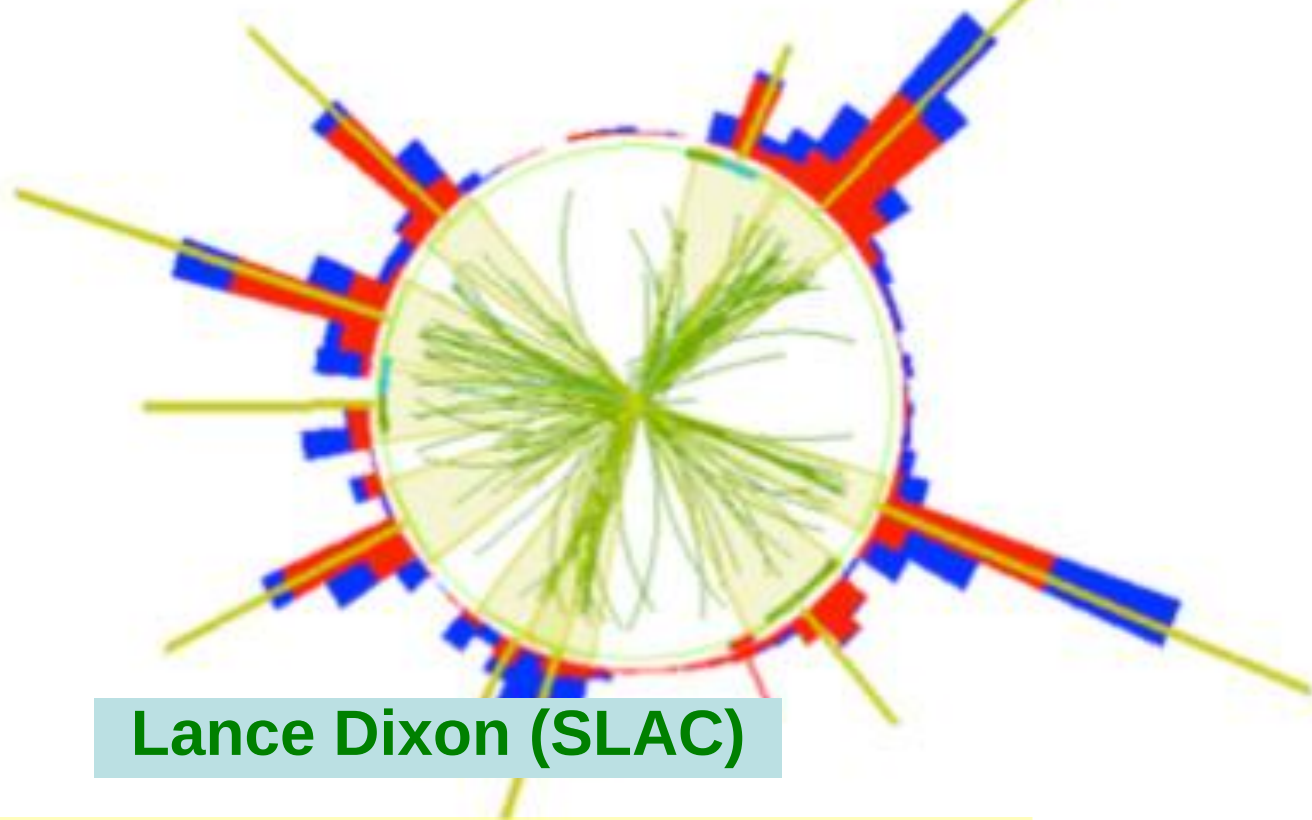
Collider Physics and Amplitudes



Lance Dixon (SLAC)

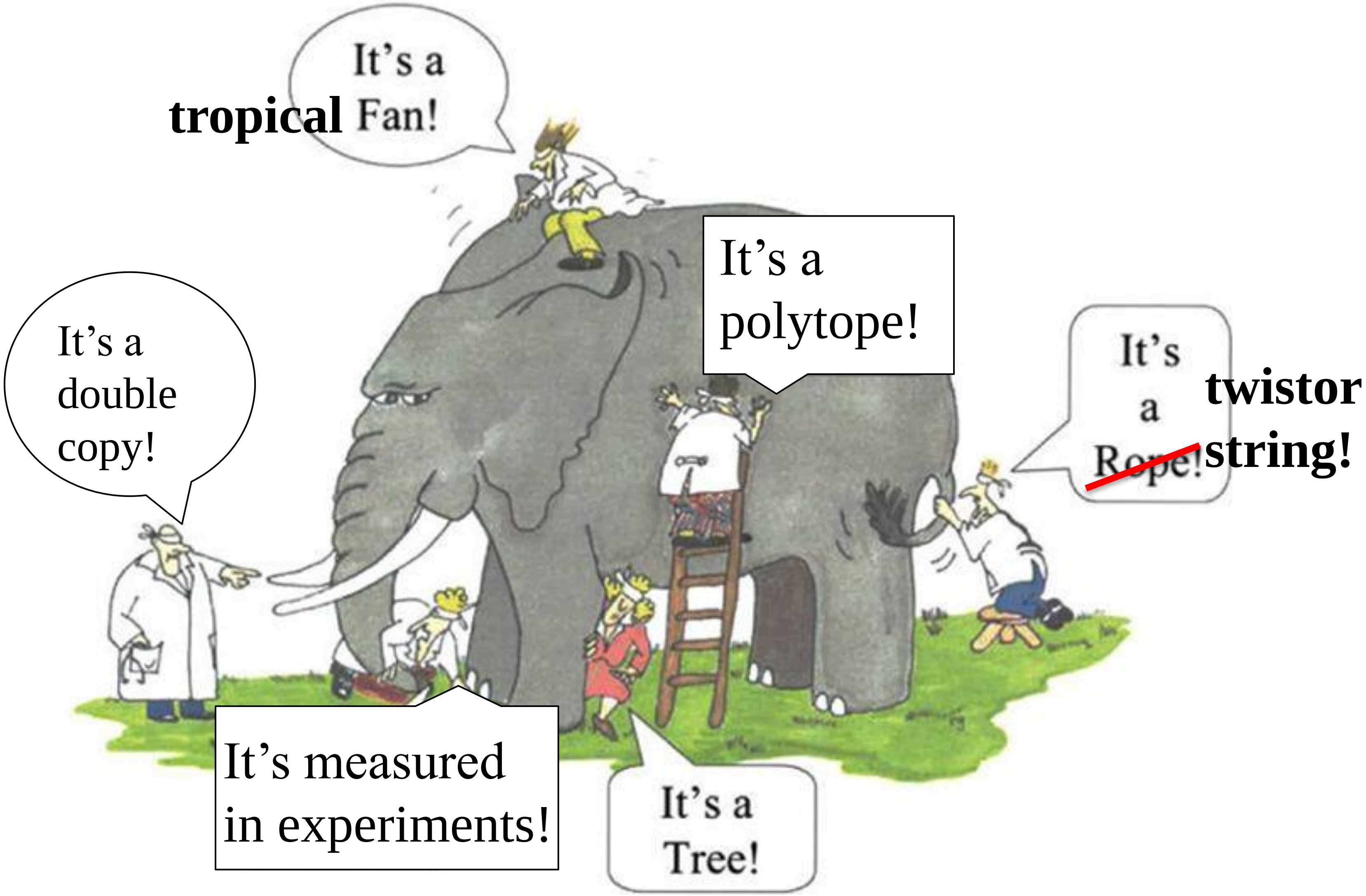
Snowmass Theory Frontier Conference
Kavli Institute for Theoretical Physics
February 25, 2022

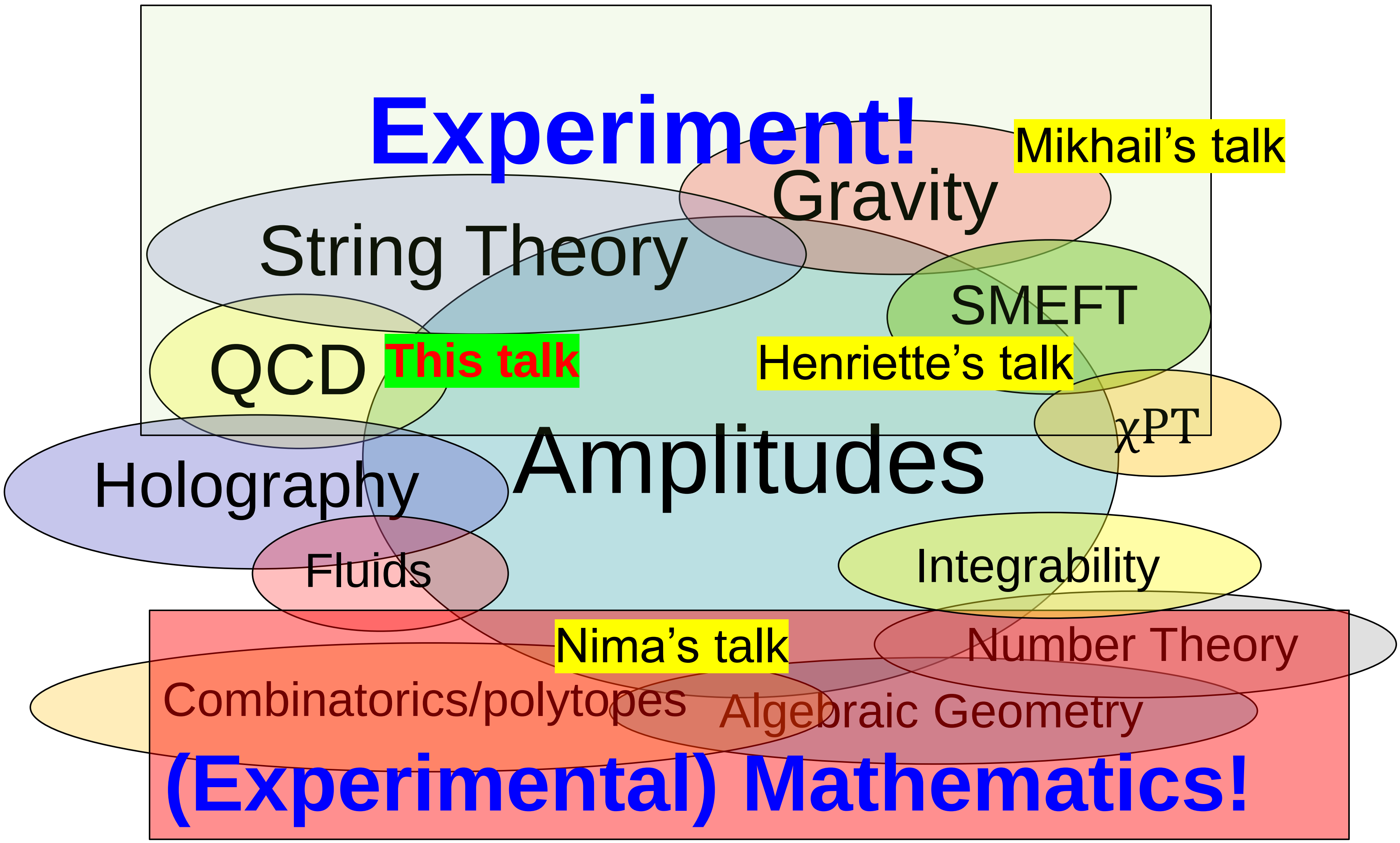
SLAC NATIONAL ACCELERATOR LABORATORY

Apologies in advance

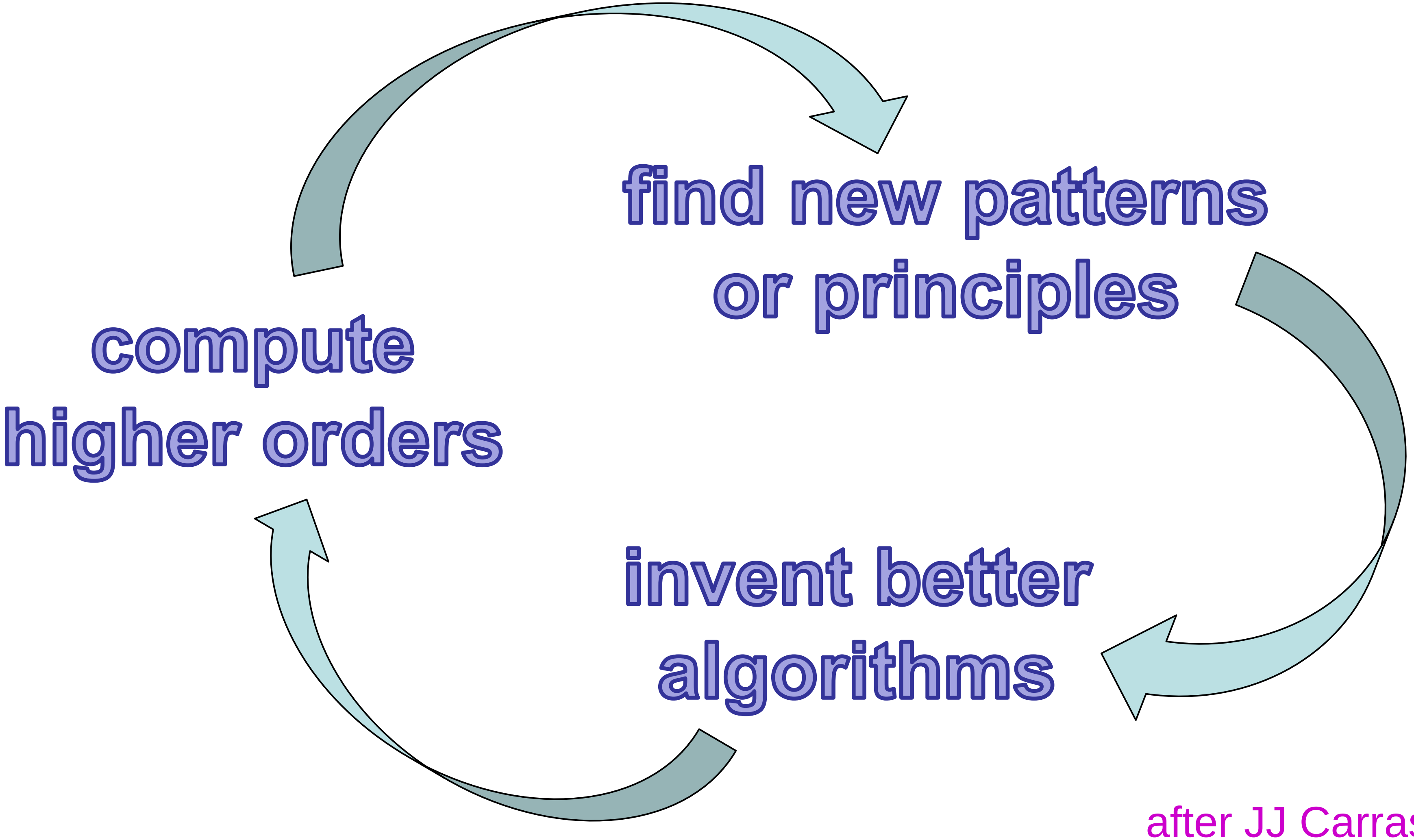
This talk will be very idiosyncratic, and is not a comprehensive review of all the impressive activity at the collider physics / amplitudes interface!

What is an amplitude?




Experiment!
Mikhail's talk
Gravity
String Theory
SMEFT
QCD
This talk
Henriette's talk
χPT
Amplitudes
Holography
Fluids
Integrability
Nima's talk
Number Theory
Combinatorics/polytopes
Algebraic Geometry
(Experimental) Mathematics!

Amplitudes virtuous circle

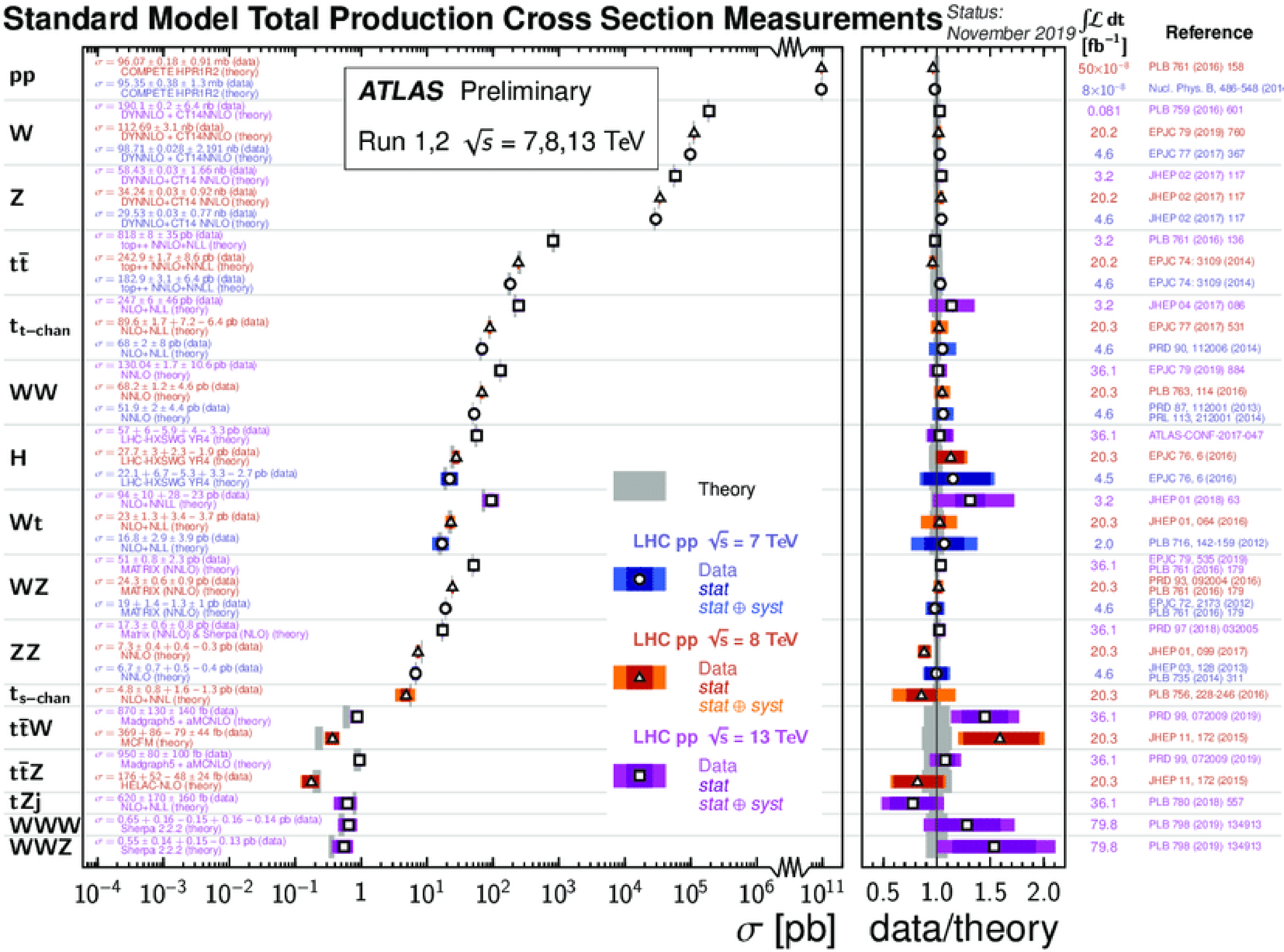


after JJ Carrasco

Challenges of precision collider theory

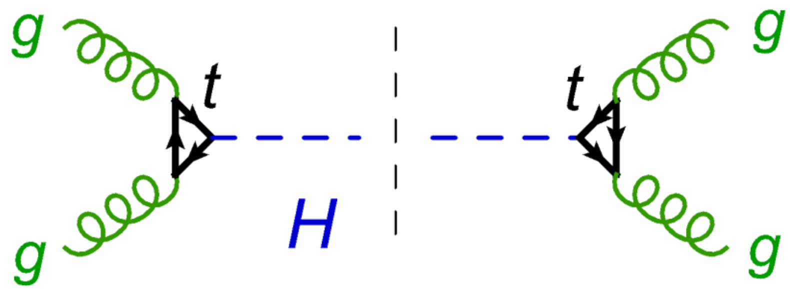
- Many processes, measured at a wide range of experimental precision
- Harder theoretically: higher loop order, higher multiplicity, massive internal lines, more partons/jets in final state
- Total cross sections for $H, W, Z, \ldots$ easier than differential, especially beyond NLO, but never actually measured because of acceptance cuts
- Resummation needed in corners of phase space, e.g. soft gluon (restricted) emission
- Some experimental cuts very stringent, e.g. b jet veto for WW in face of $t\bar{t}$ background
- Recent review Heinrich, 2009.00516

Standard Model Total Production Cross Section Measurements



Example: Total cross section for producing Higgs boson at LHC via gluon fusion

Leading Order (LO)



- Higgs production at LHC is dominantly via gluon fusion, mediated by top quark loop.
- Since $\boldsymbol{2m_{top}} = 350\ \mathrm{GeV}$
  $\gg \boldsymbol{m_{Higgs}} = 125\ \mathrm{GeV}$,

integrate out top quark to get a leading operator $H G^{a}_{\mu\nu} G^{\mu\nu\ a}$

Higgs gluon fusion cross section at LHC

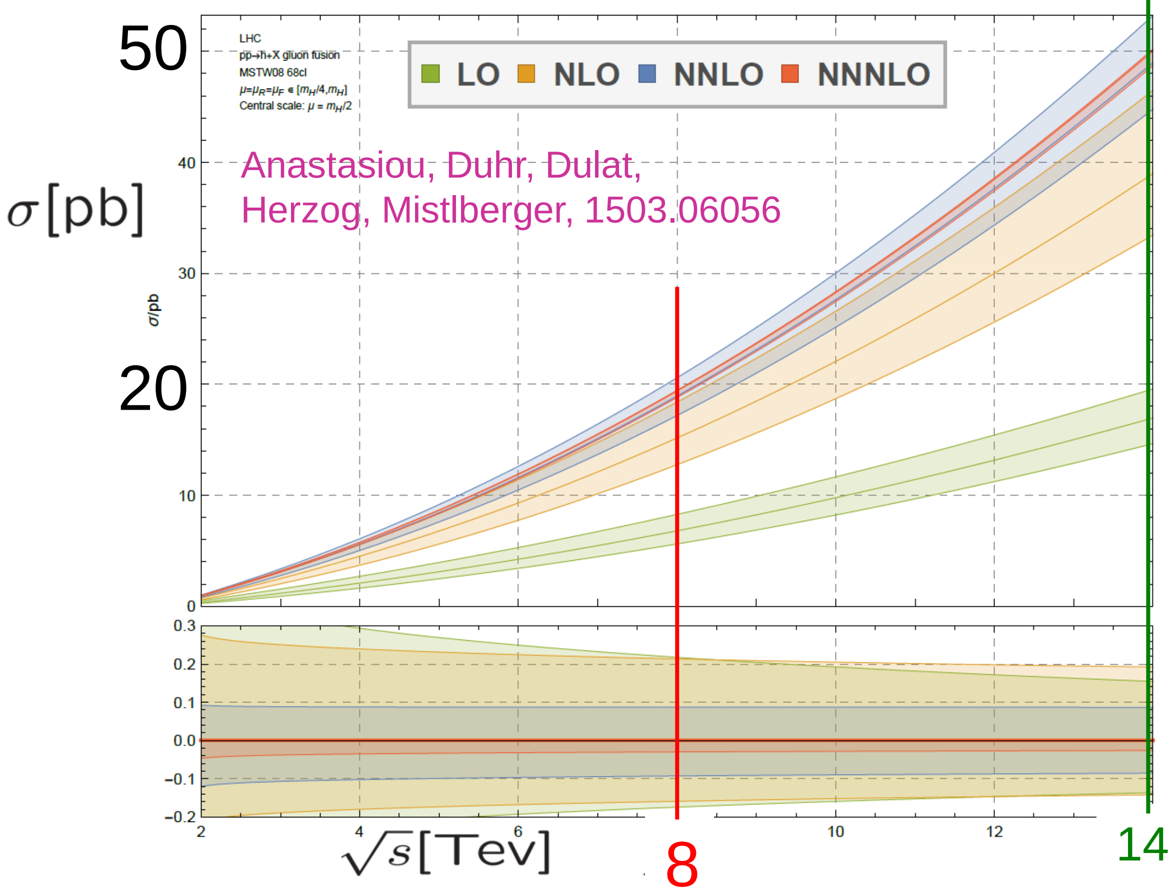
$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO

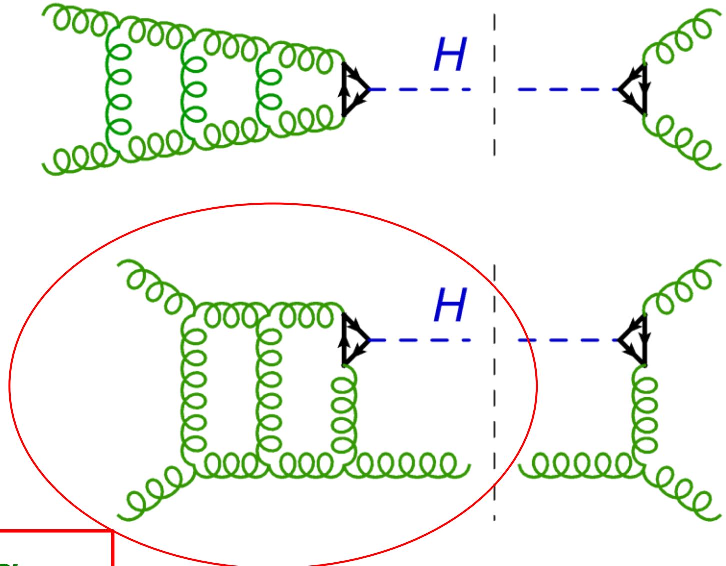
Mistlberger talk

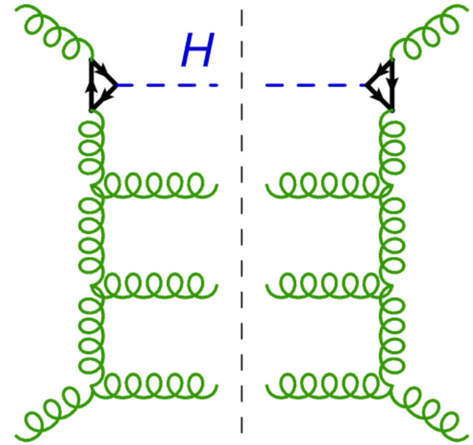
Poor convergence of expansion in $\alpha_s(\mu)$

LO → NNNLO
→ factor of 2-3 increase in cross section!

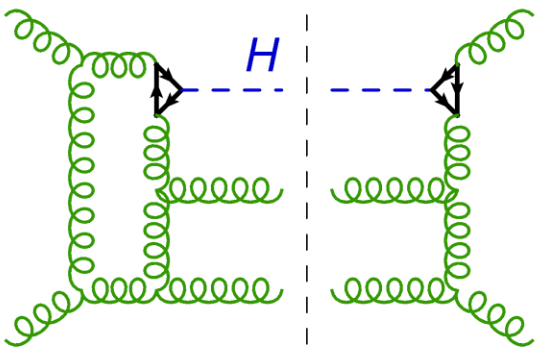


Anatomy of NNNLO QCD corrections





gg → *Hg*
@ 2 loops,
state of art
in QCD



+ …
+ quarks
+ operator renormalization
+ $1/m_t^2$ corrections
+ parton distributions

Scattering amplitudes are the underlying building blocks

Multi-loop complexities

- Multi-loop multiscale integrals typically very difficult to evaluate
- All 1 loop integrals reducible to scalar box integrals + simpler

→ combinations of

+ simpler

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1-t)$$

Brown-Feynman (1952), Melrose (1965), 't Hooft-Veltman (1974), Passarino-Veltman (1979), van Neerven-Vermaseren (1984), Bern, LD, Kosower (1992)

- At L loops, special functions with up to $2L$ integrations

  Hannesdottir, McLeod, Schwartz, Vergu, 2109.09744

- Weight $2L$ iterated integrals, multiple polylogarithms, or worse

Planar N=4 SYM as testing ground for QCD

- QCD's maximally supersymmetric cousin, N=4 super-Yang-Mills theory (SYM), gauge group $\mathrm{SU}(N_c)$, in large N_c (planar) limit, where it becomes **integrable**
- Structure very rigid:

$$\text{Amplitudes} = \sum_i rational_i \times transcendental_i$$

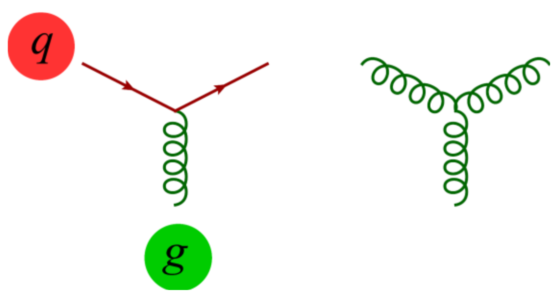
- For planar N=4 SYM, we understand rational structure quite well, can focus on the transcendental functions.
- Multiple dualities hold (virtuous circle):

1. AdS/CFT (weak-strong)
2. Amplitudes dual to Wilson loops (dual conformal symmetry)
3. New "antipodal" duality between amplitudes and form factors

QCD vs. N=4 SYM

- QCD: gluons and quarks in fundamental rep. of $SU(N_c)$
- N=4: Replace quarks with 4 copies of fermions in adjoint rep. (gluinos) and add 6 real adjoint scalars
- All in same supermultiplet
- Feynman vertices:

QCD

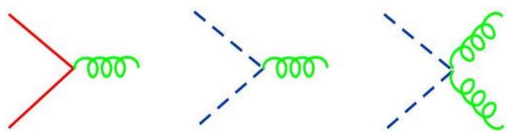


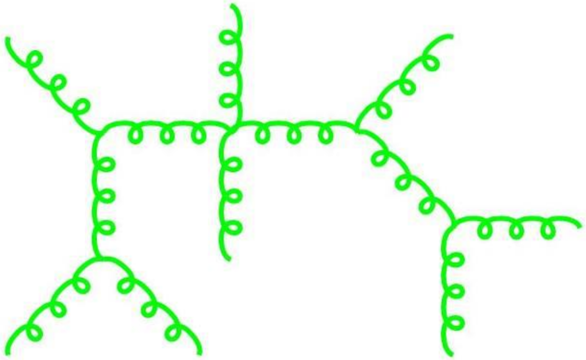
N=4 SYM

QCD vs. N=4 SYM at tree level

Essentially identical

Fermions and scalars cannot appear in tree amplitude for n gluons because produced in pairs





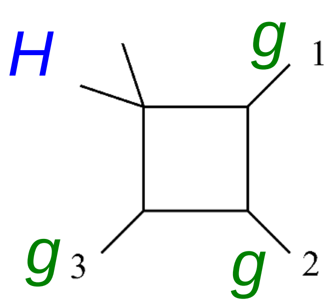
Hence amplitude is same in QCD and N=4 SYM;
QCD tree amplitude obeys all identities of N=4 supersymmetry:

No longer true at quantum (loop) level

N=4 SYM special at loop level

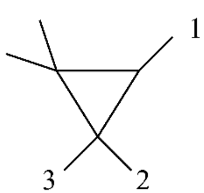
- At one loop, loop momenta cancel in numerator
  → only scalar box integrals

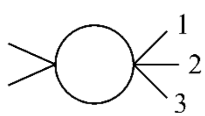
→ Weight 2 functions – dilogs. E.g., *gg* → *Hg* @ 1 loop ⊃



$$= \mathrm{Li}_2\left(1-\frac{s_{123}}{s_{12}}\right) + \mathrm{Li}_2\left(1-\frac{s_{123}}{s_{23}}\right) + \frac{1}{2}\ln^2\left(\frac{s_{12}}{s_{23}}\right) + \cdots$$

- QCD results also contain single log's and rational parts from (tensor) triangle + bubble integrals





$$= \frac{1}{\epsilon} - \ln(s_{123})$$

Higher loops

- N=4 SYM amplitudes have "uniform weight (transcendentality)" $2L$ for finite (ϵ^0) terms at loop order L
- Weight ~ number of integrations, e.g.

$$\ln(s) = \int_1^s \frac{dt}{t} = \int_1^s d\ln t \qquad 1$$

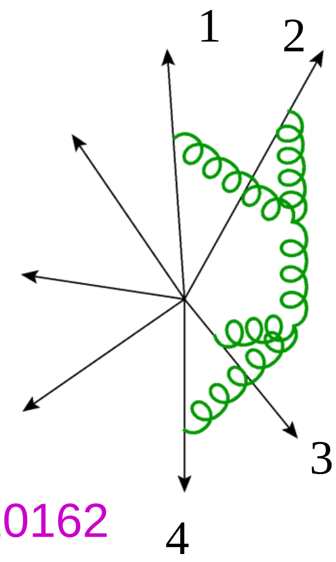
$$\mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1-t) = \int_0^x d\ln t \cdot [-\ln(1-t)] \qquad 2$$

$$\mathrm{Li}_n(x) = \int_0^x \frac{dt}{t} \mathrm{Li}_{n-1}(t) \qquad n$$

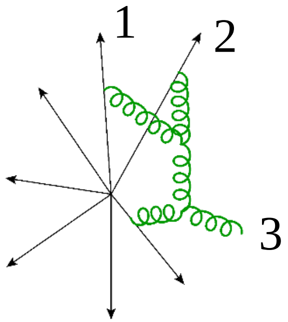
- QCD amps typically all weights from 0 to $2L$

N=4 / QCD relations at loop level

- Some infrared quantities are gluon-dominated, and therefore are exactly the same in QCD as in (nonplanar) N=4.
- Example: 3-loop soft anomalous dimension controls $\frac{1}{\epsilon}$ pole with most complicated color structure ("quadrupole") in QCD amplitudes
  Almelid, Duhr, Gardi, 1507.00047
- Uniform weight 5 function, inherited from N=4
- Bootstrappable Almelid, Duhr, Gardi, McLeod, White, 1706.10162



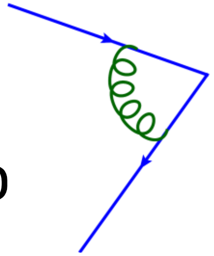
- Similar story for "tripole" terms in 2-loop soft gluon emission
  LD, Herrmann, Yan, Zhu,1912.09370



Maximal transcendentality principle

Kotikov, Lipatov, Velizhanin, hep-ph/0301021, hep-ph/0611204

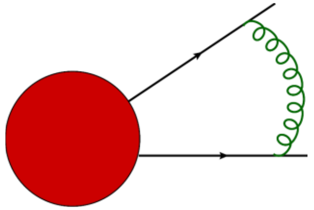
- Other quantities share a common "most complicated" piece between QCD and N=4 SYM.



- For example, light-like cusp anomalous dimension

$\Gamma_{\rm cusp}$ contributes to IR divergences in amplitudes and to soft-gluon resummation

- Known to all loop orders in planar N=4
  Beisert, Eden, Staudacher, hep-th/0610251



- QCD formula:

$$\frac{\Gamma_{{\rm cusp},R}}{4} = C_R \left\{ a\,[1] + \; a^2 \left[C_A \left(\frac{67}{9} - \frac{\pi^2}{3} \right) - \frac{10}{9} n_f \right] \right\} + \cdots$$

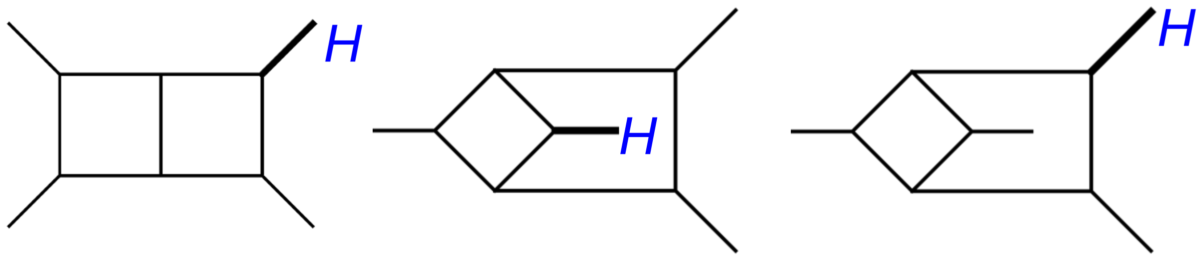
$$a = \frac{\alpha_s}{4\pi}$$

- Blue entries have "maximal transcendentality", predicted by N=4.

More loop level N=4 / QCD relations

- For massless QCD processes, integral topologies are the same
- Sometimes "amplitudes" studies reveal something about integrals also encountered in QCD.
- **Example:** *gg* → *Hg* @ 2 loops, state of art in QCD
- Integrals known 20 years ago

Gehrmann, Remiddi, hep-ph/0008287, hep-ph/0101124



+ …

- Going to higher loops in planar N=4, extra structure recently observed (adjacency restrictions on symbol letters) LD, McLeod, Wilhelm, 2012.12286; also Chicherin, Henn, Papathanasiou, 2012.12285
- Structure holds in all integrals needed for QCD, many more than needed for planar N=4. Fate at 3 loops?

Different routes to perturbative amplitudes

Route 1

Draw all Feynman graphs G_i

↓

Evaluate all Feynman rules: I_i

↓

Perform all loop integrations: A_i

↓

$$A = \sum_i A_i$$

QCD, few scales

Route 2

Evaluate all unitarity cuts C_α

↓

Construct **local** integrand I

↓

Perform all loop integrations: A_α

↓

$$A = \sum_\alpha A_\alpha$$

QCD, more scales, or N=4

Route 3

Bootstrap: Guess $A = \sum_m c_m F_m$
F_m known functions
$c_m \in \mathbb{Q}$ unknown constants

↓

Solve constraints, linear equations for $c_m \to r_m$

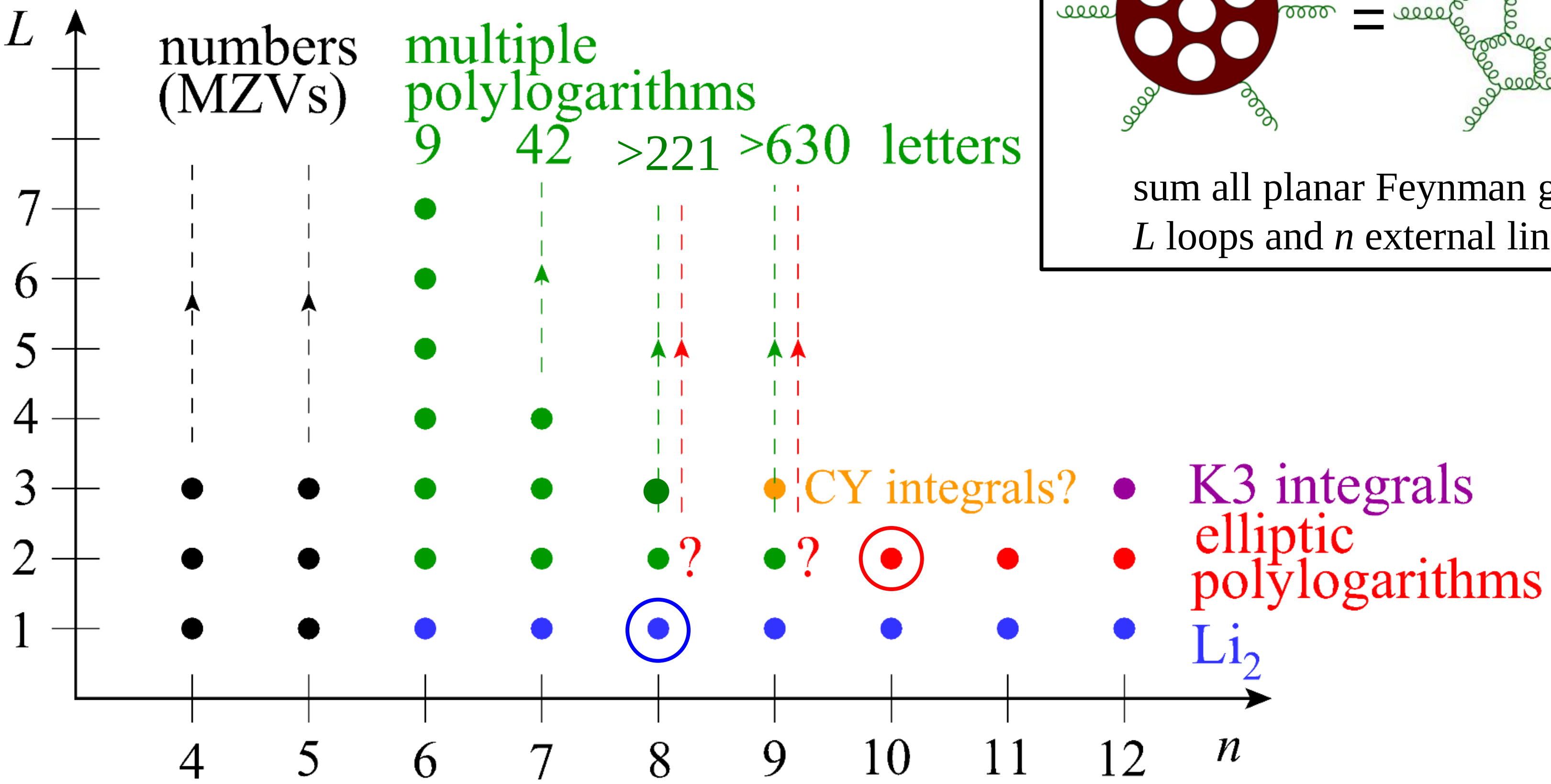
↓

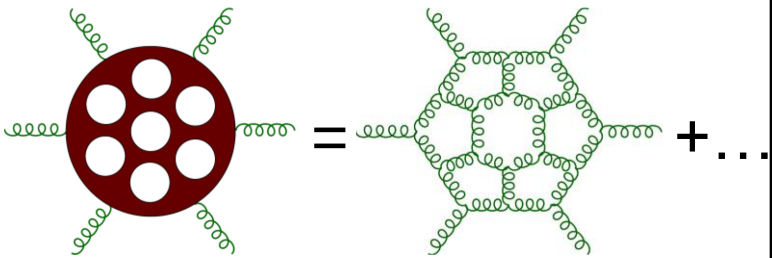
$$A = \sum_m r_m F_m$$

planar N=4 (mostly) so far

Functions entering n-point amplitudes in planar N=4 SYM

Much work has gone into this!





sum all planar Feynman graphs with L loops and n external lines

Multiple polylogarithms

Chen, Goncharov, Brown,…

- Iterated integrals, e.g.

$$G(a_1, a_2, \ldots, a_n, x) = \int_0^x \frac{dt}{t - a_1} G(a_2, \ldots, a_n, t)$$

- Or define differentially: $dF = \sum_{s_k \in \mathcal{S}} F^{s_k}\, d \ln s_k$

one weight lower

- $\boldsymbol{s_k}$ **are letters in the symbol alphabet** $\mathcal{S}$

Goncharov, Spradlin, Vergu, Volovich, 1006.5703

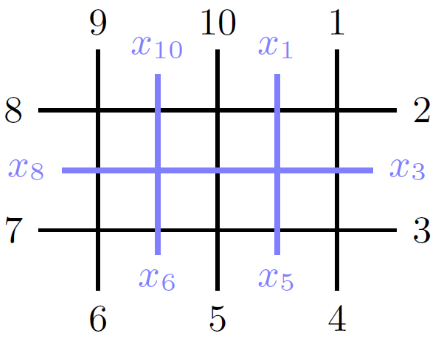
- Beyond polylogarithms → Elliptic polylogarithms

$$\sim \int_0^x \frac{dt}{\sqrt{(t-a_1)(t-a_2)(t-a_3)}} G(a_4, \ldots, a_n, t)$$

elliptic curve

Elliptic polylogarithms

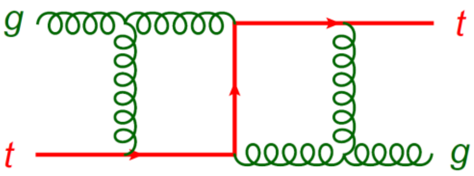
- Enter planar N=4 at $L = 2$, $n = 10$ via elliptic double box integral



Caron-Huot, Larsen 1205.0801;
Nandan, Paulos, Spradlin, Volovich, 1203.6362, 1301.2500;
Kristenson, Wilhelm, Zhang, 2106.14902

- Prototype for understanding multi-scale elliptic polylogs that enter much earlier when internal lines are **massive**, e.g.:

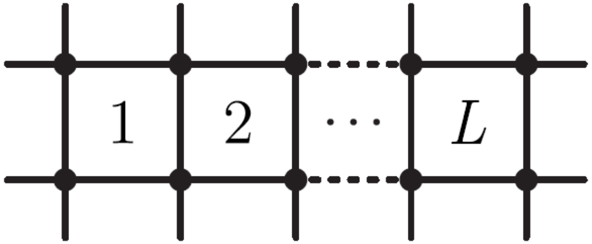
1. QCD corrections to **top quark** processes
2. electroweak corrections with virtual ***W,Z,t***

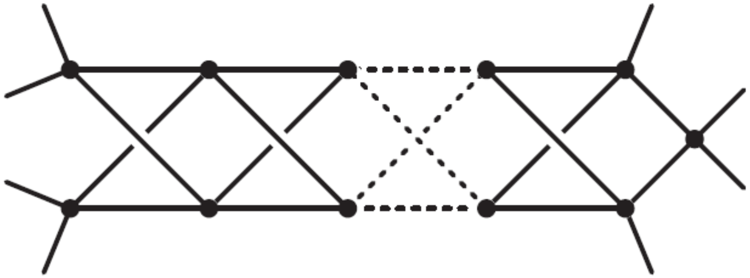


Beyond elliptic → K3, Calabi-Yau manifolds

Bourjaily, He, McLeod, Spradlin, Volk, von Hippel, Wilhelm, 2103.15423 and refs. therein

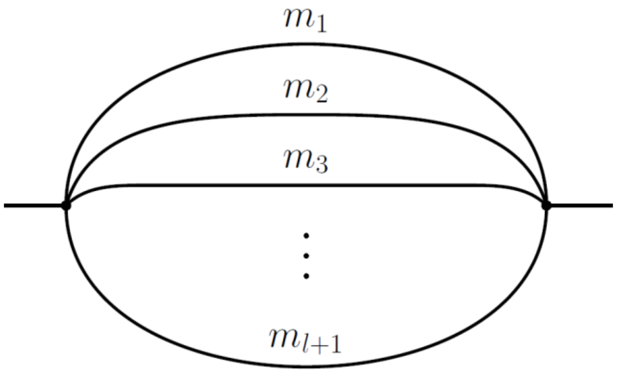
Massless examples: Train tracks, tardigrades, …



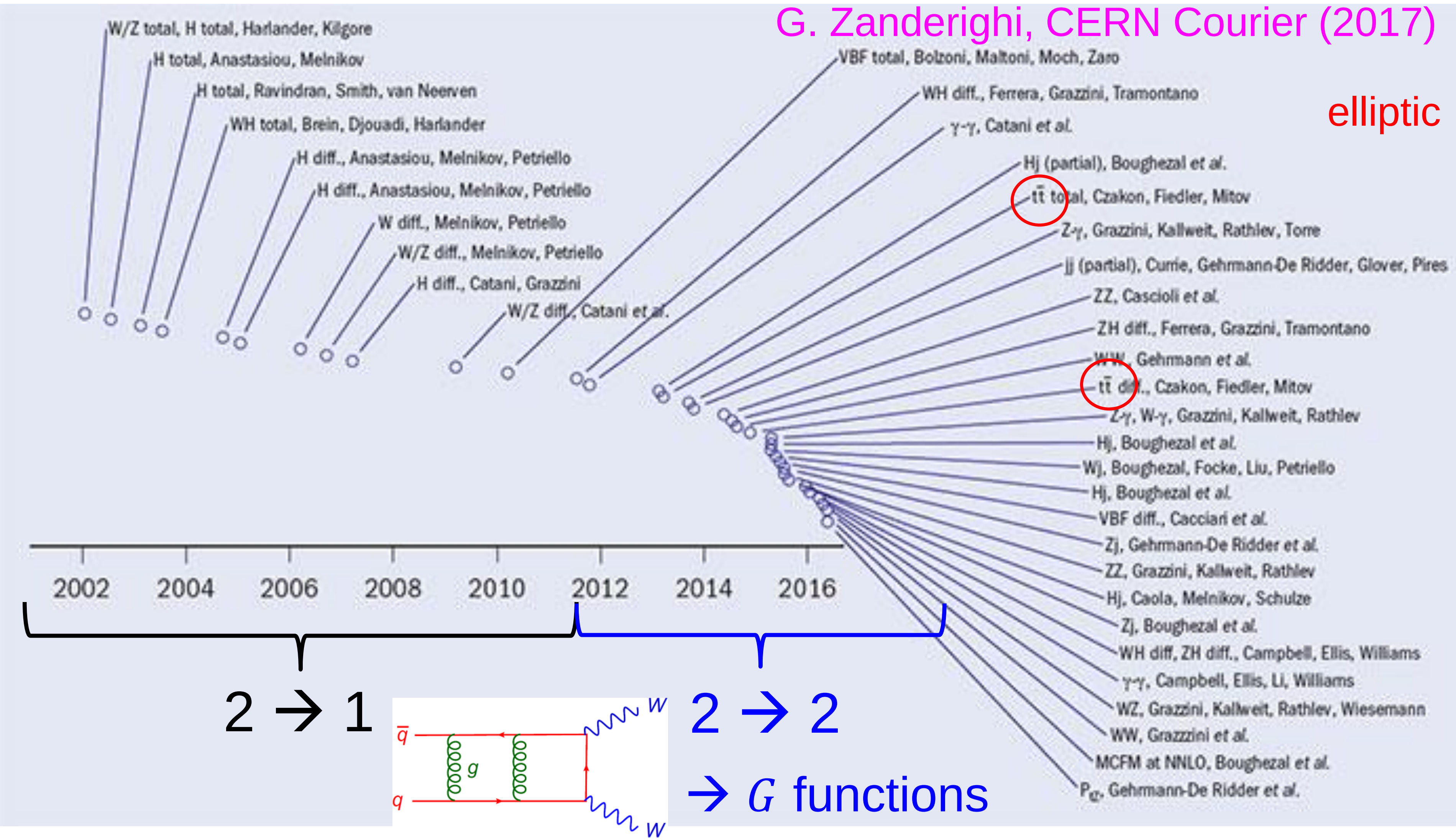


Bönisch, Duhr, Fischbach, Klemm, Nega, 2108.05310

Massive examples: Banana diagrams

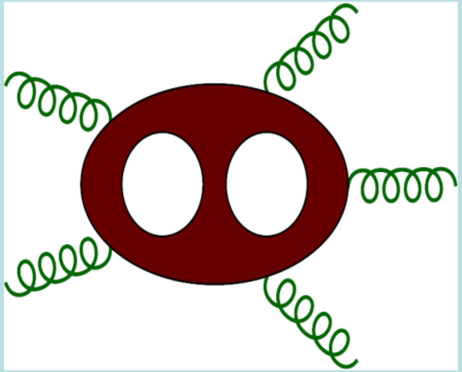


NNLO frontier circa 2017



Since 2017: First 2 loop 2 → 3 amplitudes

- All massless partons (or photons) in large N_c (planar) limit for QCD gauge group $SU(3) \to SU(N_c)$:



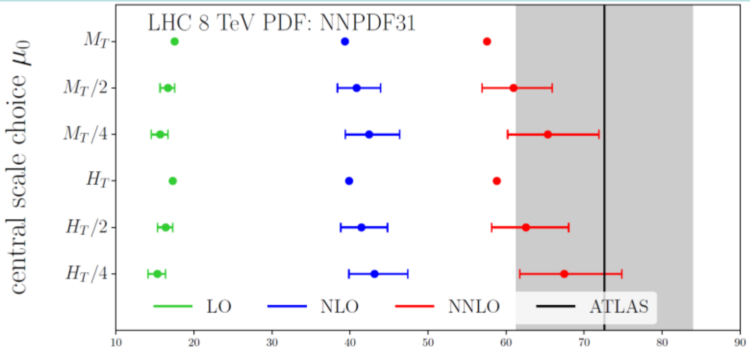
$gg \to ggg,\ qg \to qgg,\ q\bar{q} \to q\bar{q}g, \ldots$

Gehrmann, Henn, Lo Presti, 1511.05409;
Badger, Brønnum-Hansen, Hartanto, Peraro, 1712.02229, 1811.11699;
Abreu, Dormans, Febres Cordero, Ita, Page, Zeng, Sotnikov, 1712.03946, 1812.04586, 1904.00945, 2102.13609

- Last matrix elements needed for pp → 3 jets (up to $1/N_c^2$ corrections). Much use of unitarity. Still multiple polylogarithms.

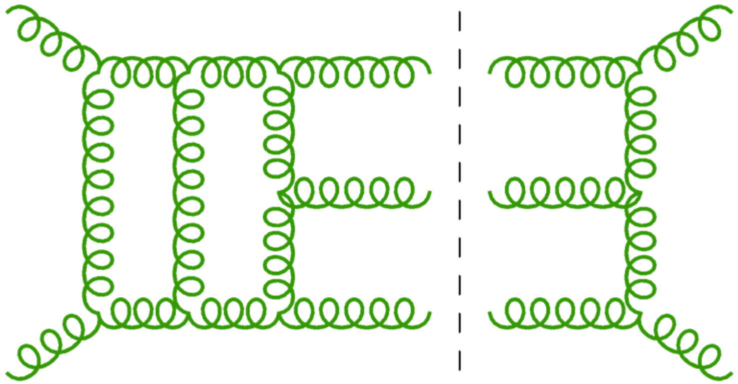
- And $q\bar{q} \to \gamma\gamma\gamma$ – already with NNLO cross section for pp → $\gamma\gamma\gamma$

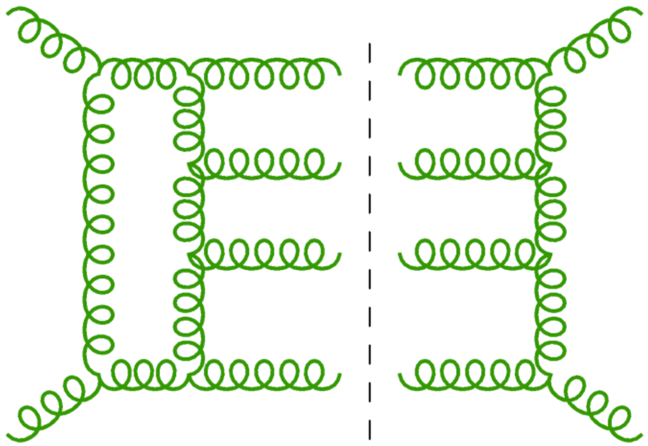
Chawdhry, Czakon, Mitov, Poncelet, 1911.00479;
Abreu, Page, Pascual, Sotnikov, 2010.15834;…

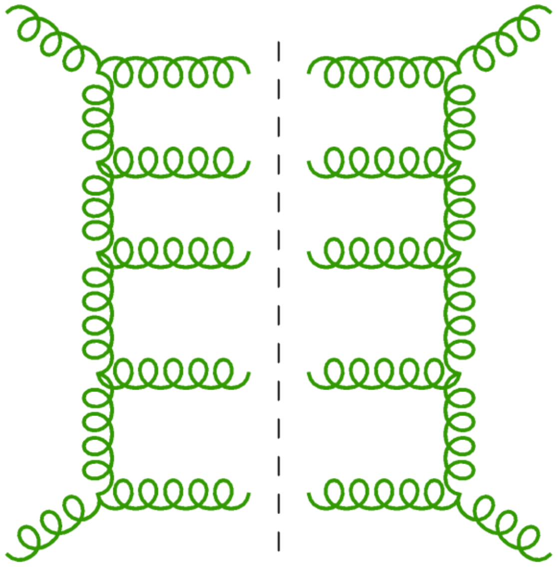


- More recently: one massive leg (Wjj), & massless nonplanar

Anatomy of NNLO pp → 3 jets

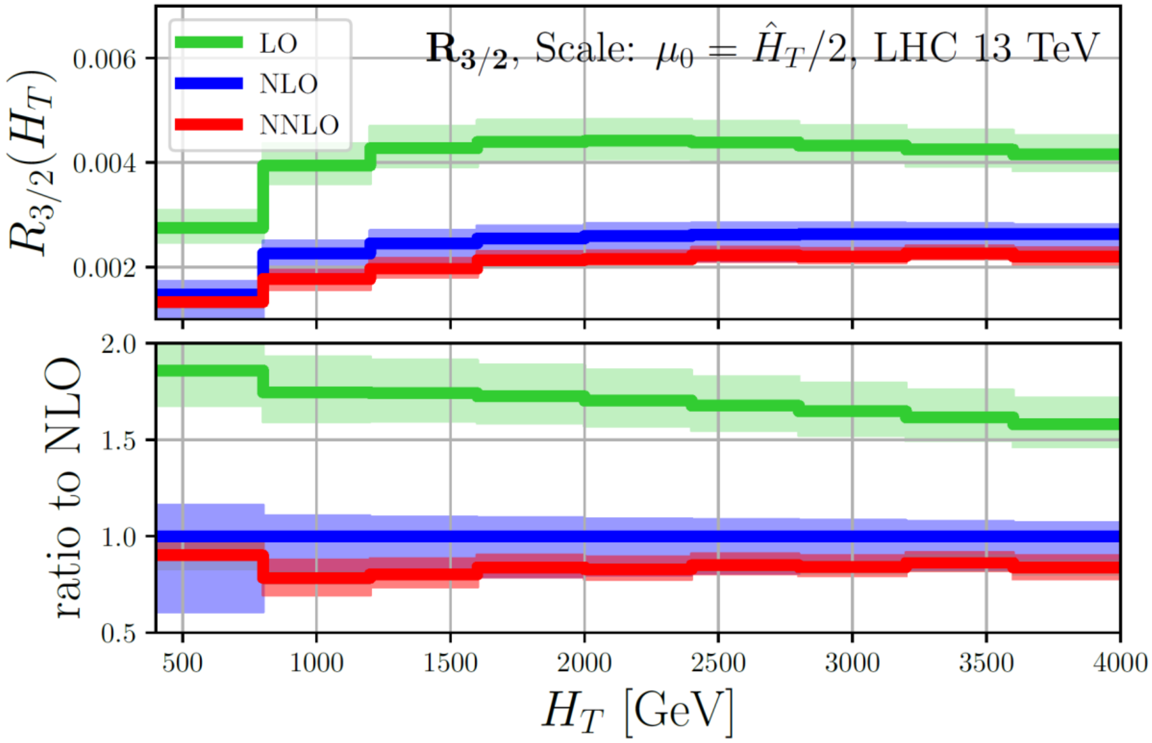






+ …
+ quarks
+ lots of tricky phase space integrals
+ parton distributions

NNLO 3-jet / 2-jet ratio



Czakon, Mitov, Poncelet, 2106.05331

- Tour de force: Great NNLO/NLO agreement (not with LO!)

→ Measure α_s more reliably at very large momentum transfer

- Foreshadows many more NNLO 2 → 3 results in near future

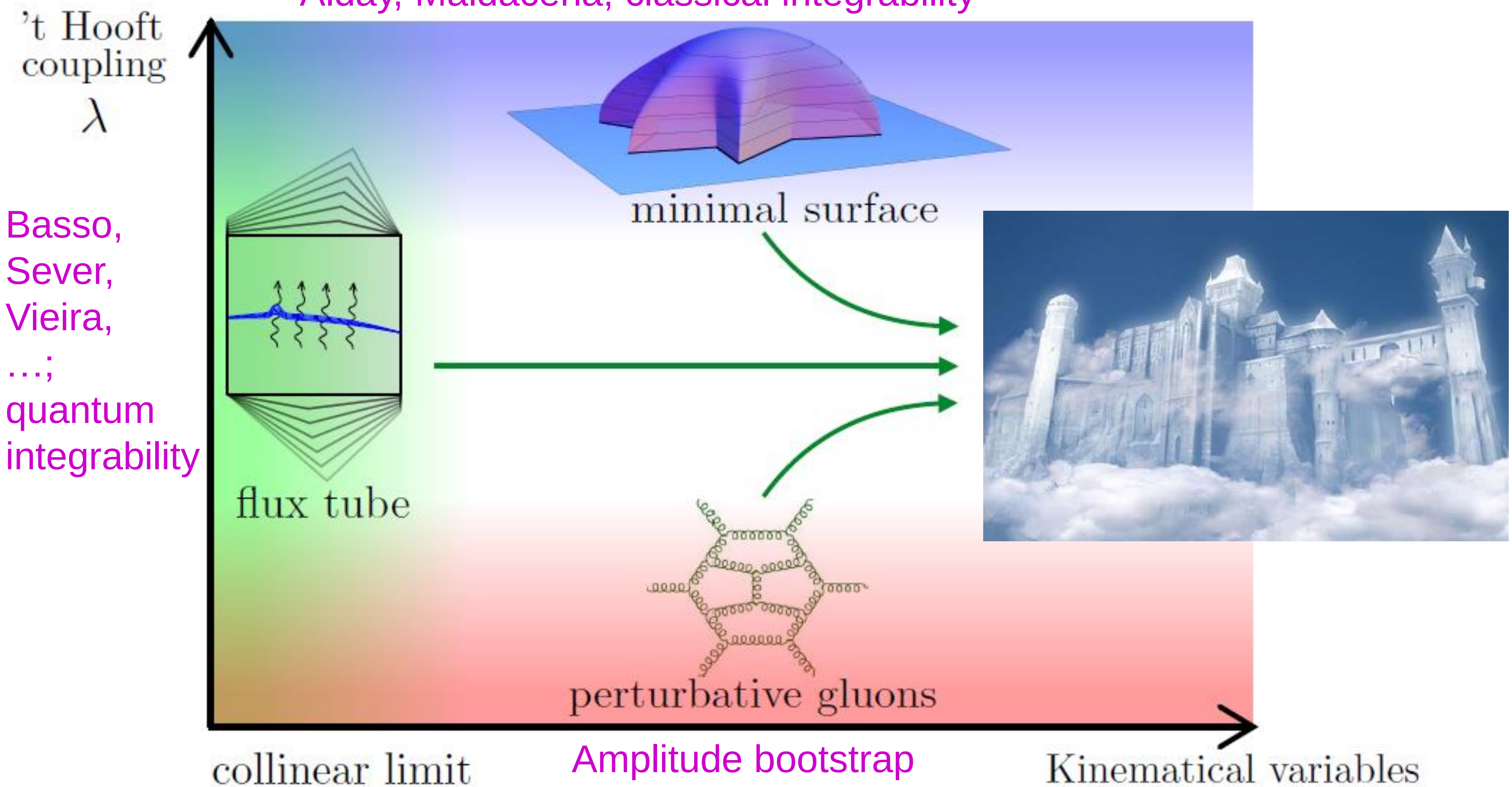
Planar N=4 SYM Dream:

Solve for amplitudes nonperturbatively for any kinematices

Images: A. Sever, N. Arkani-Hamed

Alday, Maldacena; classical integrability

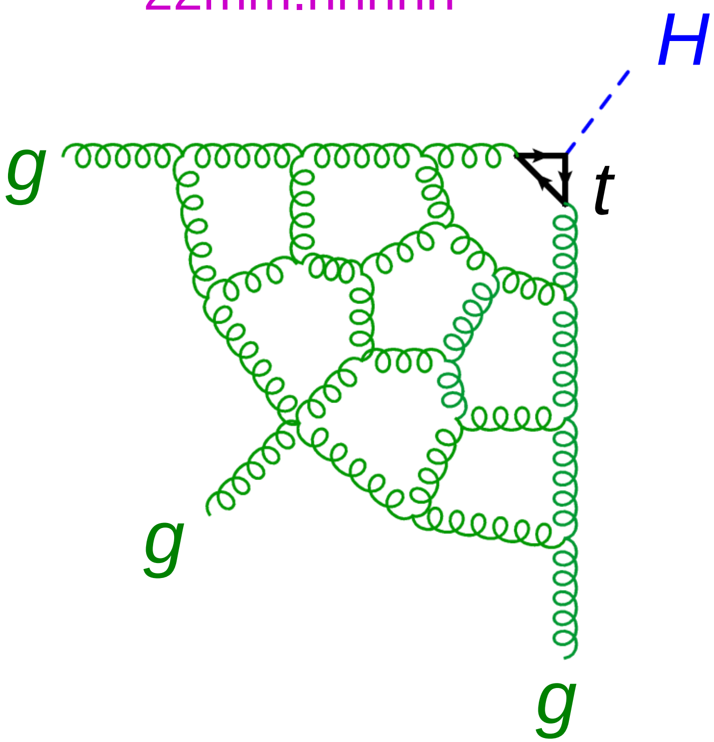
Basso, Sever, Vieira, …; quantum integrability



Amplitude bootstrap

Can push to 8 loops in a Goldilocks "Higgs" amplitude

LD, Ö. Gürdoğan, A. McLeod, M. Wilhelm, 2012.12286, 2112.06243, 22mm.nnnnn



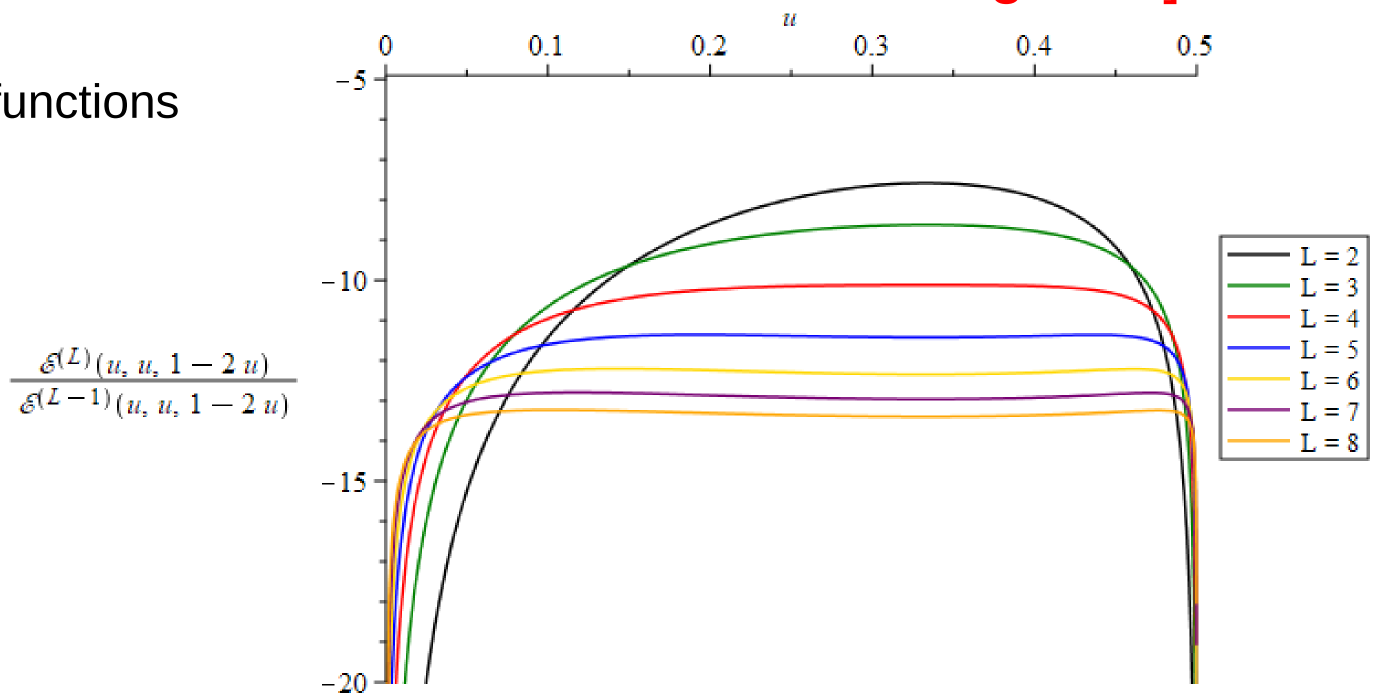
(virtuous circle)

- Matrix elements of operator $G_{\mu\nu}^{a} G^{\mu\nu\, a}$ with n gluons in planar N=4
- *Hgg* form factor ($n = 2$) "too simple", no kinematic dependence beyond overall $(-s_{12})^{-L\epsilon}$
- *Hggg* ($n = 3$) is "just right", depends on 2 dimensionless ratios
- 8 loop results for function of 2 variables → rich theoretical "data mine"
- Led to discovery of strange new antipodal duality between form factor and 6 gluon amplitude, where symbol gets reversed

Approximation Mandatory for 8 Loop Plots

Example: planar N=4 analog of *H* → *ggg,*
[Successive loop ratios reveal finite radius of convergence]

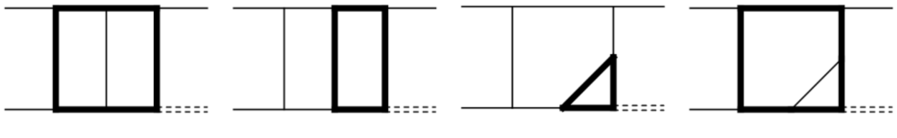
L	No. of weight $2L$ G functions
1	4
2	25
3	269
4	2,580
5	24,484
6	221,249
7	1,992,784
8	17,936,054



- On 1-dimensional lines, series expansions do the job.
- For this plot: one series around $u = 0$, one around $u = ½$

Series expansions on the fly

Moriello, 1907.13234



- Construct series expansions from differential equations
- For phenomenology, need whole phase space
- Nevertheless, elliptic problems can be handled →
- **DiffExp** package

Hidding, 2006.05510

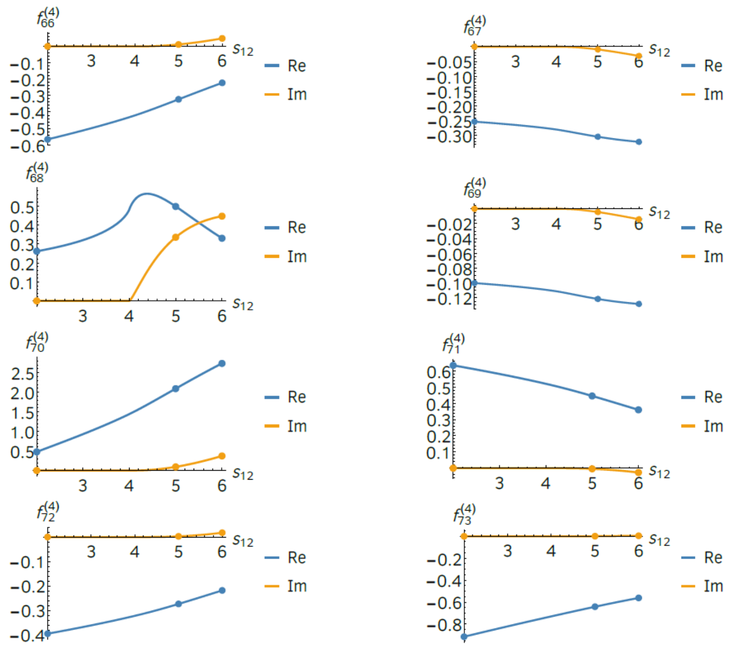


Figure 2: Plots of the elliptic integral sectors, for $s_{13} = -1$, $p_4^2 = \frac{13}{25}$, $m^2 = 1$ as a functic of s_{12}, at order ϵ^4, obtained by series expanding along the contour γ_{thr}. The solid poin represent values computed numerically with the software FIESTA [51].

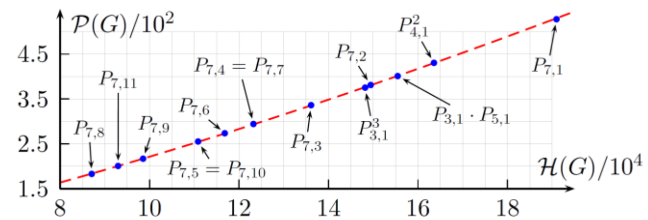
Tropical methods

Panzer, 1908.09820

- Feynman parameter polynomial for true integral $\mathcal{P}(G)$ replaced by a simple monomial (locally) using “tropicalization” (a.k.a. the Newton polytope):

$$\lim_{\xi\to\infty} p(x_1^\xi,\ldots,x_n^\xi)^{\frac{1}{\xi}} = \lim_{\xi\to\infty}\left(\sum_{\ell\in\mathrm{supp}(p)} c_\ell \boldsymbol{x}^{\xi\ell}\right)^{\frac{1}{\xi}} = \max_{\ell\in\mathrm{supp}(p)} \boldsymbol{x}^\ell = p^{\mathrm{tr}}(\boldsymbol{x})$$

- Integrate $p^{\mathrm{tr}}(x) \to$ Hepp bound $\mathcal{H}(G)$
- $\mathcal{P}(G)$ & $\mathcal{H}(G)$ strongly correlated! →



- Motivates “tropical” Monte Carlo sampling Borinsky, 2008.12310
- Remarkably efficient, even up to 17 loops!

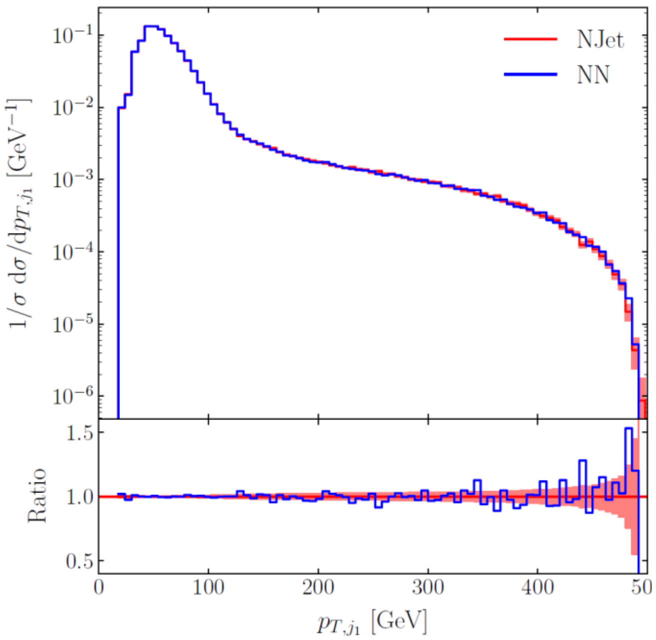
Machine learning approximations

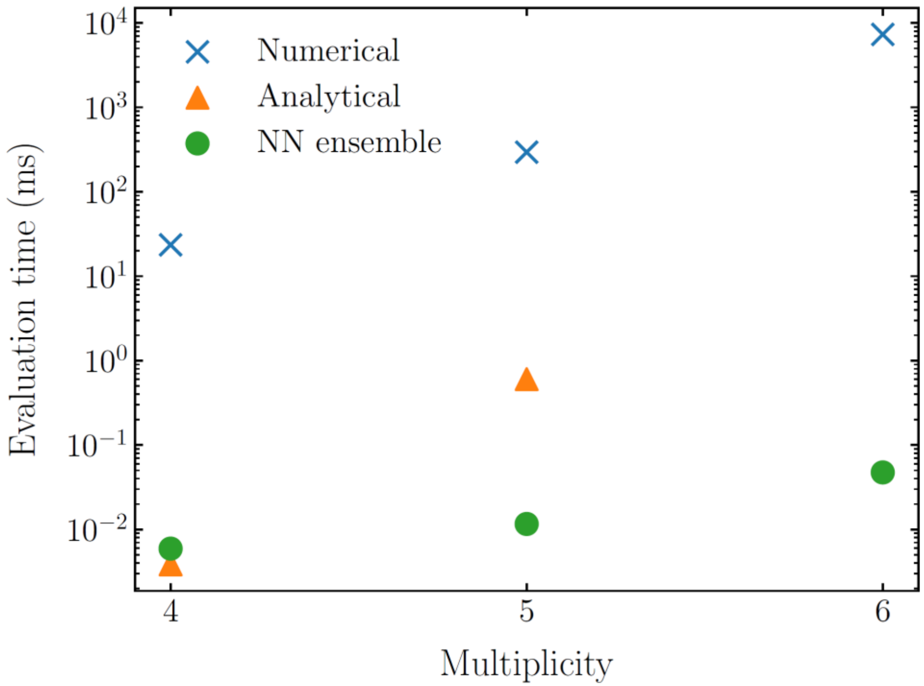
Thaler talk

- Model amplitudes that are very expensive to compute, then compute with the model

Aylett-Bullock, Badger, Moodie, 2106.09474; Moodie, 2202.04506

- Simulation time can be reduced by a large factor (~30?)





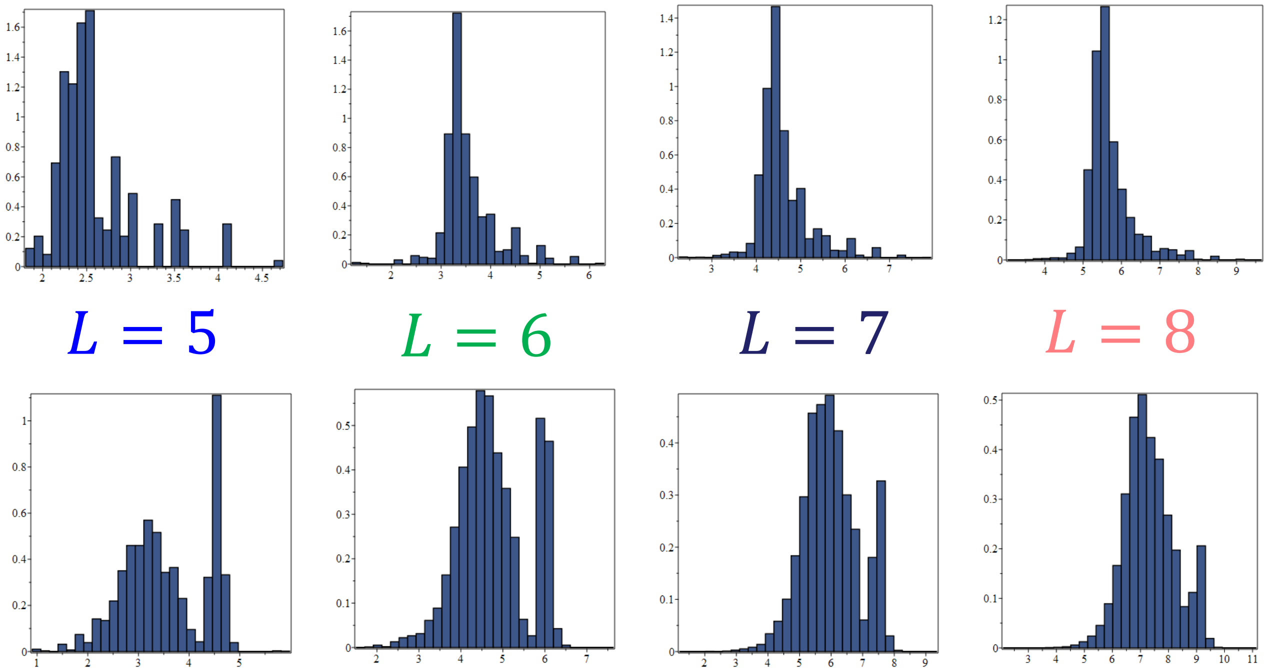
Machine learning exactly?

Statistics of the *Hggg* planar N=4 amplitude:
$\log_{10}$ of G function (HPL) coefficients (all integers!)

$v \to \infty$ line

$u = 1$ line

$L = 5$ $L = 6$ $L = 7$ $L = 8$



Grist for a machine to learn exactly $L = 9, 10, 11, \ldots$?

To the future

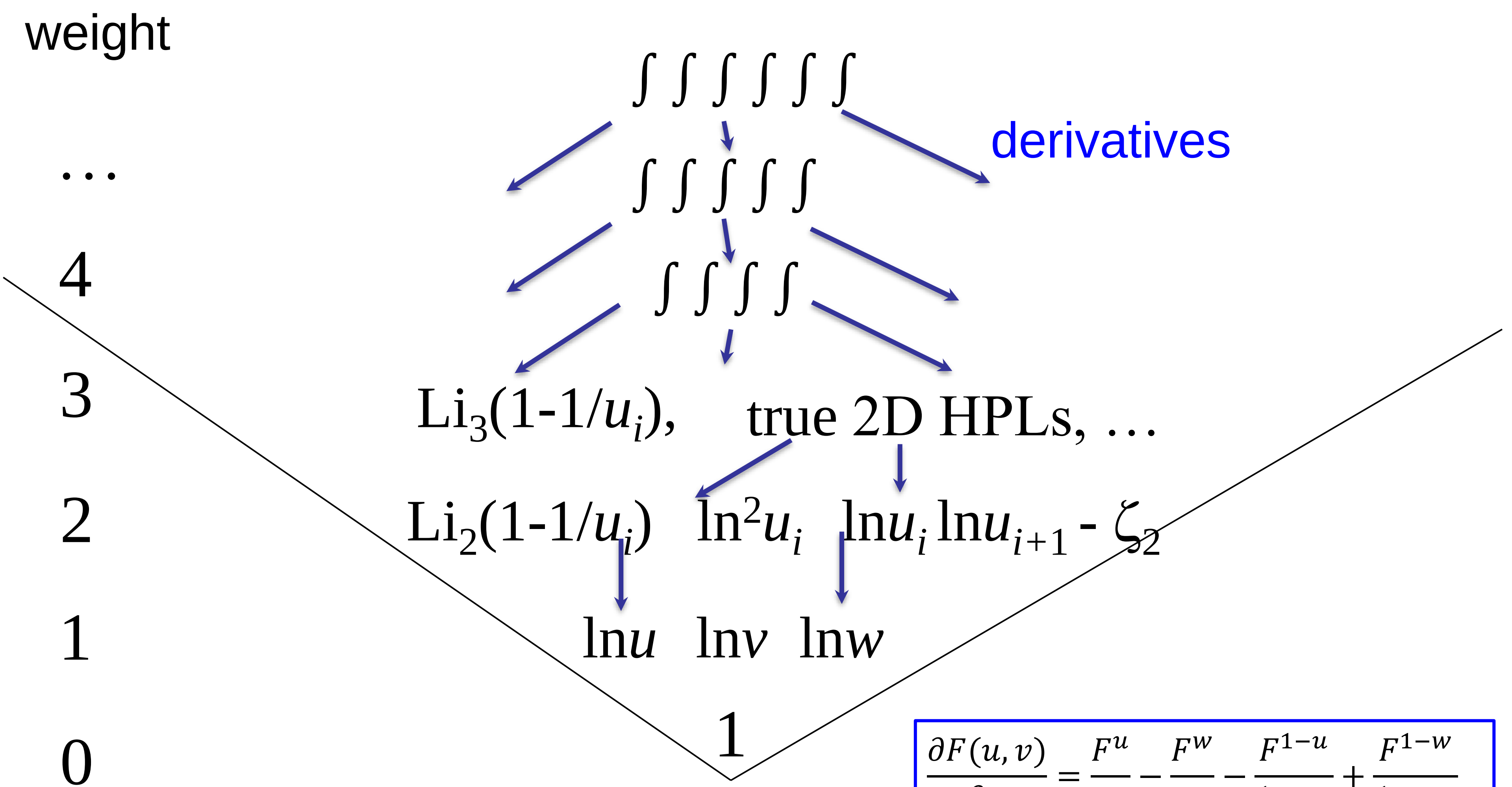
- "Amplitudes" notions and methods will continue to provide a central role in the building blocks for precision collider cross sections
- Strong interplay in both directions
- The NNLO revolution to higher multiplicity is well underway, with the aid of many of these ideas
- Also NNNLO for high-value targets   Mistlberger talk
- Deeper understanding of relevant analytic functions will help, but also novel approximation methods, and most likely more roles for machine learning
- The finite coupling solution to N=4 scattering awaits too!

Those who explore an unknown world are travelers without a map: the map is the result of the exploration. The position of their destination is not known to them, and the direct path that leads to it is not yet made.

Hideki Yukawa

Extra Slides

Heuristic view of function space



$$\frac{\partial F(u,v)}{\partial u} = \frac{F^u}{u} - \frac{F^w}{w} - \frac{F^{1-u}}{1-u} + \frac{F^{1-w}}{1-w}$$

Symbol alphabets for planar N=4 n-gluon amplitudes

parity-odd letters, algebraic in $\hat{u}, \hat{v}, \hat{w}$

$n = 6$ has 9 letters: $\mathcal{S}_6 = \{\hat{u}, \hat{v}, \hat{w}, 1-\hat{u}, 1-\hat{v}, 1-\hat{w}, \hat{y}_u, \hat{y}_v, \hat{y}_w\}$

Goncharov, Spradlin, Vergu, Volovich, 1006.5703; LD, Drummond, Henn, 1108.4461; Caron-Huot, LD, von Hippel, McLeod, 1609.00669

$n = 7$ has 42 letters

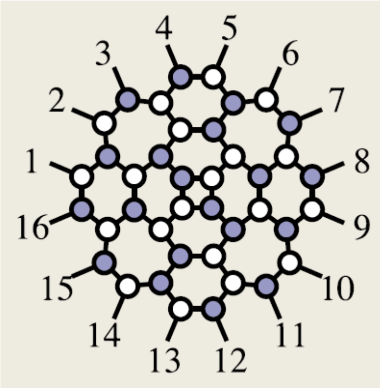
Golden, Goncharov, Paulos, Spradlin, Volovich, Vergu, 1305.1617, 1401.6446, 1411.3289; Drummond, Papathanasiou, Spradlin 1412.3763

$n = 8$ has at least 222 letters, could even be infinite as $L \to \infty$

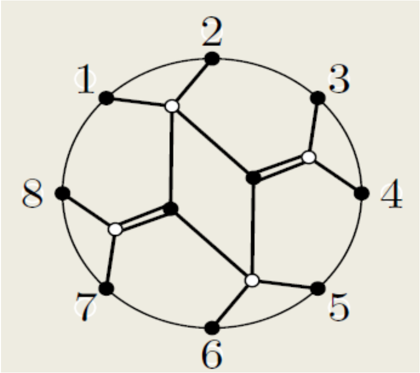
Arkani-Hamed, Lam, Spradlin, 1912.08222;
Drummond, Foster, Gürdoğan, Kalousios, 1912.08217, 2002.04624;
Henke, Papathanasiou 1912.08254, 2106.01392;
Z. Li, C. Zhang, 2110.00350

Planar N=4 SYM symbol letters from plabic graphs & tensor diagrams

Mago, Schreiber, Spradlin, Srikant, Volovich, 2007.00646, 2012.15812, 2106.01406



Ren, Spradlin, Volovich, 2106.01405

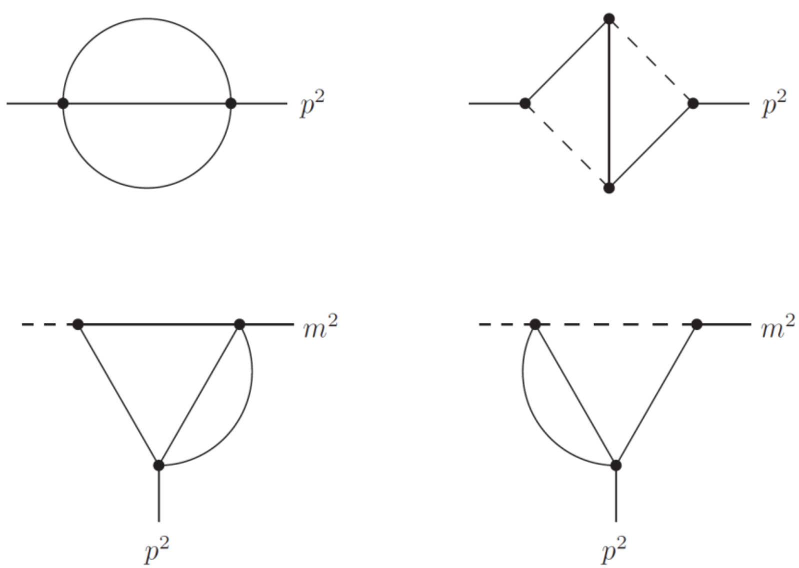


- Almost arborizable TDs can be “resummed” to give square root letters seen in $\bar{Q}$ computations
- Also predictions of symbol letters from tropical Grassmannians

Modular properties of elliptic integrals

Weinzierl, 2011.07311

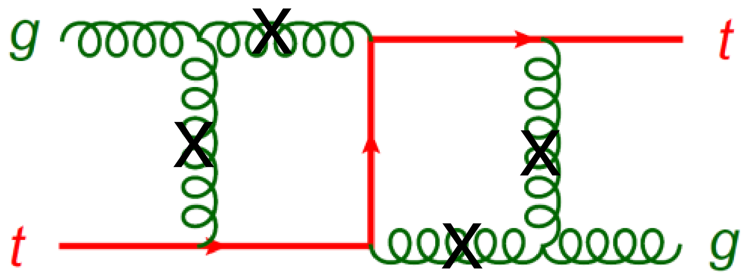
Examples of elliptic Feynman integrals depending on one kinematic
$x = \frac{-p^2}{m^2}$ (solid internal lines of mass m, dashed lines massless):



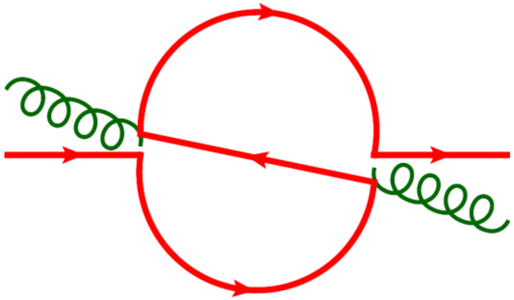
- For numerical purposes, handy to use modular transformations to move $q = e^{2\pi i\tau}$ into fundamental region for modular group, so $|q| < 0.0043$

Elliptic sunset inside top production

- At subleading color in *gg* → *t* $\bar{t}$,



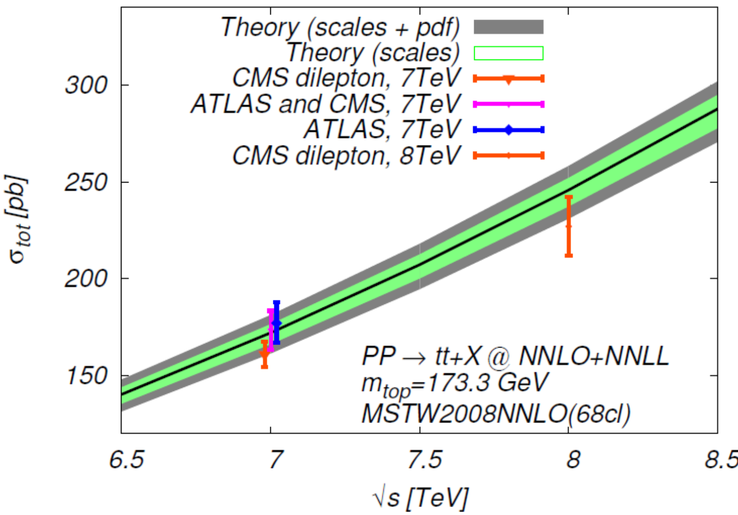
→



- Done numerically long ago

Czakon, Fiedler, Mitov, 1303.6254

- Better analytic understanding should aid computational efficiency in multiscale processes

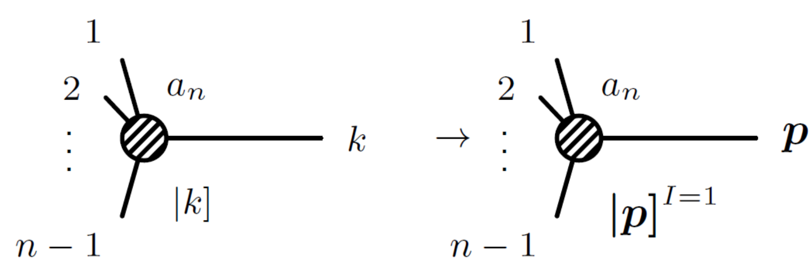


On-shell Massive Amplitudes and SMEFT

Arkani-Hamed, Huang[2], 1709.04891; …; Shadmi, Weiss, 1809.09644; …

- **Massive** external states also have a natural helicity formalism based on "**bold**" spinors (A-HH[2])
- In EFT descriptions, such as SMEFT, on-shell amplitudes take into account equations of motion naturally
- Both massless and massive versions of EFT; latter leverages A-HH[2] massive spinor technology:

bolding as higgsing



massless EFT → massive EFT

Double copy / color kinematics duality

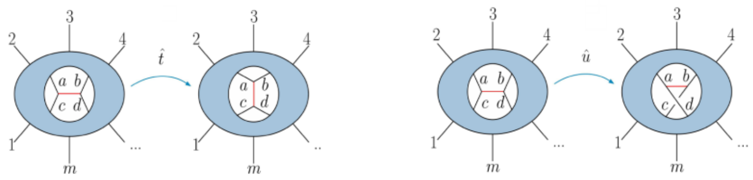
- Began with KLT relations from string theory, and a slogan:

Gravity = YM^2

- Then became graphical [Bern, Carrasco, Johansson]

$$n_i - n_j = n_k \quad \Leftrightarrow \quad c_i - c_j = c_k,$$

$$n_i \to -n_i \quad \Leftrightarrow \quad c_i \to -c_i$$



- Now connects a much broader web of theories
- Underlies much of the motivation for the "scattering amplitudes for LIGO" program [Elvang, Solon talks]

Hggg kinematics is two-dimensional

$$k_1 + k_2 + k_3 = -k_H$$

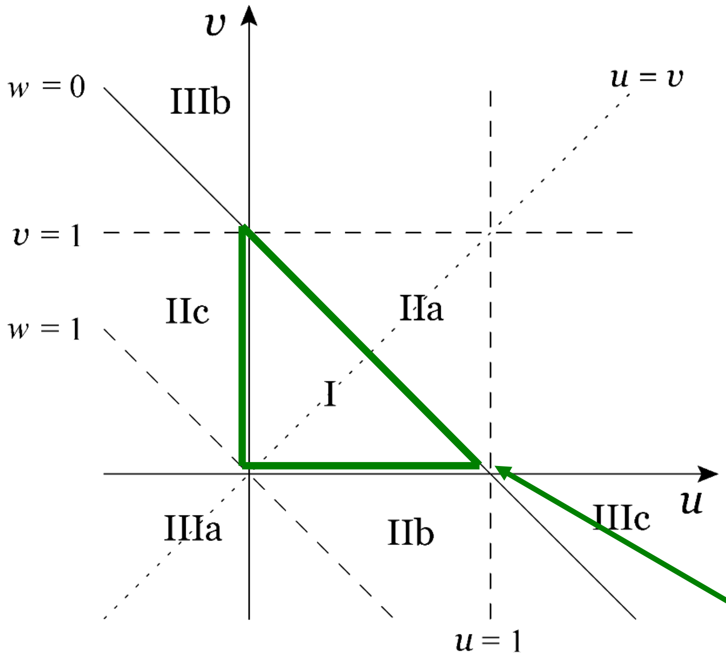
$$s_{123} = s_{12} + s_{23} + s_{31} = m_H^2$$

$$s_{ij} = (k_i + k_j)^2 \qquad k_i^2 = 0$$

$$u = \frac{s_{12}}{s_{123}} \qquad v = \frac{s_{23}}{s_{123}} \qquad w = \frac{s_{31}}{s_{123}}$$

not cross ratios!

$$u + v + w = 1$$



I = decay / Euclidean

IIa,b,c = scattering / spacelike operator

IIIa,b,c = scattering / timelike operator

Boundary data, near-collinear limit from the form factor OPE:
Sever, Tumanov, Wilhelm, 2009.11297, 2105.13367