at KITP, UC Santa Barbara ***(October 2, 2013)***

Theory of Spin Current Generation in Spintronics

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Co-workers:

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Experiment: K. Uchida and E. Saitoh (IMR, Tohoku University)

Acknowledgements:

G. Bauer, J. Heremans, B. Hillebrands, Y. Otani,…

Contents:

1) *Introduction of spin current and spin Hall effect,*
2) *The linear response theory of spin current generation,*
3) *Spin current generation by heat, i.e., Spin Seebeck effect,*
4) *Spin Seebeck effect in Ferrimagnets and Antiferromagnets*

*Refs.: * H. Adachi , K. Uchida, E. Saitoh and S. Maekawa: Rep. Prog. Phys. 76, 036501 (2013),*

**S. Maekawa, H. Adachi, K. Uchida, J. Ieda and E. Saitoh: J. Phys. Soc. Jpn. (2013).*

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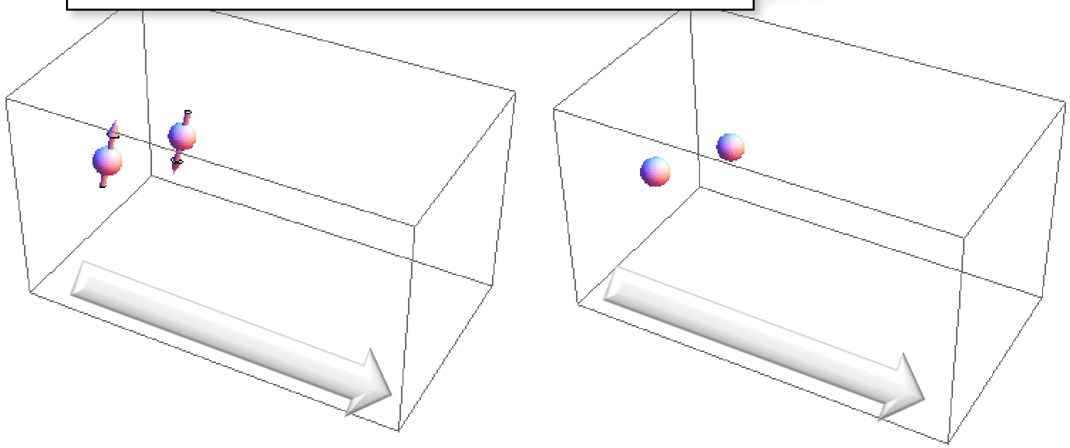
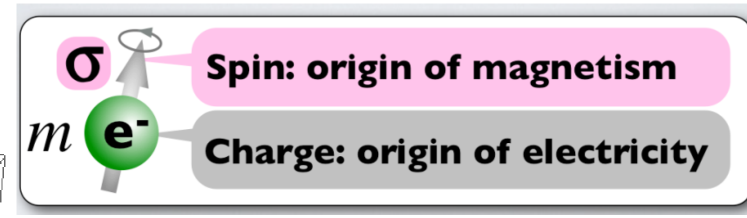
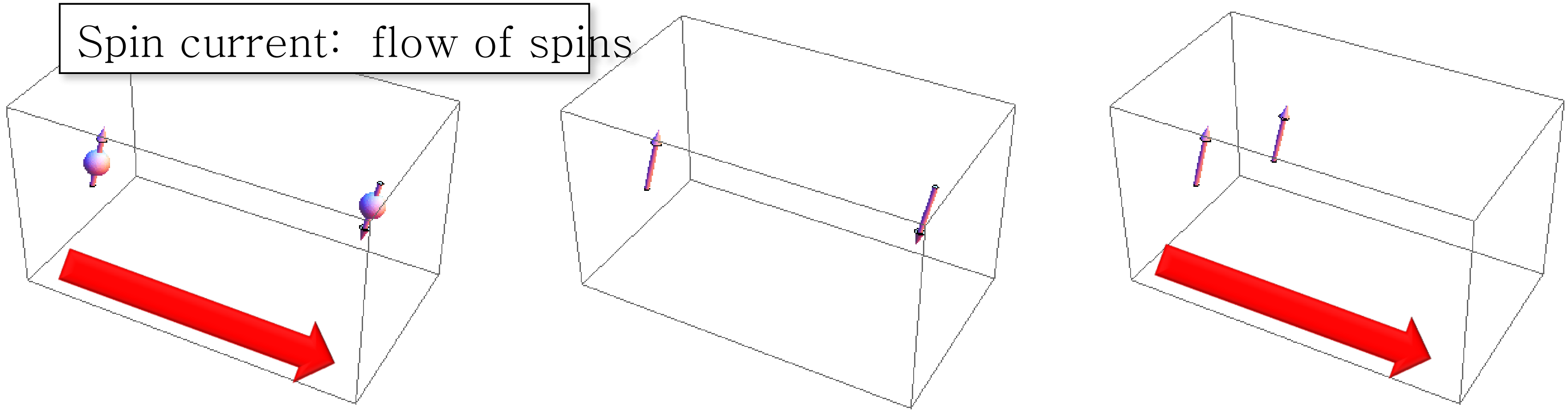
Spintronics utilizes charge and spin currents on an equal footing

Charge current: flow of charges



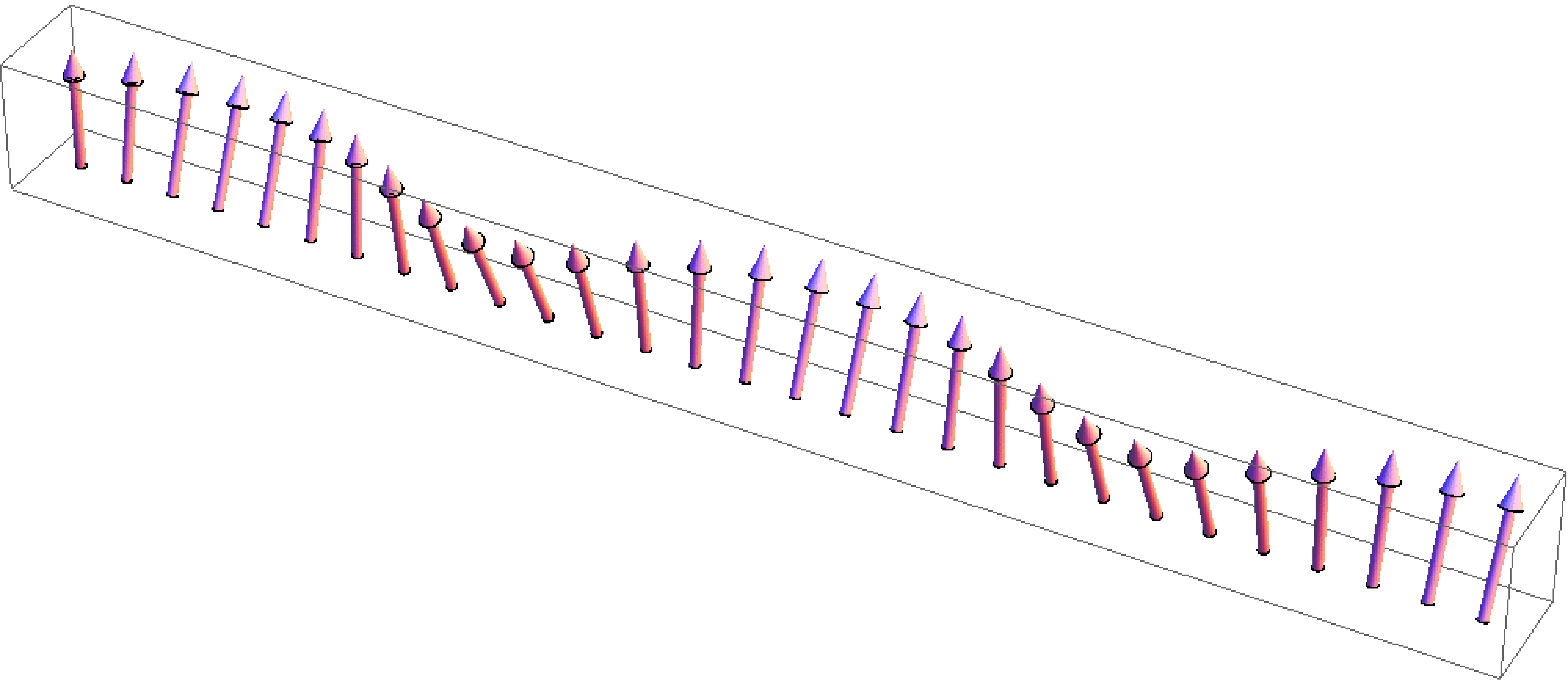


Spin current: flow of spins



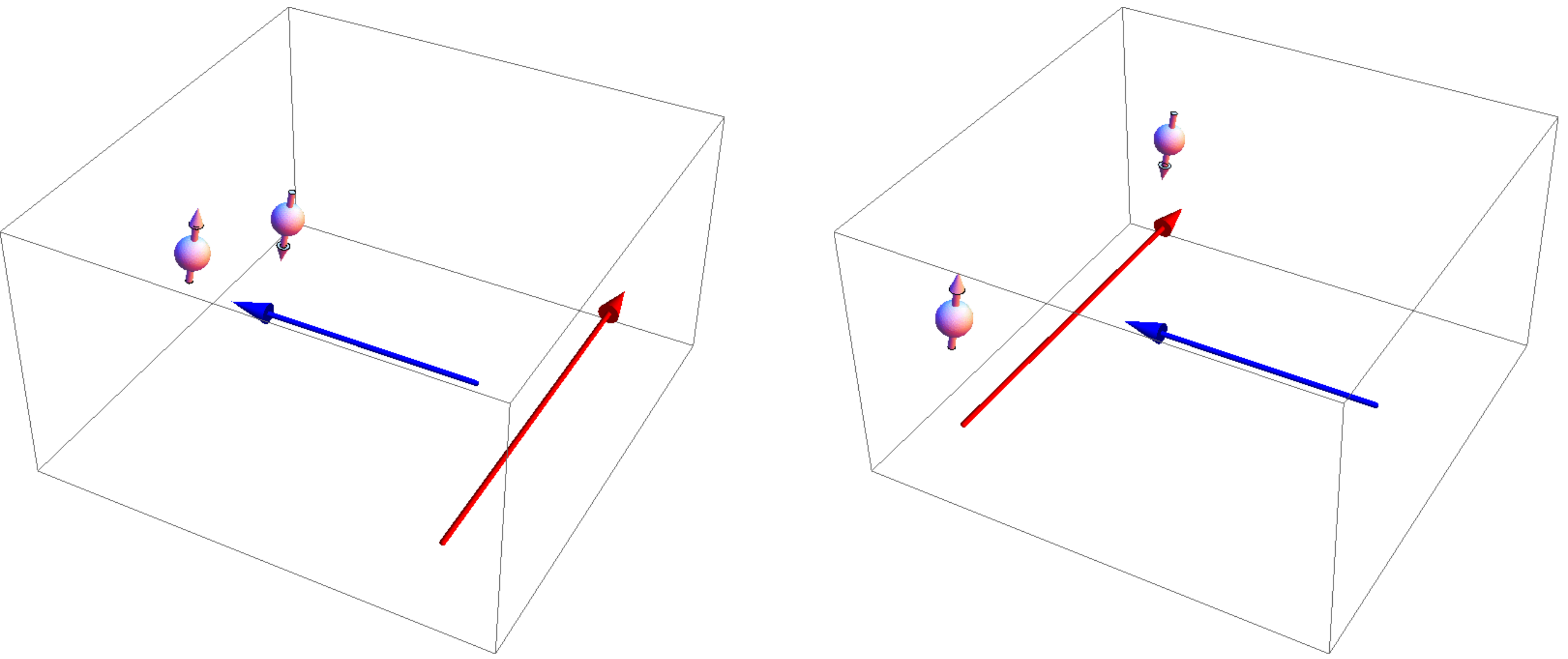
Spin current carried by conduction electrons

Spin-wave (magnon) spin current



Spin Hall Effect / Inverse SHE

Conversion between charge and spin current



Charge curent
→Spin current

Spin current
→Charge current

nature Vol 464 | 11 March 2010 | doi:10.1038/ nature08876

LETTERS

Transmission of electrical signals by spin-wave interconversion in a magnetic insulator

Y. Kajiwara[1,2], K. Harii[1], S. Takahashi[1,3], J. Ohe[1,3], K. Uchida[1], M. Mizuguchi[1], H. Umezawa[5], H. Kawai[5], K. Ando[1,2], K. Takanashi[1], S. Maekawa[1,3] & E. Saitoh[1,2,4]

Spin Hall effect

↓

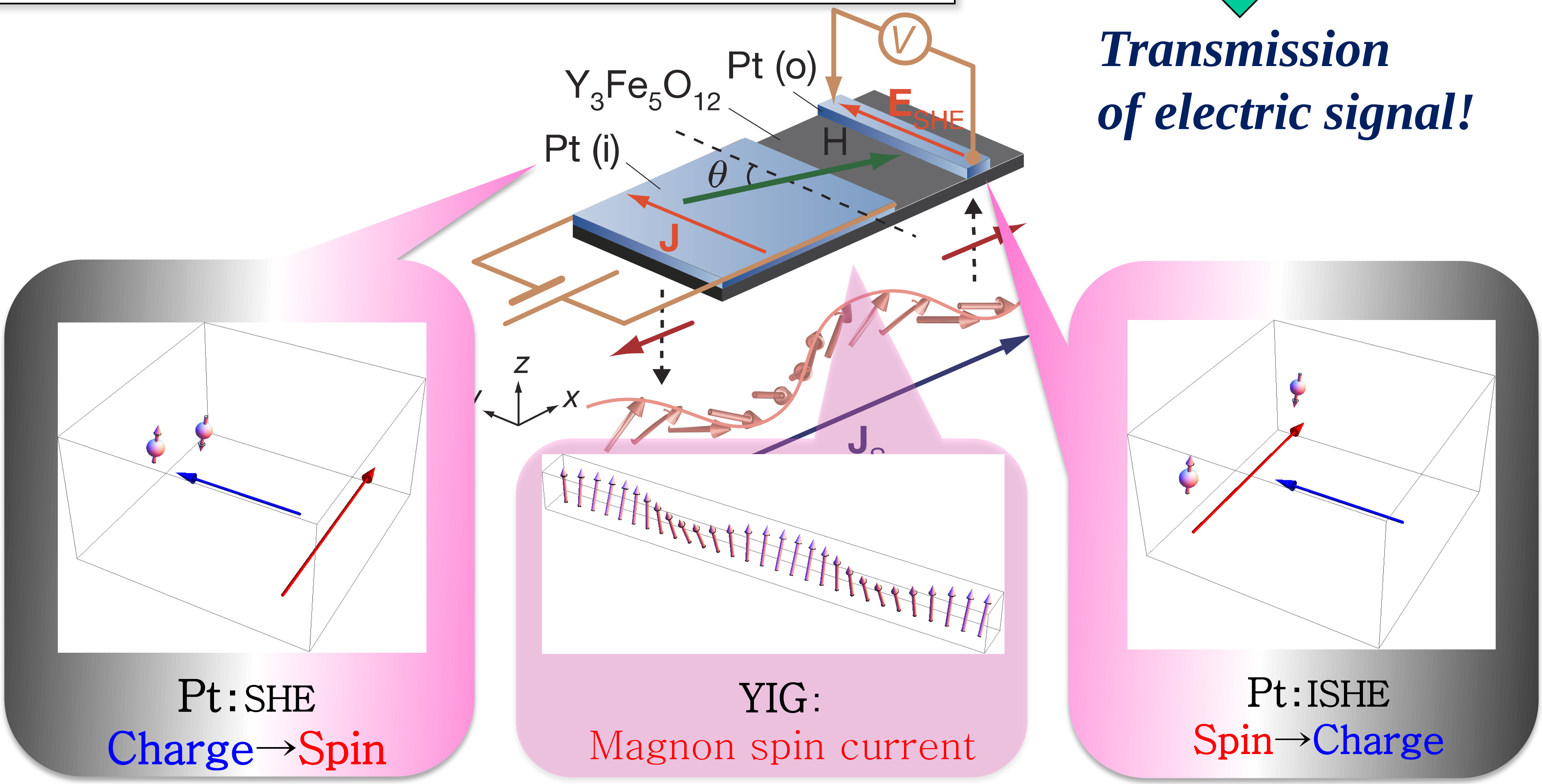
Magnon spin current

↓

Inverse spin Hall effect

⇓

Transmission of electric signal!



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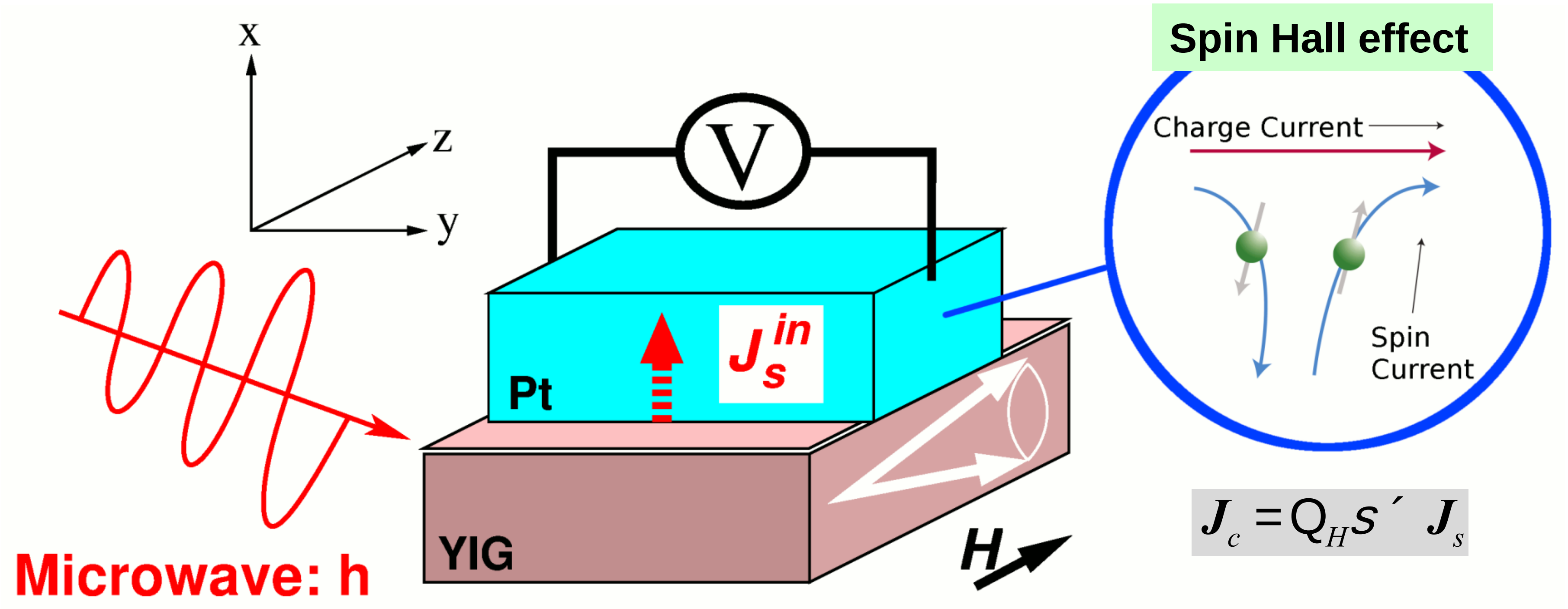
Spin current generation by FMR (Spin pumping) :

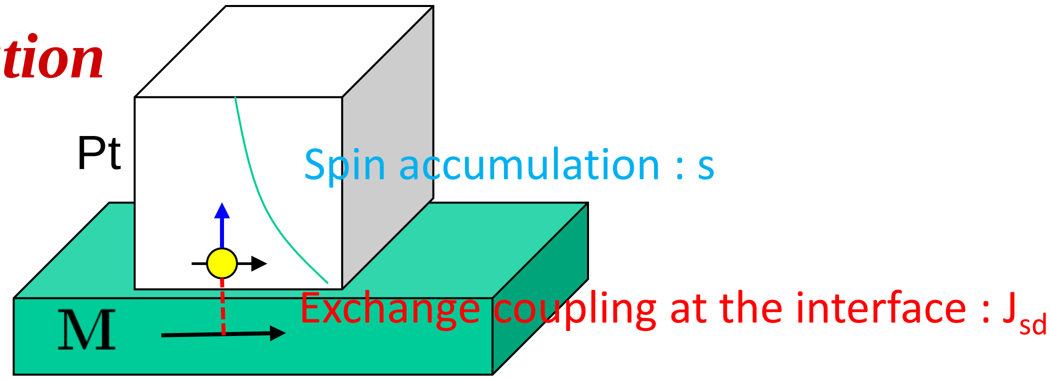


Spin Pumping; Tserkovnyak, Brataas (2002)

YIG (yttrium iron garnet): a magnetic ***INSULATOR***
→ FMR spin pumping is unaccompanied by charge transfer

FMR spin pumping is free from impedance mismatch problem
(spin injection into GaAs: Ando et al., Nature Materials 2011)

Electric and thermal generation of spin current:



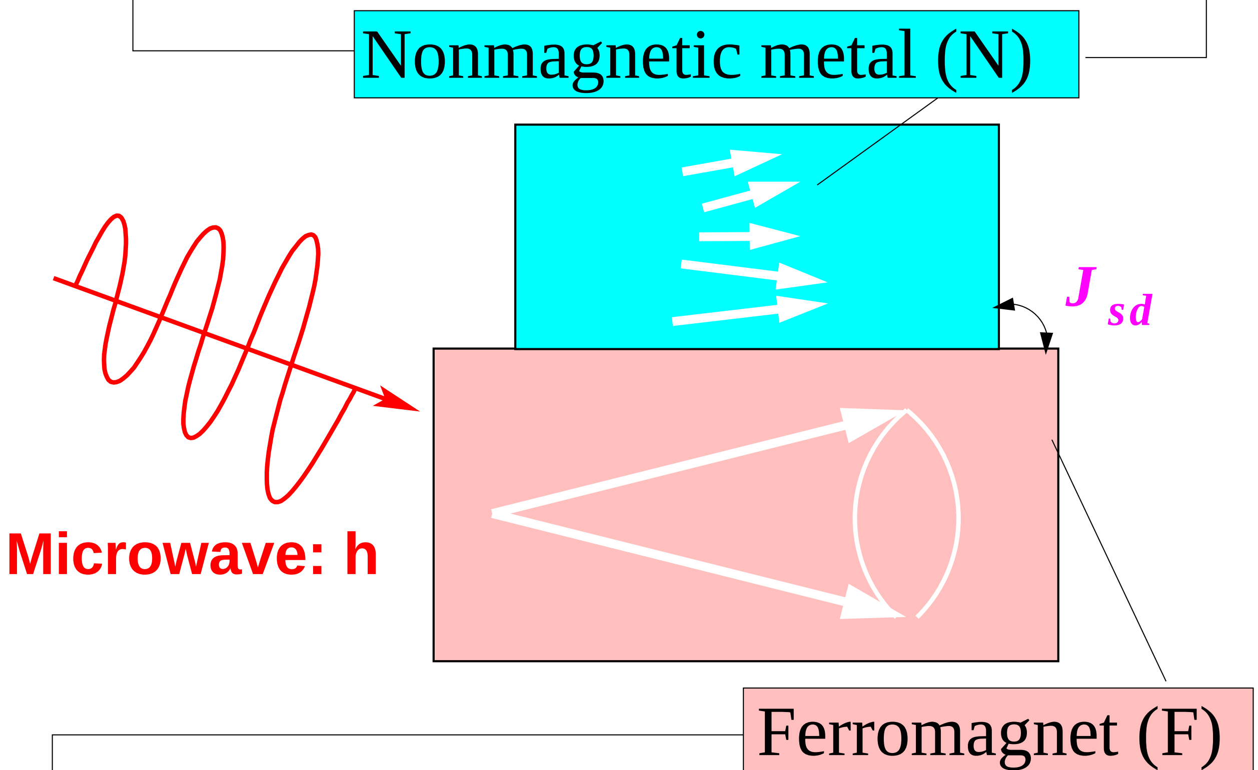
Interface exchange interaction (J_{sd})
Between normal metal (Pt) and ferromagnet

$$H_{sd} = J_{sd} \sum_{r_i \in \text{interface}} S_A(r_i) \cdot s(r_i) + J_{sd} \sum_{r_j \in \text{interface}} S_B(r_j) \cdot s(r_j)$$

Theoretical modeling of FMR spin pumping

Bloch eq. (s: spin accumulation)

$$\frac{d}{dt}\mathbf{s} = J_{sd}\,\mathbf{m}\times\mathbf{s} + (D_N\nabla^2 - \Gamma)(\mathbf{s} - s_0\mathbf{m})$$



$$\frac{d}{dt}\mathbf{m} = J_{sd}\,\mathbf{s}\times\mathbf{m} + \gamma(\mathbf{H}_0 + \mathbf{h}_1)\times\mathbf{m} + \alpha\,\mathbf{m}\times\frac{d}{dt}\mathbf{m}$$

Landau-Lifshitz-Gilbert eq. (m: localized moment)

Linear response calculation

Bloch eq.: $\partial_t \boldsymbol{s} = J_{sd}\boldsymbol{m}\times\boldsymbol{s} + (D_N\nabla^2 - \Gamma)(\boldsymbol{s} - s_0\boldsymbol{m})$ $\quad (s_0 = \chi_N S_0 J_{sd})$

LLG eq.: $\partial_t \boldsymbol{m} = J_{sd}\boldsymbol{s}\times\boldsymbol{m} + \gamma(\boldsymbol{H}_0 + \boldsymbol{h}_1)\times\boldsymbol{m} + \alpha\boldsymbol{m}\times\partial_t\boldsymbol{m}$

1) Define the spin current injected into N by $J_s^{in} = (1/A_{contact})\langle ds^z/dt\rangle$.

$$J_s^{in} \equiv \langle \partial_t s^z \rangle = (J_{sd}/A_{contact})\,\mathrm{Im}\int d\omega \langle s^+(\omega) m^-(-\omega)\rangle$$

2) Linearize above two equations with respect to s^x, s^y, m^x, m^y.

$$s^+(\omega) = \Gamma s_0 \frac{\gamma h_1^+(\omega)}{(\omega_0 + \omega - i\alpha\omega)(-i\omega + \Gamma)}$$

$$m^-(\omega) = \frac{\gamma h_1^-(\omega)}{\omega_0 - \omega - i\alpha\omega}$$

$$\chi_N(\omega) = \frac{\chi_N}{1 - i\omega/\Gamma}$$

$$X_F(\omega) = \frac{1}{\omega - \omega_0 + i\alpha\omega}$$

$(s^\pm \equiv s^x \pm i s^y)$

$(m^\pm \equiv m^x \pm i m^y)$

$(\omega_0 = \gamma H_0)$

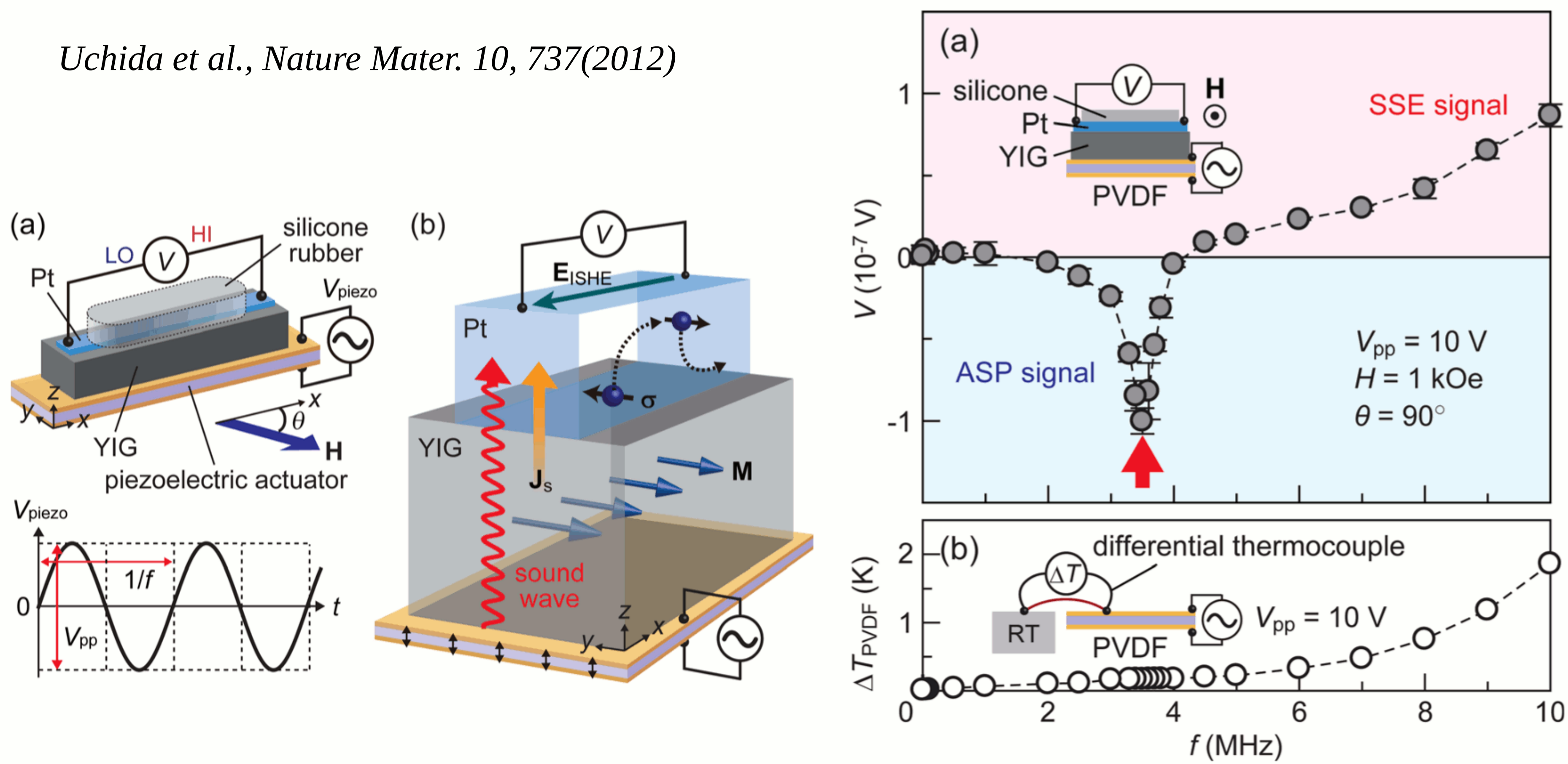
3) Substitute 2) into 1) and obtain the following result:

$$J_s^{in} = -\frac{J_{sd}^2}{A_{contact}}\int d\omega\,\mathrm{Im}\,\chi_N(\omega)\,|X_F(\omega)|^2 \langle \gamma h_1^+(\omega)\gamma h_1^-(-\omega)\rangle$$

This is the general expression valid for any types of spin pumping!

Acoustic spin pumping (experiment)

Uchida et al., Nature Mater. 10, 737(2012)



f~3.5MHz : *magnons are OFF resonance with phonons*

Volume (exchange) magnetostriction is important

$$E_{m\text{-}p} = |\nabla J_{ex}|(\nabla\cdot\boldsymbol{u})(\boldsymbol{m}\cdot\nabla^2\boldsymbol{m}) \sim g_{m\text{-}p}(\nabla\cdot\boldsymbol{u})(m^{+}\nabla^2 m^{-} + c.c.)$$

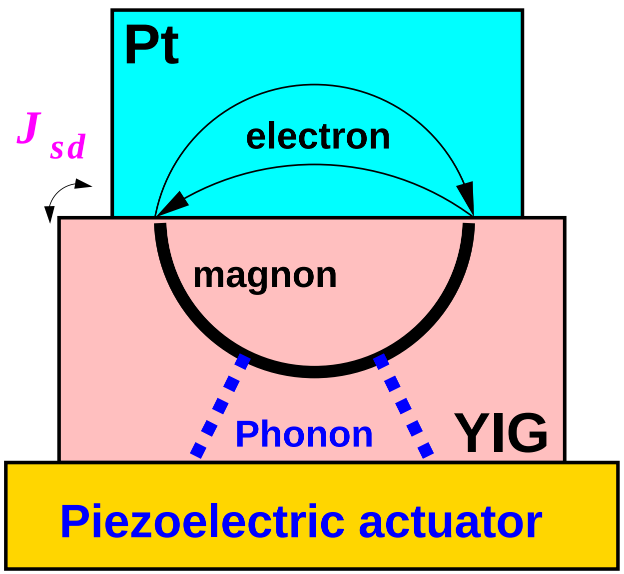
Interpretation

Start from general expression of spin pumping (derived at the beginning)

$$J_s^{in} = -(J_{sd}^2 / A_{contact}) \int d\omega \, \mathrm{Im}\, \chi_N(\omega) |X_F(\omega)|^2 \langle \delta h_1^+(\omega) \delta h_1^-(-\omega) \rangle$$

$$E_{m-p} = g_{m-p}(\nabla \cdot \boldsymbol{u})(\boldsymbol{m} \cdot \nabla^2 \boldsymbol{m}) \quad \Rightarrow \boldsymbol{h}_1 = -\delta E_{m-p} / \delta \boldsymbol{m} = \delta g_{m-p}(\nabla \cdot \boldsymbol{u}) \nabla^2 \boldsymbol{m}$$

Nonlinear with respect to ***m****, and need to employ method of many-body theory*



For details, see Adachi et al., Rep. Prog. Phys. (2013)

$$J_s^{in} = -(J_{sd}^2 / A_{contact}) B (g_{m-p} K_0 u_{K_0})^2$$

$$B = \int d\omega \, \mathrm{Im}\, \chi_N(\omega) \, \mathrm{Im}\, X_F(\omega) |X_F(\omega)|^2 \times \left[\coth\left(\frac{\omega - \delta\omega}{2T}\right) - \coth\left(\frac{\omega}{2T}\right) \right]$$

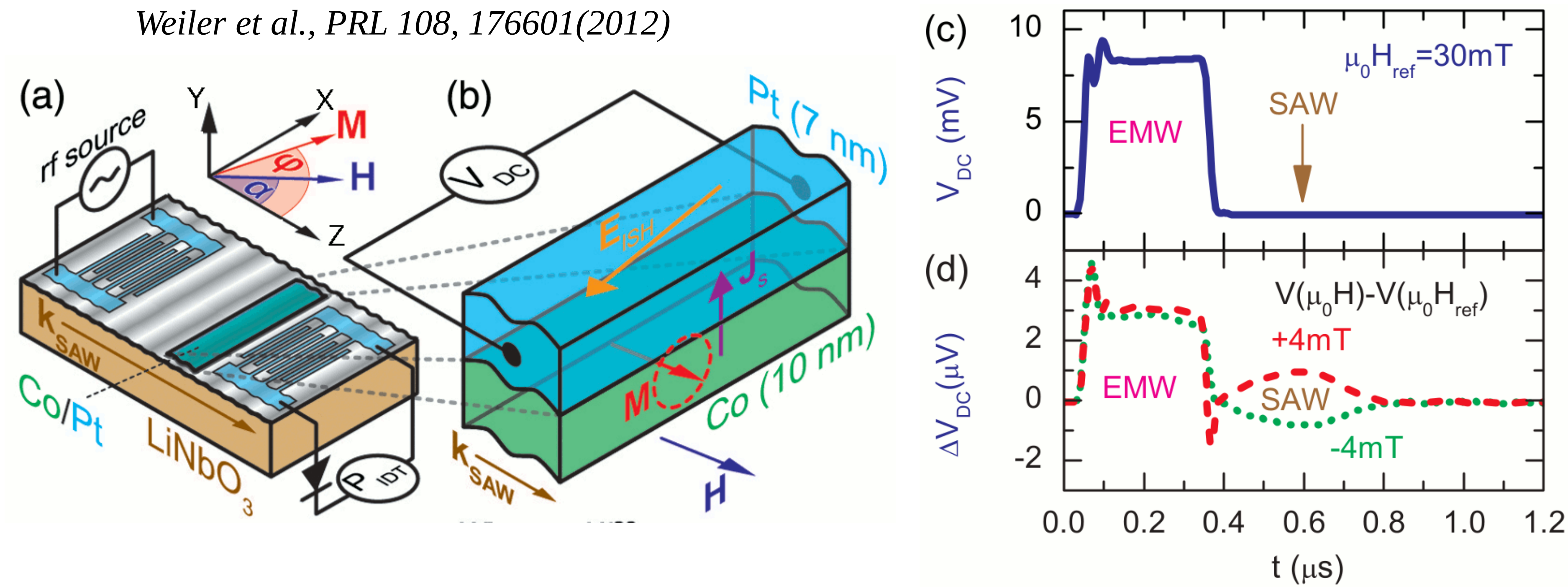
Because magnons are OFF resonance with phonons,
any phonon frequencies are allowed to excite magnons.

Analogue to the phonon-drag spin Seebeck effect

Another acoustic spin pumping
-- "$m^{x,y}$-linear" coupling --

Acoustic spin pumping ($m^{x,y}$-linear coupling)

Weiler et al., PRL 108, 176601(2012)



Surface acoustic wave (phonon) frequency~1.5GHz

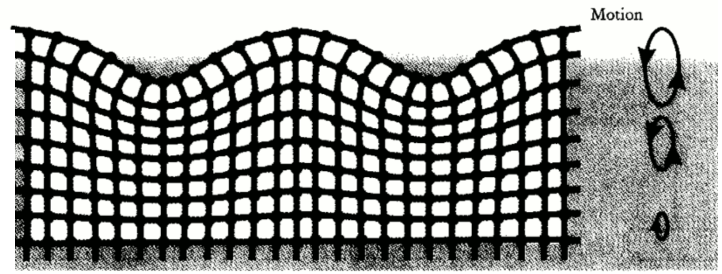
Only when magnons are in resonance with phonons, the spin injection signal by SAW is observed.

Interpretation

P. Hess, Phys. Today (2002)

SURFACE ACOUSTIC ATTENUATION DUE TO SURFACE SPIN WAVE IN FERRO- AND ANTIFERROMAGNETS

S. Maekawa
IBM Research Center, Yorktown Heights, N.Y. 10598



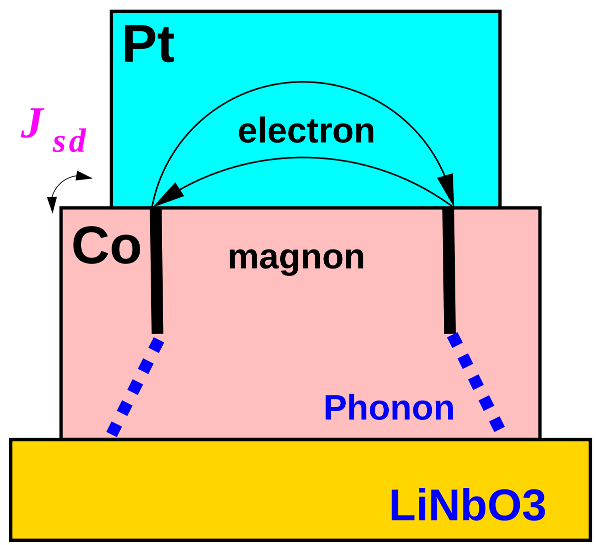
ω_{XZ}: *(local) rotation frequency*

$$-D[S_{iz}^2 + \omega_{xz}(S_{iz}S_{ix} + S_{ix}S_{iz})].$$

<-- Maekawa (1976)

M^X-linear coupling, producing $h_{rf} = -dE_{mag\text{-}ph}/dm = -D\omega_{XZ}m^Z$

Complete analogue to FMR spin pumping with a substitution $h_{rf}=-D\omega_{XZ}m^Z$



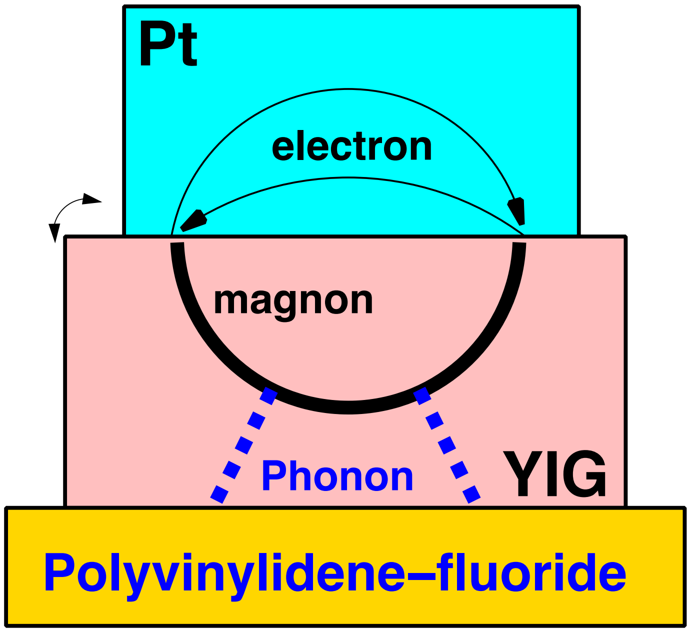
We can employ the result for FMR spin pumping

$$\backslash \quad J_s^{in} = -\frac{J_{sd}^2 \chi_N / 4G}{A_{contact}} \frac{\omega_{rf}(\gamma h_{rf})^2}{(\omega_0 - \omega_{rf})^2 + (\alpha\omega_{rf})^2}$$

$(\omega_0=\gamma H)$

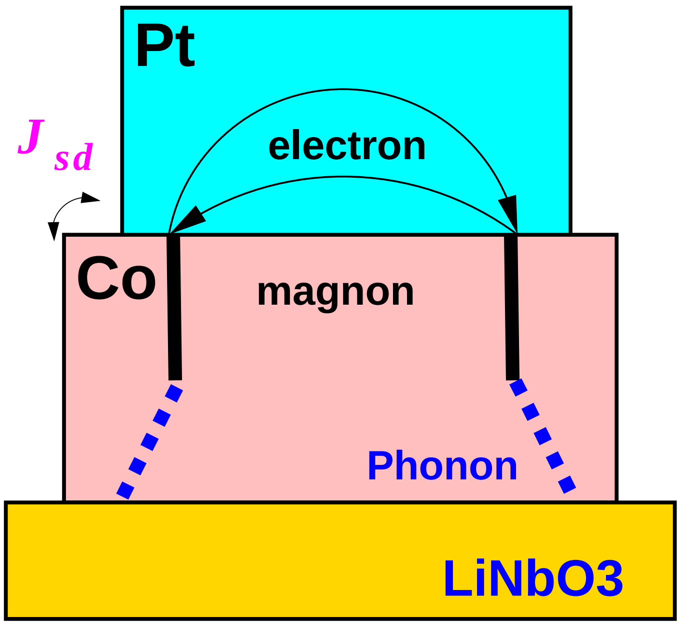
Two types of acoustic spin pumping

Phonons & magnons OFF resonance
Phonon freq. ~ MHz
-- volume (exchange) magnetostriction --



Analogue to phonon-drag SSE

Phonons & magnons IN resonance
Phonon freq. ~ GHz
-- single-ion (spin-orbit) magnetostriction --



Analogue to FMR spin pumping

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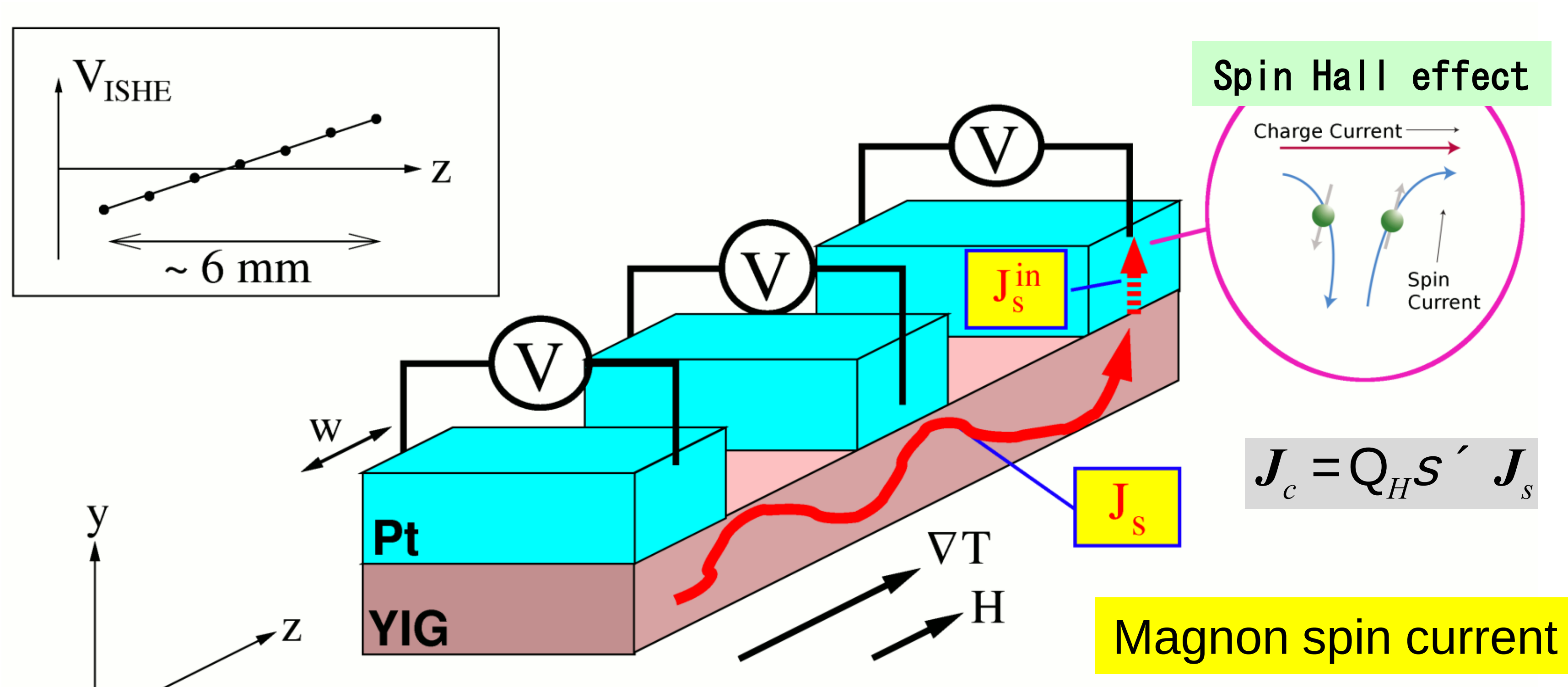
Spin Seebeck effect (SSE):

Universal phenomenon of ferromagnets

- ***Metal*** (*Ni, Fe, Ni-Fe alloy*; Uchida et al. 2008)
- ***Semiconductor*** (*GaMnAs*; Jaworski et al. 2010)
- ***Insulator*** (*Yttrium Iron Garnet, **Ferrite***; Uchida et al. 2010)



Transverse SSE device

Model for spin injection by thermal magnons

H. Adachi et al.: Phys.Rev. B83, 094410 (2011)).

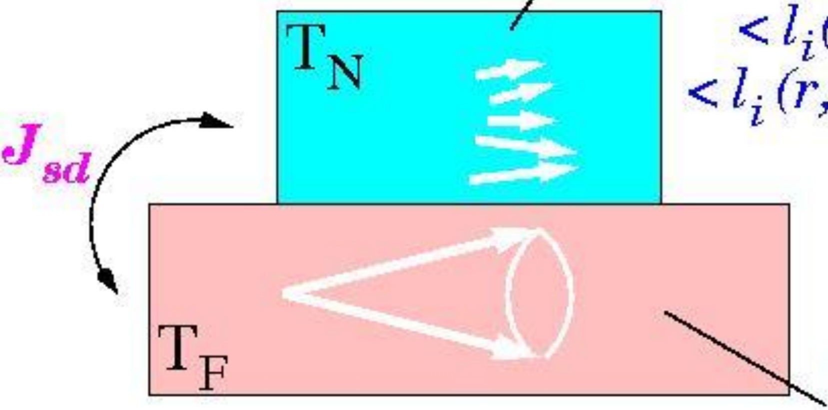
Nonmagnetic metal (N)

$$\frac{d}{dt}\mathbf{s} = \boldsymbol{J}_{sd}\,\mathbf{m}\times\mathbf{s} + (D_N\nabla^2 - \Gamma)(\mathbf{s} - s_0\mathbf{m}) + \boldsymbol{l}$$

$$< l_i(r) > = 0$$

$$< l_i(r,t)\, l_j(r',t') > = 2k_B T_N(r)\,\chi_N\,\Gamma\,\delta_{ij}\,\delta(r-r')\,\delta(t-t')$$

(Fluctuation-Dissipation theorem)



Ferromagnet (F)

$$\frac{d}{dt}\mathbf{m} = -\boldsymbol{J}_{sd}\,\mathbf{s}\times\mathbf{m} + \gamma(H_{eff} + \boldsymbol{h})\times\mathbf{m} + \alpha\,\mathbf{m}\times\frac{d}{dt}\mathbf{m}$$

$$< h_i(r) > = 0$$

$$< h_i(r,t)\, h_j(r',t') > = \frac{2k_B T_F(r)\,\alpha}{\gamma M_s}\,\delta_{ij}\,\delta(r-r')\,\delta(t-t')$$

(c.f., J. Xiao et al.: Phys. Rev. B81, 214418 (2010))

(Local) spin injection by thermal magnons

(H. Adachi et al.: Phys.Rev. B83, 094410 (2011)).

Spin diffusion eq.: $\partial_t \boldsymbol{s} = J_{sd}\boldsymbol{m}\times\boldsymbol{s} + (D_N\nabla^2 - \Gamma)(\boldsymbol{s} - s_0\boldsymbol{m}) + \boldsymbol{l}$ $\quad (s_0 = \chi_N S_0 J_{sd})$

LLG eq.: $\partial_t \boldsymbol{m} = J_{sd}\boldsymbol{s}\times\boldsymbol{m} + \gamma(\boldsymbol{H}_{eff} + \boldsymbol{h})\times\boldsymbol{m} + \alpha\boldsymbol{m}\times\partial_t\boldsymbol{m}$

Injected spin current: $J_s^{\ in} \equiv \langle \partial_t s^z \rangle = J_{sd}\,\mathrm{Im}\int d\omega \langle s^+(\omega)m^-(-\omega)\rangle$

$$s^+(\omega) = \Gamma s_0 X_F^{\ *}(-\omega)\gamma h^+(\omega) + \chi_N^{\ *}(-\omega)l^+(\omega)$$

$$m^-(\omega) = X_F(\omega)\gamma h^-(\omega) + J_{sd}X_F(\omega)\chi_N(\omega)l^-(\omega)$$

$$(a^\pm \equiv a^x \pm i a^y)$$

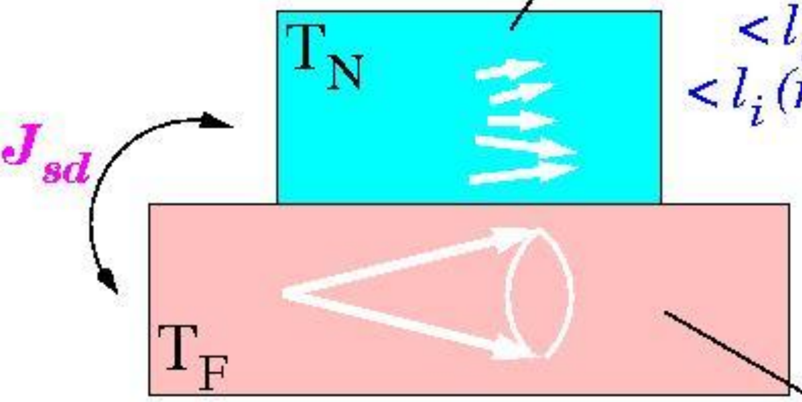
$$J_s^{\ in} = J_{sd}\int d\omega \frac{1}{\omega}\mathrm{Im}\,\chi_N(\omega)\,\mathrm{Im}\,X_F(\omega)\left[\Gamma s_0 \langle \gamma h^+(\omega)\gamma h^-(-\omega)\rangle - \alpha J_{sd}\langle l^+(\omega)l^-(-\omega)\rangle\right]$$

↑ $J_s^{\ pump}$ ↑ $J_s^{\ back}$

$$\therefore J_s^{\ in} = J_s^{\ pump} - J_s^{\ back}$$

Model for spin injection by thermal magnons

H. Adachi et al.: Phys.Rev. B83, 094410 (2011)).



Nonmagnetic metal (N)

$$\frac{d}{dt}\mathbf{s} = \boldsymbol{J}_{sd}\,\mathbf{m}\times\mathbf{s} + (D_N\nabla^2 - \Gamma)(\mathbf{s} - s_0\mathbf{m}) + \boldsymbol{l}$$

$$\langle l_i(r)\rangle = 0$$

$$\langle l_i(r,t)\, l_j(r',t')\rangle = 2k_B T_N(r)\,\chi_N \Gamma\, \delta_{ij}\,\delta(r-r')\,\delta(t-t')$$

(Fluctuation-Dissipation theorem)

Ferromagnet (F)

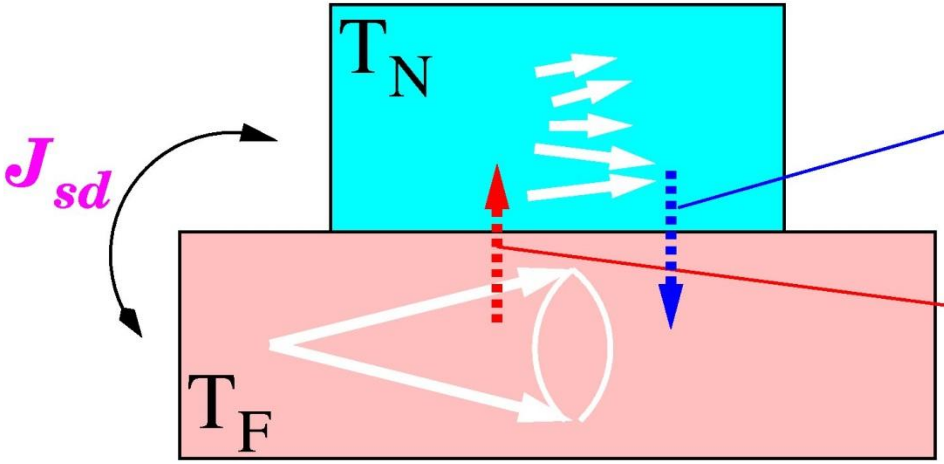
$$\frac{d}{dt}\mathbf{m} = -\boldsymbol{J}_{sd}\,\mathbf{s}\times\mathbf{m} + \gamma(H_{eff} + \boldsymbol{h})\times\mathbf{m} + \alpha\,\mathbf{m}\times\frac{d}{dt}\mathbf{m}$$

$$\langle h_i(r)\rangle = 0$$

$$\langle h_i(r,t)\, h_j(r',t')\rangle = \frac{2k_B T_F(r)\,\alpha}{\gamma M_s}\,\delta_{ij}\,\delta(r-r')\,\delta(t-t')$$

(c.f., J. Xiao et al.: Phys. Rev. B81, 214418 (2010))

Summary: local spin injection by magnons



$J_s^{back} = -A \cdot T_N$

$J_s^{pump} = A \cdot T_F$

Local non-equilibrium

$$\therefore J_s^{in} = A(T_F - T_N)$$

$$\left(A \propto J_{sd}^{\ 2} \int d\omega \frac{1}{\omega} \mathrm{Im}\,\chi_N(\omega) \mathrm{Im}\,X_F(\omega) \right)$$

$$J_s^{in} \equiv \langle \partial_t s^z \rangle = J_{sd}\, \mathrm{Im} \int d\omega \langle s^+(\omega) m^-(-\omega) \rangle$$

⟶ *(Field theoretical calculation of Green's function)*

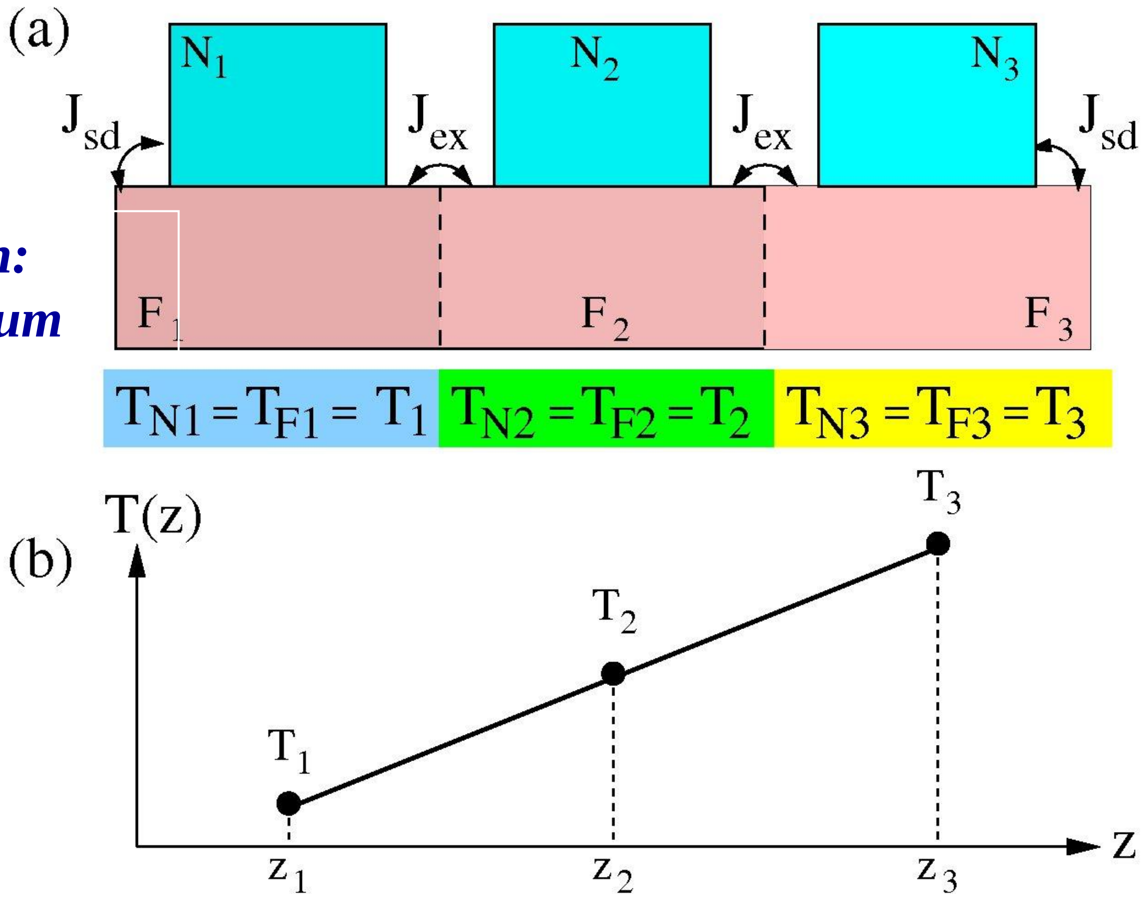
To get the non-equiliburium condition, we need heat flow!

No temp-diff. between Pt and YIG in the experiment

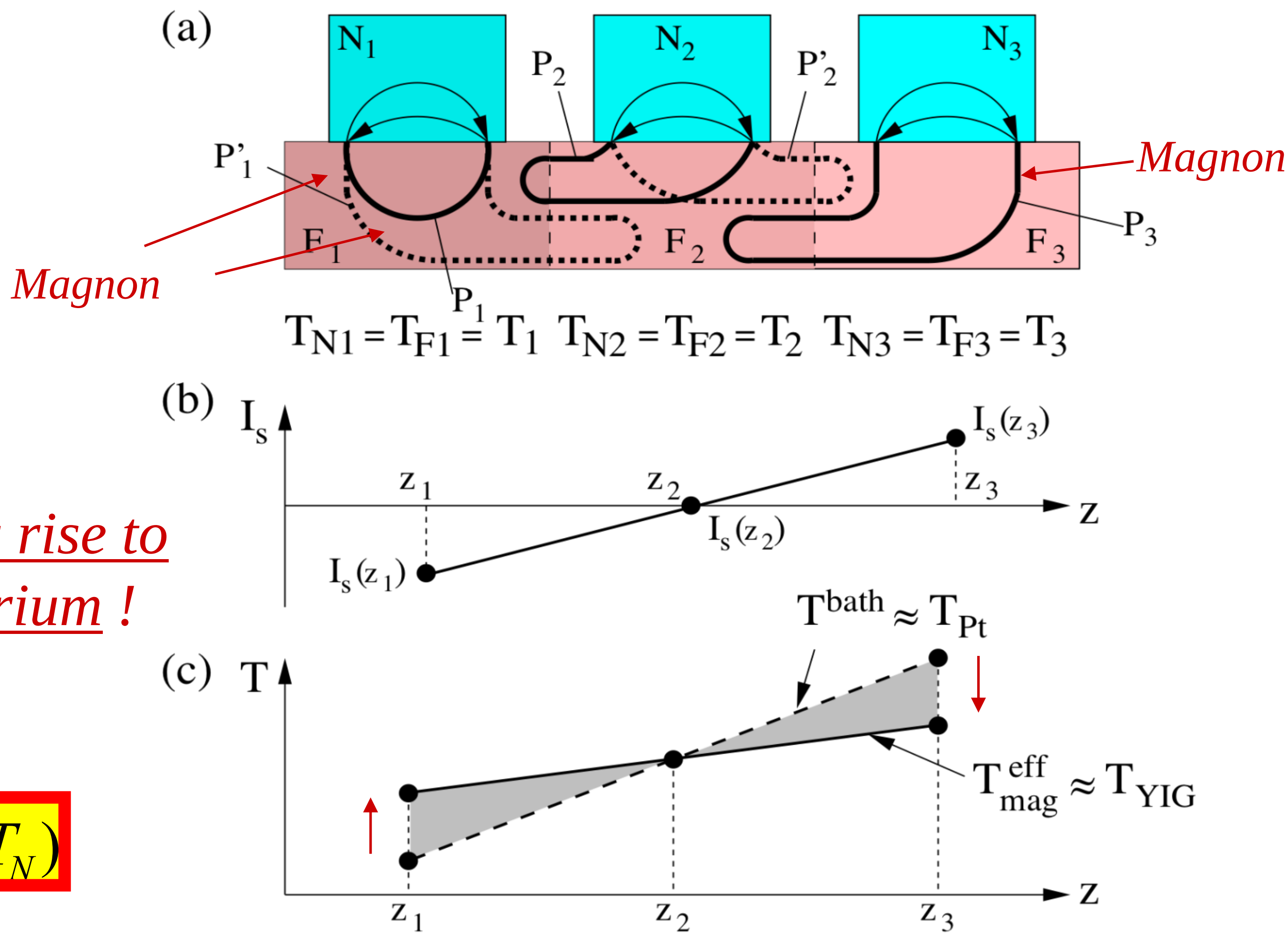
➔need to consider the effect of **temp-gradient** in YIG

Consider the following model

Static condition:
Local equilibrium



Interpretation by magnon effective temp.



Dynamics gives rise to the non-equilibrium !

$$\therefore J_s^{in} = A(T_F - T_N)$$

$$J_s^{in} \equiv < \partial_t s^z > = J_{sd}\,\mathrm{Im} \int d\omega < s^+(\omega) m^-(-\omega) >$$

⟶ *Field theoretical calculation of the Green function*

Heat transport Q:

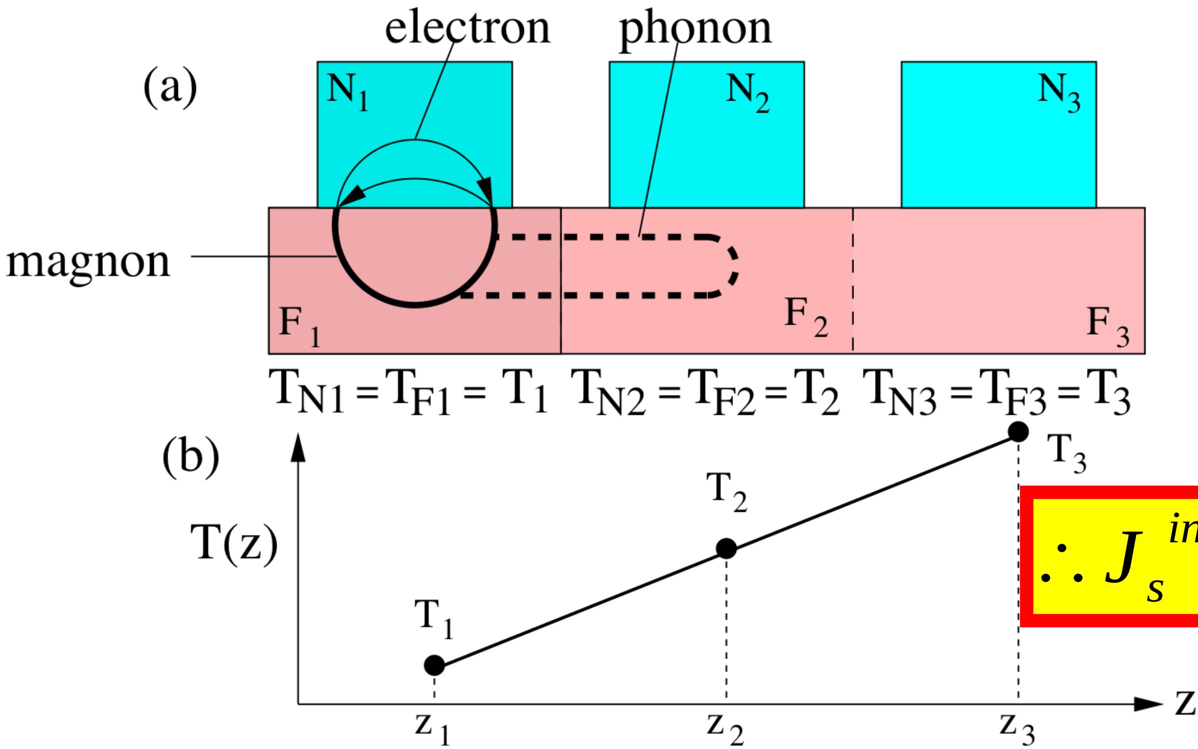
In ferromagnetic insulators,

Q = Q(magnon) + Q (phonon)

Phonon-drag contribution to SSE

Phonon drag process;

Magnons dragged by nonequilibrium phonons → spin injection



$$\therefore J_s^{in} = A(T_F - T_N)$$

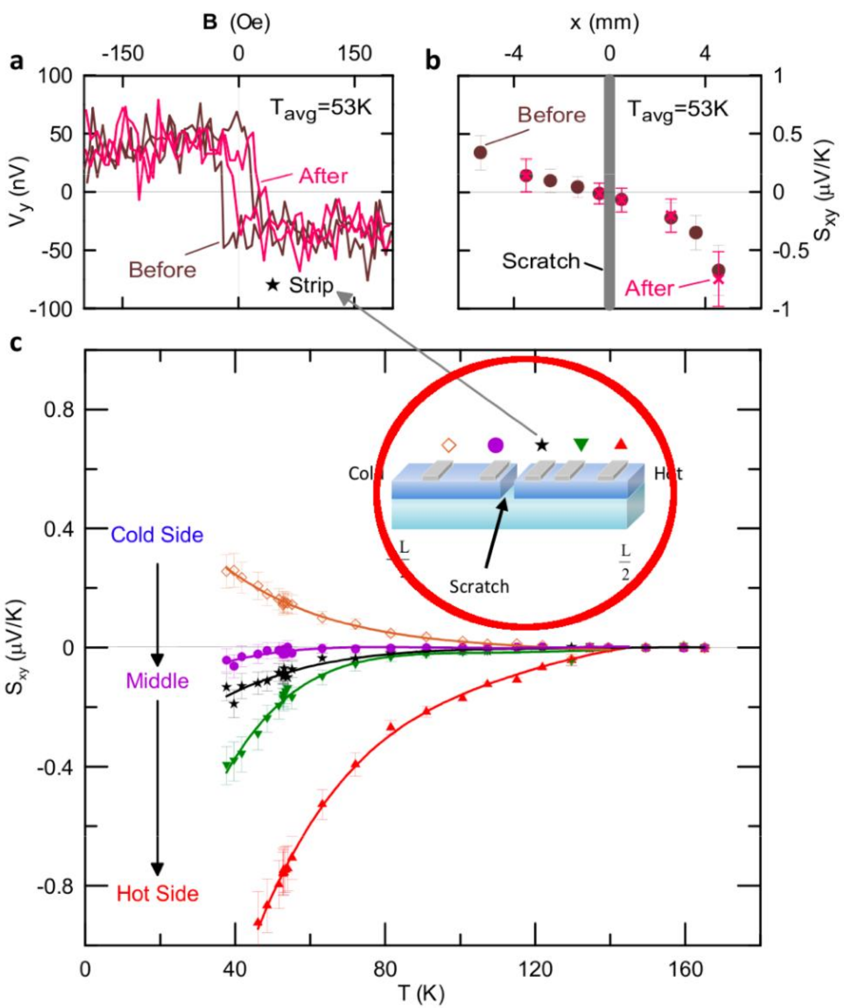
$$J_s^{ph-drag} \propto \tau_{ph} \Delta T$$

Phonon drag gives low-T enhancement of SSE due to the rapid suppression of umklapp scatt.

Spin Seebeck eff. without global spin current

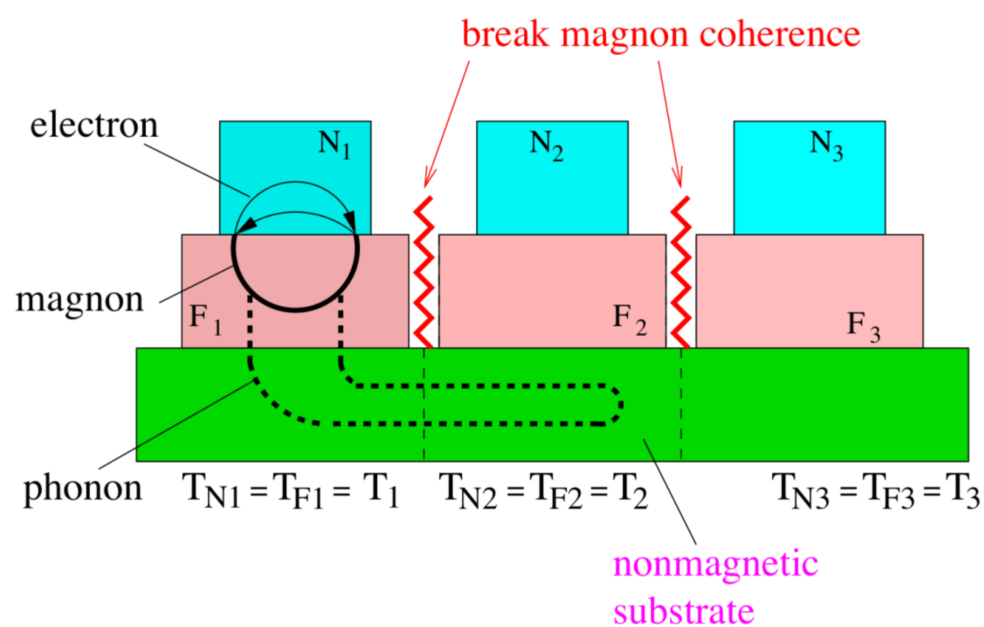
Observation of the Spin-Seebeck Effect in a Ferromagnetic Semiconductor

C. M. Jaworski[1], J. Yang[2], S. Mack[3], D. D. Awschalom[3], J. P. Heremans[1,4*], R. C. Myers[2,4*]



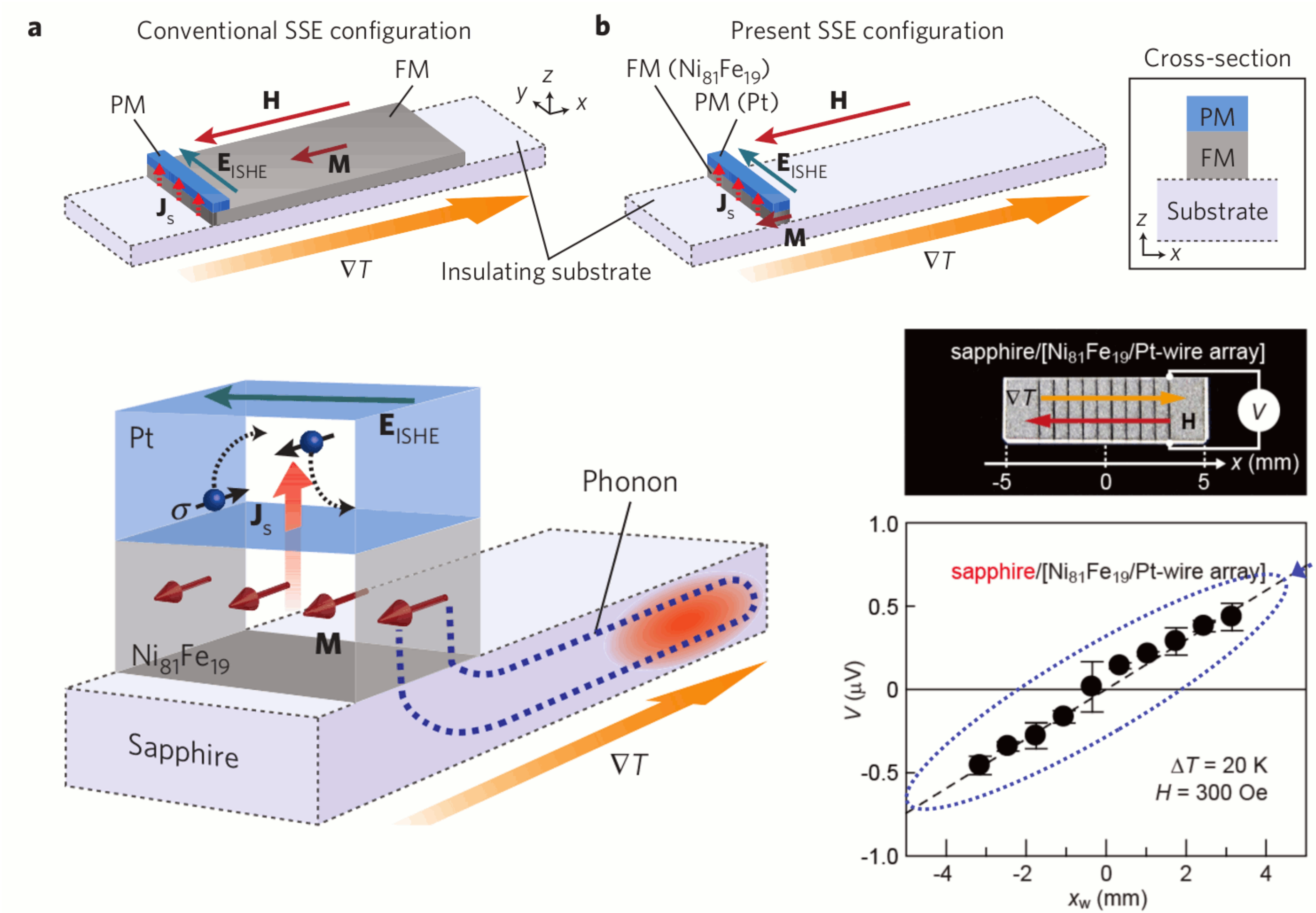
Our interpretation

Phonon drag: Effective at low-T.
GaMnAs: low Curie temp



Phonon-drag spin Seebeck effect

Uchida et al., Nature Mater. 10, 737(2012)



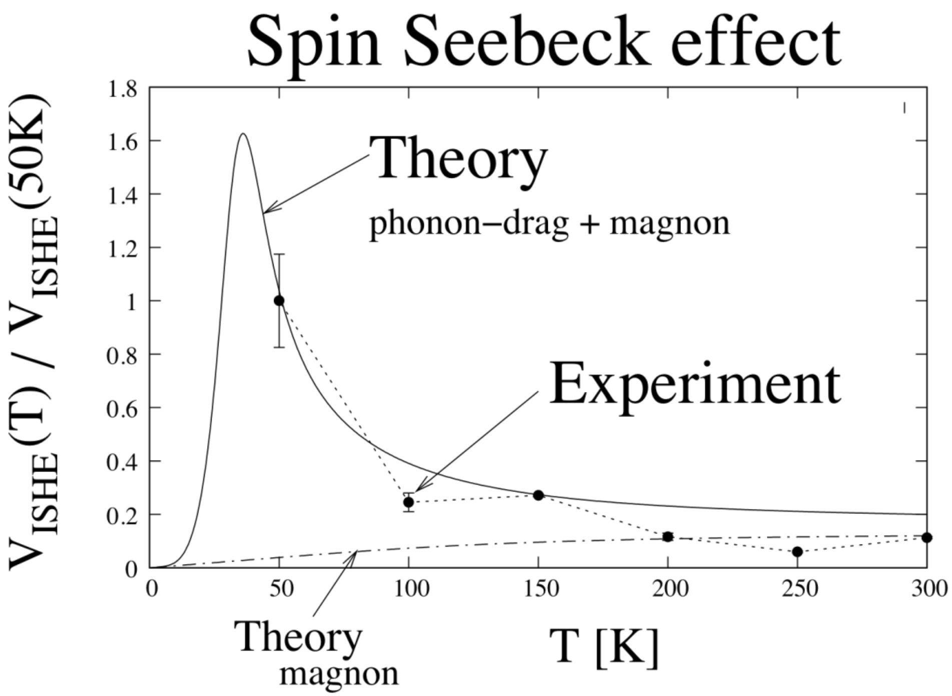
Only phonons in the nonmagnetic substrate can sense gradT!

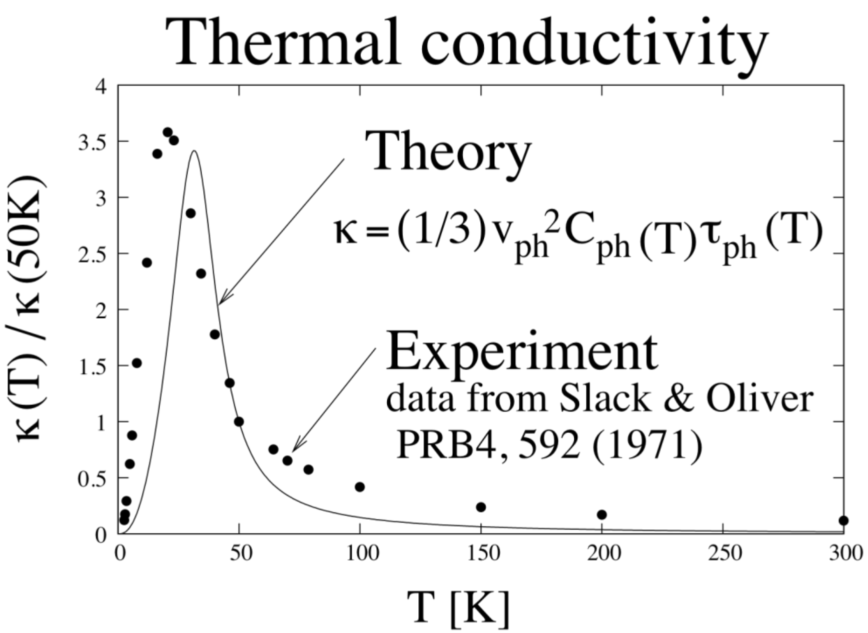
Fitting of the data by our theory

$$J_s^{ph-drag}(T) = const \cdot B_1(T) B_2(T) \tau_{ph}(T)$$

$$\tau_{ph}(T) = \tau^{(0)}{}_{ph} \left(\frac{1}{1 + (1/a)\exp[-b(T_D/T)]} \right) \left(\frac{1}{1 + (1/c)(T/T_D)} \right)$$

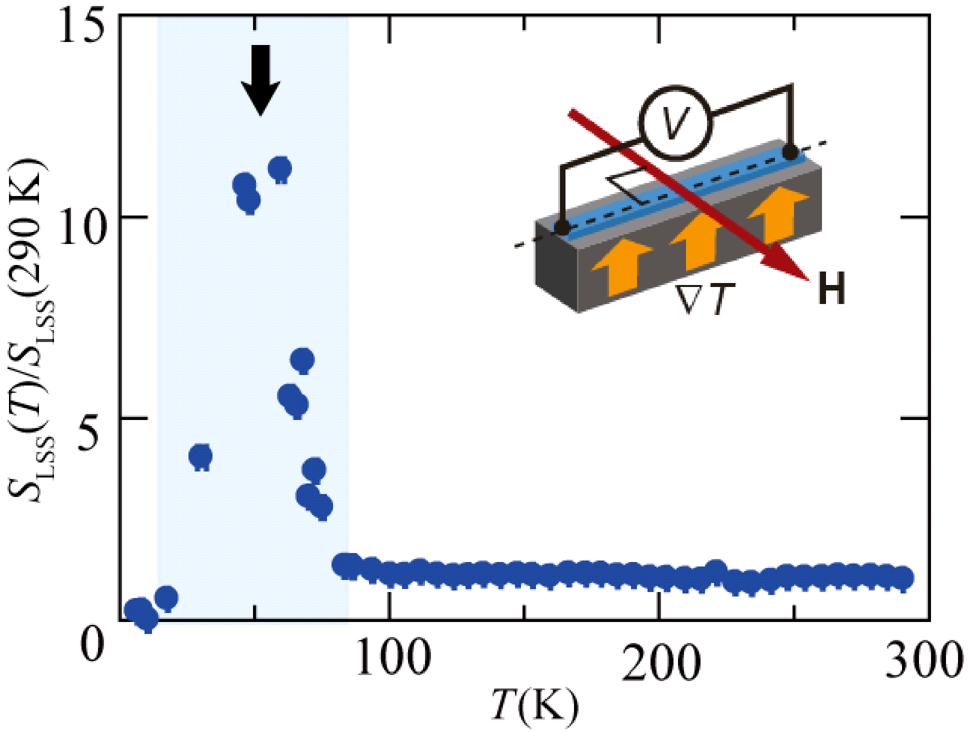
$$B_1(T) = (T/T_D)^5 \int_0^{T_D/T} \frac{du\, u^6}{sh^2(u/2)} \qquad B_2(T) = (T/T_M)^{9/2} \int_0^{T_M/T} \frac{dv\, v^{7/2}}{th(v/2)}$$





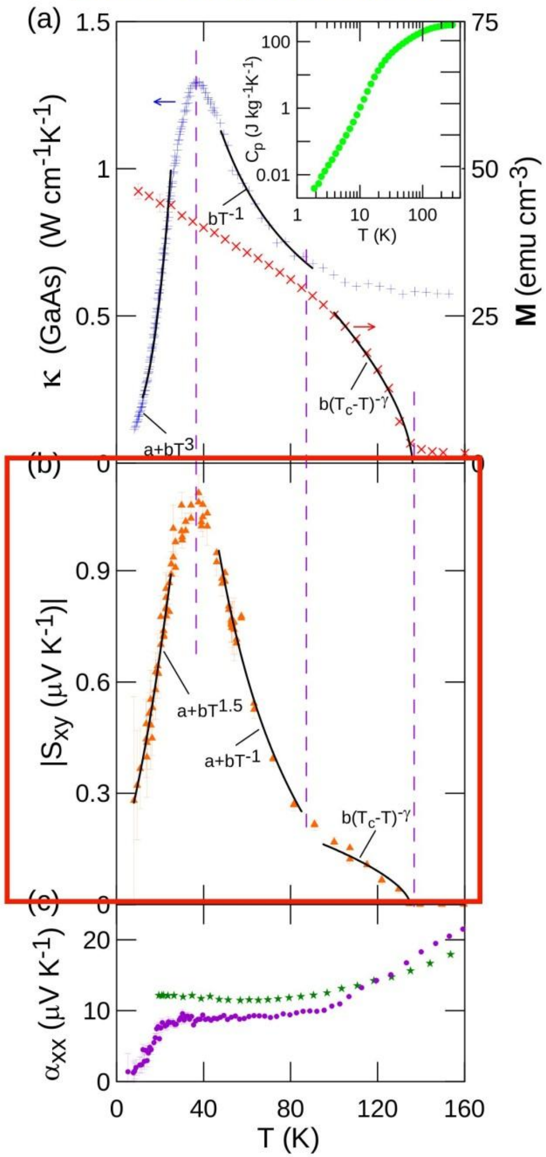
Importance of phonons

YIG



T. Ota *et al.*, (submitted).

GaMnAs



Pronounced peak consistent with our prediction!

Contents:

1) Introduction of spin current,
2) The linear response theory of spin current generation,
3) Spin current generation by heat, i.e., Spin Seebeck effect,
4) Spin Seebeck effect in Ferrimagnets and Antiferromagnets

Refs.: ** H. Adachi , K. Uchida, E. Saitoh and S. Maekawa: Rep. Prog. Phys. 76, 036501 (2013),*
**S. Maekawa, H. Adachi, K. Uchida, J. Ieda and E. Saitoh: J. Phys. Soc. Jpn. (2013).*

SSE in AFM & ferrimagnet

Metal

$$H_{\mathrm{N}} = \sum_{i\in \mathrm{N}} c_i^+ \left\{-\nabla^2 + V\left(r_i\right) + \lambda_{so}\left[\nabla V\left(r_i\right)\times\left(-i\hbar\nabla\right)\right]\cdot\sigma_i\right\} c_i$$

Ferri/NM interface

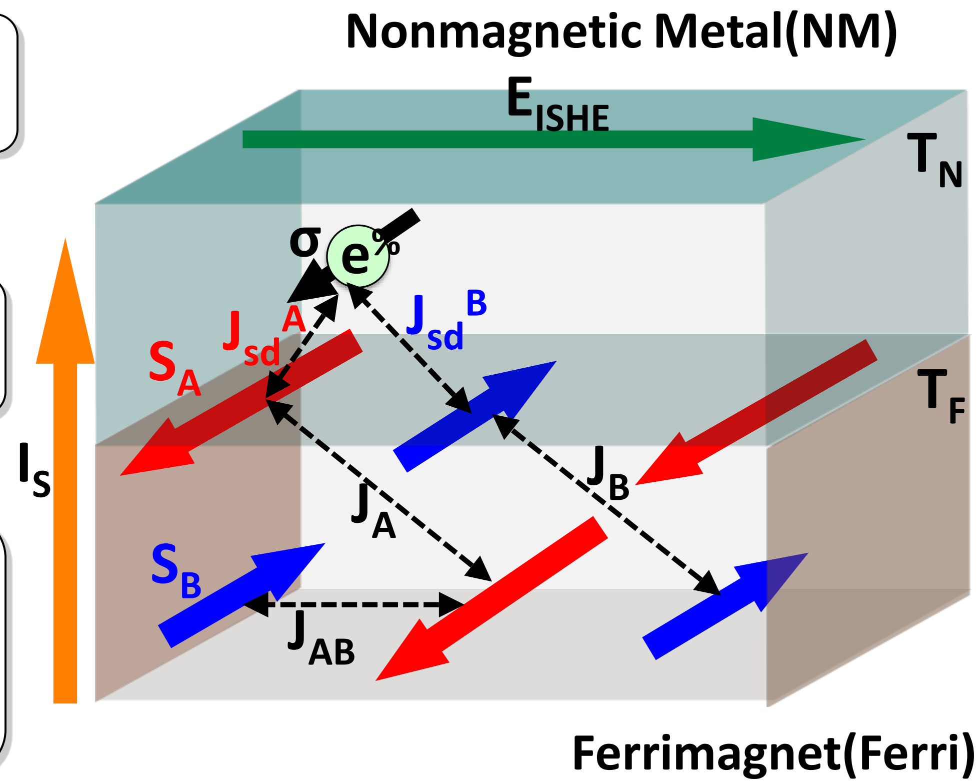
$$H_{\mathrm{sd}} = J_{sd}^{A} \sum_{i\in \text{interface-A}} \mathbf{S}_i^A \cdot \sigma_i + J_{sd}^{B} \sum_{j\in \text{interface-B}} \mathbf{S}_j^B \cdot \sigma_j$$

Ferrimagnet

$$H_{Ferri} = -J_A \sum_{\langle i,i'\rangle\in A} \mathbf{S}_i^A \cdot \mathbf{S}_{i'}^A - J_B \sum_{\langle j,j'\rangle\in B} \mathbf{S}_j^B \cdot \mathbf{S}_{j'}^B + J_{AB} \sum_{\langle ij\rangle} \mathbf{S}_i^A \cdot \mathbf{S}_j^B$$

Spin current injected into NM:

$$I_S = \sum_{i\in NM} \left\langle \partial_t \sigma_i^z\left(t\right)\right\rangle \qquad \sigma_i^z = \sum \sigma c_{i\sigma}^+ c_{i\sigma}$$



SSE in uniaxial AFM

Spin current injected into NM:

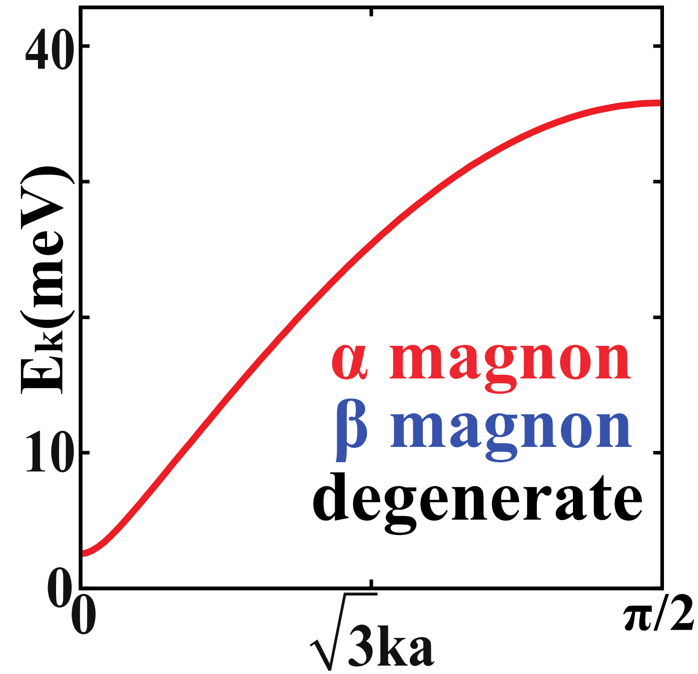
$$I_S = \sum_{i \in NM} \langle \partial_t s_i^z(t) \rangle$$

$$\boxed{I_S = I_S^{\alpha} - I_S^{\beta}}$$

$H_0 \neq 0$

$$I_S \approx \frac{g\mu_B H_0 \tau_{sf}}{\hbar} \frac{SJZ\tau_{sf}}{\hbar} \left(\frac{N_{int} J_{sd}^2 S c_N \tau_{sf}}{2\sqrt{2}\pi^4 \hbar^3 (l_N / a_{Pt})^3} \right) \Gamma_1 k_B (T_{AF} - T_N)$$

$H_0 = 0$ $\quad \boxed{I_S = 0}$



SSE = 0 because magnons are degenerate.

Each spin wave branch carries different spin angular momentum,

Compensation effects

Magnetization(M_S)

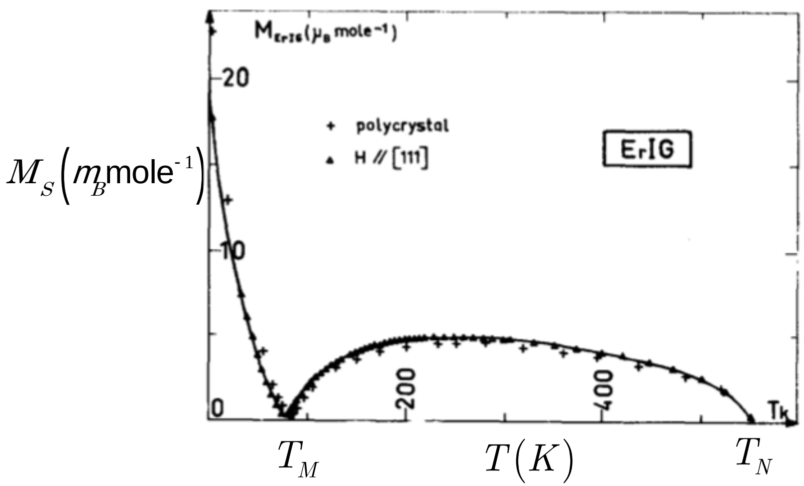
$$M_S = M_A + M_B$$

$$T = T_M \qquad \boxed{M_S = 0}$$

Angular momentum(S_{tot})

$$S_{tot} = S_A + S_B$$

$$T = T_A \qquad \boxed{S_{tot} = 0}$$



$Er_3Fe_5O_{12}$

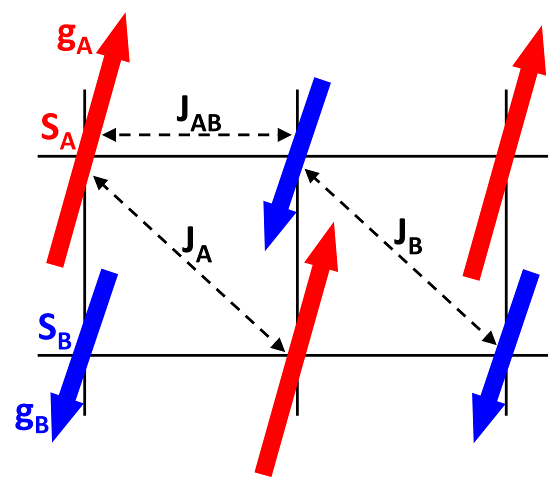
M. Guillot(1979)

Magnetic moments

$$M_A = \frac{g_A m_B}{N_A}\sum_{i\in A}\langle S^A_{iz}\rangle \qquad M_B = \frac{g_B m_B}{N_B}\sum_{j\in B}\langle S^B_{jz}\rangle$$

Angular moments

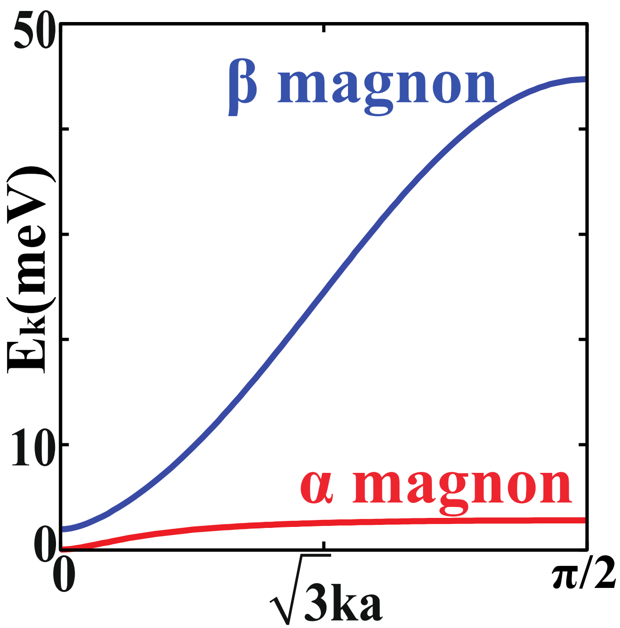
$$S_A = \frac{1}{N_A}\sum_{i\in A}\langle S^A_{iz}\rangle \qquad S_B = \frac{1}{N_B}\sum_{j\in B}\langle S^B_{jz}\rangle$$



Why is SSE finite at T_M & T_A ?

Spin current is driven by magnons

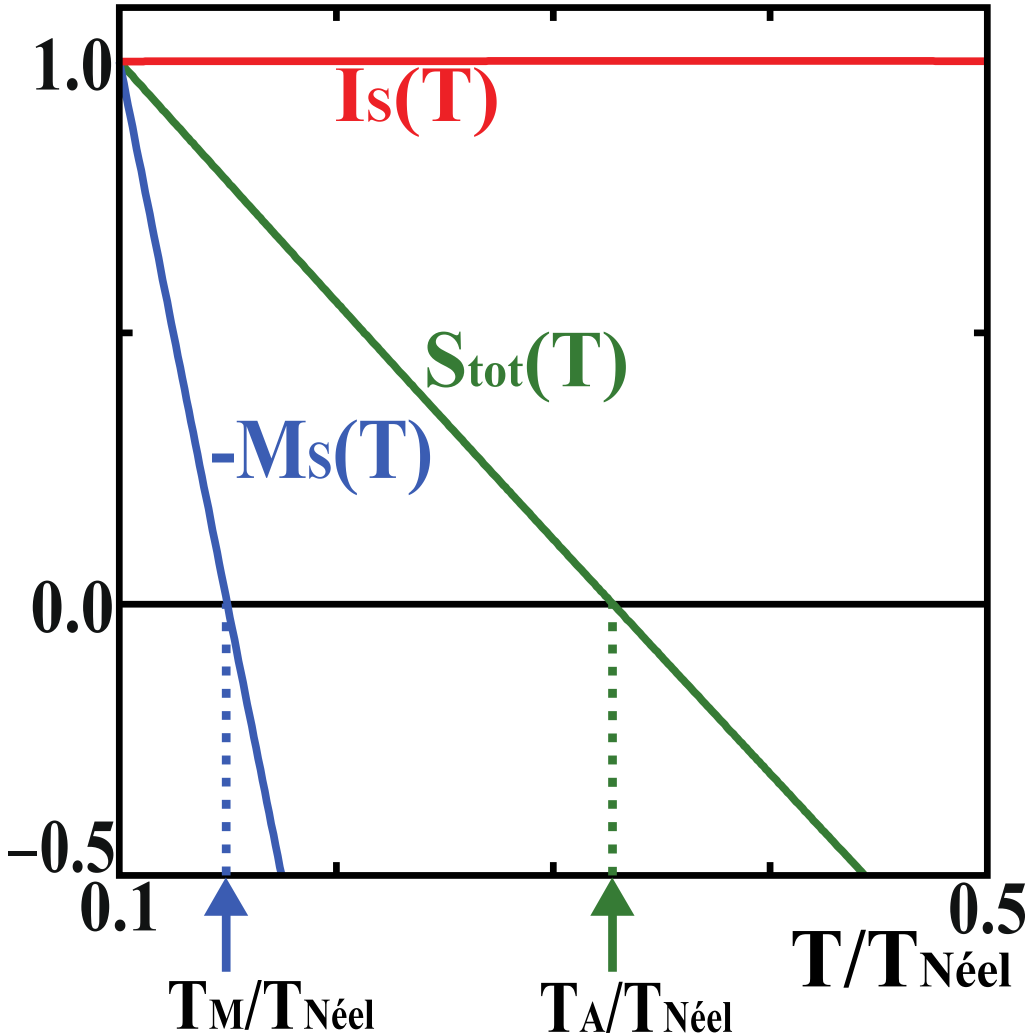
$$I_S^{\alpha,\beta} = G\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{k,q}\left(J_{sd}^{\alpha,\beta}(\mathbf{q})\right)^2 \mathrm{Im}\,G_{\alpha,\beta}^R(\mathbf{q},\omega)\,\mathrm{Im}\,\chi^R(\mathbf{k},\omega)\frac{2k_B}{\hbar\omega}\left(T_N - T_F\right)$$



Nondegenerate magnons exit even when S_{tot}^z & M_S^z vanishes.

$$I_S \neq 0$$

Numerical Results of the injected spin current



SSE from vanishing saturation magnetization!

***In Summary*:**

Spin current generation:

by heat :Spin Seebeck Effect

In ferromagnetic insulators,
Q = Q(magnon) + Q (phonon)

At the Interface,
Spin flow, not Charge flow!!
(different from spin-dependent Seebeck effect)

Phonons

ferromagnets, substrates, piezoelectric actuators,…

→ ***A variety of spin Seebeck devices!***