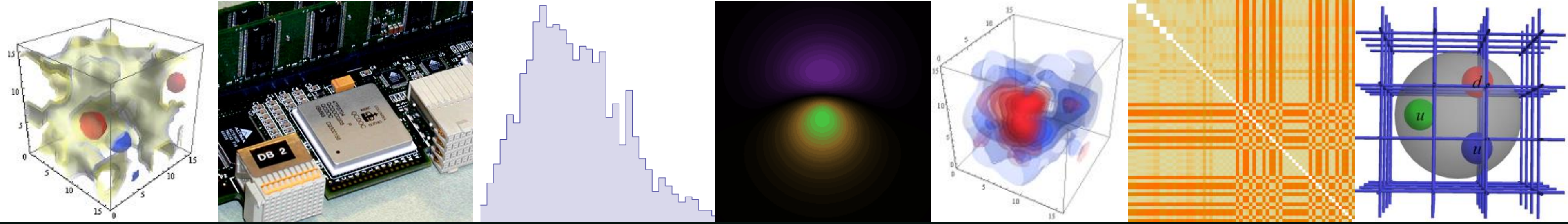
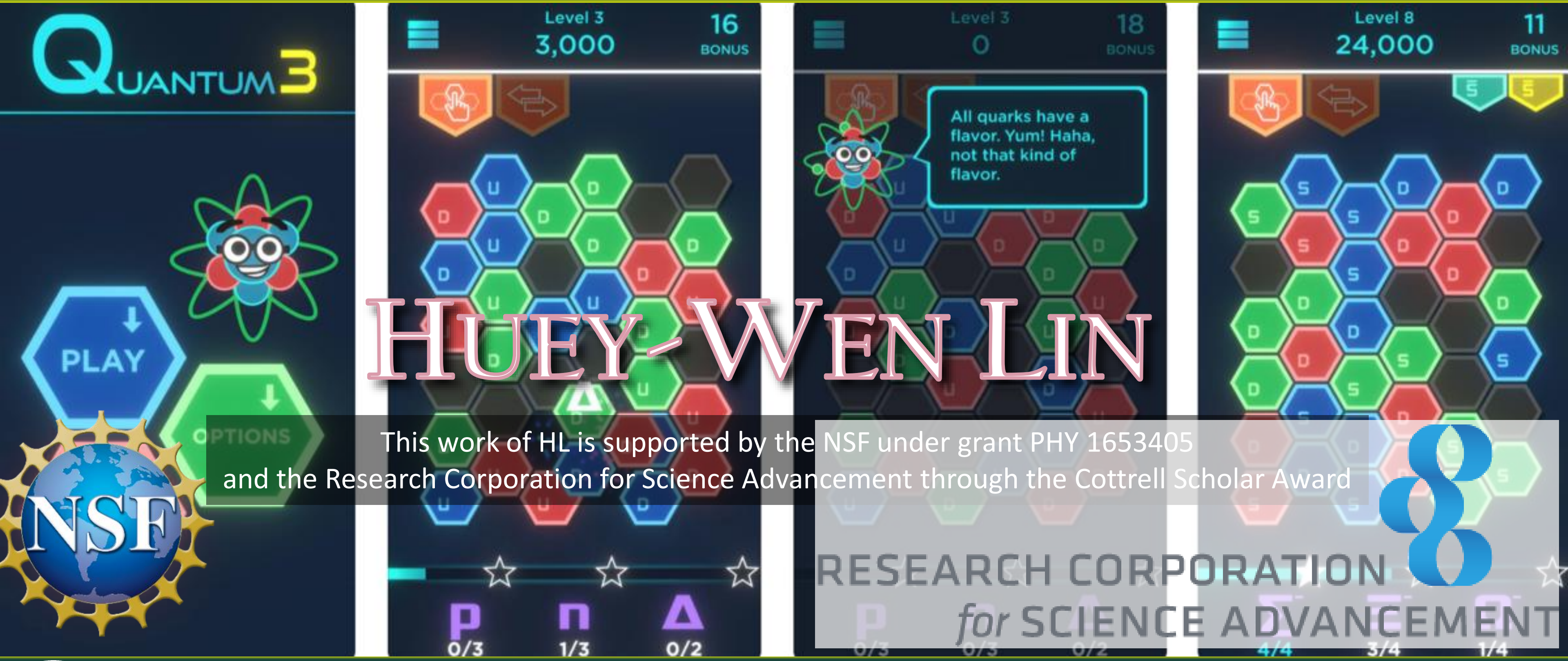


Parton Distribution Functions



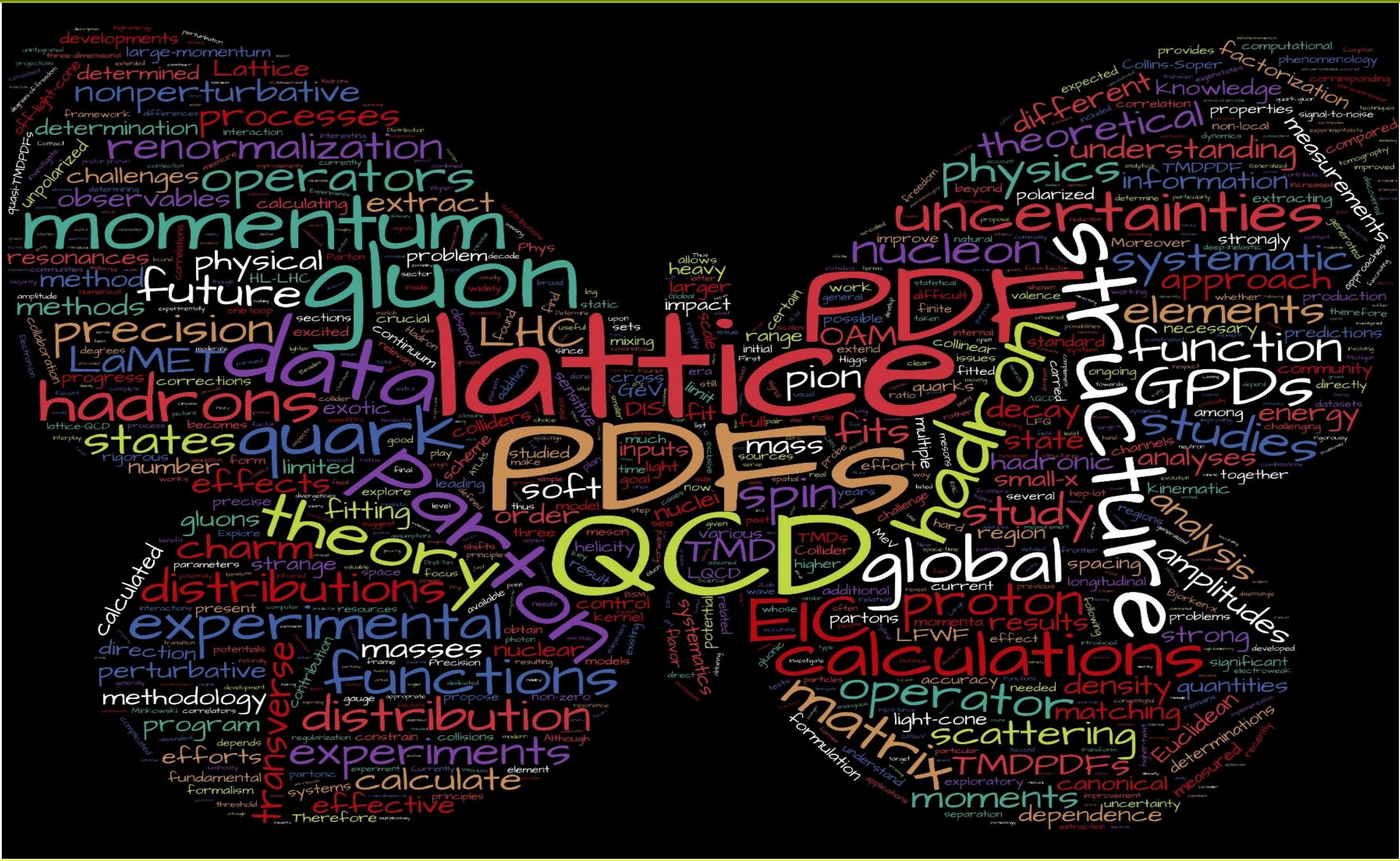
HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

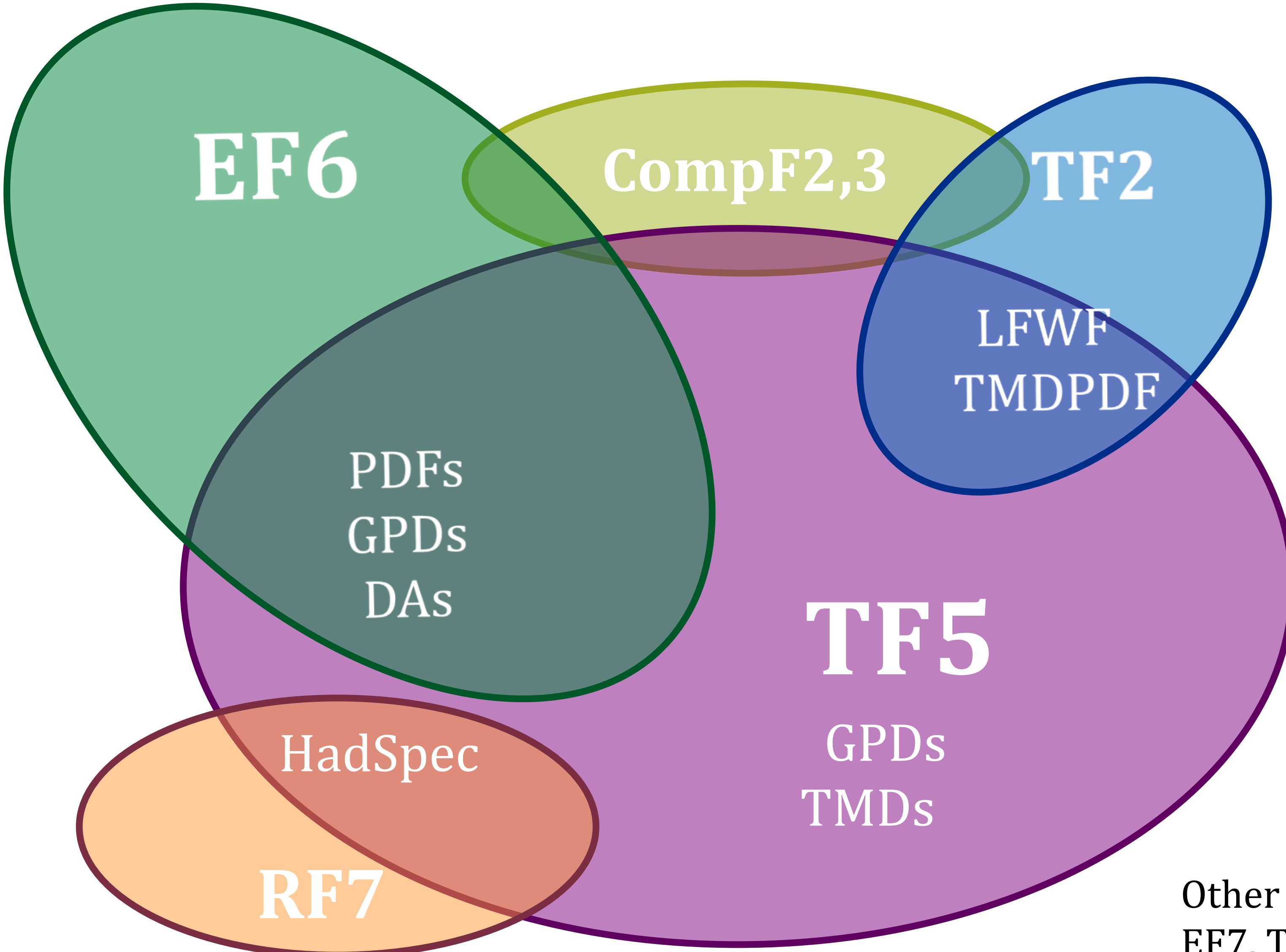
NSF

RESEARCH CORPORATION *for* SCIENCE ADVANCEMENT

EF06: Hadronic structure & forward QCD



EF06: Hadronic structure & forward QCD



Other TGs:
EF7, TF4, TF6

PDFs Whitepapers

- § Proton Structure at the Precision Frontier
  - Coordinated by Maria Ubiali, Radja Boughezal, Stefan Hoeche, Pavel Nadolsky & HL
- § Electron-Ion Collider for High-Energy Physics
  - Coordinated by EF05/EF06/EF07 conveners, led by Swagato Mukherjee
- § **Lattice-QCD Calculations of Parton Physics**
  - Coordinated by Zohreh Davoudi and HL (2202.07193)

M. Constantinou, L. Del Debbio, X. Ji, H. Lin, K. Liu, C. Monahan, K. Orginos, P. Petreczky, J. Qiu, D. Richards, N. Sato, P. Shanahan, C.-P. Yuan, J. Zhang, Y. Zhao

Open Questions

- What is the best approach to reduce systematic uncertainties in LHC measurements to achieve the **accuracy of PDFs** envisioned by electroweak precision studies at the high-luminosity LHC?
- What is the feasible strategy for obtaining **accurate PDFs** for N3LO QCD computations? Which theoretical advances and computational tools will be necessary?
- How does the knowledge of **hadron structure** affect measurements of the QCD coupling constant in various processes?
- When do power-suppressed contributions to the **hadron structure** become important in NXLO QCD calculations? What are the best approaches to predict or measure them?
- How can the LHC, LHeC, and FCC improve our knowledge of the 3-dimensional **structure of nucleons and nuclei**?
- Complete list on https://snowmass21.org/energy/forward_qcd

PDFs from Global Fits

- Experiments cover diverse kinematics of parton variables
  - Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

- Some choices made for the analysis
  - Choice of data sets and kinematic cuts
  - Strong coupling constant $\alpha_s(M_Z)$
  - How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

  - Assumptions imposed

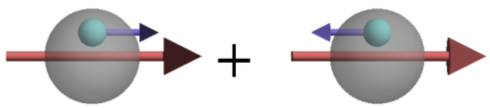
    SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

Lattice PDFs 2013

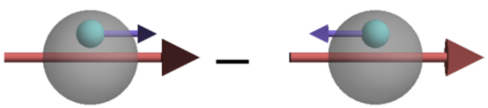
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\left\langle x^{n-1} \right\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\left\langle x^{n-1} \right\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent
transversely polarized

$$\left\langle x^{n-1} \right\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

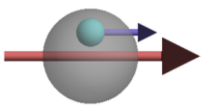
§ True distribution can only be recovered with all moments

Lattice PDFs 2013

§ Traditional lattice calculations rely on operator product expansion, only provide moments

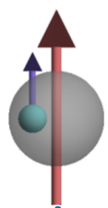
 +

spin-averaged

 −

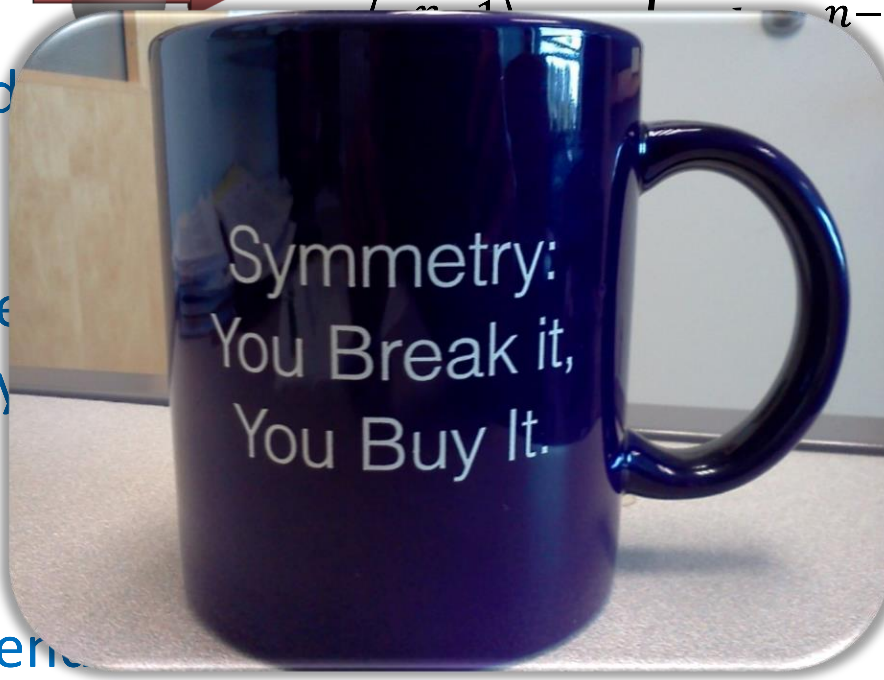
spin-depe

longitudinally

 −

spin-depen

transversely polarized



$\int^1 \ldots x^{n-1} q(x)$

$\ldots^1 \Delta q(x)$

$\ldots^1 \delta q(x)$

most well known

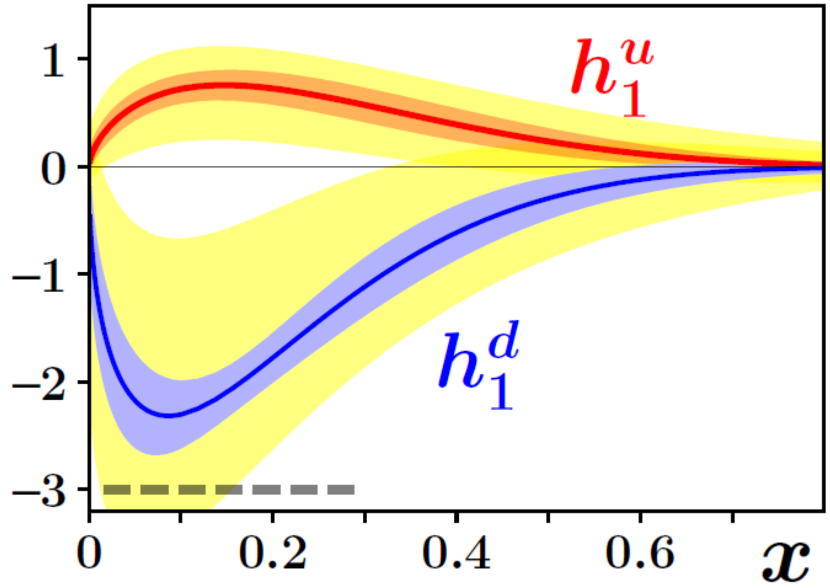
very poorly known

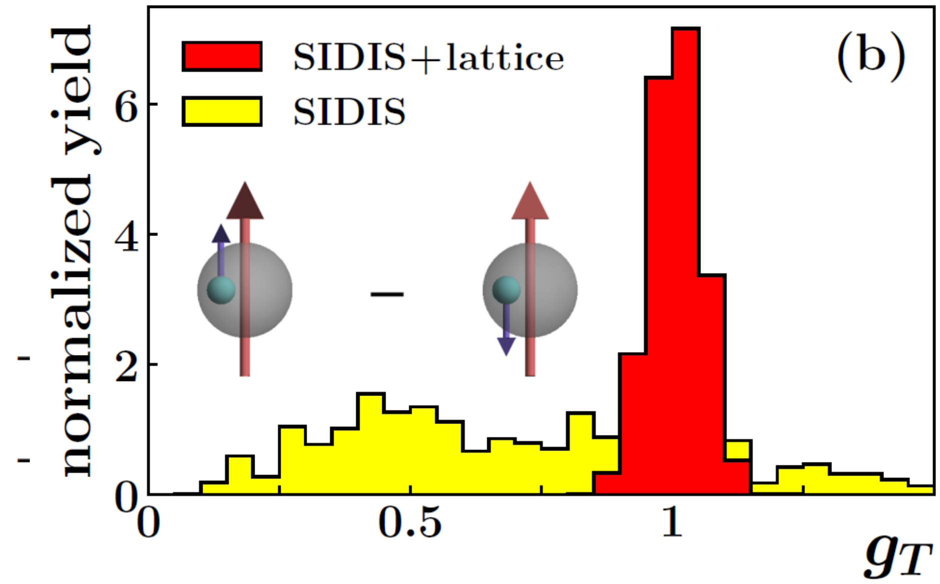
§ True distribution can only be recovered with all moments

From Charges to PDFs

§ Improved **transversity** distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global-analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

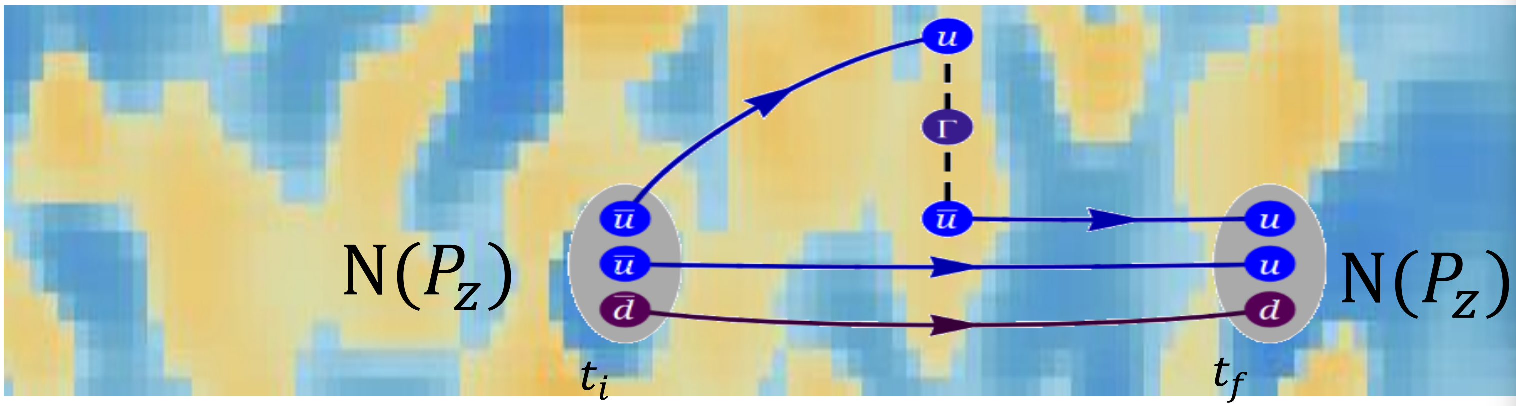


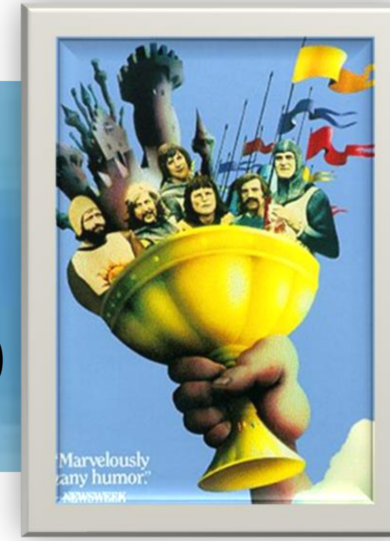


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/Quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)





§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

§ Recover true distribution (take Pz → ∞ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{P_z^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(xP_z)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)P_z)^2}\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

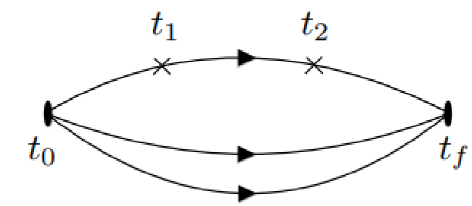
§ Short-distance factorization (SDF)

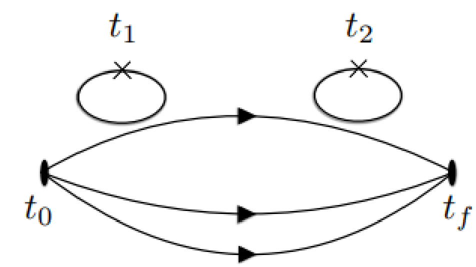
- Pseudo-PDF method (A. Radyushkin, 2017)
- Hadronic tensor currents

  (Liu et al., hep-ph/9806491, … 1603.07352)
- Lattice cross-section method (LCS)

  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions

  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)

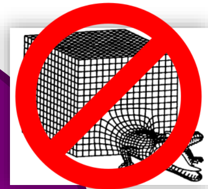




Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

Lattice Parton Method

§ Differences and similarity



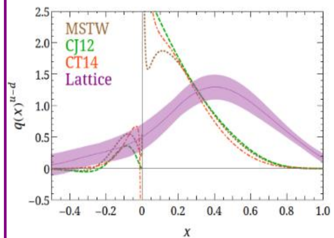
Quantities that can be calculated on the lattice today $=$ $\sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

- Large momentum is needed in the lattice calculations in all methods to reach small-x region
  - Current projects focus on $x \in [0.3, 0.8]$ (for 2-GeV boosted hadron)
- Kernel is a complicated object;
  mostly current calculations used up to one-loop level
- SDF suffers Inverse problem to extract the wanted distribution
- LaMET requires to reach large Wilson-line displacement

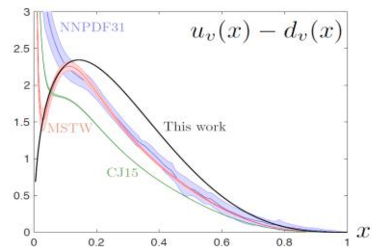
Lattice Parton Calculations

§ Rapid developments!

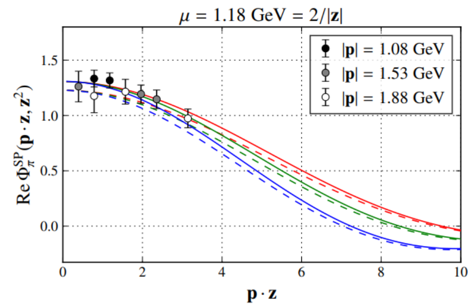
First unpol. PDF lattice calculation



First lattice pseudo-PDFs

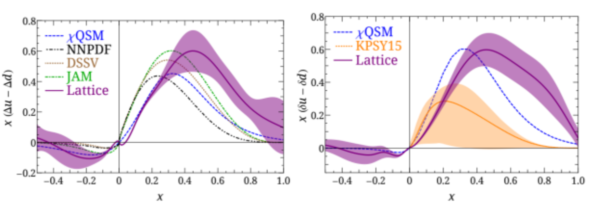


Euclidean correlation functions

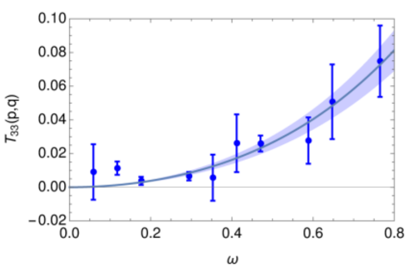


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

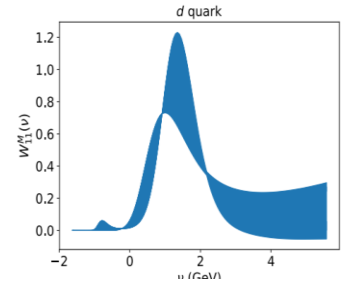
Pol. PDFs and mass corrections



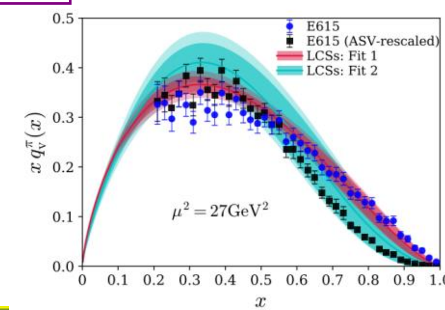
Compton amplitude



Hadronic tensor



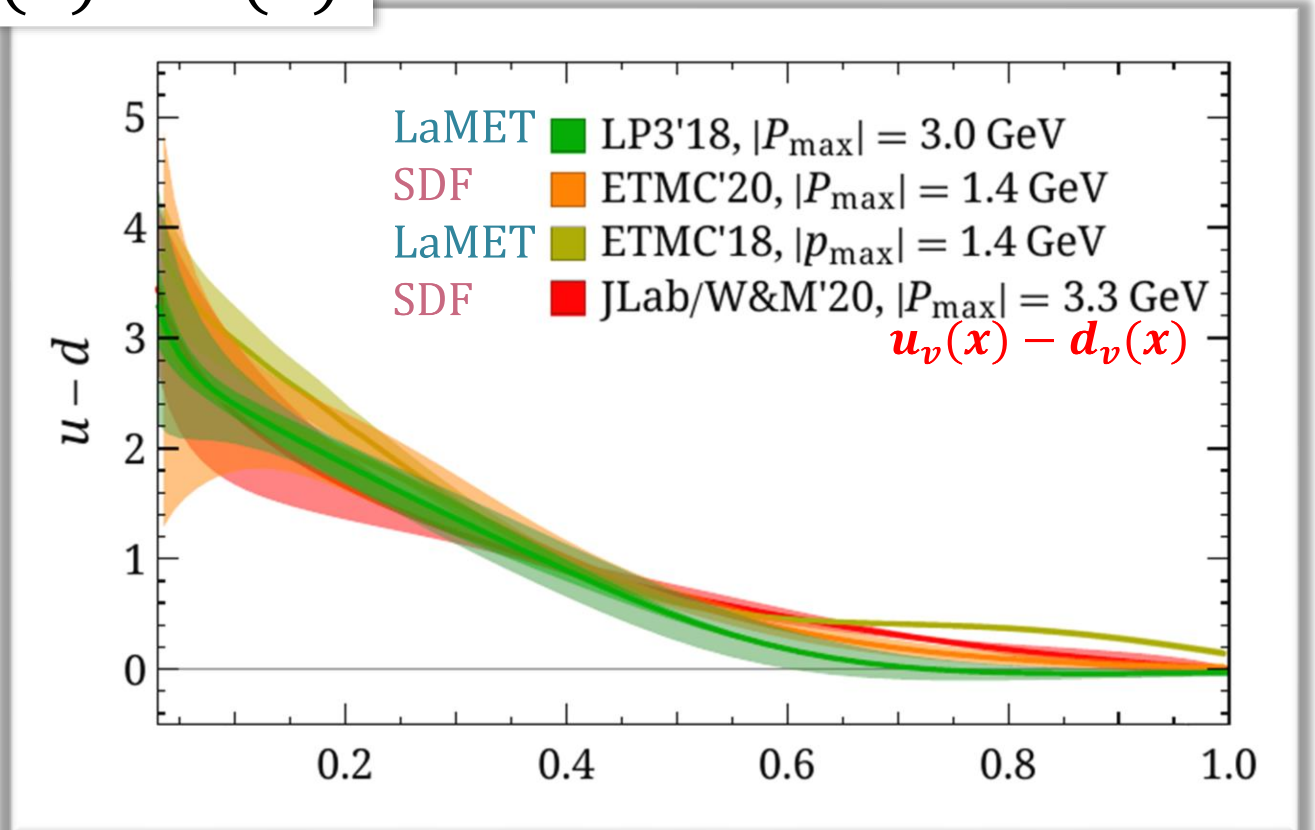
LCS



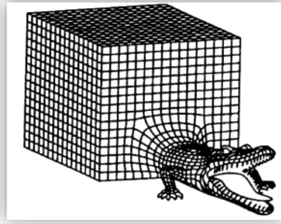
Results at Physical Pion Mass

§ Summary of results at physical pion mass

$\boldsymbol{u(x) - d(x)}$



2006.08636, PDFLattice2019 report

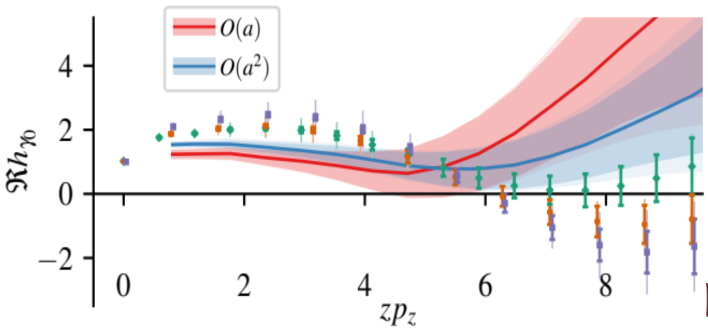


Finite volume,
Discretization,
…

Systematics Study

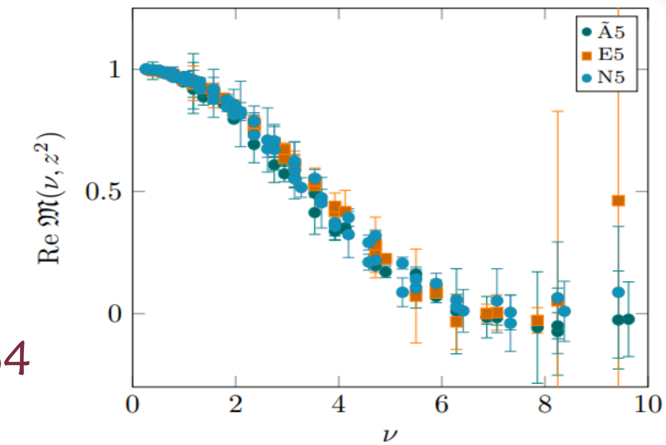
§ Lattice discretization study examples

$M_\pi \approx$ **370** MeV (2+1+1f)
$a \approx [0.064, 0.093]$ fm



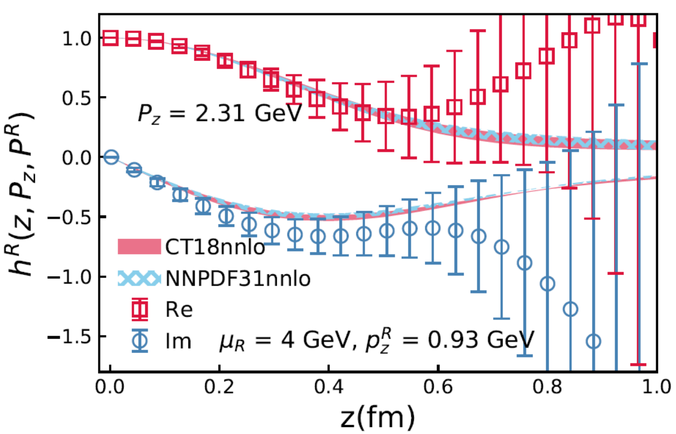
ETMC, 2011.00964

$M_\pi \approx$ **440** MeV (2f)
a≈[0.048, 0.075] fm



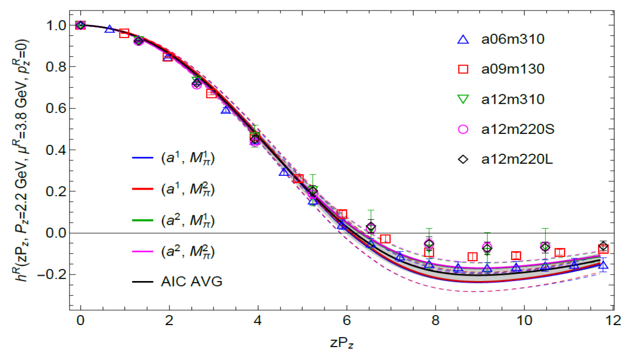
HadStruc, 2105.13313

$M_\pi \approx$ **310** MeV (2+1+1f)
$a \approx 0.042$ fm



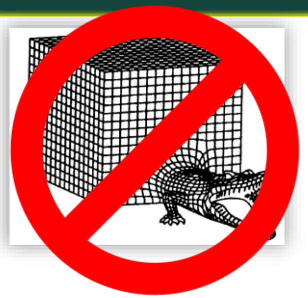
BNL/MSU, 2005.12015

$M_\pi \approx$ **[135, 310]** MeV (2+1+1f)
$a \approx [0.06, 0.12]$ fm



MSULat, 2011.14971

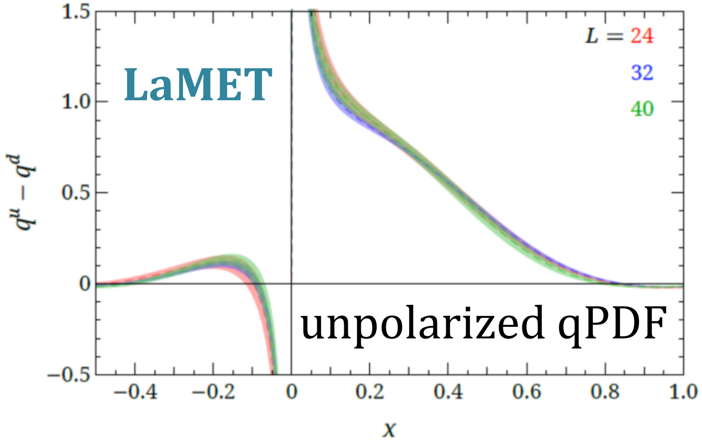
Systematics Study



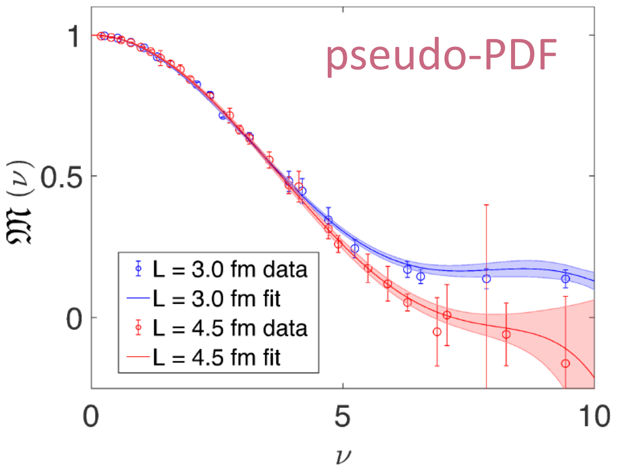
§ Finite-volume study

$M_\pi \approx \mathbf{220}$ MeV (2+1+1f)
$L \approx 2.9, 3.8, 4.8$ fm

$M_\pi \approx \mathbf{415}$ MeV (2+1f)
$L \approx 3, 4.5$ fm



MSULat, PRD 100 (2019) 7, 074502



HadStruc, JHEP 12 (2019) 081

§ Lattice artifacts are sensitive to the simulated QCD vacuum

- Each group will have to check their own systematics carefully

Other Lattice Progress

- Exploratory study on strange, charm and gluon PDFs
- Many approaches are moving to the NNLO level
  - Expect to see more improved lattice calculations
- Beyond the standard twist-2 collinear PDFs
  - Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
  - Transverse-momentum– dependent distributions (TMDs)
    - Collins-Soper kernel, soft function and wavefunctions
  - Twist-3 PDFs and GPDs

For more details and references, refer to 2202.07193

Challenges

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Renormalization of linear divergence
  - Wilson-line ops have linear divergences that must be subtracted
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Inverse problems PDF extraction in SDF
  - Remove the model/preconditioner-choice dependence
- Reaching long-range correlations in LaMET
  - For small-x physics, new methods for calculating longer-range correlations must be developed

Computational Challenges

- Wanted lattice calculations in the next few years for isovector nucleon PDFs
  - a = 0.05 fm (corresponding to $a^{-1} \approx$ 4 GeV)
  - $M_\pi \approx$ 139 MeV with at least M_πL = 3
    (L = 4.5 fm , L/a ≈ 90)
  - We need nucleon momenta of P ≈ 2.6 GeV
- Flavor-dependent PDFs more challenging to overcome

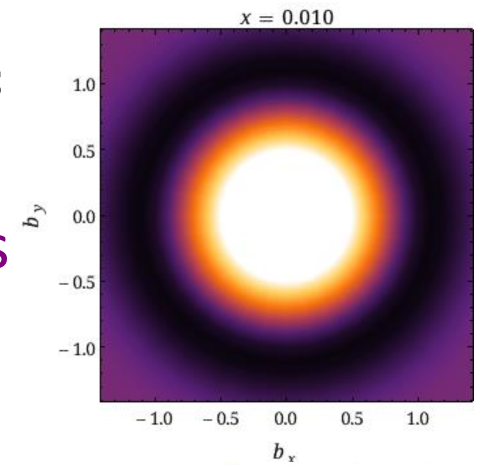
Interplay with Experimental Data

- Flavor separation of unpolarized and polarized PDFs
  - Strange contribution, d/u ratio at large x, …
- Spin-dependent PDFs
  - Helicity PDFs and transversity PDFs provide unique insights on the spin structure of nucleons.
- Complementary to TMD phenomenology
  - LQCD to provide fundamental quantities such as the CS kernel in the nonperturbative regime
- Realistic three-dimensional images of nucleon
  - Lattice calculations, in fact, allow us to explore all of the kinematic dependence, x, ξ, and t

And more…

Summary and Outlook

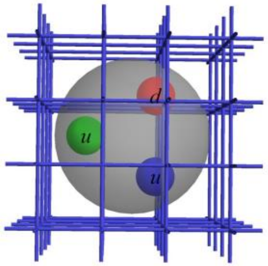
- § Exciting era using LQCD to study PDFs
- § Overcoming longstanding limitations of moment method
  - Bjorken-*x* dependence of parton distributions are widely studied
  - More study of systematics planned for the near future
  - Start to address neglected disconnected contributions obtaining flavor-dependent quantities



- § Precision and progress are limited on resources
  - Challenges = new opportunities quantities
- § Until next Snowmass for precision PDFs

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

+



Backup Slides

Parton Distribution Functions

M. UBIALI – LOI "NEW FRONTIERS IN PDF ANALYSES IN THE HL-LHC ERA"

M.ubiali@damtp.cam.ac.uk

SNOWMASS21-TF5_TF6-040.pdf

- Precision physics frontier at HL-LHC opens up new fascinating challenges also in the field of PDF determination
- Precise and accurate understanding of the proton structure is key to achieve accurate theoretical predictions
- HL-LHC projection: reduction of PDF uncertainties by factor 2-3, but to achieve this goal benchmark among PDF sets and thorough scrutiny of each PDF analysis is a must.

Global PDF point of view…

- Need: robust methodology (e.g. closure tests) and increased precision in theoretical predictions in PDF fits (N3LO, estimate of missing higher order uncertainties, EW corrections, photon and lepton PDFs)
- News: estimate of theoretical uncertainties associated with missing higher order and nuclear models in PDF fits, fit of the methodology, new tools to quantify the effects of new data.
- Longer-term aim is to build up technologies that allow to perform global fits of all parameters that enter LHC analyses (PDFs + α_S, PDFs + EW parameters) and also of PDFs + BSM EFT parametrisation, to prevent PDFs from absorbing signs of new physics
- Broad effort and cross-talk essential to advance and face these challenges

Slide by M. Ubiali

PDF-Related Topics in Snowmass

Topic	Status, 2013	Status and plans, 2020
Benchmarking of PDFs for the LHC	Before PDF4LHC'2015 recommendation	In progress toward PDF4LHC'2X recommendation
PDFs with NLO EW contributions	MSTW'04 QED, NNPDF2.3 QED	Needs an update using LuXQED and other photon PDFs; PDFs with leptons and massive bosons
PDFs with resummations	Small x (in progress)	Needs an update for PDFs with small-x and threshold resummations
Parton luminosities at 14, 33, 100 TeV	CT10, MSTW2008, NNPDF2.3 Update at 100 TeV in CERN YR (1607.01831)	Need an update based on the latest PDFs
LHC processes to measure PDFs	W/Z, single-incl. jet, high-p_T Z, $t\bar{t}$, $W+c$ production	updates on these processes + $Q\bar{Q}$, dijet, $\gamma/W/Z$ +jet, low-Q DY, …
Future experiments to probe PDFs	LHC Run-2 DIS: LHeC	LHC Run-3 DIS: EIC, LHeC, …

NEW TASKS in THE HL-LHC ERA:

Obtain complete NNLO and N3LO predictions for PDF-sensitive processes	Improve models for correlated systematic errors	Find ways to constrain large-x PDFs without relying on nuclear targets
Develop and benchmark fast NNLO interfaces	Estimate NNLO theory uncertainties	Develop an agreement on comparing and combining PDF fits

TABLE I. PDF-related topics in Snowmass'2013 [18] and '2021 studies.

Slide by P. Nadolsky

Connection with EIC

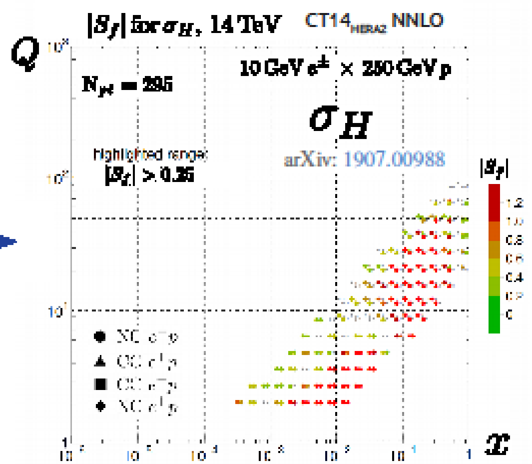
EIC@Snowmass21 LOI: Hadronic Tomography at the EIC and the Energy Frontier

October 2020 **Editors: Salvatore Fazio, Tim Hobbs, Alexei Prokudin, Alessandro Vicini** **160+ coauthors/signers**

- tomography encompasses a wide range of EIC ↔ HEP topics
- EIC determinations of partonic distributions (PDFs, TMDs, GPDs)

- high-energy QCD (DIS measurements; heavy quarks/masses, jets, α_s)
- gluonic structure/Higgs (gluon PDF/GPD; improvements to gg→ h production)
- QED effects (photon PDF; improved EW corrections)
- TMD measurements, precision EW physics (TMDs and M_W extractions)
- nuclear structure (nuclear PDFs; connections to heavy-ion UPCs)

select topics



EIC pseudodata sensitivity

- progress will depend on various methods
  - → phenomenological studies; global analyses
  - → continuum QCD approaches
  - → lattice QCD input
  - → AI/machine-learning and MCEGs

Slide by Tim Hobbs; check more details for his talks at #92

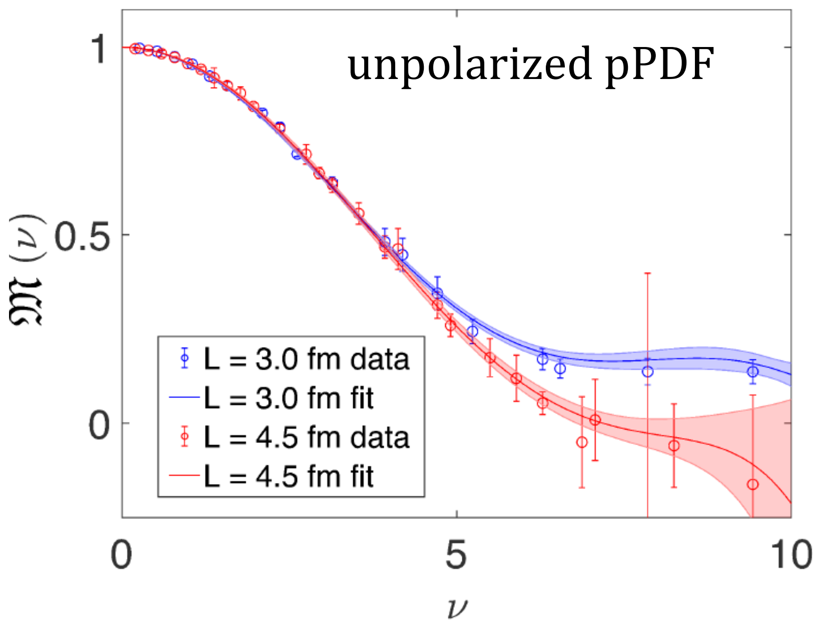
- completed LoI available here

Systematics Study

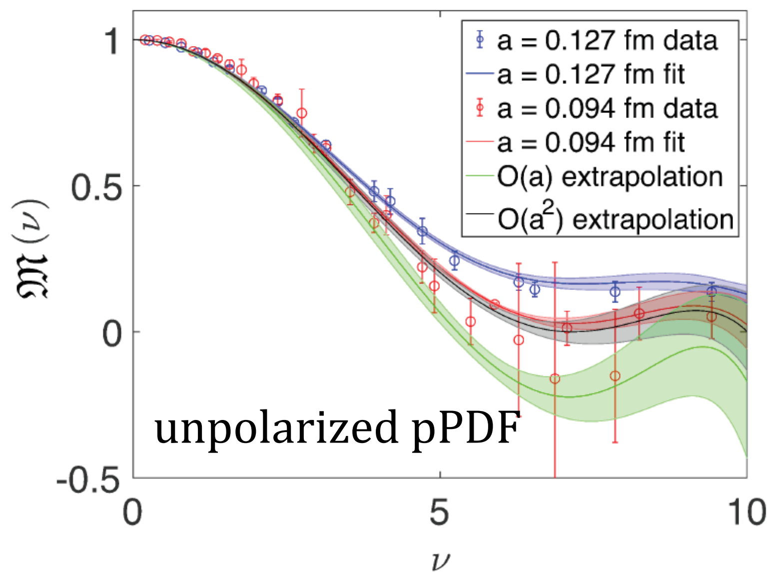
§ Finite-volume study in unpolarized pseudo-PDFs

- 2+1f clover, $M_\pi \approx \mathbf{415}$ MeV, $a \approx 0.127$ fm
- Two volumes used: $L \approx 3, 4.5$ fm

B. Joo et al (Jlab/W&M) 1908.09771,



§ Also see strong lattice-spacing dependence



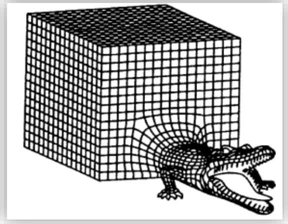
§ Lattice artifacts are sensitive to the simulated QCD vacuum

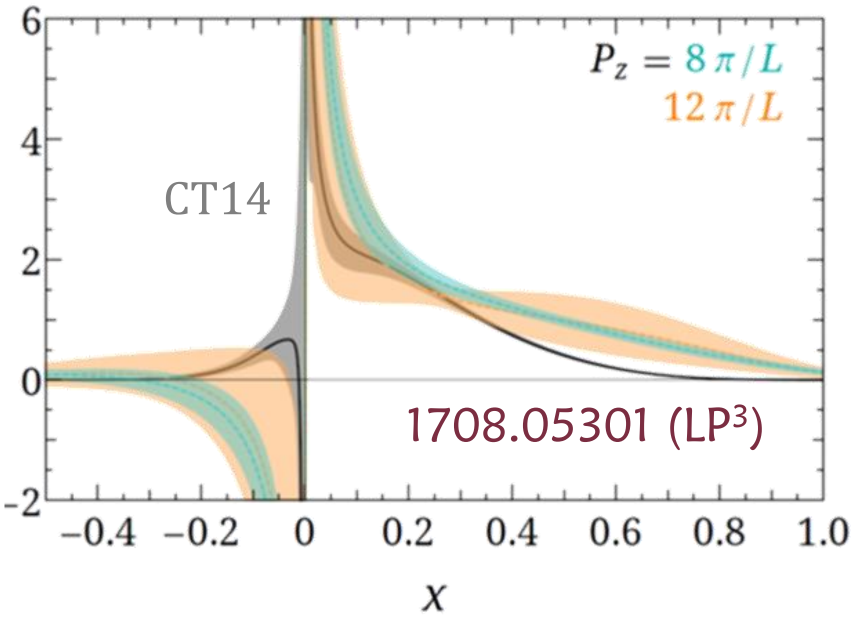
- Each group will have to check their own systematics carefully

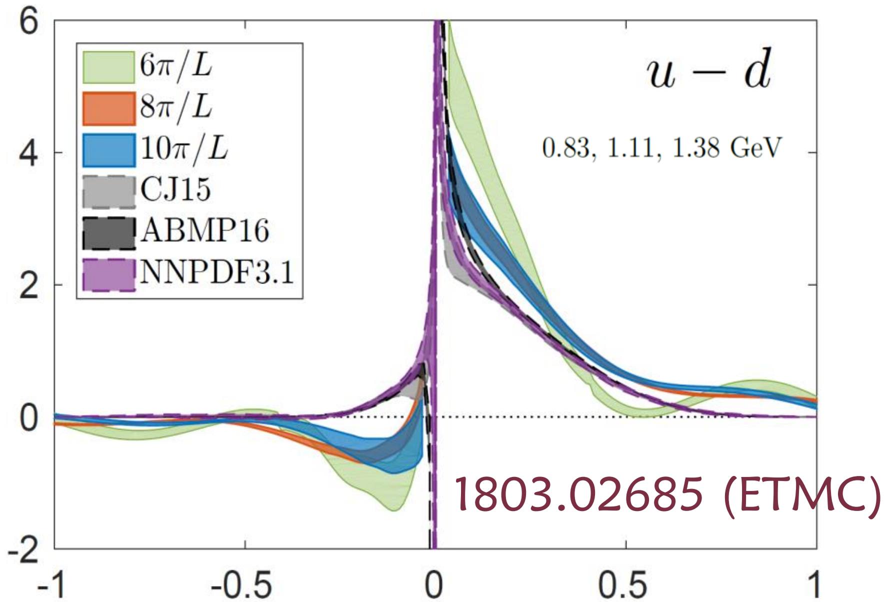
Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





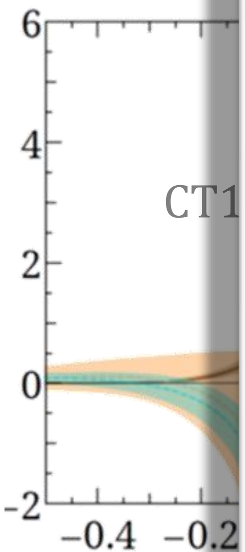


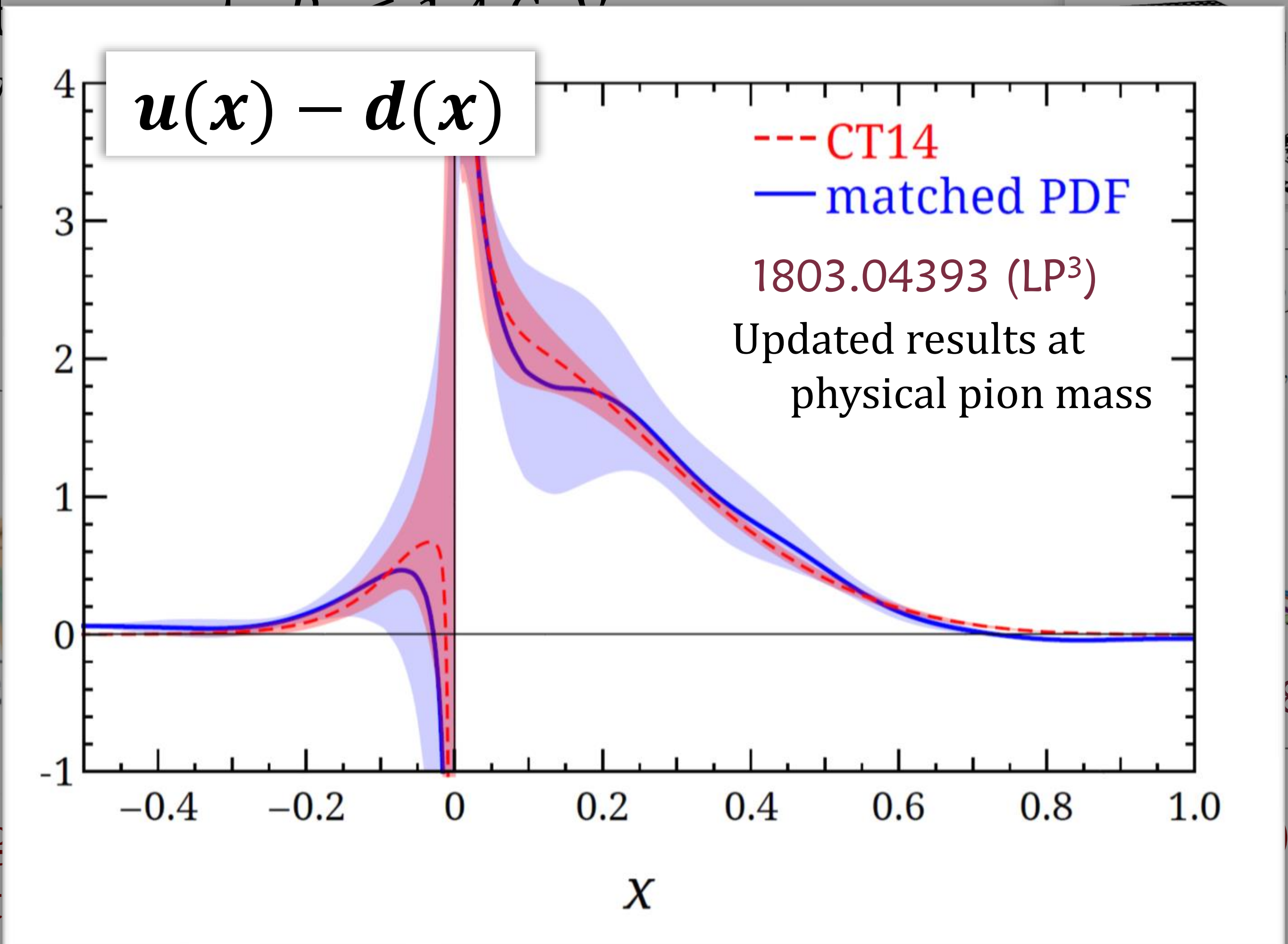
Not using parametrization (e.g. $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.

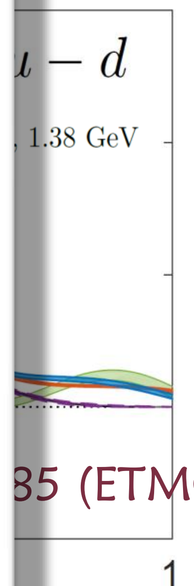
Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost
- Study







35 (ETMC)

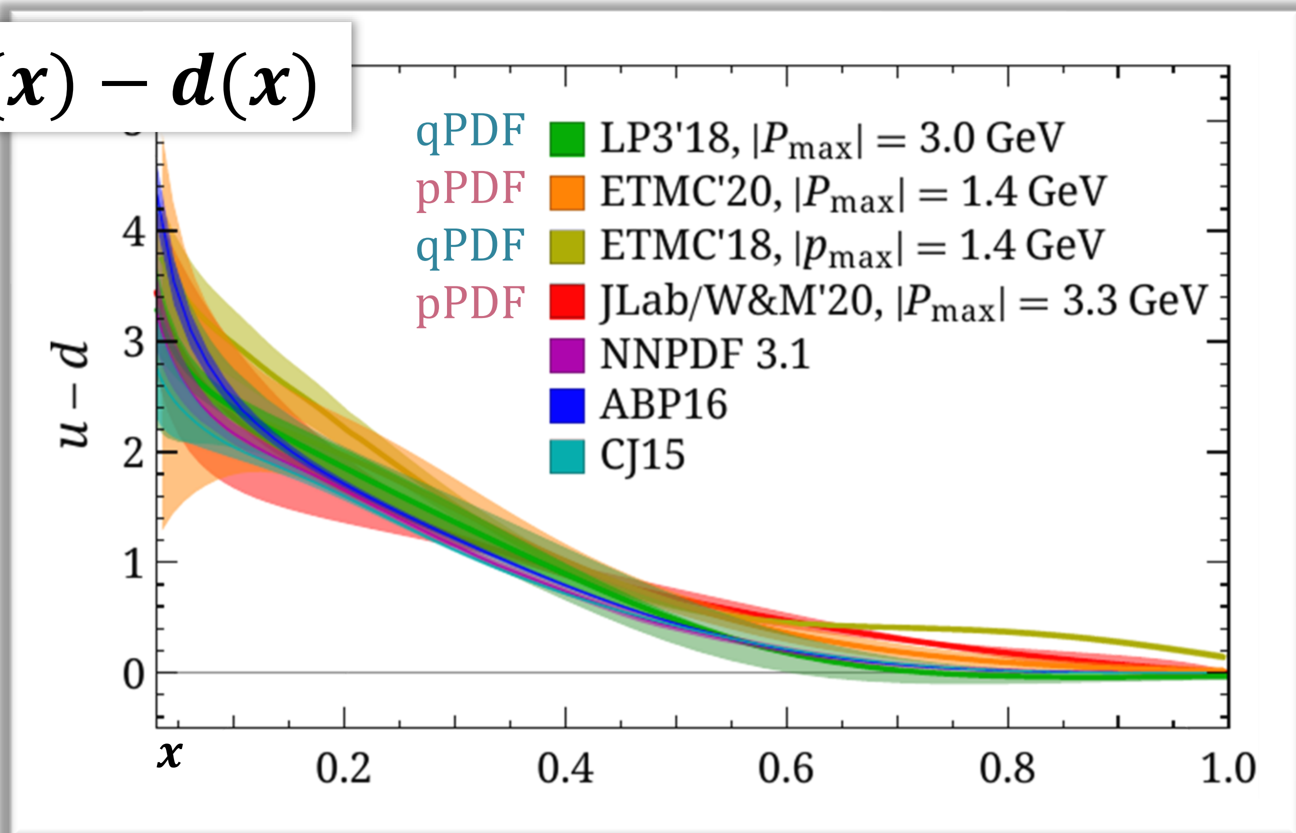
Not using
Less pret
less likely to exactly coincide with global fits.

Physical Pion Mass Results

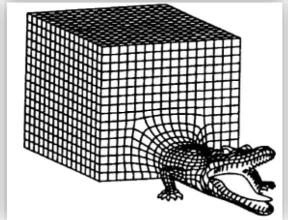
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$u(x) - d(x)$



2006.08636, PDFLattice2019 report



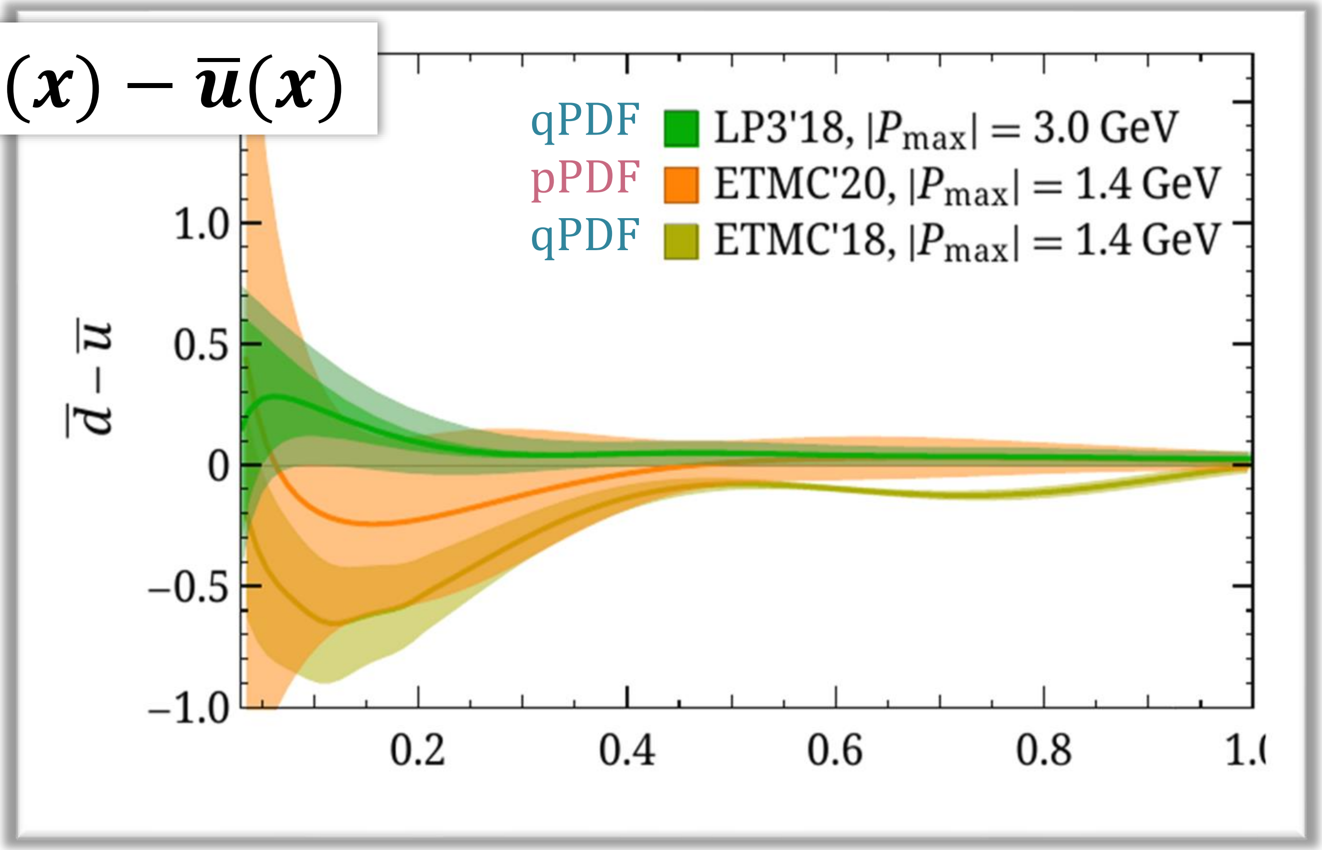
Finite volume,
Discretization,
…

Physical Pion Mass Results

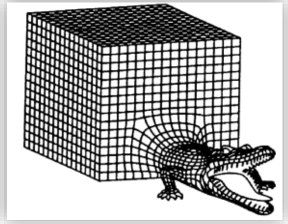
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



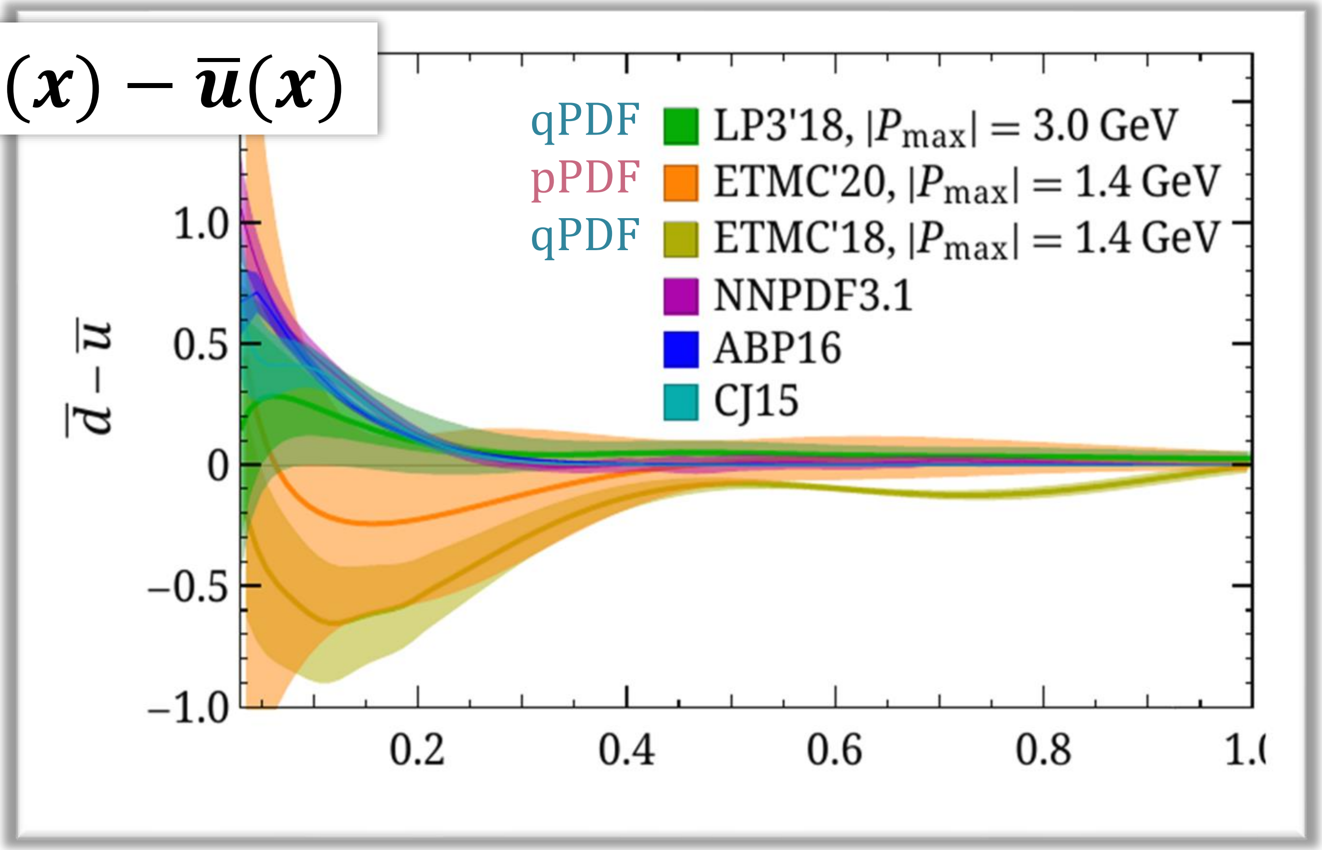
Finite volume, Discretization, …

Physical Pion Mass Results

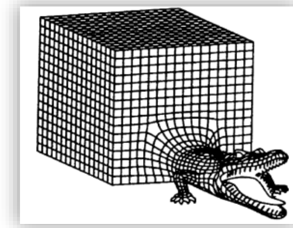
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



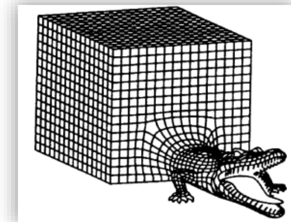
Finite volume,
Discretization,
…

Polarized PDFs

§ Summary of physical pion mass results

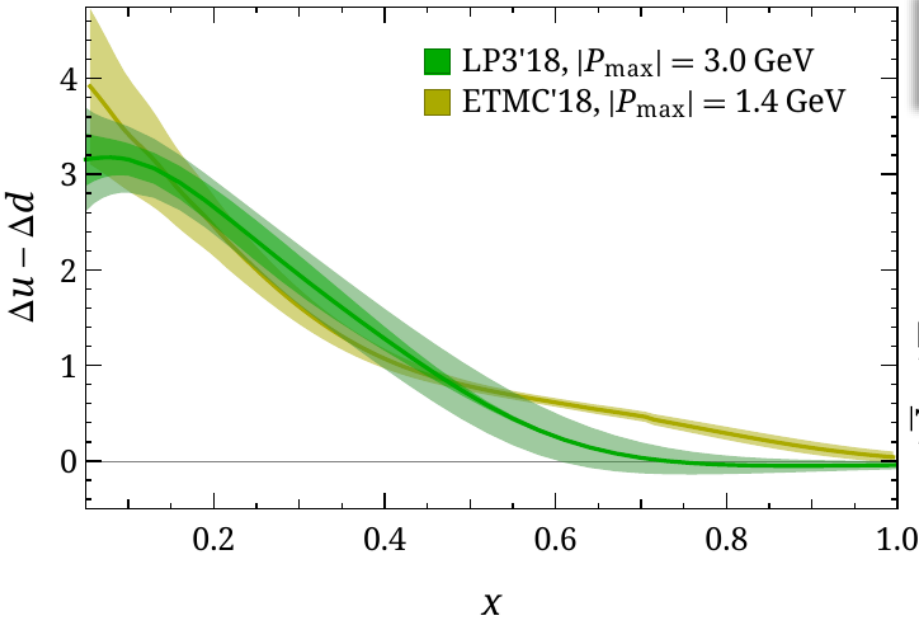
- Quasi-PDF method only

Helicity
long. polarized

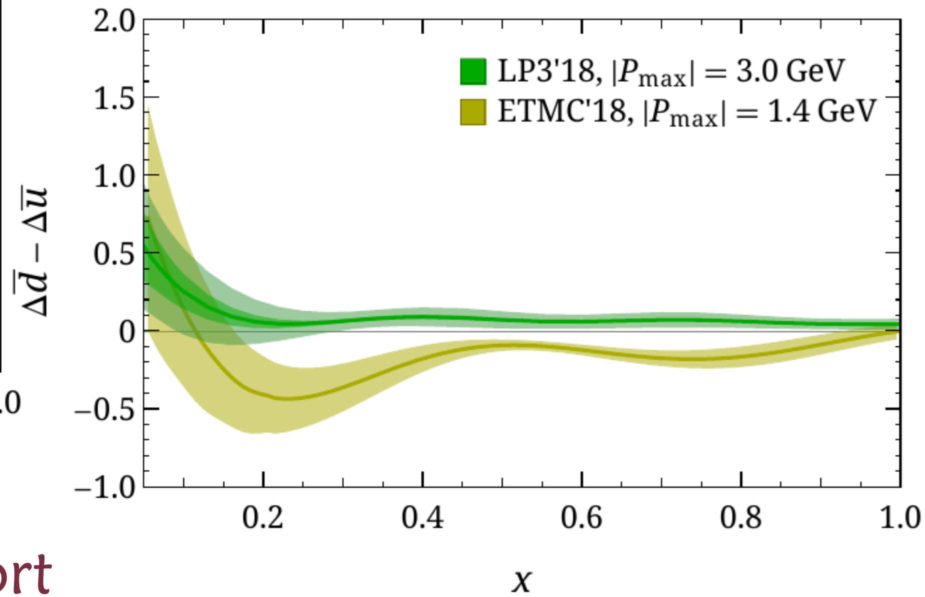


Finite volume,
Discretization,
…

$\Delta u(x) - \Delta d(x)$



$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$



2006.08636, PDFLattice2019 report

Polarized PDFs

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- Quasi-PDF method only

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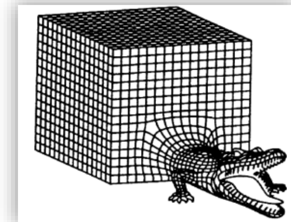
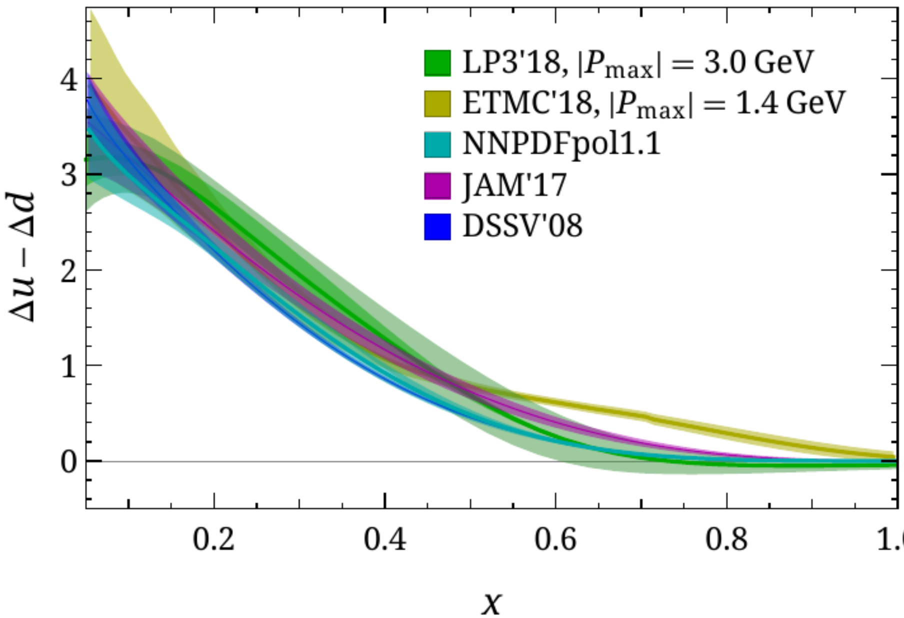
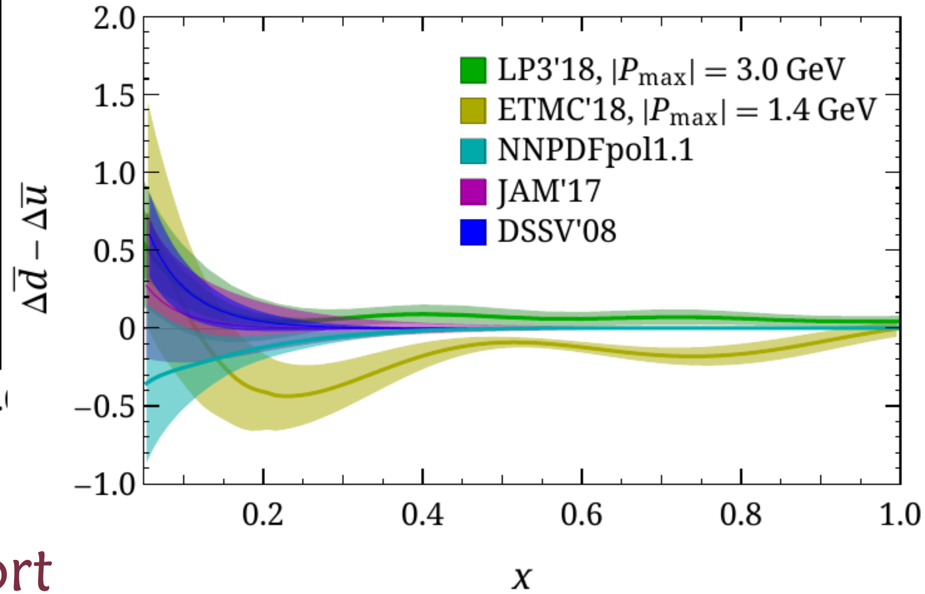


Finite volume,
Discretization,
…

$$\Delta u(x) - \Delta d(x)$$



$$\Delta \bar{u}(x) - \Delta \bar{d}(x)$$



2006.08636, PDFLattice2019 report

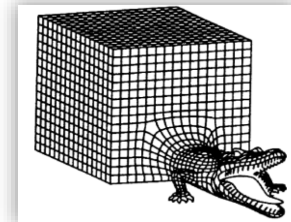
Polarized PDFs

§ Summary of physical pion mass results

- Quasi-PDF method only

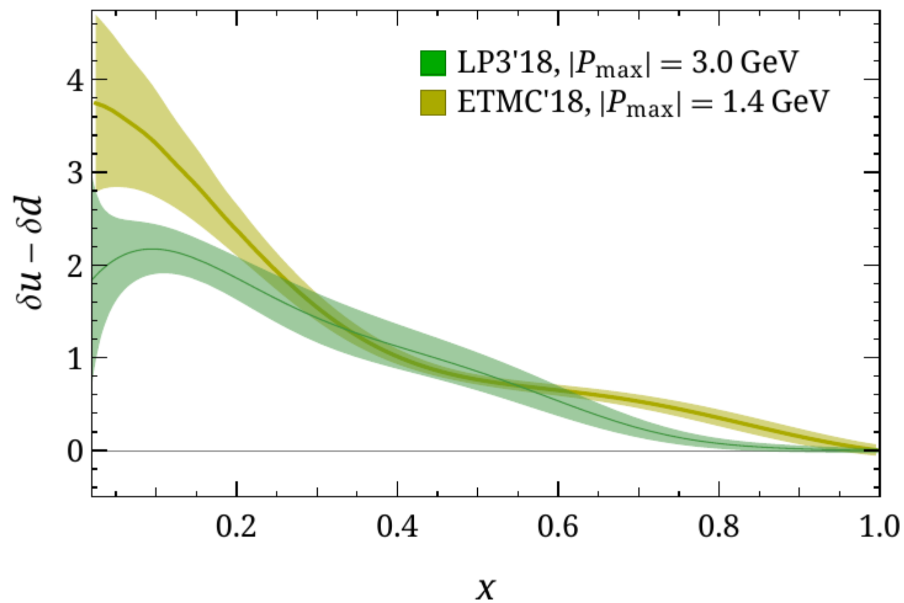
$\boldsymbol{\delta u(x) - \delta d(x)}$

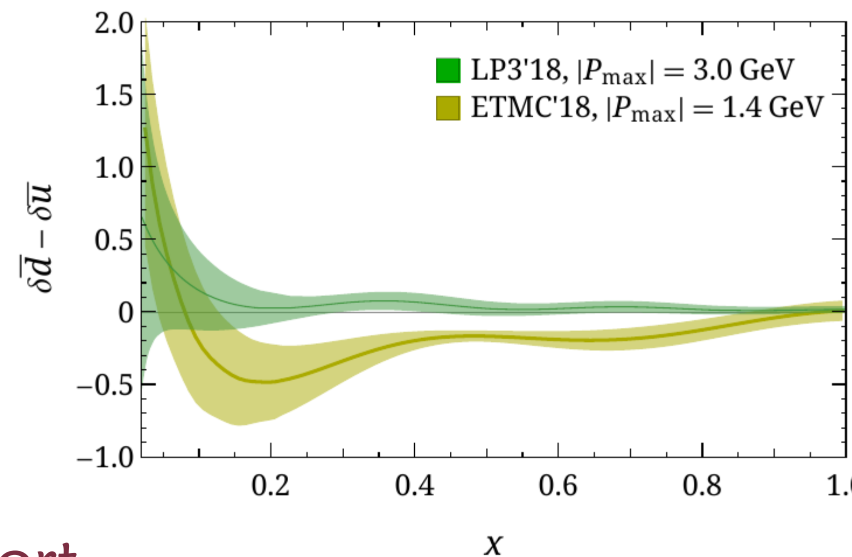
Transversity



Finite volume,
Discretization,
…

$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$





2006.08636, PDFLattice2019 report

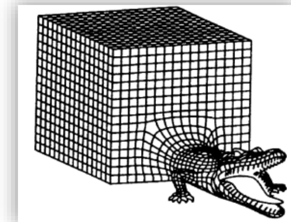
Polarized PDFs

§ Summary of physical pion mass results

- Quasi-PDF method only

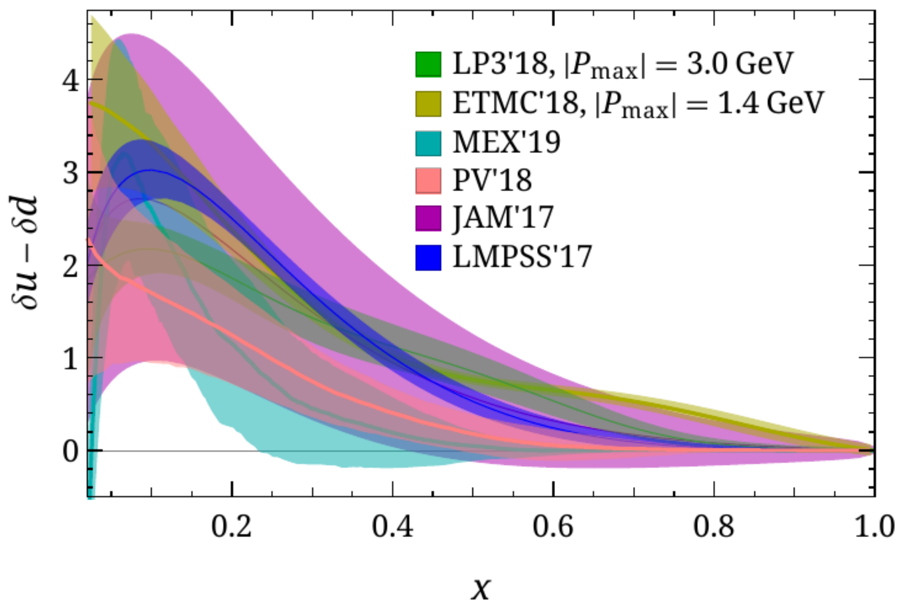
$$\boldsymbol{\delta u(x) - \delta d(x)}$$

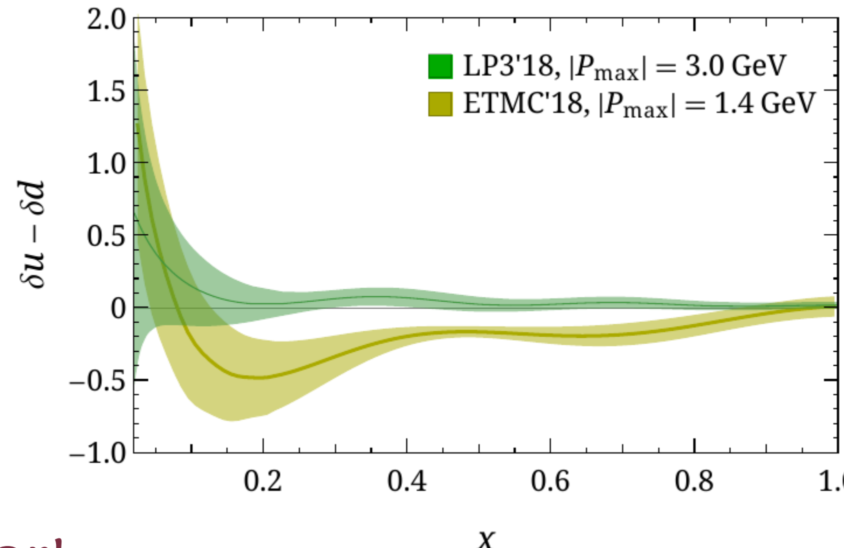
Transversity



Finite volume,
Discretization,
...

$$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$$



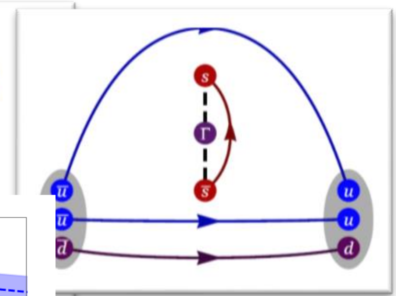


2006.08636, PDFLattice2019 report

First Lattice Strange PDF

§ Large uncertainties in global PDFs

$$h^R(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx e^{ixzPz} \int_{-1}^{1} \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu R}{\mu}, \frac{\mu}{yPz}, \frac{p_z^R}{yPz}) q(y,\mu = 2\text{ GeV})$$

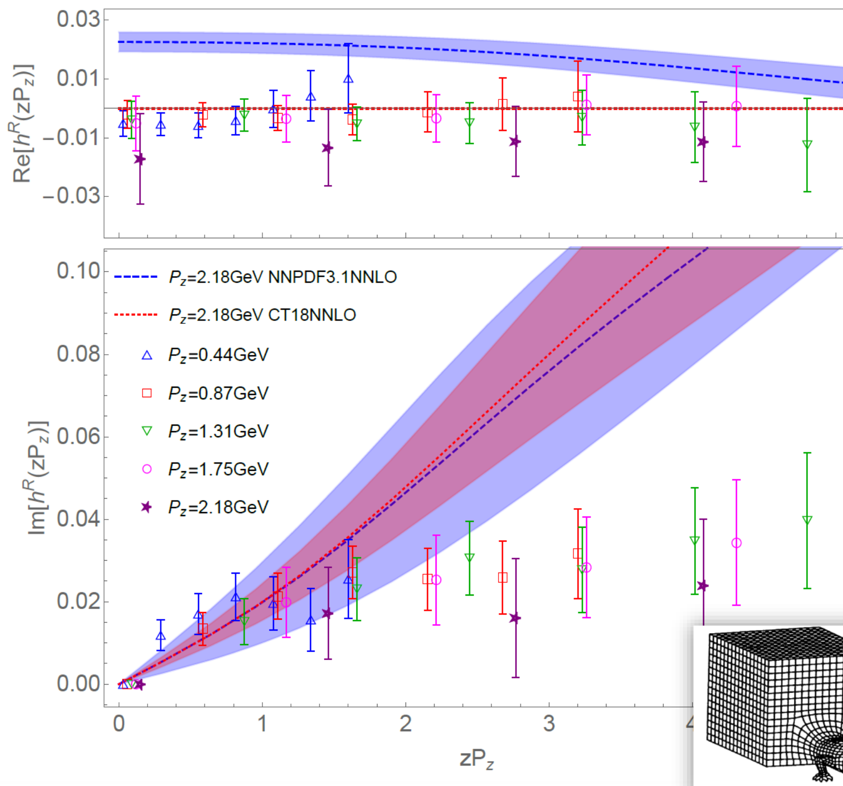


$$\text{Re}[h(z)] \propto$$

$$\int dx\, (s(x) - \bar{s}(x)) \cos(xzP_z)$$

$$\text{Im}[h(z)] \propto$$

$$\int dx\, (s(x) + \bar{s}(x)) \sin(xzP_z)$$



- symmetric $s - \bar{s}$ distribution.
- smaller momentum fraction.

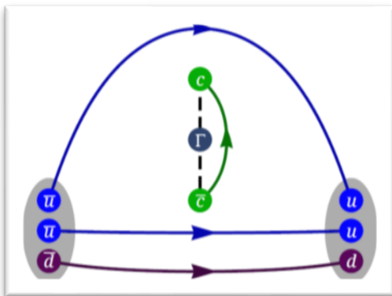
Rui Zhang (MSU)

Slide by Rui Zhang @ DNP2020

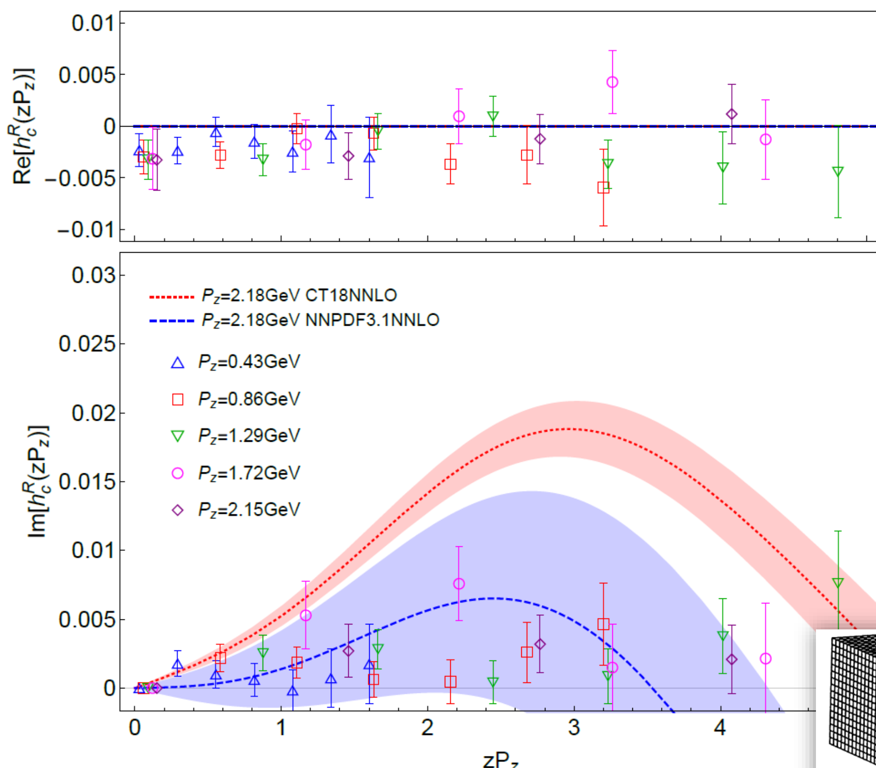
First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method

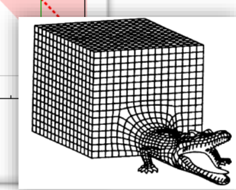
2005.12015, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Rui Zhang (MSU)



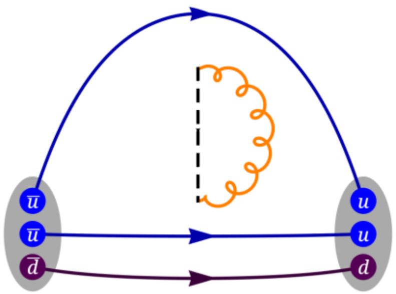
Gluon PDF in Nucleon

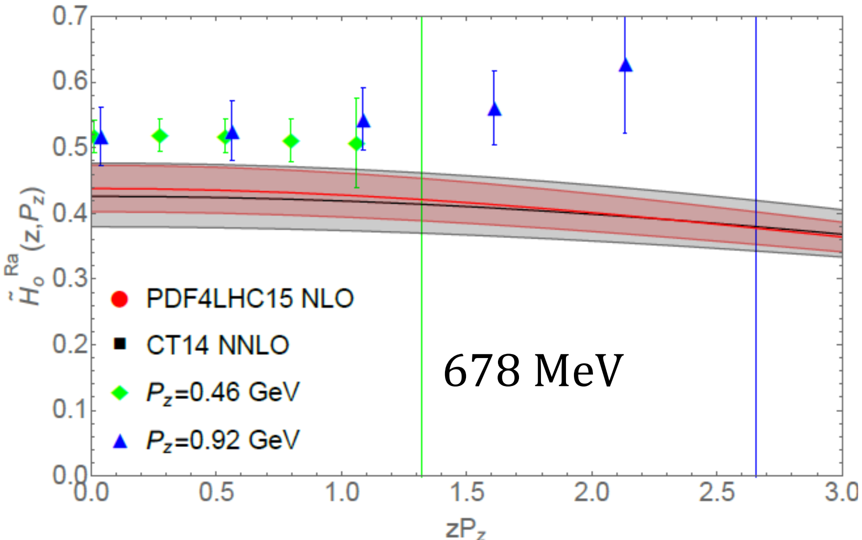
§ Pioneering first glimpse into gluon PDF using LaMET

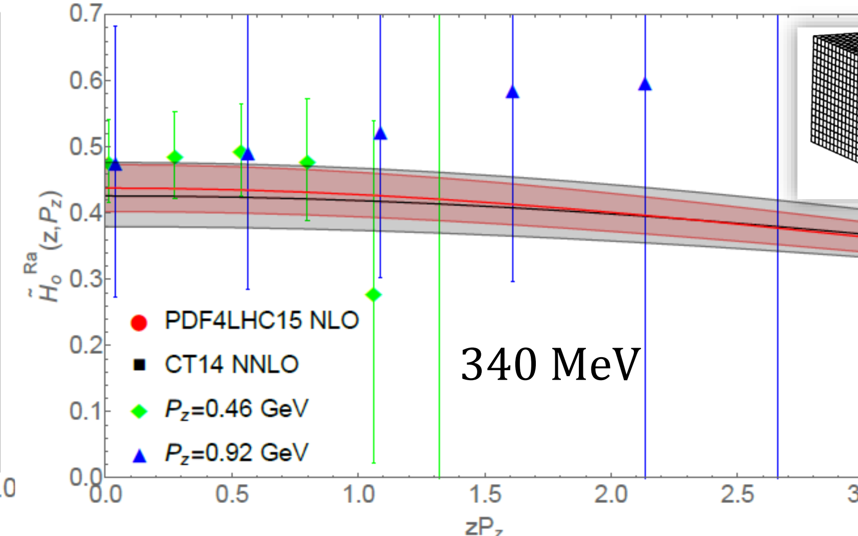
- Lattice details: overlap/2+1DWF, <u>0.16</u>fm, 340-MeV sea pion mass
- Study strange/light-quark

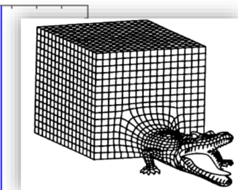
Fan. et al, Phys.Rev.Lett. 121, 242001 (2018)

- Promising results using coordinate-space comparison, but signal does not go far in *z*
- Hard numerical problem to be solved









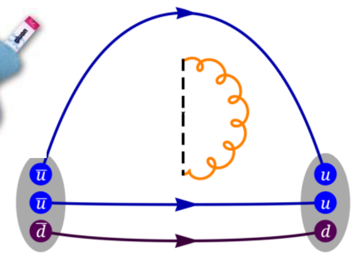
(plot by Zhouyou Fan)

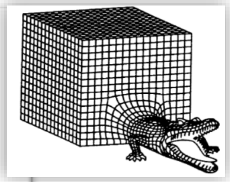
Gluon PDF in Nucleon

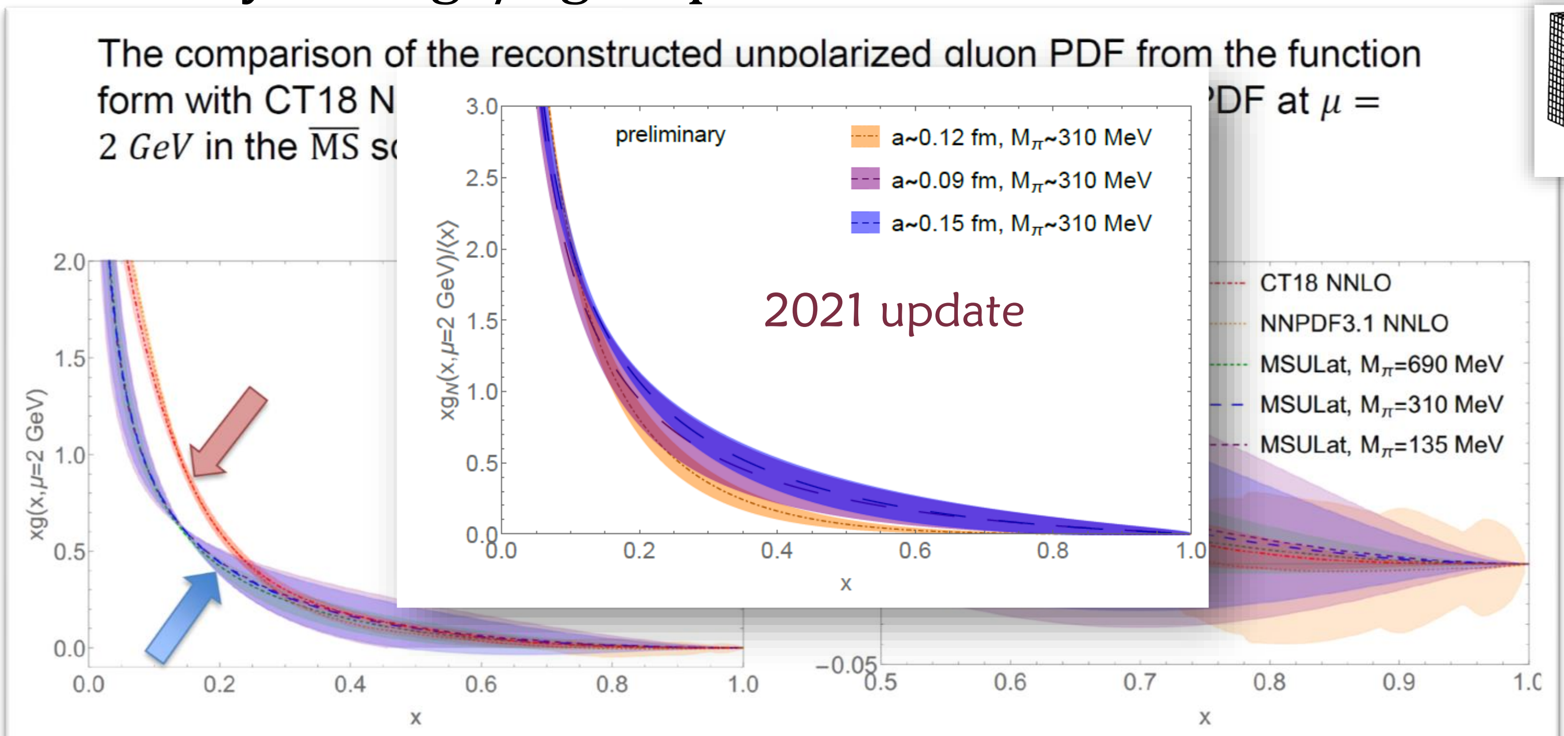
§ Gluon PDF using pseudo-PDF

- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

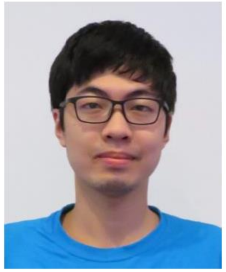
Z. Fan. et al (MSULat), 2007.16113







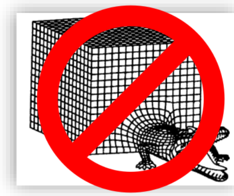
Slide by Zhouyou Fan @ DNP 2020



G: Zhouyou Fan

First Continuum PDF

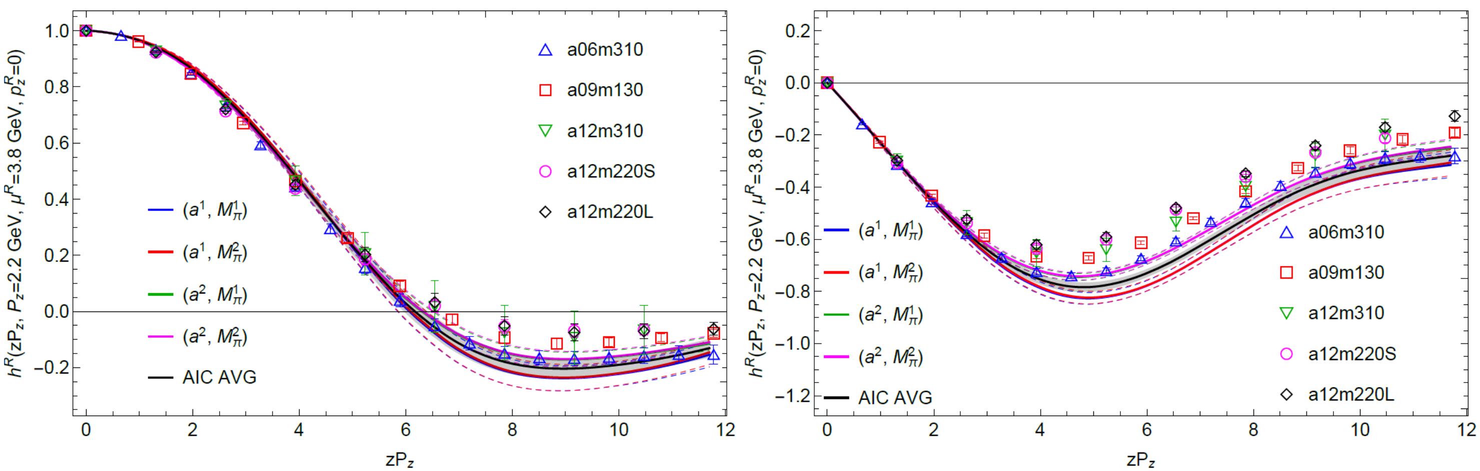
§ Nucleon PDFs using quasi-PDFs in the continuum limit



- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV

2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit



First Continuum PDF

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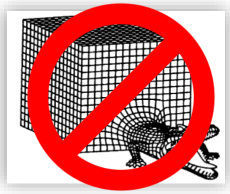

2011.14971, HL et al (MSULat)

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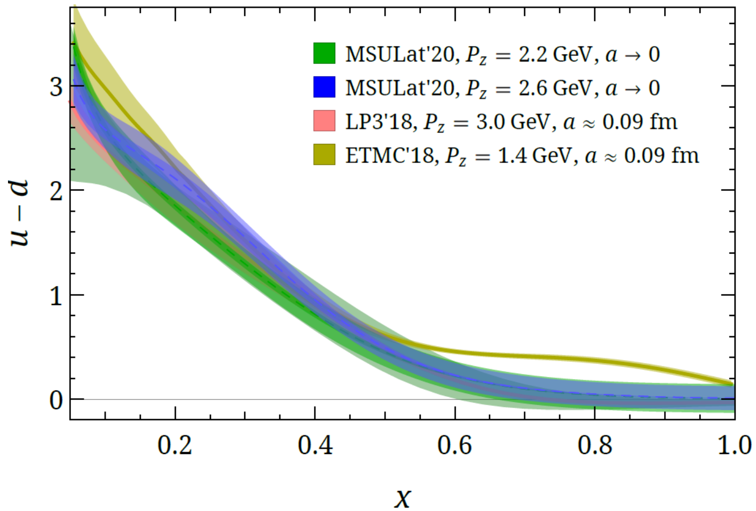
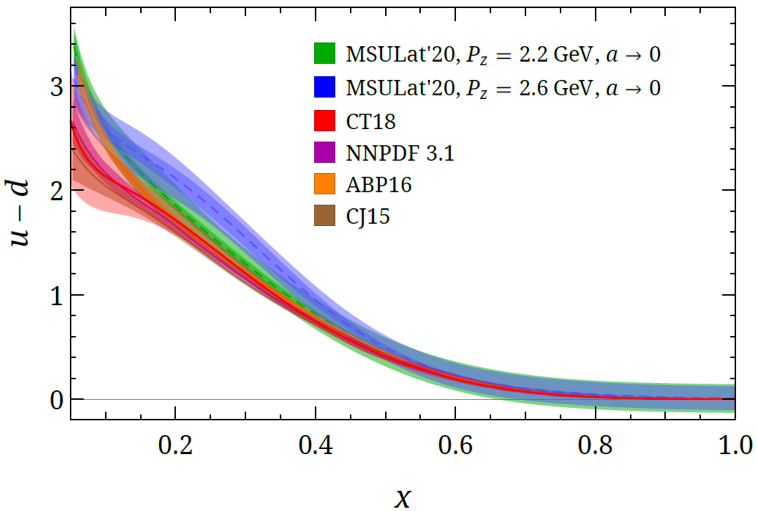
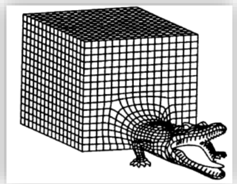




Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

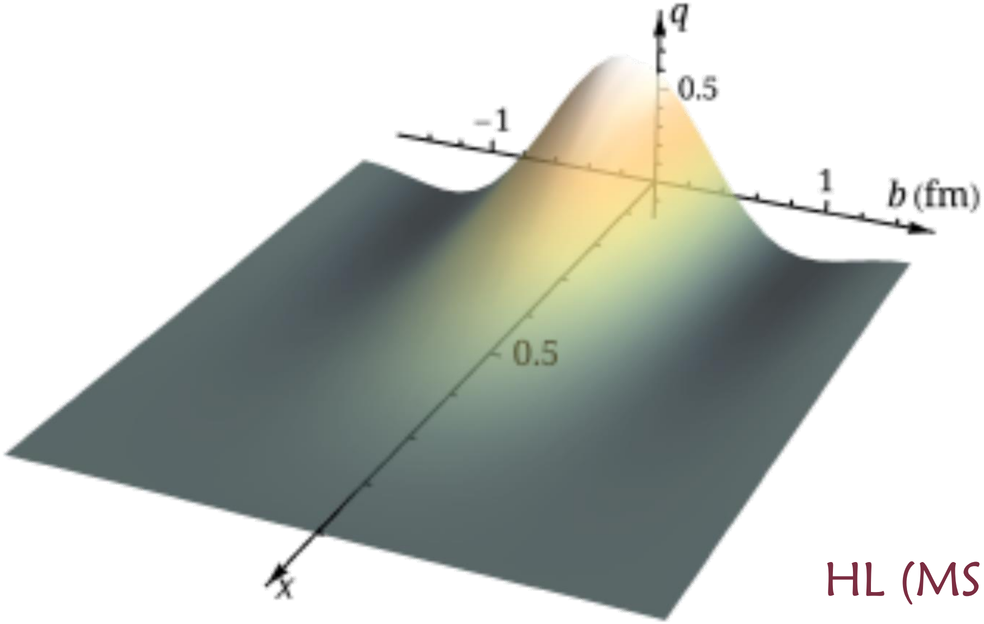
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



finite-volume,
discretization,
…

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$

M. Burkardt, hep-ph/0207047

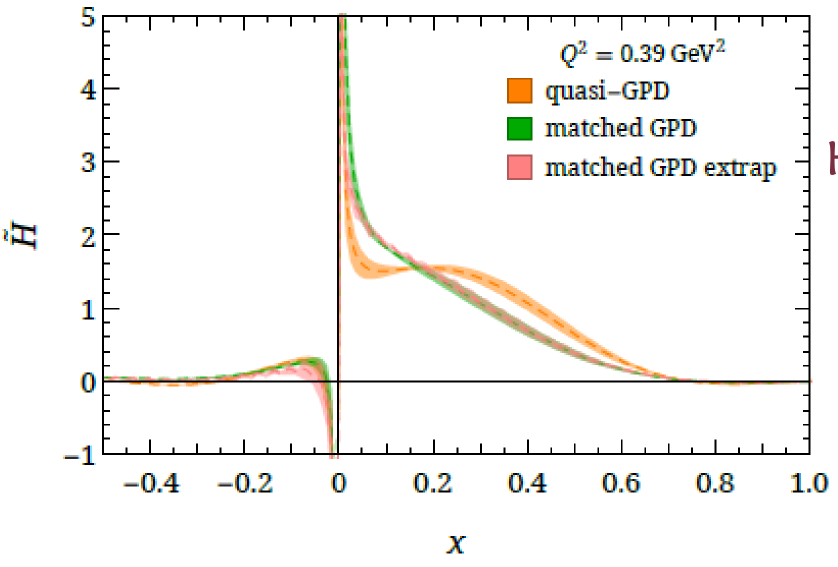


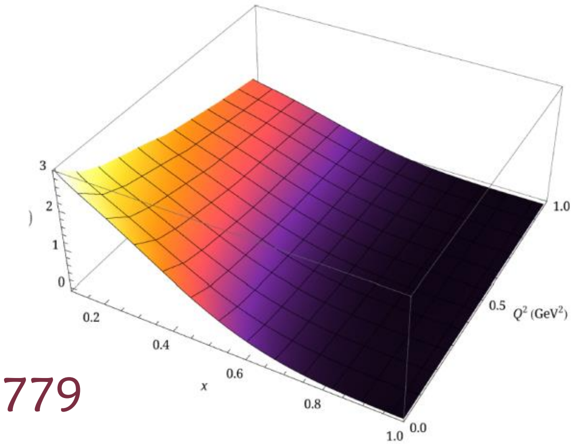
HL (MSULat), 2008.12474

Nucleon Polarized GPDs

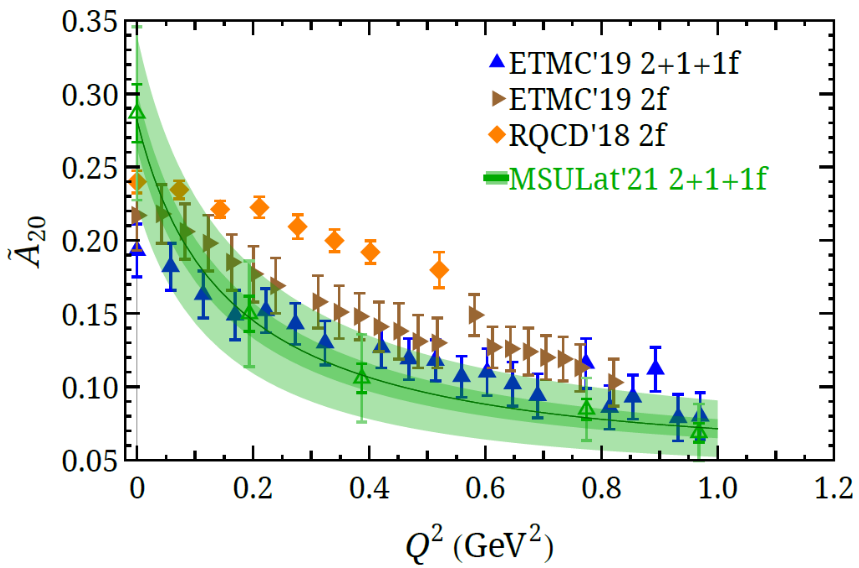
§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results





HL (MSULat), 2110.06779



"Extrap": $c_1(-izP_z)^{-d_1} + c_2 e^{izP_z}(izP_z)^{-d_2}$

X. Ji et al, 2008.03886