New platforms for 1D topological superconductors: spin polarisation and transport properties

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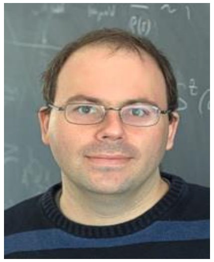
Comprendre le monde, construire l'avenir®

Collaborators:

1st part

Bernd Braunecker, St Andrews, UK



Daniel Loss, Uni Basel

S. Gangadhariah, Uni Basel-> Bhopal, India

Majorana fermions in condensed matter

Ordinary fermions $\{c_i^+, c_j\} = \delta_{ij}$

Write in terms of Majorana fermions $c_j = (\gamma_{j1} + i\gamma_{j2})/2$

$$\gamma_{i\alpha}^+ = \gamma_{i\alpha}$$

Majorana fermions are their own anti-particles

Majorana fermions can occur as collective excitations in solids with unconventional SC pairing

→ Obey non-Abelian statistics: potential platform for fault-tolerant quantum computation

Any fermionic Hamiltonian can be recast in terms of Majorana operators but very few can support solutions with **isolated localized** Majorana fermions

Where to look for Majorana fermions ?

Collective excitations in solids with unconventional (triplet) SC pairing: **why ?**

s-wave Bogoliubov qps: $\gamma_n^+ = \sum \left(u_{ni} a_{i\uparrow}^+ + v_{ni} a_{i\downarrow} \right)$

p-wave Bogoliubov qps: $\gamma_n^+ = \sum_i \left(u_{ni} a_{i\uparrow}^+ + v_{ni} a_{i\uparrow} \right)$

zero energy:
- ABS states
- localized states

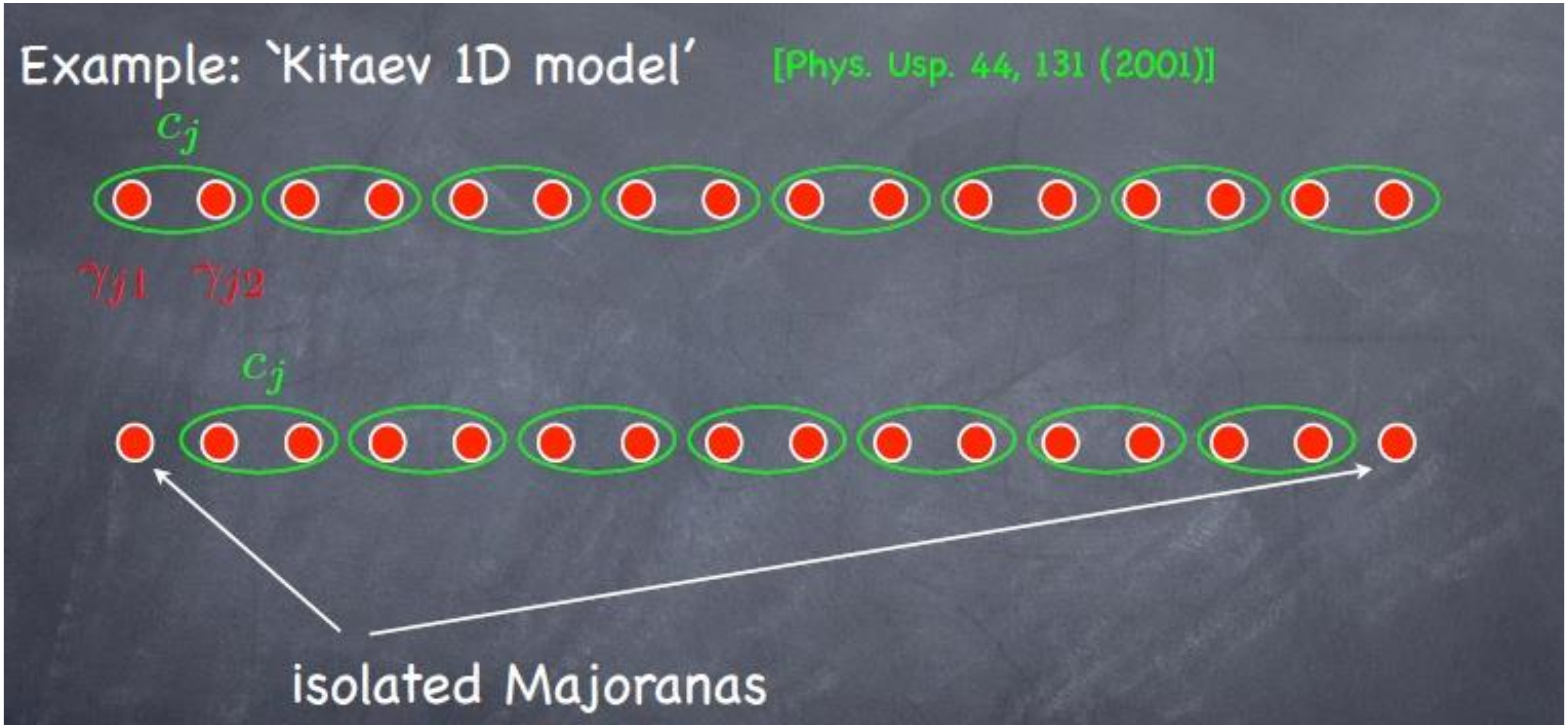
$$\gamma^+ = \sum_i u_{0i} a_{i\uparrow}^+ + u_{0i}^* a_{i\uparrow} = \gamma .$$

$\gamma = \gamma^+ \Rightarrow$ its own antiparticle

These zero-modes are Majorana fermions !

Majorana fermions: in short

Certain Hamiltonian can support solutions with **isolated** localized Majorana fermions



They also encode one complex fermion but in a way robust

to any local perturbation → Ideal for quantum bit

Proposed realizations

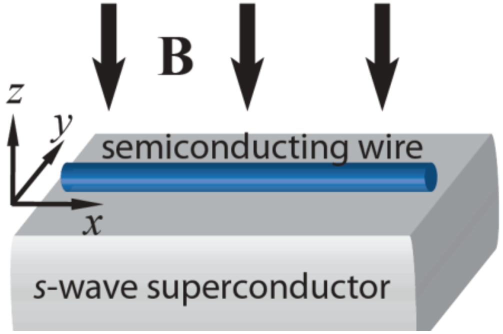
1- Moore-Read FQHE

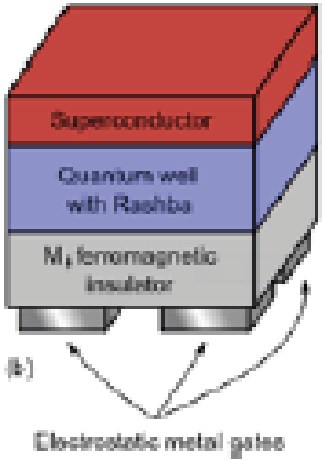
2- Spin polarized p+ip superconductor

3- Topological/superconductor interface

4- Rashba coupled semiconductors+SC+Magnetic insulators

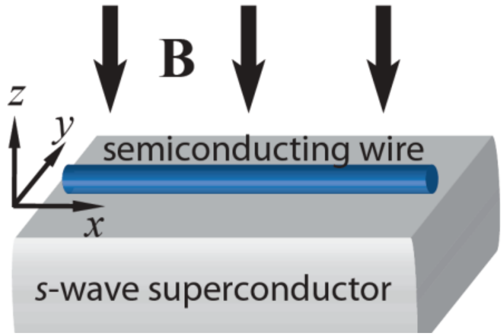
5- 1D quantum wires+SC





Proposed realizations

5- 1D quantum wires+SC and related systems



Outline

I) 1D effective topological superconductors using RKKY systems

II) A local order parameter for Majorana fermions:
spin and Majorana polarization: application to transport
in SN and SNS junctions

I) 1D effective topological superconductors using RKKY systems

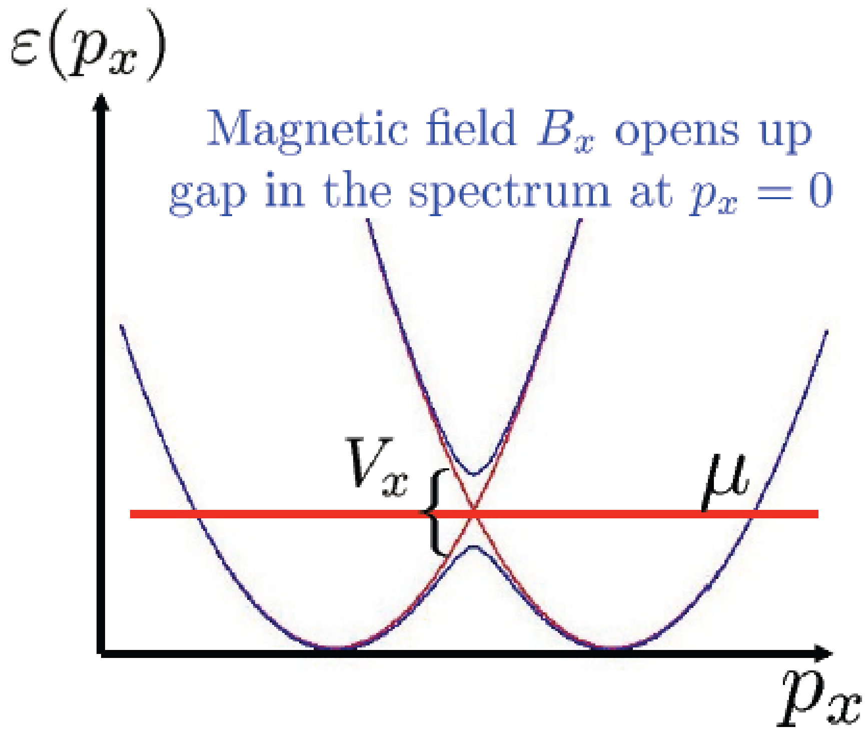
Band structure of a quantum wire

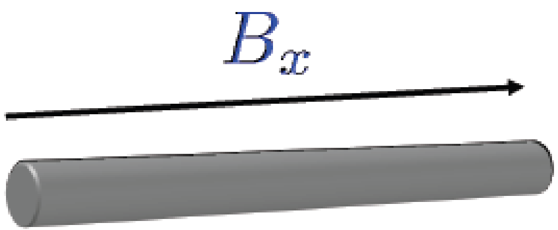
$$H_0=\int_{-L}^{L} dx \psi_\sigma^\dagger(x)\left(-\frac{\partial_x^2}{2m^*}-\mu+i\alpha\sigma_y\partial_x+V_x\sigma_x\right)_{\sigma\sigma'}\psi_{\sigma'}(x)$$

single channel nanowire

spin-orbit coupling

Zeeman splitting





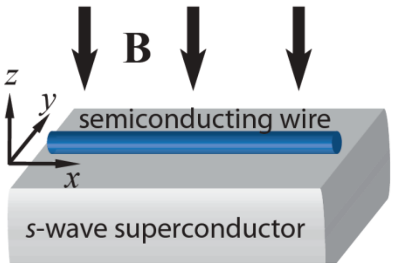
InAs, InSb nanowires

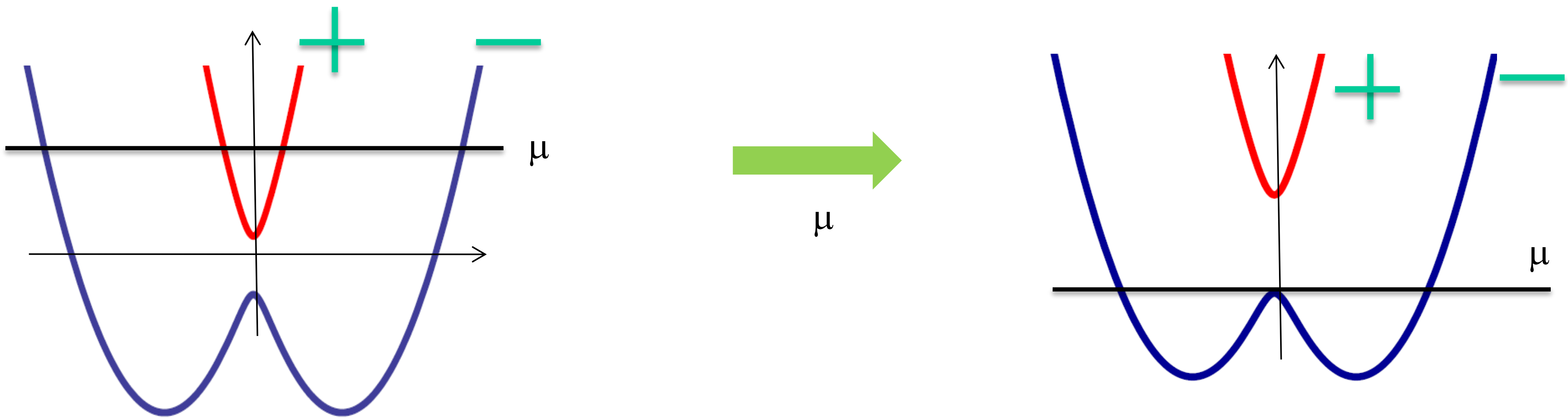
large spin-orbit ($\alpha \sim 0.1 eV\mathring{A}$)

large g-factor ($g \sim 10-50$)

good contacts with metals

Equivalent system: Rashba nanowire in B-field





Effective Pairing: + +

− −

− +

Effective Pairing: − −

TRIPLET

Sau et al., PRL 104, (2010).

Oreg et al., PRL 105, (2010).

Majorana fermions in 1D : Quantum wires

1-Dimensional Semiconducting quantum-wires

- Spin-orbit interaction : $\alpha_R k_F$
- Magnetic field : Δ_Z
- s-wave superconductivity : Δ_S

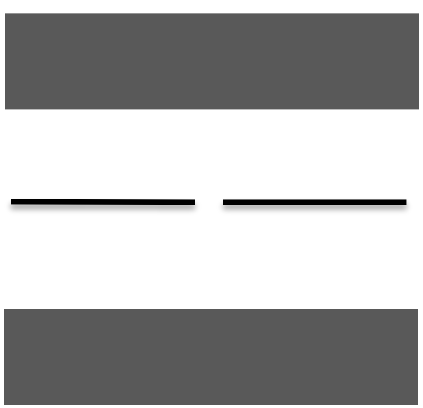
- Topological phase

$$\Delta_Z > \sqrt{\Delta_S^2 + \mu^2}$$

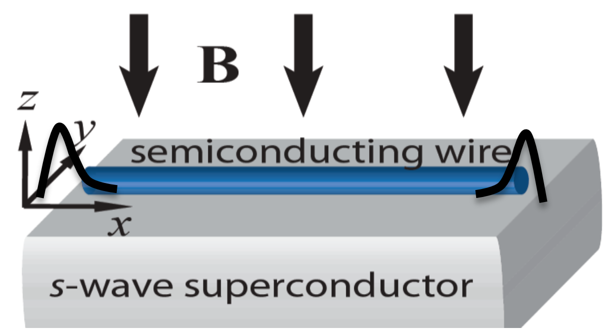
Sato et al., PRB (2009)

Sau et al., PRL 104, (2010).

Oreg et al., PRL 105, (2010).

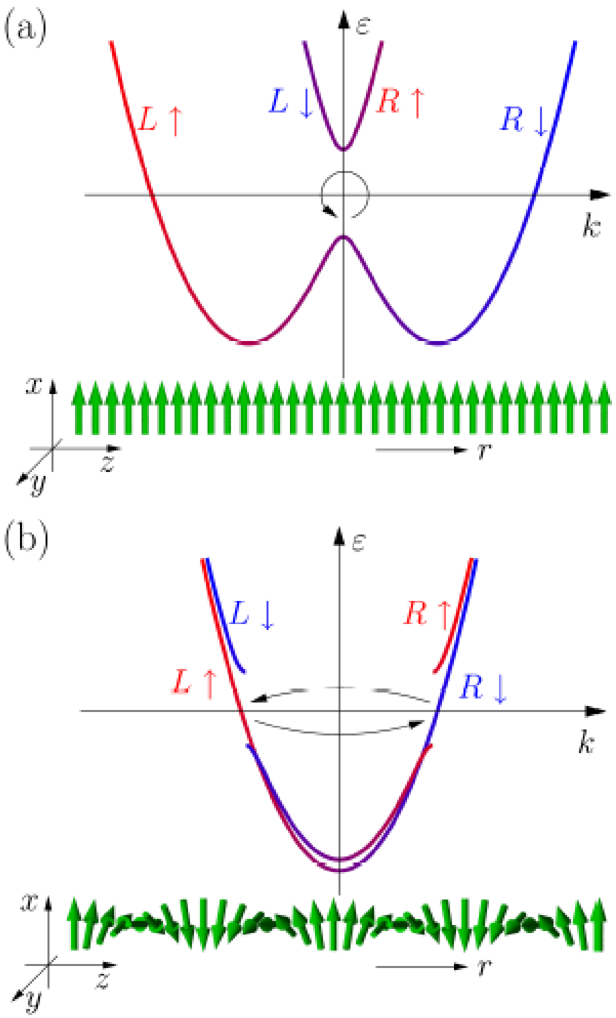


$\epsilon = 0$



Majorana: $\gamma_0 = \gamma_0^\dagger$

Can we find other equivalent realization ?



Change of frame: Gauge transform

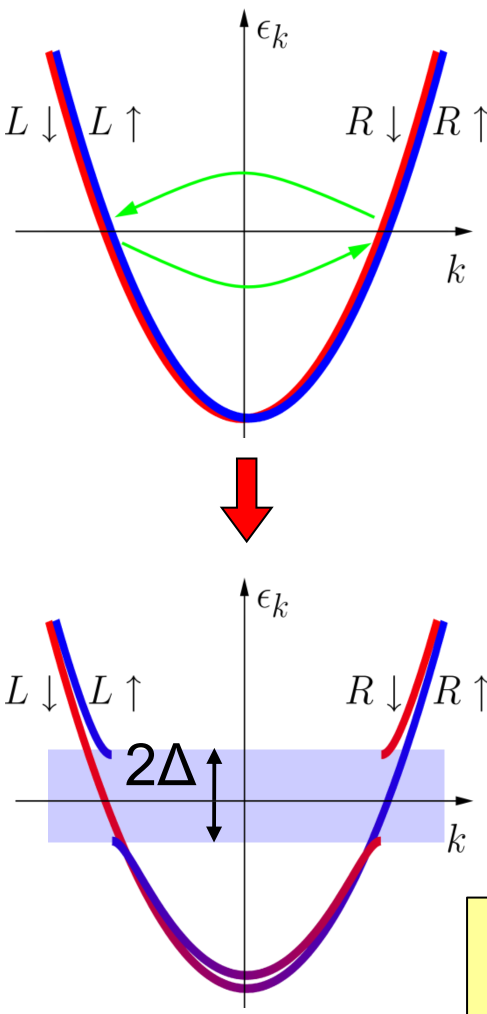
$$\psi_\sigma(r) \rightarrow e^{i\sigma k_{so} r}\psi_\sigma(r)$$

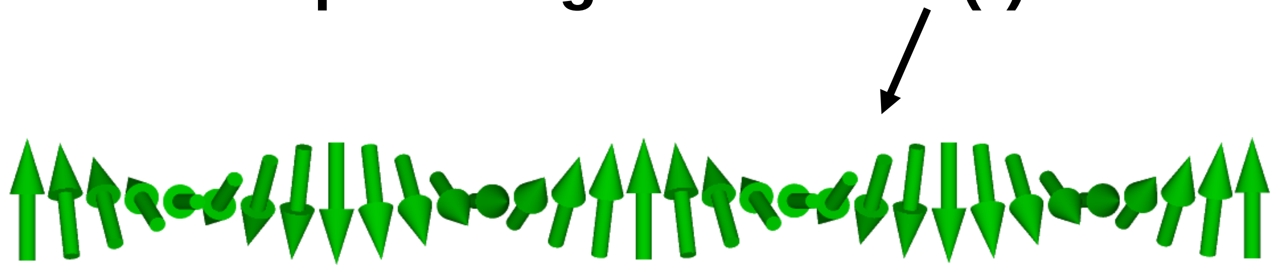
Equivalent to a spiral magnetic field

B. Braunecker, G. Japaridze, J. Klinovaja, D. Loss, PRB 82, 045127 (2010)

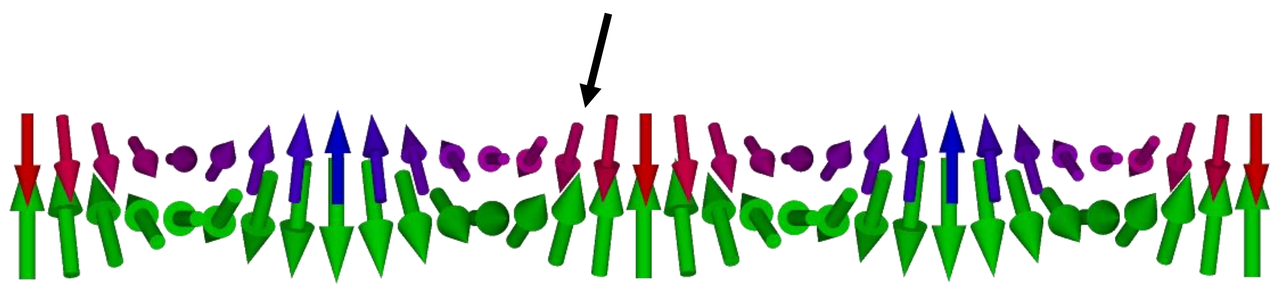
The spin selective Peirls transition

external periodic potential: spiral magnetic field B(r)





backscattering on B(r): **gap Δ** for **one-half** of the conducting modes with **opposite spin** by forming a **helical spin/charge density wave**



- **one-half** of the system becomes **insulating**
- **one-half** remains conducting and forms a **spin-filter**

B. Braunecker, G. Japaridze, J. Klinovaja, D. Loss, PRB 82, 045127 (2010)

Where to find a spiral field ?

→ Quantum wires with nuclear spins

Basic Hamiltonian

$$H = H_{\mathrm{el}} + \sum_i A\mathbf{S}_i^{el} \cdot \mathbf{I}_i^{nucl}$$

Below some critical temperature T*, electrons and nuclear spins can be tightly bound into an new ordered phase in 1D

B. Braunecker, PS, D. Loss, PRL 2009, PRB 2009

How does it work ?

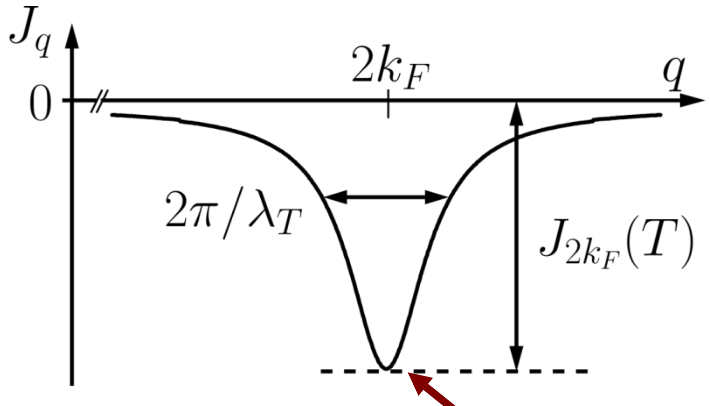
$$H = H_{\rm el} + \sum_i A\mathbf{S}_i^{el} \cdot \mathbf{I}_i^{nucl}$$

- Integrate out electrons (separation of time-scales)

$$H_{\rm nucl}^{\rm eff} = \frac{A^2}{n_n} \sum_{ij} \chi_s(r_i - r_j)\mathbf{I}_i \cdot \mathbf{I}_j$$
$$= \frac{A^2}{n_n}\frac{1}{N_s} \sum_q \chi_s(q)\mathbf{I}_{-q} \cdot \mathbf{I}_q$$

static electron spin susceptibility

RKKY interaction long ranged
Interaction J_q



minimum at $2k_F$

B. Braunecker, PS, D. Loss, PRL 2009, PRB 2009

Separation of time scales

$$H = H_{\mathrm{el}} + \sum_i A\mathbf{S}_i^{el} \cdot \mathbf{I}_i^{nucl}$$

effective electron Hamiltonian

$$H_{\mathrm{el}}^{\mathrm{eff}} = H_{\mathrm{el}} + \sum_i A\mathbf{S}_i^{el} \cdot \langle \mathbf{I}_i^{nucl} \rangle$$

effective nuclear spin Hamiltonian

$$H_{\mathrm{nucl}}^{\mathrm{eff}} = \sum_{ij} J_{ij} \mathbf{I}_i^{nucl} \cdot \mathbf{I}_j^{nucl}$$

$$= \frac{1}{N_s} \sum_q J_q \mathbf{I}_{-q}^{nucl} \cdot \mathbf{I}_q^{nucl}$$

$J_{ij} \propto A^2 \chi_s(r_i - r_j)$ $\quad J_q \propto A^2 \chi_s(q)$

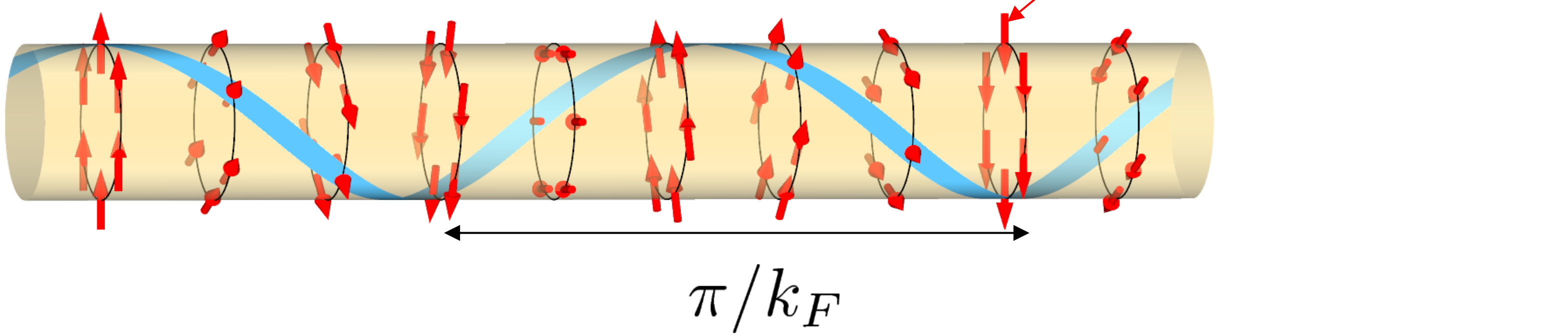
mutual dependence

self-consistency

novel physics

Effect of nuclear helical field

Helical order of nuclear spins :



Nuclear order generates a spiral Overhauser field:

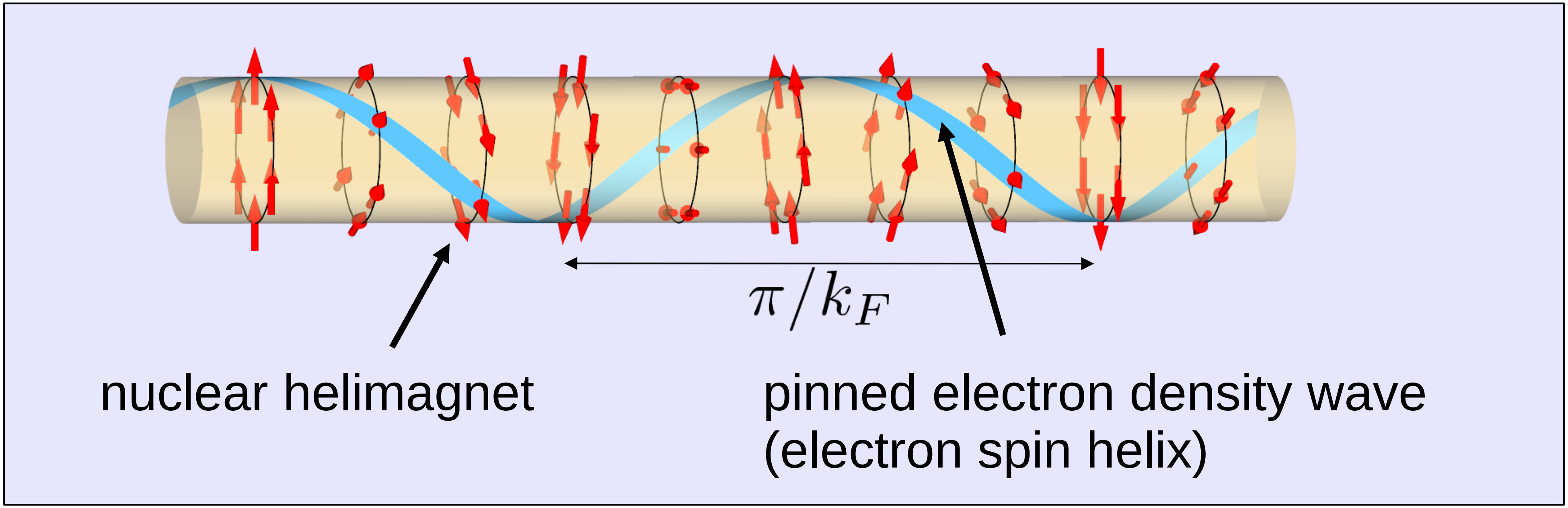
$$\langle \mathbf{I}_i \rangle = I m_{2k_F} \left[\cos(2k_F r_i)\mathbf{e}_x + \sin(2k_F r_i)\mathbf{e}_y \right]$$

Nuclear magnetization

This Overhauser field drives the spin-Peierls **instability** and gap **half of electronic degrees of freedom**

B. Braunecker, PS, D. Loss, PRL 2009, PRB 2009

Combined electron / nuclear spin order



Below *T** the ordered phases depend on each other:

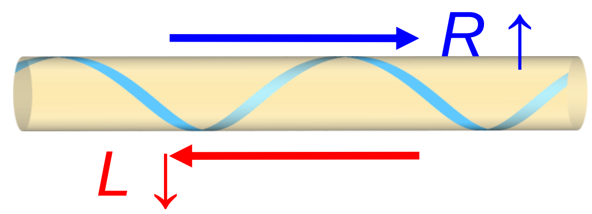
Phase of tightly bound electron & nuclear spin degrees of freedom

- independent of system size for realistic samples
- huge renormalization due to electron-electron interactions
- experimentally (challenging but) accessible temperatures

B. Braunecker, PS, D. Loss, PRL 2009, PRB 2009

Experimental consequences

1. Reduction of conductance



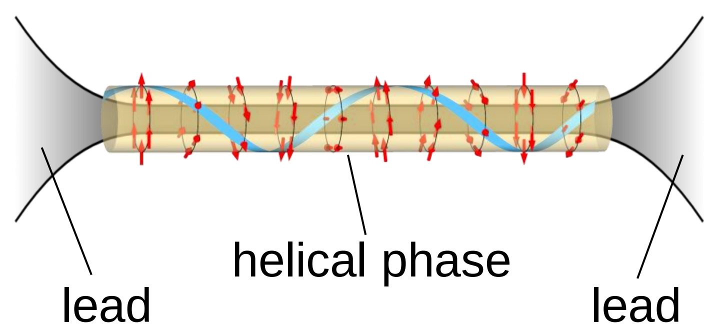
Through feedback: Pinning of ϕ_+ channels

- ➡ Blocking of ½ of the conducting channels
- ➡ **Universal reduction of conductance by factor 2**
- ➡ Remaining channels are **spin-filtered**

Example: Luttinger liquid adiabatically connected to metallic leads

Conductance: $$G = \frac{e^2}{h}\, n$$

number of conducting channels reduced to $n \rightarrow n / 2$

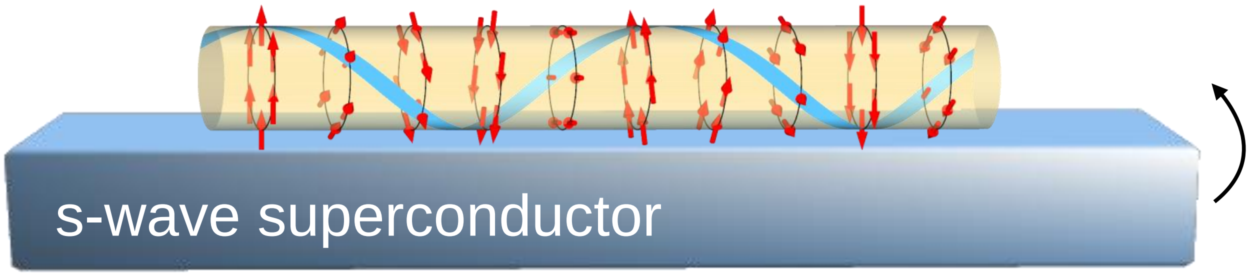


Maslov & Stone, Ponomarenko, Safi & Schulz (all in PRB 1995)

B. Braunecker, PS, D. Loss, PRL 2009, PRB 2009

In proximity of a superconductor

Superconductivity induced by proximity effect is of **p-wave type**
(projection onto spin-filtered conducting modes)



proximity induced superconductivity

for nanotubes cf. Le Hur, Vishveshvara, Bena PRB (2008)

Expect Majorana edge states ?

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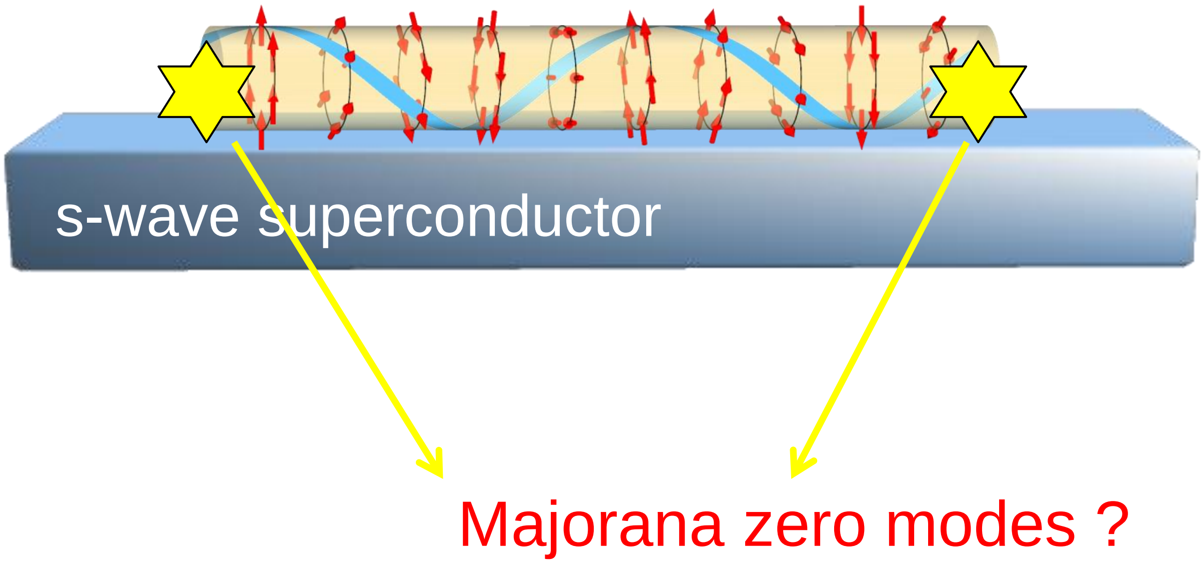
(projection onto spin-filtered conducting modes)



S Gangadharaih, B Braunecker, PS, Loss, PRL 2011

How does SC modifies the whole locking scenario ?

$$H = H_{\text{el}} + \sum_i A\mathbf{S}_i^{el} \cdot \mathbf{I}_i^{nucl}$$

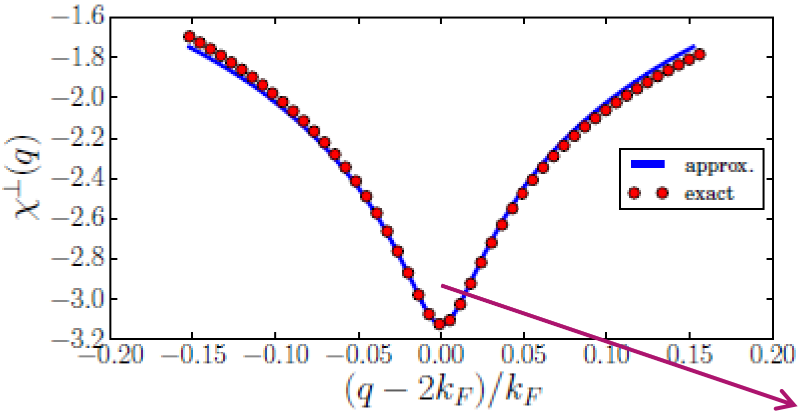
Same as before **but** now H_{el} contains the proximity induced SC gap

$$H_{el}^{\text{eff}} = H_{el} + \sum_{x_i} \mathbf{B}_{\text{eff}}(x_i) \cdot \mathbf{S}_{x_i},$$

$$H_m^{\text{eff}} = \sum_{x_i, x_j} J(x_i - x_j)\mathbf{I}_{x_i} \cdot \mathbf{I}_{x_j}.$$

Mean field decoupling as before

→ **How is the susceptibility/RKKY modified in a 1D SC?**



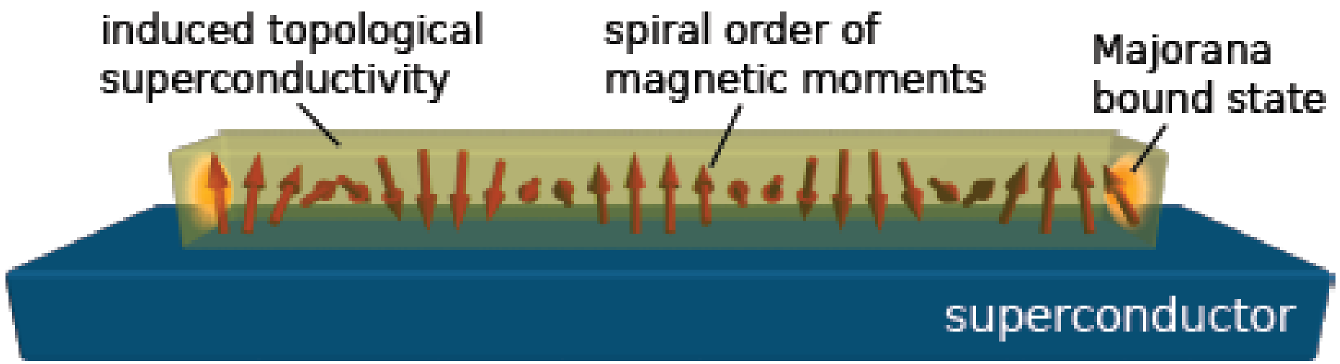
minimum at $2k_F$

→ **Therefore helical magnetic structure survives**

Majorana zero modes expected !

Braunecker, PS, PRL (2013)
Klinovaja et al., PRL (2013)
Vazifeh, Franz, PRL (2013)

Minimal ingredients and robustness



Recipe ingredients: - Magnetic moments interacting via RKKY interactions

- 1D-like electronic band

- s-wave superconductor

One can include also take electron-electron interactions into account:

This enhances T^* but reduces the proximity induced gap (compromise)

This whole locking scenario is also robust versus point-like non-magnetic disorder !

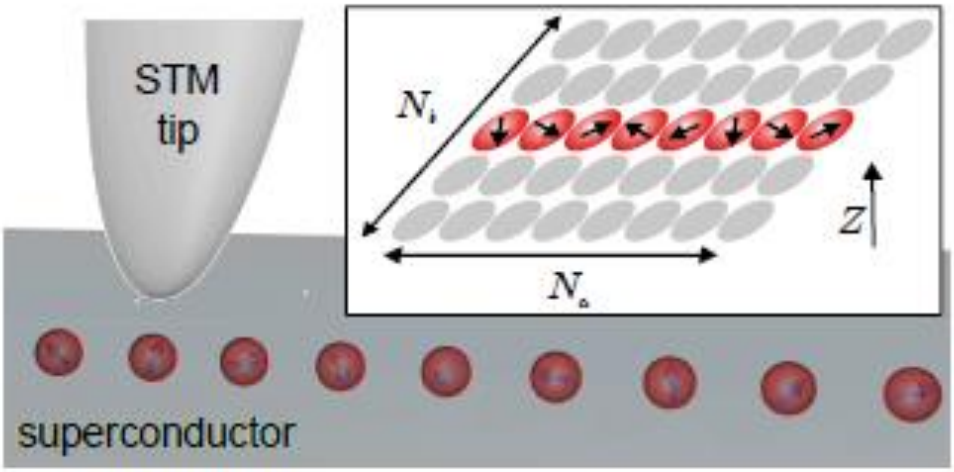
B. Braunecker, PS, PRL (2013)

A natural platform

Recipe ingredients:

- Magnetic moments interacting via RKKY interactions
- 1D-like electronic band
- s-wave superconductor

Proposal: magnetic atoms with some **pre-existing** spin texture on a SC surface



S. Nadj-Perge, I. Drozdov, A. Bernevig, A. Yazdani PRB (2013)

See also T.-P. Choy, J. M. Edge, A. R. Akhmerov, C. W. J. Beenakker, Phys. Rev. B (2011)

How about the 1D band ?

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{sd}$$

$$\mathcal{H}_0 = \sum_{k\sigma} \varepsilon_k a^\dagger_{k\sigma} a_{k\sigma} - \Delta_0 \sum_k (a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow} + a_{-k\downarrow} a_{k\uparrow}) \qquad \mathcal{H}_{sd} = -\frac{J}{2N} \sum_{kk'} a_k{}^\dagger \boldsymbol{\sigma} a_{k'} \cdot \boldsymbol{S},$$

"classical limit" $J \to 0, \quad S \to \infty, \quad JS = \text{finite}$

Shiba bound state: $$\omega = \pm \omega_B = \pm \Delta_0 \frac{1 - ((J/2) S\pi\rho)^2}{1 + ((J/2) S\pi\rho)^2} .$$

Yu Lu (1965), Shiba (1968), Rusinov (1968)

Several classical spins ➡ impurity band in the SC gap : Shiba band

Shiba (1968)

For a chain of magnetic impurities, an effective more complicated tight binding Kitaev-like Hamiltonian can be actually derived

F. Pientka, L. Glazman, F. vonOppen, PRB (2013)

Experimental predictions?

Helical magnetic phase is the ground state: self-sustained helical phase

1) Semi-conducting wires with nuclear spins

GaAs wire: $\Delta_S \sim 0.2\text{meV}$ $A_0 = 90\mu\text{eV}$ and $I = 3/2$.

$$B_{\text{eff}} \sim 0.2\text{meV} \text{ and } T^* \lesssim 1\text{mK}$$

Forget it !

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Forget it !

InAs wire: $\Delta_S \sim 0.2\mathrm{meV}$ $A_0 \sim 110\mu\mathrm{eV}$ $I = 9/2$

For $K_c = 0.8$, $\Rightarrow$ $T^* \sim 40\mathrm{mK}$

$B^*_{\mathrm{eff}} \sim 0.5\mathrm{meV} > \Delta^* \sim 0.1\mathrm{meV}$

Maybe !

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$B^*_{\text{eff}} \sim 0.5\text{meV} > \Delta^* \sim 0.1\text{meV}$

Maybe !

2) Magnetic Co atoms on a Nb surface

$\Delta_S \sim 1\text{meV}$. $I = 5/2$, $a_I \sim 3\text{Å}$.

➡ $B_{\text{eff}} \sim I A_0 > \Delta_S$ provides a constraint on A_0:

A_0 on the order of 0.5meV

Maybe !

Conclusion of this first part

- New platform for Majorana fermions: RKKY systems

  - Wires with nuclear spins (however only at very low T= 0(10mK)

  - Magnetic atoms on a SC surface

→ **Advantage: self-sustained Majorana phase**

(no fine-tuning)

B. Braunecker, PS, PRL (2013)