KITP, UCSB
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Berry curvature and topological modes for magnons

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Magnon thermal Hall effect

- Matsumoto, Murakami, Phys. Rev. Lett. 106, 197202 (2011)
- Matsumoto, Murakami, Phys. Rev. B 84, 184406 (2011)

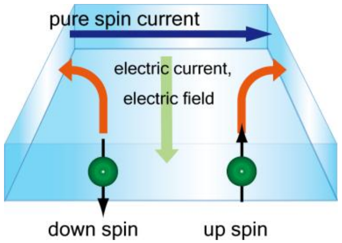
Topological magnonic crystals

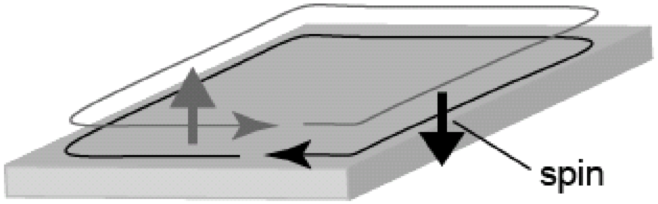
- Shindou, Matsumoto, Ohe, Murakami, Phys. Rev. B 87,174427 (2013)
- Shindou, Ohe, Matsumoto, Murakami, Saitoh, Phys. Rev. B87, 174402 (2013)

Phenomena due to Berry curvature in momentum space

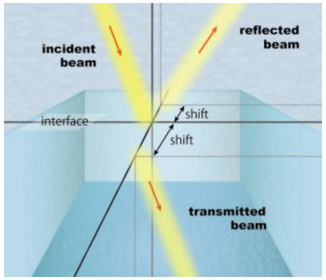
Gapless: Various Hall effects		Gapped: Topological edge/surface modes in gapped systems
• Hall effect	→	Quantum Hall effect *chiral edge modes*
• Spin Hall effect (of electrons)	→	Topological insulators *helical edge/surface modes*
•Spin Hall effect of light	→	one-way waveguide in photonic crystal
• Magnon thermal Hall effect	→	topological magnonic crystal

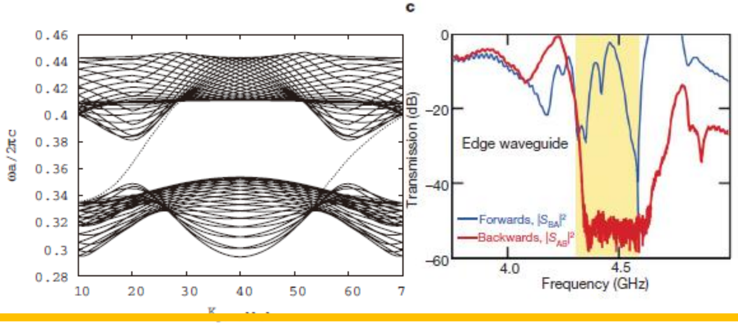
Fermions

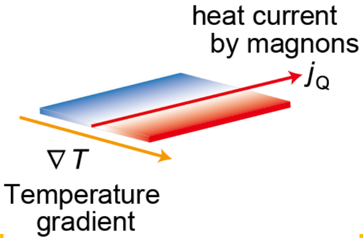


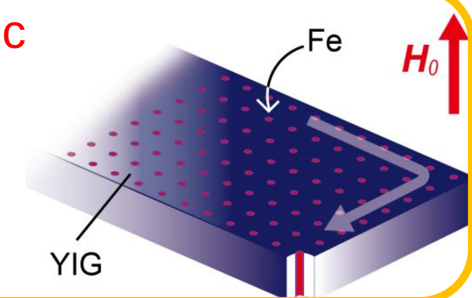


Bosons



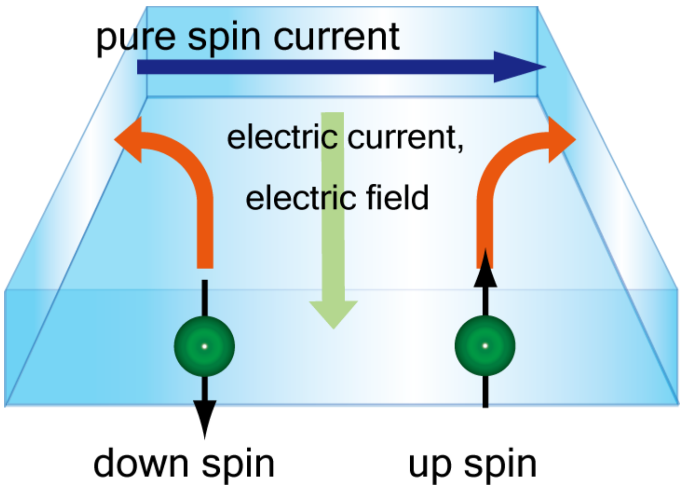






Present work

Intrinsic spin Hall effect in metals& semiconductors



- SM, Nagaosa, Zhang, Science (2003)
- Sinova et al., Phys. Rev. Lett. (2004)

semiclassical eq. of motion for wavepackets

$$\begin{cases} \dot{\vec{x}} = \dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -e\vec{E} \end{cases}$$

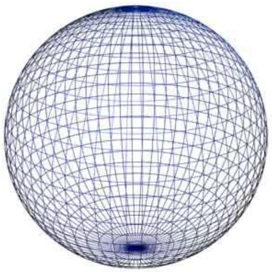
Force // electric field

Adams, Blount; Sundaram,Niu, …

$$\vec{\Omega}_n(\vec{k}) = i \left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$ **: Berry curvature**

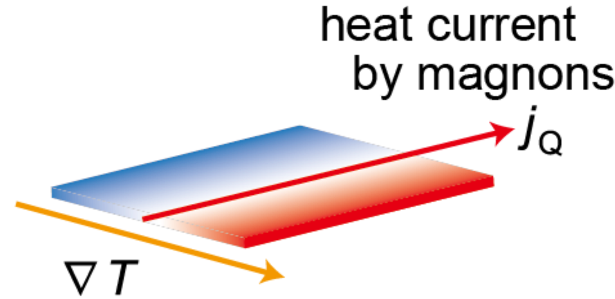
$u_{n\vec{k}}$: periodic part of the Bloch wf.

$\psi_{n\vec{k}}(\vec{x}) = u_{n\vec{k}}(\vec{x}) e^{i\vec{k}\cdot\vec{x}}$ (n : band index)



Spin-orbit coupling → Berry curvature depends on spin

Magnon Thermal Hall conductivity



Temperature gradient $\left(j_Q\right)_x = \kappa_{xy}\left(\nabla T\right)_y$

$$\kappa_{xy} = -\frac{k_B^2 T}{\hbar V}\sum_{n,k} \boxed{c_2\ \ \rho(\varepsilon_{nk})}\boxed{\Omega_n^z\ \ \vec{k}}$$

Berry curvature

$$c_2(\rho) = \int_0^\rho \left[\log\left(\frac{1+t}{t}\right)\right]^2 dt = (1+\rho)\left[\log\left(\frac{1+\rho}{\rho}\right)\right]^2 - (\log\rho)^2 - 2\mathrm{Li}_2(-\rho)$$

ρ: Bose distribution

R. Matsumoto, S. Murakami, Phys. Rev. Lett. 106, 197202 (2011)
T. Qin, Q. Niu and J. Shi,Phys. Rev. Lett. 107, 236601 (2011)

Cf: different from previous works

Katsura, Nagaosa, and Lee, PRL.104, 066403 (2010).
Onose, et al., Science 329, 297 (2010);

(1) Semiclassical theory

Eq. of motion

$$\begin{cases} \dot{\vec{x}} = \dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}} - \dot{\vec{k}}\times\vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -\nabla U \end{cases}$$

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

: Berry curvature

(2) Linear response theory (Kubo formula)

Density matrix

$$g(H) = f_0(H) + \boxed{f_1(H)}$$

equilibrium

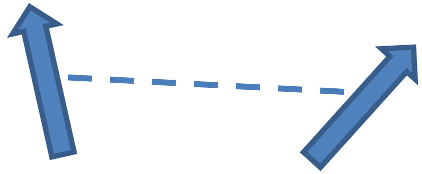
deviation by external field

Current

$$\boldsymbol{j}(\boldsymbol{r}) = \boldsymbol{j}^{(0)}(\boldsymbol{r}) + \boldsymbol{j}^{(1)}(\boldsymbol{r})$$

$$\boldsymbol{j}_E(\boldsymbol{r}) = \boldsymbol{j}_E^{(0)}(\boldsymbol{r}) + \boldsymbol{j}_E^{(1)}(\boldsymbol{r})$$

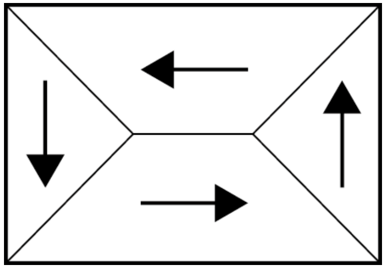
Magnetic dipole interaction



$$H_{\text{dipole}} = \frac{\mu_0}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|^3} \left\{ 3 \frac{\boldsymbol{S_r} \cdot (\boldsymbol{r} - \boldsymbol{r}')\ \boldsymbol{S_{r'}} \cdot (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^2} - \boldsymbol{S_r} \cdot \boldsymbol{S_{r'}} \right\}.$$

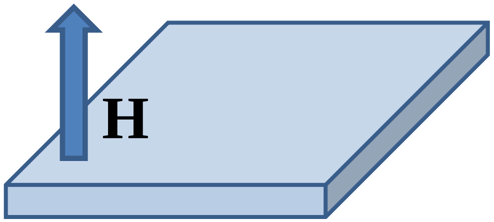
- Dominant in long length scale (microns)
- Similar to spin-orbit int.
  → Berry curvature

- Long-ranged → nontrivial, controlled by shape



Magnetic domains

Magnetostatic modes in ferromagnetic films (YIG)



- Landau-Lifshitz (LL) equation $\dfrac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H})$
- Maxwell equation $\nabla\cdot\mathbf{B}=0 \quad , \quad \nabla\times\mathbf{H}=0$
- Boundary conditions $\mathbf{B}_{1\perp} = \mathbf{B}_{2\perp} \quad , \quad \mathbf{H}_{1//} = \mathbf{H}_{2//}$

Generalized eigenvalue eq. : MSFVW mode

B. A. Kalinikos and A. N. Slavin, *J. Phys. C* **19**, 7013 (1986)

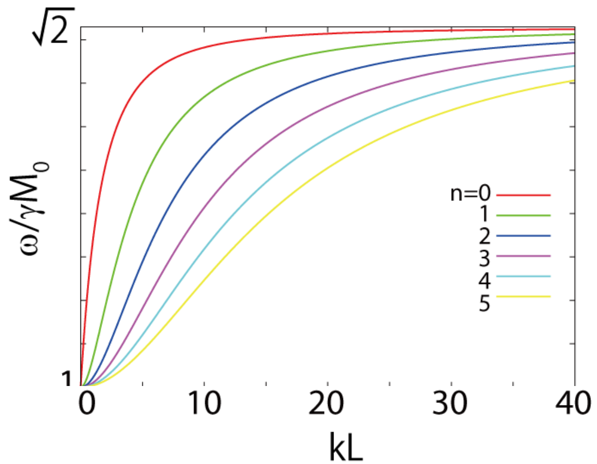
$$\hat{H}\mathbf{m}(z) = \omega\sigma_z\mathbf{m}(z) \qquad \left(\hat{H}\mathbf{m}(z) \equiv \omega_H\mathbf{m}(z) - \omega_M\int_{-L/2}^{L/2} dz'\hat{G}(z,z')\mathbf{m}(z')\right)$$

$\omega_H = \gamma H_0, \quad \omega_M = \gamma M_0, \quad L:$ thickness of the film, $\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, $\mathrm{m}(z) = \begin{pmatrix}m_x + im_y\\ m_x - im_y\end{pmatrix}$

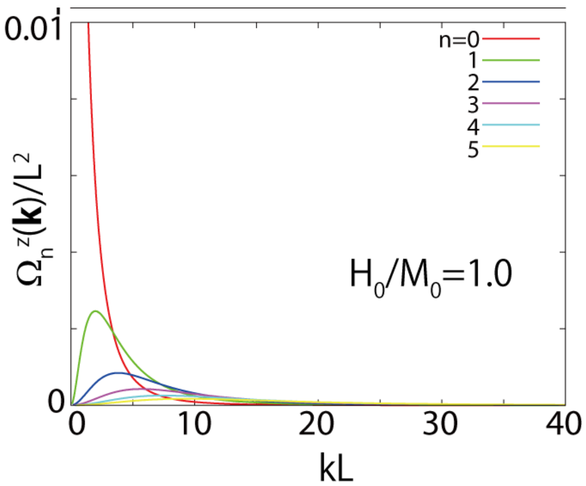
$M_0:$ saturation magnetization, $H_0:$ static magnetic field, $z\perp$ film,

$\hat{G}:$ 2×2 matrix of the Green's function, $\omega:$ frequency of the spin wave

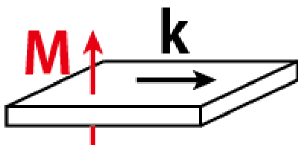
Berry curvature for magnetostatic forward volume wave mode



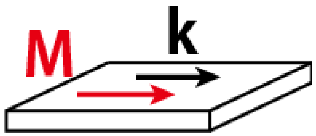
: Band structure for magnetostatic forward volume-wave mode

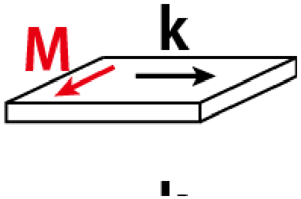


: Berry curvature



Berry curvature is zero for backward volume wave and surface wave





- R. Matsumoto, S. Murakami, PRL 106,197202 (2011), PRB84, 184406 (2011)

Bosonic BdG eq. and Berry curvature

Generalized eigenvalue eq. $H_k \psi = \omega_k \sigma_z \psi$

→ bosonic Bogoliubov-de Gennes Hamiltonian

$$\mathcal{H} = \frac{1}{2} \sum_{\boldsymbol{k}} \left(\boldsymbol{\beta}_{\boldsymbol{k}}^{\dagger} \boldsymbol{\beta}_{-\boldsymbol{k}} \right) H_{\boldsymbol{k}} \begin{pmatrix} \boldsymbol{\beta}_{\boldsymbol{k}} \\ \boldsymbol{\beta}_{-\boldsymbol{k}}^{\dagger} \end{pmatrix}$$

Diagonalization

$$\mathcal{E}_{\boldsymbol{k}} = T_{\boldsymbol{k}}^{\dagger} H_{\boldsymbol{k}} T_{\boldsymbol{k}} = \begin{pmatrix} E_{\boldsymbol{k}} & \\ & E_{-\boldsymbol{k}} \end{pmatrix}$$

$$T_k^{+} \sigma_z T_k = \sigma_z$$

$$T_k \sigma_z T_k^{+} = \sigma_z$$

T: paraunitary matrix

Berry curvature for *n*-th band

$$\Omega_{n\boldsymbol{k}} \equiv i\epsilon_{\mu\nu} \left[\sigma_z \frac{\partial T_{\boldsymbol{k}}^{\dagger}}{\partial k_{\mu}} \sigma_z \frac{\partial T_{\boldsymbol{k}}}{\partial k_{\nu}} \right]_{nn}$$

Linear response theory →

$$\kappa_{\mu\nu} = -\frac{k_{\mathrm{B}}^2 T}{\hbar V} \sum_{\boldsymbol{k}} \sum_{n=1}^{N} \left(c_2 \left(g \left(\varepsilon_{n\boldsymbol{k}} \right) \right) - \frac{\pi^2}{3} \right) \Omega_{n\boldsymbol{k}}.$$

Berry curvature

Bosonic BdG eq. and Berry curvature

Generalized eigenvalue eq. $H_k \psi = \omega_k \sigma_z \psi$

→ bosonic Bogoliubov-de Gennes Hamiltonian

$$\mathcal{H} = \frac{1}{2} \sum_{\boldsymbol{k}} \left(\beta^{\dagger}_{\boldsymbol{k}} \beta_{-\boldsymbol{k}} \right) H_{\boldsymbol{k}} \begin{pmatrix} \beta_{\boldsymbol{k}} \\ \beta^{\dagger}_{-\boldsymbol{k}} \end{pmatrix}$$

Diagonalization

$$\mathcal{E}_{\boldsymbol{k}} = T^{\dagger}_{\boldsymbol{k}} H_{\boldsymbol{k}} T_{\boldsymbol{k}} = \begin{pmatrix} E_{\boldsymbol{k}} & \\ & E_{-\boldsymbol{k}} \end{pmatrix}$$

$$T_k^{+} \sigma_z T_k = \sigma_z$$

$$T_k \sigma_z T_k^{+} = \sigma_z$$

T: paraunitary matrix

Berry curvature for *n*-th b

$$\Omega_{n\boldsymbol{k}} \equiv i\epsilon_{\mu\nu} \left[\sigma_z \frac{\partial T}{\partial k} \right.$$

Linear response theory -

$$\kappa_{\mu\nu} = -\frac{k_B^2 T}{\hbar V} \sum_{\boldsymbol{k}} \sum_{n=1}$$

Berry curvature

(Example):

$\gamma = 2.8\text{MHz/Oe}, M_s = 1750\text{gauss}, T = 300\text{K}$

$H_{ex} = 3000\text{Oe}, l_{ex} = 17.2\text{nm}$ for YIG,

$$\kappa_{xy} \approx 5.8 \times 10^{-8} \text{W/Km}$$

Topological chiral modes
in magnonic crystals

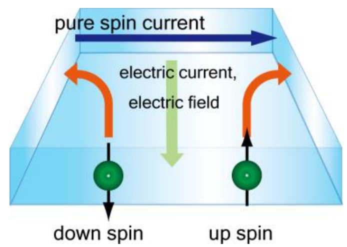
- R. Shindou, R. Matsumoto, J. Ohe, S. Murakami, Phys. Rev. B 87,174427 (2013)
- R. Shindou, J. Ohe, R. Matsumoto, S. Murakami, E. Saitoh,Phys. Rev. B87, 174402 (2013)

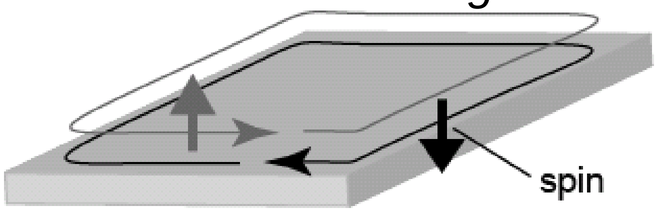
Phenomena due to Berry curvature in momentum space

Gapless	Various Hall effects	Gapped	Topological edge/surface modes in gapped systems

Fermions

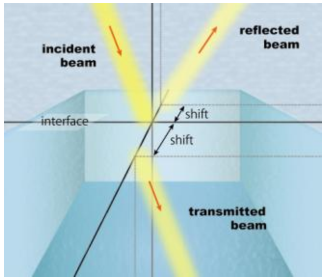
- Hall effect → Quantum Hall effect
  chiral edge modes
- Spin Hall effect (of electrons) → Topological insulators
  helical edge/surface modes

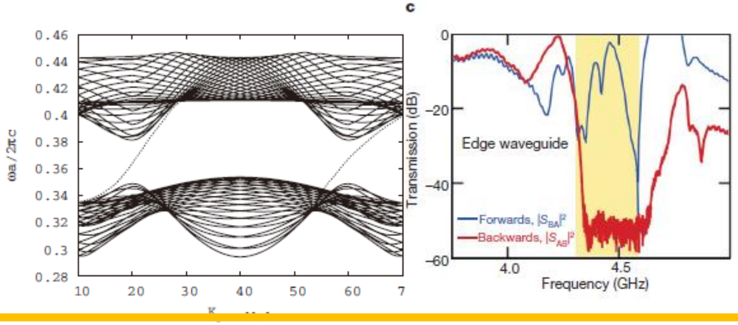




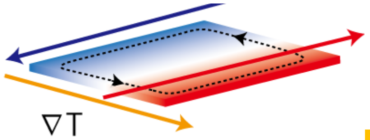
Bosons

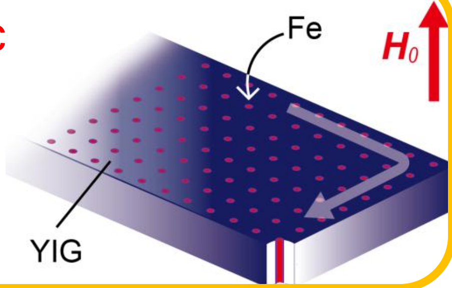
- Spin Hall effect of light → one-way waveguide in photonic crystal





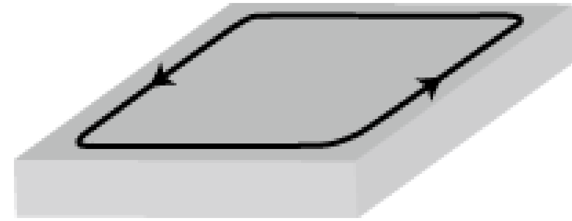
- Magnon thermal Hall effect → topological magnonic crystal





Present work

Chern number & topological chiral modes



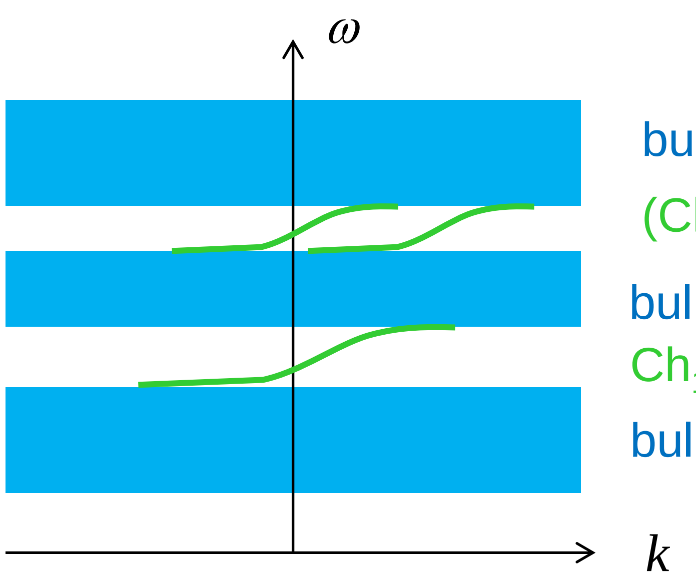
Band gap → Chern number for n-th band = integer

$$\mathrm{Ch}_n = \int_{BZ} \frac{d^2k}{2\pi} \Omega_n \; \vec{k} \qquad \vec{\Omega}_n(\vec{k}) = i \left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

Berry curvature

$$\sum_{n \in \text{bands below } E} \mathrm{Ch}_n = N \equiv \#(\text{clockwise chiral edge states in the gap at } E)$$

- Analogous to chiral edge states of quantum Hall effect.
- N>0 → cw, N<0: ccw mode



bulk mode: Chern number= Ch_3

$(\mathrm{Ch}_1+\mathrm{Ch}_2)$ topological edge modes

bulk mode: Chern number= Ch_2

Ch_1 topological edge modes

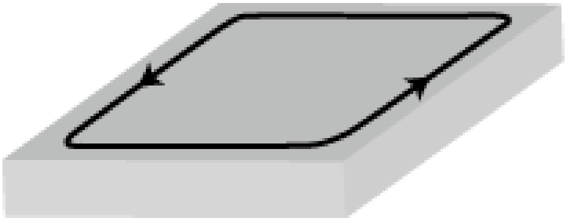
bulk mode: Chern number= Ch_1

Chern number & topological chiral modes

Band gap → Chern number for n-th band = integer

$$\mathrm{Ch}_n = \int_{BZ} \frac{d^2k}{2\pi} \Omega_n \quad \vec{k} \qquad \vec{\Omega}_n(\vec{k}) = i \left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

Berry curvature

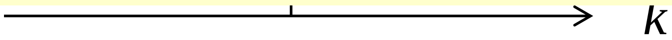


topological chiral edge modes

$$\sum_{n \in \text{bands below } E} \mathrm{Ch}_n = N \equiv \#(\text{clockwise chiral edge states in the gap at } E)$$

Why the Chern number is related
with the number of edge states within the gap?

→ Mathematics : Index theorem
Physics : Analogy to quantum Hall effect
(in electrons)



Quantum Hall effect of electrons:
-- Chern number & edge states --

Hall conductivity for 2D systems (of electrons)

Electric field //y

$$|\alpha\rangle \rightarrow |\alpha'\rangle = |\alpha\rangle + \sum_{\beta(\neq\alpha)} \frac{\langle\beta|eEy|\alpha\rangle}{E_\alpha - E_\beta}\ |\beta\rangle$$

Current along x

$$\langle j_x\rangle = \frac{1}{L^2}\sum_\alpha f\ \ E_\alpha\ \ \langle\alpha'|j_x|\alpha'\rangle = \frac{1}{L^2}\sum_\alpha f\ \ E_\alpha\ \sum_{\beta(\neq\alpha)} \frac{\langle\alpha|(-ev_x)|\beta\rangle\langle\beta|eEy|\alpha\rangle}{E_\alpha - E_\beta}\ |\beta\rangle$$

Hall conductivity (Kubo formula)

$$\sigma_{xy} = -ie^2 h \frac{1}{L^2}\sum_\alpha f\ \ E_\alpha\ \sum_{\beta(\neq\alpha)} \frac{\langle\alpha|v_y|\beta\rangle\langle\beta|v_x|\alpha\rangle}{E_\alpha - E_\beta\ {}^2}\ |\beta\rangle$$

$\Downarrow$

$$\sigma_{xy} = -\frac{e^2}{\hbar}\sum_{E_{n\vec{k}}<E_F} B_{nz}(\vec{k})$$ at T=0

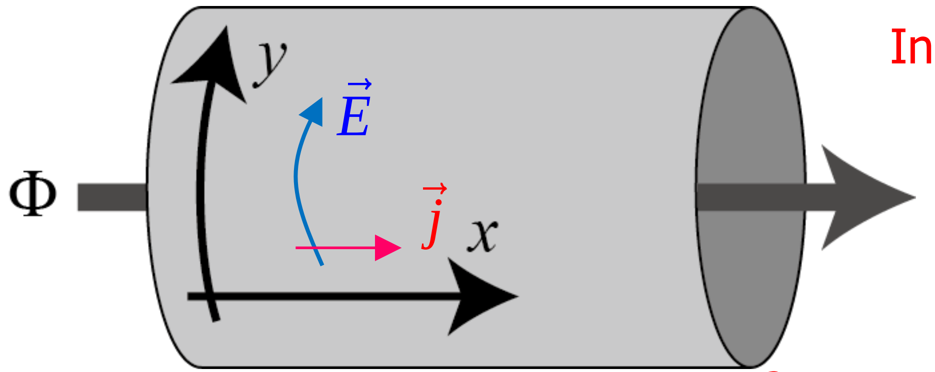
$$\vec{B}_n(\vec{k}) = i\left\langle \frac{\partial u_{n\vec{k}}}{\partial\vec{k}}\right| \times \left|\frac{\partial u_{n\vec{k}}}{\partial\vec{k}}\right\rangle$$: Berry curvature

Example: insulator

$$\sigma_{xy} = -\frac{e^2}{h}\sum_{n:\text{filled band}} \mathrm{Ch}_n \qquad \mathrm{Ch}_n = \int_{BZ}\frac{d^2k}{2\pi}B_{nz}(\vec{k})$$: Chern number = integer

Hall conductivity is expressed as a sum of Chern numbers→ integer QHE

Laughlin gedanken experiment: Chern number & edge states



Increase the flux
→ charge transport along x

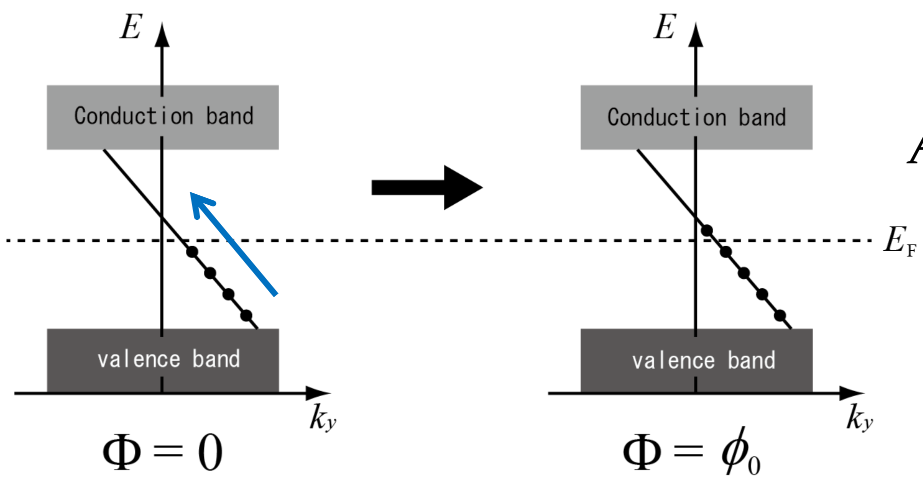
$$\Phi = 0 \text{ at } t = 0$$

$$\Phi = \phi_0 \text{ at } t = T$$

$$E_y = \frac{\phi_0}{TL_y} \Rightarrow j_x = \sigma_{xy}\frac{\phi_0}{TL_y} = \frac{-e^2}{h}\mathrm{Ch}\frac{\phi_0}{TL_y} = -\frac{e}{TL_y}\mathrm{Ch}$$

$$\Rightarrow Q = -e\cdot\mathrm{Ch}$$

Number of electron carried from left end to right end = Ch



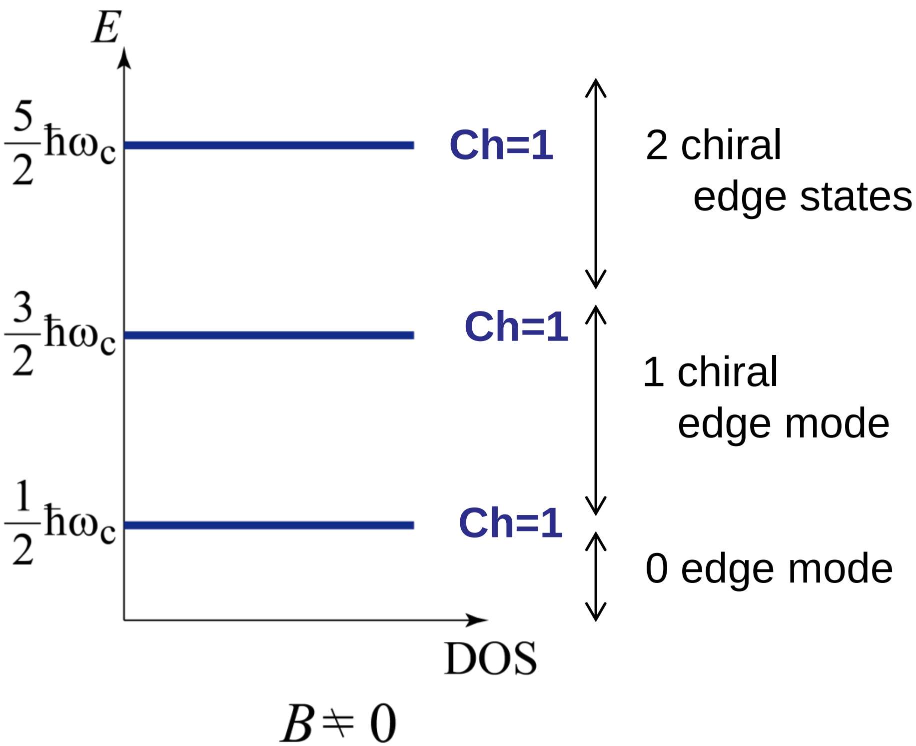
Edge states at the right end

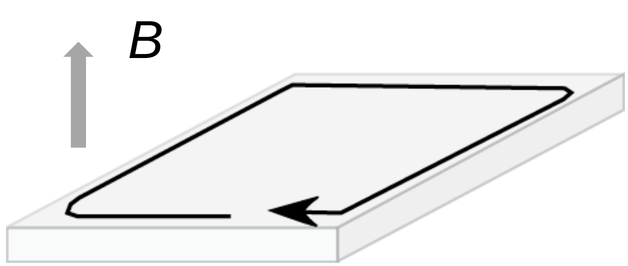
$$A_y = -\frac{\Phi}{L_y}$$

Gradual change of vector pot.
= change of wavenumber

This charge transport is between the edge states on the left and right ends.
→ gapless edge modes exist.

Integer quantum Hall effect

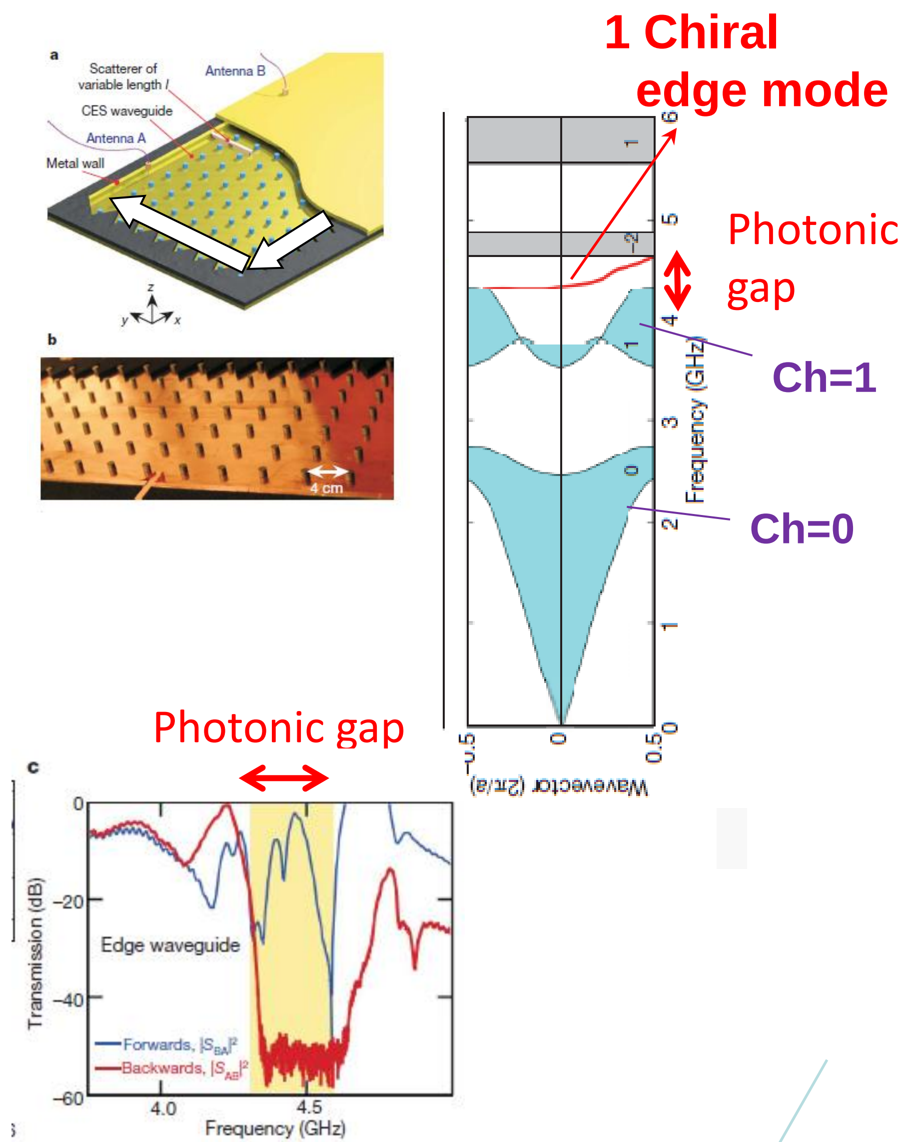




Topological photonic crystals

Theory: Haldane, Raghu, PRL100, 013904 (2008)

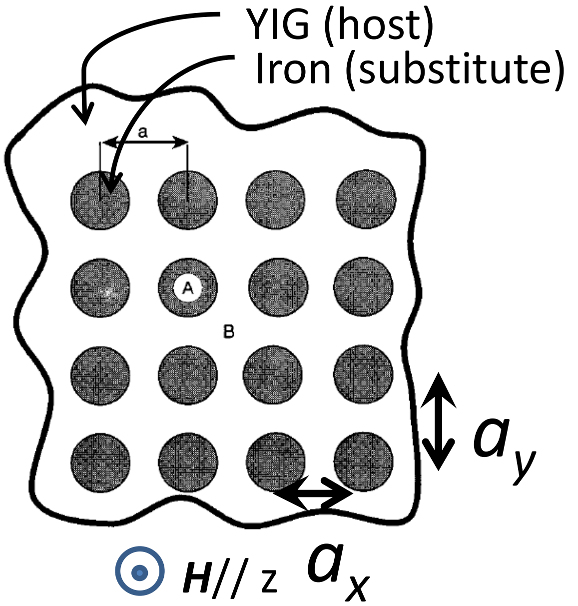
Experiment: Wang et al., Nature 461, 772 (2009)



2D Magnonic Crystal : periodically modulated magnetic materials

- Landau-Lifshitz equation $\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma \boldsymbol{M} \times \boldsymbol{H}_{\text{eff}}$
- Maxwell equation (magnetostatic approx.)

$$\nabla \times \boldsymbol{H} = \boldsymbol{0},$$
$$\nabla \cdot (\boldsymbol{H} + 4\pi \boldsymbol{M}) = 0.$$



- Saturation magnetization M_s
- exchange interaction length Q

} modulated

- Linearized EOM

$$\frac{1}{|\gamma|\mu_0}\frac{\partial m_\pm}{\partial t} = \mp i H_0 m_\pm \pm 2iM_s(\nabla \cdot Q\nabla)m_\pm \mp 2im_\pm(\nabla \cdot Q\nabla)M_s \pm ih_\pm M_s.$$

$$m_\pm \equiv m_x \pm i m_y. \quad h_\pm \equiv h_x \pm i h_y.$$

External field

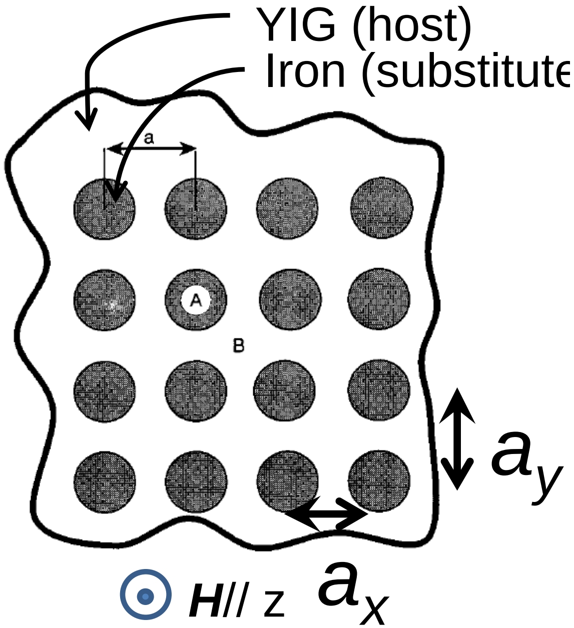
exchange field (quantum mechanical short-range)

Dipolar field (classical, long range)

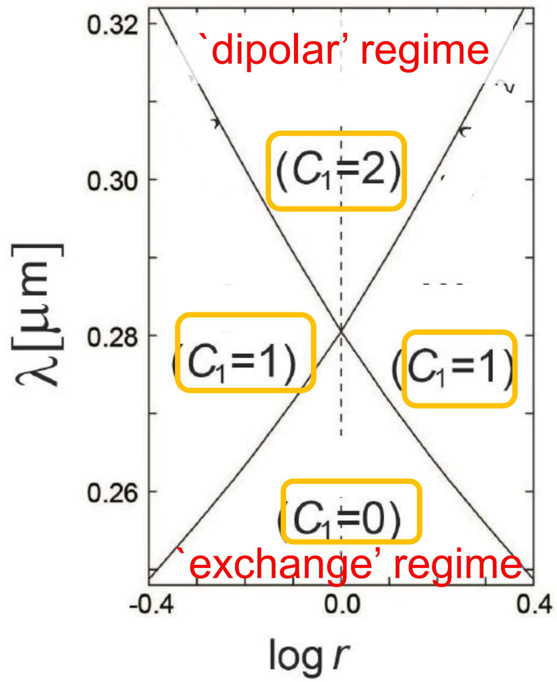
$$\mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}} [\,\beta_{\boldsymbol{k}}^\dagger \;\; \beta_{-\boldsymbol{k}}\,] \cdot \boldsymbol{H}_{\boldsymbol{k}} \cdot \begin{bmatrix} \beta_{\boldsymbol{k}} \\ \beta_{-\boldsymbol{k}}^\dagger \end{bmatrix}.$$

bosonic Bogoliubov – de Gennes eq.

chiral magnonic band in magnonic crystal



• Phase diagram



$\lambda = \sqrt{a_x a_y}$: unit cell size

$r = a_y / a_x$: aspect ratio of unit cell

Magnonic gap between 1st and 2nd bands

→C_1 (Chern number) for 1st band

$$C_1 = \int_{BZ} \frac{d^2k}{2\pi} \Omega_1(\vec{k})$$

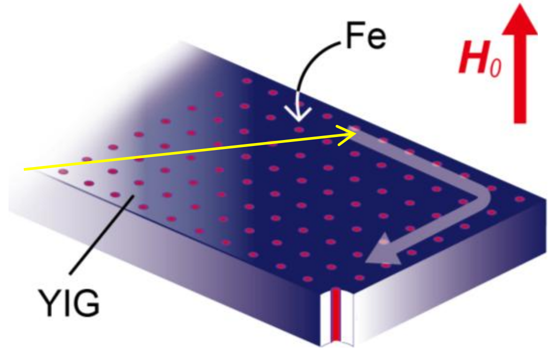
=Number of topological chiral modes within the gap

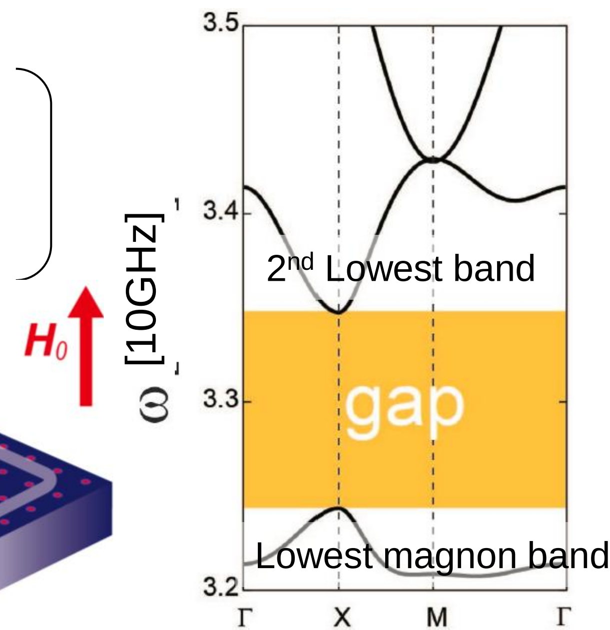
dipolar interaction

➔ non-trivial Chern integer

➔ topological chiral edge modes

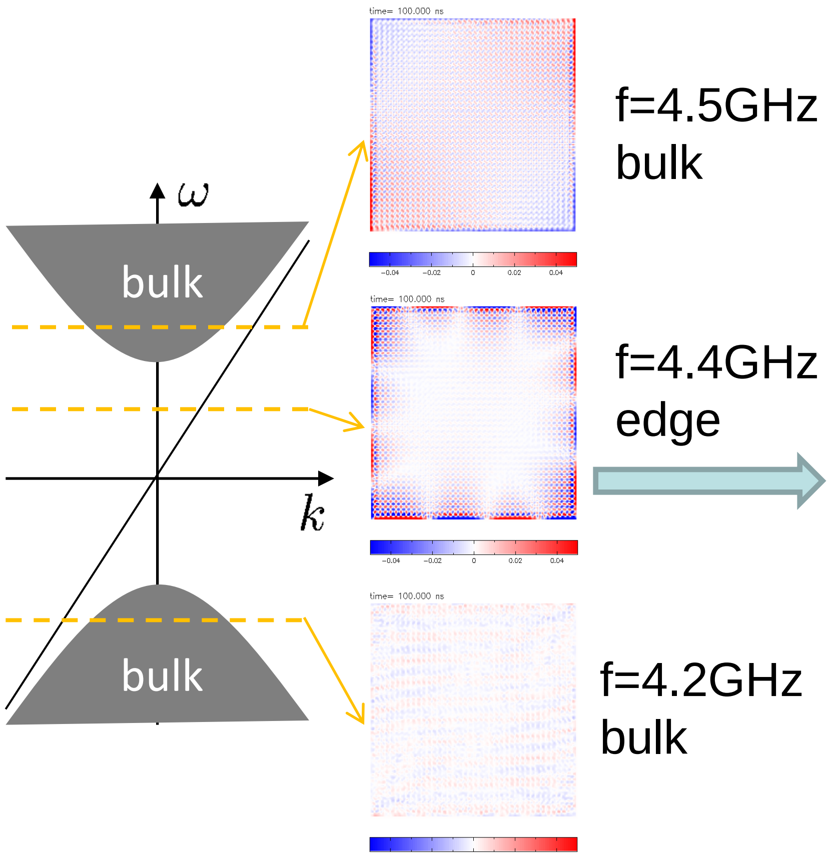
(like in quantum Hall effect)

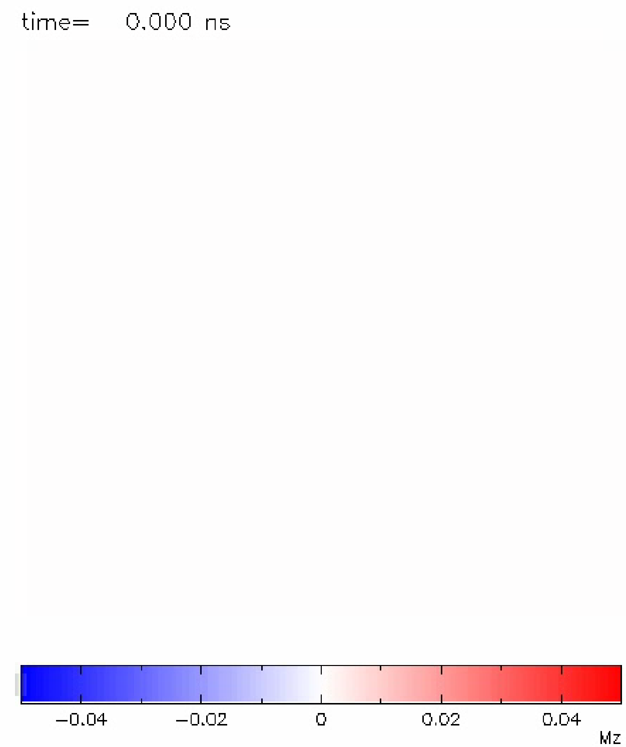




Simulation (by Dr. Ohe)

DC magnetic field : out-of-plane
AC magnetic field : in-plane





Magnonic crystals with ferromagnetic dot array

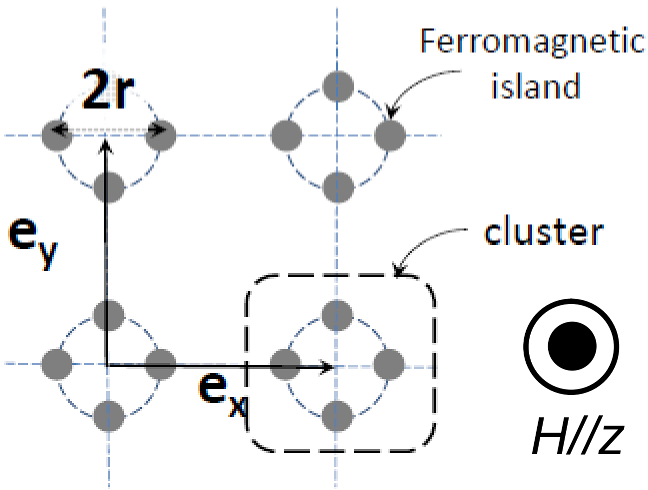
R. Shindou, J. Ohe, R. Matsumoto, S. Murakami, E. Saitoh, PRB (2013)

dot (=thin magnetic disc) → cluster: forming "atomic orbitals"

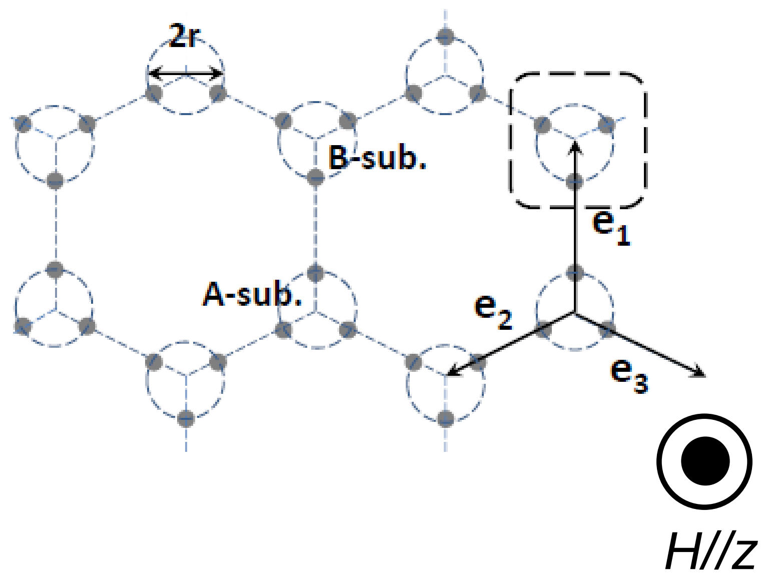
➔ convenient for (1) understanding how the topological phases appear

(2) designing topological phases

decorated square lattice

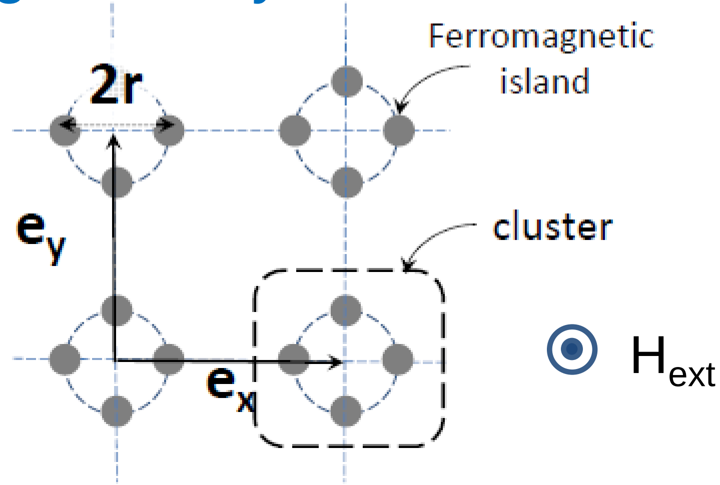


decorated honeycomb lattice



Each island is assumed to behave as monodomain

Magnonic crystals: decorated square lattice



$$|\vec{e}_x| = |\vec{e}_y| = 2.4,\ 2r = 1.2,\ \Delta V = 1.70,\ M_s = 1.0$$

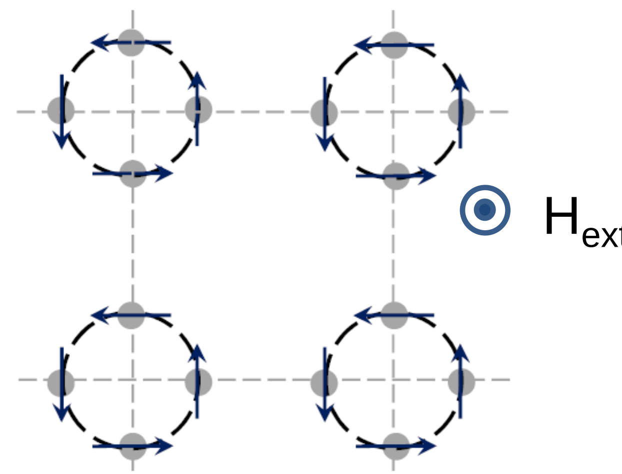
Magnetostatic energy

$$E = -\frac{1}{2}(\Delta V)^2 \sum_{i,j}^{i \neq j} M_a(r_i) f_{ab}(r_i - r_j) M_b(r_j) + H \Delta V \sum M_z(r_j).$$

$$f_{ab}(r) = \frac{1}{4\pi}\left(\frac{\delta_{a,b}}{|r|^3} - \frac{3 r_a r_b}{|r|^5}\right)$$

Equilibrium spin configuration

$H_{ext} < H_c = 1.71$

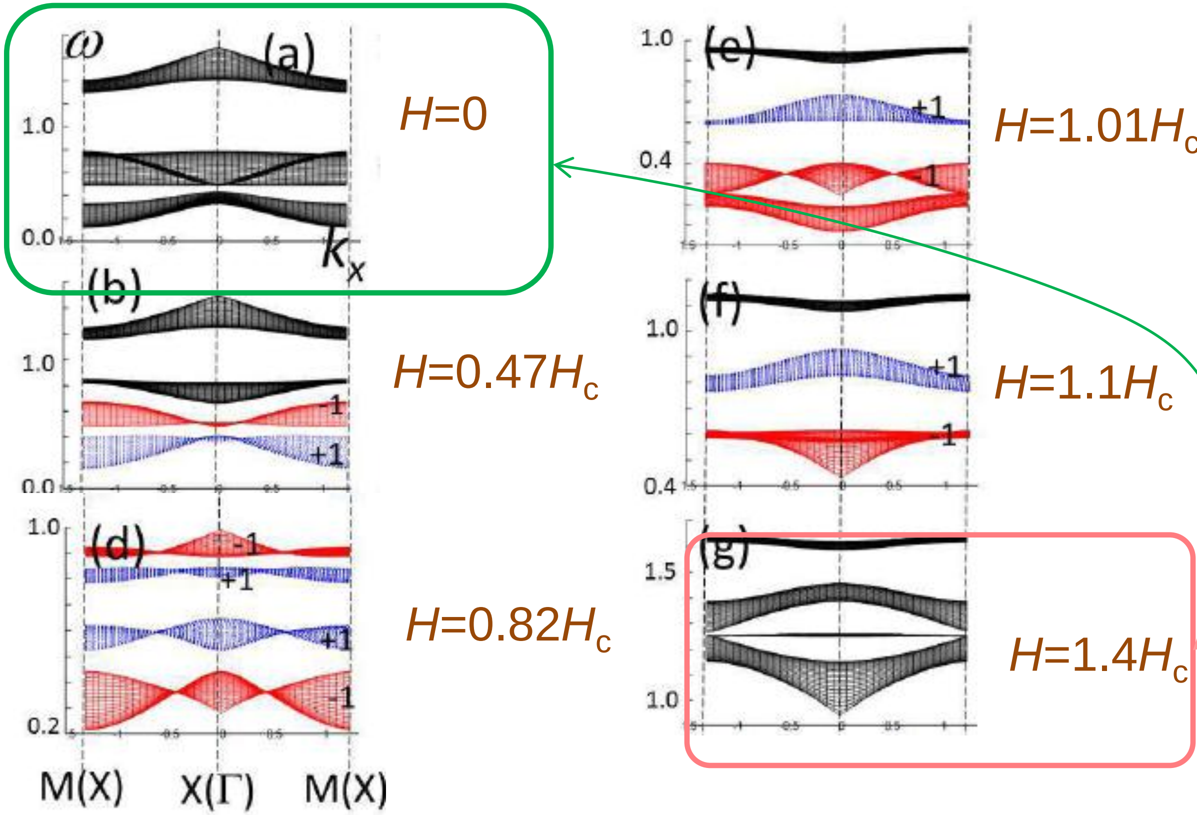


Tilted along H_{ext}

$H_{ext} > H_c$

Collinear // H_{ext}

- <u>Spin-wave bands and Chern numbers</u>



Red: Ch=-1
Blue: Ch=+1
→ <u>Topologically nontrivial</u>
<u>= chiral edge modes</u>

Time-reversal symmetry

- Small H<<H_c
- Large H>>H_c

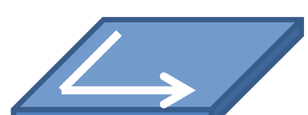
→ Topologically trivial

Dipolar interaction is weak

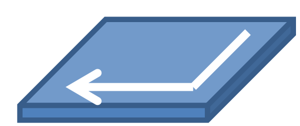
→ <u>Nontrivial</u> phases (i.e. nonzero Chern number) in the intermediate magnetic field strength

<u>topological chiral edge modes</u>

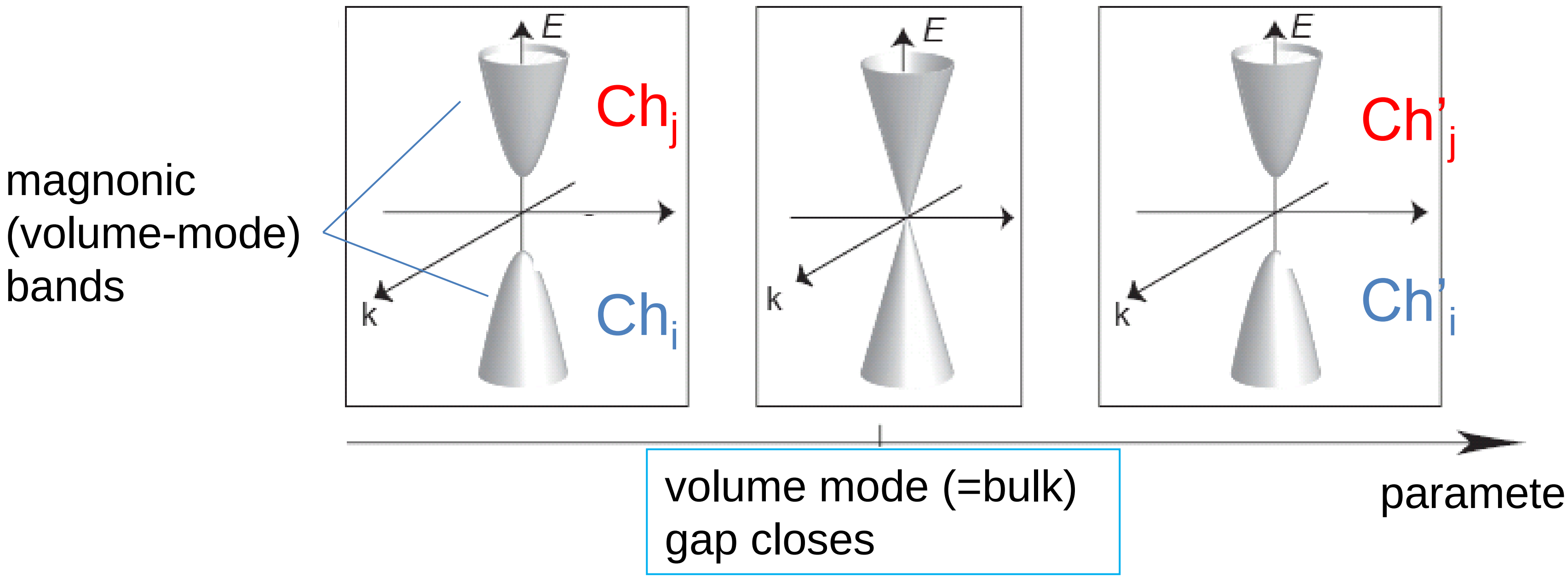
-1 chiral mode



+1 chiral mode



Change of Chern number at gap closing event

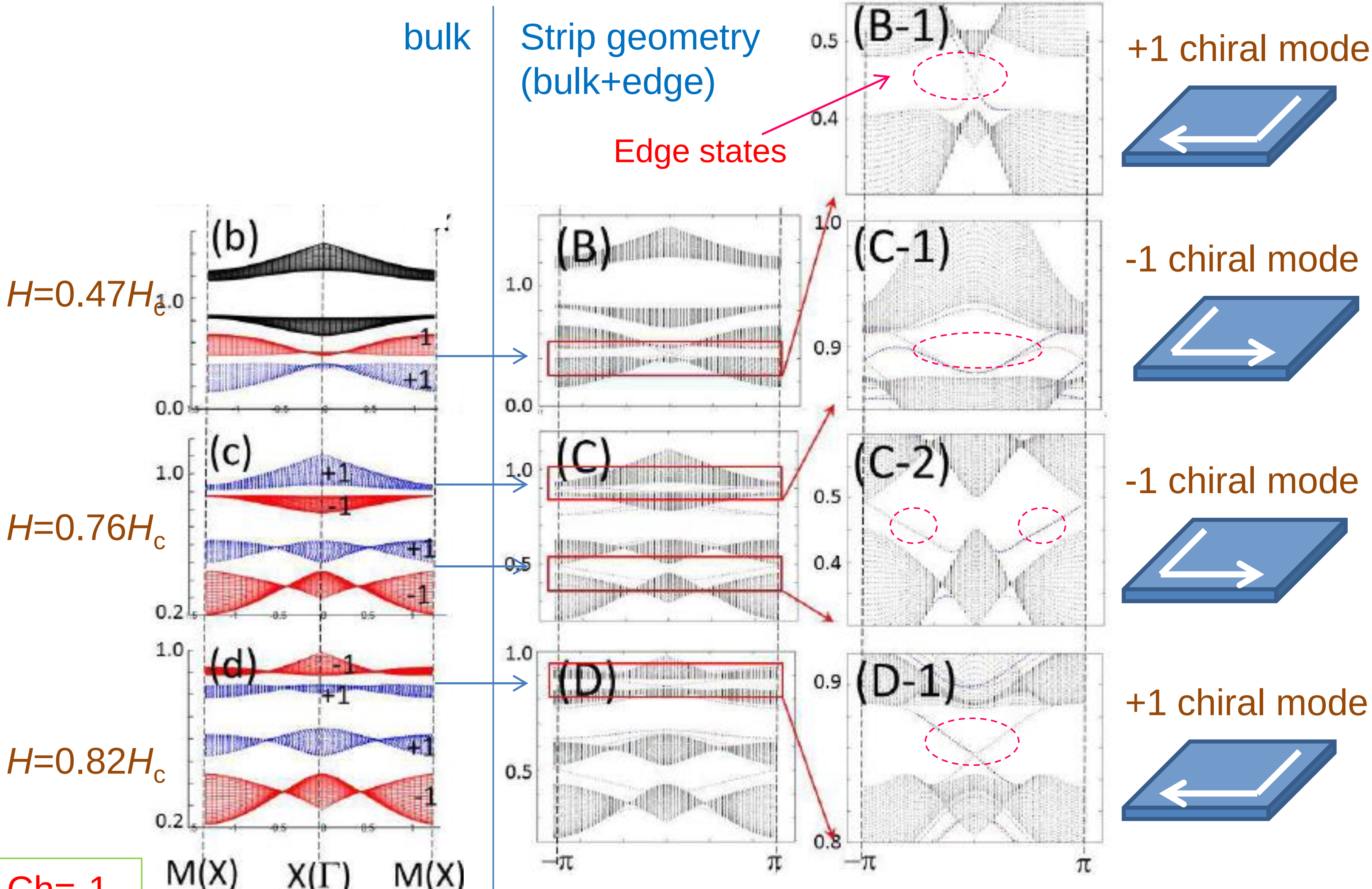


$\Delta Ch_i = -\Delta Ch_j = \pm 1$

: Dirac cone at gap closing

($\rightarrow$ $Ch_i + Ch_l = Ch'_i + Ch'_l$: sum is conserved)

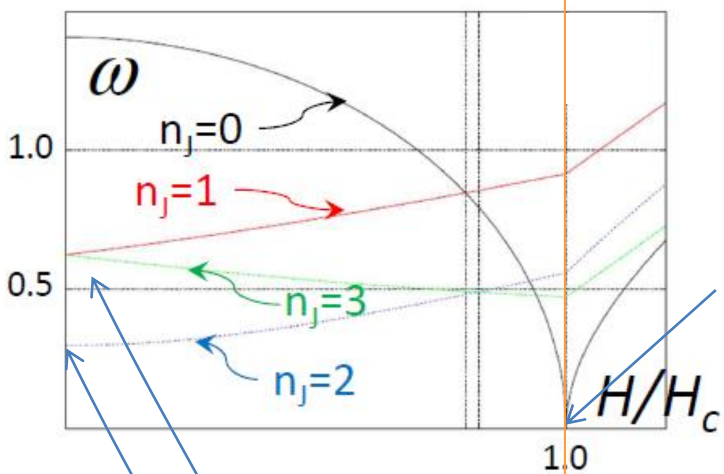
Magnonic crystals: edge states and Chern numbers (1)



Red: Ch=-1
Blue: Ch=+1

"atomic orbitals" within a single cluster

Energy levels of atomic orbitals



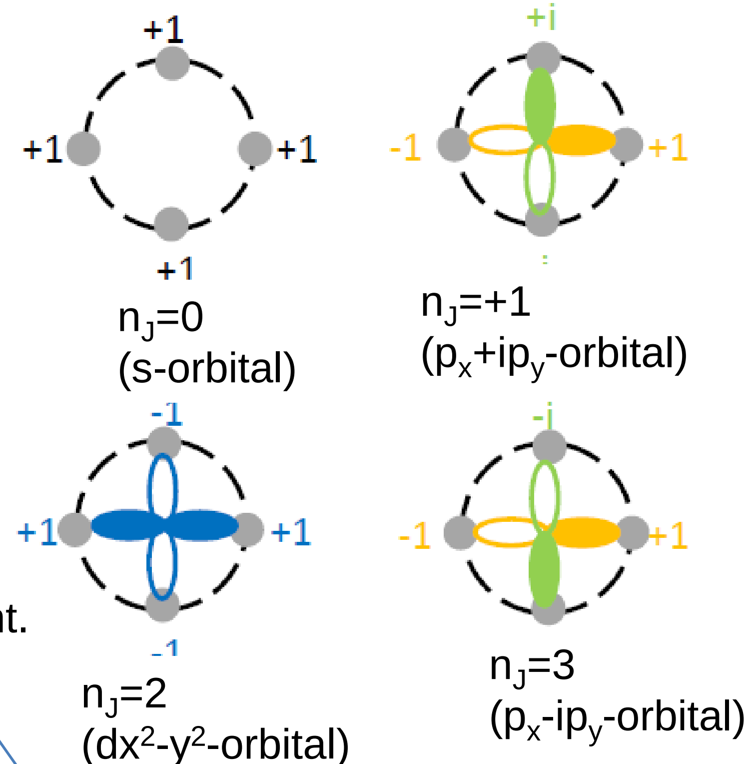
n_J=0 softens at H=H_c

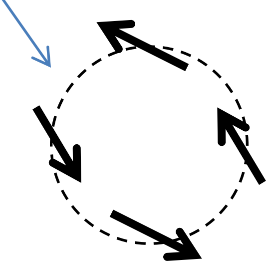
n_J=1 and n_J=3 degenerate at H=0

n_J=2 is lowest at H=0: favorable for dipolar int.

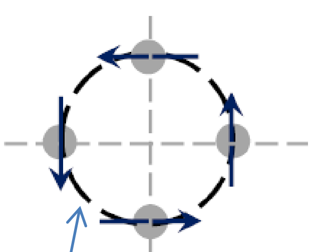
Spin wave excitations: "atomic orbitals"

relative phase for precessions





Equilibrium configuration



H<H_c: noncollinear

H>H_c: collinear // z

Magnonic crystals: tight-binding model with atomic orbitals

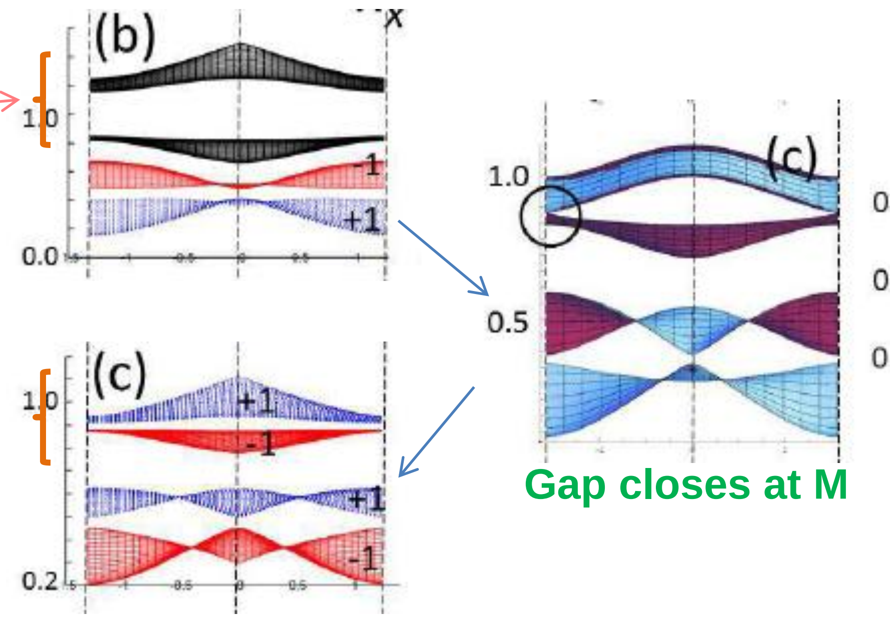
(example) *:*

$H=0.47H_c \rightarrow H=0.82H_c$

gap between 3rd and 4th bands

retain only n_J=0 and n_J=1 orbitals

→ tight binding model



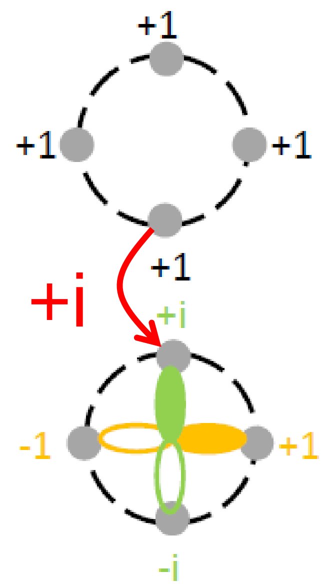
$$H_{01,\boldsymbol{k}} = \begin{pmatrix} \varepsilon_0 + 2a_{00}(c_{k_x} + c_{k_y}) & -2ib_{01}(s_{k_x} - is_{k_y}) \\ 2ib_{01}(s_{k_x} + is_{k_y}) & \varepsilon_1 + 2a_{11}(c_{k_x} + c_{k_y}) \end{pmatrix}$$

complex phase for hopping

← p_x+ip_y orbitals

= Model for quantum anomalous Hall effect

(e.g. Bernevig et al., Science 314, 1757 (2006))



n_J=0
(s-orbital)

n_J=+1
(p_x+ip_y-orbital)

Design of topological magnonic crystals

- Metal or insulator ? → either is OK
  (for metal: damping might be a problem for observation of edge modes.)
- Symmetry restrictions:
  - Usually, the magnonic crystals with in-plane magnetic field is non-topological by symmetry reasons.
  - Out-of-plane magnetic field is OK.
    (cf.: Thin-film with out-of-plane magnetic field has nonzero Berry curvature.)
- Magnonic gap is required
  Strong contrast within unit cell is usually required.
  1 ferromagnet and vacuum
  2 ferromagnets with very different saturation magnetization
- Multiband & sublattice structure:
  nonzero Berry curvature requires multiband structure
  Sublattice structure is helpful for designing topological magnonic crystal

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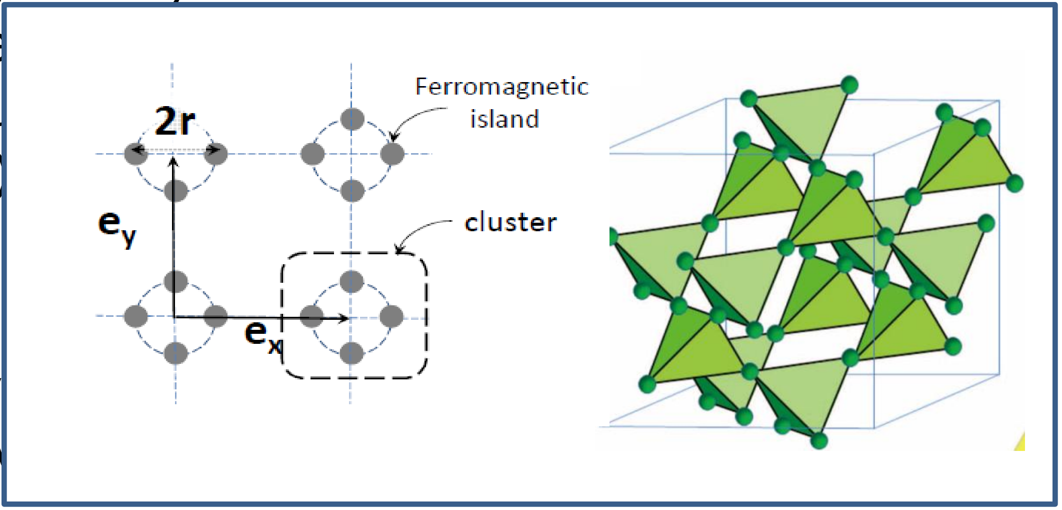
  1 ferromagnet and v
  2 ferromagnets with v



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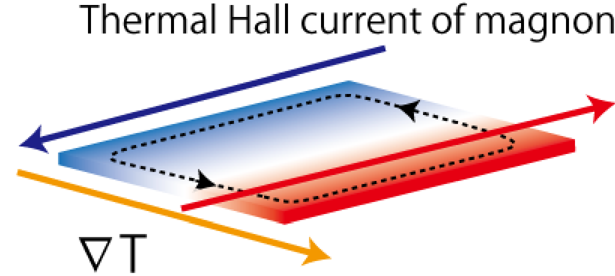
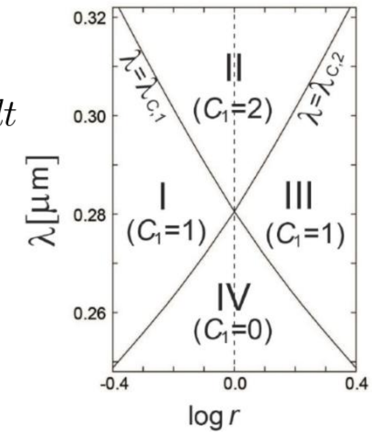


Summary

- Magnon thermal Hall effect (Righi-Leduc effect)

$$\kappa^{xy} = \frac{2k_B^2 T}{\hbar V} \sum_{n,\boldsymbol{k}} c_2(\rho(\varepsilon_{n\boldsymbol{k}})) \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial k_x} \middle| \frac{\partial u_{n\boldsymbol{k}}}{\partial k_y} \right\rangle \quad c_2(\rho) = \int_0^\rho \left(\log\frac{1+t}{t}\right)^2 dt$$



- Topological chiral modes in magnonic crystals
  - ✓ magnonic crystal with dipolar int.
    → Berry curvature & Chern number

Array of columnar magnet in other magnet

phases with different Chern numbers by changing lattice constant

Array of disks

non-zero Chern numbers

atomic orbitals → tight-binding model → reproduce spin-wave bands

- Matsumoto, Murakami, Phys. Rev. Lett. 106,197202 (2011)
- Matsumoto, Murakami, Phys. Rev. B 84, 184406 (2011)
- Shindou, Matsumoto, Ohe, Murakami, PRB87, 174427 (2013)
- Shindou, Ohe, Matsumoto, Murakami, Saitoh, PRB87, 174402 (2013)