Snowmass conference
Feb 23 2022

Generalized Global Symmetries

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Snowmass white paper with
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Symmetry

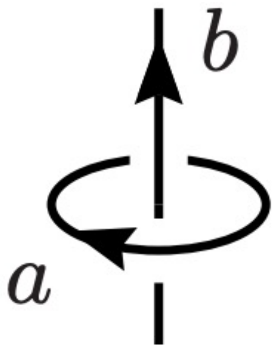
- Symmetry has proven, from time and again, to be of fundamental importance for describing Nature.
- In recent years, there has been a revolution in our understanding of global symmetries.
- The notion of global symmetry has been generalized in different directions.
- These generalized global symmetries are some of the few **universally applicable** tools to analyze general quantum systems, not limited to supersymmetric or solvable models.

Generalized global symmetries

- These new symmetries lead to several surprising consequences:
  - generalized ‘t Hooft anomaly matching conditions
  - new implications for the phase diagram of gauge theories
  - new organizing principles of topological phases in condensed matter physics
  - new dualities
- Active collaboration between experts from high energy physics, condensed matter physics, quantum gravity, and mathematics.
- In this talk I’ll discuss only some of these developments. Please see the upcoming white paper for more references. I apologize in advance for the variety of fascinating papers that are not discussed below.

Many other generalizations of global symmetries not discussed here, e.g. dipole symmetry, asymptotic symmetry,…

Generalizations



Higher-form symmetries

e.g. center symmetry in gauge theory

Subsystem symmetries

e.g. fractons

Non-invertible symmetries

e.g. Ising model, 4d Maxwell theory, Yang-Mills,…

$$a \times b = \sum_c N_{ab}^c c$$

Noether current

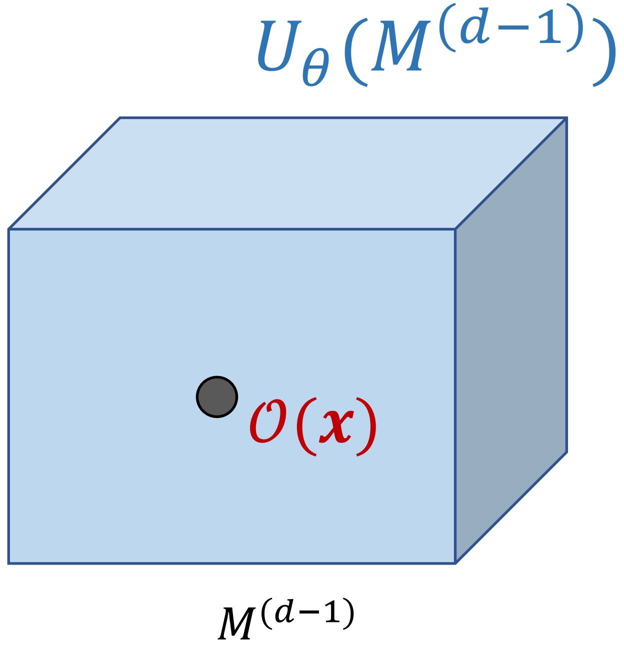
- Consider a relativistic QFT in d spacetime dimensions. Suppose it has an ordinary $U(1)$ global symmetry with a $d-1$ form Noether current $j^{(d-1)}(x)$ satisfying the conservation equation:

$$dj^{(d-1)} = 0$$

- The conserved, unitary symmetry operator is an integral over a codimension-1 manifold $M^{(d-1)}$ in spacetime (e.g. the entire space at a fixed time)

$$U_\theta(M^{(d-1)}) = \exp\left(i\theta \oint_{M^{(d-1)}} j^{(d-1)} \right)$$

- Thanks to the conservation equation, the dependence on $M^{(d-1)}$ is topological: it is invariant under small deformations.
- It acts on a charged local operator $\mathcal{O}(x)$ by enclosing the latter.



Ordinary global symmetry

Properties of symmetry op.	Ordinary symmetry $U_g\left(M^{(d-1)}\right)$	**Example**: $U(1)$ $\exp\left(i\theta \oint_{M^{(d-1)}} j^{(d-1)}\right)$
Codimension in spacetime	1	$j^{(d-1)}$ is a $d-1$-form
Topological	yes	$j^{(d-1)}$ is closed, $dj^{(d-1)} = 0$
Fusion rule	group $U_{g_1}U_{g_2} = U_{g_1 g_2}$	$U(1)$ $U_{\theta_1}U_{\theta_2} = U_{\theta_1+\theta_2}$

Next, we generalize the ordinary global symmetry by modifying the above conditions.

Generalized global symmetries

Properties of symmetry op.	Ordinary symmetry	Higher-form symmetry	Subsystem symmetry	Non-invertible symmetry
Codimension in spacetime	1	> 1	> 1	≥ 1
Topological	yes	yes	not completely but conserved in time	yes
Fusion rule	group $g_1 \times g_2 = g_3$	group $g_1 \times g_2 = g_3$	group $g_1 \times g_2 = g_3$	fusion ring $a \times b = \sum_c N_{ab}^c c$

Higher-Form Symmetry

Global symmetries and generalizations

Properties of symmetry op.	Ordinary symmetry	Higher-form symmetry	Subsystem symmetry	Non-invertible operator
Codimension in spacetime	1	> 1	> 1	≥ 1
Topological	yes	yes	not completely but conserved in time	yes
Fusion rule	group $g_1 \times g_2 = g_3$	group $g_1 \times g_2 = g_3$	group $g_1 \times g_2 = g_3$	fusion ring $a \times b = \sum_c N_{ab}^c c$

Higher-form global symmetry

[Gaiotto-Kapustin-Seiberg-Willett 2014,…]

Properties of symmetry op.	q-form symmetry $U_g\left(M^{(d-q-1)}\right)$	**Example**: $U(1)$ $\exp\left(i\theta \oint_{M^{(d-q-1)}} j^{(d-q-1)}\right)$
Codimension in spacetime	$q+1$	$j^{(d-q-1)}$ is a $d-q-1$-form
Topological	yes	$j^{(d-q-1)}$ is closed, $dj^{(d-q-1)} = 0$
Fusion rule	group $U_{g_1}U_{g_2} = U_{g_1 g_2}$	$U(1)$ $U_{\theta_1}U_{\theta_2} = U_{\theta_1+\theta_2}$

The charged objects are q-dimensional.

Higher-form symmetries and anomalies

[Gaiotto-Kapustin-Seiberg-Willett 2014,…]

- The simplest example of higher-form symmetries is the one-form center symmetry in gauge theory. E.g. $\mathbb{Z}_N$ center symmetry in $SU(N)$ Yang-Mills theory. It acts on the Wilson lines, rather than the local operators.
- Higher-form global symmetries can have anomalies, which prevent us from gauging them. These anomalies lead to generalized 't Hooft anomaly matching conditions. Nontrivial constraints on renormalization group flows.
- E.g. $SU(2)$ pure gauge theory at $\theta = \pi$ has a mixed anomaly between CP and the $\mathbb{Z}_2$ one-form center symmetry. The low energy phase cannot be trivially gapped with a non-degenerate ground state. (Contrast with the expectation at $\theta = 0$.) [Gaiotto-Kapustin-Komargodski-Seiberg 2017]

Higher-groups

- Higher-group symmetry: mixture of higher-form symmetries of different degrees [Kapustin-Thorngren 2013, Tachikawa 2017, Cordova-Dumitrescu-Intriligator 2018-2020, Benini-Cordova-Hsin 2018,…].
- Similar to group extensions, but for symmetries of different form degrees.
- Higher-groups exist in many quantum systems in diverse dimensions: 2+1d Chern-Simons matter theories, 3+1d gauge theories, 5+1d supersymmetric theories…
- Dynamical consequences. E.g. Constraints on the 3+1d axion-Yang-Mills theory [Hidaka-Nitta-Yokokura 2020-2021, Brennan-Cordova 2020].

Subsystem Symmetry

Generalized global symmetries

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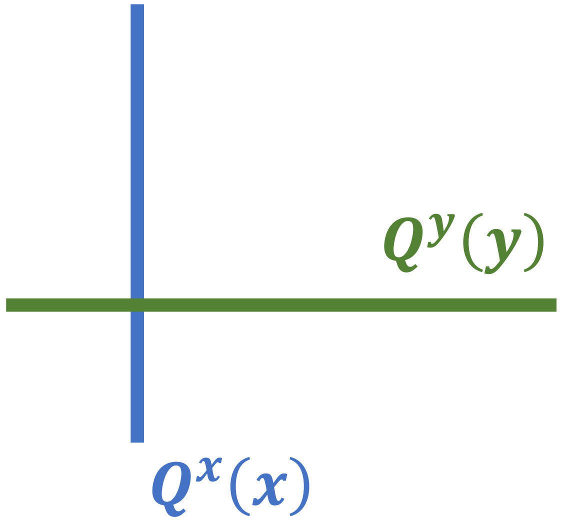
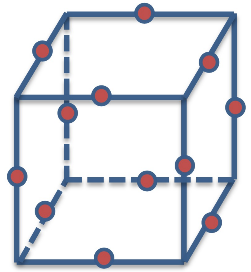
Subsystem symmetry

- There are many interesting lattice models, such as fractons, exhibiting subsystem symmetries.
- The subsystem symmetry charges are supported on certain higher-codimensional manifolds L in space (E.g. straight lines on a plane) [..., Paramekanti-Balent-Fisher 2002, ...]. They depend NOT only on the topology of the manifolds.
- The number of subsystem symmetry charges generally depends on the number of lattice points.
- Low energy observables are sensitive to short distance details: UV/IR mixing [Gorantla-Lam-Seiberg-SHS 2021].



Fractons



- Fractons [Chamon 2005, Haah 2011, …] are a large class of 3+1d gapped lattice spin models with many peculiar features.
- They do **not** admit a conventional continuum field theory limit. Challenge the canonical paradigm that QFT describes low energy phases.
- Large ground state degeneracy $\sim 2^{\# L}$, where L is the number of lattice sites in every direction.
- The peculiarities of fractons can be universally captured by the underlying subsystem symmetries. For example, the large ground state degeneracy is a direct consequence of the anomalies of the subsystem symmetries [Seiberg-SHS 2020, Burnell-Devakul-Gorantla-Lam-SHS 2021].
- Many fracton models can also be realized as the gauge theory of subsystem symmetries [Vijay-Haah-Fu 2016, Williamson 2016, Slagle-Kim 2017, Shirley-Slagle-Chen 2018,…]

Non-invertible Symmetries

Generalized global symmetries

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Wilson lines for finite gauge groups

- Consider a QFT with a finite gauge group G (e.g. $\mathbb{Z}_N, S_N$, etc.).
- The topological Wilson lines W_R are labeled by the irreducible representations R of G.
- The fusion of the Wilson lines is generally **NOT** a group! (E.g. the representation ring of S_3: $2 \otimes 2 = 1 \oplus 1_- \oplus 2$)

$$W_{R_a} \times W_{R_b} = \sum_{c \in irreps} N_{ab}^c \, W_{R_c}$$

More than one term on RHS

- Do these Wilson lines generate a global symmetry?

Non-invertible symmetries

- More generally, a topological operator L is called non-invertible if there is no inverse L^{-1} such that $L \times L^{-1} = 1$.
- It has been advocated that the non-invertible topological operators should be viewed as generalizations of ordinary global symmetries [Bhardwaj-Tachikawa 2017, Chang-Lin-SHS-Wang-Yin 2018,…].
- Non-invertible symmetries in many familiar systems:
    - 1+1d Ising model [Frohlich-Fuchs-Runkel-Schweigert 2006,…]
    - 3+1d gauge theories (Maxwell, Yang-Mills, $\mathcal{N} = 4$ super Yang-Mills) [Choi-Cordova-Hsin-Lam-SHS 2021, Kaidi-Ohmori-Zheng 2021]
    - 3+1d $\mathbb{Z}_N$ lattice gauge theories [Koide-Nagoya-Yamaguchi 2021]

Non-invertible symmetries

Why should we think of the non-invertible topological operators as generalized symmetries?

- Some non-invertible operators can be gauged [Brunner-Carqueville-Plencner 2014].
- They can have generalized anomalies, which lead to generalized 't Hooft anomaly matching conditions. They result in nontrivial constraints on the renormalization group flows [Chang-Lin-SHS-Wang-Yin 2018, Thorngren-Wang 2019, 2021, Komargodski-Ohmori-Roumpedakis-Seifnashri 2020, …].
  - Analytic obstruction to a trivially confining phase in 3+1d gauge theories [Choi-Cordova-Hsin-Lam-SHS 2021].

Conclusion

- We have discussed three generalizations of global symmetries, higher form symmetries, subsystem symmetries, and non-invertible symmetries. Many other generalizations.
- This more general perspective of global symmetry unifies many known phenomena into a coherent framework.
  - Generalized global symmetries and their anomalies provide an invariant characterization of many topological phases of matter such as fractons.
- More importantly, they lead to new dynamical consequences that are otherwise obscured.
  - Generalizations of the ‘t Hooft anomaly matching condition lead to nontrivial constraints on renormalization group flows.
- **New** symmetries in **new** and **old** QFTs!

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Thank you for listening!