Instanton Effects in Semi-Realistic MSSM Quivers and F-theory

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Based on:
[0905.3379, 0909.4292, 0910.2239] with M. Cvetič and R. Richter
[1001.3148] with M. Cvetič, P. Langacker and R. Richter
and [1003.5337] with M. Cvetič and I. García-Etxebarria

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Strings and Particle Physics: Motivation

- String vacua from different corners of the landscape have been fruitful in building semi-realistic models of particle physics.
- Heterotic string compactifications on Calabi-Yau threefolds:
  - Exceptional gauge symmetry and chiral matter.
  - Theory of closed strings only.
- Type II orientifold compactifications localize gauge degrees of freedom on D-branes, on which open strings can end.
  - "Geometric" picture of particle physics.
  - Can realize $U(N)$, $Sp(2N)$ and $SO(2N)$ gauge symmetry.
  - Chiral matter at the intersection of gauge D-branes.

$\rightsquigarrow$ Realistic gauge symmetry and matter content, but what about Yukawa couplings, mass hierarchies, moduli stabilization, etc?

Many of these details can be accounted for by stringy instanton corrections to the superpotential!

Outline

Type II: Phenomenology and the Quiver Landscape

- Lightning Review: Intersecting branes and D-instantons.
- Mass Hierarchies:
  - ⇝ Segregation of MSSM matter fields.
  - ⇝ Factorizable Yukawa textures.
- The Quiver Landscape: Systematic Analyses
  - At the level of couplings.
  - Impose many "top-down" and phenomenological constraints.
  - ⇝ Three-stack and four-stack w/ MSSM $+ 3N_R$
  - ⇝ Four-stack and five-stack w/ exact MSSM

F-theory GUTs, Lifts and Instanton Effects

- What is F-theory, and why are F-theory GUTs nice?
- Fourfold geometry: a lift of a IIb GUT
- Instanton effects: counting zero modes

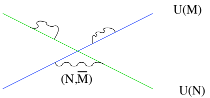
Summary and Outlook

Intersecting D6-braneworlds

D6-branes fill out spacetime and wrap three-cycles π_a in CY_3.

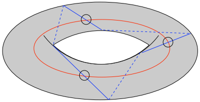
- Gauge Group: a stack of N D-branes on a generic three-cycle gives rise to U(N) = SU(N) x U(1) gauge theory in 4D.

- Chirality: presence of chiral matter at the intersection of two D6-brane stacks.



Representation	Multiplicity
$(\overline{a}, b)$	$\pi_a \circ \pi_b$
$(\overline{a}, \overline{b})$	$\pi_a \circ \pi'_b$
$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a\right)$
$\Box\Box_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a\right)$

- Family Replication: In the compact space two stacks of D6-branes may intersect multiple times



Intersecting D-brane Worlds

- Must introduce orientifold planes to cancel the RR charges of the D-branes. Requires an antiholomorphic involution.
  - $\rightsquigarrow$ The involution also defines image branes wrapping $\pi_a^{'}$.
- Generalized Green-Schwarz mechanisms lifts many of the U(1)'s, but might leave a linear combination massless.
  - $\rightsquigarrow$ important for phenomenology!

Conditions on Homology of D-branes and O-planes

Tadpole cancellation: Ramond-Ramond charges cancelled

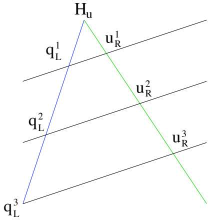
$$\leftrightarrow \sum_x N_x(\pi_x + \pi_x') = 4\pi_{O6}$$

Massless U(1): $U(1) = \sum_x q_x U(1)_x$ remains massless

$$\leftrightarrow \sum_x q_x N_x(\pi_x - \pi_x') = 0$$

Superpotential

- Yukawa couplings can be extracted from string amplitudes.
- Suppressed by worldsheet instantons $e^{-\frac{Area}{2\pi\alpha'}}$
  $\rightsquigarrow$ may give rise to observed hierarchies



But . . .

- fine-tuning to get MSSM hierarchies and realistic CKM
- various couplings are forbidden due to global $U(1)$'s.
  - $10\,10\,5_H$ couplings in $SU(5)$ GUTs
  - Majorana neutrino masses
  - μ-term

D-instantons

[Blumenhagen, Cvetič, Weigand], [Ibañez,Uranga],[Florea, Kachru, McGreevy, Saulina]

In IIa, interested in E2 instantons:

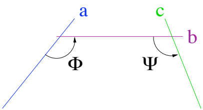
- Pointlike in spacetime, wrap a 3-cycle in CY_3.
- "Uncharged" open string zero modes in E2-E2 sector.
  - x^μ from breakdown of Poincaré invariance.
  - θ_α and $\overline{\tau}_{\dot\alpha}$ from breakdown of $\mathcal{N}=2$ preserved by CY_3
  - Deformation modes if cycle is not rigid.
- "Charged" open string zero modes in E2-D6 sector.
- Must lift or project out extra fermionic zero modes in order to have a superpotential contribution.

$\rightsquigarrow$ Rigid $O(1)$ instanton carries the correct zero modes to contribute to the superpotential. $\int d^4x d^2\theta$

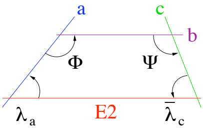
An Example

Consider three D6-brane stacks, each with $U(1)$ gauge group.

Note that the coupling $\Phi_{(-1,1,0)}\,\Psi_{(0,-1,1)}$ has non-zero charge and is thus perturbatively forbidden.



However, an $O(1)$ instanton with $[\Xi \cap \pi_a]^+ = 1, \;\; [\Xi \cap \pi_c]^- = 1$ can non-perturbatively generate the coupling.



$$M_s \int d^4x\, d^2\theta d\lambda_a\, d\overline{\lambda}_c e^{-\frac{2\pi}{l_s^3\, g_s} Vol_{E2} + \lambda_a\, \Phi\, \Psi\, \overline{\lambda}_c} \rightsquigarrow M_s\, e^{-\frac{2\pi}{l_s^3\, g_s} Vol_{E2}} \int d^4x\, d^2\theta\; \Phi\, \Psi$$

Mass Hierarchies

Observation:

- three different mass scales for up- and down-flavor quarks.
- three different mass scales for the charged leptons.
- mass hierarchy between top-quark mass and all other fermionic MSSM matter fields.
- mass hierarchy between neutrino masses and all other MSSM matter field masses.

Can D-brane compactifications naturally account for these?

Segregation of MSSM Matter Fields

[Anastasopolous, Kiritsis, Lionetto]

Idea: The families do not have to come from the same D-brane sector!

Example: Four-stack quiver with $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$
$\rightsquigarrow SU(3) \times SU(2) \times U(1)_Y$ via GS mechanism with

$$U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d$$

$q_L^{1,2} : (a, \overline{b})$

$q_L^3 : (a, b)$

$u_R^{1,2} : (\overline{a}, \overline{c})$

$u_R^3 : (\overline{a}, \overline{d})$

$H_u : (b, c)$

With matter transforming as specified to the left, the up-flavor quark Yukawa texture takes the form

$$Y_{q_L H_u u_R} = \begin{pmatrix} A & B & B \\ C & D & D \\ C & D & D \end{pmatrix}$$

Same letters denote entries which have the same global $U(1)$ charge and are of the same order

Factorizable Yukawa Matrices

[Cvetic, J.H, Richter arXiv:0909.4292, arXiv:0910.2239]

Idea: In the presence of additional vector-like zero modes, the Yukawa texture is sometimes rank one.

Example: 3 branes a, b, c wrapping π_a, π_b, π_c with $\pi_a \cap \pi_b = K \qquad \pi_b \cap \pi_c = K$.
Want to induce the perturbatively forbidden coupling $\Phi^I_{(1,-1,0)} \Psi^J_{(0,1,-1)}$.

Factorizable

Key: If the instanton has charged modes λ_b, $\overline{\lambda}_b$ present:

$\rightsquigarrow$ mass matrix of the form $M^{IJ} \sim M_s \, e^{-S^{cl}_{E2}} \, Y^I_\Phi \, Y^J_\Psi$

Point: Only one non-zero mass, of order $M_s e^{-S^{cl}_{E2}}$. Need an additional $K-1$ instantons to induce masses for all families, with the ratio of the masses being given by $e^{-S^{cl}_{E2}} : e^{-S^{cl}_{E2'}} : e^{-S^{cl}_{E2''}} : \ldots$, which depends on the volume of the cycles which the instantons wrap.

Neutrino Masses

Mechanisms for neutrino masses of the observed order:

- Type I Seesaw Mechanism:

  $$M\, N_R N_R \qquad L H_u N_R \qquad \rightsquigarrow \qquad \frac{\langle H_u \rangle^2}{M}$$

  In type II with instanton induced Majorana masses.

  [Cvetič, Richter, Weigand]

- "Stringy" Dirac Neutrino Mass: [Cvetič, Langacker]

  $e^{-S^{cl}_{E2}}\, L H_u N_R$ suppression factor $e^{-S^{cl}_{E2}} \sim 10^{-13}$

- "Stringy" Weinberg operator:

  [Cvetič, J.H., Langacker, Richter. arXiv:1001.3148]

  $$e^{-S^{cl}_{E2}} \frac{L\, H_u\, L\, H_u}{M_s} \qquad \rightsquigarrow \qquad M_s \leq 10^{14} GeV$$

The Standard IIa Approach to Model-Building

These ingredients suggest a way to build semi-realistic models:

1. type IIa compactified on CY_3 with orientifold projection:
   - $\rightsquigarrow$ $O6$-planes carry RR-charge
2. introduce D6-branes which :
   - preserve common SUSY
   - cancel RR-charge of $O6$-planes
   - have massless sector similar to MSSM
3. apply Green-Schwarz mechanism $\rightsquigarrow$massless $U(1)$'s
   - $\rightsquigarrow$ Interpretation as hypercharge
4. compute the whole chiral spectrum
5. determine the perturbative superpotential

investigated extensively for toroidal orbifolds.

Systematic Search: The Bottom-Up Approach

Idea:

Transformation behavior of matter $\Longrightarrow$ Knowledge of physics (e.g. Yukawa couplings)

Suggests a "bottom-up" approach to model building:

- Specify the matter content and transformation behavior (a quiver) under the D-branes.
- Investigate physics, possibly including D-instanton effects.
- Assume that the quiver can eventually be embedded in a global model.

Why bother?

- Can get a lot of physics out of a small amount of data.
  - $\rightsquigarrow$ Effecient search of the "quiver landscape".

Example Quiver:

$$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d \times U(1)_e$$

$\rightsquigarrow SU(3)_C \times SU(2)_L \times U(1)_Y$
with $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d + \frac{1}{2}U(1)_e$

Sector	Matter Fields	Transformation	Multiplicity	Hypercharge
ab	q_L^*	$(a,\bar{b})$	1	$\frac{1}{6}$
ab'	q_L	(a,b)	2	$\frac{1}{6}$
ac'	u_R^*	$(\bar{a},\bar{c})$	2	$-\frac{2}{3}$
ad'	u_R	$(\bar{a},\bar{d})$	1	$-\frac{2}{3}$
aa'	d_R	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$	3	$\frac{1}{3}$
bc'	H_u	(b,c)	1	$\frac{1}{2}$
bd'	L	$(\bar{b},\bar{d})$	3	$-\frac{1}{2}$
be'	H_d	$(\bar{b},\bar{e})$	1	$\frac{1}{2}$
ce'	E_R	(c,e)	2	1
ce	N_R	$(\bar{c},e)$	1	0
dd'	E_R	$\square\square_d$	1	1
de	N_R	$(\bar{d},e)$	2	0

Physics: Can immediately see $q^*_{L_{(1,-1,0,0,0)}} H_{u_{(0,1,1,0,0)}} u^*_{R_{(-1,0,-1,0,0)}}$ is perturbatively present, but other combinations of families are forbidden!

Systematic Search: Constraints

"Top-Down" Constraints: [Cvetič,J.H., Richter]

- Constraints on the matter content which are necessary for tadpole cancellation.

$$\#(a) - \#(\overline{a}) + (N_a - 4)\#(\begin{smallmatrix}\square\\\square\end{smallmatrix}_a) + (N_a + 4)\#(\square\square_a) = 0$$

- Similar constraints on the matter content exist for the masslessness of $U(1)_Y$.

$$q_a\, N_a\left(\#(\square\square_a) + \#(\begin{smallmatrix}\square\\\square\end{smallmatrix}_a)\right) = \sum_{x\neq a} q_x\, N_x\,(\#(a,\overline{x}) - \#(a,x))$$

$\rightsquigarrow$ The topological intersection number constraints can be derived in a simple manner from conditions on homology required by tadpole and masslessness.

Recall . . .

Conditions on Homology of D-branes and O-planes

Tadpole cancellation: Ramond-Ramond charges cancelled

$$\leftrightarrow \sum_x N_x(\pi_x + \pi'_x) = 4\pi_{O6}$$

Massless U(1): $U(1) = \sum_x q_x U(1)_x$ remains massless

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$\square\!\square_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a\right)$

Systematic Search: Constraints

Bottom-up Constraints: [Cvetič,J.H., Richter]

- exact MSSM spectrum + 3 N_R.
  - $\rightsquigarrow$ No chiral exotics charged under MSSM gauge groups.
- MSSM superpotential is realized perturbatively or non-perturbatively. $\rightsquigarrow$ Yukawas for all three families.
- The top-quark Yukawa coupling and μ-term are of the correct order.
- R-parity couplings are absent, perturbatively and non-perturbatively.
- Proton decay operators $q_L q_L q_L L$ and $u_R u_R d_R E_R$ are absent or highly suppressed.
- Presence of some mechanism for small neutrino masses.

Systematic Search: Results

[Cvetič, J.H., Richter. arXiv:0905.3379]

- Around 10000 four-stack D-brane quivers with the exact MSSM + $3N_R$ satisfy the necessary "top-down" constraints.
- Around 30 of these satisfy all the bottom-up constraints.

  $\rightsquigarrow$ No quiver found which gives rise to desired hierarchies.

- Five-stack quivers in the extended Madrid embedding

$$U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d + \frac{1}{2}U(1)_e$$

  $\rightsquigarrow$ Found five five-stack quivers (out of a million) which satisfy all constraints and can give rise to the desired hierarchies.

- General Lessons:
  - Different families tend to arise from the same sector.
  - Right-handed quarks tend to not be realized as $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a$.

Another Systematic Search

[Cvetič, J.H., Langacker, Richter. arXiv:1001.3148]

- spectrum: exact MSSM
- neutrino mass via instanton induced Weinberg operator.
- allow for lower string scale
  - $\rightsquigarrow$ μ-term constraint is relaxed.

Results:

- around 20 viable setups, out of 20,000.
- family splitting is still suppressed.
- right-handed quarks realized as $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$ still suppressed.
- generically μ-term is generated by an instanton which induces a Yukawa coupling:
  - $\rightsquigarrow$ M_s forced to be in $10^3 - 10^7\, GeV$ range.

GUTs: Problems and Solutions

Problem: In $SU(5)$ GUT models in type IIb, the $10\,10\,5_H$ top-quark Yukawa coupling is perturbatively forbidden. It can be generated by a D-instanton, but would generically be suppressed.

Features of F-theory GUTs:

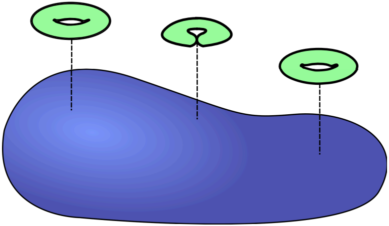
- Combines nice features of heterotic and type IIb
  - Localization of gauge theory on seven-branes.
  - Exceptional gauge structures.
- Exceptional group structure allows for perturbative realization of $10\,10\,5_H$ Yukawa coupling.
- Decoupling limit of gravity for GUT branes wrapped on del Pezzo surfaces?

What is F-theory?

- F-theory can be thought of as type IIB with the axio-dilation

$$\tau = C_0 + ie^{-\phi} = C_0 + \frac{i}{g_s}$$

fibered over the internal dimensions, where τ is interpreted as the complex structure modulus of an auxiliary torus. [Vafa]



Weierstrass Form:

$$y^2 = x^3 + fxz^4 + gz^6$$

Discriminant Locus:

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

where f and g are functions of the base.

- Seven-branes located at the discriminant locus, where the elliptic fiber degenerates.

What is F-theory?

Implications:

- Location of seven-branes is geometrized.
- Non-abelian gauge group of the seven-brane can be read off from the structure of singularities

  $\rightsquigarrow$ can get E_6, E_7, E_8 enhancement!
- D7 brane tadpole cancellation is automatic.

GUT Model Building: [Donagi, Wijnholt] [Beasley, Heckman, Vafa] [Blumenhagen, Grimm, Jurke, Weigand] [Watari et al] [Grimm, Krause, Weigand] [Marsano, Saulina, Schafer-Nameki]

- Local model building can be near the GUT brane.
- Global model building requires the explicit construction of an elliptically fibered Calabi-Yau fourfold.

Questions and Direction

- What can be said about instanton effects in F-theory?
  - $\leadsto$ Can our knowledge of instantons in type II help?
- Are there practical geometric tools to aid in constructing elliptically fibered Calabi-Yau fourfolds?

Direction We're Going:

- Understand useful geometric tools for fourfolds.
- Consider an interesting IIb GUT geometry with an instanton induced $10\,10\,5_H$ coupling.
- Lift the IIb geometry to F-theory CY_4.
- Calculate zero modes of the lifted instanton.
- Offer direct calculational techniques.

Toric Geometry: The General Idea

Very roughly: A generalization of weighted projective spaces.

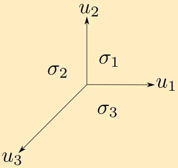
$$\rightsquigarrow \quad (x_1, \ldots, x_r) \;\sim\; (\lambda^{Q_1^a} x_1, \ldots, \lambda^{Q_r^a} x_r)$$

Illustrative Examples:

$\mathbb{P}^2$

Coords	Q^1
x_1	1
x_2	1
x_3	1

$$1u_1 + 1u_2 + 1u_3 = 0$$

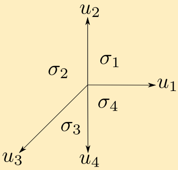


dP_1

Coords	Q^1	Q^2
x_1	1	0
x_2	1	1
x_3	1	0
x_4	0	1

$$1u_1 + 1u_2 + 1u_3 = 0$$

$$1u_2 + 1u_4 = 0$$



Can define a toric variety in terms of lattice data.

The point: Can realize Calabi-Yau's as hypersurfaces or complete intersections in an ambient toric variety. $\rightsquigarrow$(e.g. the quintic)

Instantons in a IIb GUT Model

[Blumenhagen, Braun, Grimm, Weigand]

- The model: a globally consistent two family GUT model where a Euclidean D3 instanton generates the $10\,10\,5_H$ Yukawa coupling.
- Geometric specifics: The IIb CY_3 is the Calabi-Yau hypersurface in the toric variety below, with orientifold involution $\sigma : x_3 \to -x_3$.

Coords / Vertices	Q^1	Q^2	Q^3	Q^4	Divisor Class
$x_1 = (1,0,0,0)$	3	0	0	0	$3M$
$x_2 = (0,1,0,0)$	2	0	0	0	$2M$
$x_3 = (0,0,1,0)$	0	1	0	0	N
$x_4 = (0,0,0,1)$	0	0	1	0	O
$x_5 = (-9,-6,-1,-1)$	0	1	1	-1	$N + O - P$
$x_6 = (-3,-2,0,0)$	1	-1	-1	-1	$M - N - O - P$
$x_7 = (-6,-4,-1,0)$	0	0	-1	1	$-O + P$
$x_8 = (-6,-4,0,-1)$	0	-1	0	1	$-N + P$
$\sum_i [D_i]$	6	0	0	0	$6M$

- Instanton wraps the rigid orientifold invariant divisor $D_5 = \{x_5 = 0 \cap CY_3\}$.
- Question: Can we lift this geometry to F-theory, and what happens to the instanton if we do?

F-theory Lift

[Cvetič, J.H., García-Etxebarria. arXiv:1003.5337]

- It has been recently shown how to lift broad classes of IIb orientifolds to F-theory. [Collinucci] [Blumenhagen, Grimm, Jurke, Weigand]
- Geometric Specifics: An elliptically fibered Calabi-Yau fourfold as the intersection of two hypersurfaces in the following toric variety

	Coords / Vertices	Q^1	Q^2	Q^3	Q^4	Q^5	Divisor Class
Base	$\widetilde{x}_1 = (1,0,0,0,0,0)$	3	0	0	0	0	$3I$
	$\widetilde{x}_2 = (0,1,0,0,0,0)$	2	0	0	0	0	$2I$
	$\widetilde{x}_3 = (0,0,1,0,0,0)$	0	2	0	0	0	$2J$
	$\widetilde{x}_4 = (0,0,0,1,0,0)$	0	0	1	0	0	K
	$\widetilde{x}_5 = (0,0,0,0,1,0)$	0	1	1	-1	0	$J + K - L$
	$\widetilde{x}_6 = (-3,-2,0,0,0,0)$	1	-1	-1	-1	0	$I - J - K - L$
	$\widetilde{x}_7 = (6,4,1,1,1,0)$	0	0	-2	2	0	$-2K + 2L$
	$\widetilde{x}_8 = (-6,-4,0,-1,0,0)$	0	-1	0	1	0	$-J + L$
Fiber	$x = (0,0,2,1,1,3)$	0	2	-2	2	2	$2J - 2K + 2L + 2M$
	$y = (-3,-2,-2,-1,-1,-2)$	0	3	-3	3	3	$3J - 3K + 3L + 3M$
	$z = (9,6,2,1,1,0)$	0	0	0	0	1	M
	$\sum_i [D_i]$	6	6	-6	6	6	$6I + 6J - 6K + 6L +$ 6M

- Instanton wraps the $D_5 = \{\widetilde{x}_5 = 0 \cap CY_4\}$.
- Question: What can be said about the zero modes of this instanton?

Uncharged Zero Modes in F-theory

It has been shown how to count uncharged instanton zero modes in F-theory via cohomological techniques.

From duality with M-theory: [Witten, '96]

arithmetic genus: $\chi(D, \mathcal{O}_D) = \sum_{n=0}^{3} (-1)^n h^{n,0}(D) = 1$

From relation to IIb: [Blumenhagen, Collinucci, Jurke]

The counting was shown to hold in a number of examples where the IIb orientifold and its F-theory lift are known.

zero modes	statistics	Type IIB	F-theory
(X_μ, θ_α)	(bose, fermi)	$H_+^{0,0}(E)$	$H^{0,0}(\mathcal{M})$
$\overline{\tau}_{\dot\alpha}$ γ_α	fermi fermi	$H_-^{0,0}(E)$ $H_+^{1,0}(E)$	$H^{1,0}(\mathcal{M})$
$(w, \overline{\gamma}_{\dot\alpha})$ χ_α	(bose, fermi) fermi	$H_-^{1,0}(E)$ $H_+^{2,0}(E)$	$H^{2,0}(\mathcal{M})$
$(c, \overline{\chi}_{\dot\alpha})$	(bose, fermi)	$H_-^{2,0}(E)$	$H^{3,0}(\mathcal{M})$

For our lift:

- $h^{1,0}(D_5) = 0$, and thus the $\overline{\tau}$ modes are absent.
- This is expected, since this an instanton on D_5 is the lift of the instanton which generated the $10\,10\,5_H$ coupling in IIb.

Performing the Calculations: Čech Cohomology

How difficult is it to perform these cohomology calculations?

- In general, it can be quite difficult!
- BUT it is straightforward when the fourfold is a hypersurface or complete intersection in a toric variety.

Fact: From the Dolbeault theorem, we know that $H^{q,0}(\mathcal{M}) \cong H^q(\mathcal{M}, \mathcal{O}_{\mathcal{M}})$, and we can therefore calculate sheaf cohomology instead.

Fourfold $\subset$ Toric Variety $\Longrightarrow$ Calculate sheaf cohomology of line bundles on toric varieties by Čech cohomology.

This is straightforward and, though tedious, has been computerized.
[Cvetič, J.H., García-Etxebarria]

An efficient alternative method for computing these Čech cohomology groups has been conjectured. [Blumenhagen, Jurke, Rahn, Roschy]

Summary

Type II

- Account for detailed features of semi-realistic models via non-perturbative D-instanton effects.
  - ⇝ Non-zero masses for all MSSM matter fields.
  - ⇝ Mass hierarchies via MSSM matter field segregation or factorizable Yukawa matrices.
  - ⇝ Neutrino masses via seesaw mechanism, highly suppressed Dirac mass, or “stringy” Weinberg operator.

- Systematic search of semi-realistic MSSM quivers which satisfy necessary constraints for tadpole cancellation and a massless hypercharge, as well as extensive phenomenological constraints..
  - ⇝ 30 quivers with 3 N_R's.
  - ⇝ Five-stack: 5 quivers with 3 N_R's with nice mass hierarchies.
  - ⇝ Around 20 quivers without N_R's which allow for neutrino mass from the stringy Weinberg operator.

Summary and Outlook

F-theory

- Specific F-theory lift with $10\,10\,5_H$ from an $O(1)$ instanton.
- Algebraic geometry techniques applied to an elliptically fibered CY_4 realized as a complete intersection in a toric variety.
- Count instanton zero modes via Čech cohomology.
  - $\rightsquigarrow$ computerized for efficiency.

Outlook

- Continuation of the bottom-up program
  - $\rightsquigarrow$ More stacks? Could be nice, but there are some issues . . .
  - $\rightsquigarrow$ Different spectra? Perhaps there are some lessons to learn about what is favored or disfavored.
  - $\rightsquigarrow$ Attempt to embed nice quivers in global models.
- Building theoretical and technical foundations for systematic searches of global F-theory models and the role of instantons there.

Segregation of MSSM Matter Fields

Potential quark Yukawa textures in the Madrid embedding:

$$Y^1 = \begin{pmatrix} A & A & A \\ A & A & A \\ A & A & A \end{pmatrix} \quad Y^2 = \begin{pmatrix} B & A & A \\ B & A & A \\ B & A & A \end{pmatrix} \quad Y^3 = \begin{pmatrix} A & A & A \\ B & B & B \\ B & B & B \end{pmatrix}$$

$$Y^4 = \begin{pmatrix} A & B & B \\ C & D & D \\ C & D & D \end{pmatrix} \quad Y^5 = \begin{pmatrix} A & B & C \\ A & B & C \\ A & B & C \end{pmatrix} \quad Y^6 = \begin{pmatrix} A & B & C \\ D & E & F \\ D & E & F \end{pmatrix}$$

Textures in blue can have three different mass scales.

The charged leptons can have these textures, or, in the case where the three L and E_R families all come from different sectors, the lepton texture can have all entries of different orders.

Non-Factorizable Yukawa Matrices

Non-factorizable

Key: No charged modes for the b brane.

$[\pi_{E2} \circ \pi_a]^+ = 1 \qquad [\pi_{E2} \circ \pi_c]^- = 1$

$M_s \int d^4x\, d^2\theta\, d\overline{\lambda}_a\, d\lambda_c\; e^{-S^{cl}_{E2} + Y^{IJ}_{\Phi\Psi}\; \overline{\lambda}_a\, \Phi^I\, \Psi^J\, \lambda_c} \quad \rightsquigarrow \quad M^{IJ} \sim M_s\, e^{-S^{cl}_{E2}}\, Y^{IJ}_{\Phi\Psi}$

Point: There are K mass eigenvalues of order $M_s\, e^{-S^{cl}_{E2}}$.

Factorizable

Key: There are a pair of charged modes for the b brane, λ_b, $\overline{\lambda}_b$.

$[\pi_{E2} \cap \pi_a]^+ = 1 \quad [\pi_{E2} \cap \pi_b]^{\pm} = 1 \quad [\pi_{E2} \cap \pi_c]^- = 1$

$S^{int}_{E2} = Y'^{IJ}_{\Phi\Psi}\; \overline{\lambda}_a\, \Phi^I\, \Psi^J \lambda_c + Y^I_{\Phi}\; \overline{\lambda}_a\, \Phi^I\, \lambda_b + Y^I_{\Psi}\; \overline{\lambda}_b \Psi^J \lambda_c$

$M_s \int d^4x\, d^2\theta\, d\overline{\lambda}_a\, d\lambda_b d\overline{\lambda}_b\, d\lambda_c\; e^{-S^{cl}_{E2} + S^{int}_{E2}} \quad \rightsquigarrow \quad M^{IJ} \sim M_s\, e^{-S^{cl}_{E2}}\, Y^I_{\Phi}\, Y^J_{\Psi}$

Point: Only one mass eigenvalues of order $M_s e^{-S^{cl}_{E2}}$. Need an additional $K-1$ instantons to induce masses for all families, with the ratio of the masses being given by $e^{-S^{cl}_{E2}} : e^{-S^{cl}_{E2'}} : e^{-S^{cl}_{E2''}} : \ldots$, which depends on the volume of the cycles which the instantons wrap.

Gauge Groups of the Seven Branes

IIb:

- Tadpole cancellation is a condition on the homology cycles which the D7 branes wrap.
- One configuration which cancels tadpoles gives rise to $SO(10)$, $Sp(2)$, and $SO(6)$ on D_7, D_5, and D_3.
- $SO(10)$ broken to $SU(5)$ by magnetic fluxes.

F-theory:

- Information about seven-branes encoded in the structure of the elliptic fiber, which can be put in either Weierstrass or Tate form.

The Tate Form, Explicitly

The Tate form for an elliptic curve can be written:

$$y^2 + a_1\, xyz + a_3\, yz^3 = x^3 + a_2\, x^2 z^2 + a_4\, xz^4 + a_6\, z^6$$

For our lift this takes the explicit form

$$\begin{gathered} a_1 = c_0\widetilde{x}_5\widetilde{x}_7 \qquad a_2 = c_1\widetilde{x}_3\widetilde{x}_7 + c_2\widetilde{x}_5^2\widetilde{x}_7^2 \\ a_3 = c_3\widetilde{x}_5^3\widetilde{x}_7^3 + c_4\widetilde{x}_3\widetilde{x}_5\widetilde{x}_7^2 \qquad a_4 = c_5\widetilde{x}_5^4\widetilde{x}_7^4 + c_6\widetilde{x}_3\widetilde{x}_5^2\widetilde{x}_7^3 + c_7\widetilde{x}_3^2\widetilde{x}_7^2 \\ a_6 = c_8\widetilde{x}_5^6\widetilde{x}_7^6 + c_9\widetilde{x}_3\widetilde{x}_5^4\widetilde{x}_7^5 + c_{10}\widetilde{x}_3^2\widetilde{x}_5^2\widetilde{x}_7^4 + c_{11}\widetilde{x}_3^3\widetilde{x}_7^3. \end{gathered}$$

Fixing some complex structure moduli this becomes

$$\begin{gathered} a_2 = c_1\widetilde{x}_3\widetilde{x}_7 \qquad a_3 = 4c_1c_3\widetilde{x}_3\widetilde{x}_5\widetilde{x}_7^2 \qquad a_1 = a_4 = a_6 = 0 \\ \Delta = -256c_1^4c_3^2\widetilde{x}_3^4\widetilde{x}_5^2\widetilde{x}_7^7(27c_3^2\widetilde{x}_5^2\widetilde{x}_7 + c_1\widetilde{x}_3), \end{gathered}$$

recovering the $SO(10)$ and $Sp(2)$ enhancements along the D_7 and D_5. The $SO(6)$ enhancement is recovered precisely in Sen's limit, when $c_3 \mapsto 0$.

Instanton Zero Modes: Details

Instanton wraps three-cycle Ξ in internal CY_3.

Uncharged Modes: E2-E2 sector

- x^μ from breakdown of Poincaré invariance.
- θ_α and $\overline{\tau}_{\dot{\alpha}}$ from breakdown of $\mathcal{N} = 2$ preserved by CY_3
- Deformation modes if Ξ is not rigid.
- Modes at intersection of Ξ and Ξ'

Charged Modes: E2-D6 sector

- chiral fermionic zero modes λ_a or $\overline{\lambda}_a$

Zero modes	Reps	Number
$\lambda_{a,I}$	$(-1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \pi_a]^+$
$\overline{\lambda}_{a,I}$	$(1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \pi'_a]^+$
$\overline{\lambda}_{a',I}$	$(1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \pi'_a]^-$