Josephson-controlled topological superconductivity

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With

1. Anna Keselman (Station Q), Erez Berg (Weizmann)
   Falko Pientka, Bert Halperin, Amir Yacoby (Harvard) - PRX 2017

2. Arbel Haim (Cal-Tech) – PRL 2019

3. Erez Berg – PRL 2019

4. Setiawan Wenming and Erez Berg – PRB Rapid 2019

5. Collaboration with the QDEV experimentalists –
   A. Fornieri, F. Nichele, A. Drachmann, A. Whiticar, E. Porteles, C. Marcus, S. Wenming, A. Keselman, Erez Berg (Nature)

Funding by: Microsoft's Station Q, ERC, ISF, CRC183

Relevant papers by Hell et al., and Akhmerov et al.

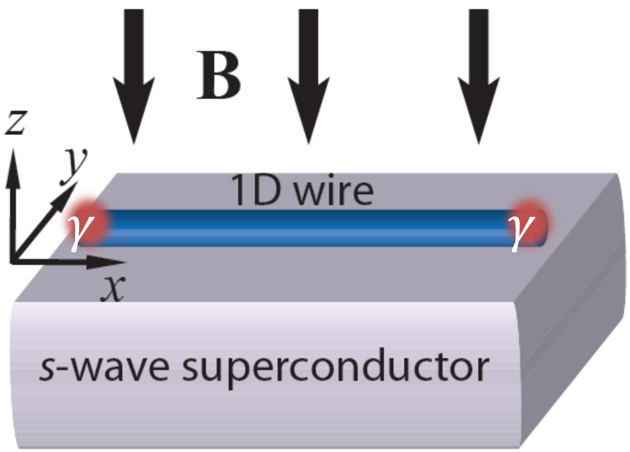
Outline:

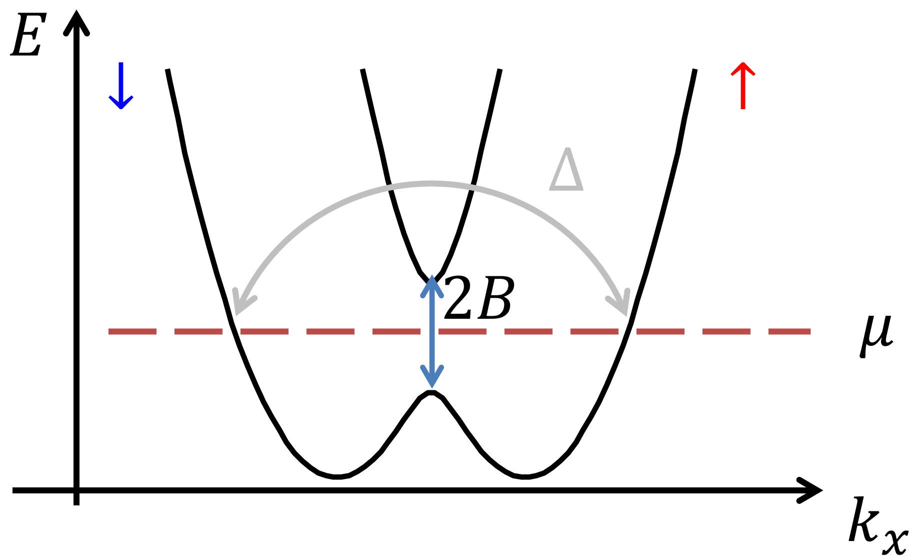
1. Background – 1D topological superconductivity
2. Going half a dimension higher – Planar Josephson junction
3. The limit of a narrow junction and the relation to quantum wires
4. The (friendly!) effect of disorder
5. Braiding Majorana zero modes in Josephson junctions

Background I: Topological Superconductivity in Nanowires

Quantum wire with SOC:

$$H_0 = \frac{k_x^2}{2m} - \mu + \alpha k_x \sigma_y + B\sigma_z$$





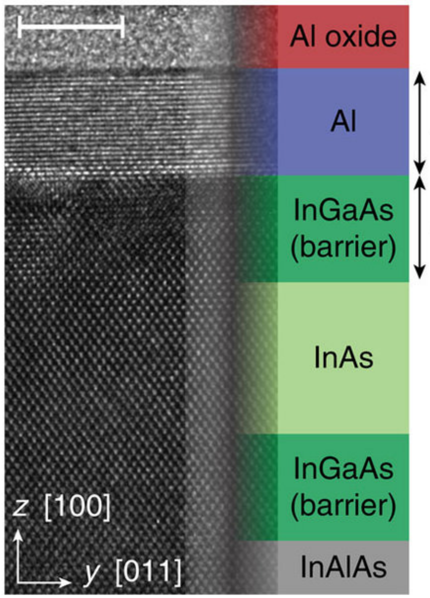
Lutchyn et al. PRL 2010
Oreg et al. PRL 2010

Requires fine-tuning of μ!

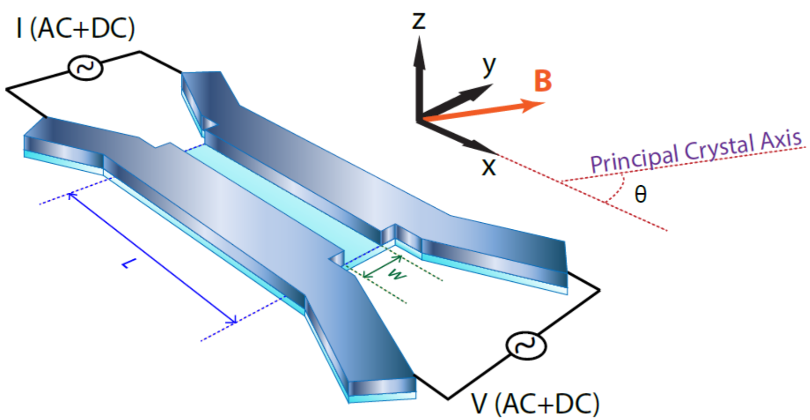
Compelling experimental evidence

figure taken from
Alicea, Rep. Prog. Phys. (2012)

Background 2: Proximity-coupled 2DEGs



Kjaergaard et al.,
Nat. Commun. (2016)

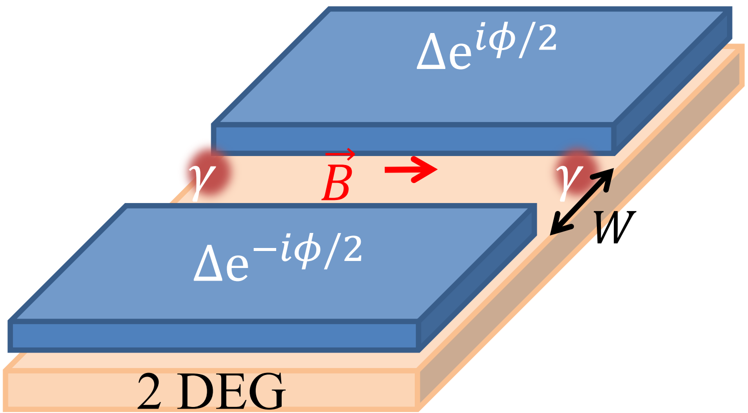


Hart et al., arXivπ1509.02940
(2015)

Shabani et al. Phys. Rev. B (2016), Wan et al., Nat. Commun. (2015)

Topological superconductivity in planar Josephson junctions

Going half a dimension higher –
1D topological superconductor in a 2D setting



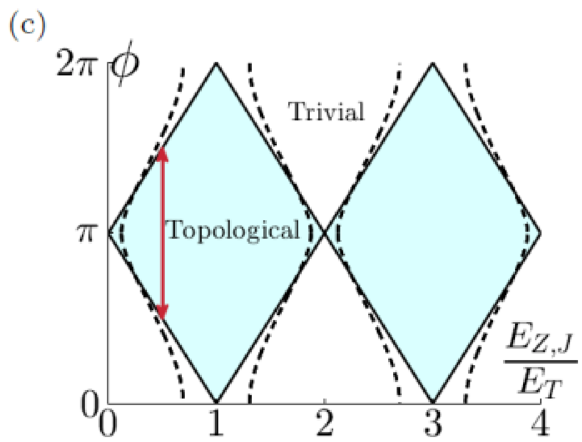
Ingredients:
- ✓ 1D
- ✓ Spin-orbit
- ✓ Superconductivity
- ✓ Magnetic field

New knobs to tune – phase difference, Josephson current, enclosed magnetic flux

New features:
- Robust topological phase with no fine-tuning (for $\phi \approx \pi$)
- Can tune itself the topological phase!

Robust topological phase with no fine-tuning (for $\phi \approx \pi$)

Phase difference

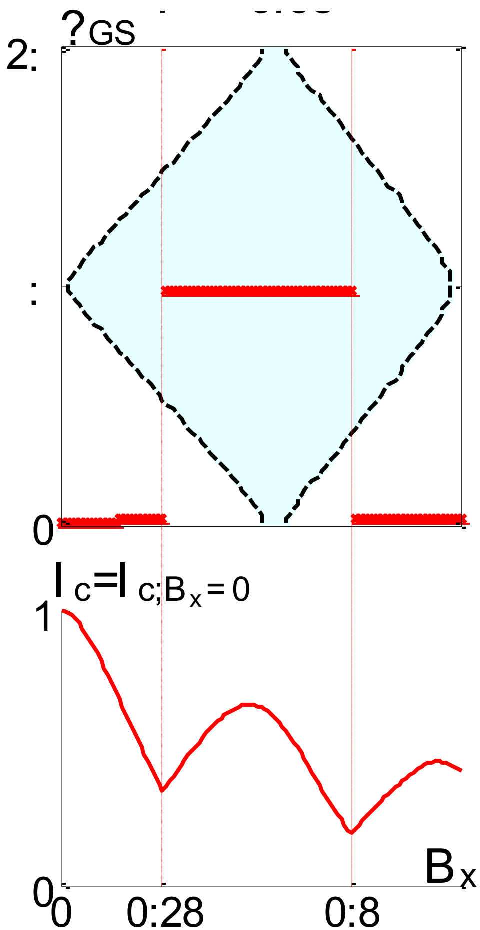


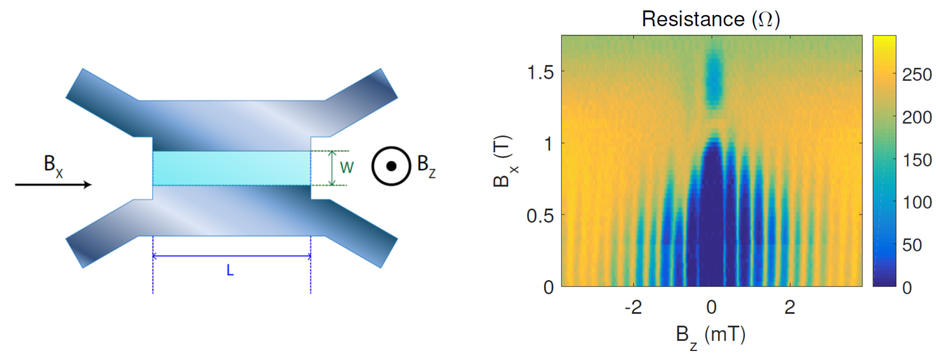
Zeeman energy * traversal time

Only weak dependence on the chemical potential
(Spin orbit energy >> Zeeman energy, wide superconductors)

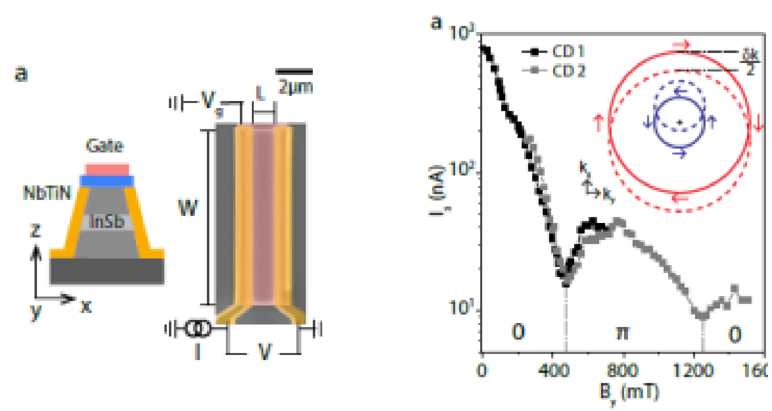
First order phase transition between trivial and topological state – the system self tunes to the topological regime

The phase difference at the ground state:





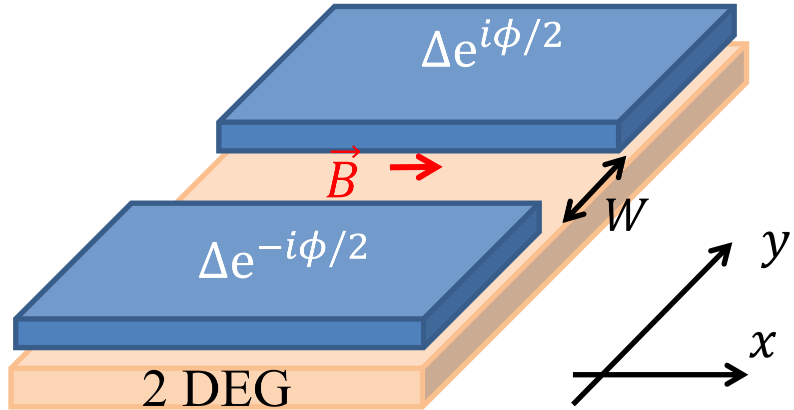
Hart et al. (2015)

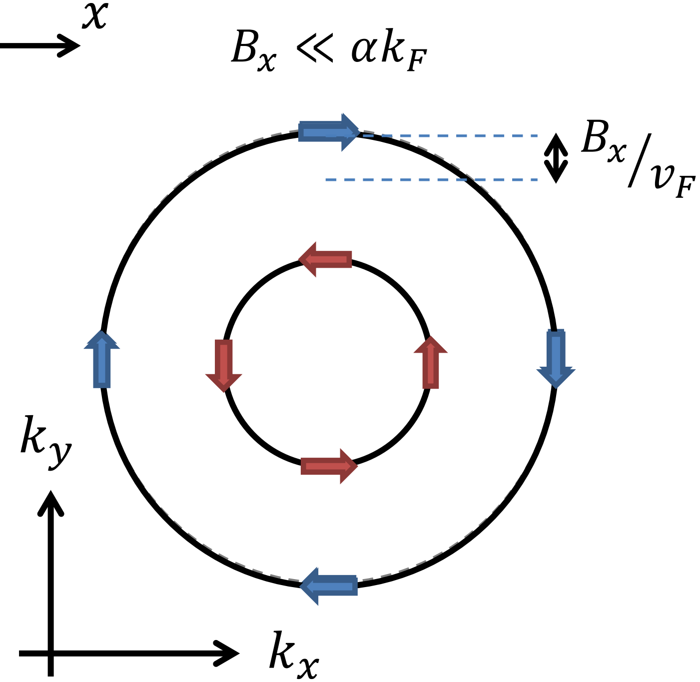


The transition coincides with a minimum of the critical current.

Ke et al. (2019)

Setup and Model



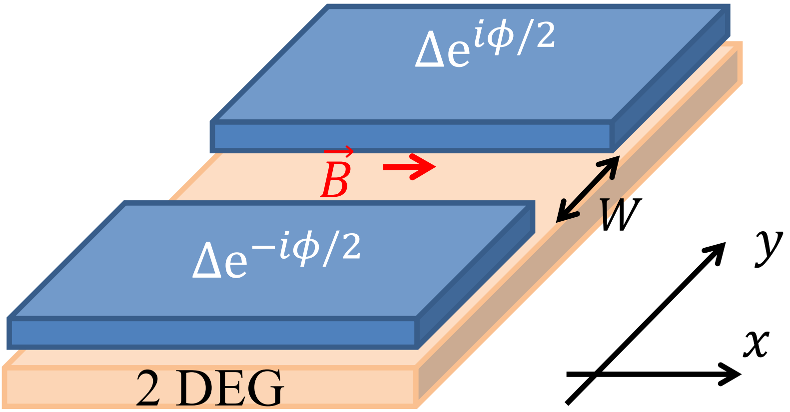


Hamiltonian in the normal region:

$$H_0 = \frac{k_x^2 + k_y^2}{2m} - \mu + \alpha\left(k_x\sigma_y - k_y\sigma_x\right) + B_x\sigma_x$$

$$H_0 = \frac{k_x^2 - \partial_y^2}{2m} - \mu + \alpha\left(k_x\sigma_y + i\partial_y\sigma_x\right) + B_x\sigma_x$$

We are looking for states within the gap, bound between the two superconductors



Almost the particle in the box problem, except the boundary conditions - Andreev processes

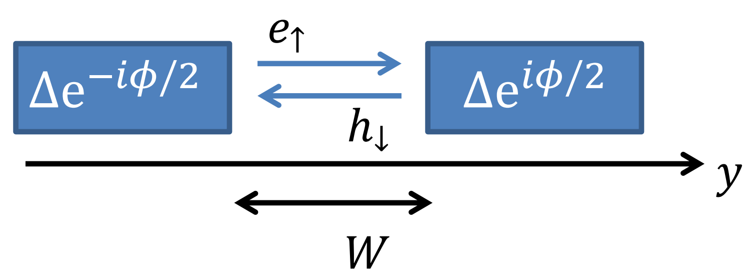
For distinguishing topological from trivial, we need to look at k_x=0

For the ground state energy and energy gap, we need all k_x

Topological invariant = fermion parity at k_x=0 Kitaev (2001)

$\Rightarrow$ Look for single gap closing at k_x=0

Andreev bound state spectrum, $\Delta \ll \mu$:



$$2\cos^{-1}\frac{E_n}{\Delta} + \phi + 2\frac{E_n}{v_F}W \pm 2\frac{B_x}{v_F}W = 2\pi n$$

phase acquired upon Andreev reflection

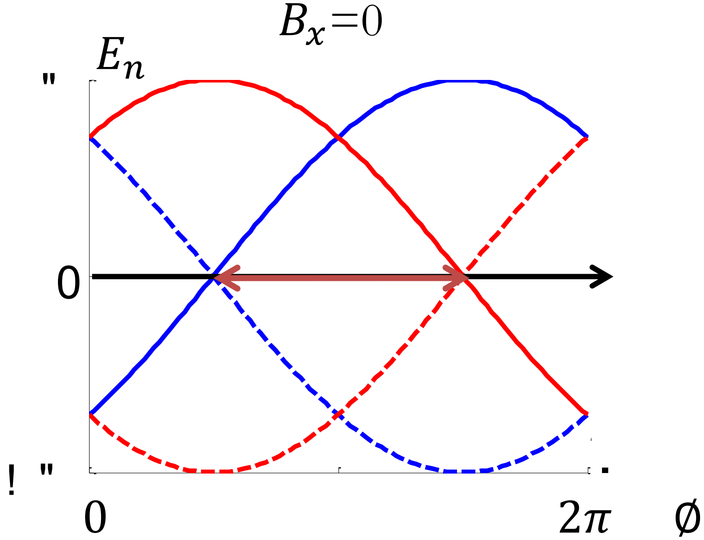
phase acquired upon traversing the junction

Narrow junction, i.e. $\Delta \ll {}^{v_F}/_{W}$: $E_n = \Delta\cos\left(\frac{\phi}{2} \pm \frac{B_x}{v_F}W\right) = \Delta\cos(\frac{\phi}{2} \pm \phi_B)$

Phase Diagram

k_x=0 bound states:

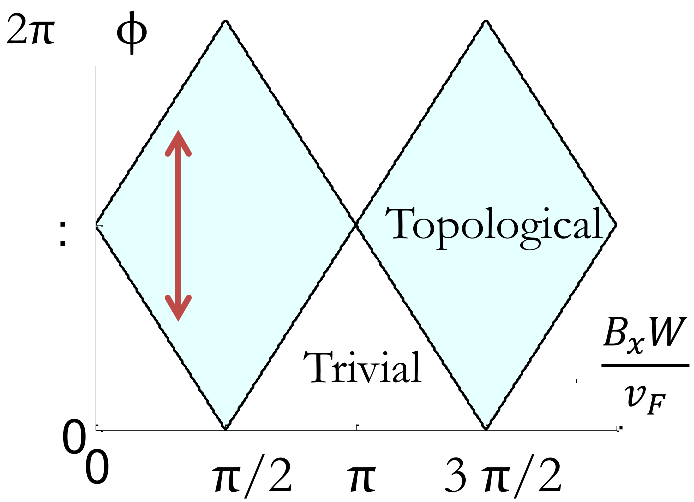
$$E_n = \Delta\cos\left(\frac{\phi}{2} \pm \frac{B_x}{v_F}W\right)$$



State is doubly degenerate!

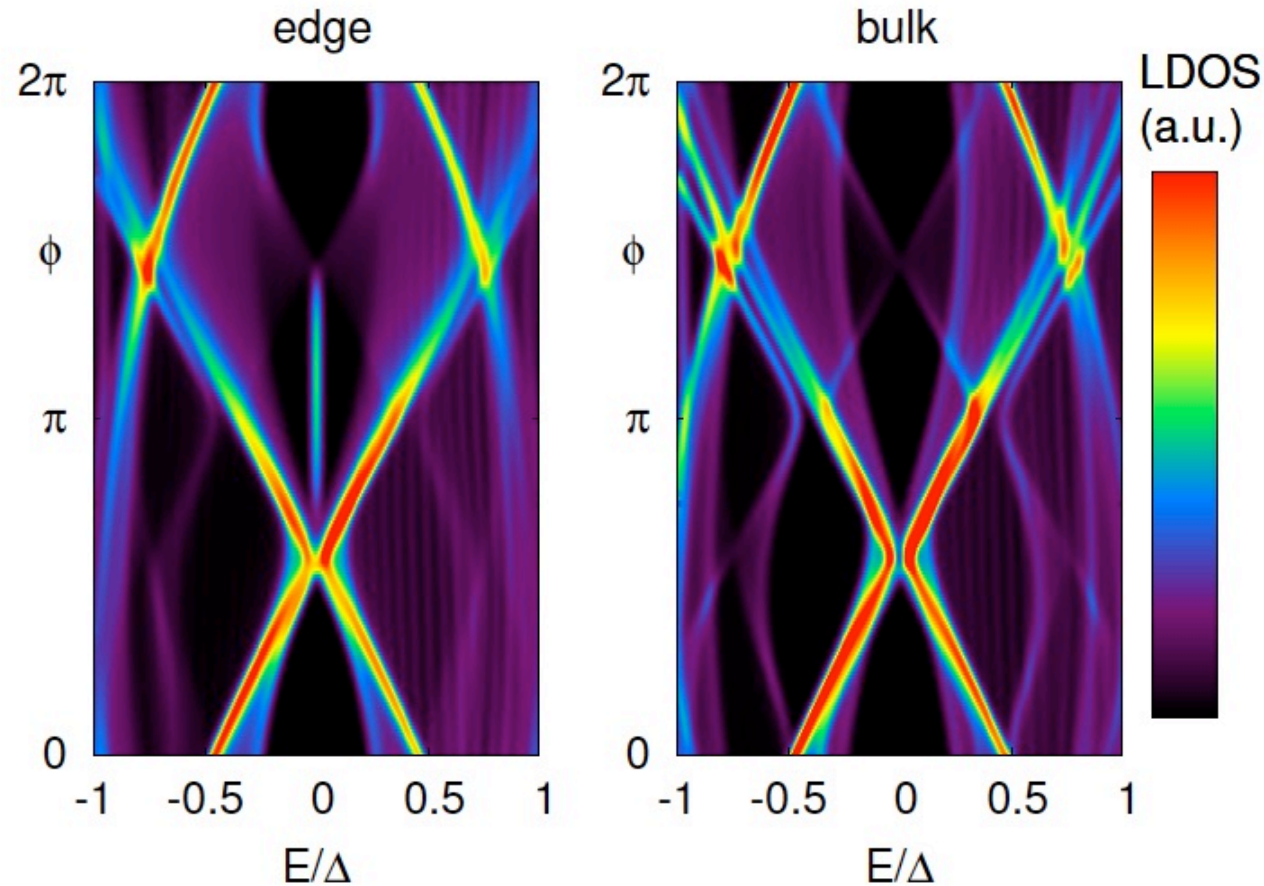
Gap closing lines (for any W):

$$\phi \pm 2\frac{B_x}{v_F}W = (2n+1)\pi$$

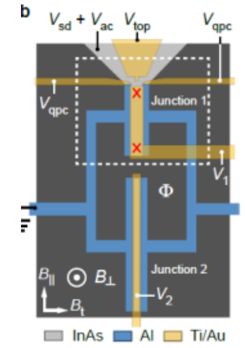


Insensitive to μ!

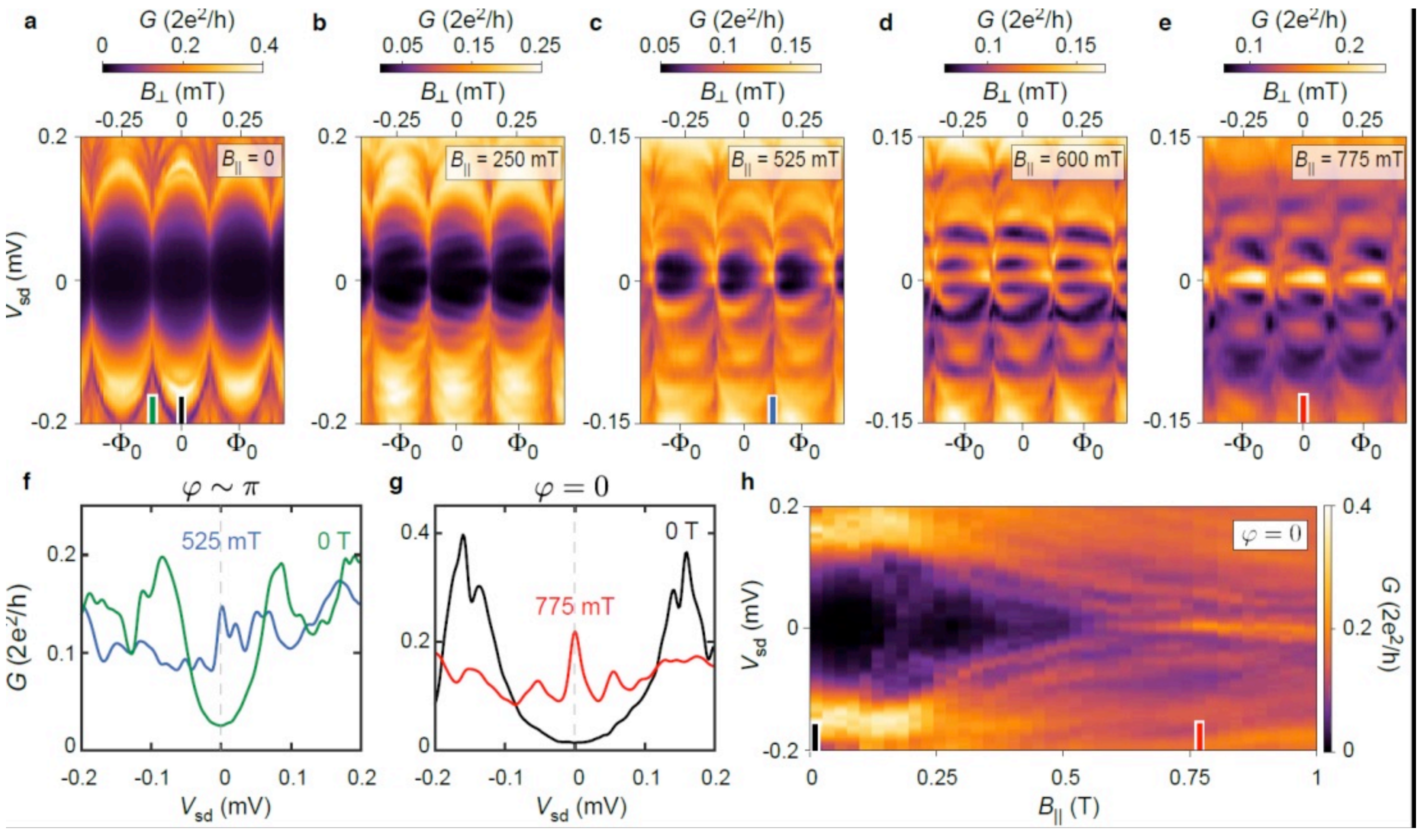
Majorana end states



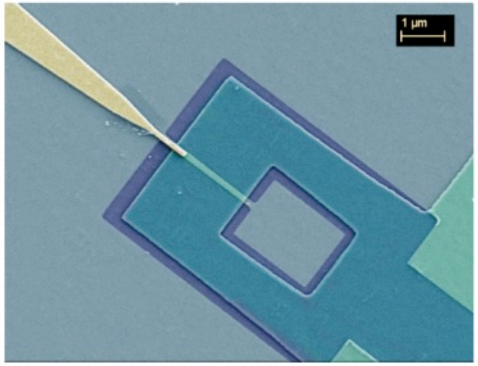
Experiments – Nichele - Marcus group (NBI)



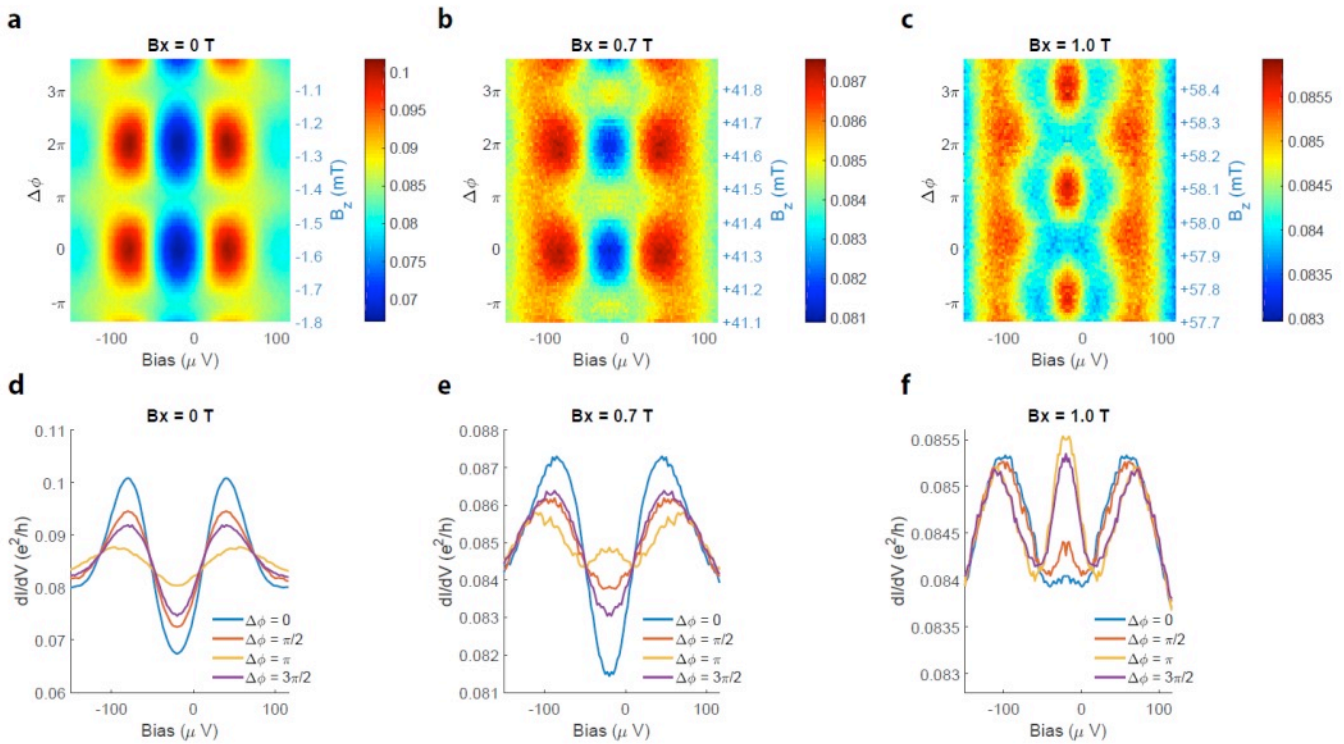
Measuring the tunneling density of states
at the end of the junction



Experiments – Yacoby group (Harvard)



Measuring the tunneling density of states at the end of the junction



Experiments – Goswami group (Delft)

Measurement of the recovery of the critical current with increasing parallel magnetic field

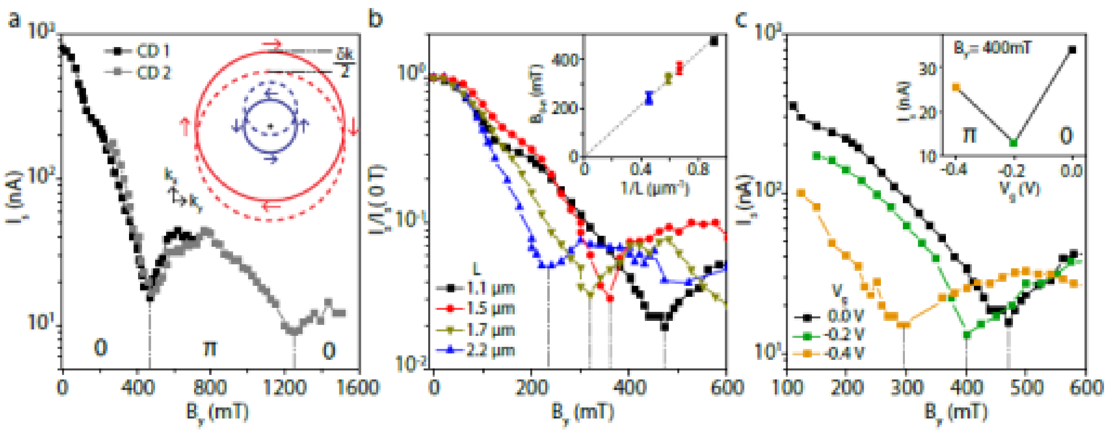
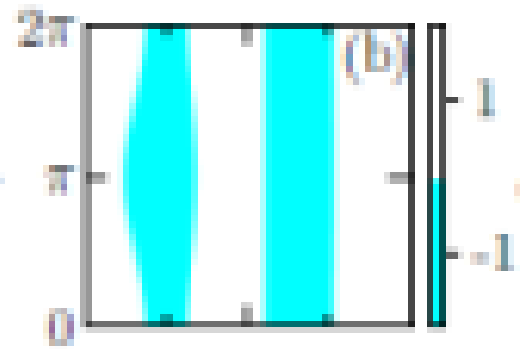


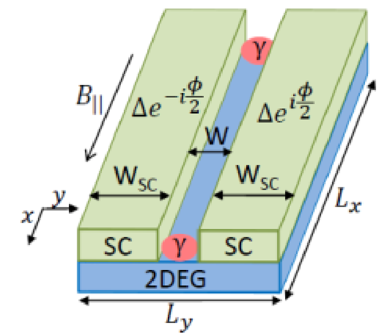
FIG. 2. | **Magnetic field-driven $0-\pi$ transitions.** **a,** Variation of the switching current, I_s, with in-plane magnetic field, B_y, at $V_g = 0$ V for the same JJ as in Fig. 1b,c. Two distinct revivals of I_s are visible at $B_y = 470$ mT and 1250 mT, associated with $0-\pi$ transitions. The data is from two cool downs (CDs). The momentum shift, $\delta k/2$, of the Fermi surfaces due to the Zeeman field is sketched in the inset. The solid (dashed) lines depict the situation at zero (finite) magnetic field, and the arrows represent the spin orientation. **b,** I_s as a function of B_y at $V_g = 0$ V for four JJs with different lengths. For better visibility, I_s is normalized with respect to I_s at $B_y = 0$ T. Dashed lines indicate $B_{0-\pi}$, the field at which the transition occurs for each length. The inset shows a linear dependence of $B_{0-\pi}$ on $1/L$, in agreement with ballistic transport. **c,** I_s vs. B_y at three different V_g for the JJ with $L = 1.1$ µm. $B_{0-\pi}$ shifts to lower values of B_y with more negative gate voltages. I_s vs. V_g at $B_y = 400$ mT shows a non-monotonic behavior as displayed in the inset. The length and gate dependence of panel **b** and **c** are in qualitative agreement with Eq. 1.

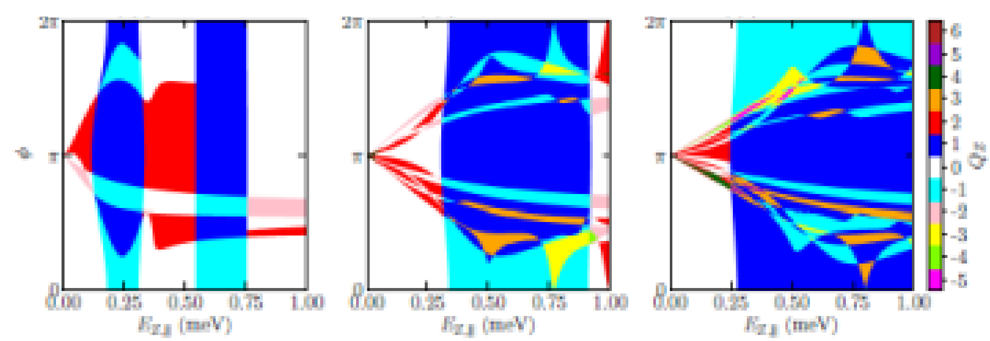
What if the superconductors are narrow (Copenhagen experiment)?

(with Setiawan Wenming and Erez Berg, 2019)

Width of superconductor $\ll$ induced coherence length hv_F/Δ.







The effect of disorder on the localization of the zero modes

With Arbel Haim (Cal-Tech)
PRL 2019

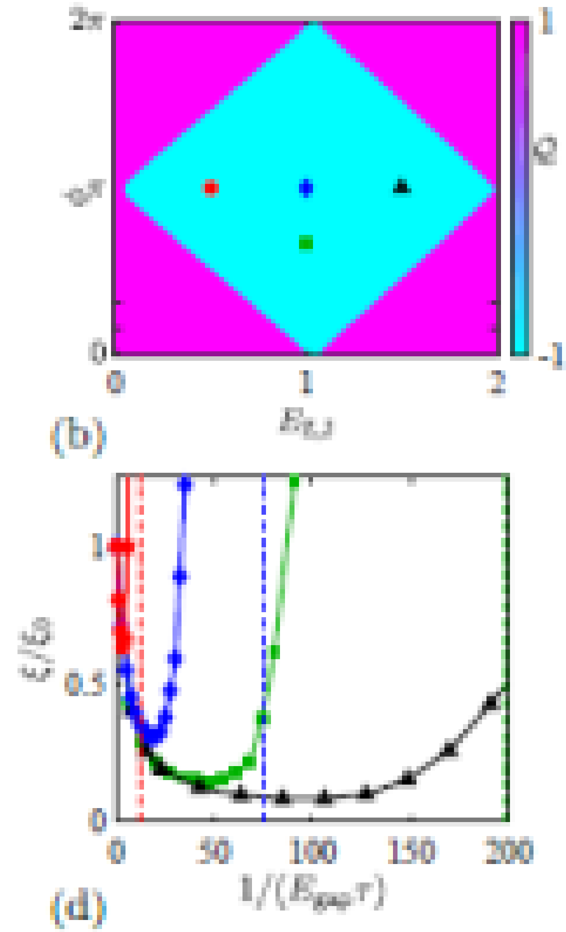
Numerically -

For weak disorder, Majoranas get (significantly) better localized.

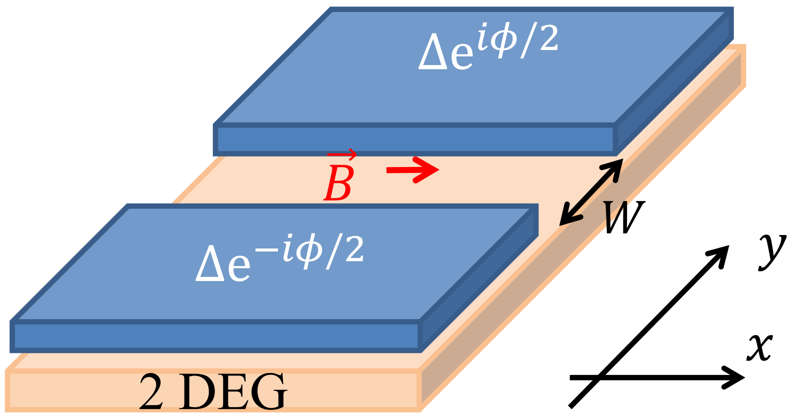
In contrast to 1D p-wave superconductors.

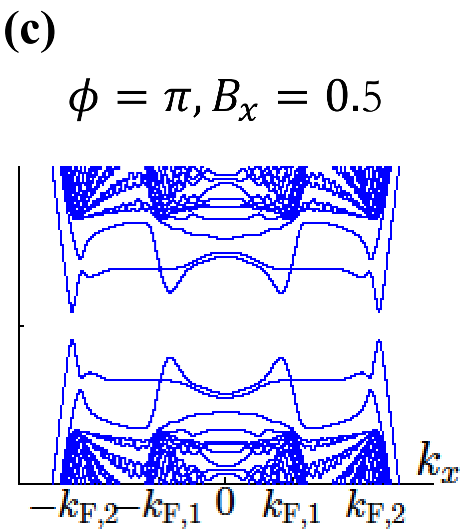
Why –

1. Identify the culprit – large k gap
2. The effect of disorder on that gap – combination of selection rules and pairing phases.



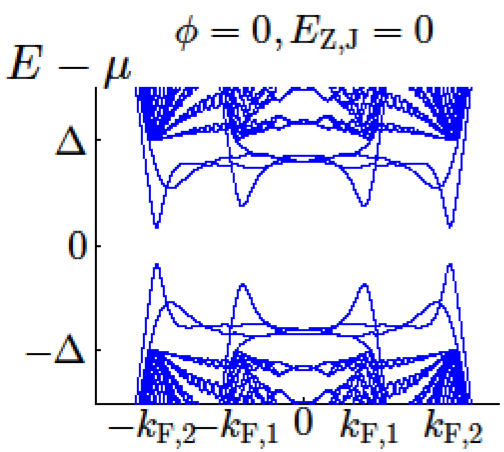
Spectrum of excitations in the topological phase

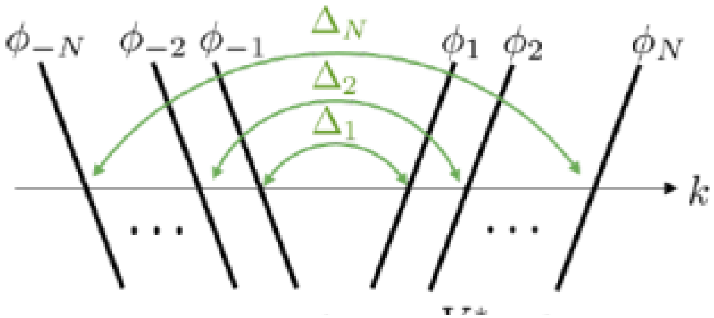




Smallest gap at the two Fermi momenta

Think about the spectrum as coming from pairing of several modes





Effect of disorder - perturbative calculation:

Localization length - $\frac{hv_F}{\Delta_{eff}}$

$$\Delta_{eff,m} \approx \Delta_m + \sum_{n \neq m} \frac{1}{\tau_{mn}} e^{i \arg(\Delta_n) + i\alpha_{mn}} \qquad \frac{1}{\tau_{mn}} = \frac{V_{mn}^2}{|v|}$$

$$\Delta_{eff,m} \approx \Delta_m + \sum_{n \neq m} \left(\frac{1}{\tau}\right)_{mn} e^{i \arg(\Delta_n)}$$

- To be affected by a channel, need to be able to scatter into it
- Once scattered into it, the phase of its pairing potential matters.

Particular cases:

- Disorder scattering into the pairing partner – necessarily reduces Δ_{eff} (phase difference of π).

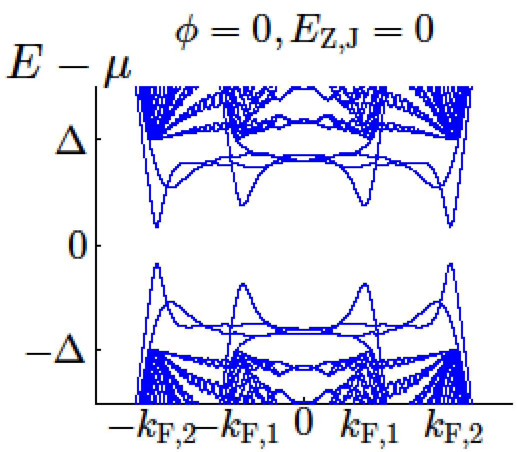
$$\Delta_m c_m^+ c_{-m}^+ \quad \Rightarrow \quad \Delta_m = -\Delta_{-m}$$

- Delocalizes Majorana modes in p-wave superconductors.

- Different situation for s-wave superconductors
  - Selection rule – disorder does not flip spin, so no scattering to pairing partner.
  - No phase difference of pairing potentials.

  - Disorder enhances localization

In our case, large k behaves like s-wave, small k behaves like p-wave

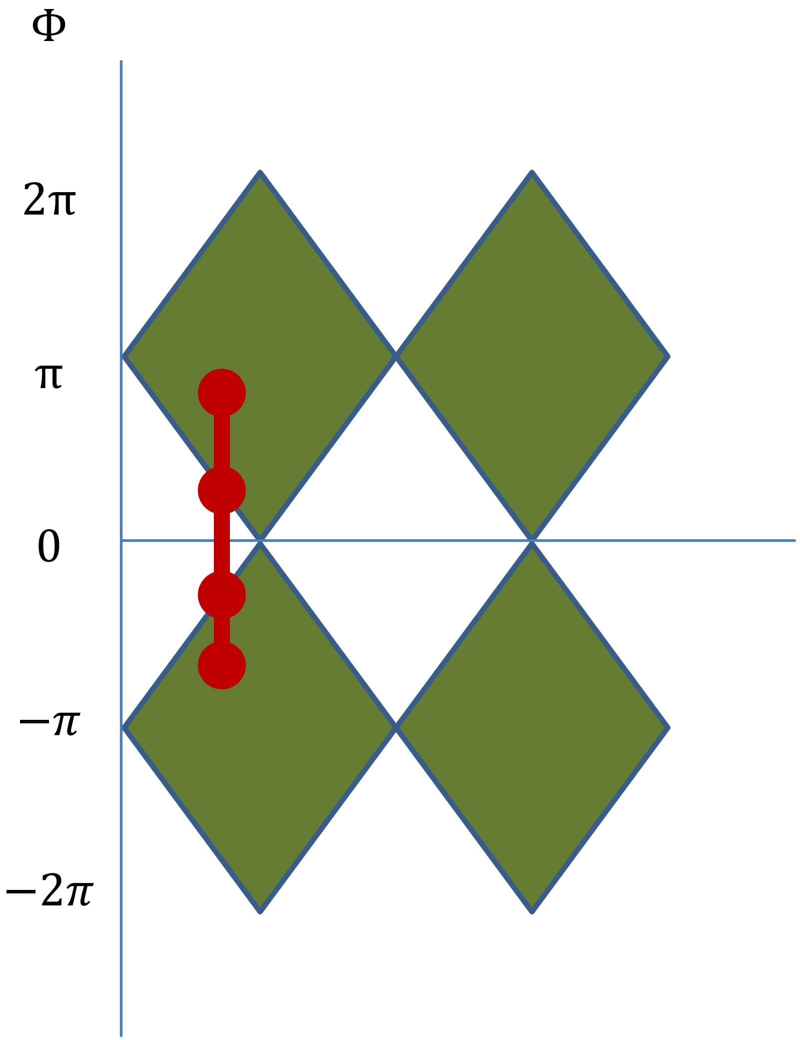


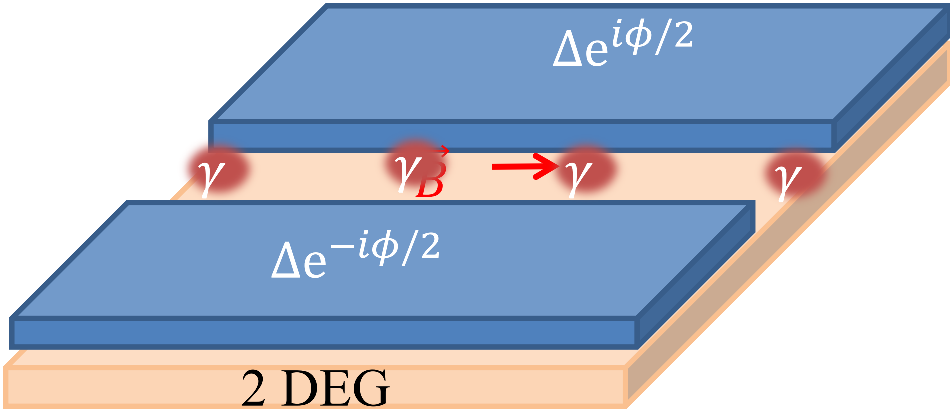
The small k determines topology, the large k determines localization.

Manipulating and braiding the Majorana zero modes

With Erez Berg (WIS)

Perpendicular magnetic field makes the phase vary along the junction

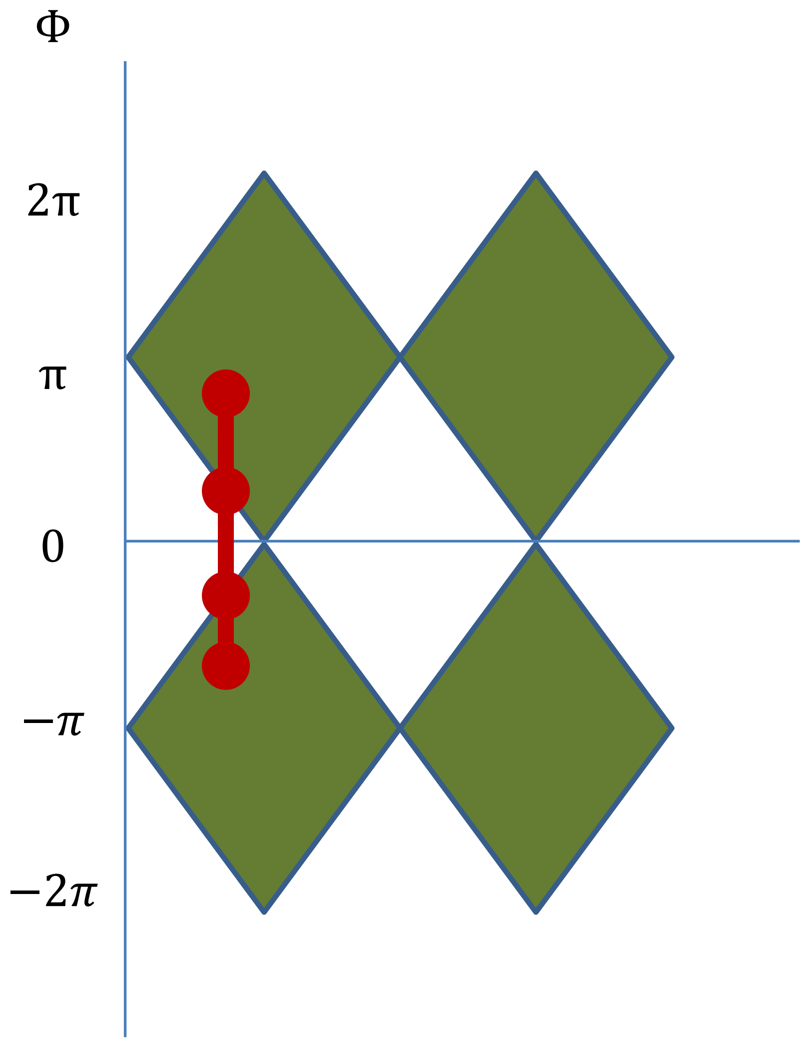




Four Majorana zero modes –

two at the junction ends, two movable by $B_\perp$

The two zero modes at the interior may be moved by varying the magnetic field, or by driving a supercurrent through the junction.



$B_\perp$ stretches the covered range of ϕ, while supercurrent shifts it.

The current-driven tri-junction braiding scheme:

Braiding in $1+\epsilon$ dimensions
Alicea et al., many follow-ups

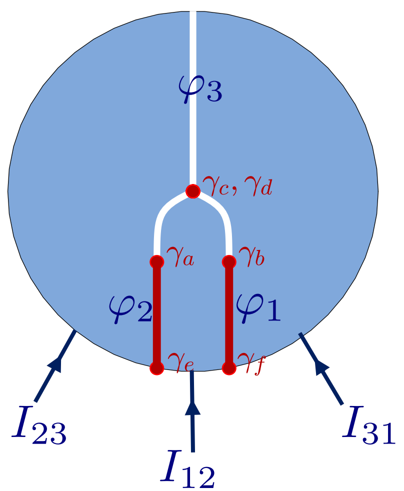
For the planar Josephson junction –
Hell, Flensberg, Leijnse

The knobs:

- Parallel magnetic field
- Perpendicular magnetic field
  (assume $L \ll \lambda_J$, no screening currents)

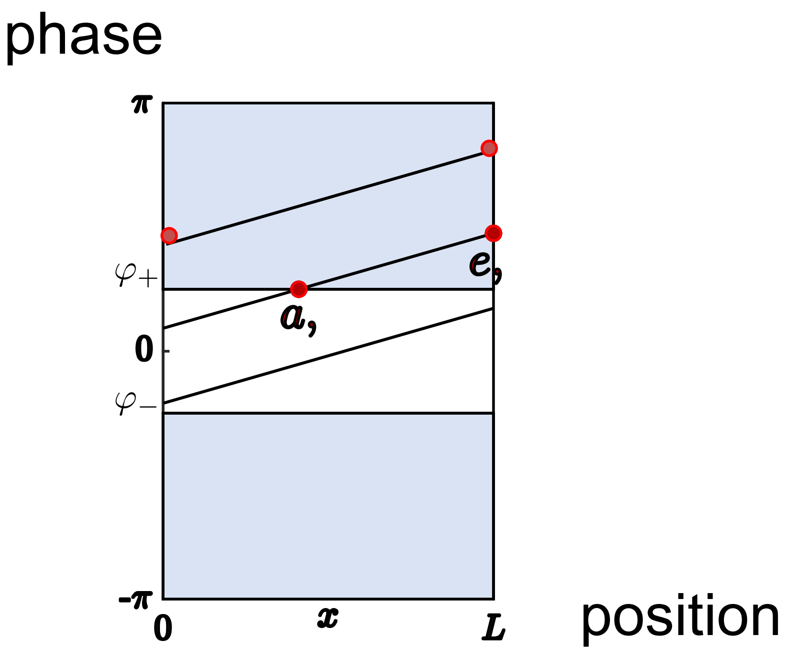
} fixed

- Currents – time dependent



The single junction:

- In the absence of screening currents the phase varies linearly with position, with the slope determined by the perpendicular magnetic field.
- Limit ourselves to 0-2 MZMs per arm



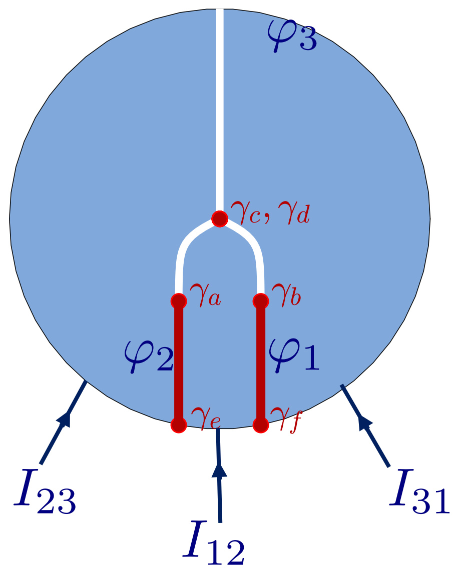
The tri-junction –

1. Quantization of the vorticity at the center

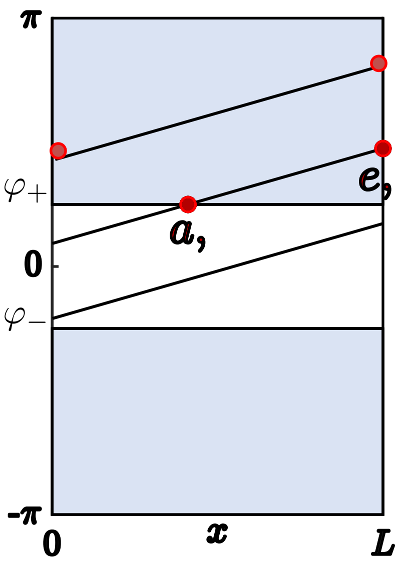
$$\varphi_1(x_1 = 0) + \varphi_2(x_2 = 0) + \varphi_3(x_3 = 0) = 2\pi n$$

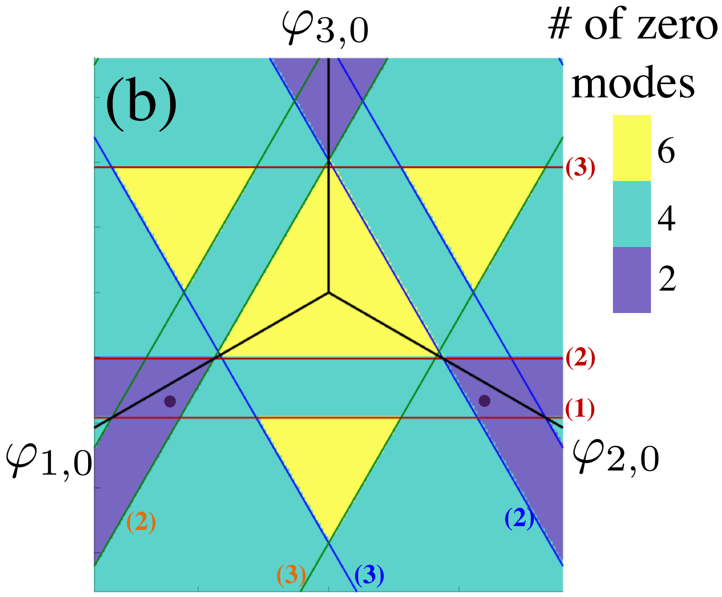
2. Continuity of the magnetic field at the center

$$\partial_x \varphi_1(x_1 = 0) = \partial_x \varphi_2(x_2 = 0) = \partial_x \varphi_3(x_3 = 0)$$

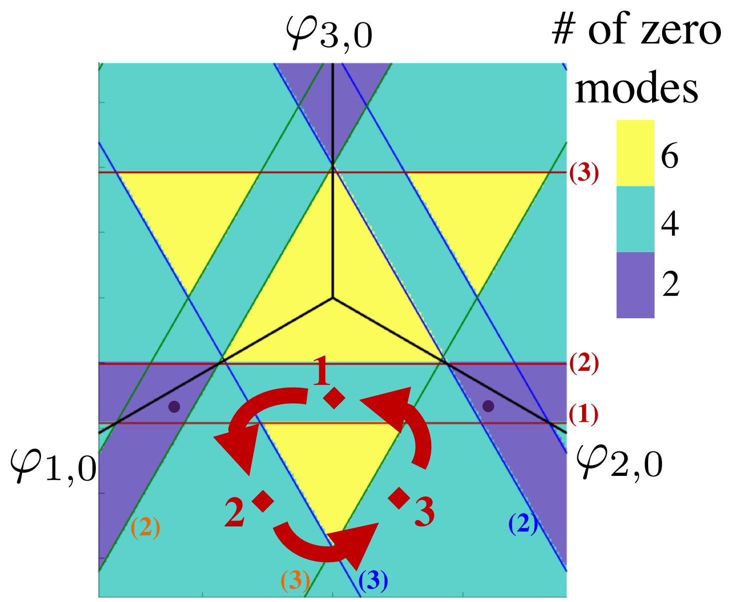


The plane of no-vortex at the center

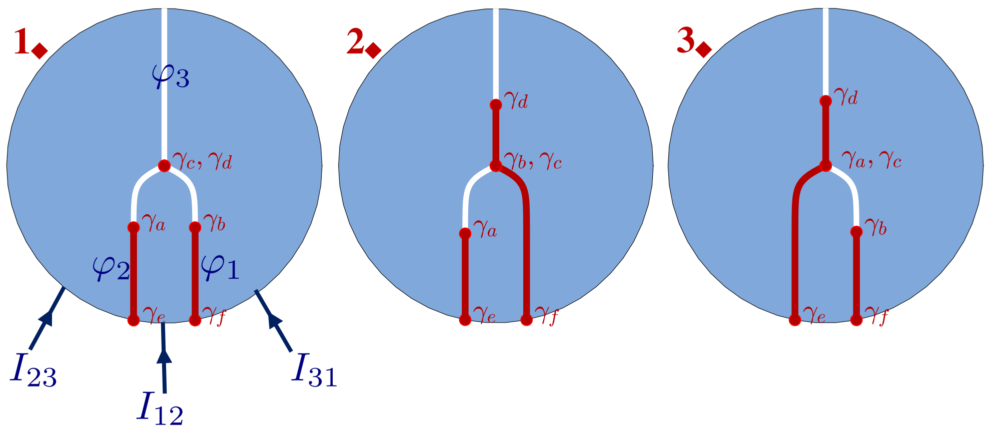


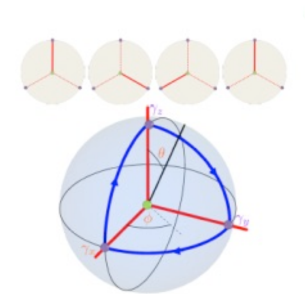


A topological manipulation – motion along a trajectory that cannot be contracted to a point



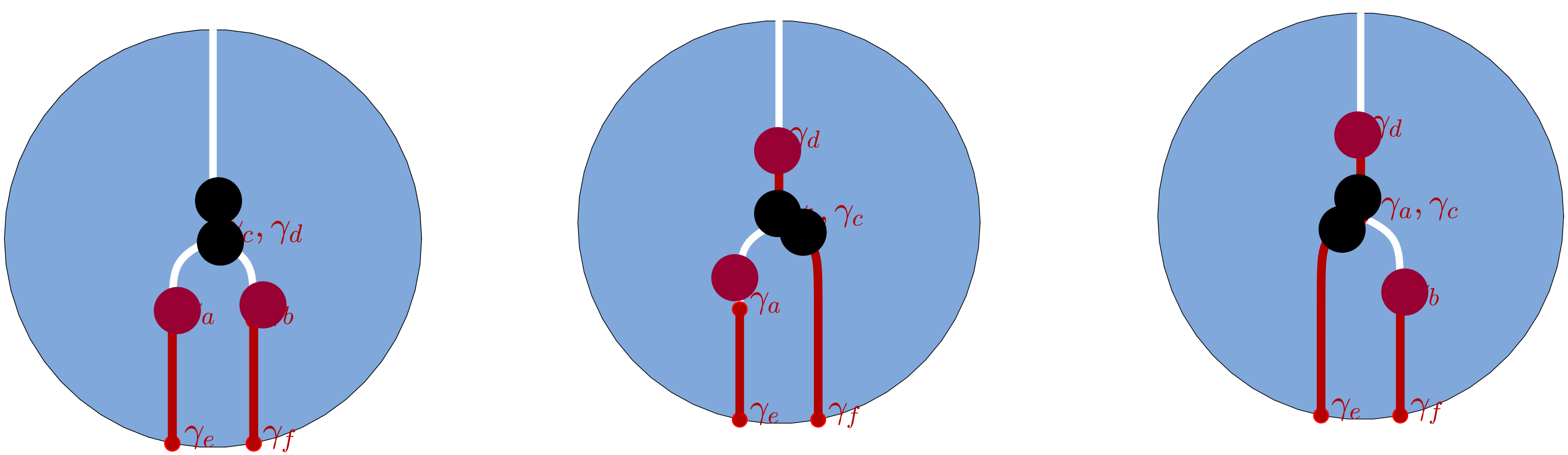
Current minimizes $I_i\varphi_i + V(\varphi_i)$





Karzig et al. 2016

Can be looked at as six zero modes, out of which two are coupled



Summary

1. The relative phase is a user-friendly parameter to use on the way to topological superconductivity.
2. First-order phase transition where the system self tunes itself to the topological regime.
3. When the superconductors are narrow – relation to a multimode wire
4. Disorder localizes the Majorana modes.
5. Scheme for Majorana braiding.