



# Instability of Walker Propagating Domain-Wall Mode in Magnetic Nanowires

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**Concepts in Spintronics,  
KITP, Oct. 1, 2013**



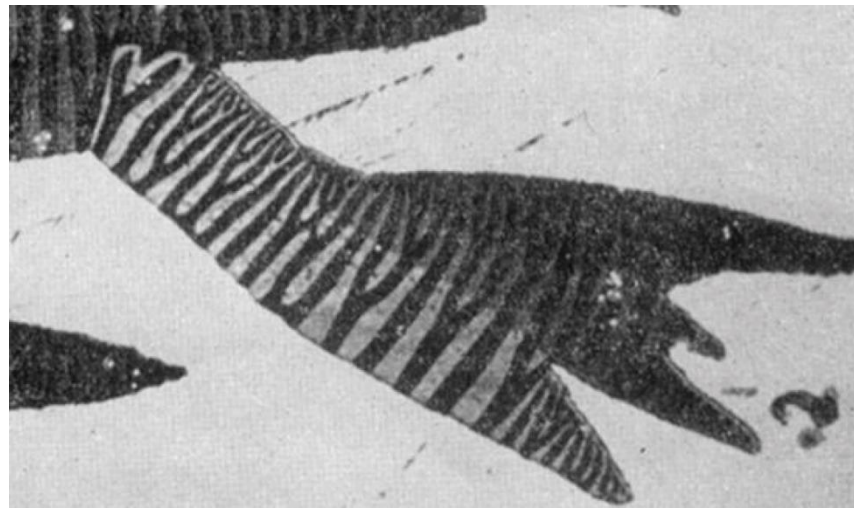
# Outline

- Introduction
  - Magnetization Dynamics and Applications
  - Landau-Lifshitz-Gilbert Equation and Walker Solution
  - The Issue
- Instability of Walker Solution in Magnetic Nanowires
  - Essential Spectrum and Domain Instability
  - Absolute Spectrum and DW-Profile Instabilities
    - Transient Instability
    - Convective/Absolute Instability
- Conclusion

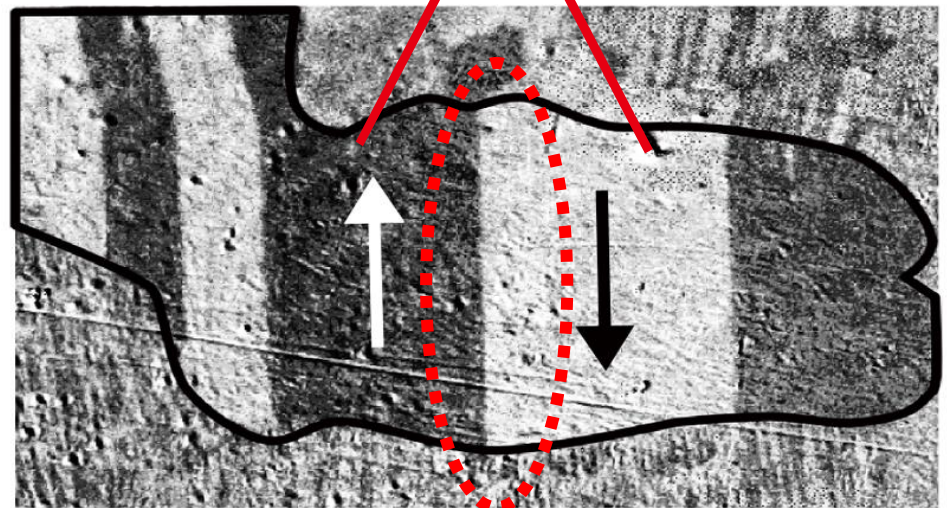


# Magnetic Domain & Domain Wall

Magnetic Domains



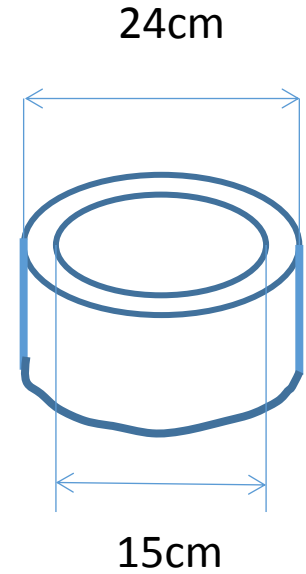
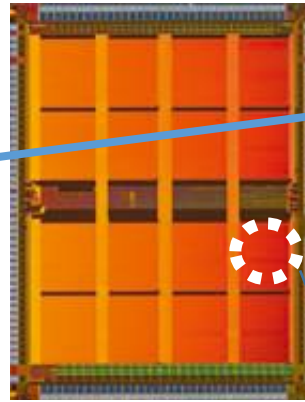
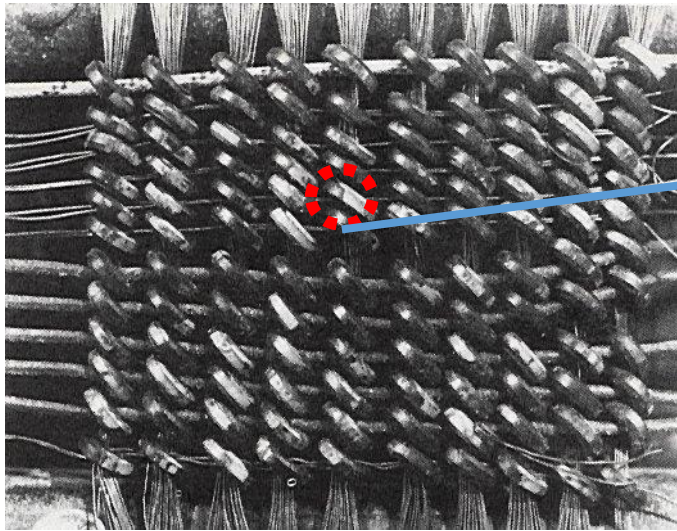
Roberts et al., Phys. Rev. 96, 1494 (1954).



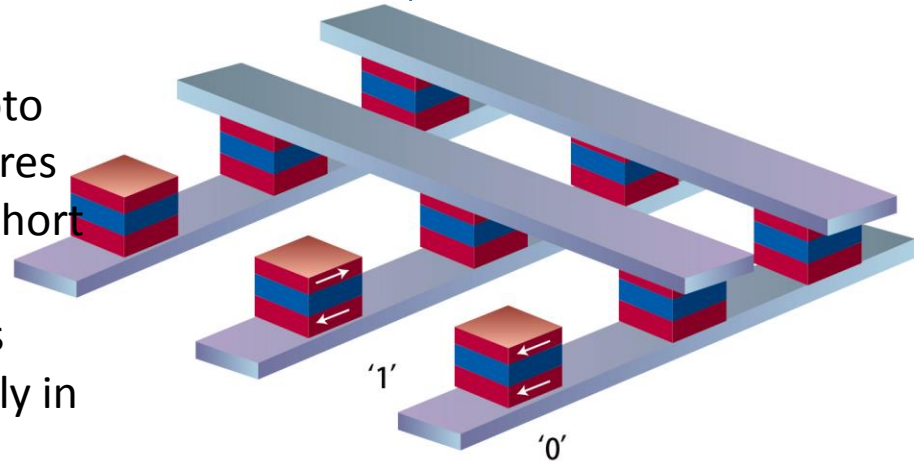
Zureks, Chris Vardon, Wikipedia, 2008.

Domain-Wall (DW)

# Domain Applications



Magneto-resistance Random Access Memory



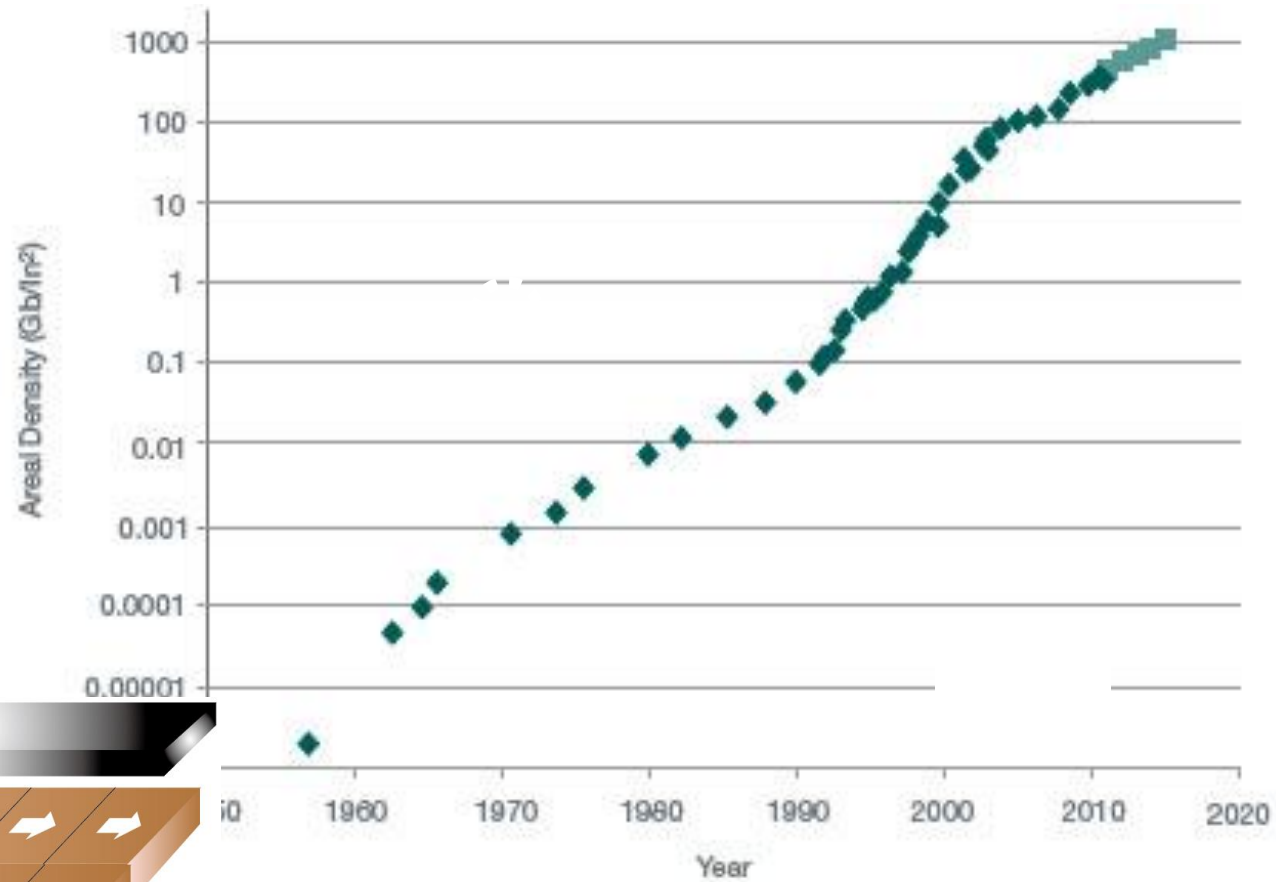
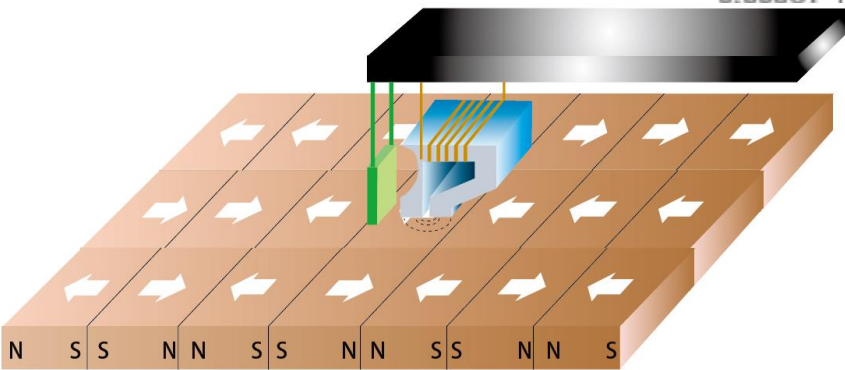
The 1st magnetic core memory, IBM 405 Alphabetical Accounting Machine. The photo shows the single drive lines through the cores in the long direction and fifty turns in the short direction. The cores are 150 mil inside diameter, 240 mil outside, 45 mil high. This experimental system was tested successfully in April 1952.



# Applications



Hard Disk Drive

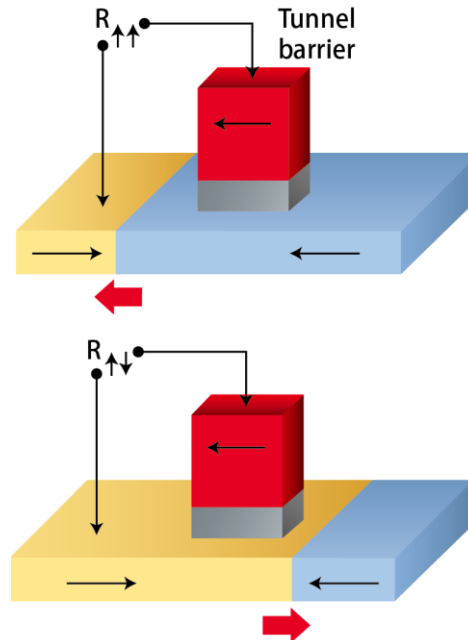


# Domain Wall Propagation: Applications

Symbol	CMOS Circuit	Domain Wall Logic Circuit
Vdd (+5 V)	Charge	Magnetization
0 V	No charge	Magnetization
Fan-out	Input, Output A, Output B	Input, Output A, Output B
Cross-over	Input A, Input B, Output A, Output B, Vias	Input A, Input B, Output A, Output B
NOT	Input, Output, Vdd	Input / Output, 500 nm, 225 nm, 100 nm, 500 nm, 20°
AND	Input A, Input B, Output, NAND, Inverter, Vdd	Input A, Input B, Output, 1 μm, 125 nm, 200 nm

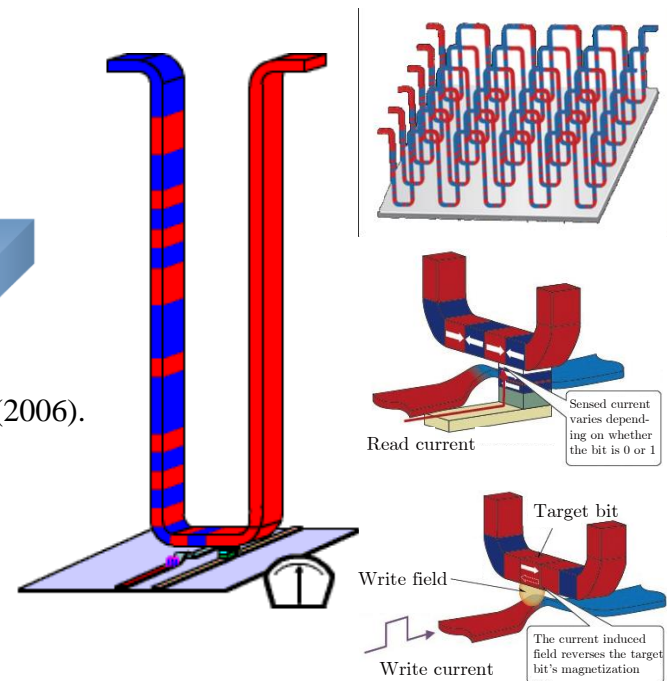
D. A. Allwood *et al.*, Science **309**, 1688 (2005).

## DW Logic Circuit



Cros, V *et al.*, Patent WO, 064022 (2006).

## DW-RAM



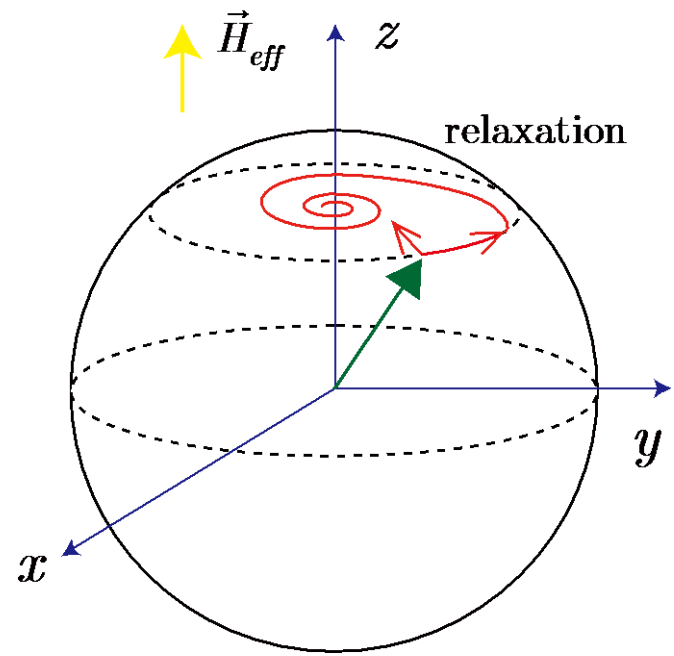
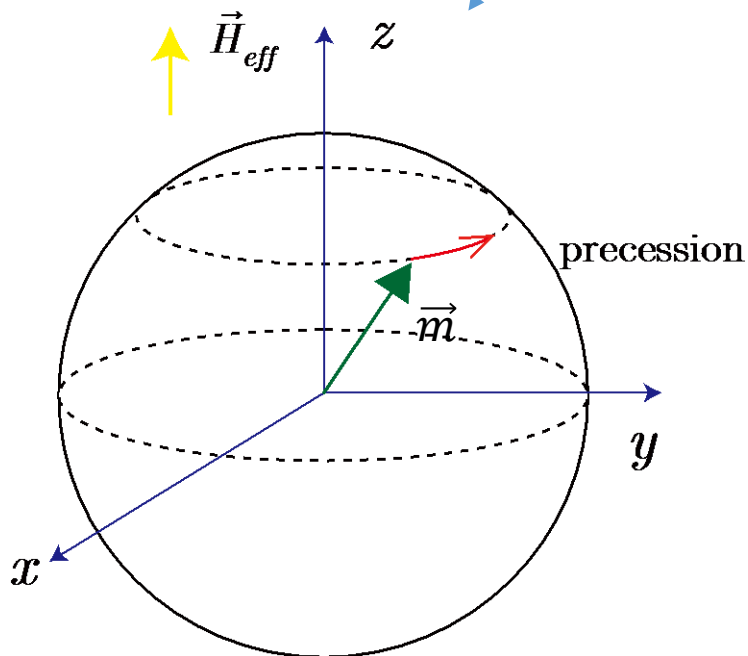
S. Parkin *et al.*, Science **320**, 190 (2008).

## Racetrack Memory

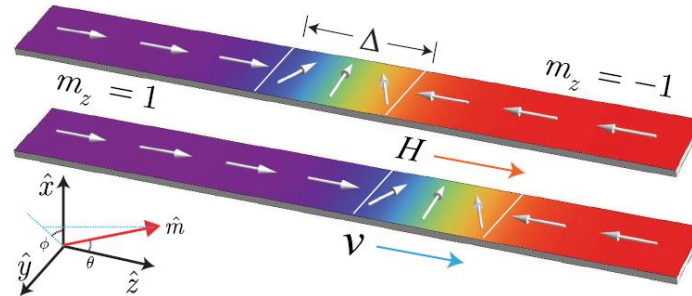
# Landau-Lifshitz-Gilbert (LLG) Equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t}$$



# Walker DW Solution of the LLG Eq.

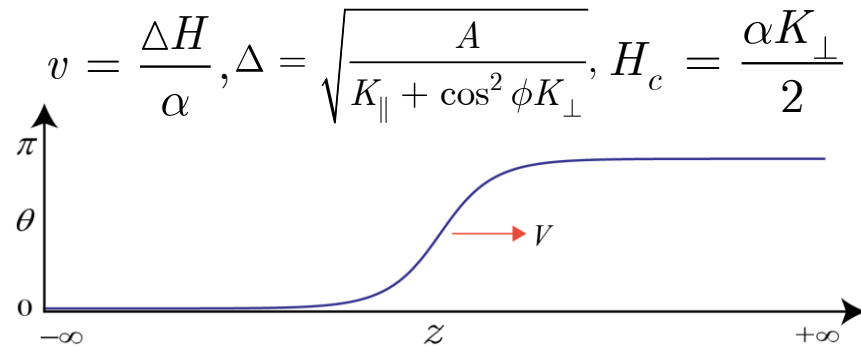
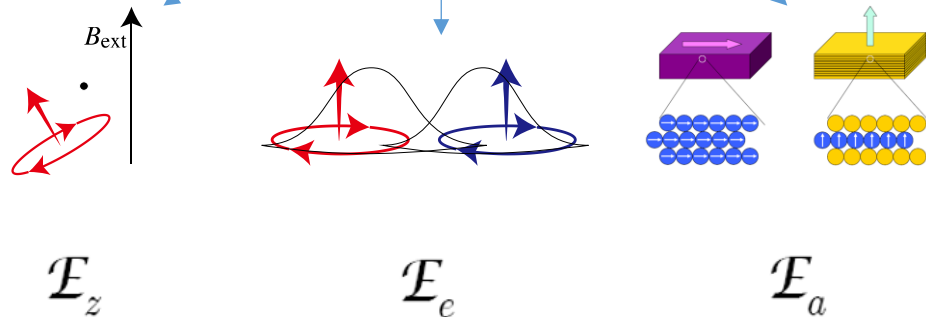


$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t},$$

$$\vec{H}_{eff} = H \hat{z} + A \frac{\partial^2 \vec{m}}{\partial z^2} + K_{\parallel} m_z \hat{z} - K_{\perp} m_x \hat{x}$$

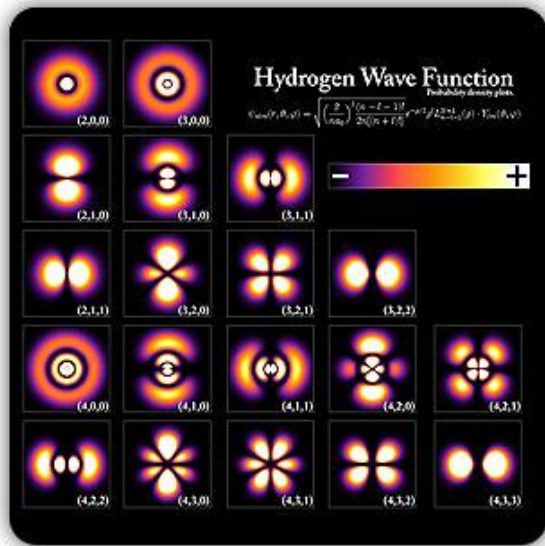
$$\theta(z, t) = 2 \tan^{-1} \exp\left(\frac{z - vt}{\Delta}\right),$$

$$\phi = \pm \frac{\pi}{2} - \frac{1}{2} \sin^{-1} \frac{H}{H_c}.$$





# The Walker DW Solution-**A Starting Point!**



Hydrogen Wave function

S. Zhang and Z. Li, "Roles of nonequilibrium conduction electrons on the magnetization dynamics of ferromagnets," *Physical Review Letters*, vol. **93**, p. 127204, 2004.

K. Yamada, S. Kasai, Y. Nakatani, K. Kobayashi, H. Kohno, A. Thiaville, *et al.*, "Electrical switching of the vortex core in a magnetic disk," *Nature materials*, vol. **6**, pp. 270-273, 2007.

D. Ralph and M. D. Stiles, "Spin transfer torques," *Journal of Magnetism and Magnetic Materials*, vol. **320**, pp. 1190-1216, 2008.

Z. Li and S. Zhang, "Domain-wall dynamics and spin-wave excitations with spin-transfer torques," *Physical review letters*, vol. **92**, p. 207203, 2004.

M. Hayashi, L. Thomas, C. Rettner, R. Moriya, and S. S. Parkin, "Direct observation of the coherent precession of magnetic domain walls propagating along permalloy nanowires," *Nature Physics*, vol. **3**, pp. 21-25, 2006.

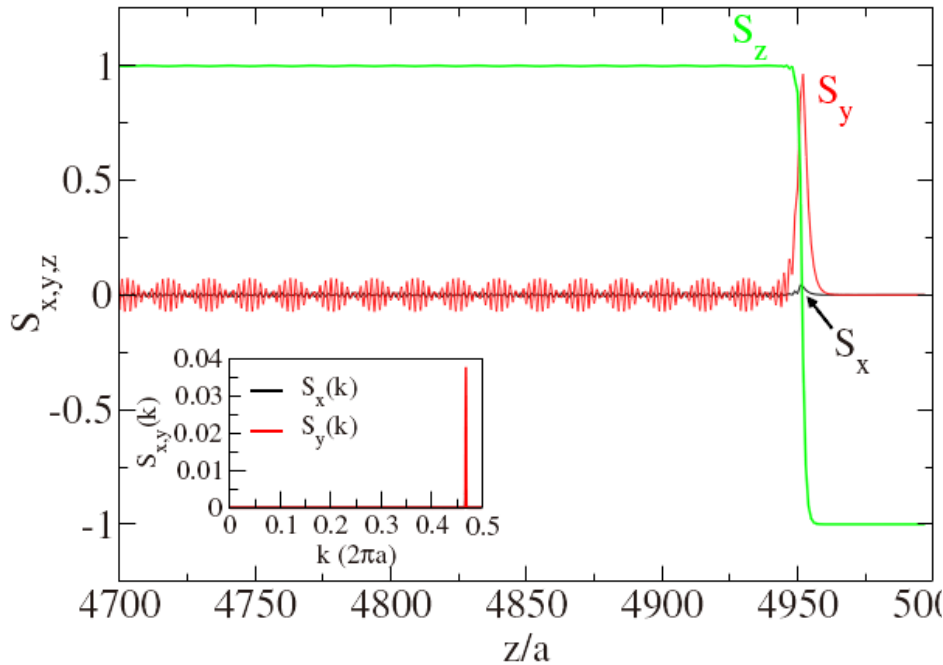
...



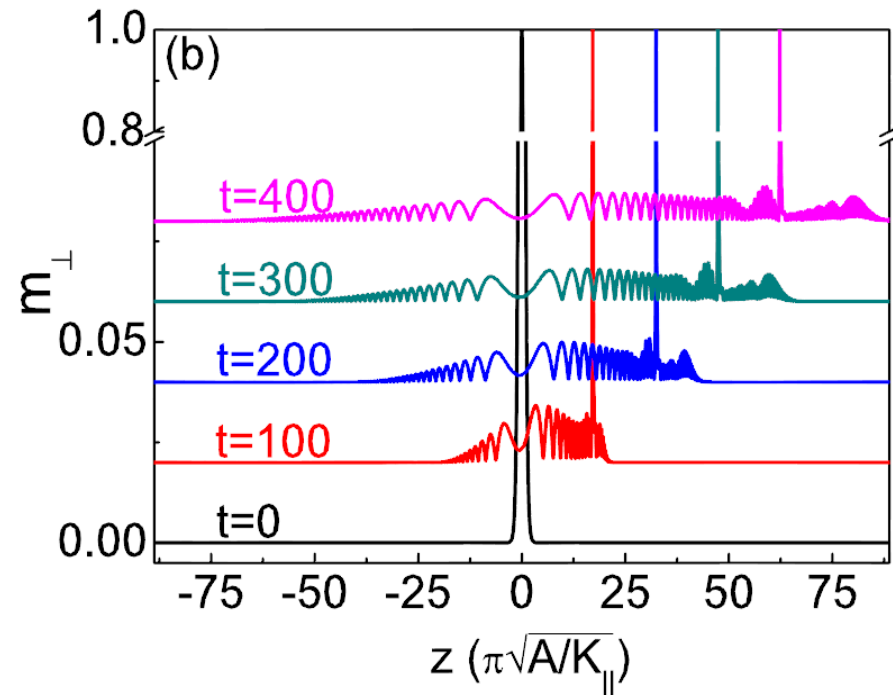
Issue:

**Is the Walker Solution Stable?**

# Signs of Instability of Walker DW Mode



R. Wieser, et al. ,  
Phys. Rev. B **81**, 024405 (2010).

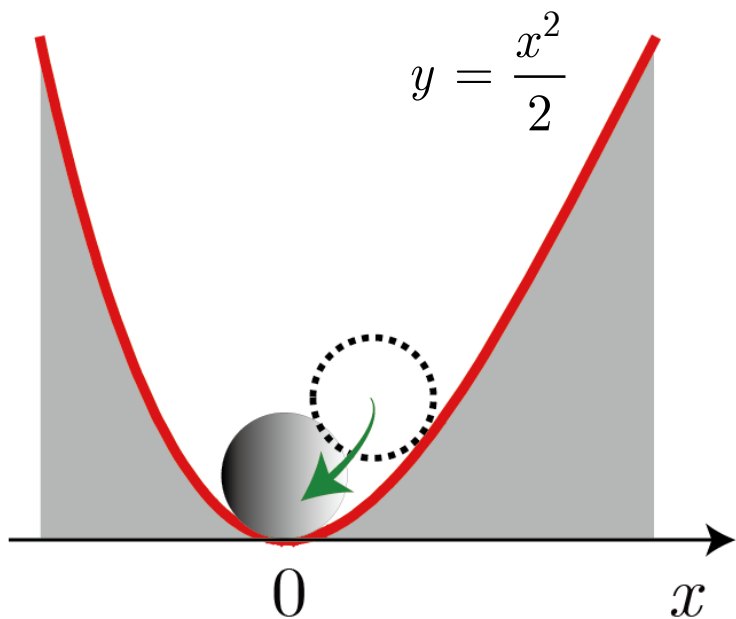


X. S. Wang, *et al.*,  
Phys. Rev. Lett. **109**, 167209 (2012).

Overlooked! Attributed to Quasi-1D Nature



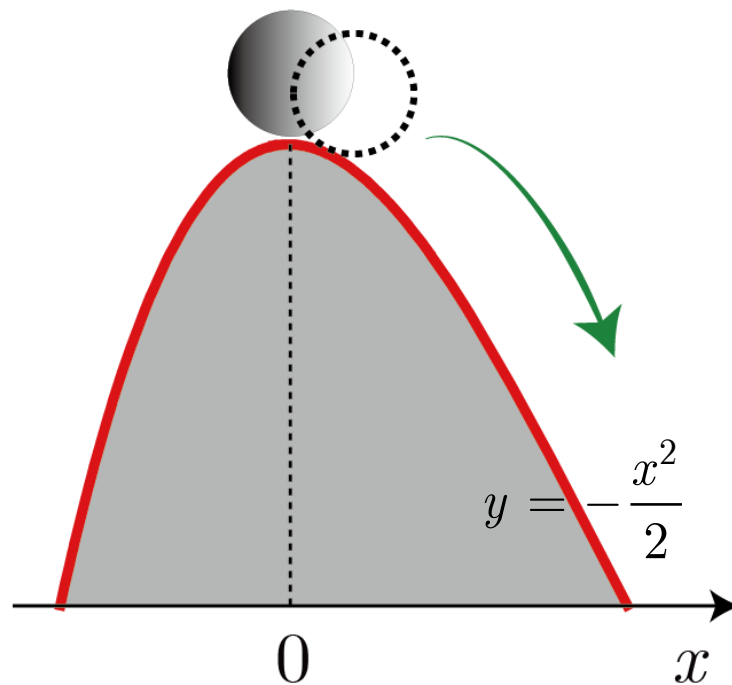
# Issue: Instability of a Walker DW mode?



$$\ddot{x} = -x - \eta \dot{x},$$

$$x_0 = 0.$$

Stable



$$\ddot{x} = x - \eta \dot{x},$$

$$x_0 = 0.$$

Unstable



# Stability Analysis Revisit

## Linear ODE

$$\dot{x} = Ax,$$

$$x \in R^n, A \in R^n \times R^n.$$

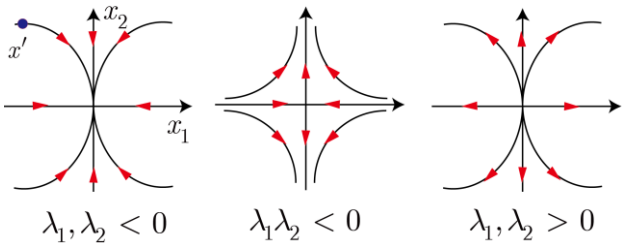
$$\dot{x} = 0 \Rightarrow x_0 = 0$$

*eigenvalues of A*

example:

$$A = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

$$x(t) = \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix} x', x(0) = x'.$$



## Nonlinear ODE

$$\dot{x} = f(x),$$

$$x \in R^n,$$

$$\dot{x} = 0 \Rightarrow f(x_0) = 0,$$

$$\Downarrow x \rightarrow x + \delta$$

$$\dot{\delta} = A' \cdot \delta,$$

$$A' = \nabla f(x) |_{x=x_0}.$$

*eigenvalues of A'*

## Nonlinear PDE ?

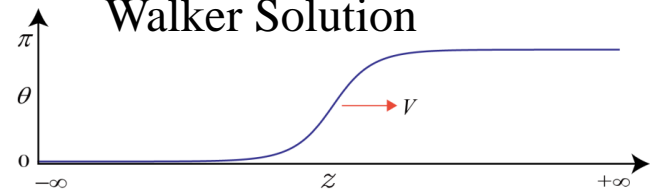
$$LLG : \vec{m}(z, t)$$

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t},$$

$$\vec{H}_{eff} = H \hat{z} + A \frac{\partial^2 \vec{m}}{\partial z^2} + K_{\parallel} m_z \hat{z} - K_{\perp} m_x \hat{x}$$

Recent Progress (2001)  
of Stability Analysis of  
**Traveling Front**

Walker Solution



Stable

Unstable

**Lyapunov analysis**

# Modus Operandi: Linearization

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t},$$



$$\begin{aligned} \dot{\theta} - \alpha \sin \theta \dot{\varphi} &= -2K_{\perp} \sin \theta \sin \varphi \cos \varphi + 4A\theta' \varphi' \\ &\quad + 2A \sin \theta \varphi'', \\ \sin \theta \dot{\varphi} + \alpha \dot{\theta} &= -2K_{\perp} \sin \theta \cos \theta \cos^2 \varphi + 2K_{//} \sin \theta \cos \theta \\ &\quad + H \sin \theta + 2A \sin \theta \cos \theta \varphi'^2 - 2A\theta'', \end{aligned}$$

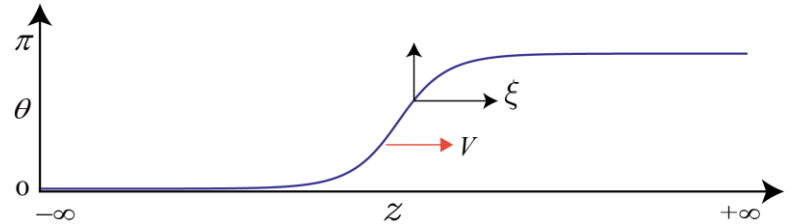
$$\sin 2\varphi_w = \frac{H}{H_c},$$

$$\ln \tan \frac{\theta_w(z,t)}{2} = \frac{z-vt}{\Delta}.$$

$$\xrightarrow{\xi = z-vt}$$

$$\sin 2\varphi_w = \frac{H}{H_c},$$

$$\ln \tan \frac{\theta_w(\xi)}{2} = \frac{\xi}{\Delta}.$$



$$\theta_w + \theta$$

$$\varphi_w + \varphi$$

$$|\theta|, |\varphi| \ll 1$$

substitute with  $\theta_w, \varphi_w \rightarrow LLG$

$$\frac{d\Lambda}{dt} = L_0 \Lambda + L_1 \frac{\partial \Lambda}{\partial \xi} + L_2 \frac{\partial^2 \Lambda}{\partial \xi^2}$$

$$\Lambda \equiv (\theta, \varphi)^T, L_{0,1,2}(\theta_w)$$

expand w.r.t.  $\theta, \varphi$   
keep only 1st terms



$$L_{0,11} = \frac{1}{1 + \alpha^2}$$

$$L_{0,12} = \frac{K_{\perp} \sinh \xi}{1 + \alpha^2} (-\sqrt{1 - \rho^2} + \alpha \rho \tanh \xi)$$

$$L_{0,21} = \frac{1}{1 + \alpha^2}$$

$$L_{0,22} = \frac{K_{\perp}}{1 + \alpha^2} (\alpha \sqrt{1 - \rho^2} + \rho \tanh \xi)$$

$G(\xi)$  is the Gudermannian function;  $\rho = \frac{H}{H_c}$ .

$$L_1 = \begin{pmatrix} v - \frac{2A}{(1 + \alpha^2) \cosh \xi} \\ 0 \quad v + \frac{2A\alpha}{1 + \alpha^2} \end{pmatrix}$$

$$L_2 = \frac{1}{1 + \alpha^2} \begin{pmatrix} A\alpha & -\frac{A}{\cosh \xi} \\ A \cosh \xi & A\alpha \end{pmatrix}$$



$$\frac{d\Lambda}{dt} = L_0\Lambda + L_1 \frac{\partial\Lambda}{\partial\xi} + L_2 \frac{\partial^2\Lambda}{\partial\xi^2},$$

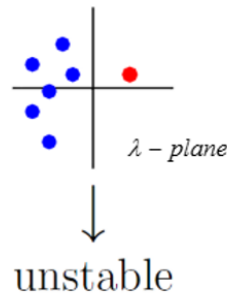
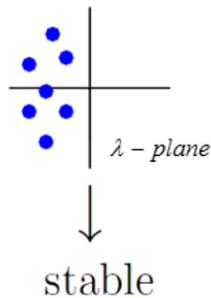
$$\Lambda(\xi, t) = \sum_i e^{\lambda_i t} \Lambda_i(\xi).$$

**Spectrum:** any  $\lambda_i$  such that the equation (\*) has nontrivial solution  $\Lambda_i$

$$(L - \lambda_i)\Lambda_i = 0, \quad *$$
$$L := L_0 + L_1 \frac{\partial}{\partial\xi} + L_2 \frac{\partial^2}{\partial\xi^2}.$$

iff.  $\forall \lambda_i, \operatorname{Re}(\lambda) < 0$  : all  $\Lambda$  exponentially decay  $\rightarrow$  **stable**

iff.  $\exists \lambda_i, \operatorname{Re}(\lambda_i) > 0$  : some  $\Lambda$  exponentially grows  $\rightarrow$  **unstable**





## In the 1<sup>st</sup> Order ODEs form:

$$(L - \lambda_i)\Lambda_i = 0, \quad *$$
$$L := L_0 + L_1 \frac{\partial}{\partial \xi} + L_2 \frac{\partial^2}{\partial \xi^2}.$$

$$\Rightarrow \frac{d}{d\xi} \Lambda' = \Gamma(\lambda, \theta_w) \Lambda',$$

$$\Gamma(\lambda, \theta_w) = \begin{pmatrix} 0 & I \\ L_2^{-1}(\lambda - L_0) & -L_2^{-1}L_1 \end{pmatrix},$$

$$\Lambda' = \left( \theta, \varphi, \frac{\partial \theta}{\partial \xi}, \frac{\partial \varphi}{\partial \xi} \right)^T$$

$$\text{Spec}(L - \lambda) \xleftrightarrow{\text{iden.}} \text{Spec}\left[ \frac{d}{d\xi} - \Gamma(\lambda, \theta_w) \right]$$

How to find  $(\lambda, \Lambda')$  for  $\frac{d}{d\xi} \Lambda' = \Gamma(\lambda, \theta_w) \Lambda'$  ?

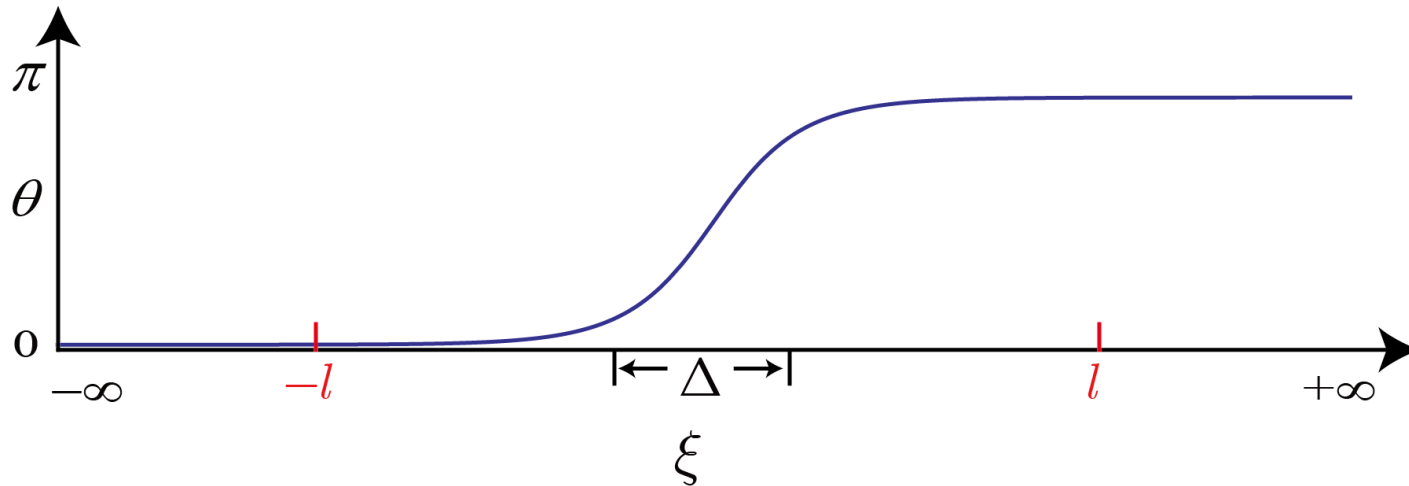


$$\Gamma = \begin{array}{ccccc} & 0 & 0 & 0 & 1 \\ & \Gamma_{31} & & & \\ & & \frac{\lambda - K_{\perp} \rho \tanh \xi}{A \cosh \xi} & -\frac{v\alpha}{A} & -\frac{v}{A \cosh \xi} \\ -\cosh \xi & & \alpha\lambda - K_{\perp} \sqrt{1 - \rho^2} & v \cosh \xi & v\alpha \end{array}$$

$$\Gamma_{31} = \frac{\alpha\lambda - H \tanh \xi}{A} - \frac{1}{2A} [K_{\perp} - 2K_{\parallel} - K_{\perp} \sqrt{1 - \rho^2}] \cos[2G(\xi)]$$

$G(\xi)$  is the Gudermannian function;  $\rho = \frac{H}{H_c}$ .

## Utilize the Property of a Front:



$$\theta_w(\xi) = 2 \arctan e^{\frac{\xi}{\Delta}}$$

$$\lim_{\xi \rightarrow \pm\infty} \theta_w = 0, \pi, \quad \Rightarrow$$

$$\lim_{\xi \rightarrow \pm\infty} \Gamma(\lambda, \theta_w) = \Gamma^\pm$$

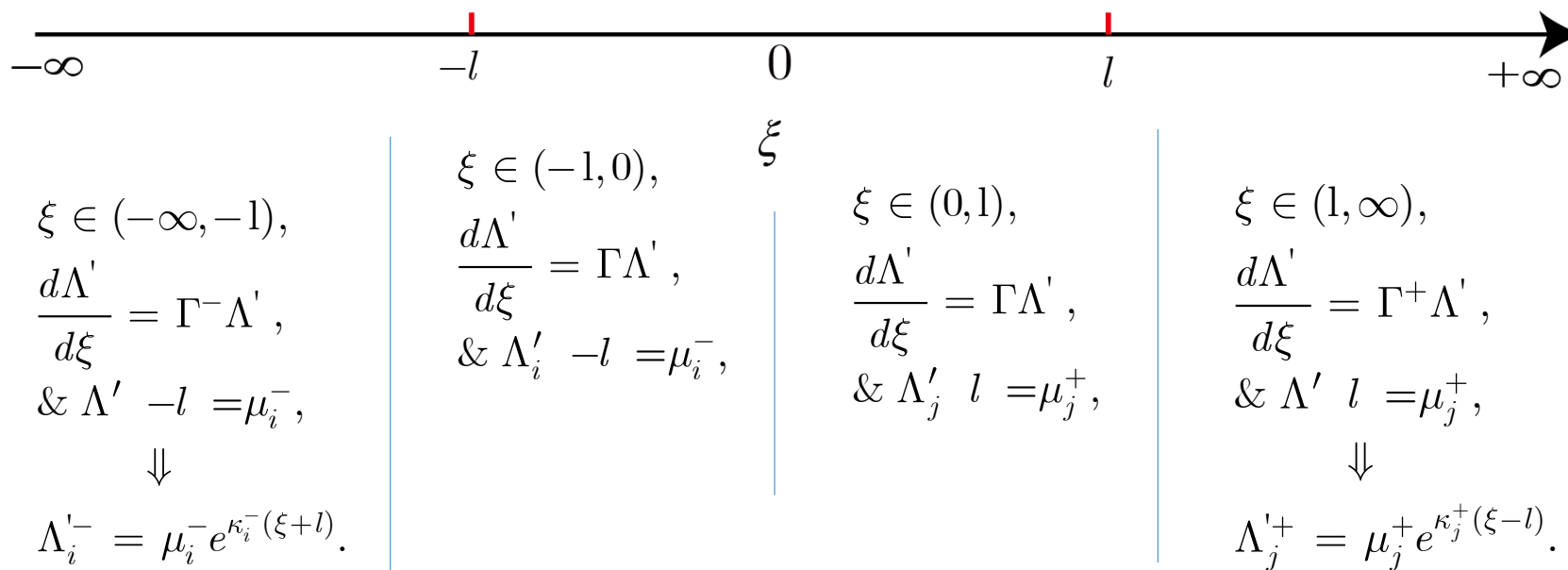


$$\exists l \gg 1, \quad \frac{d\Lambda'}{d\xi} = \Gamma \Lambda' \quad \text{is} \quad \frac{d\Lambda'}{d\xi} = \Gamma^\pm \Lambda' \quad \text{for} \quad |\xi| > l$$

# Solve for $\Lambda'$ (in principle)

Denote (

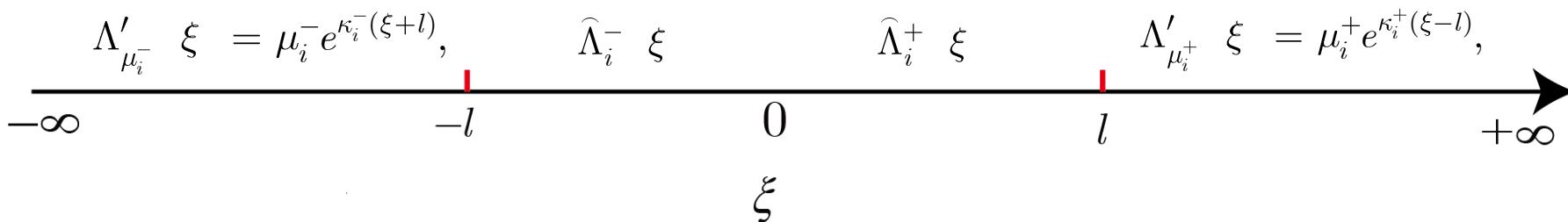
$\lambda$  can be classified by two integers  $n_{\pm}^{\pm}$  ( $n_{\pm}^{\pm}$ ), denoting the number of  $\kappa_j^{\pm}$  whose real part is positive (negative).



for  $|\xi| < l$ , shoot towards 0 from  $\xi = \pm l$ , with  $\Lambda'_i \Big|_{\pm l} = \mu_i^{\pm}$ , denoted as  $\widehat{\Lambda}_i^{\pm}(\xi)$ .



# Solve for $\Lambda'$ (in principle)



$\lambda \in spec \Leftrightarrow$  for  $\lambda$ ,  $\exists (a_i, b_j)$ , *s.t.*

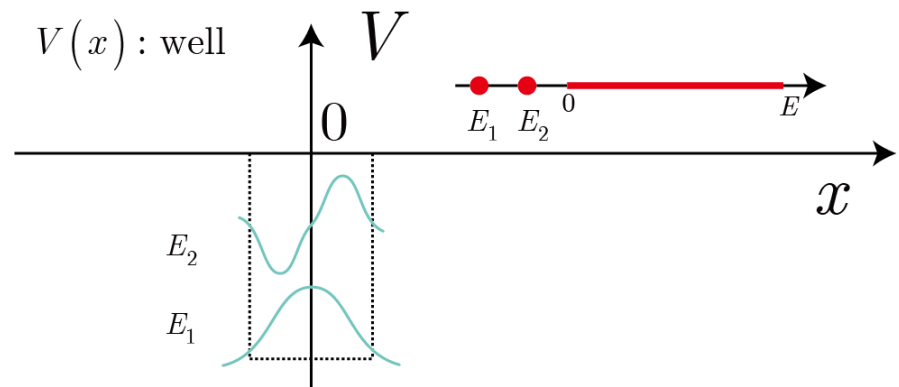
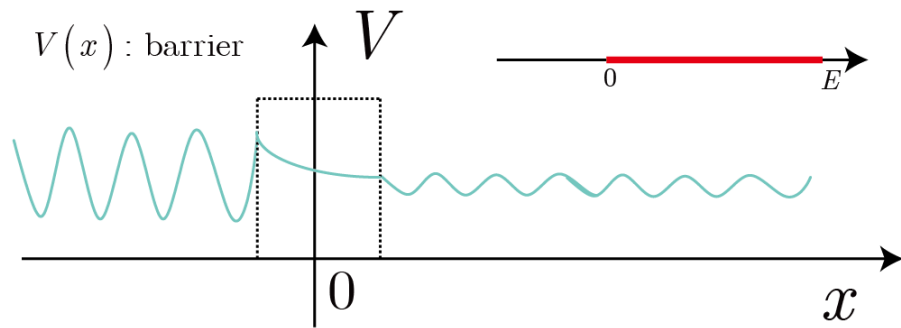
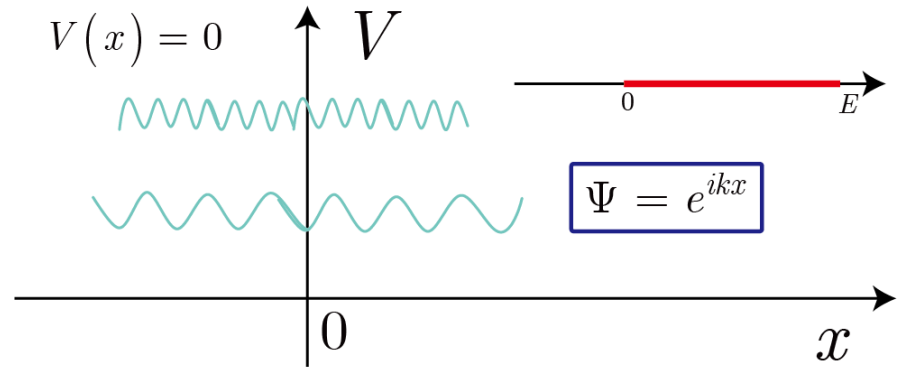
$$\sum_i a_i \widehat{\Lambda}_i^+(0) = \sum_j b_j \widehat{\Lambda}_j^-(0).$$

for each  $(a_i, b_j)$ ,  $\Lambda' \xi = \begin{cases} \sum_i a_i s \widehat{\Lambda}_i^+ \xi + \sum_i a_i (1-s) \Lambda_i^+(\xi), & \xi \geq 0 \\ \sum_j b_j s \widehat{\Lambda}_j^- \xi + \sum_j b_j (1-s) \Lambda_j^-(\xi). & \xi \leq 0 \end{cases}$   $s = \begin{cases} 1, & |\xi| < l, \\ 0, & |\xi| > l. \end{cases}$

How to dodge the heavy workload in finding  $\widehat{\Lambda}$ ?

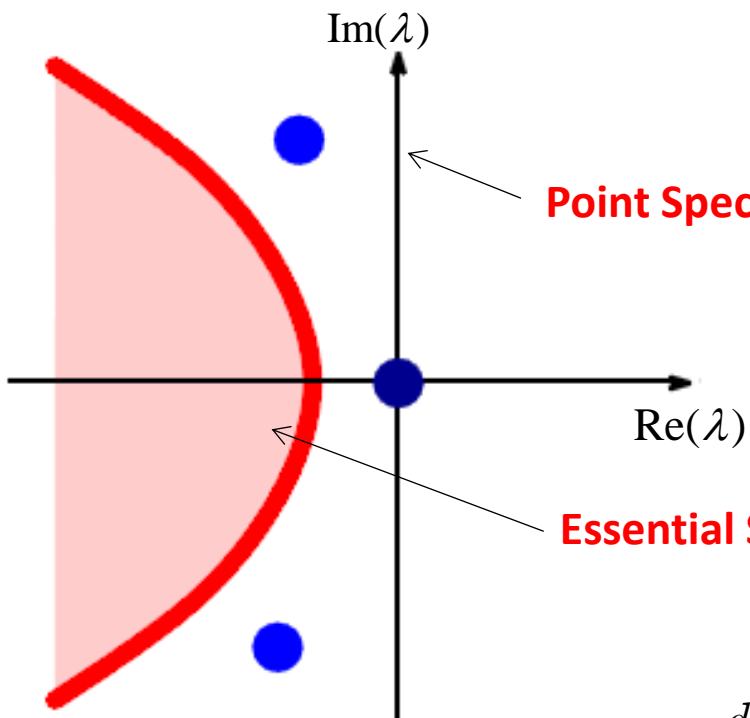
# A Clue from the Schrödinger Eq.

$$\left( -\frac{d^2}{dx^2} + V(x) \right) \Psi = E\Psi$$



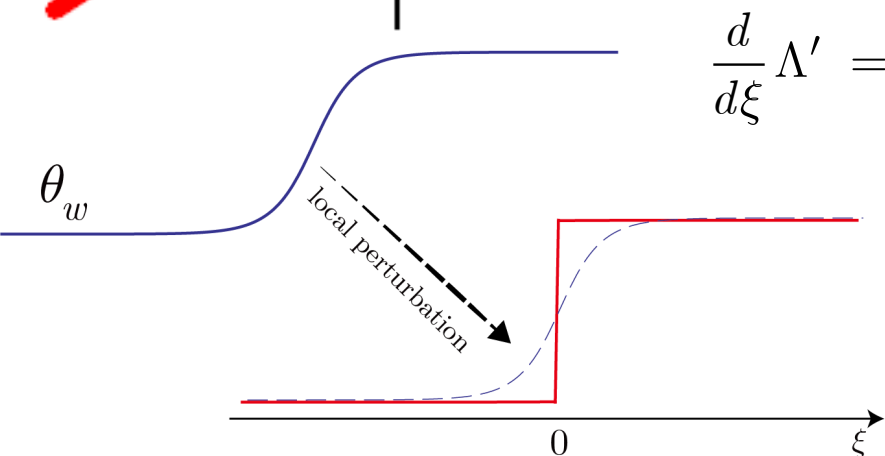
- 1 Spectrum can be decomposed into continuum and discrete.
- 2 Local perturbation does not change the continuum spectrum.

# Decomposition of the Spectrum



**Point Spectrum:** all  $\Lambda'$  for each  $\lambda$ , s.t.  $\|L(\lambda)\Lambda'\| < \varepsilon$   
 expand **finite** dimensional space

**Essential Spectrum:** all  $\Lambda'$  for each  $\lambda$ , s.t.  $\|L(\lambda)\Lambda'\| < \varepsilon$   
 expand **infinite** dimensional space



$$\frac{d}{d\xi} \Lambda' = \Gamma(\lambda, \theta_w) \Lambda', \quad \xleftrightarrow{\text{id. } \lambda_{ess}} \frac{d}{d\xi} \Lambda' = \Gamma^\infty \Lambda',$$

$$\Gamma^\infty \begin{cases} \Gamma^+, & \xi > 0, \\ \Gamma^-, & \xi < 0. \end{cases}$$

$$\lim_{\xi \rightarrow \pm\infty} \Gamma(\lambda, \theta_w) = \Gamma^\pm$$



# Essential Spectrum of $d\Lambda' / d\xi = \Gamma^\infty \Lambda'$

$$\frac{d}{d\xi} \Lambda' = \Gamma(\lambda, \theta_w) \Lambda', \xrightarrow{\text{identical } \lambda_{ess}} \frac{d}{d\xi} \Lambda' = \Gamma^\infty \Lambda'$$



$$\sum_i a_i \widehat{\Lambda}_{\mu_i^+}(0) = \sum_j b_j \widehat{\Lambda}_{\mu_j^-}(0)$$



$$\sum_i a_i \mu_i^+ = \sum_j b_j \mu_j^-$$

$n_-^+$ : number of  $\kappa^+$  with

$$\text{Re}(\kappa^+) < 0,$$

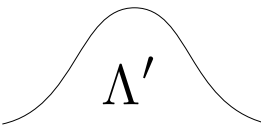
$n_+^-$ : number of  $\kappa^-$  with

$$\text{Re}(\kappa^-) > 0.$$

Is  $\lambda_{ess}$  related  
with  $n_-^+$  &  $n_+^-$  ?

**Example:**

$$\sum_{i=1}^{n_-^+} a_i \mu_i^+ = \sum_{j=1}^{n_+^-} b_j \mu_j^-$$



$$a_1 \begin{pmatrix} \square \\ \square \\ \square \\ \square \end{pmatrix} + \dots + a_{n_-^+} \begin{pmatrix} \square \\ \square \\ \square \\ \square \end{pmatrix} = b_1 \begin{pmatrix} \square \\ \square \\ \square \\ \square \end{pmatrix} + \dots + b_{n_+^-} \begin{pmatrix} \square \\ \square \\ \square \\ \square \end{pmatrix}$$

$n_-^+ + n_+^- \Rightarrow$  number of variables  
4  $\Rightarrow$  number of equations

$n_-^+ + n_+^- \begin{cases} > 4 & \text{many solutions} \\ = 4 & \text{unique solution} \\ < 4 & \text{no solution} \end{cases}$



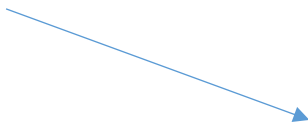
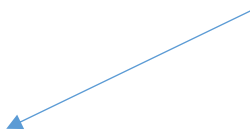
Conclusion:  $\lambda \in \lambda_{ess}$



closed region on complex plane



boundaries+inner area



two curves

$$\lambda_{1,2}(k) := \det[\Gamma^\pm(\lambda) - ik] = 0$$



$\Lambda' \propto e^{ik\xi}$ , plane wave

$$v_{1,2} = \text{Im}\left(\frac{d\lambda_{1,2}(k)}{dk}\right)$$

$v > 0$  **bow** wave

$v < 0$  **stern** wave

all  $\lambda$ , s.t.  $n_-^+ + n_+^- \neq 4$ .



a group of  $\Lambda'$ , wave packet





## YIG Parameter

Damping

$$\alpha = 0.001$$

Exchange

$$A = 3.84 \times 10^{-12} \text{ J/m}$$

Saturation Magnetization

$$M_s = 1.94 \times 10^5 \text{ A/m}$$

Gyromagnetic Ratio

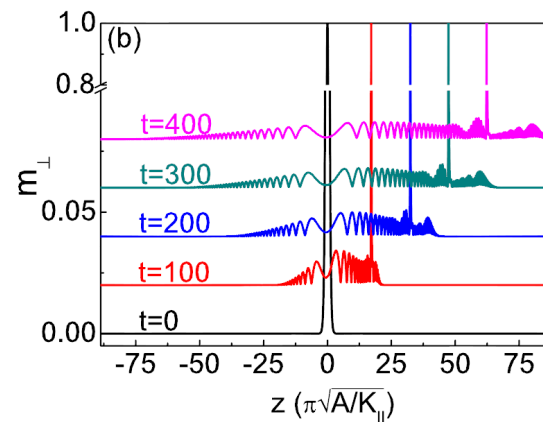
$$\gamma = 3.51 \text{ /kHz/(A/m)}$$

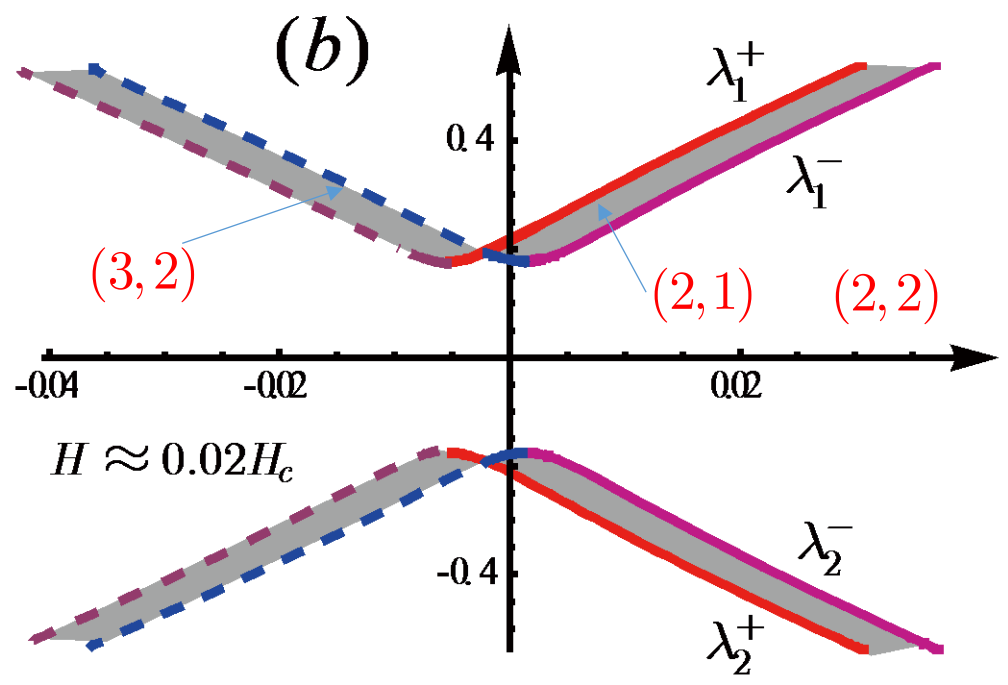
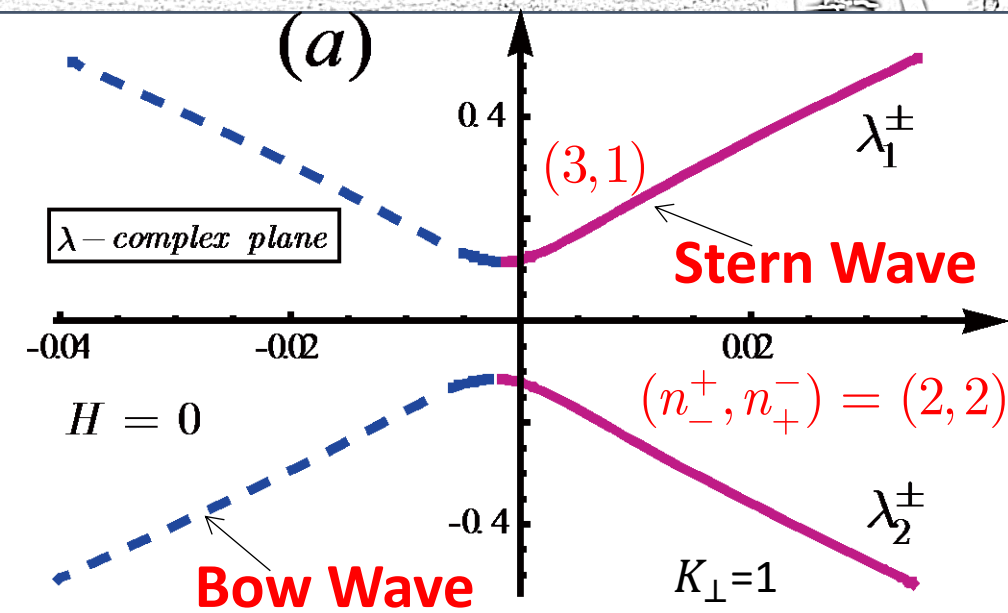
Easy Axis Anisotropy

$$K_{\parallel} = 2 \times 10^3 \text{ J/m}^3$$

Hard Axis Anisotropy

$$K_{\perp} = 1$$

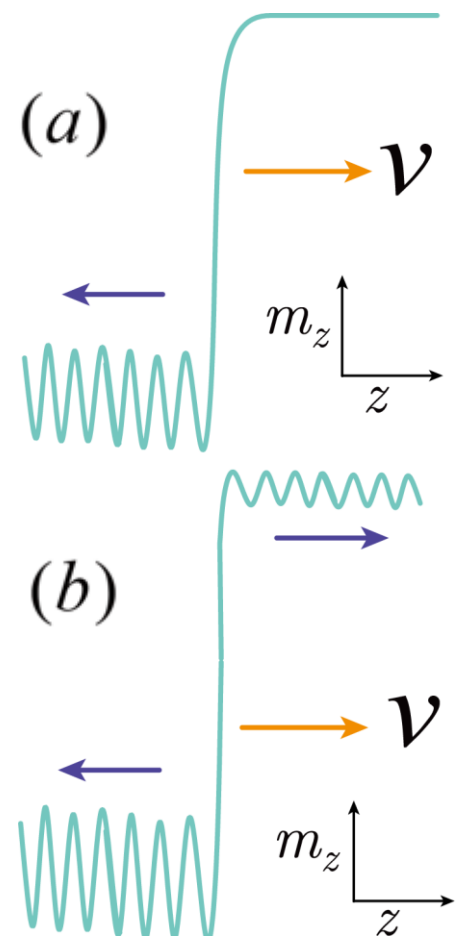


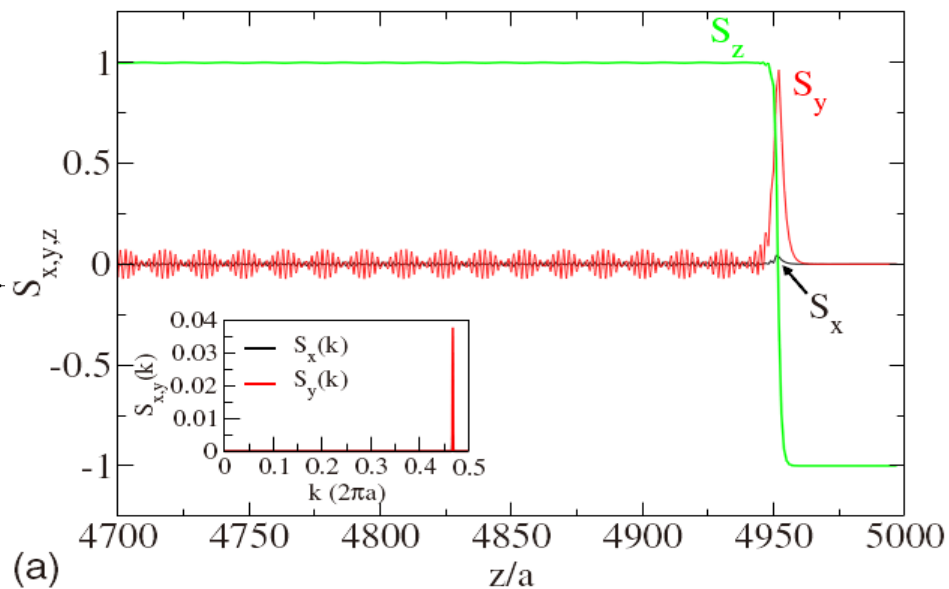
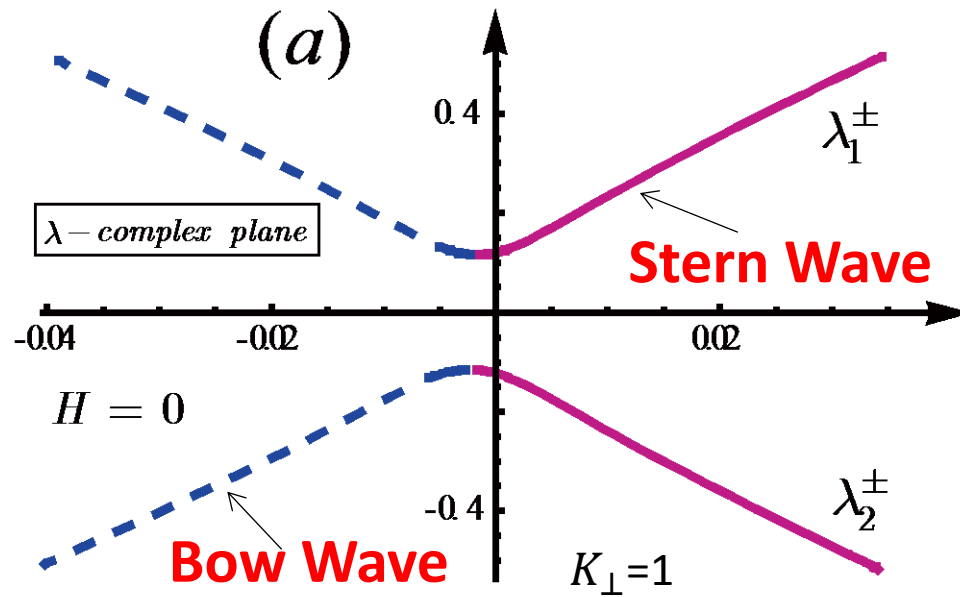


$$v_{1,2} = \text{Im}\left(\frac{d\lambda_{1,2}(k)}{dk}\right)$$

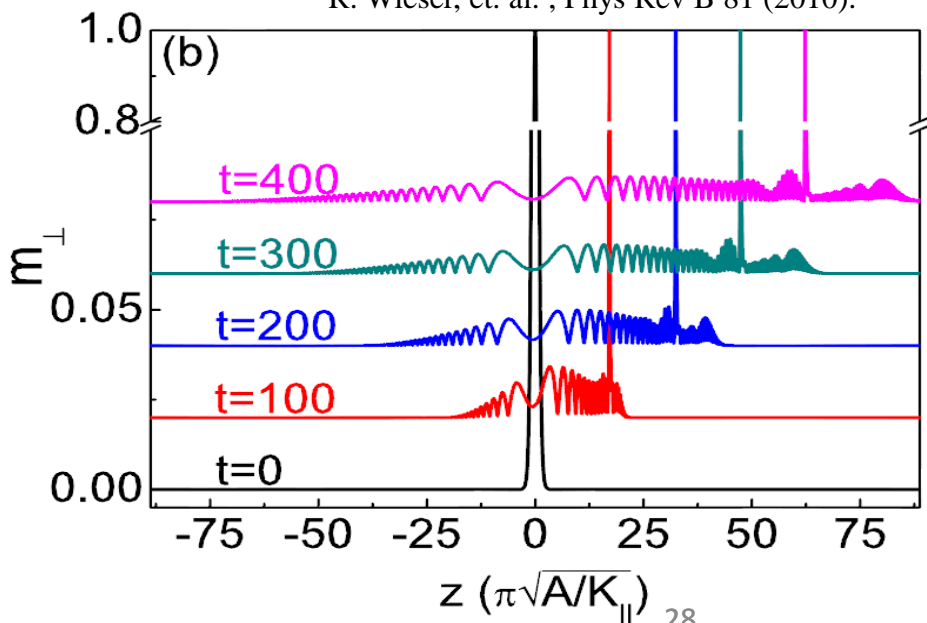
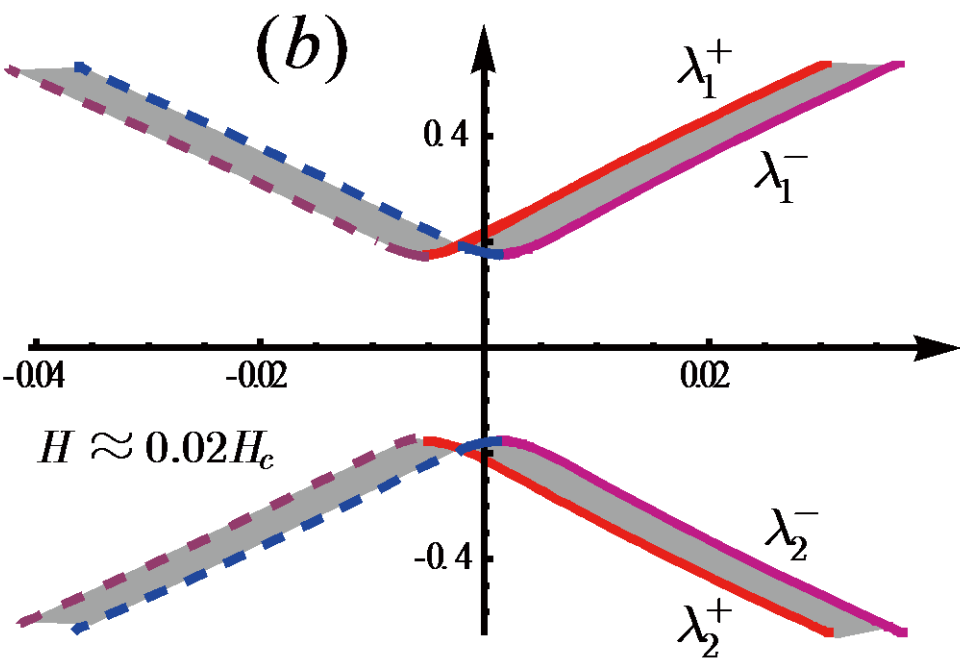
$v > 0$  bow wave

$v < 0$  stern wave

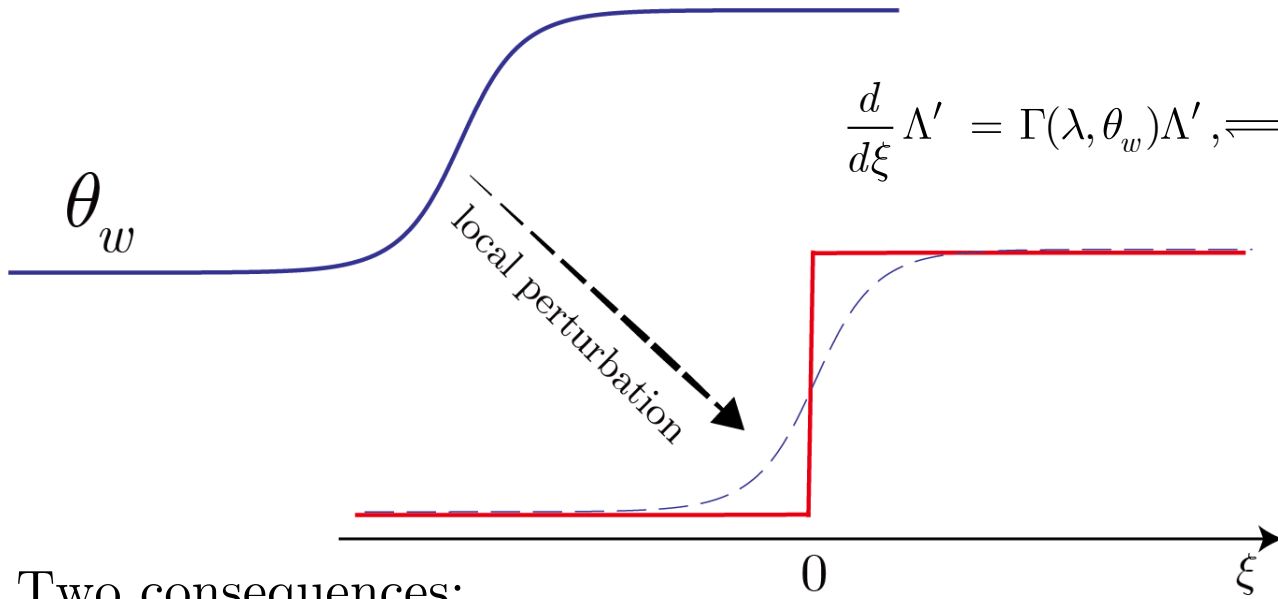




R. Wieser, et. al., Phys Rev B 81 (2010).



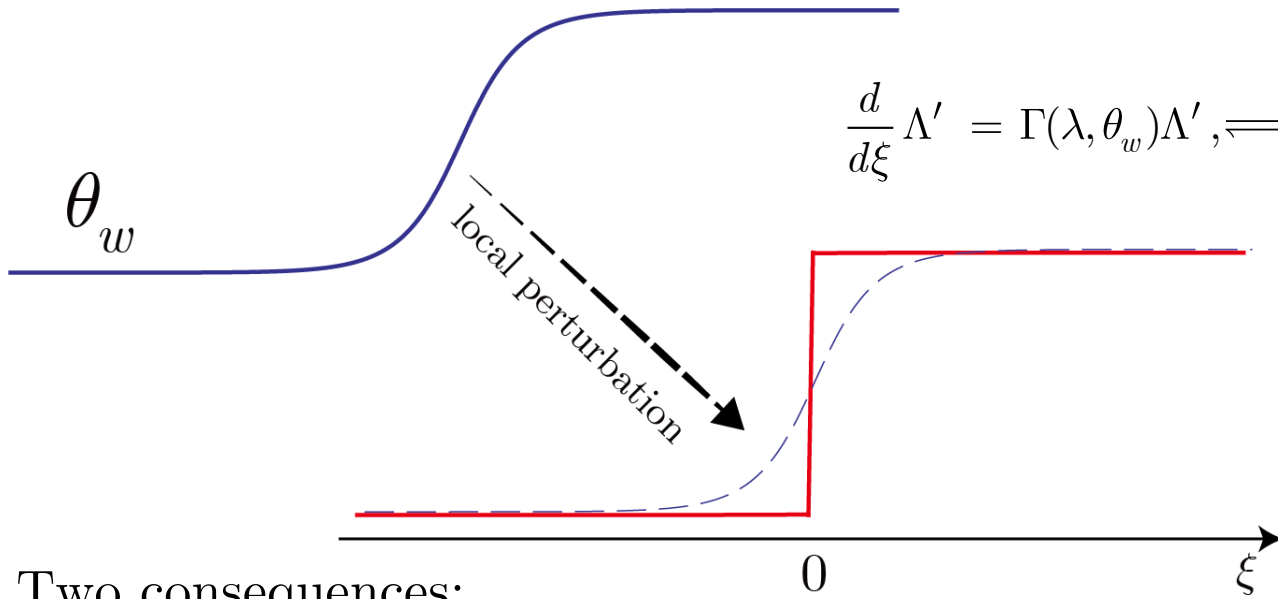
X. S. Wang et al., Phys Rev Lett 109 167209 (2012).



$$\frac{d}{d\xi} \Lambda' = \Gamma(\lambda, \theta_w) \Lambda', \xrightarrow{\text{id. } \lambda_{ess}} \frac{d}{d\xi} \Lambda' = \Gamma^\infty \Lambda',$$

Two consequences:

- 1 Spin wave emission is not sensitive to local deformation of DW profile.
- 2 Essential spectrum cannot capture instability of DW profile.



$$\frac{d}{d\xi} \Lambda' = \Gamma(\lambda, \theta_w) \Lambda', \xrightarrow{\text{id. } \lambda_{ess}} \frac{d}{d\xi} \Lambda' = \Gamma^\infty \Lambda',$$

Two consequences:

- 1 Spin wave emission is not sensitive to local deformation of DW profile.
- 2 Essential spectrum cannot capture instability of DW profile.

## Why and how to capture DW's instability?

Many quantities,  
e.g. DW speed, are  
sensitive to profile  
deformation

growth rate  $\lambda$  ( $e^{\lambda t}$ ),  
group velocity  $V$ ,  
wave packet **profile**.



# Absolute Spectrum

$$\lambda \in \lambda_{abs} \text{ iff. } \operatorname{Re}(\kappa_2^+) = \operatorname{Re}(\kappa_3^+) \text{ or } \operatorname{Re}(\kappa_2^-) = \operatorname{Re}(\kappa_3^-)$$

Consider:  $\lambda \in$  branching set  $\lambda_{sd}$  iff.  $\kappa_2^+ = \kappa_3^+$  or  $\kappa_2^- = \kappa_3^-$

branching set:  $\lambda_{sd}$



nontraveling modes

Proof :  $v = \operatorname{Im}\left[\frac{d\lambda(\kappa)}{d\kappa}\right]$

$$\frac{d\lambda}{d\kappa} = -\frac{\partial F(\lambda, \kappa)}{\partial \kappa} / \frac{\partial F(\lambda, \kappa)}{\partial \lambda}$$

$$\lambda \in Sd \Leftrightarrow \kappa_2 = \kappa_3 = \bar{\kappa}$$

$$F(\lambda, \kappa) \equiv \det(\Gamma(\lambda) - \kappa I)$$

$$F(\lambda_{sd}, \kappa)$$

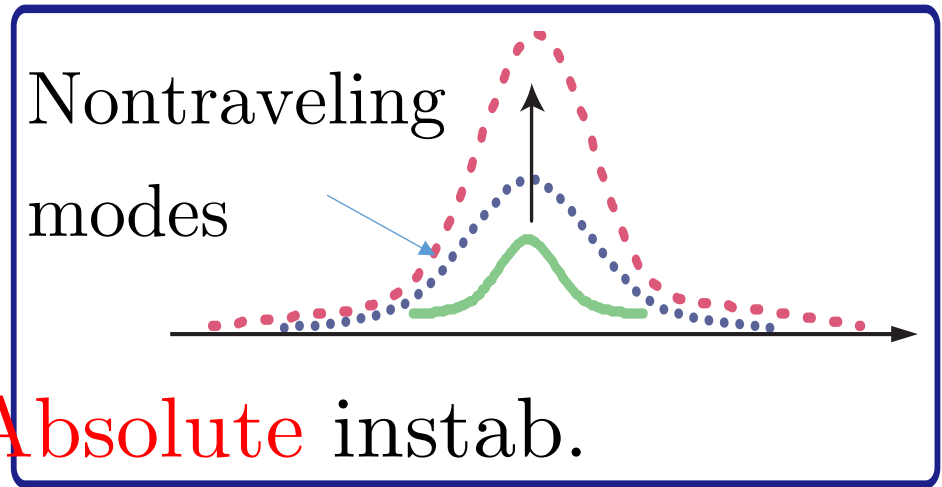
$$= (\kappa - \bar{\kappa})^2 (\kappa - \kappa_1) (\kappa - \kappa_4)$$

$$\Rightarrow \left. \frac{\partial F(\lambda, \kappa)}{\partial \kappa} \right|_{\kappa=\bar{\kappa}, \lambda=\lambda_{sd}}$$

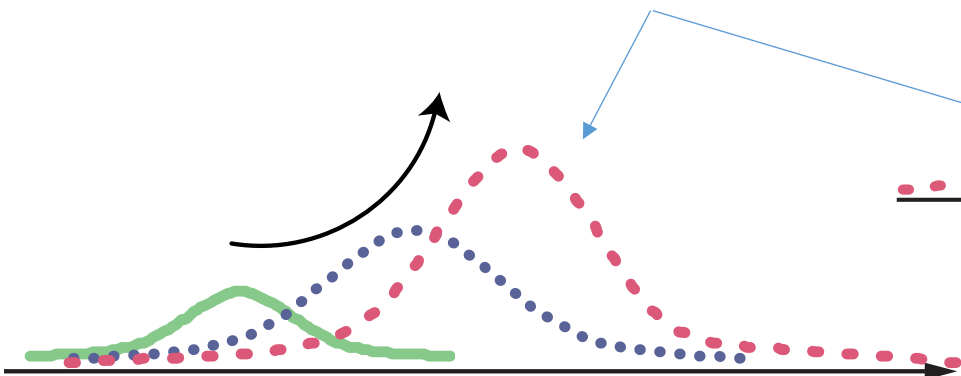
$$= 0 = v!$$



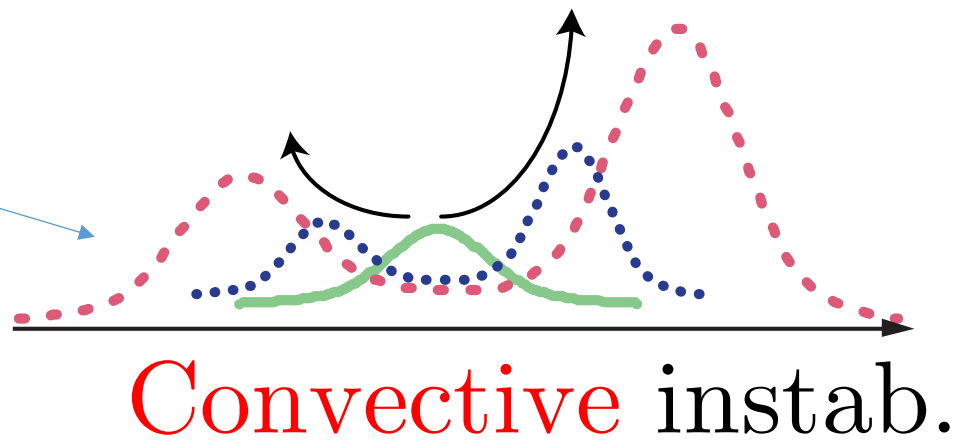
# Classifications of Instabilities with Unstable $\lambda_{ess}$



Pointwise  $\left\{ \begin{array}{l} \text{Grow} \rightarrow \text{Absolute instab.} \\ \text{Decay} \end{array} \right.$



Transient instab.

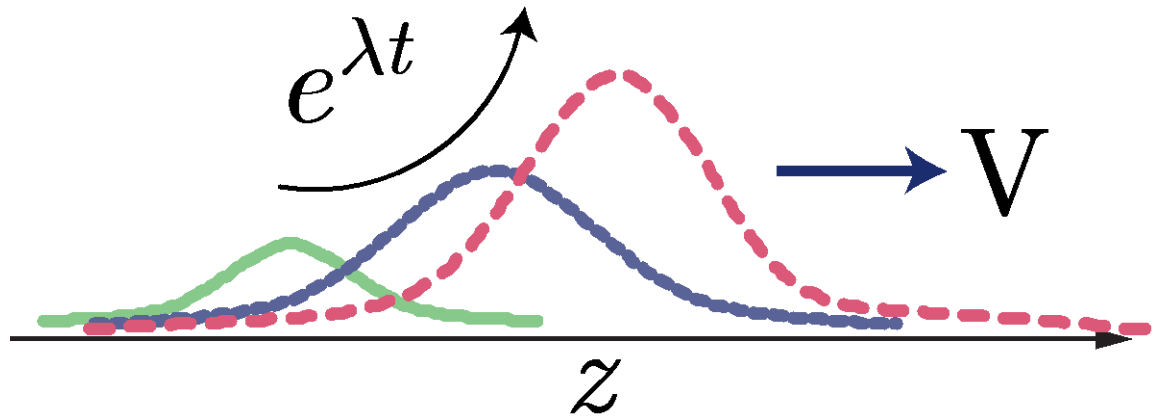


Convective instab.



## An Example:

growth rate  $\lambda$  ( $e^{\lambda t}$ ),  
group velocity  $V$ ,  
wave packet **profile**.



$$\Lambda(z, t) = e^{\lambda t} \operatorname{sech}(z - vt)$$

$\forall$  fixed  $z_0$ ,

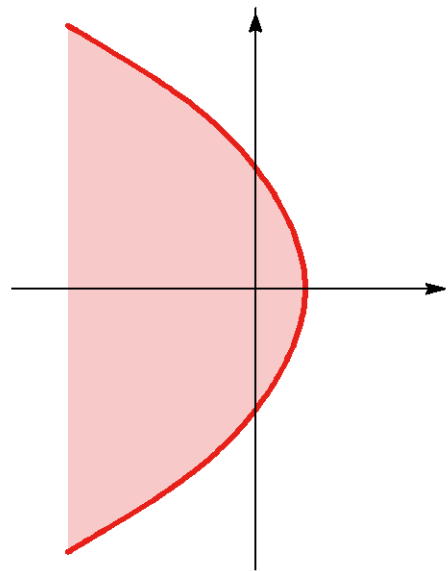
$$\lim_{t \rightarrow \infty} \Lambda(z_0) = \begin{cases} 0, & \text{if } V > \operatorname{Re}(\lambda) \rightarrow p.w. \text{ decay} \\ \infty. & \text{if } V < \operatorname{Re}(\lambda) \end{cases}$$

↓

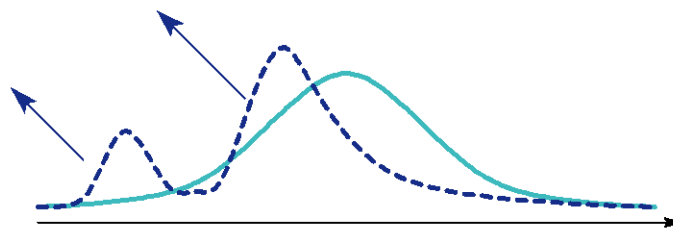
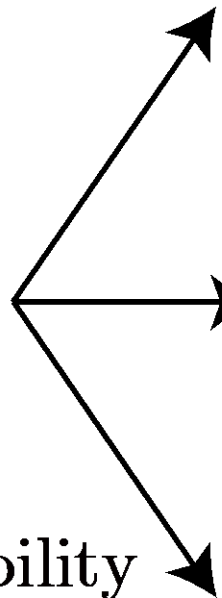
*p.w.* **growth**



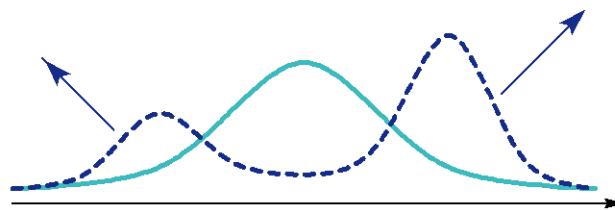
# Summary of Instab.



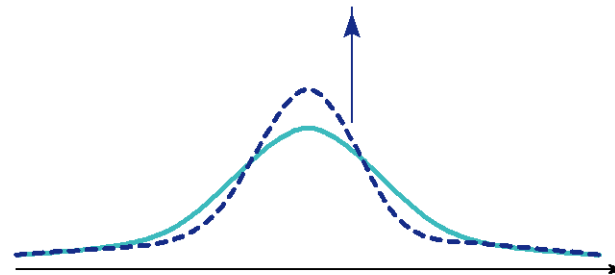
Essential Instability  
unstable  $\lambda_{ess}$



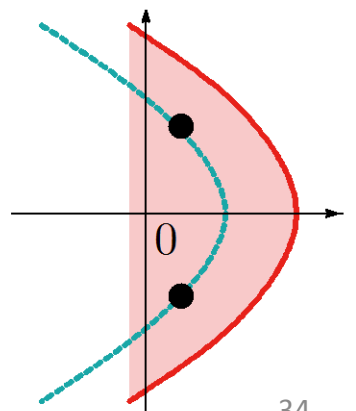
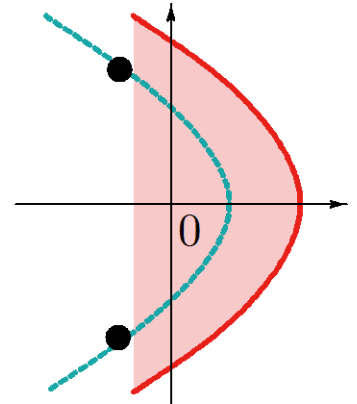
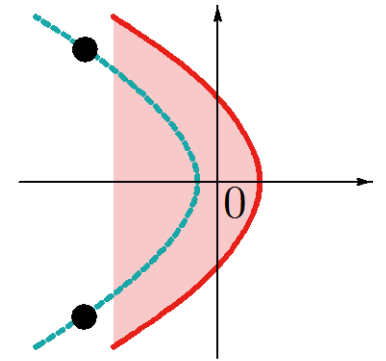
Transient Instability:  
unstable  $\lambda_{ess}$  + stable  $\lambda_{abs}$



Convective Instability  
unstable  $\lambda_{abs}$  + stable  $\lambda_{sd}$



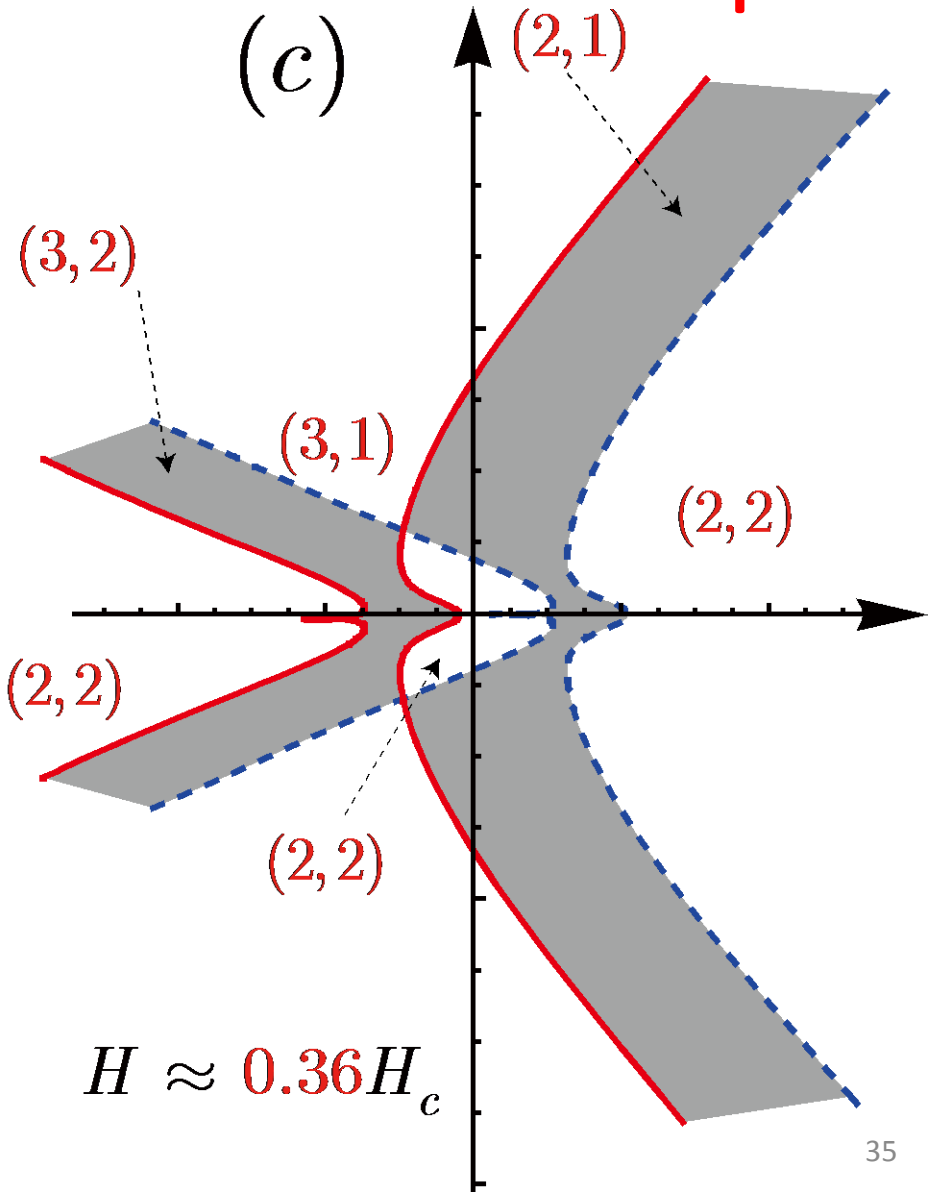
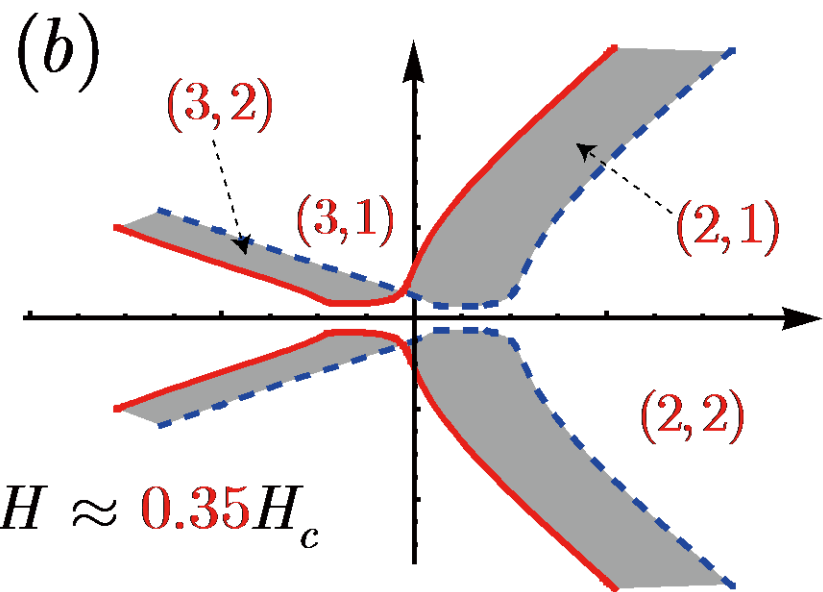
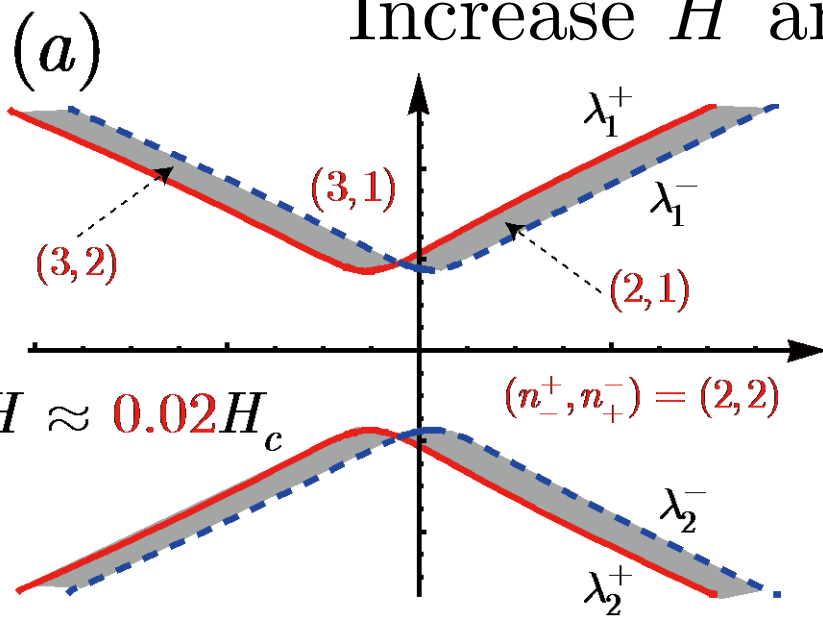
Absolute Instability  
unstable  $\lambda_{sd}$





Increase  $H$  and solve  $\lambda_{ess}$

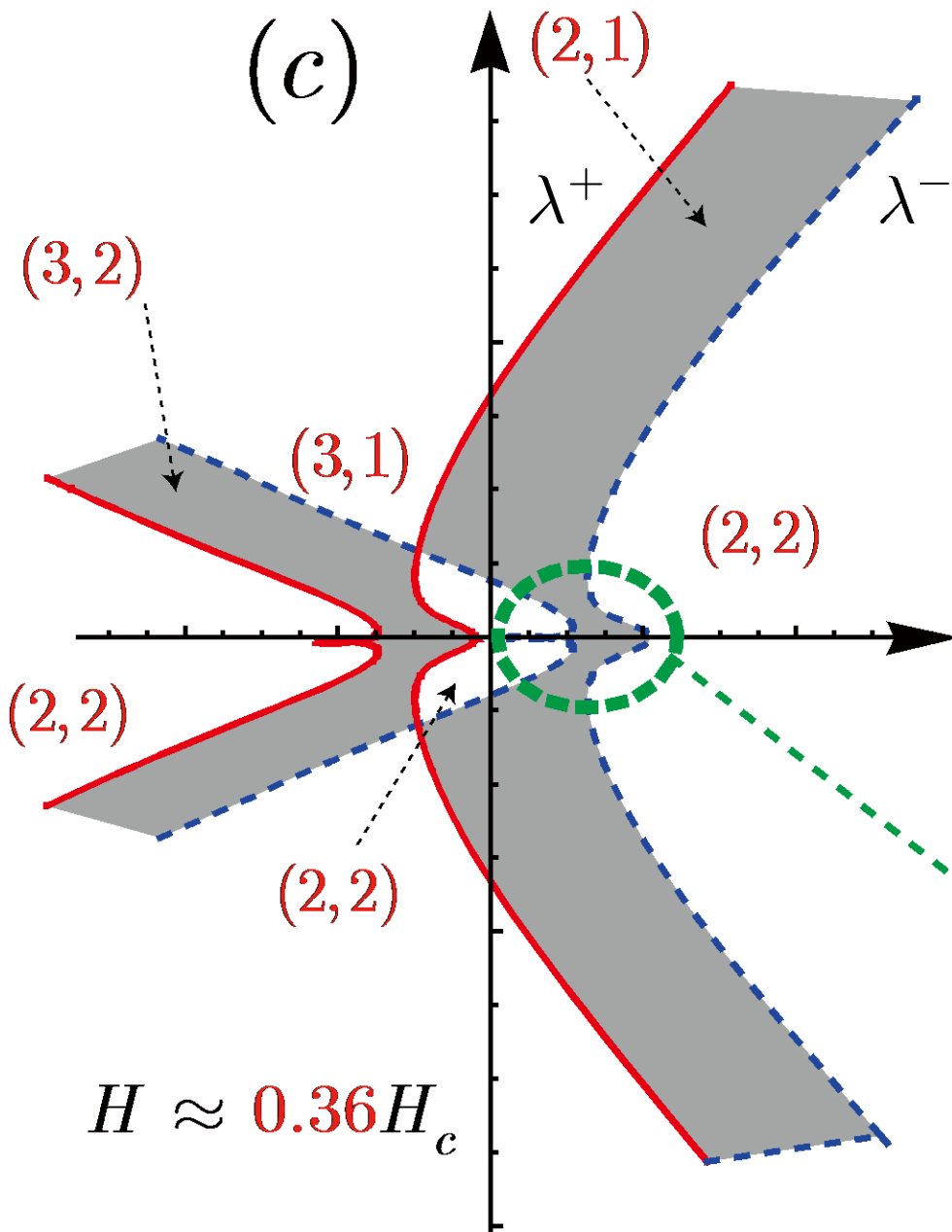
$\lambda$  plane





(c)

$\lambda$  plane

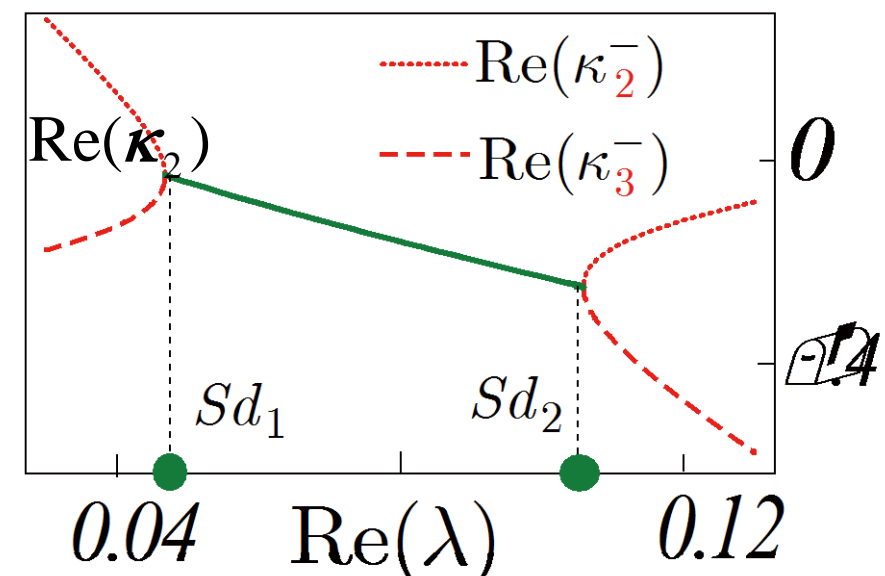
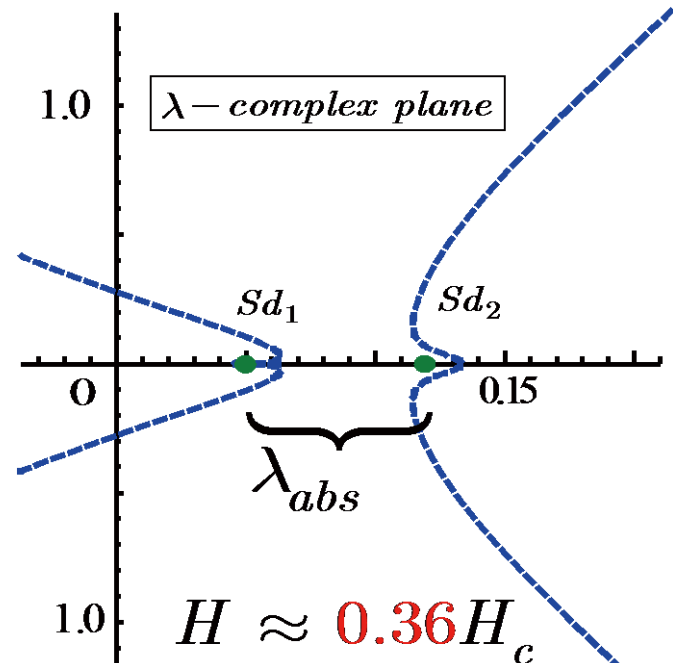
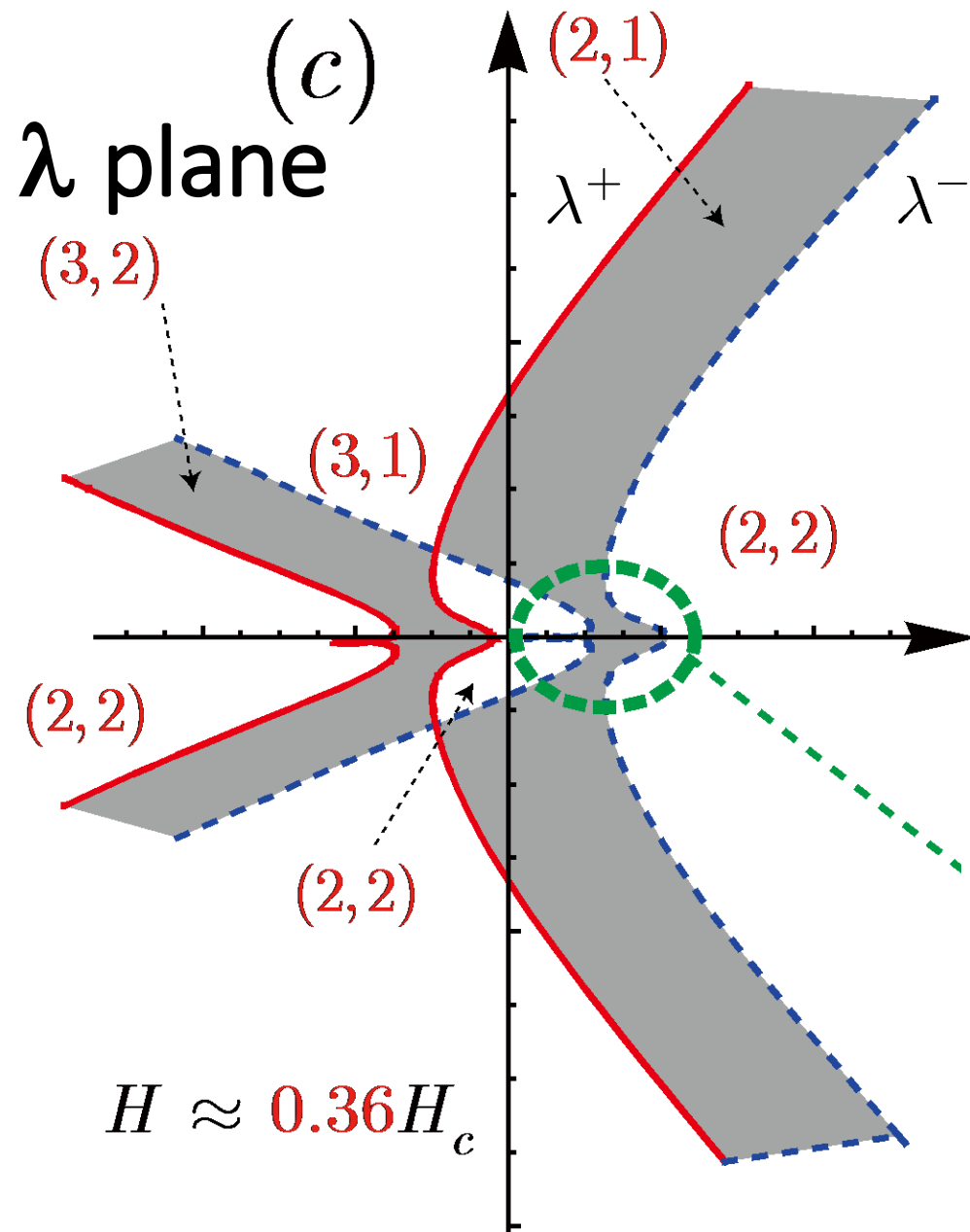


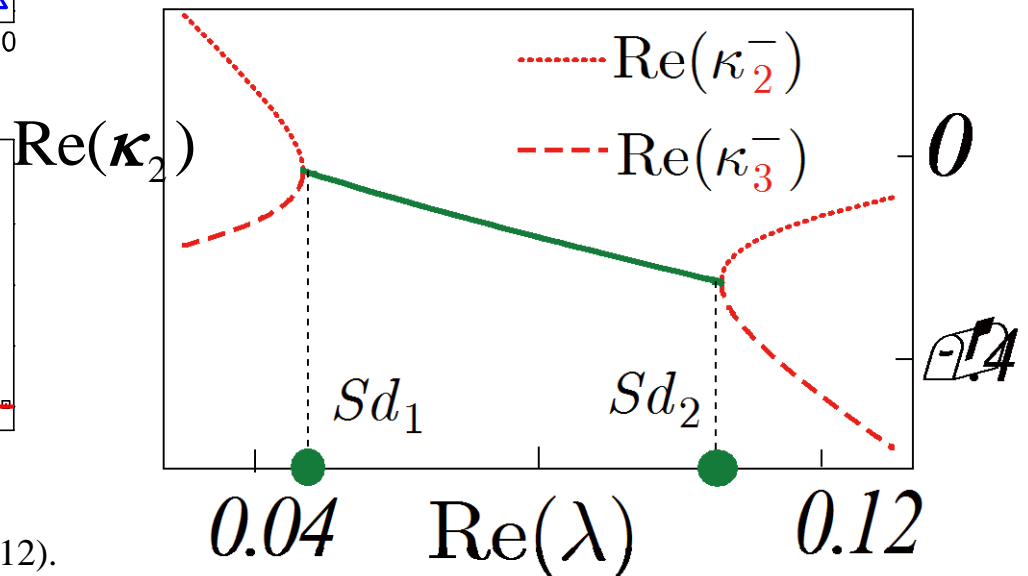
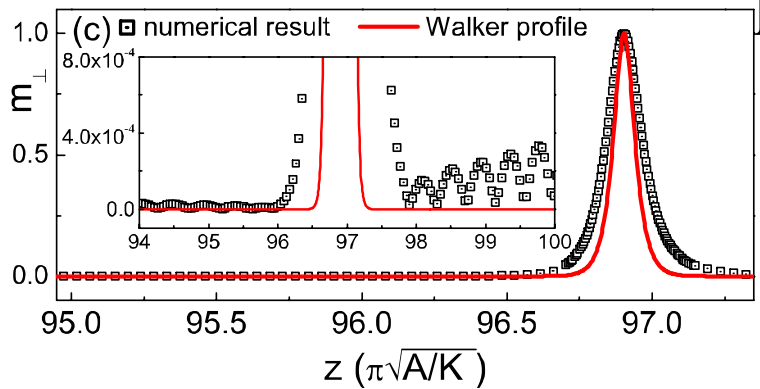
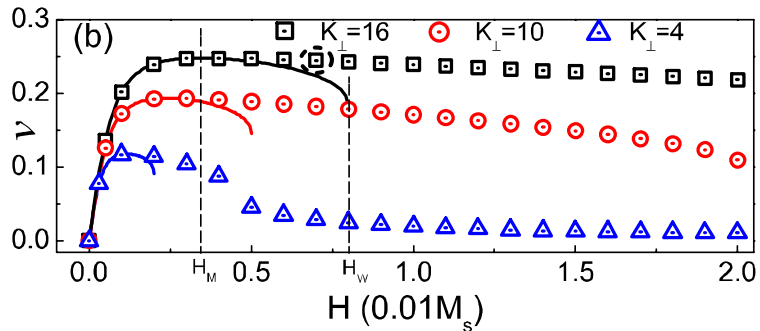
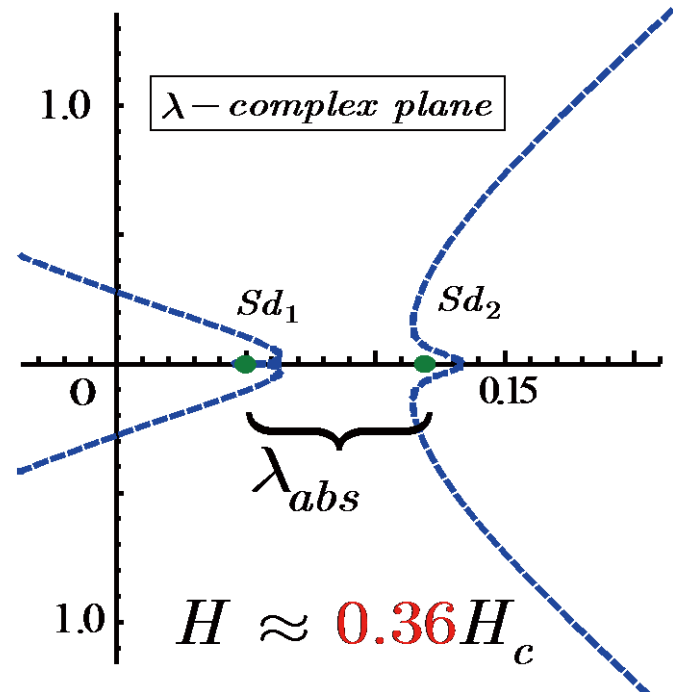
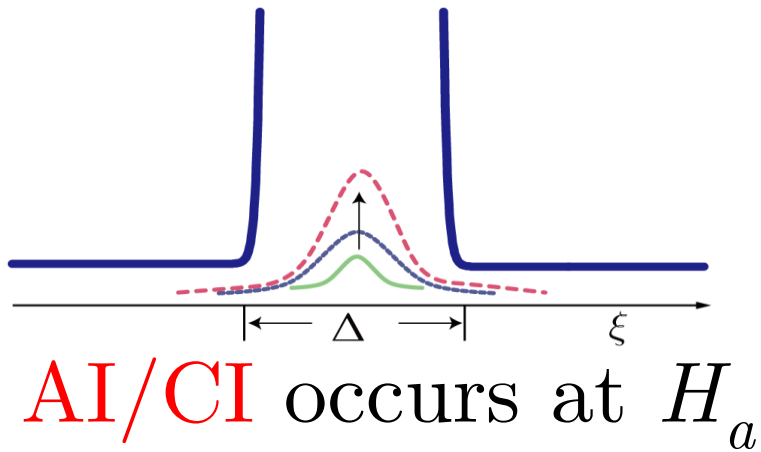
No  $\lambda_{abs}$  found for  $H < H_a \approx 0.36 H_c$ ,  
Transient Ins.

$\lambda_{abs}$  emerges if  $H > H_a \approx 0.36 H_c$

$H \approx 0.36 H_c$







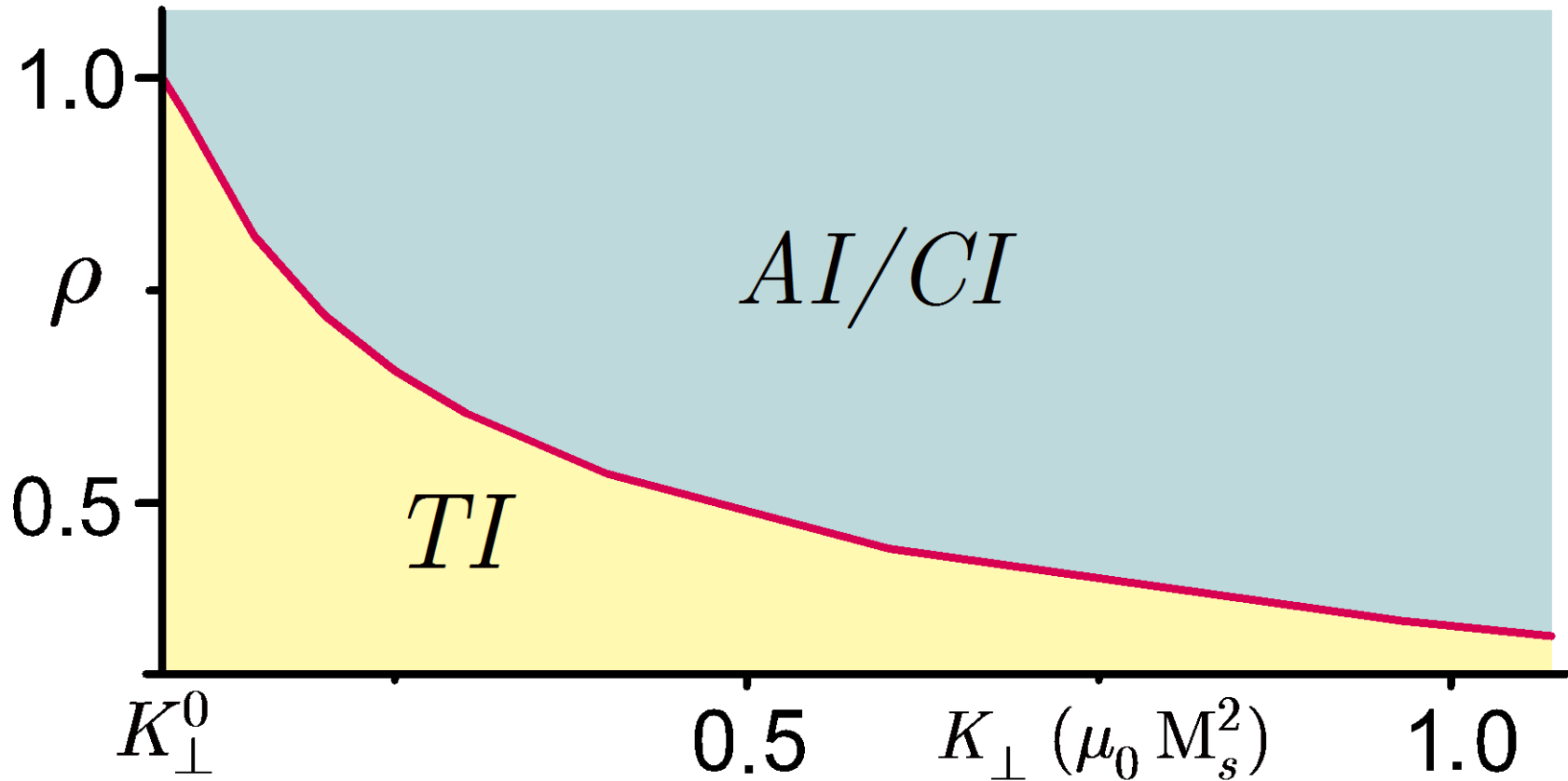


YIG Parameter	
Damping	$\alpha = 0.001$
Exchange	$A = 3.84 \times 10^{-12} \text{ J/m}$
Saturation Magnetization	$M_s = 1.94 \times 10^5 \text{ A/m}$
Gyromagnetic Ratio	$\gamma = 3.51 \text{ /kHz/(A/m)}$
Easy Axis Anisotropy	$K_{\parallel} = 2 \times 10^3 \text{ J/m}^3$
Hard Axis Anisotropy	$K_{\perp}$ : <b>Varying</b>

$$\rho = H_a / H_c,$$

$$\rho = 1 \text{ at},$$

$$K_{\perp}^0 \approx 0.085$$





# Conclusion

- It is shown that a Walker propagating DW will always emit stern waves in a low field, and both stern and bow waves in a higher field.
- The true propagating DW is always dressed with spin waves.
- For a realistic wire with its transverse magnetic anisotropy larger than a critical value and when the applied external field is large enough, a propagating DW may undergo simultaneous convective and absolute instabilities, leading to DW deformation and velocity deviation.