# Fractional Quantum Hall Effect in Weyl Semimetals

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CATS
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# Outline

- Introduction: topological insulators, topological metals and quantum anomalies.

- Gapping symmetry-protected topological surface states by strong interactions.

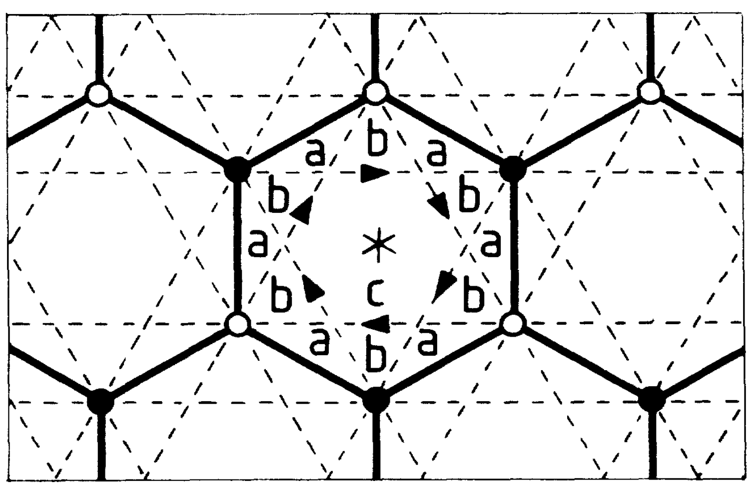
- Chiral anomaly in strongly-interacting Weyl semimetals and 3D FQHE.

arXiv:1907.02068

# Topological insulators

- Insulators, characterized by quantized topological invariants.

- In the weakly-interacting case, the invariant may be evaluated using the noninteracting band eigenstates.




- Chern number: $C = \frac{1}{2\pi} \int d^2k \, \Omega_z(\mathbf{k}) = \pm 1$

# Interacting topological insulators

- Band picture is not applicable with strong interactions.

- More general picture is in terms of response.

- Chern-Simons action describes electromagnetic response of the Haldane Chern insulator:

$$S = \frac{e^2 C}{4\pi} \int d^2x \, dt \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

# Edge states and anomalies

- In the presence of a boundary this leads to a non-conserved current:

$$S = \frac{e^2 C}{4\pi} \int d^2x \, dt \, \Theta(x_1) \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

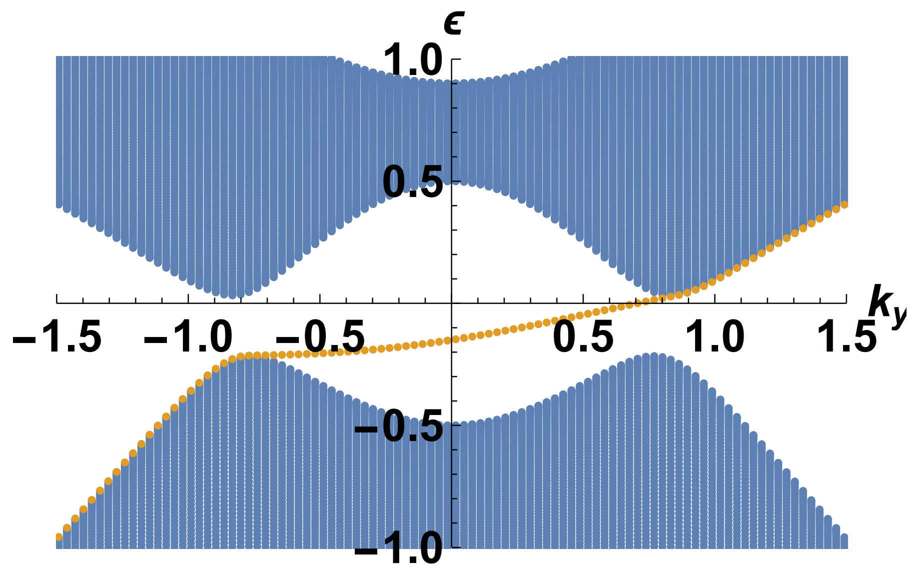
$$\partial_\mu j^\mu_{bulk} = \frac{e^2 C}{4\pi} \delta(x_1) \epsilon^{1\nu\lambda} \partial_\nu A_\lambda$$

# Edge states and anomalies

- This is cancelled by the chiral anomaly of the edge state:

$$\partial_\mu j^\mu_{bulk} = \frac{e^2 C}{4\pi}\delta(x_1)\epsilon^{1\nu\lambda}\partial_\nu A_\lambda$$

$$\partial_\mu j^\mu_{edge} = -\frac{e^2 C}{4\pi}\delta(x_1)\epsilon^{1\nu\lambda}\partial_\nu A_\lambda$$




$$\partial_\mu (j^\mu_{edge} + j^\mu_{bulk}) = 0$$

# Higher dimensions

- 3D time-reversal-invariant TI:

$$S = \frac{\theta e^2}{8\pi^2} \int d^3x \, dt \, \epsilon^{\mu\nu\lambda\rho} \partial_\mu A_\nu \partial_\lambda A_\rho$$

$$\theta = \pi$$

- In a sample with boundary this may be written as boundary action:

$$S = \frac{\theta e^2}{8\pi^2} \int d^2x \, dt \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \quad \Rightarrow \quad \sigma_{xy} = \frac{e^2}{2h}$$

# Higher dimensions

- This violates time-reversal symmetry and is cancelled by the corresponding action of the gapless 2D Dirac surface state (parity anomaly):

$$S_{bulk} = \frac{\theta e^2}{8\pi^2} \int d^2x \, dt \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$S_{surf} = -\frac{\theta e^2}{8\pi^2} \int d^2x \, dt \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

# Edge topological metal

- Edge states of the Haldane Chern insulator and 3D TR-invariant TI are the simplest examples of a topological metals: metals whose physical properties are determined by topology.

- Topological metal is very robust: immune to effects of disorder and interactions.

- Protected as long as bulk electronic structure topology is intact.

# Bulk topological metals

- Can bulk metals be topologically nontrivial?

# Bulk topological metals

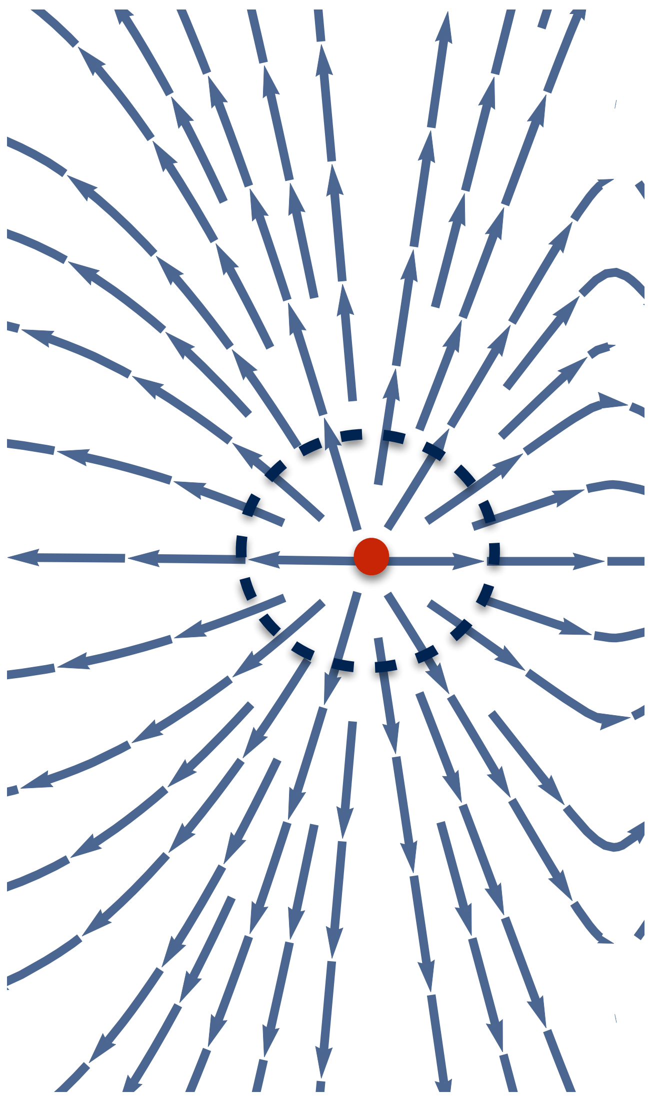
- Can bulk metals be topologically nontrivial?

Yes!

# Fermi surface invariants

- Flux of the Berry curvature through the 2D Fermi surface of a 3D metal is a topological invariant (a Chern number).

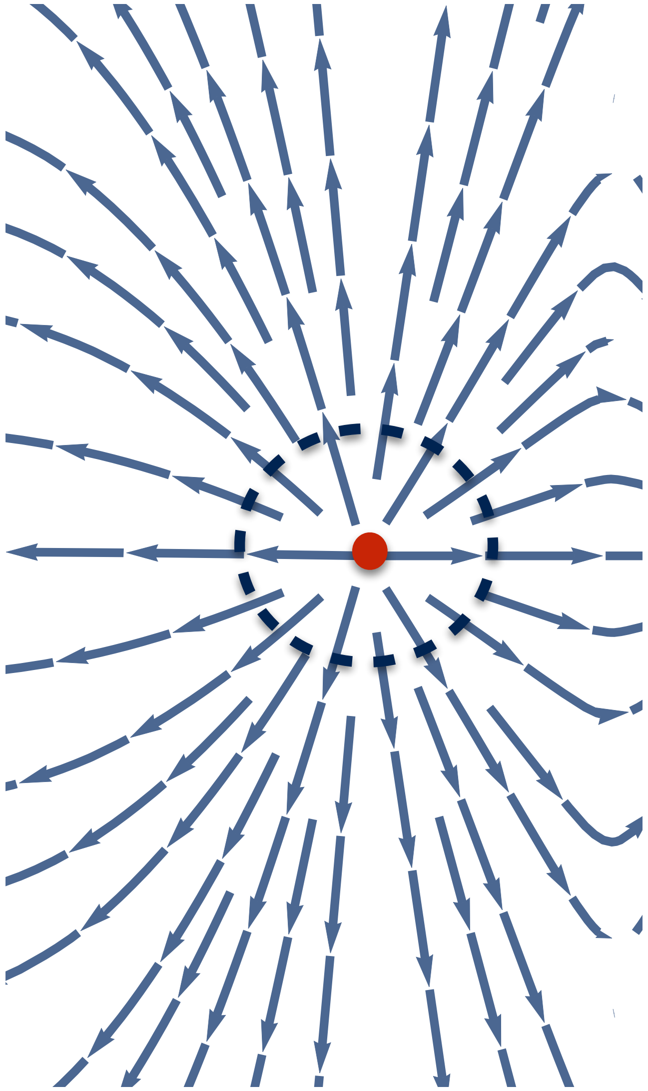
$$\frac{1}{2\pi}\int \mathbf{\Omega}(\mathbf{k})\cdot d\mathbf{S} = C$$



# Fermi surface invariants

- Gauss' theorem: there must be a point source of Berry curvature, enclosed by the Fermi surface.

$$\mathbf{\Omega}(\mathbf{k}) = \pm \frac{\mathbf{k}}{2k^3}$$



- Such a point source arises as a result of touching of two bands.

# Weyl fermions

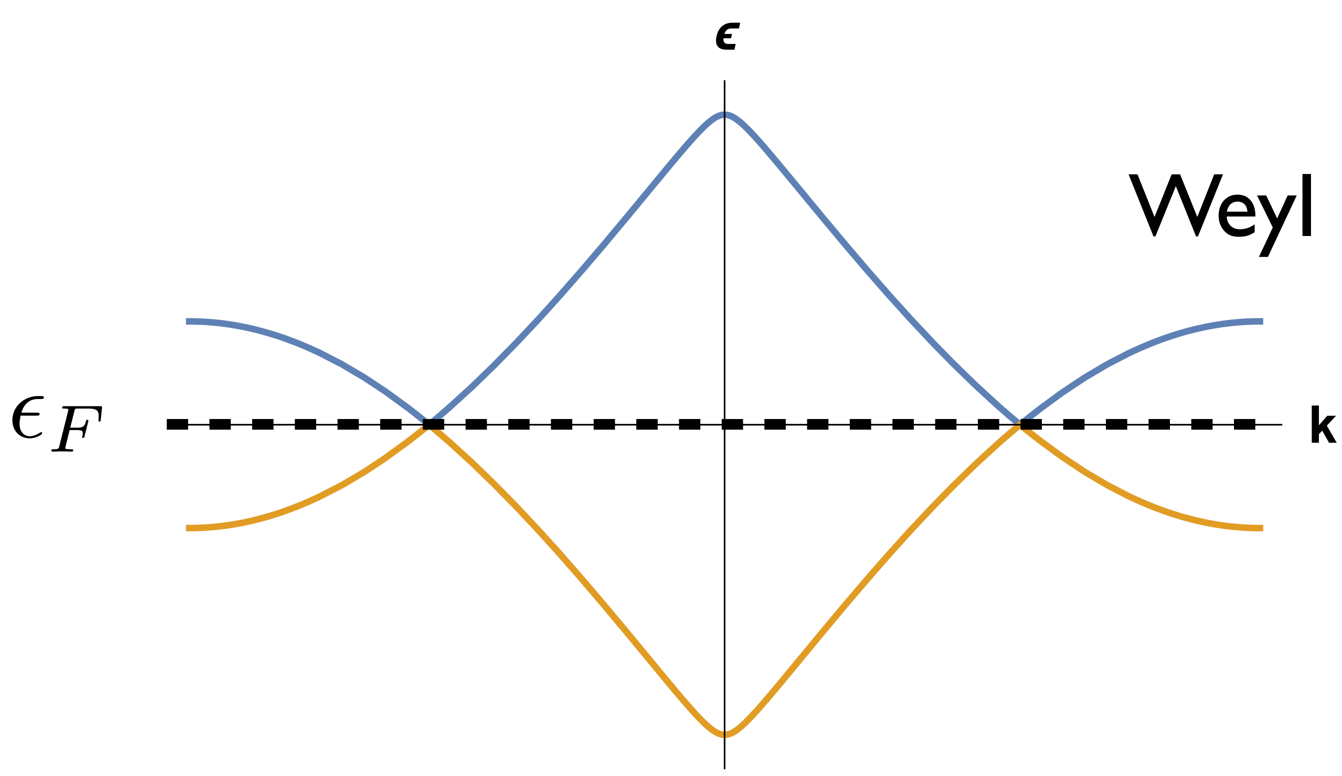
- The band Hamiltonian in the vicinity of a band-degeneracy point has a universal from:

$$H = \pm \boldsymbol{\sigma} \cdot \mathbf{k}$$

- This coincides with the Hamiltonian for relativistic massless chiral fermions, first proposed by Hermann Weyl in 1929.

# Weyl semimetal

- Weyl node: point contact between two nondegenerate bands, which acts as a point-like source of Berry curvature ("magnetic monopole" in momentum space).




Weyl semimetal

Murakami, 2007

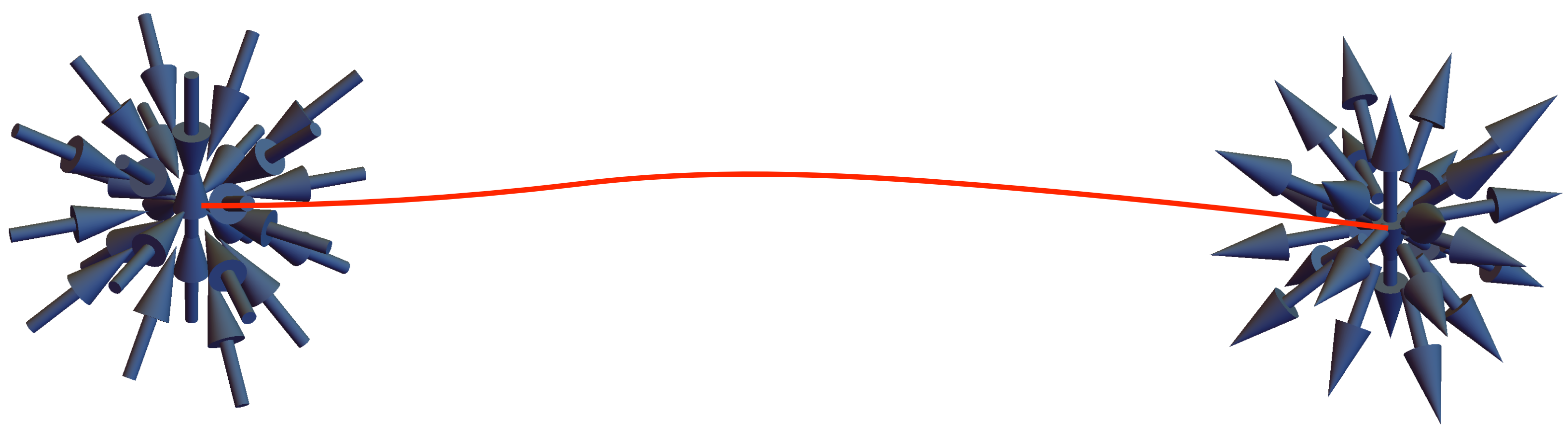
Wan et al., 2011

AAB & Balents, 2011

- This arises naturally as an intermediate phase between a topological and ordinary insulator in 3D.

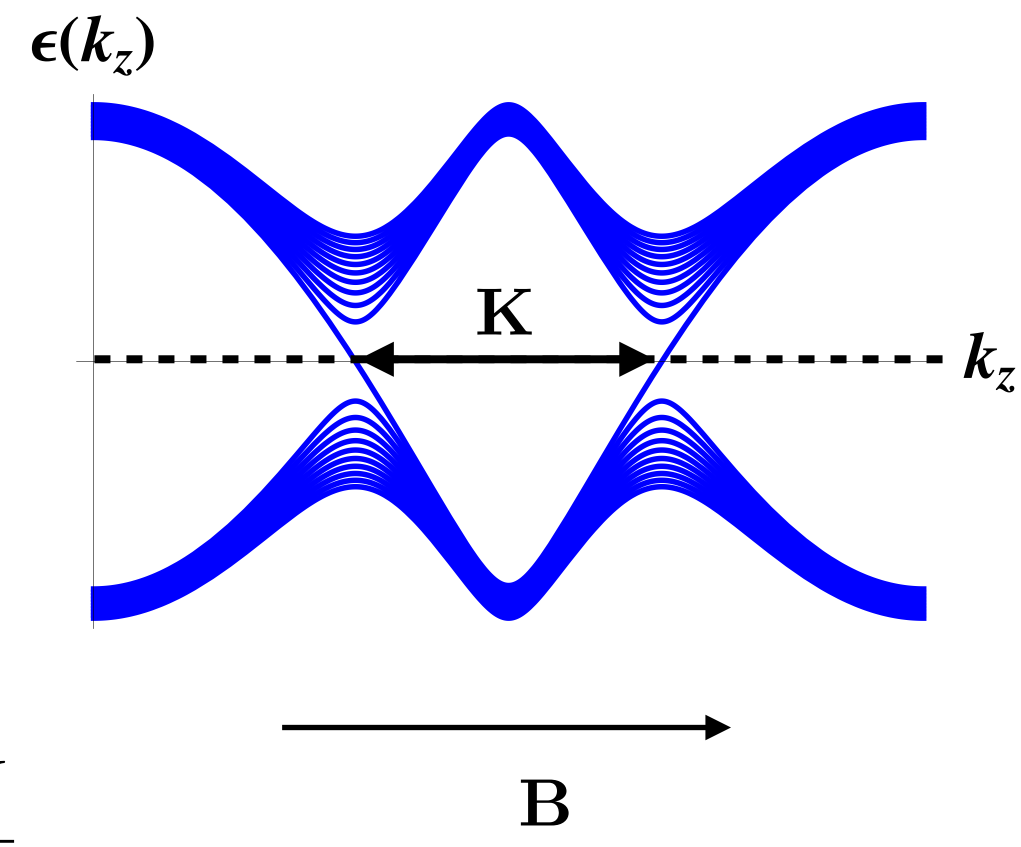
# Chiral anomaly

- Anomaly in this context means that Weyl nodes can only appear in pairs of opposite chirality, "chiral symmetry" can not be realized in a 3D lattice Hamiltonian.



- This leads to various observable penomena in response.

# Chiral anomaly

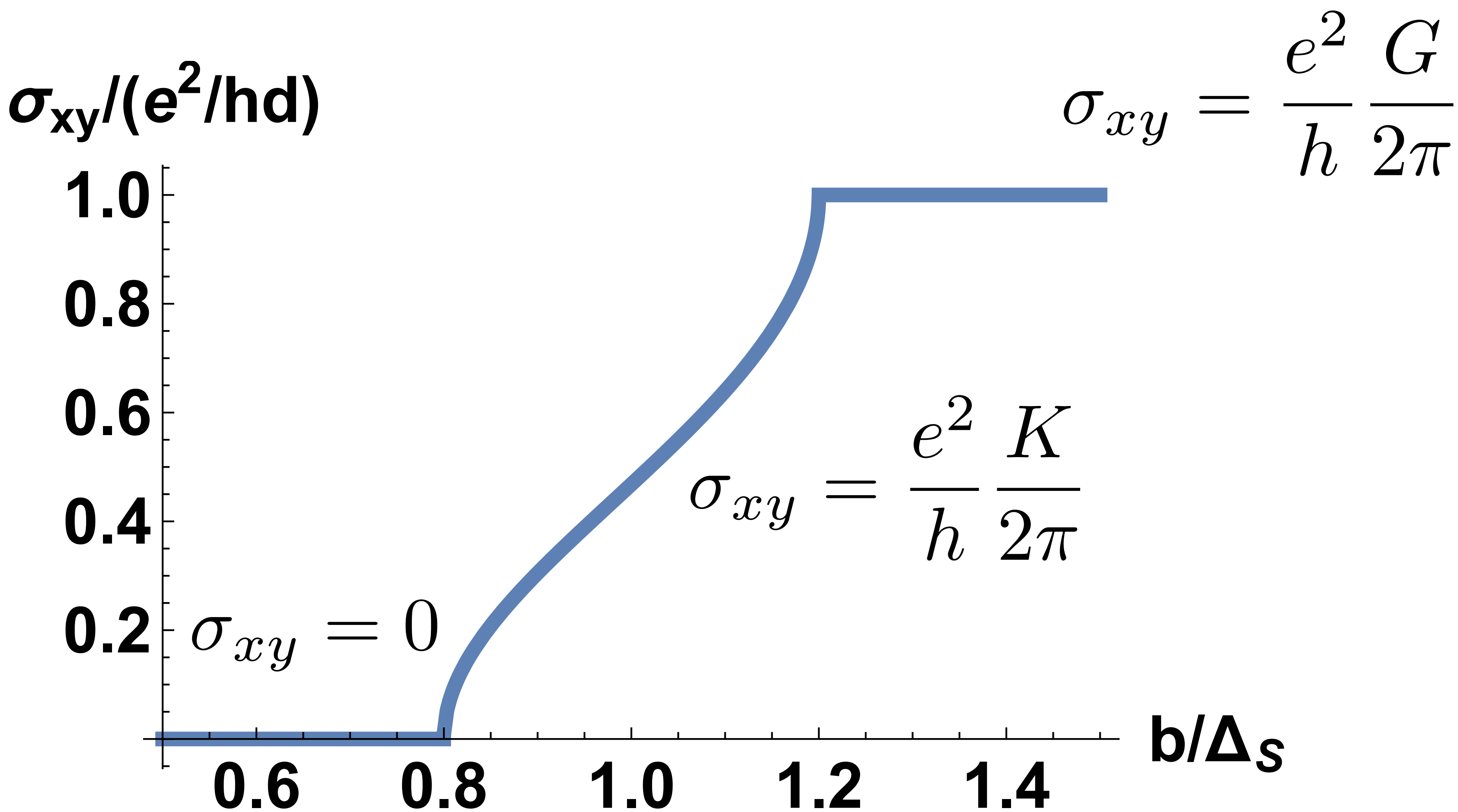



- Extra Landau level below the Fermi energy in between the Weyl nodes.

$$\sigma_{xy} = \sigma_{xy}^{II} = e\frac{\partial n}{\partial B} = eK2\pi\hbar\frac{\partial}{\partial B}\frac{1}{2\pi\ell_B^2} = \frac{e^2}{h}\frac{K}{2\pi}$$

- "Fractional" Hall conductivity in the absence of a Fermi surface inevitably implies Weyl nodes.

# “Plateau transition” in 3D



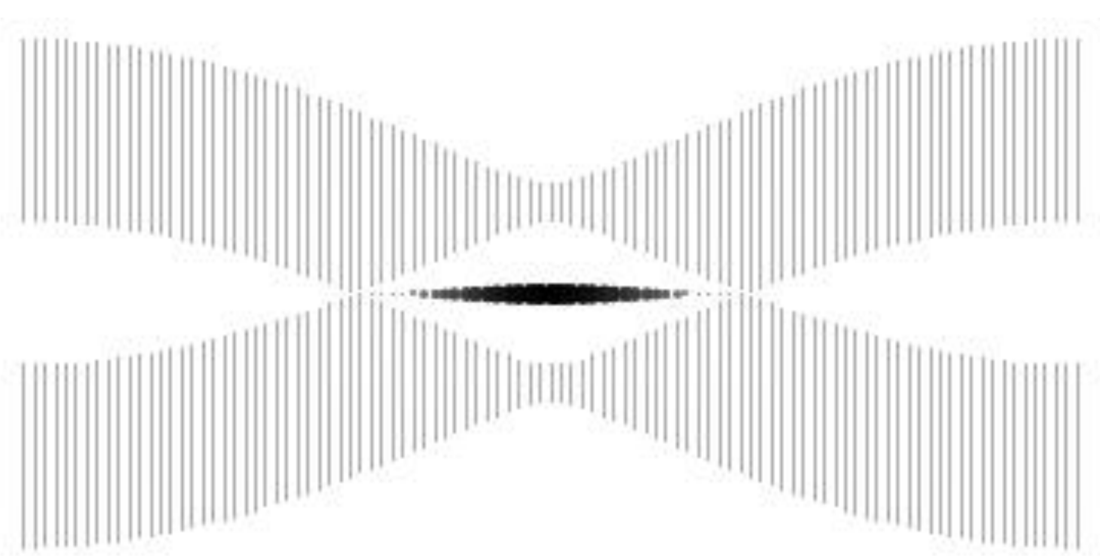

- Plateau transition is sharp in 2D, but broadens into Weyl semimetal phase in 3D.

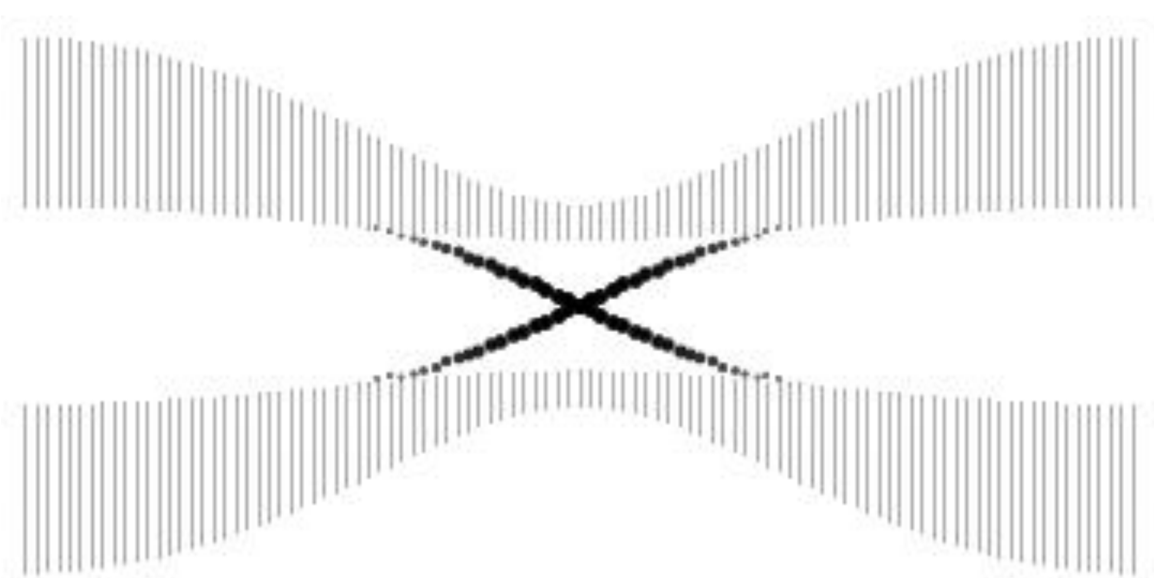
# Fermi arcs

- Equilibrium Hall conductivity leads to a "Chern-Simons" term:

$$\mathcal{L} = \frac{e^2}{4\pi h} K_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$$

- This is not gauge invariant in a sample with boundaries, which means chiral surface states must exist to compensate.





# Chiral anomaly and interactions

- Chiral anomaly inevitably implies Weyl nodes in case of weak interactions.
- Does this remain true when the interactions are not weak?
- In other words, can we gap out the Weyl nodes while preserving the chiral anomaly and while not breaking any symmetries?

# Analogous question for 3D TI surface states

- Parity anomaly in 3D TR-invariant TI implies the existence of gapless 2D Dirac surface states.

- Can we gap out the surface state without breaking TR and while preserving the parity anomaly?

Wang, Potter, Senthil

Metlitskii, Kane, Fisher

Chen, Fidkowski, Vishwanath

Bonderson, Nayak, Qi

# Vortex condensation

- Induce superconductivity on the surface: p+ip topological SC.

- Destroy SC coherence by condensing vortices while keeping the pairing gap: this produces an insulator.

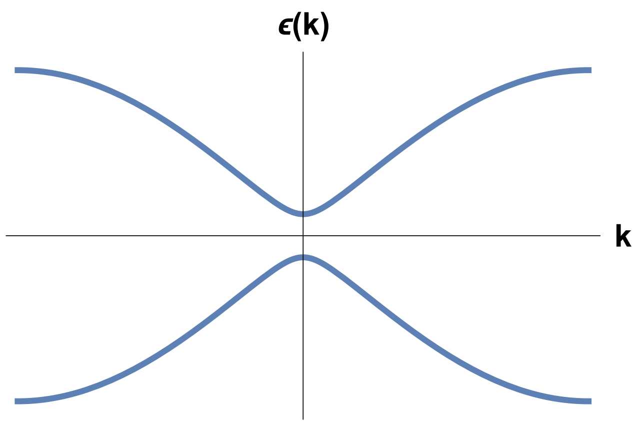
- Parity anomaly places strong restrictions on this procedure.

# 2D Dirac fermions and parity anomaly

- Convenient to deal with actual 2D Dirac fermions, not TI surface.

- This describes a QH plateau transition at which $\sigma_{xy}$ jumps by $e^2/h$

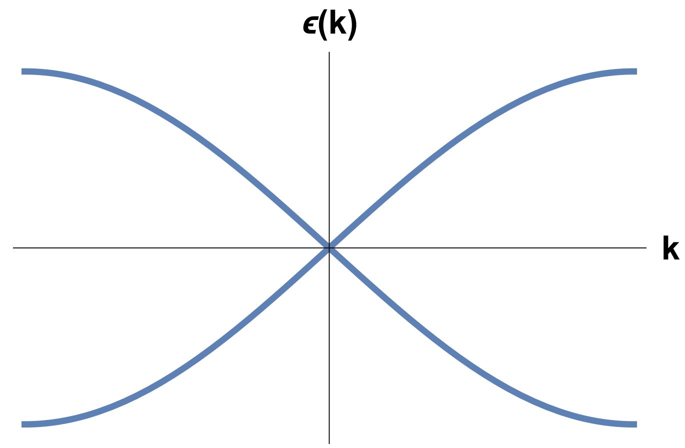
$m > 0$




$\sigma_{xy} = 0$

Haldane

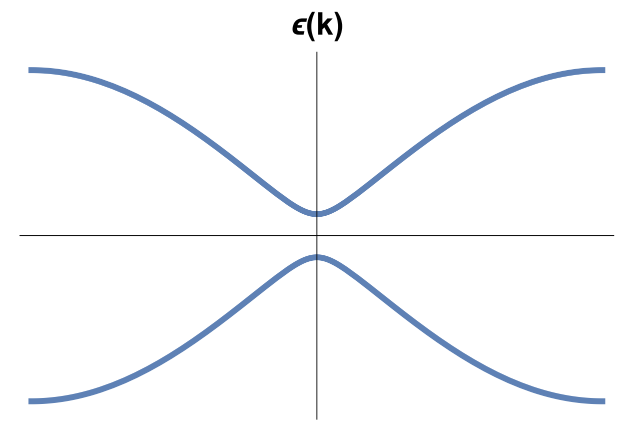
$m = 0$




$$\sigma_{xy} = \frac{e^2}{2h}$$

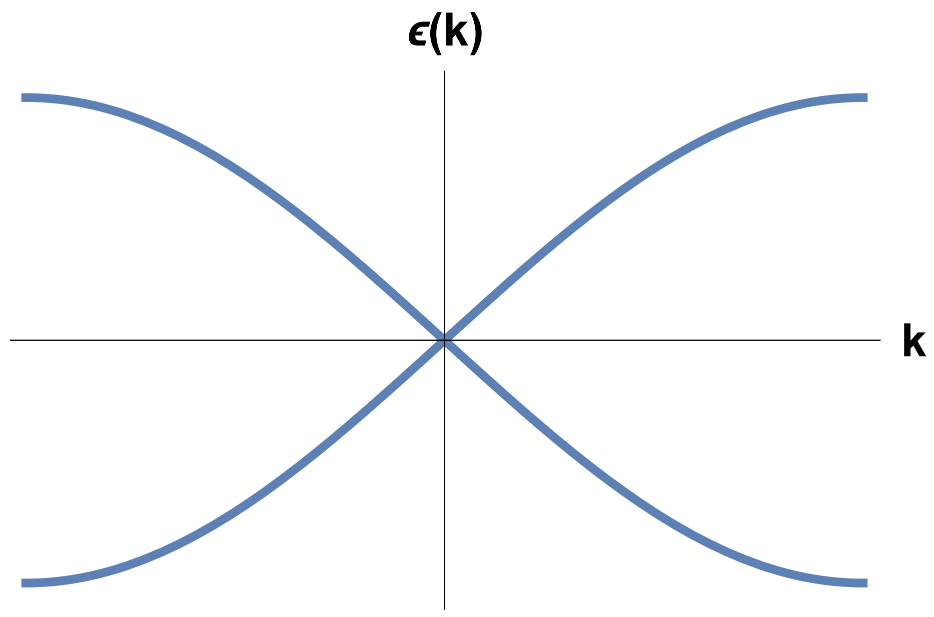
Ludwig, Fisher, Shankar, Grinstein

$m < 0$




$$\sigma_{xy} = \frac{e^2}{h}$$

# 2D Dirac fermions and parity anomaly

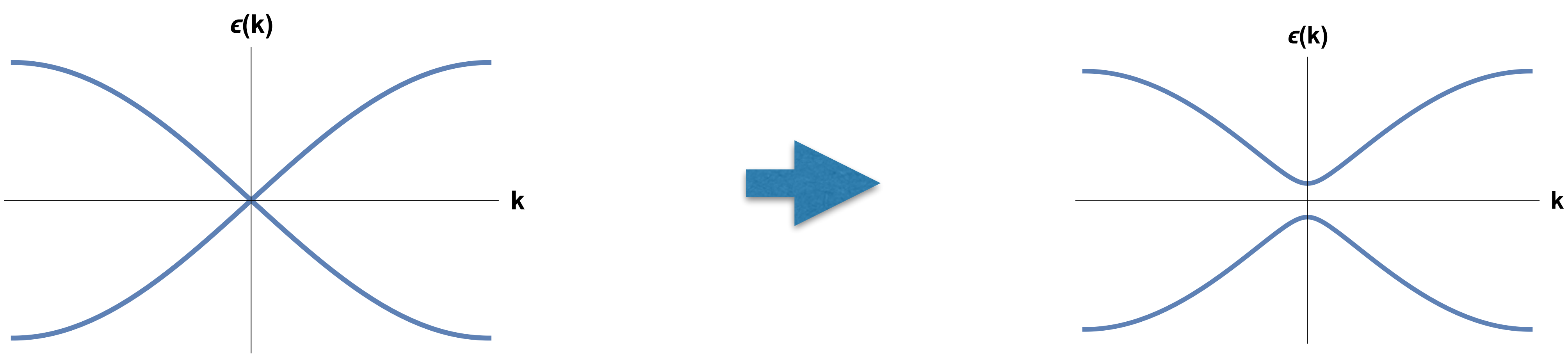



$$\sigma_{xy} = \frac{e^2}{2h}$$

- Massless 2D Dirac fermion thus has $\sigma_{xy} = \frac{e^2}{2h}$
- This is a manifestation of the parity anomaly.

# 2D Dirac fermions and parity anomaly

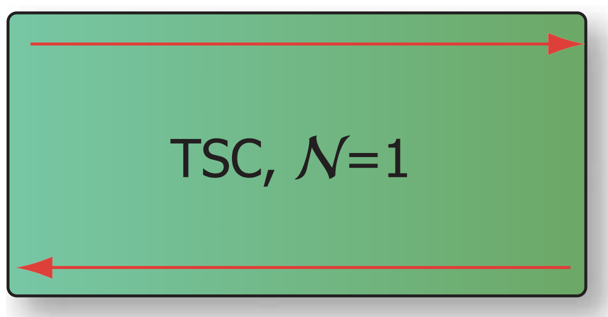
- We want to open a gap while keeping $\sigma_{xy} = \frac{e^2}{2h}$




- This is clearly impossible without creating a fractionalized state with topological order.

# Superconducting 2D Dirac fermion

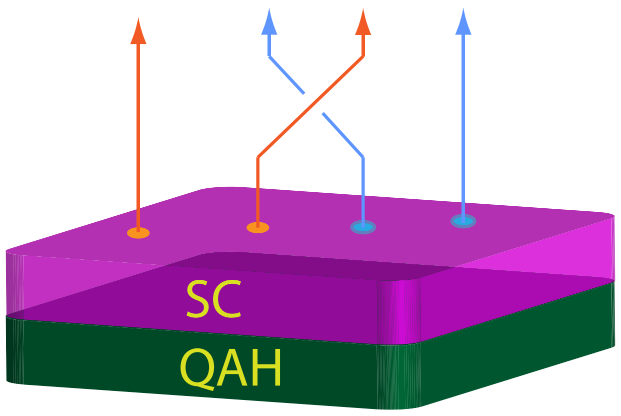
- Adding s-wave pairing get a fully gapped topological SC (analog of the Read-Green p+ip SC).




Qi, Hughes, Zhang

- Φ=hc/2e=π vortex then carries a Majorana zero-energy mode in its core.

Fu & Kane



# Condensing vortex composites

- Double $\Phi=hc/e=2\pi$ vortices have a pair of Majorana modes, which always get gapped out by perturbations (pairing, finite Fermi energy, etc.)

- Naively, such vortices may be condensed.

# Condensing vortex composites

- However, $\Phi$ = hc/e = $2\pi$ vortex binds a half-electron charge, due to the half-quantized Hall conductivity:

$$\mathcal{L} = \frac{\sigma_{xy}}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

$$n = \frac{\Phi}{4\pi} = \frac{1}{2}$$

- Two such charges will have semion exchange statistics:

$$\theta = 2\pi^2\sigma_{xy} = \frac{\pi}{2}$$

# Condensing vortex composites

- Two such charges will have semion exchange statistics:

$$\theta = 2\pi^2 \sigma_{xy} = \frac{\pi}{2}$$

- Semions can not be condensed directly.

- They may transform themselves into bosons by flux attachment, but this will also change the Hall conductivity.

# Condensing vortex composites

- The smallest vortex that is a boson and may be condensed is quadruple vortex $\Phi = 2hc/e = 4\pi$.

- Condensation of composite vortices implies charge fractionalization and topological order.

# Composite vortices and fractionalization

- Superconductor-insulator (charge-flux) duality:

Flux is quantized in a superconductor, but not well-defined (condensed) in an insulator.

Charge is quantized in an insulator, but not well-defined (condensed) in a supercoductor.

# Composite vortices and fractionalization

- Flux quantum in an ordinary superconductor with paired electrons:

$$\Phi = \frac{hc}{2e}$$

- If electrons are condensed in quartets instead of pairs, the flux quantum is halved:

$$\Phi = \frac{hc}{4e}$$

- The dual picture: pairing vortices leads to charge fractionalization.

# Composite vortices and fractionalization

- The dual picture: pairing vortices leads to charge fractionalization.

$$Q\Phi = hc$$

Senthil & Fisher

# Composite vortices and fractionalization

- If we condense vortices with the smallest vorticity, we get an ordinary insulator, electron is unfractionalized and gapped.

$$\Phi = \frac{hc}{2e}$$

- Condensing double vortices fractionalizes electron into "spinon" and "chargon":

$$\Phi = \frac{hc}{e}$$

$$c = bf$$

- Condensing quadruple vortices fractionalizes the chargon:

$$\Phi = \frac{2hc}{e}$$

$$c = b_1 b_2 f$$

# Pfaffian-antisemion state

$$c = b_1 b_2 f$$

- b is a charge-1/2 boson, while f is a neutral fermion.

- f inherits the bandstructure of a massive Dirac fermion and provides the thermal conductivity:

$$\kappa_{xy} = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Wang, Potter, Senthil

Metlitski, Kane, Fisher

# Pfaffian-antisemion state

$$c = b_1 b_2 f$$

- b is a charge-1/2 boson, while f is a neutral fermion.
- b exists in a bosonic IQH state:

$$\mathcal{L} = \frac{1}{4\pi} \sum_{IJ} \epsilon^{\mu\nu\lambda} a_{I\mu} K_{IJ} \partial_\nu a_{J\lambda} - \frac{e}{4\pi} \sum_{I} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_{I\lambda} \qquad K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_{xy} = 2(1/2)^2/2\pi = 1/4\pi \qquad \kappa_{xy} = 0$$

Lu & Vishwanath

Senthil & Levin

# Pfaffian-antisemion state

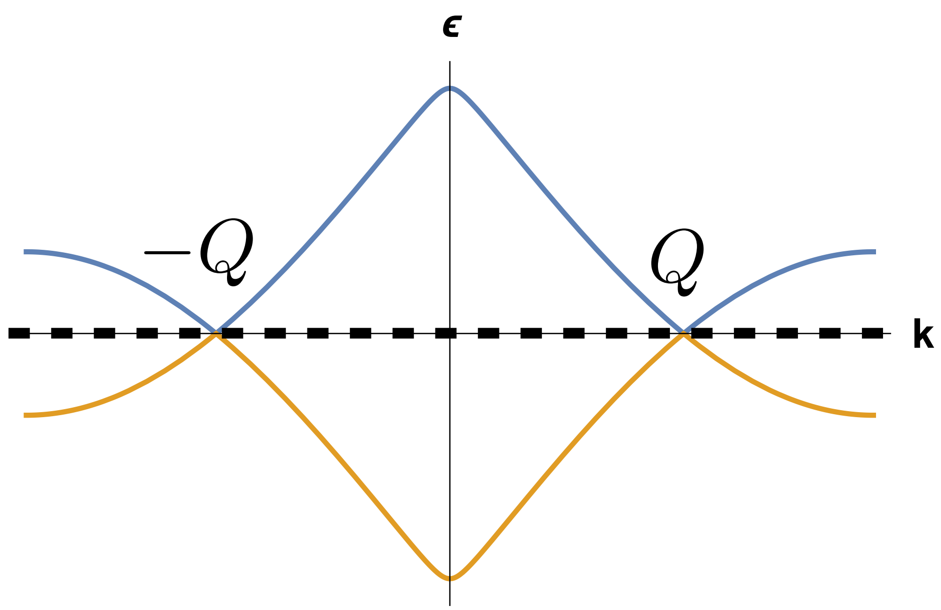
- b exists in a bosonic IQH state:

$$\mathcal{L} = \frac{1}{4\pi}\sum_{IJ}\epsilon^{\mu\nu\lambda}a_{I\mu}K_{IJ}\partial_\nu a_{J\lambda} - \frac{e}{4\pi}\sum_{I}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_{I\lambda} \qquad K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_{xy} = 2(1/2)^2/2\pi = 1/4\pi \qquad \kappa_{xy} = 0$$

# Chiral anomaly and interactions

- Can we gap out the Weyl nodes while preserving the chiral anomaly?




$$\sigma_{xy} = \frac{e^2}{h}\frac{2Q}{2\pi}$$

Wiedemann-Franz law: $$\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

May also view this as a consequence of the gravitational anomaly.

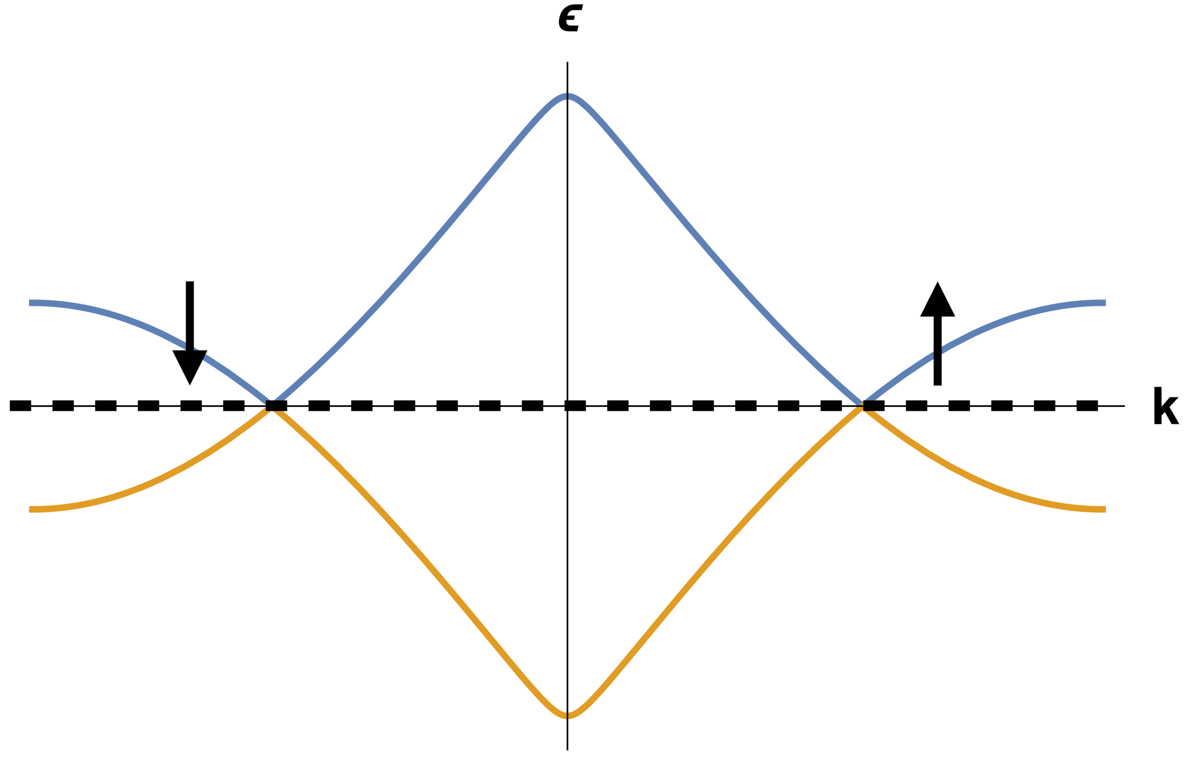
# Vortex condensation

- Start from a superconducting Weyl semimetal and attempt to destroy superconductivity by condensing vortices and preserving the chiral anomaly.

- If successful, this leads to an insulator with the same chiral anomaly as Weyl semimetal.

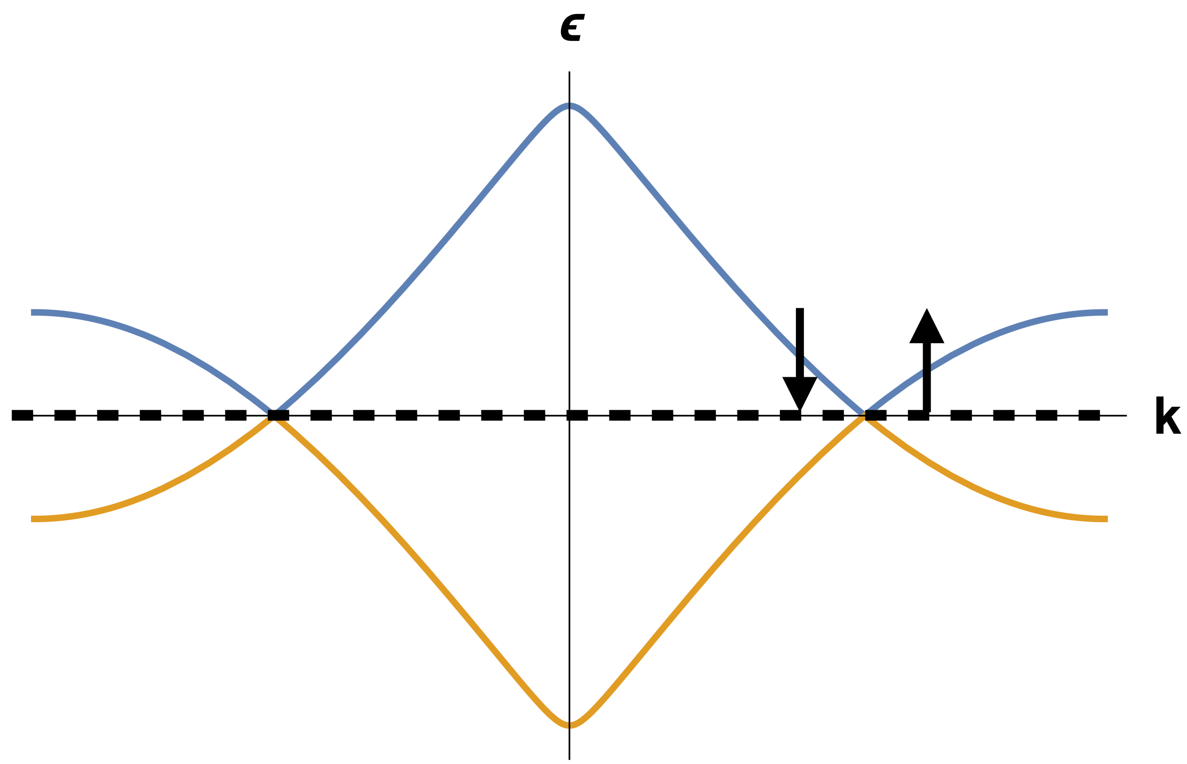
# Weyl superconductor

- BCS: pairing k and -k states, i.e. internodal pairing.



# Weyl superconductor

- FFLO: pairing states on the opposite side of each Weyl point, i.e. intranodal pairing.



# BCS pairing

- BCS pairing can not open a gap, since the two chiralities are not mixed by the pairing term:

$$H = v_F \sum_{\mathbf{k}} c_{\mathbf{k}}^\dagger \tau^z \boldsymbol{\sigma} \cdot \mathbf{k} c_{\mathbf{k}} + \Delta \sum_{\mathbf{k}} (c_{\mathbf{k}R}^\dagger i\sigma^y c_{-\mathbf{k}L}^\dagger + h.c.)$$

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}R\uparrow}, c_{\mathbf{k}R\downarrow}, c_{-\mathbf{k}L\downarrow}^\dagger, -c_{-\mathbf{k}L\uparrow}^\dagger)$$

$$H = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger (v_F \boldsymbol{\sigma} \cdot \mathbf{k} + \Delta \tau^x) \psi_{\mathbf{k}}$$

Meng & Balents

# FFLO pairing

- FFLO does open a gap, but breaks translational symmetry:

$$H = v_F \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \boldsymbol{\sigma} \cdot \mathbf{k}\, c_{\mathbf{k}} + \Delta \sum_{\mathbf{k}} (c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow})$$

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow}, c_{\mathbf{k}\downarrow}, c^\dagger_{-\mathbf{k}\downarrow}, -c^\dagger_{-\mathbf{k}\uparrow})$$

$$H = \frac{1}{2} \sum_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} (v_F \tau^z \boldsymbol{\sigma} \cdot \mathbf{k} + \Delta \tau^x) \psi_{\mathbf{k}}$$

# FFLO pairing

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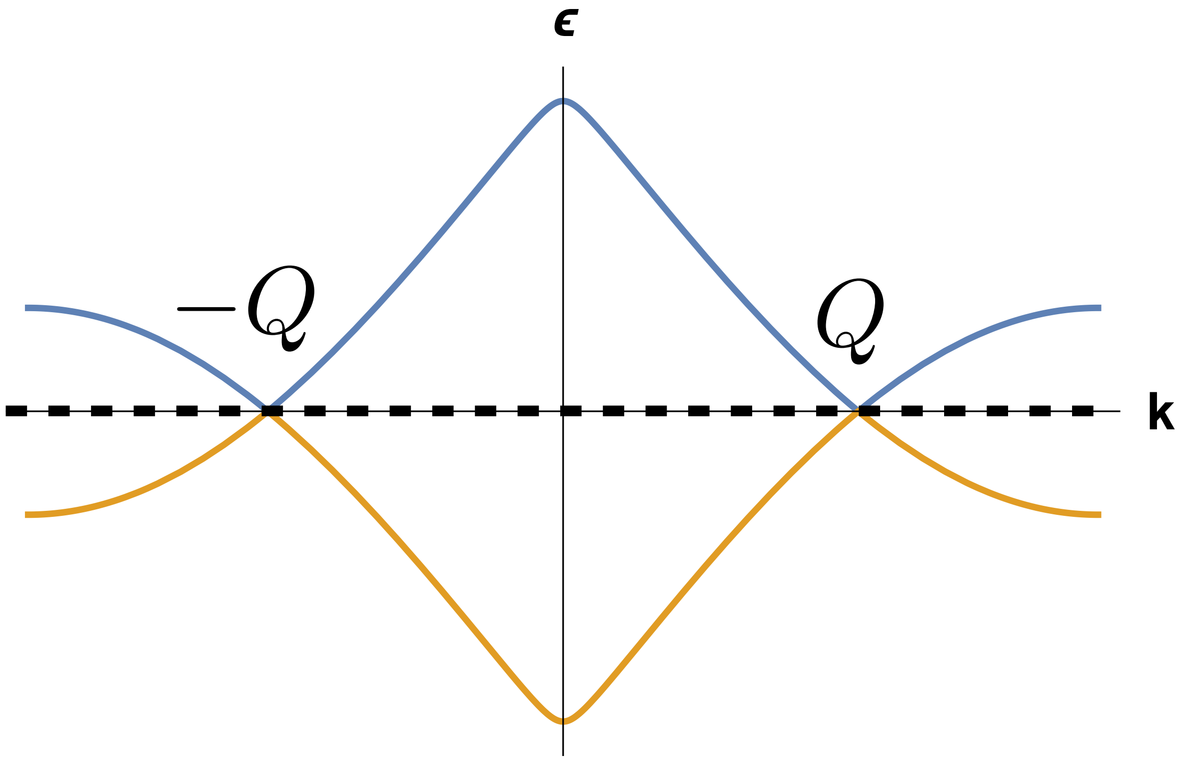
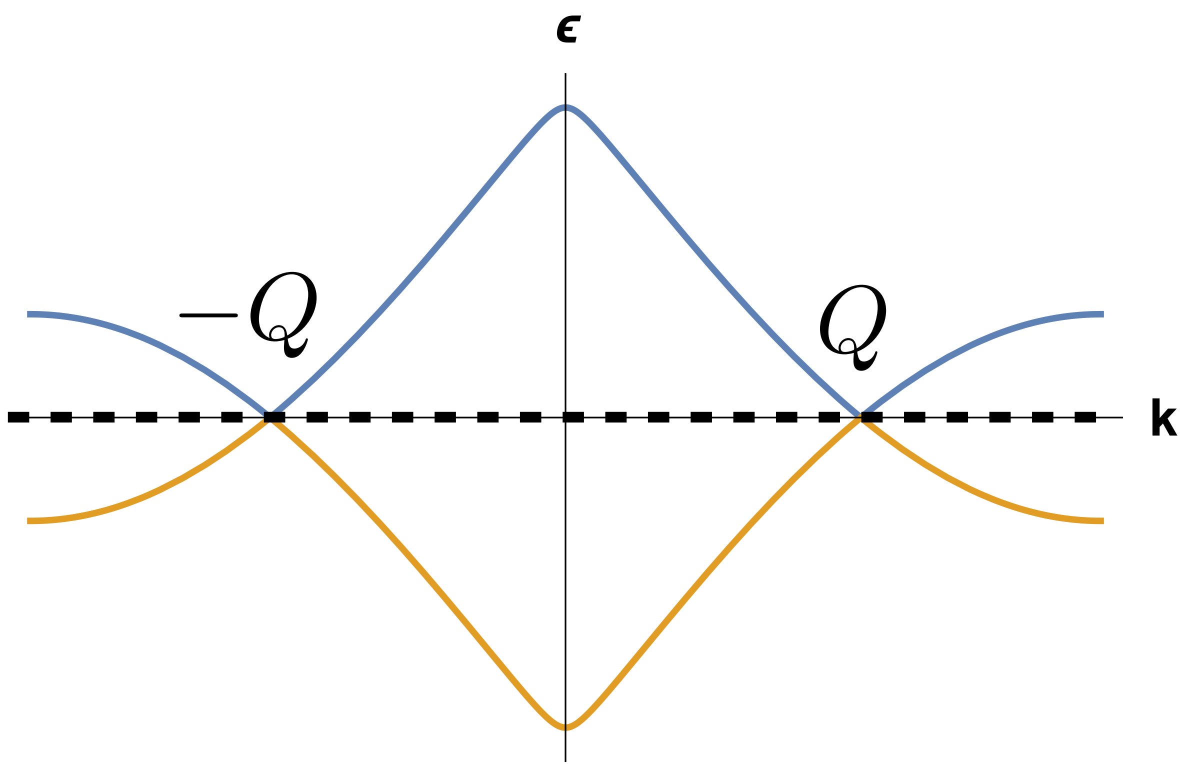



$\Delta(\mathbf{Q}) \sim \sum_{\mathbf{k}} \langle c^{\dagger}_{\mathbf{Q}+\mathbf{k}} c^{\dagger}_{\mathbf{Q}-\mathbf{k}} \rangle$

carries momentum 2Q.

$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$ carries momentum 4Q.

# FFLO pairing




$$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$$

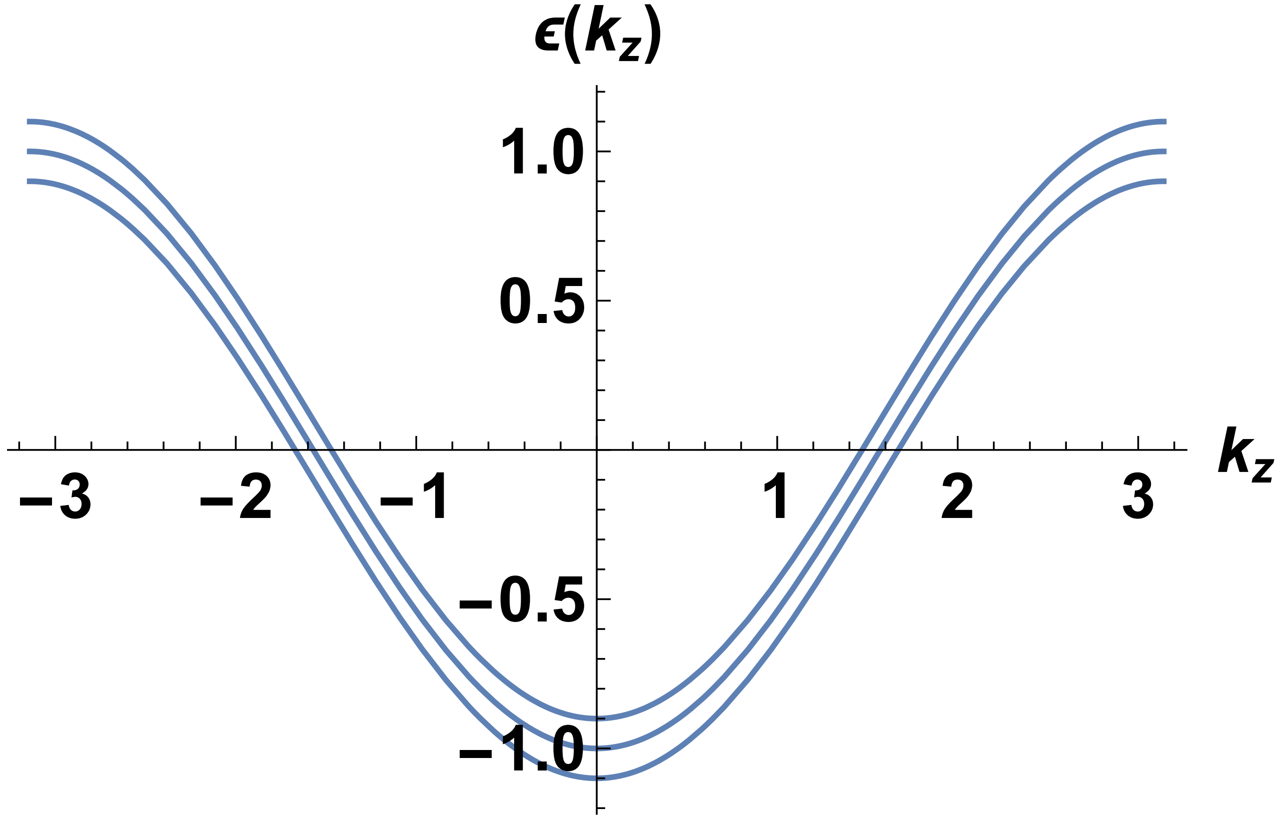
carries momentum 4Q.

- This breaks translational symmetry, unless $\mathbf{Q} = \mathbf{G}/4$
- In other words, FFLO does not break translational symmetry when Weyl node separation is exactly half the BZ size.

# Vortex condensation in FFLO state

- n-fold vortex (Φ=nhc/2e) in FFLO paired state: get n chiral Majorana modes in the vortex core.

$$\epsilon_p(k_z) = \epsilon_F\left(1 - \frac{2p}{n+1}\right) + v_F k_z. \qquad p = 1, \ldots, n.$$

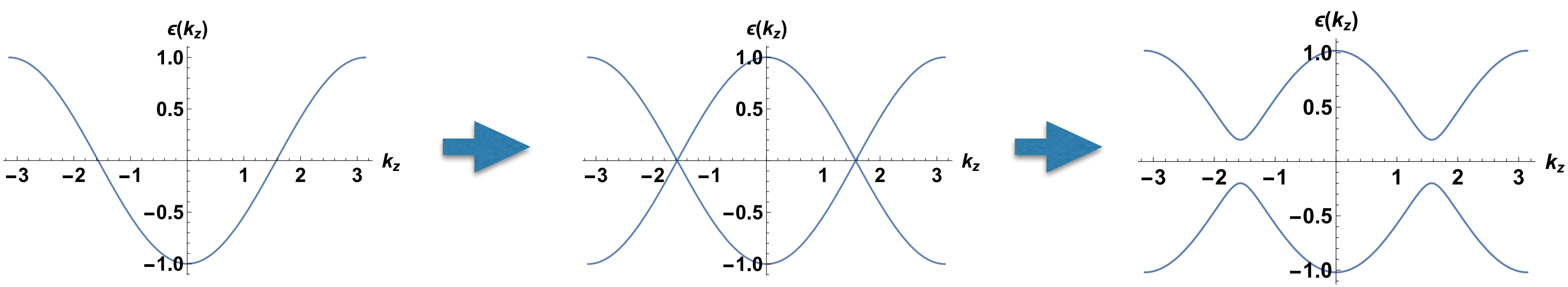



Callan & Harvey

Jackiw & Rossi

# Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:



# Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:

$$H = v_F \sum_{k_z} [k_z c^\dagger_{k_z} \tau^z c_{k_z} + \Delta(c^\dagger_{k_z} i\tau^y c^\dagger_{-k_z} + \text{h.c.})/2]$$

- An odd number of Majorana modes can not be eliminated without breaking translational symmetry, thus a fundamental SC vortex may not be condensed.

$$\Phi = \frac{hc}{2e} = \pi \qquad \hbar = c = e = 1$$

# Vortex condensation in FFLO state

- A double vortex does not have Majorana modes, but may still not be condensed.

- This follows from the fact that the insulating state we want to obtain must preserve the chiral anomaly, i.e. must have a Hall conductivity of half conductivity quantum per atomic plane:

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi}$$

# Vortex condensation in FFLO state

- A vortex will induce a charge when intersecting an atomic plane:

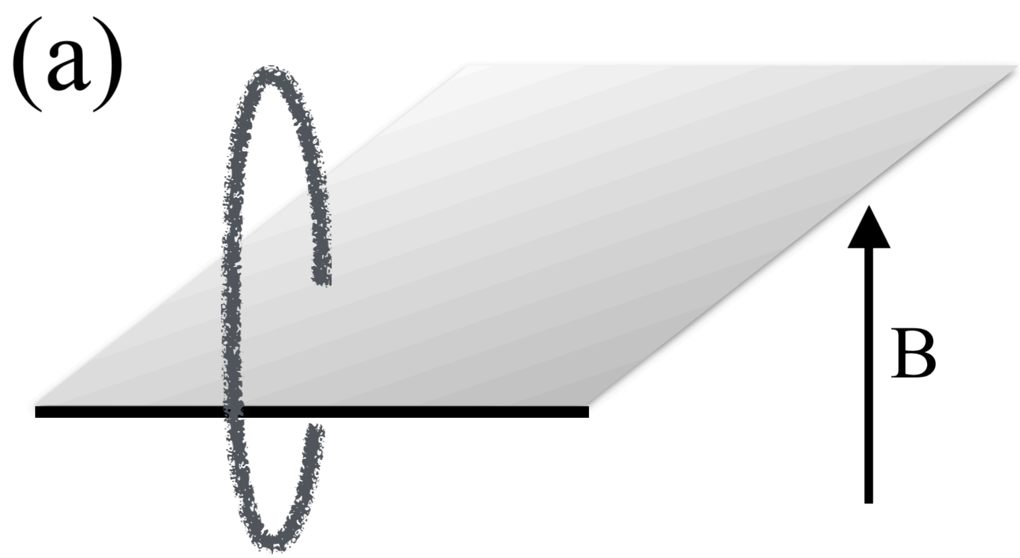
$$\mathcal{L} = \frac{\sigma_{xy}}{2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$n = \frac{\Phi}{4\pi} = \frac{1}{2}$$

- Since vortex is a loop, it will always intersect any atomic plane twice, inducing a pair of opposite charges, whose effect will thus cancel.

# Vortex condensation in FFLO state

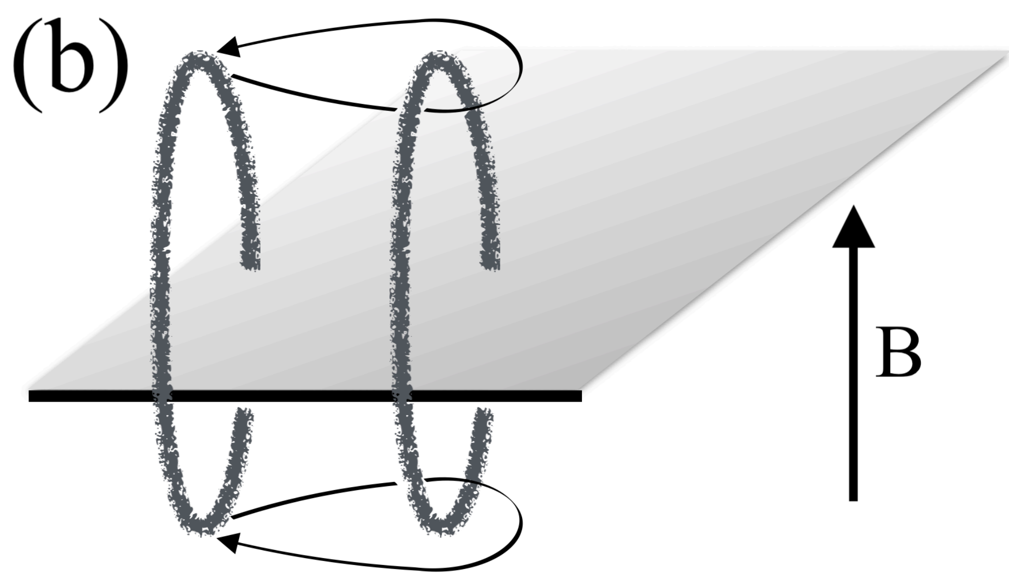
- But consider a crystal with a dislocation.




- In this case vortex loop may intersect the extra half-plane only once, inducing uncompensated 1/2 charge.

# Vortex condensation in FFLO state

- In this case vortex loop may intersect the extra half-plane only once, inducing uncompensated 1/2 charge.

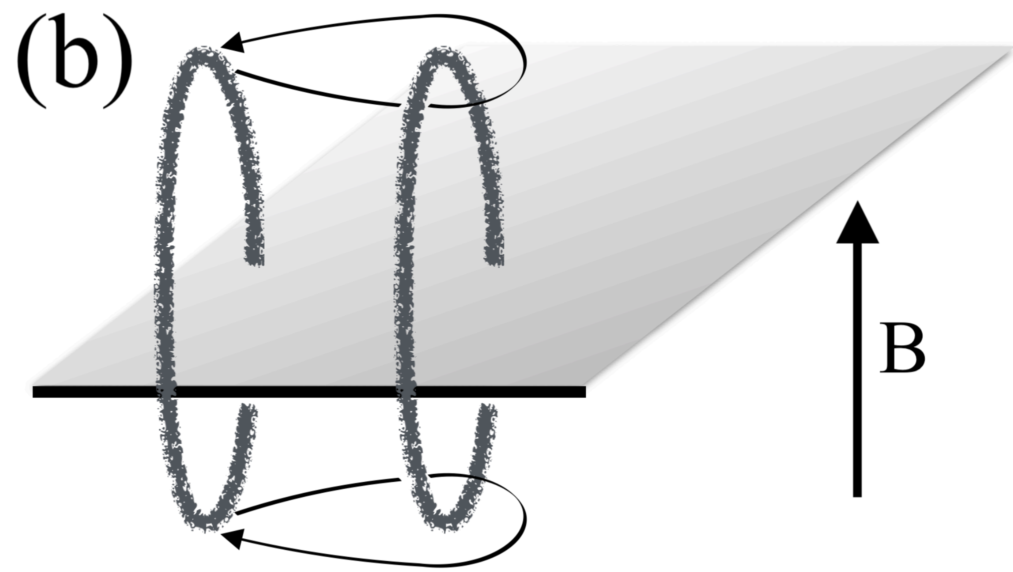



Wang & Levin 3-loop braiding

- Two such charges will have semion exchange statistics:

$$\theta = 2\pi^2 \sigma_{xy} = \frac{\pi}{2}$$

# Vortex condensation in FFLO state




- Two such charges will have semion exchange statistics.

- This means that, inserting a dislocation in a crystal with condensed vortices will cost an energy $O(L^2)$

- This implies broken translational symmetry.

# Vortex condensation in FFLO state

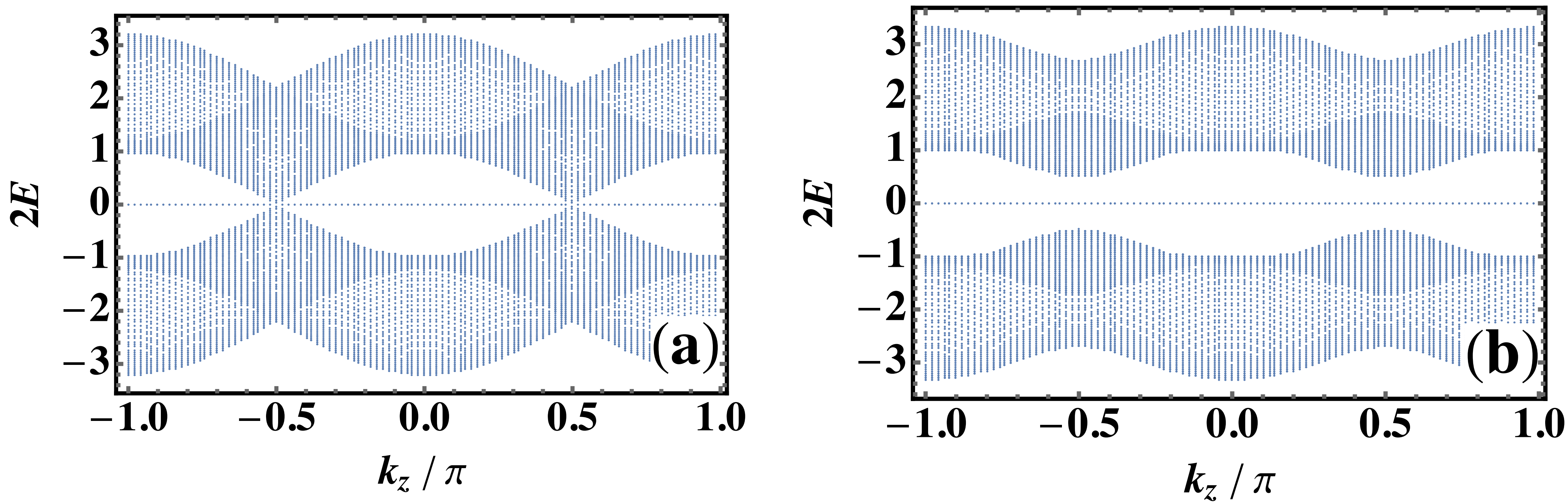
- Following the same logic, quadruple vortices have bosonic statistics and thus may be condensed without breaking any symmetries.

- This is an insulating state that preserves the chiral anomaly and does not break any symmetries.

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi} \qquad \kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

# Nature of the insulating state

- f experiences the same electronic structure as the Weyl FFLO superconductor.




$$\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

$$\sigma_{xy} = 0$$

# Nature of the insulating state

$$c = b_1 b_2 f$$

- b is a charge-1/2 boson, while f is a neutral fermion.

- b exists in a layered bosonic IQH state:

$$\sigma_{xy} = 2(1/2)^2/2\pi = 1/4\pi \qquad \kappa_{xy} = 0$$

# Nature of the insulating state

$$c = b_1 b_2 f$$

- b is a charge-1/2 boson, while f is a neutral fermion.
- b exists in a layered bosonic IQH state.
- b and f are coupled to a $Z_4$ gauge field which results from the 4-fold vortex condensation.
- This state is the 3D analog of the "Pfaffian-antisemion" state, described before.

# Nontrivial generalization of FQHE to 3D

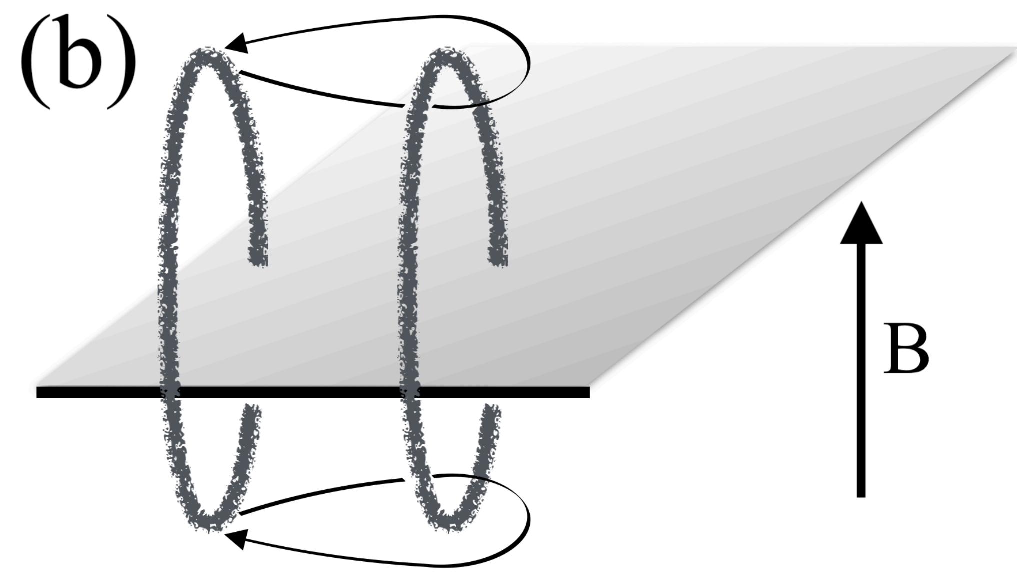
$$\mathcal{L} = \frac{m}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + a_\mu j^\mu$$

$$\sigma_{xy} = \frac{1}{2\pi m}$$

- Quasiparticles carry flux of 2π/m and thus are anyons with the braiding phase:

$$\theta = \frac{2\pi}{m} = 4\pi^2\sigma_{xy}$$

# Nontrivial generalization of FQHE to 3D

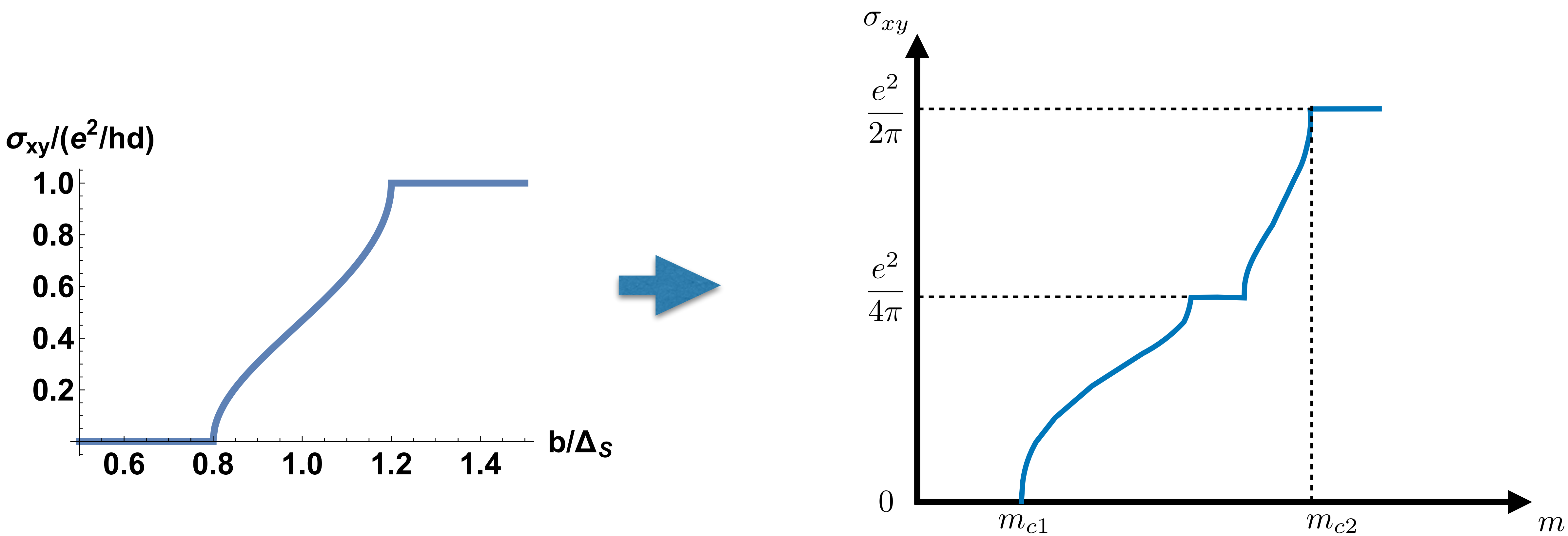



- In our case the analog of this is that 2π vortices, when linked with a dislocation, have the abelian braiding phase:

$$\theta = 4\pi^2 \sigma_{xy} = \pi$$

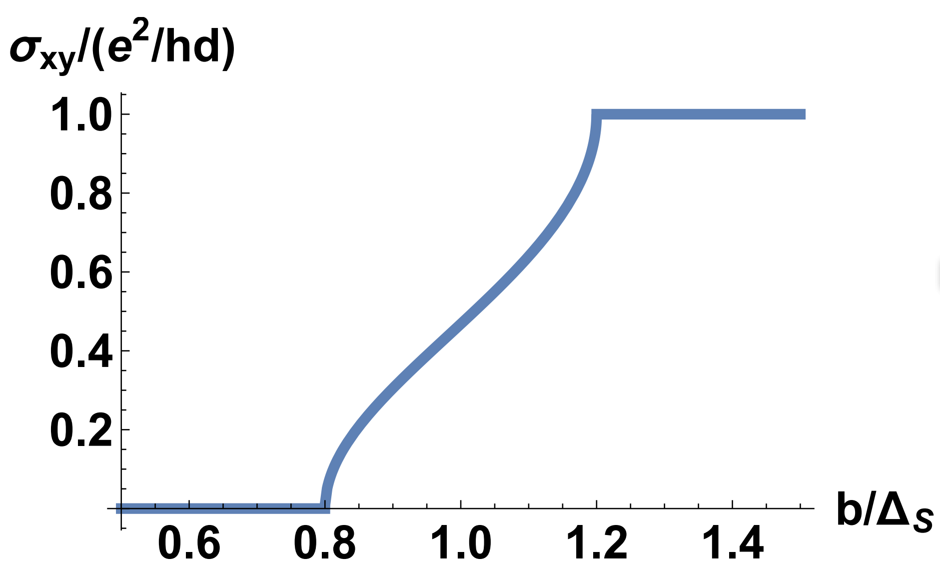
where $\sigma_{xy}$ is the Hall conductivity per atomic plane.

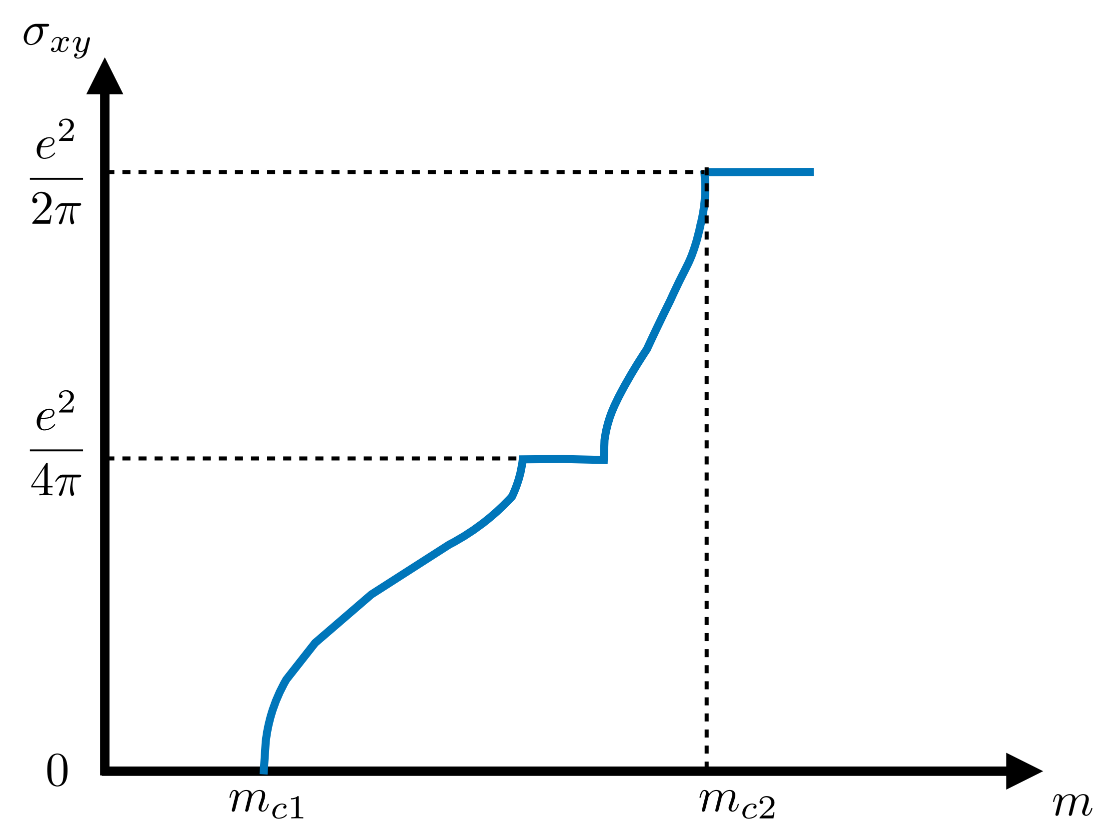
# Nontrivial generalization of FQHE to 3D




- In the presence of interactions, smooth evolution of the Hall conductivity with the magnetization in a Weyl semimetal may be interrupted by a half-quantized plateau.

# Nontrivial generalization of FQHE to 3D



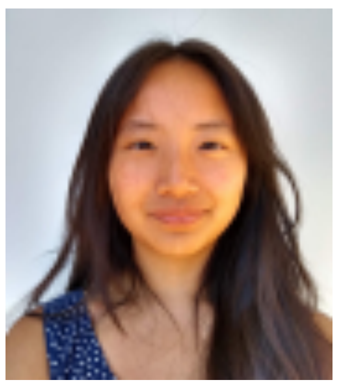

- Changing magnetization in this case is analogous to changing the magnetic field and moving the Landau levels through the Fermi energy in the case of the ordinary 2D FQHE.

# Conclusions

- Concepts of nontrivial electronic structure topology may be extended to metals.
- Gaplessness may be mandated by topology, which may be understood from the viewpoint of quantum anomalies.
- In weakly-interacting Weyl semimetals these lead, in particular, to novel macroscopic quantum transport phenomena, such as intrinsic AHE, strong AMR and spectral weight transfer in optical conductivity.
- With strong interactions, gap may be opened without destroying the chiral anomaly, but only at the expense of introduing nontrivial topological order, which may be regarded as nontrivial generalization of FQHE to 3D.

# Thanks

## LEI GIOIA YANG



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