



Inverse spin Hall effect as a means to study non-linear spin fluctuation

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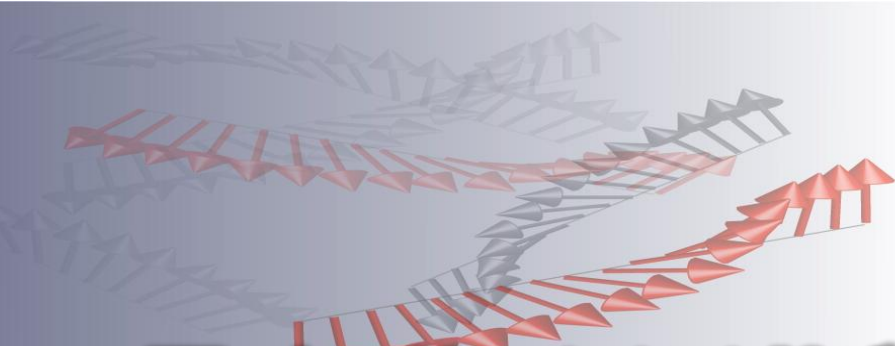
¹ISSP, Univ. of Tokyo, Japan

²ASRC, JAEA, Japan

³CREST, JST, Japan

⁴ILL-CNRS, France

⁵RIKEN-CEMS, Japan



Applications of spin Hall effect (SHE)

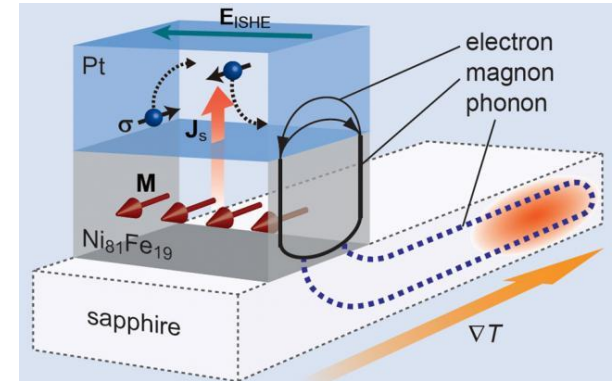
↑ Detection of spin Seebeck effect via ISHE

Pt/Py/sapphire

K. Uchida *et al.* Nature (2008); Nat. Mat.

Pt/YIG

K. Uchida

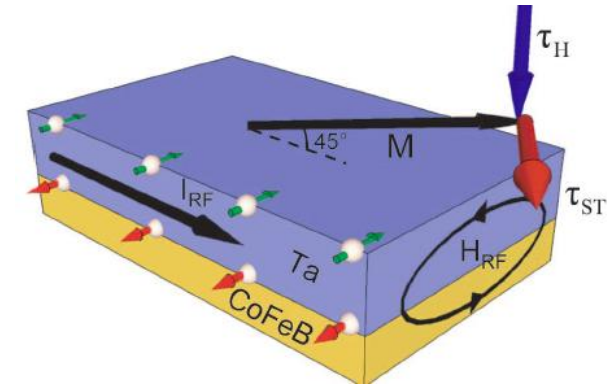


↑ Spin magnetization via SHE

Pt/CoFeB ($\alpha_H = -15\%$),

Pt/Co ($\alpha_H = 7\%$)

L. Liu *et al.* Science (2012); Phys. Rev. Lett. (2012).



The other application of SHE has not been demonstrated yet.

↑↓ Detection of non-linear spin fluctuation via ISHE

Outline

↑ Introduction

- ↓ *spin Hall effect (SHE) and anomalous Hall effect (AHE)*
- ↓ *AHE of pure Ni and Fe*
- ↓ *Kondo's model for AHE*

↑ SHE in NiPd alloys

- ↓ *experimental setup (spin absorption technique)*
- ↓ *anomaly near Curie temperature T_C*

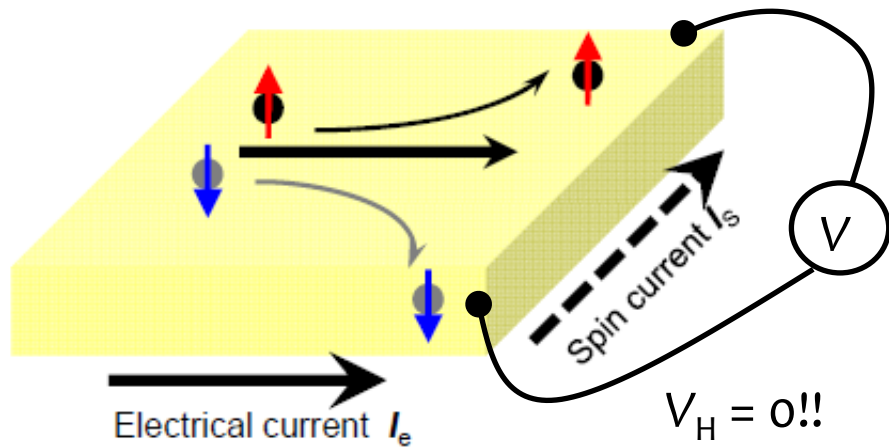
↑ Comparison with theory

- ↓ *extended Kondo's model*

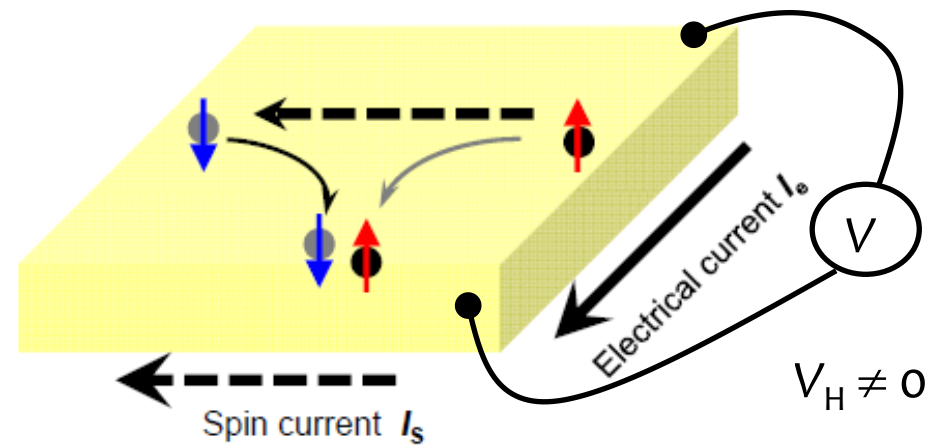
↑ Summary

Direct & Inverse Spin Hall effect

Spin-orbit interaction



Direct spin Hall effect (DSHE)



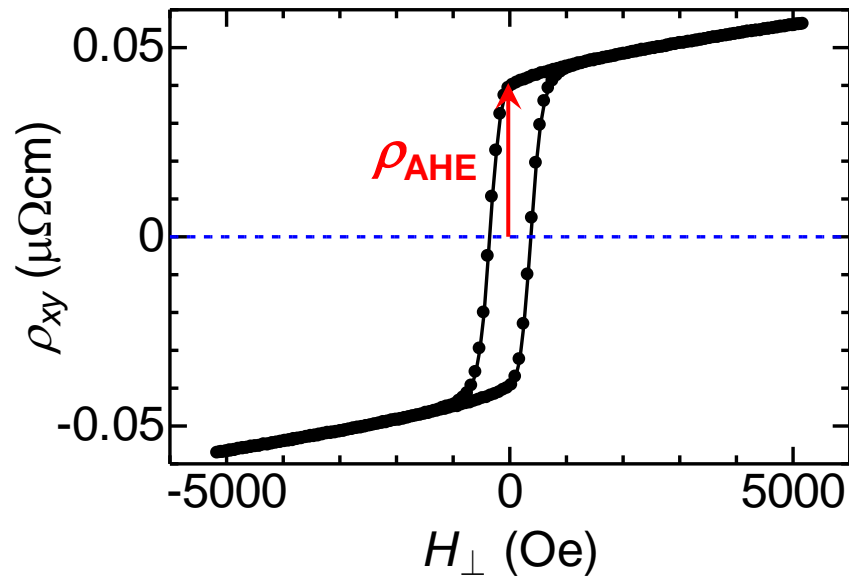
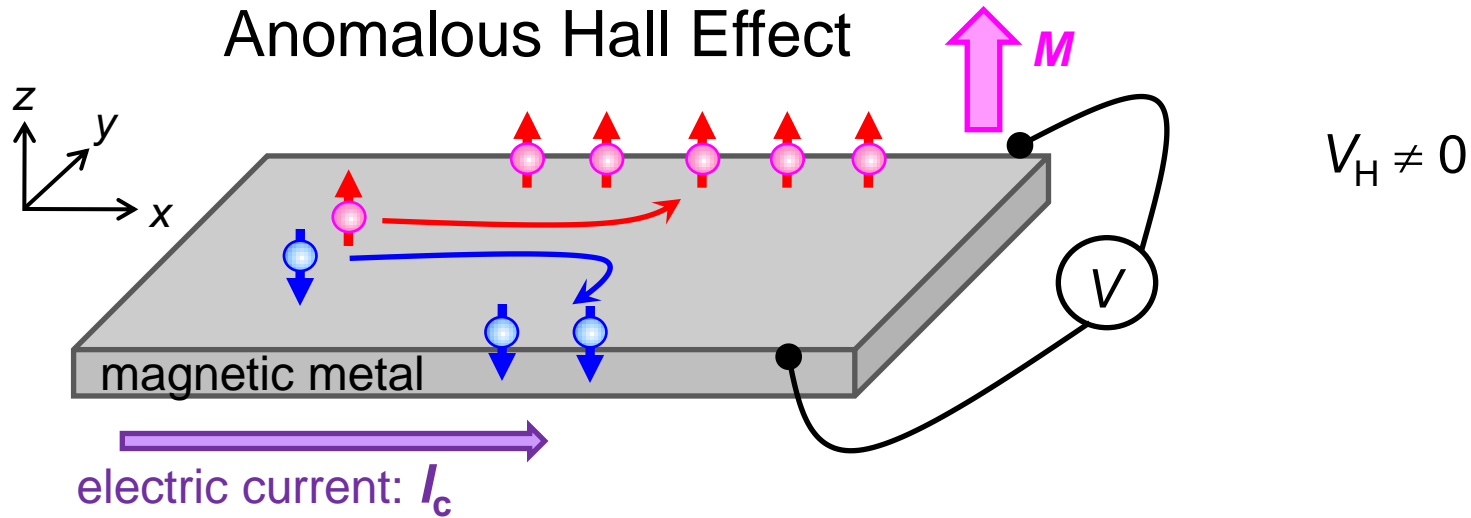
Inverse spin Hall effect (ISHE)


Un-polarized charge current \Leftrightarrow Transverse spin current

Y. K. Kato *et al.* Science **306**, 1910 (2004).

J. Wunderlich *et al.* Phys. Rev. Lett. **94**, 047204 (2005)

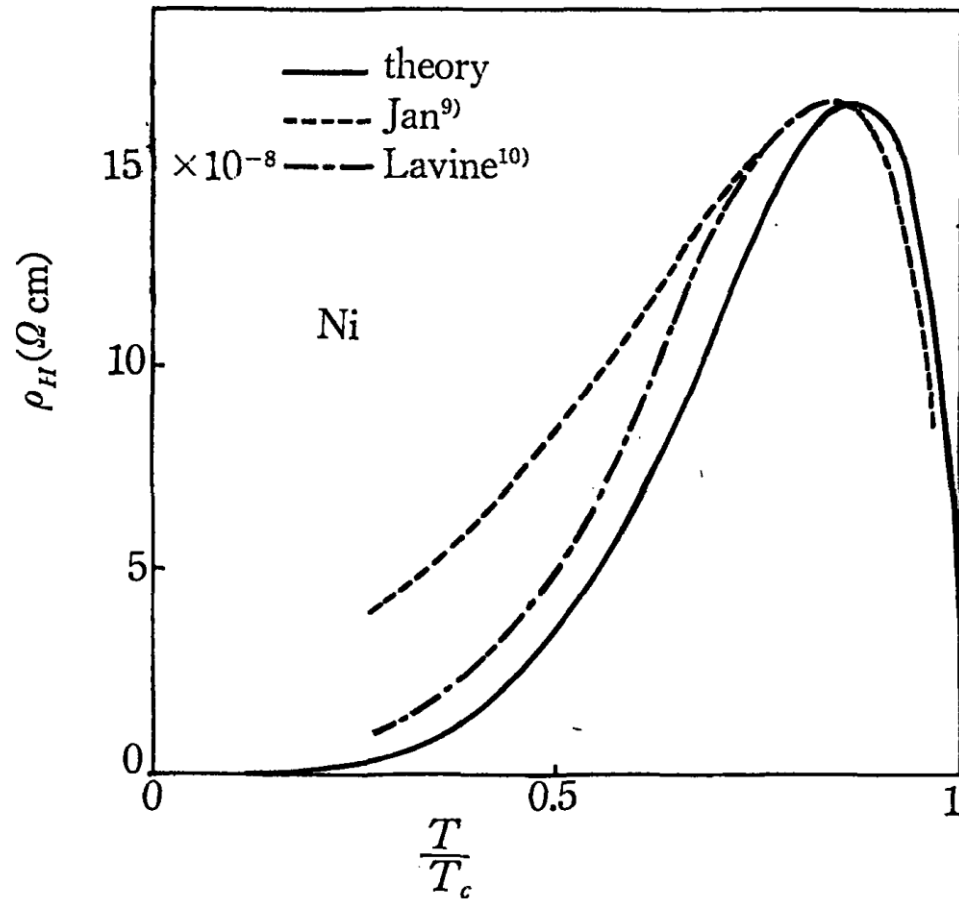
Anomalous Hall effect (AHE)



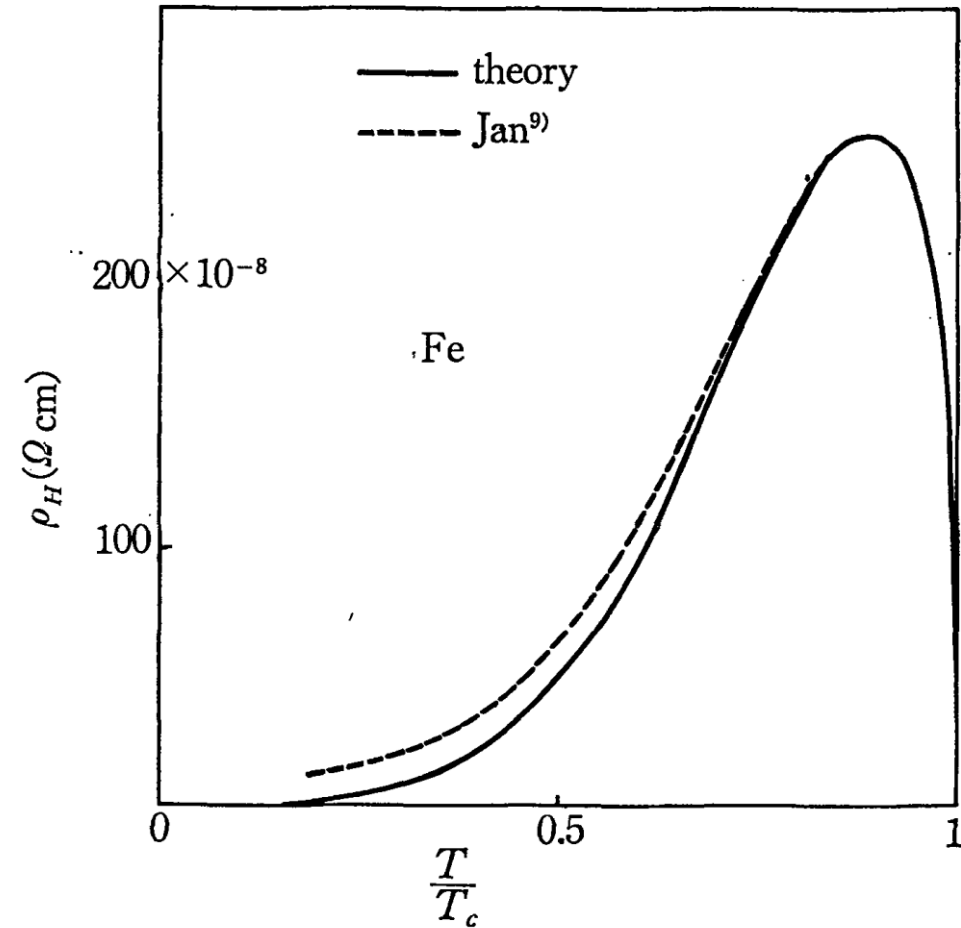
 What is an essential difference between SHE and AHE?

AHE of pure Ni & Fe

intrinsic AHE for pure ferromagnetic metals



J. P. Jan, *Helv. Phys. Acta* **25**, 677 (1952).
J. M. Lavine, *Phys. Rev.* **123**, 1273 (1961).



J. P. Jan, *Helv. Phys. Acta* **25**, 677 (1952).

To explain the peak in ρ_{xy} below T_c ...

- Karplus & Luttinger's theory

$$\rho_{xy} \propto \rho_{xx}^2 \langle m \rangle$$

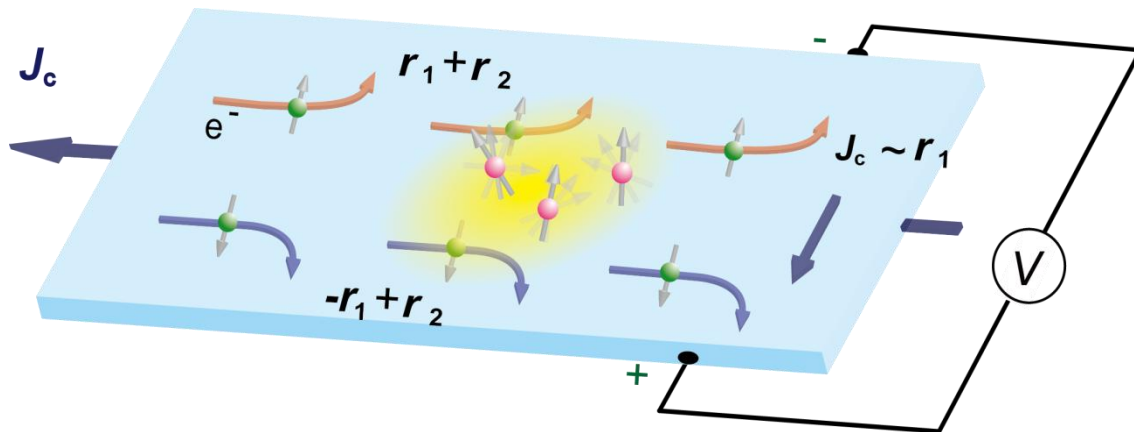
R. Karplus & J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).

- Kondo's theory

$$\rho_{xy} \propto \langle (m - \langle m \rangle)^3 \rangle$$

J. Kondo, Prog. Theor. Phys. **27**, 772 (1962).

Interaction between conduction electron and localized moment

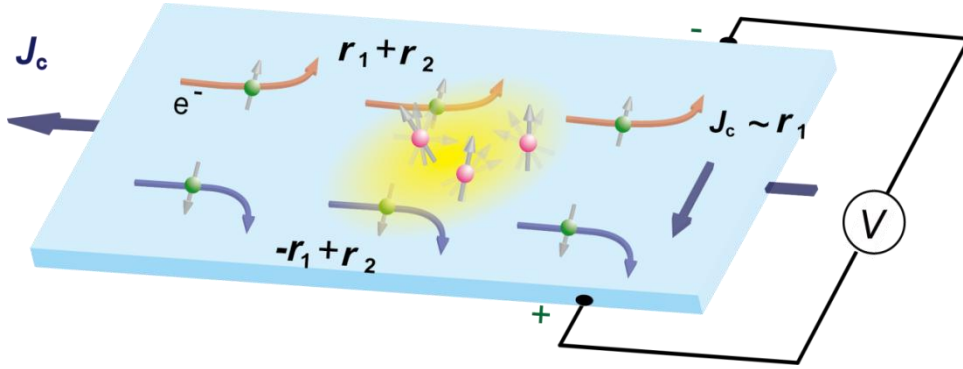


$$r_1 : \langle (m - \langle m \rangle)^3 \rangle$$

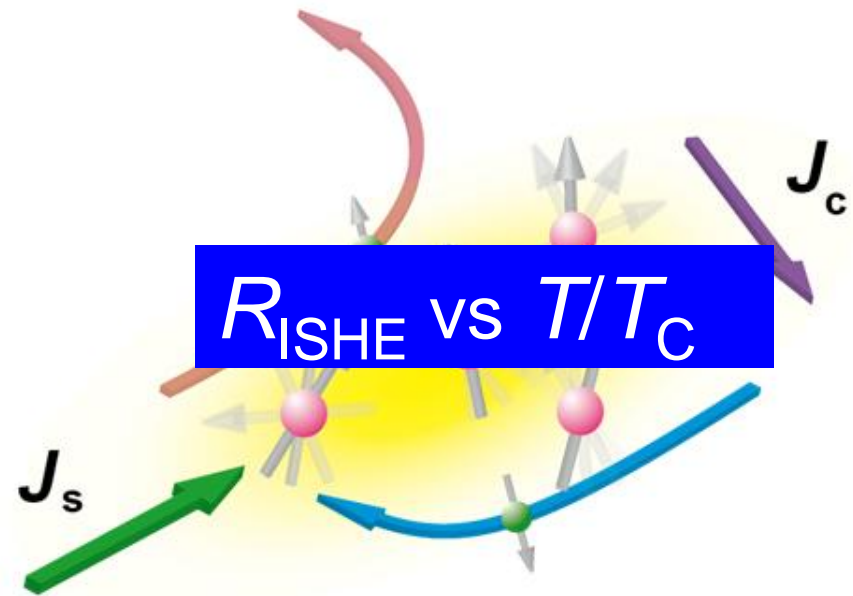
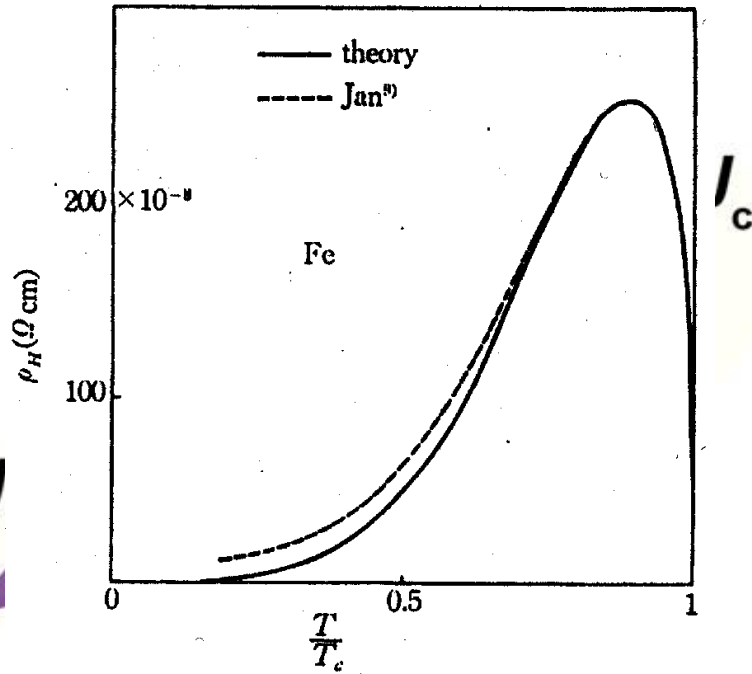
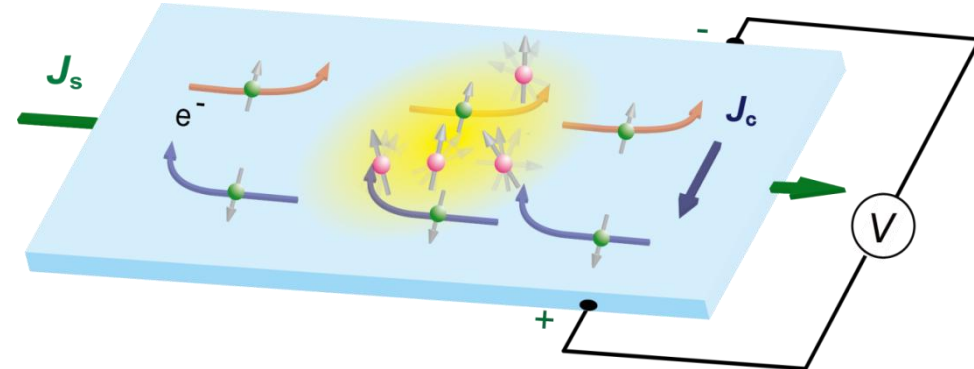
$$r_2 : 2C_2 \langle (m - \langle m \rangle)^2 (m^2 - \langle m^2 \rangle) \rangle$$

Kondo's work for ISHE?

AHE



ISHE



Outline

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- ↓ *AHE of pure Ni and Fe*
- ↓ *Kondo's model for AHE*

↑ SHE in NiPd alloys

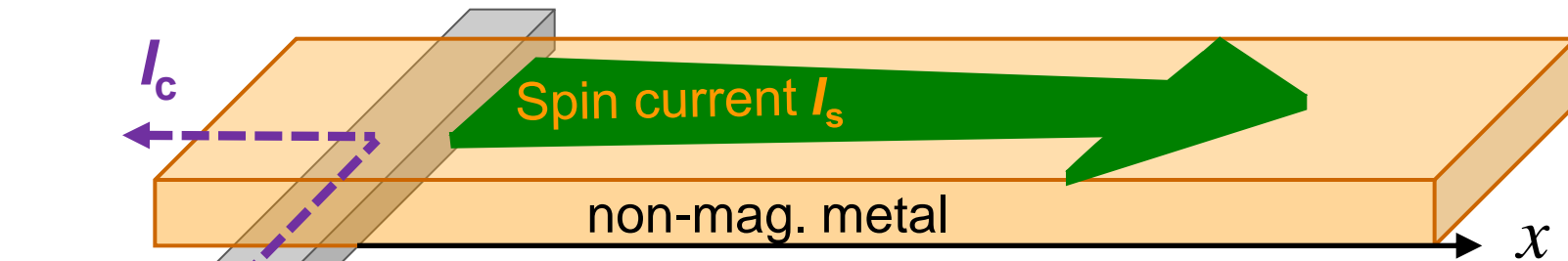
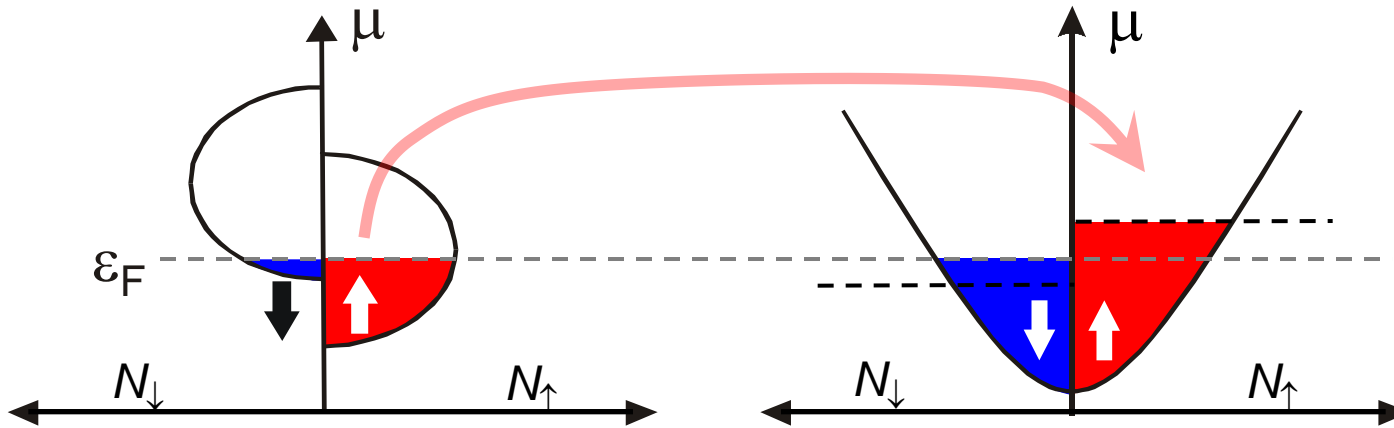
- ↓ *experimental setup (spin absorption technique)*
- ↓ *anomaly near Curie temperature T_C*

↑ Comparison with theory

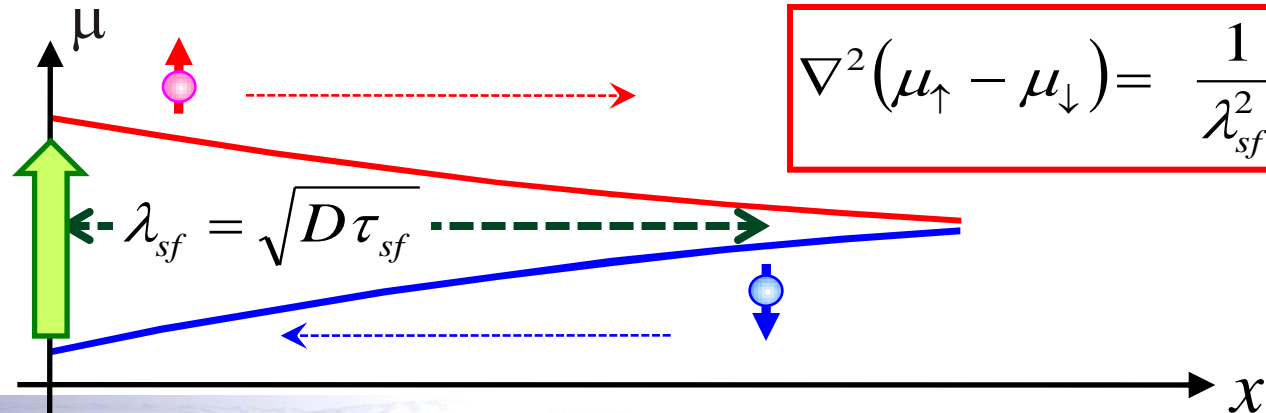
- ↓ *extended Kondo's model*

↑ Summary

Non-local spin injection



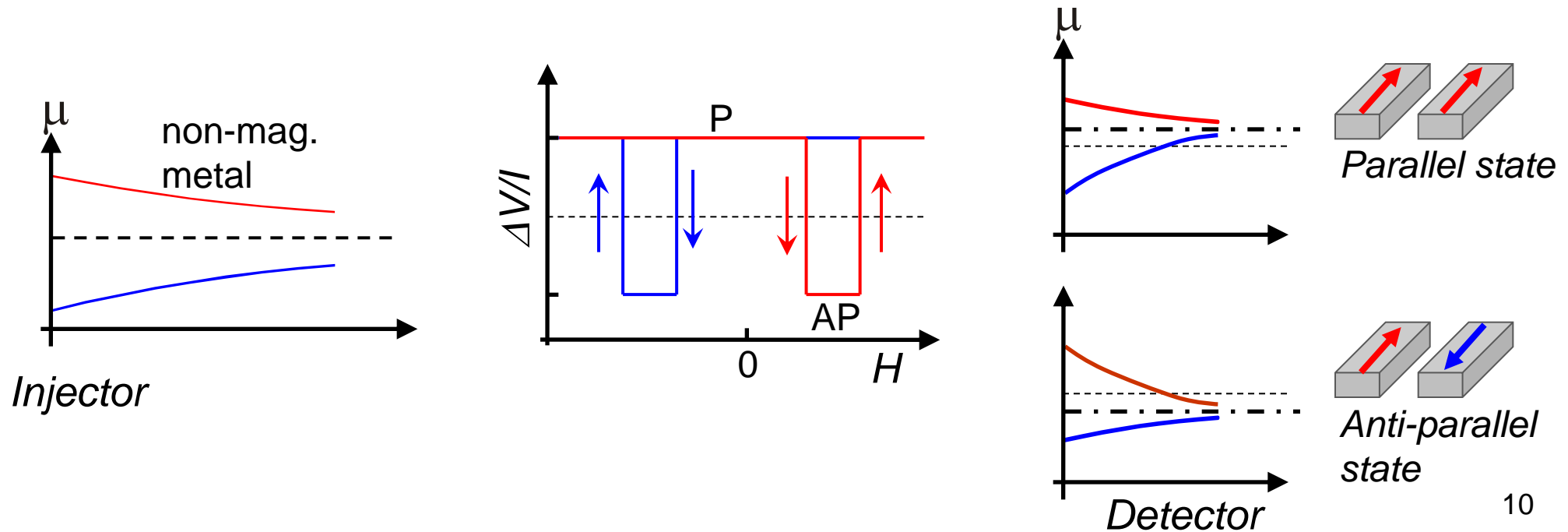
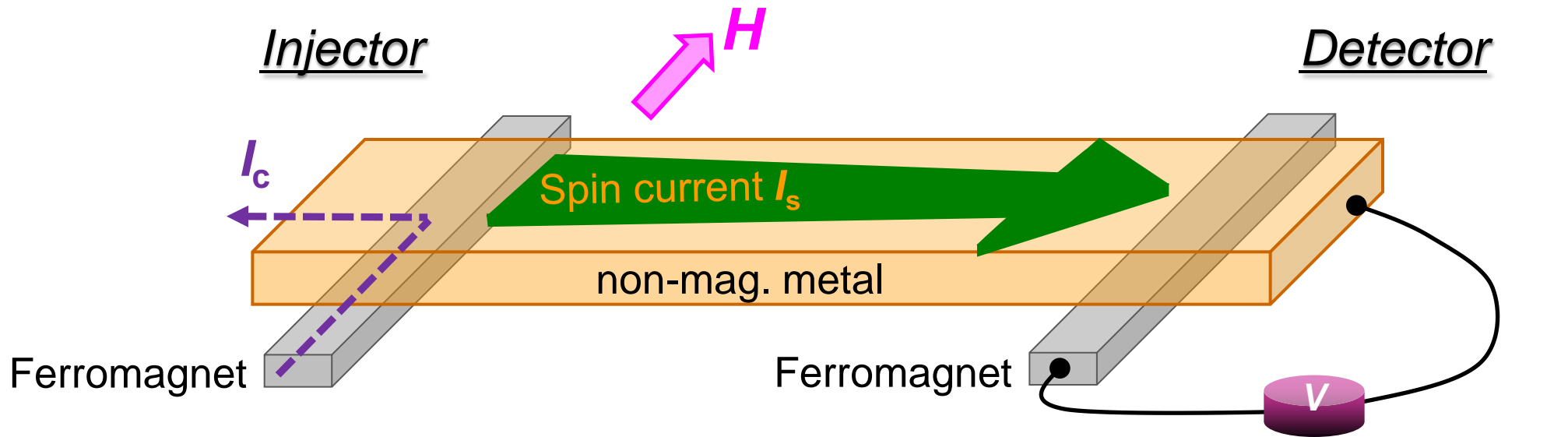
Ferro magnet: Py



$$\nabla^2(\mu_{\uparrow} - \mu_{\downarrow}) = \frac{1}{\lambda_{sf}^2}(\mu_{\uparrow} - \mu_{\downarrow})$$

$$\lambda_{sf} = \sqrt{D\tau_{sf}}$$

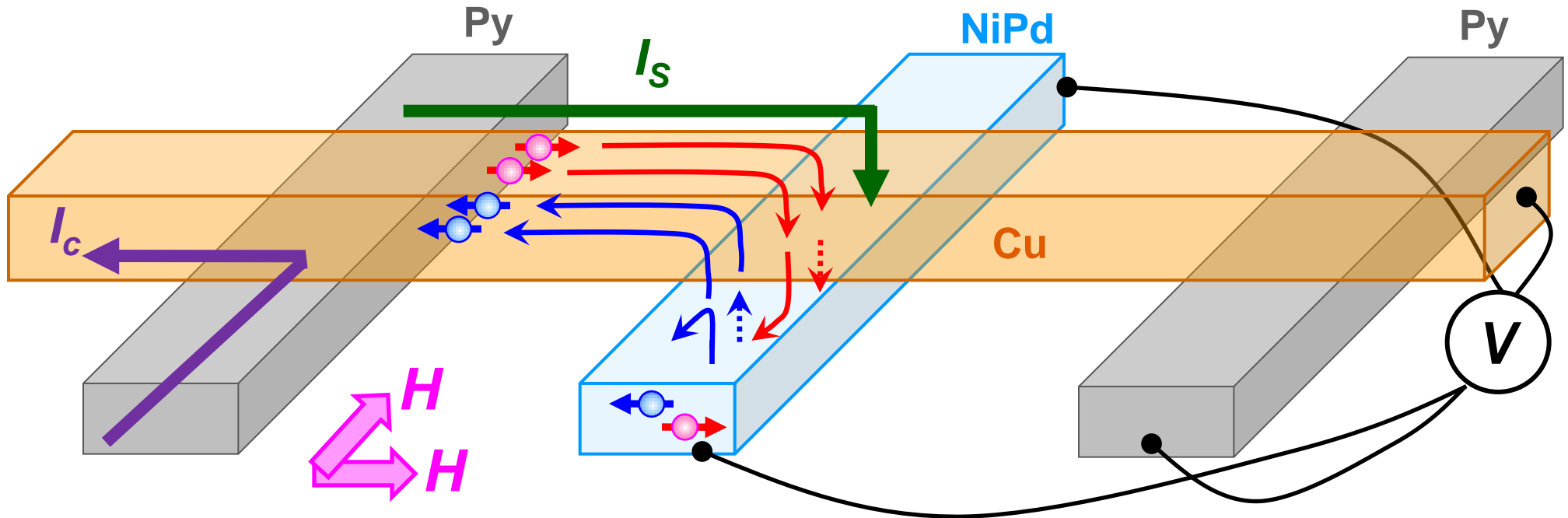
Electrical detection of non-local spin signal



Spin absorption method

Spin absorption technique & ISHE

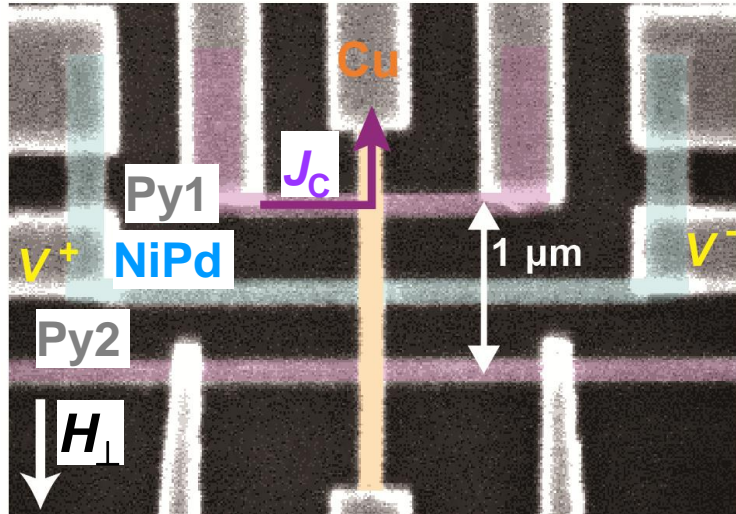
Y. Niimi *et al.*, Phys. Rev. Lett. **106**, 126601 (2011).



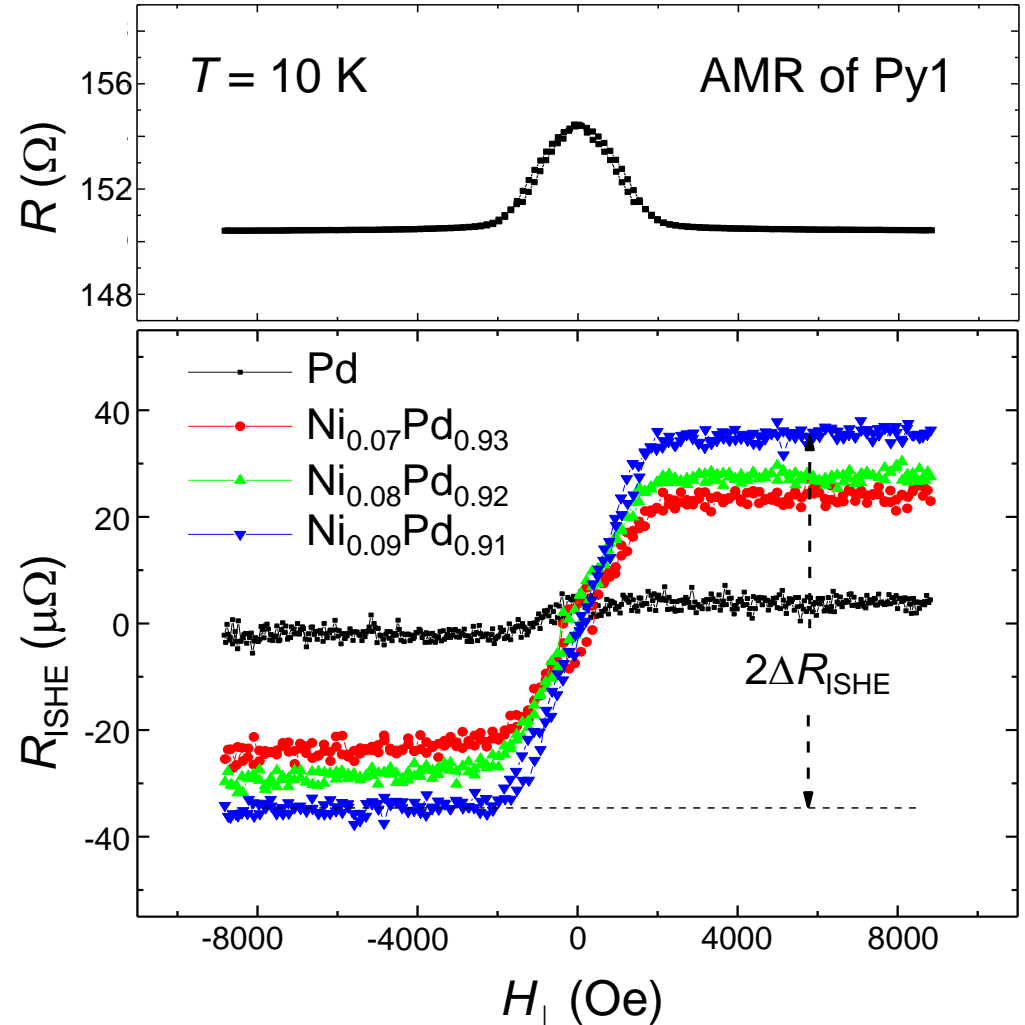
Spin current I_s into NiPd can be experimentally determined by measuring NLSV.

ISHE for $\text{Ni}_x\text{Pd}_{1-x}$ alloys

$\text{Ni}_x\text{Pd}_{1-x}$: weakly ferromagnetic alloy

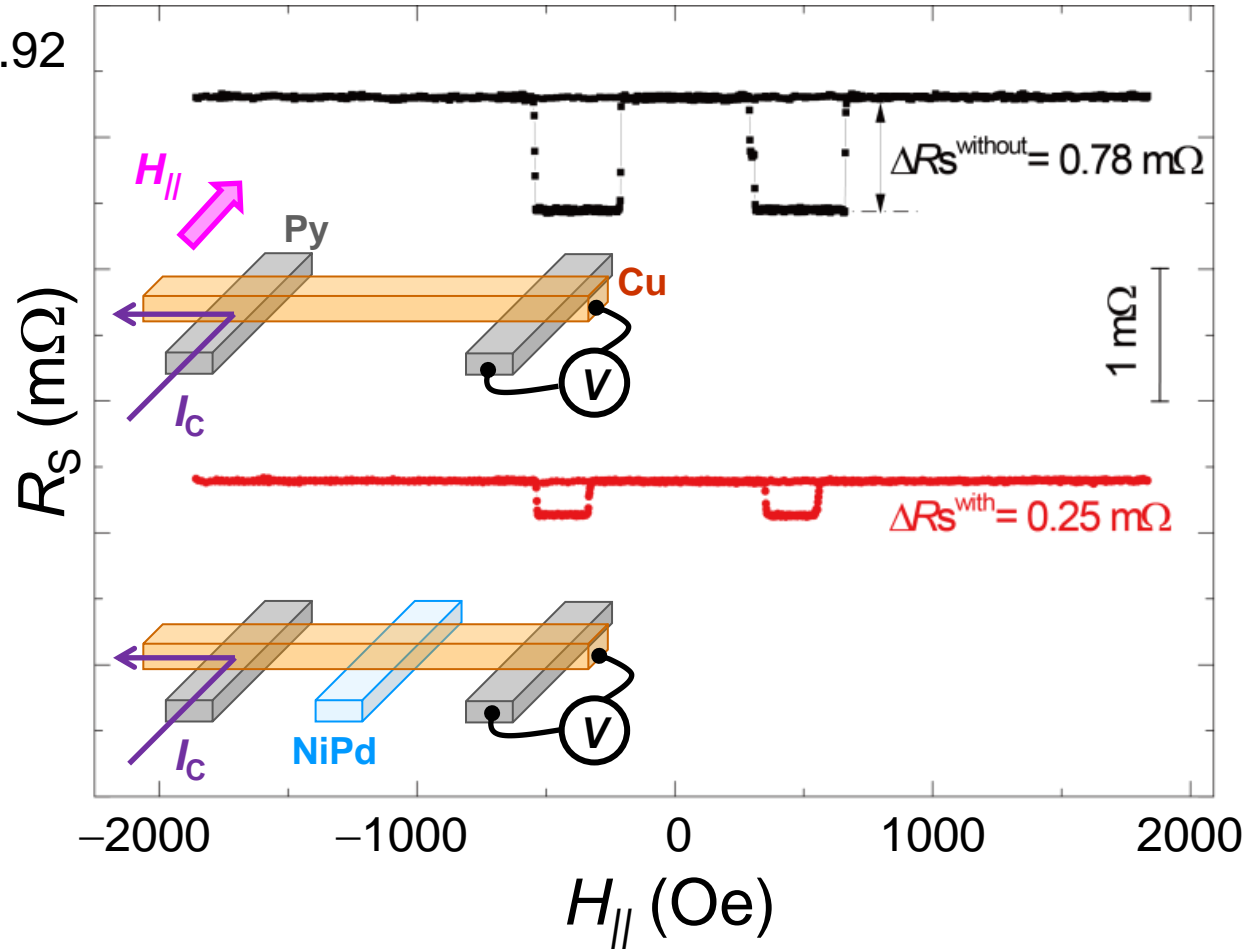


size: $w = 100 \text{ nm}$, $t = 20 \text{ nm}$



Spin absorption into NiPd alloy

Ni_{0.08}Pd_{0.92}



$$\eta \equiv \frac{\Delta R_S^{\text{with}}}{\Delta R_S^{\text{without}}} = 0.32$$

$$\lambda_{\text{Ni}_{0.08}\text{Pd}_{0.92}} = 10 \text{ nm}$$

$$\eta \equiv \frac{\Delta R_S^{\text{with}}}{\Delta R_S^{\text{without}}} \approx \frac{2 \frac{R_{\text{NiPd}}}{R_{\text{Cu}}} \sinh(L / \lambda_{\text{Cu}})}{\tanh(t_{\text{NiPd}} / \lambda_{\text{NiPd}}) \{ \cosh(L / \lambda_{\text{Cu}}) - 1 \} + 2 \frac{R_{\text{NiPd}}}{R_{\text{Cu}}} \sinh(L / \lambda_{\text{Cu}})}$$

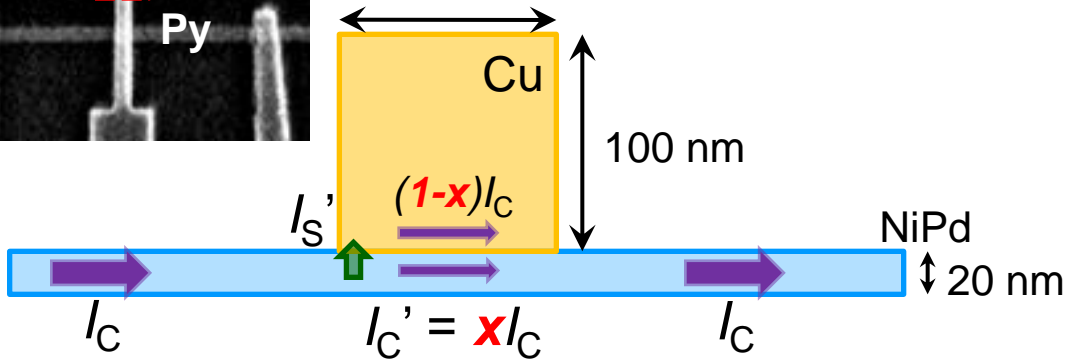
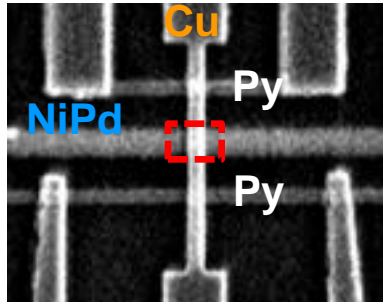
S. Takahashi and S. Maekawa,
PRB **67**, 052409 (2003).

Spin Hall resistivity as a function of ρ_{imp}

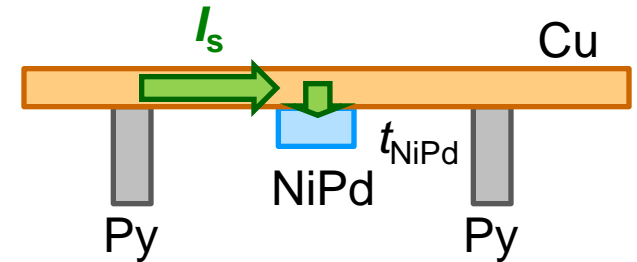
$$\rho_{\text{SHE}} = \frac{w_{\text{NiPd}}}{x} \frac{I_C}{\bar{I}_S} \Delta R_{\text{SHE}}$$

Correction factor for shunting by Cu bridge

Effective pure spin current (\bar{I}_S)

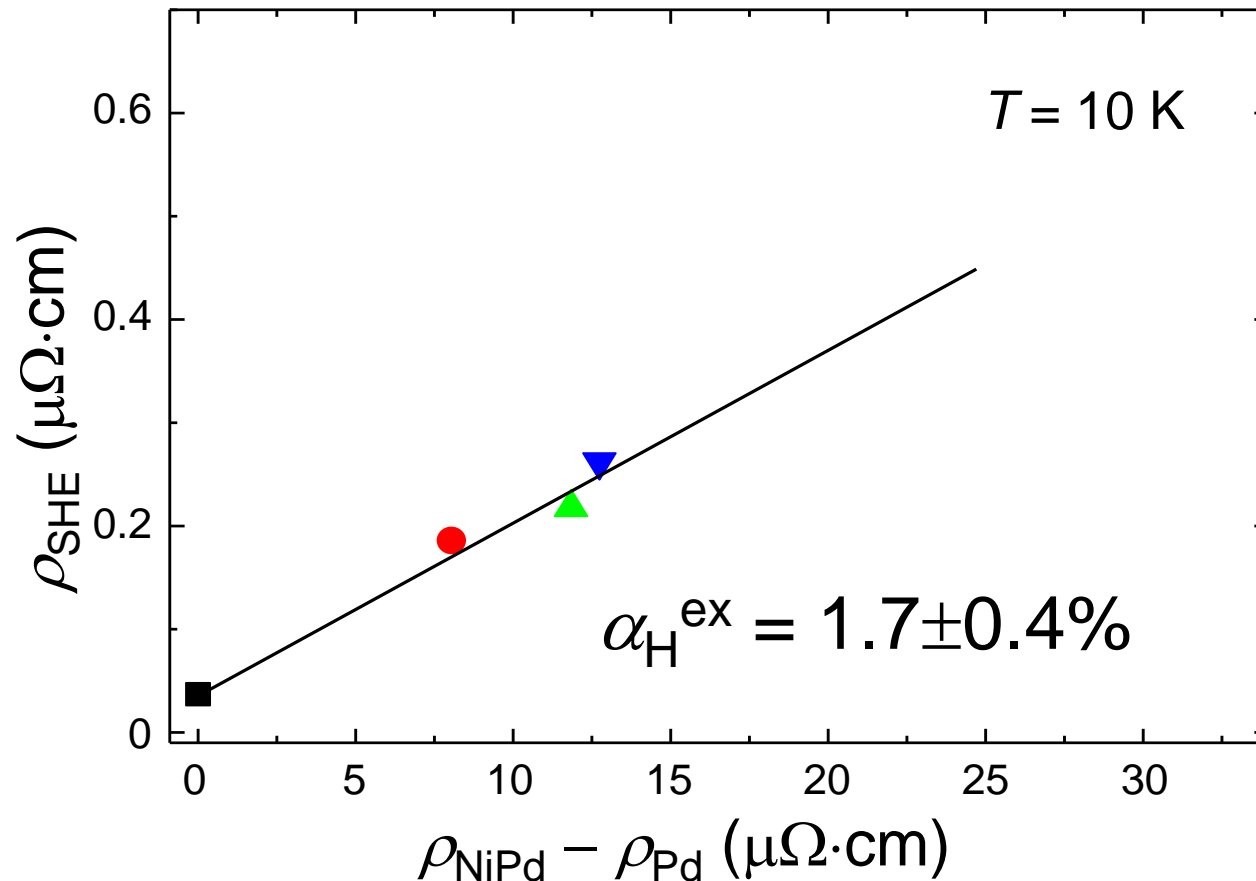


$x = 0.36 \pm 0.08$ which is not so sensitive to ρ_{NiPd}



$$\begin{aligned} \frac{\bar{I}_S}{I_C} &= \frac{\int_0^{t_{\text{NiPd}}} I_S(z) dz}{t_{\text{NiPd}} I_C} \\ &= \frac{\lambda_{\text{NiPd}}}{t_{\text{NiPd}}} \frac{(1 - e^{-2t_{\text{NiPd}}/\lambda_{\text{NiPd}}})^2}{1 - e^{-t_{\text{NiPd}}/\lambda_{\text{NiPd}}}} \frac{I_S(z=0)}{I_C} \end{aligned}$$

Spin Hall resistivity of NiPd as a function of ρ_{imp}

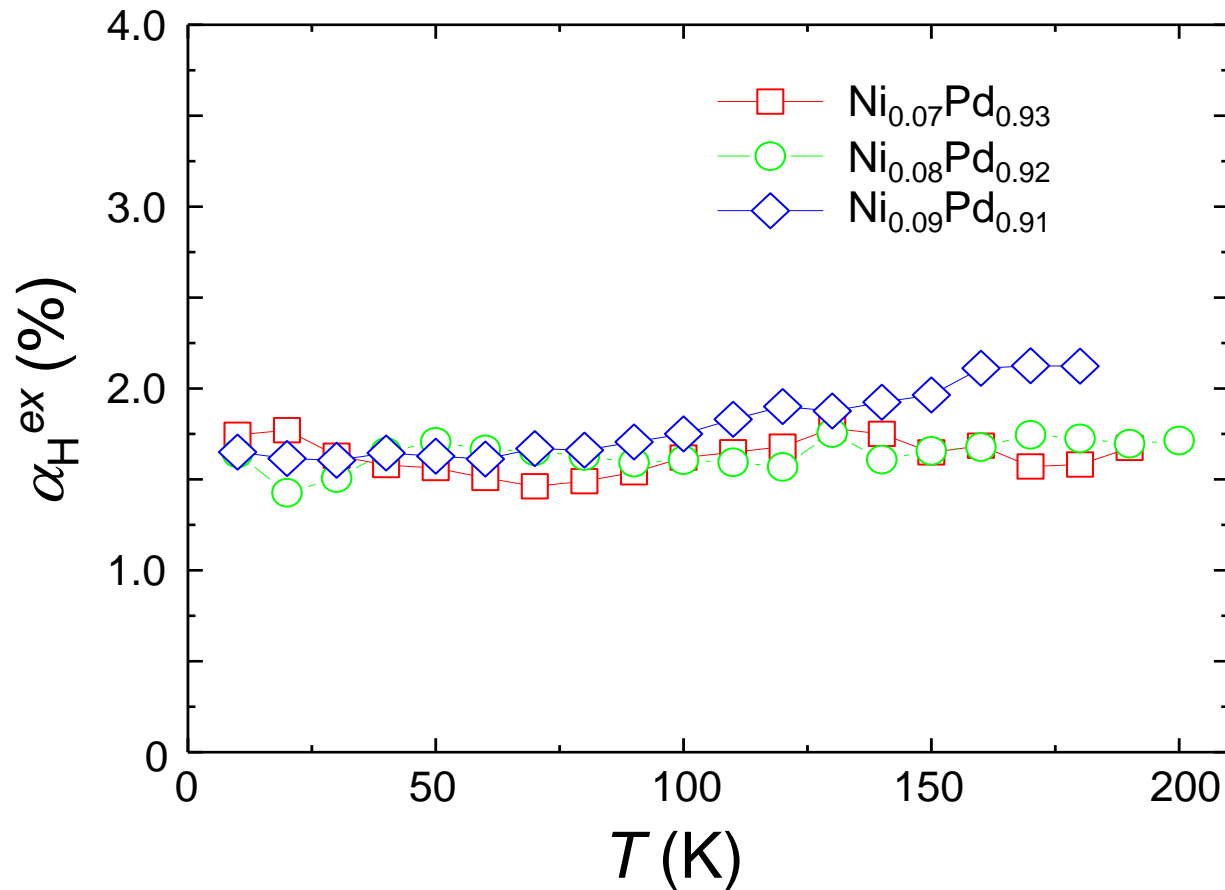


$$\rho_{\text{SHE}} = \rho_{\text{SHE}}^{\text{in}} + \alpha_{\text{H}}^{\text{ex}} \cdot (\rho_{\text{NiPd}} - \rho_{\text{Pd}})$$

$$\alpha_{\text{H}}^{\text{in}} \equiv \rho_{\text{SHE}}^{\text{in}} / \rho_{\text{Pd}} \sim 0.6\%$$

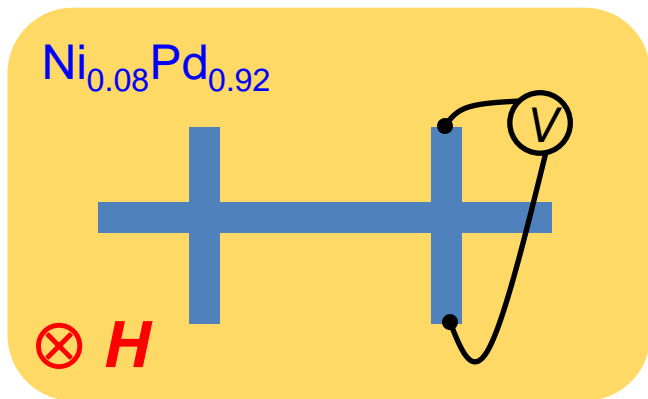
M. Morota, Y. N. *et al.*, PRB **83**, 174405 (2011).

Temperature dependence of SH angle

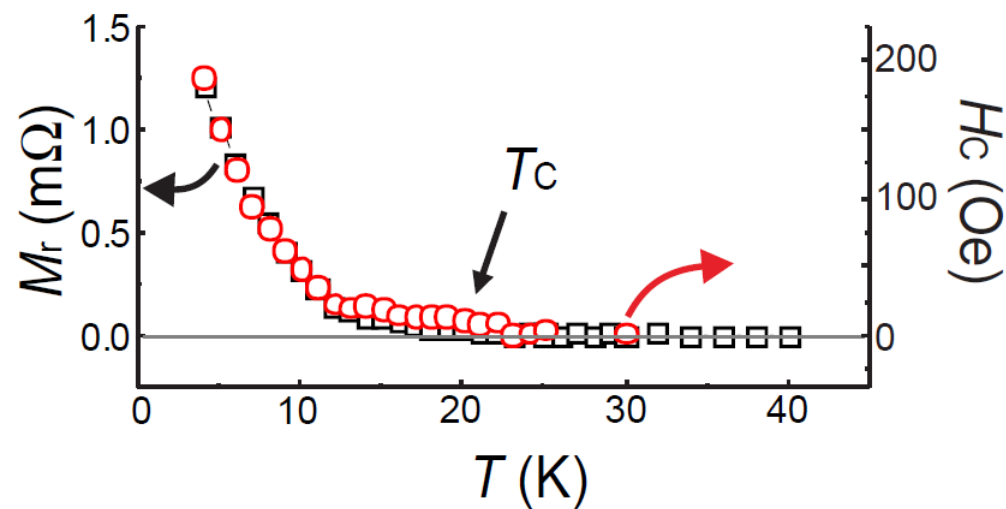
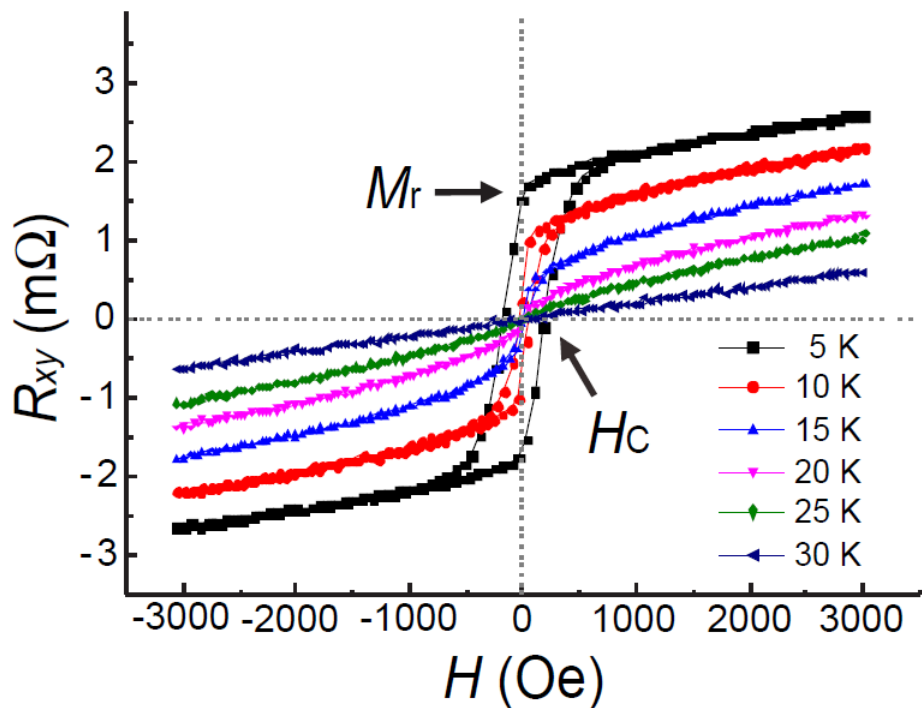


 almost independent of T  skew scattering mechanism

AHE for $\text{Ni}_x\text{Pd}_{1-x}$ alloys

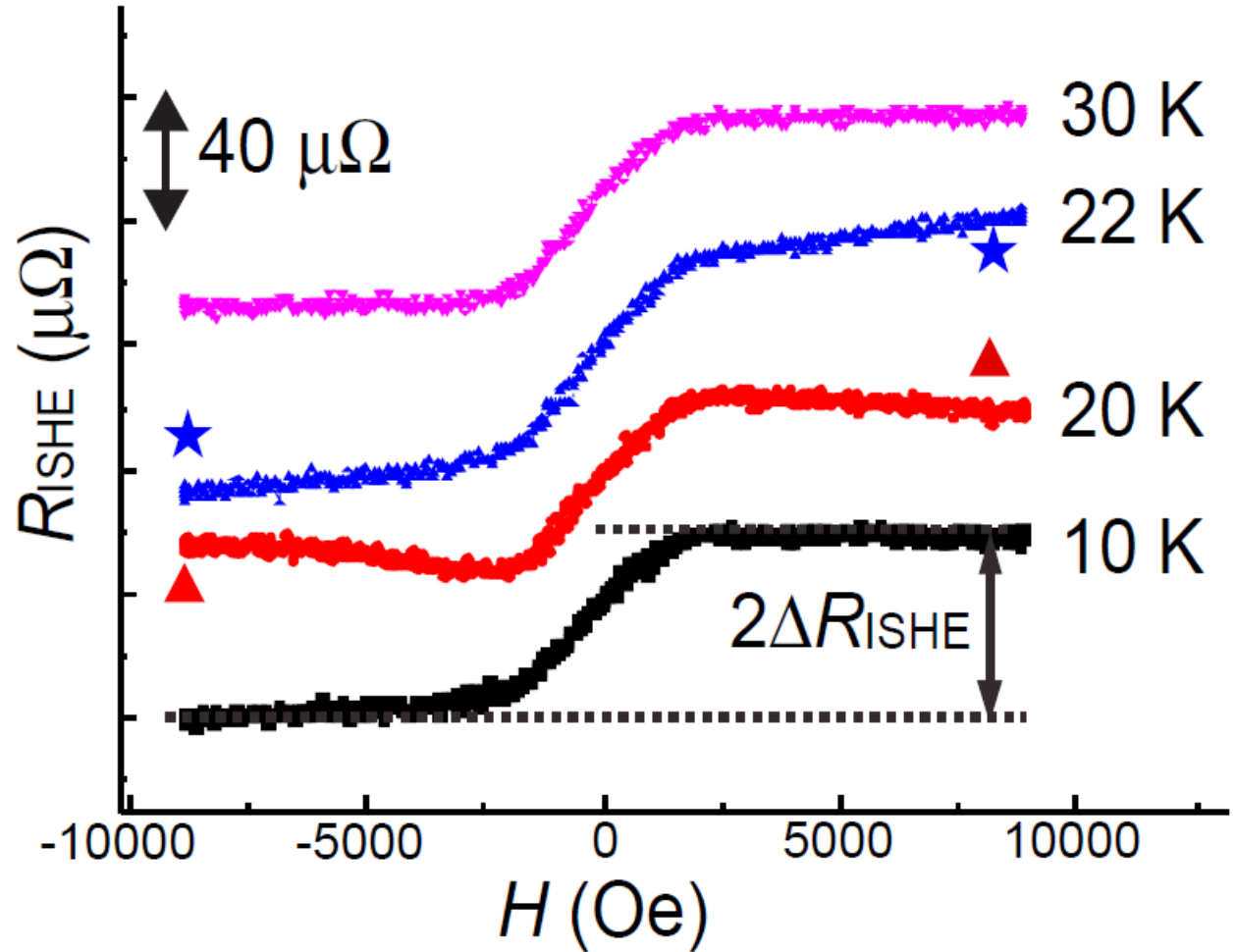
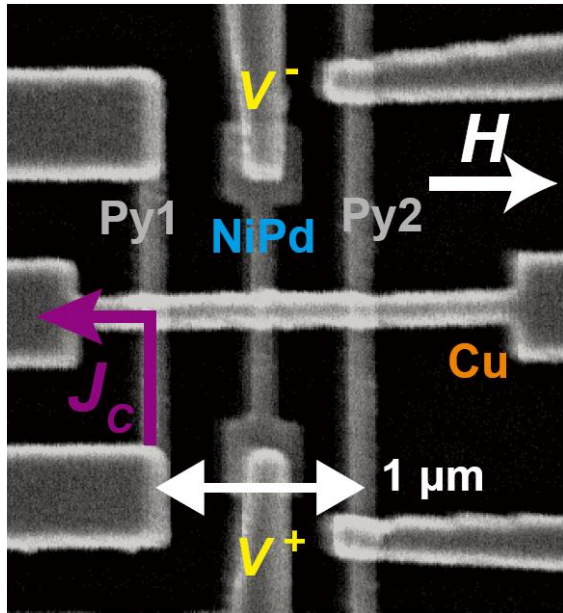


$\text{Ni}_x\text{Pd}_{1-x}$	T_C
$\text{Ni}_{0.07}\text{Pd}_{0.93}$	16 K
$\text{Ni}_{0.08}\text{Pd}_{0.92}$	21 K
$\text{Ni}_{0.09}\text{Pd}_{0.91}$	32 K



ISHE of NiPd near T_c

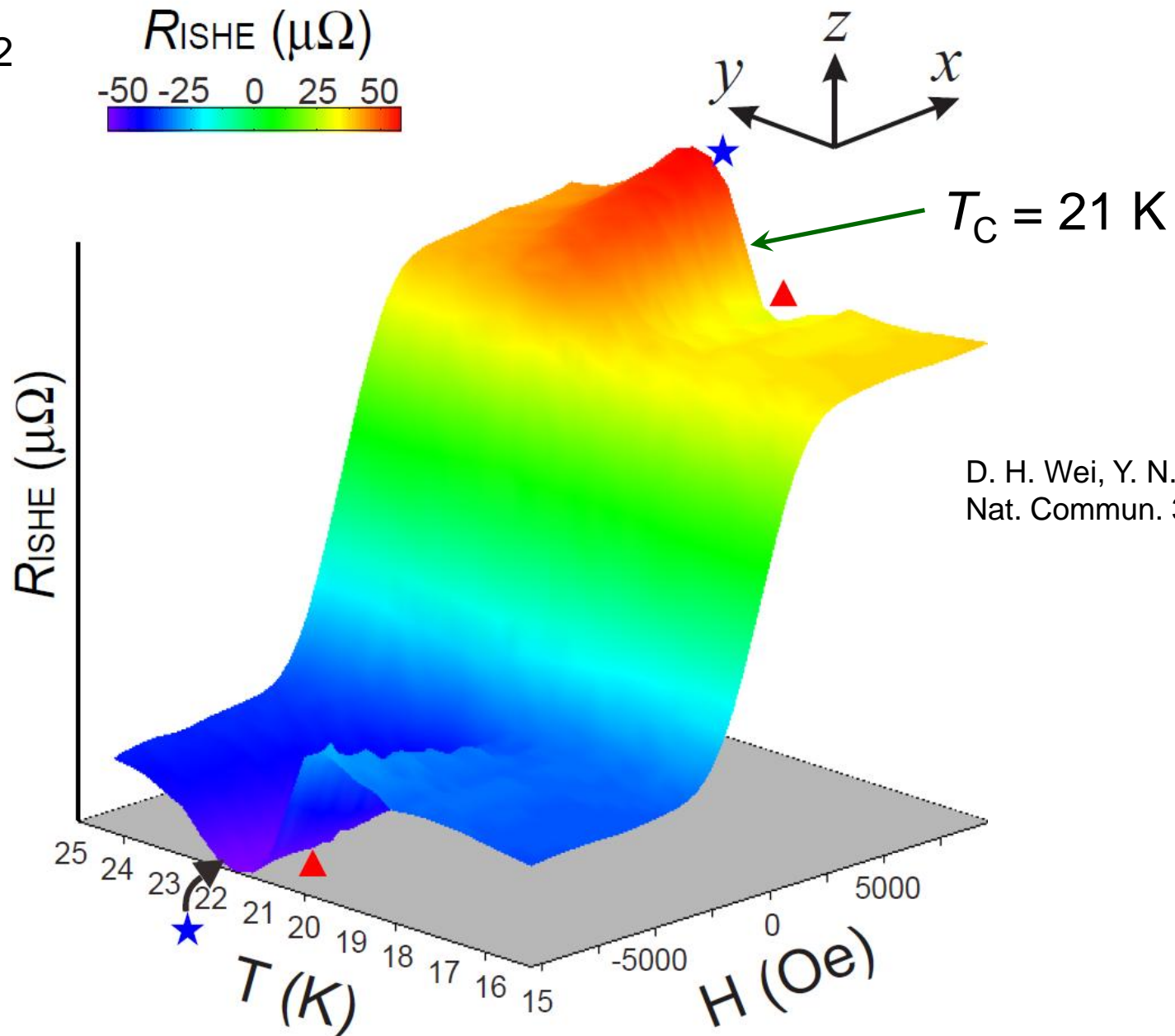
$\text{Ni}_{0.08}\text{Pd}_{0.92}$



D. H. Wei, Y. N. *et al.*,
Nat. Commun. **3**, 1038 (2012).

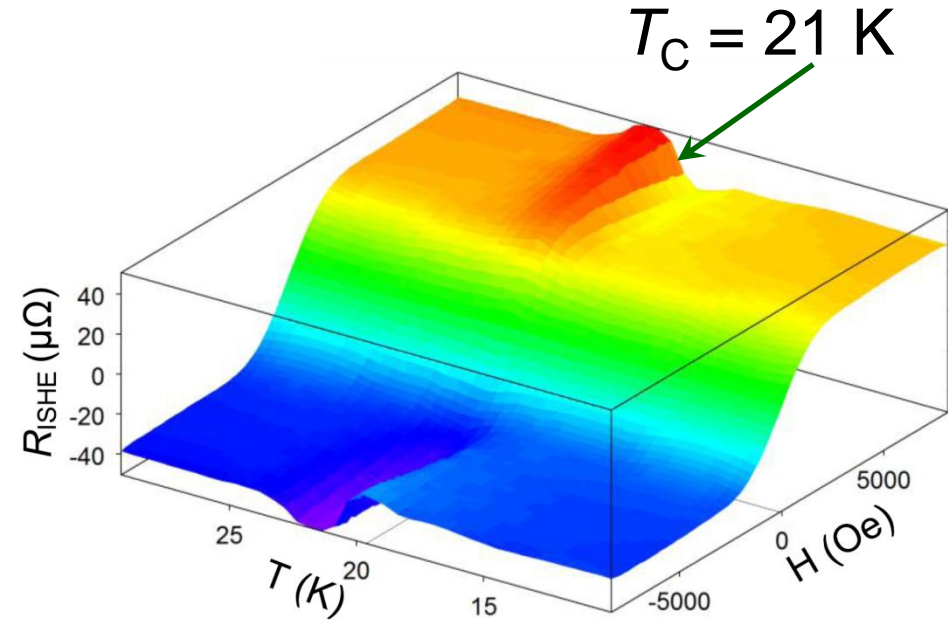
3D plot of R_{ISHE} near T_C

$\text{Ni}_{0.08}\text{Pd}_{0.92}$

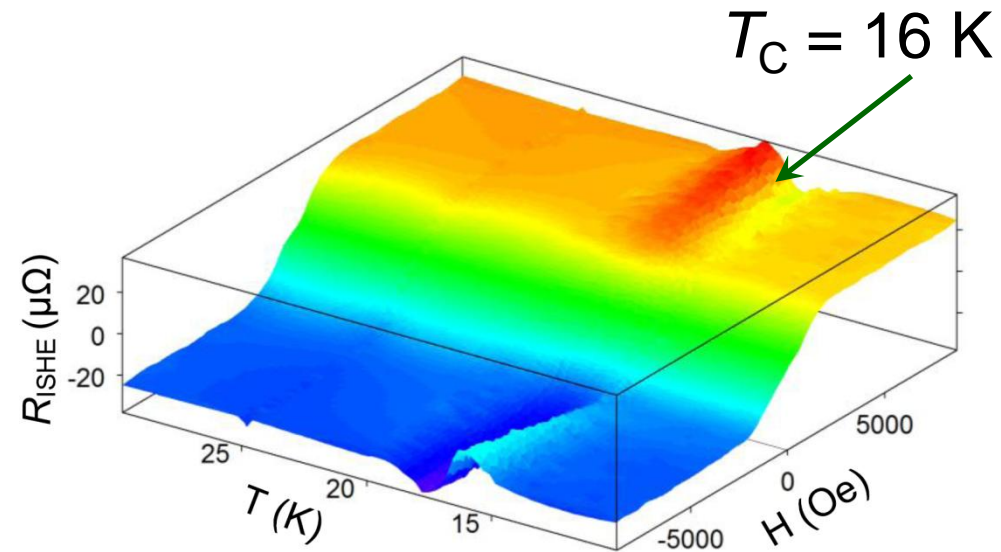


3D plot of R_{ISHE} near T_C

$Ni_{0.08}Pd_{0.92}$



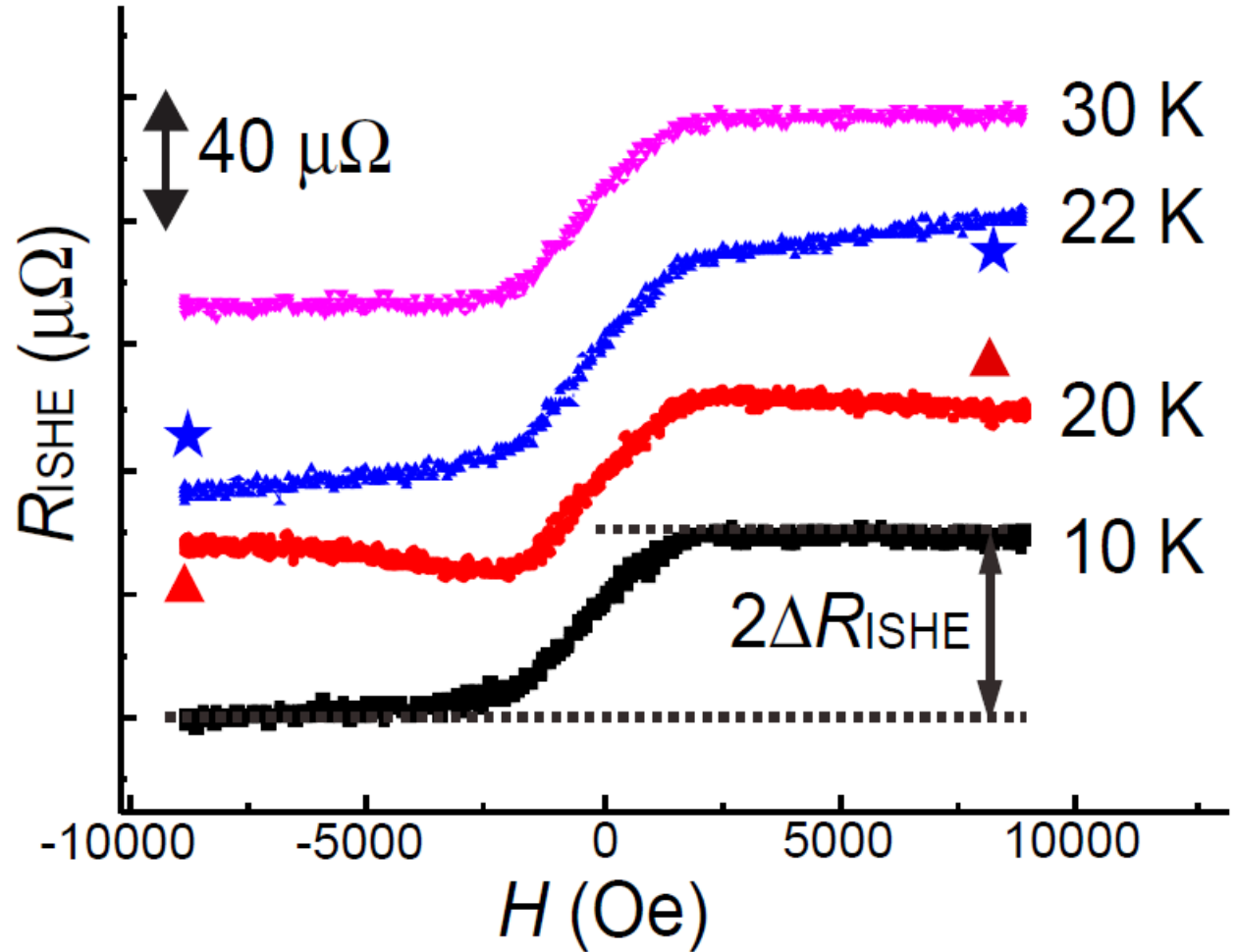
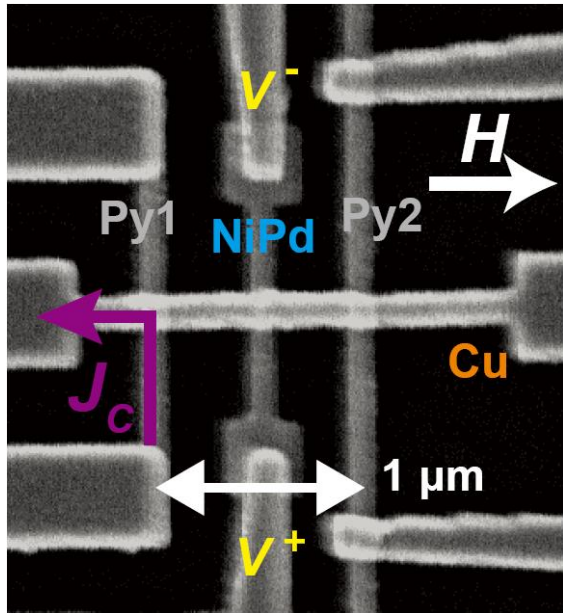
$Ni_{0.07}Pd_{0.93}$



↑ The anomalous behavior near T_C is quite reproducible!

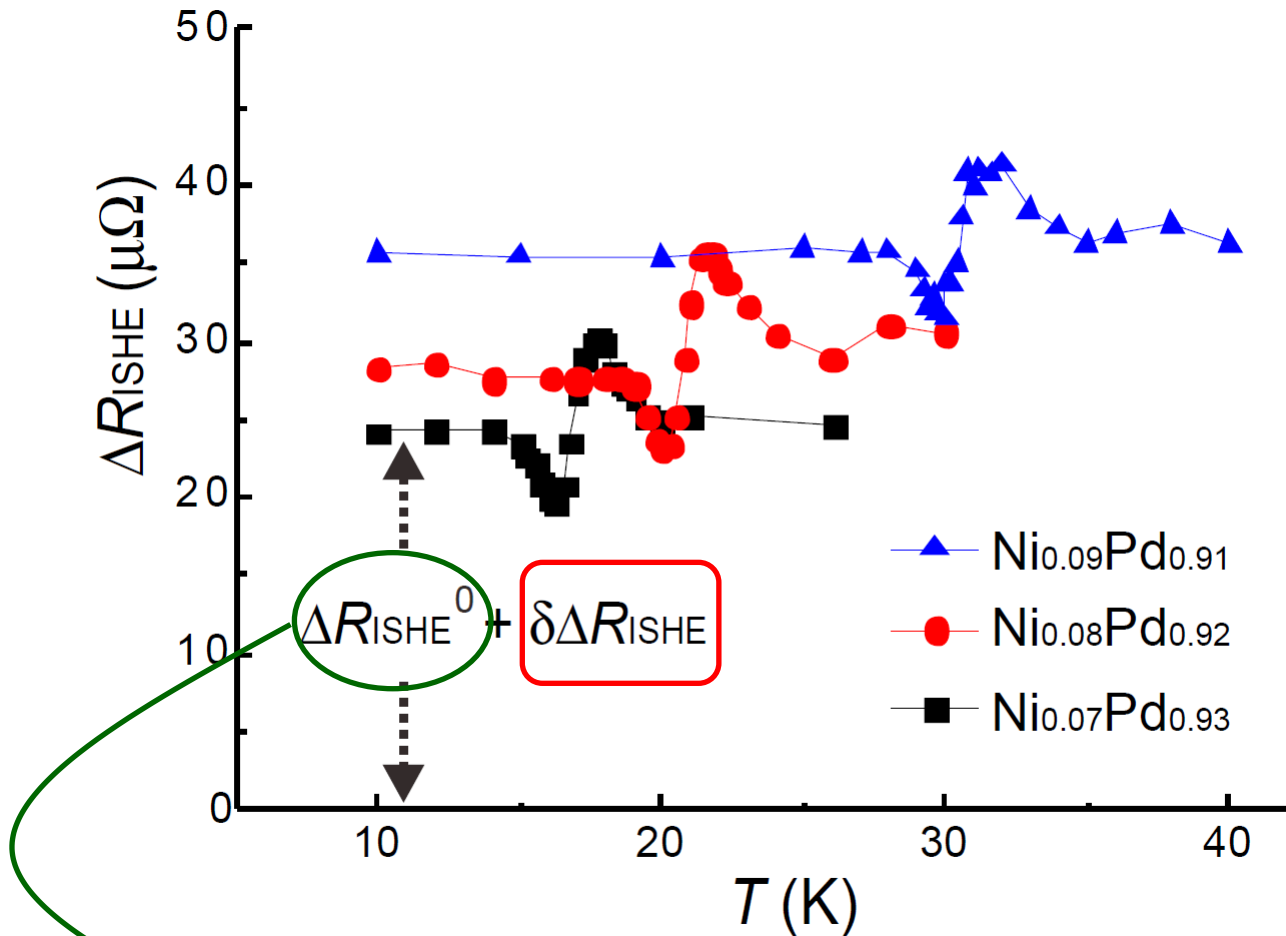
ISHE of NiPd near T_c

$\text{Ni}_{0.08}\text{Pd}_{0.92}$



D. H. Wei, Y. N. *et al.*,
Nat. Commun. **3**, 1038 (2012).

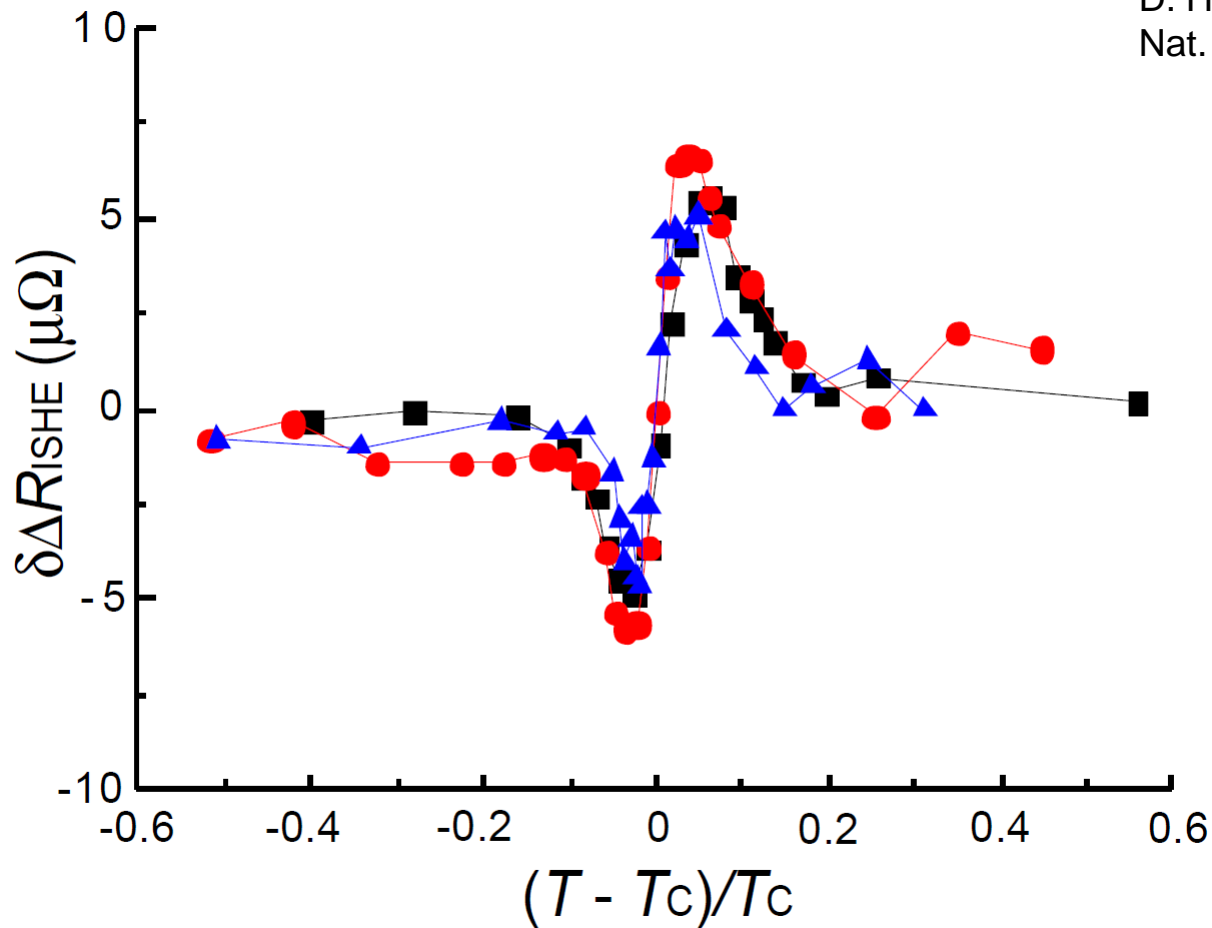
Anomalous part of R_{ISHE} near T_C



Extrinsic SHE induced by impurities

$\delta\Delta R_{ISHE}$ as a function of reduced temperature

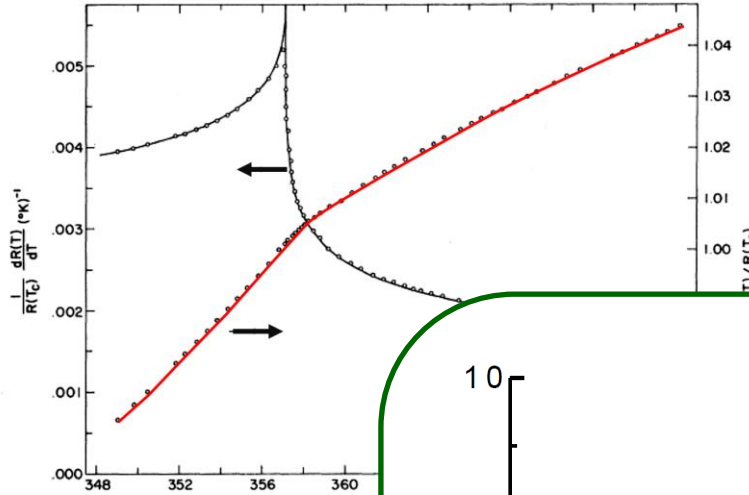
D. H. Wei, Y. N. *et al.*,
Nat. Commun. **3**, 1038 (2012).



- The anomalous part is almost independent of the Ni concentration.

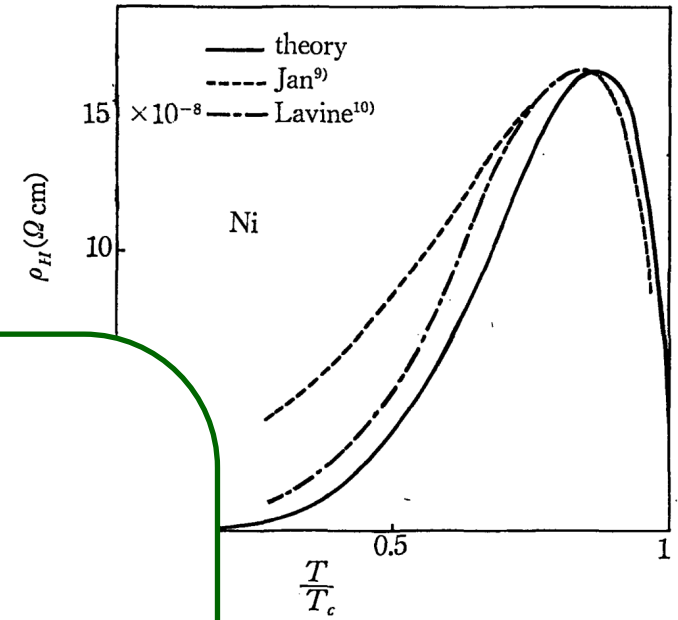
Anomalies near T_c

Anomaly in ρ_{xx} of pure Ni

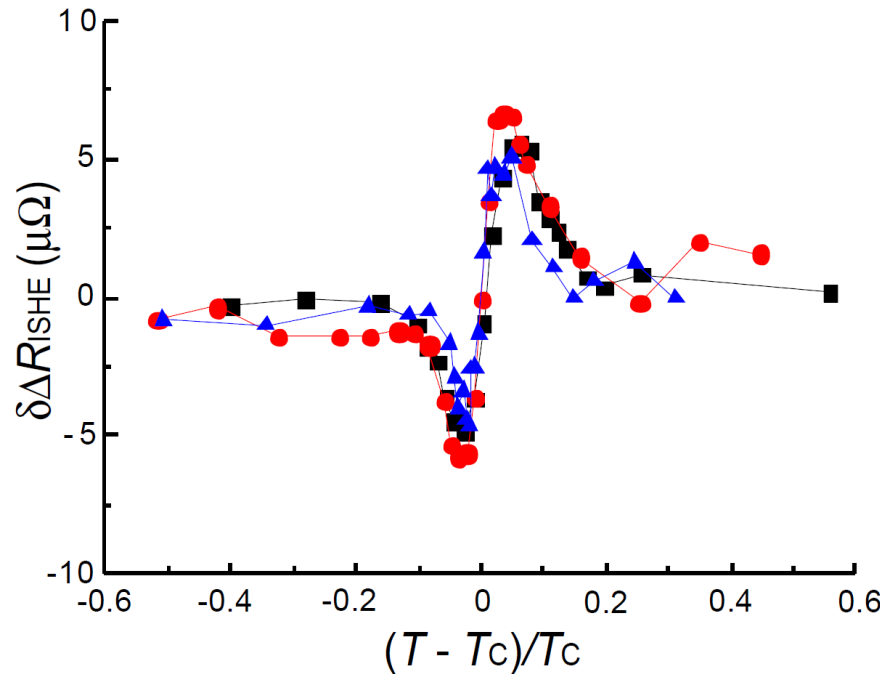


F. C. Zumsteg *et al.*,

Anomaly in ρ_{xy} of pure Ni



iv. Phys. Acta **25**, 677 (1952).
Phys. Rev. **123**, 1273 (1961).



Anomaly in R_{ISHE} of NiPd alloys

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↑ Comparison with theory

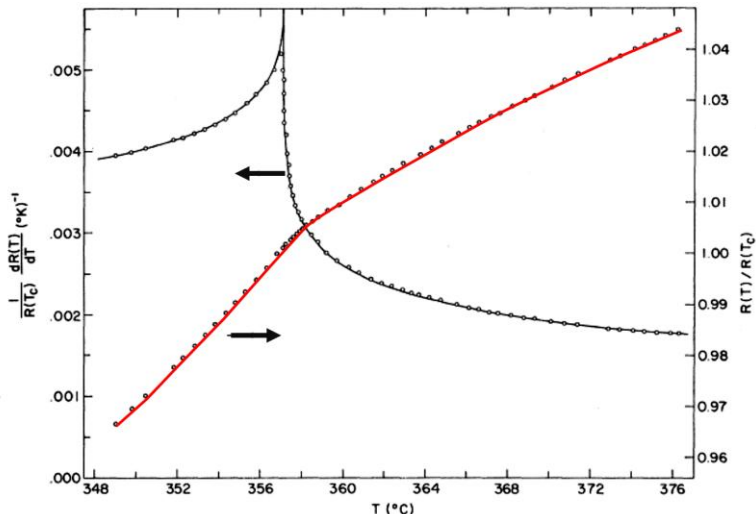
- ↓ *extended Kondo's model*

↑ Summary

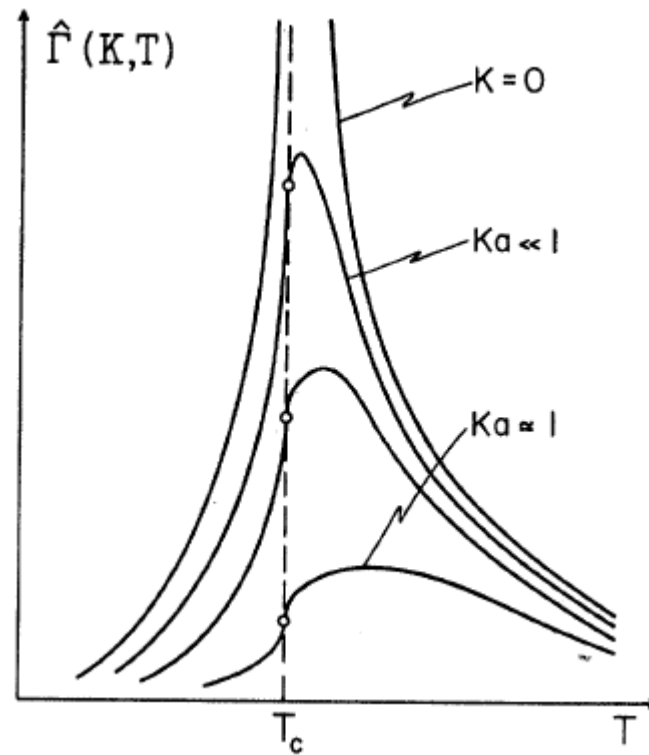
To explain the anomaly in ρ_{xx} near T_C ...

M. E. Fisher and J. S. Langer, PRL **20**, 1344 (1967).

Anomaly in ρ_{xx} of pure Ni



F. C. Zumsteg *et al.*, PRL **24**, 520 (1967).



$$\rho_{xx} \propto \langle (m - \langle m \rangle)^2 \rangle \propto \chi_0$$

first Born approximation!!

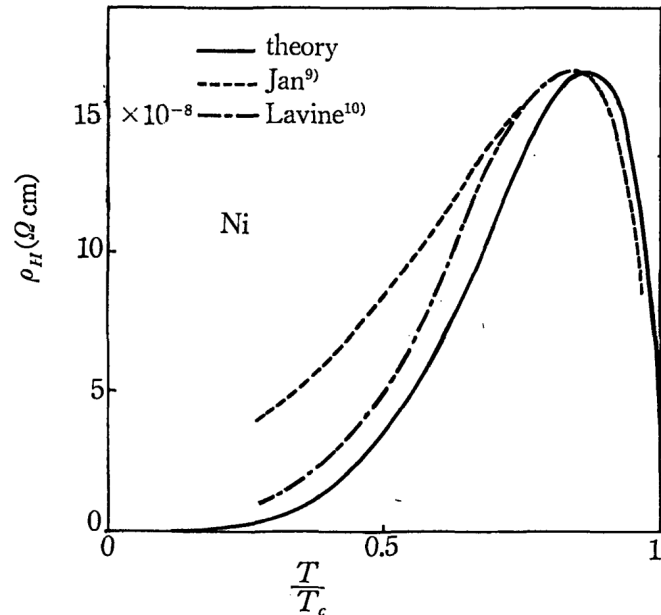
Smearing of ρ_{xx} just on T_C

$$\rho_{xx} \propto \langle (m_i - \langle m_i \rangle)(m_j - \langle m_j \rangle) \rangle \propto \chi_0^{\text{loc}}$$

contributions come only when i and j are within a certain cutoff distance.

To explain the anomaly in ρ_{xy} below T_C ...

Anomaly in ρ_{xy} of pure Ni



J. P. Jan, Helv. Phys. Acta **25**, 677 (1952).
J. M. Lavine, Phys. Rev. **123**, 1273 (1961).

➤ Karplus & Luttinger's theory

R. Karplus & J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).

$$\rho_{xy} \propto \rho_{xx}^2 \langle m \rangle$$

➤ Kondo's theory

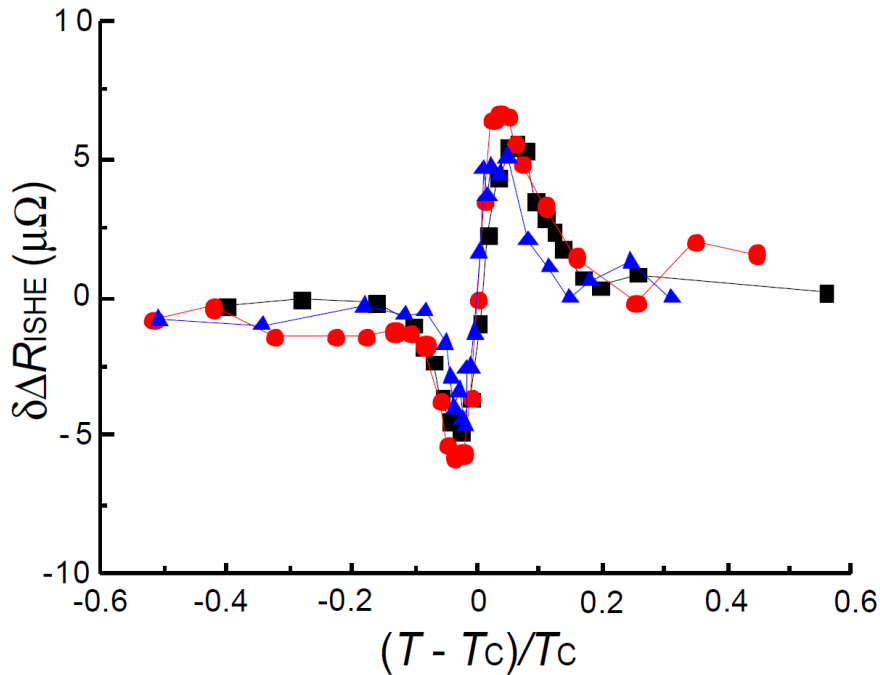
J. Kondo, Prog. Theor. Phys. **27**, 772 (1962).

$$\rho_{xy} \propto \langle (m - \langle m \rangle)^3 \rangle$$

second Born approximation!!

To explain the anomaly in R_{ISHE} near T_C ...

Anomaly in R_{ISHE} of NiPd alloys



D. H. Wei, Y. N. et al., Nat. Commun. **3**, 1038 (2012).

➤ Karplus & Luttinger's theory

R. Karplus & J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).

$$\rho_{xy} \propto \rho_{xx}^2 \langle m \rangle$$

For ISHE, $\langle m \rangle$ vanishes above T_C .

➤ Kondo's theory

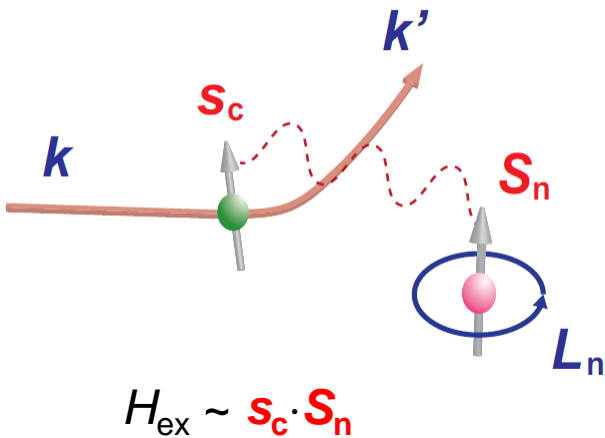
J. Kondo, Prog. Theor. Phys. **27**, 772 (1962).

$$\rho_{xy} \propto \langle (m - \langle m \rangle)^3 \rangle$$

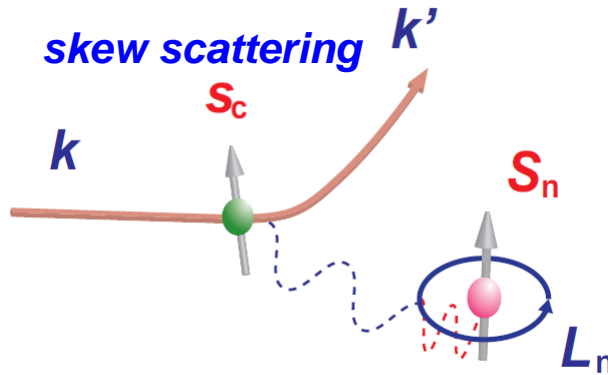
The Kondo's theory has to be extended for the ISHE configuration.

Kondo's theory

s-d exchange interaction

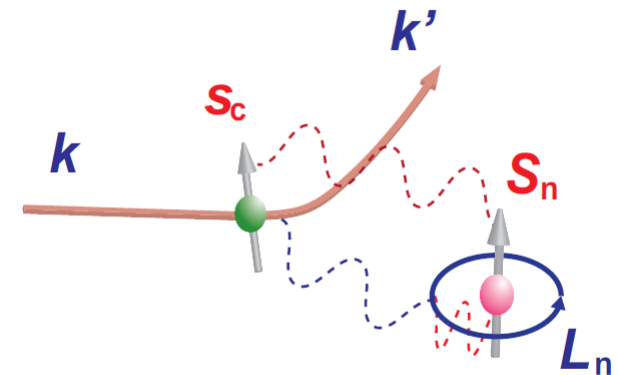


spin-orbit interaction (1)



$$H_{\text{SO}}^{(1)} \sim (\mathbf{L}_n \cdot (\mathbf{k} \times \mathbf{k}')) (\mathbf{S}_n \cdot \mathbf{L}_n) \\ \sim (\mathbf{k} \times \mathbf{k}') \cdot \mathbf{S}_n$$

spin-orbit interaction (2)



$$H_{\text{SO}}^{(2)} \sim H_{\text{SO}}^{(1)} \cdot H_{\text{ex}} \\ \sim (\mathbf{L}_n \cdot (\mathbf{k} \times \mathbf{k}')) (\mathbf{S}_n \cdot \mathbf{L}_n) (\mathbf{S}_n \cdot \mathbf{s}_c) \\ \sim (\mathbf{k} \times \mathbf{k}') \cdot \mathbf{s}_c \cdot \mathbf{S}_n \mathbf{S}_n$$

To obtain the transition probability from \mathbf{k} to \mathbf{k}'

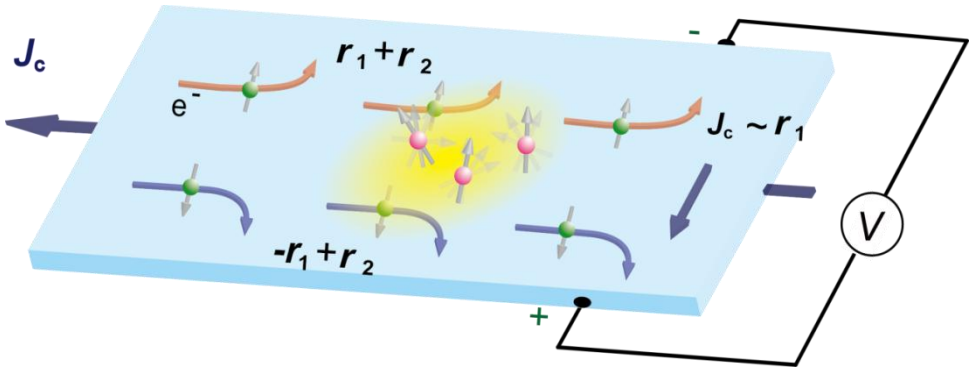
$$SO(1): \langle H_{\text{ex}} H_{\text{SO}}^{(1)} H_{\text{ex}} \rangle \sim \langle \mathbf{S}_n^3 \rangle \langle \mathbf{s}_c^2 \rangle \equiv r_1 \langle \mathbf{s}_c^2 \rangle \longrightarrow r_1 : \text{third-order spin correlation}$$

$$SO(2): \langle H_{\text{ex}} H_{\text{SO}}^{(2)} H_{\text{ex}} \rangle \sim \langle \mathbf{S}_n^4 \rangle \langle \mathbf{s}_c^3 \rangle \equiv r_2 \langle \mathbf{s}_c^3 \rangle \longrightarrow r_2 : \text{fourth-order spin correlation}$$

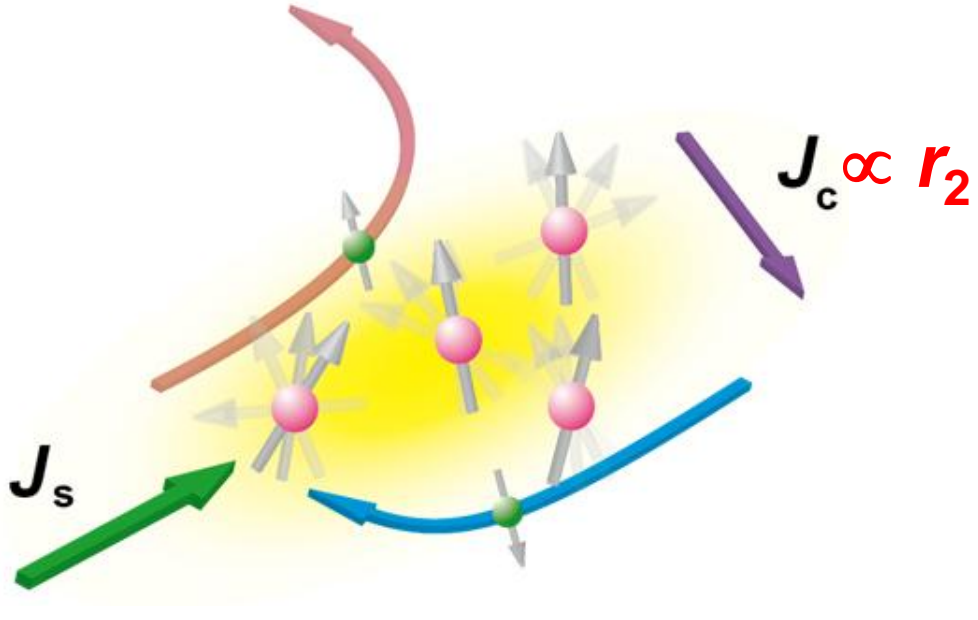
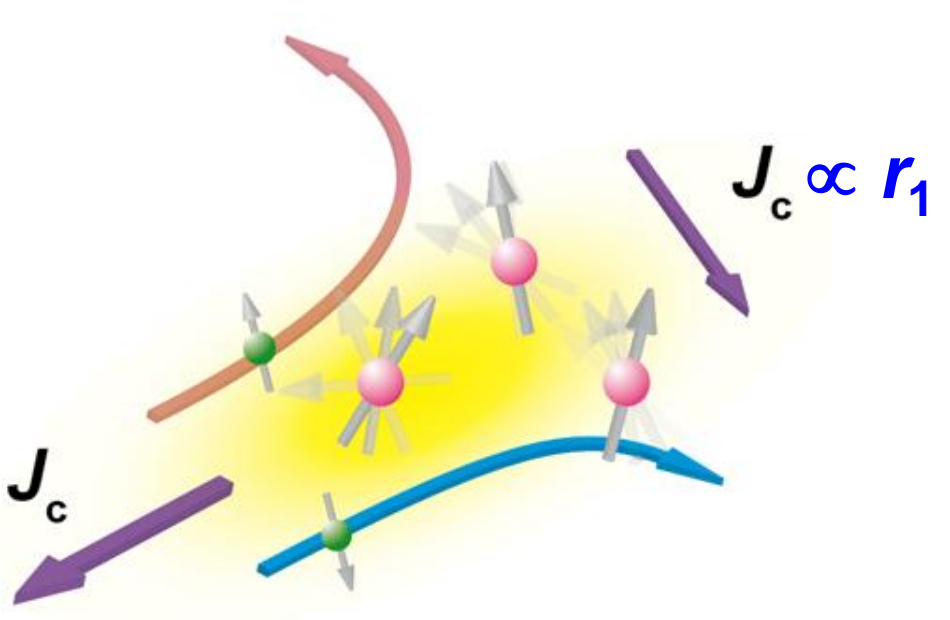
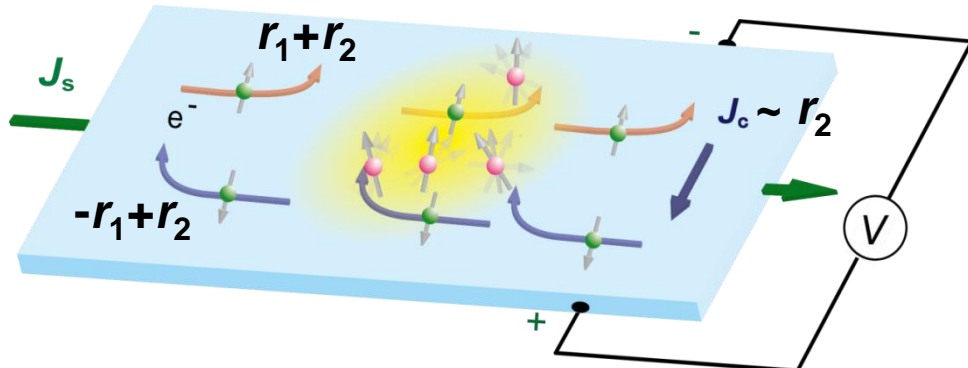
** $H_{\text{SO}}^{(1)}$ and $H_{\text{SO}}^{(2)}$ appear in the s-d Hamiltonian to the same order with respect to the SO coupling constant λ of the localized moment.

AHE vs ISHE

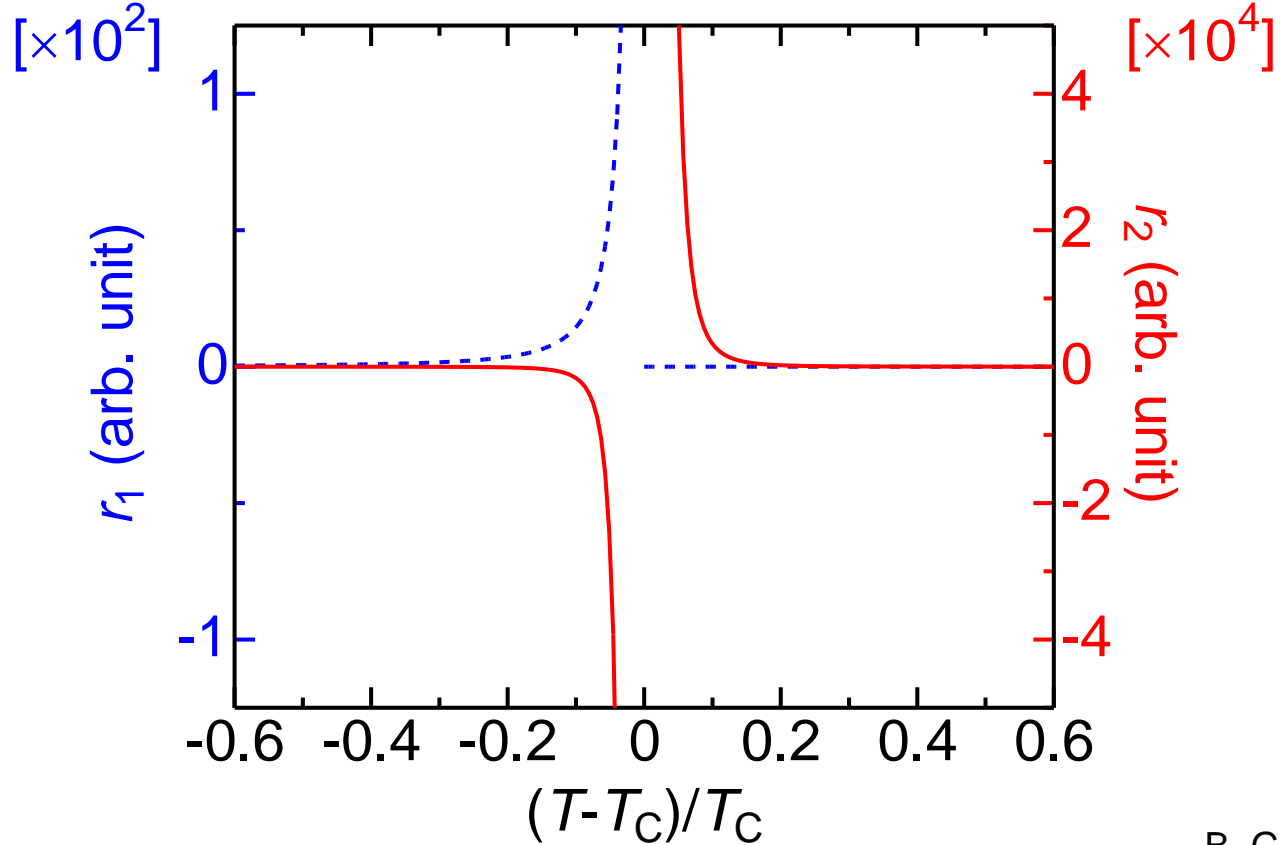
AHE



ISHE



Temperature dependence of r_1 and r_2



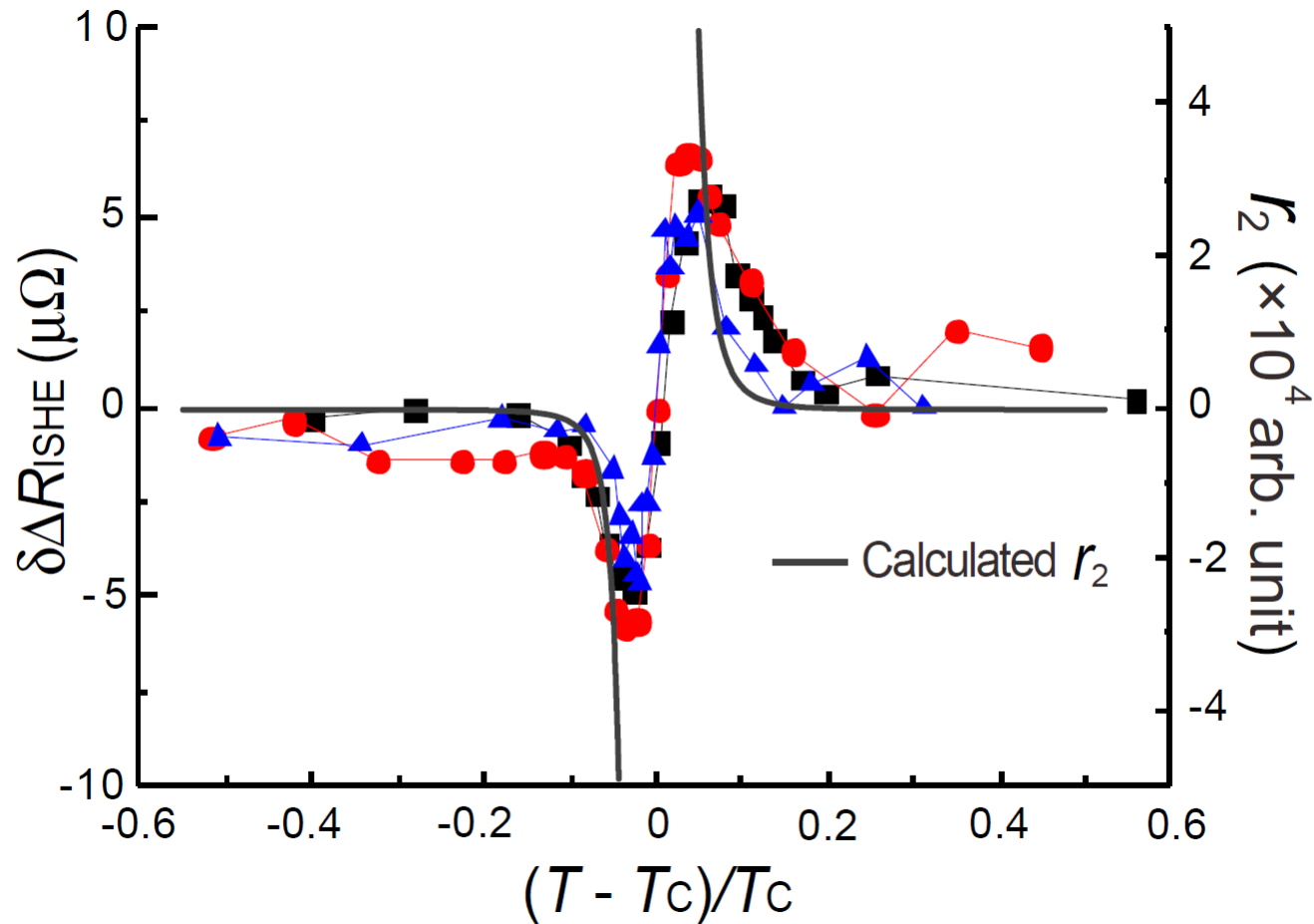
B. Gu, T. Ziman, and S. Maekawa,
PRB **86**, 241303(R) (2012).

$$r_1 = \langle (m - \langle m \rangle)^3 \rangle \propto \chi_1$$

$$r_2 = 2c_2 \langle (m - \langle m \rangle)^2 (m^2 - \langle m^2 \rangle) \rangle \propto 2c_2 \chi_2$$

$c_2 = -1/2$ for Ni

Anomalous part of R_{ISHE} near T_C



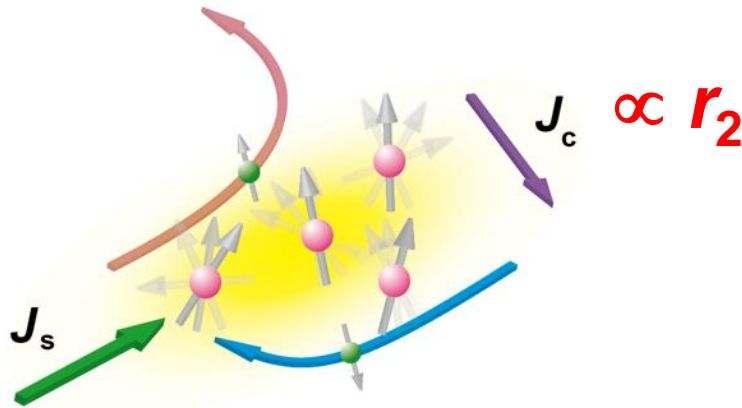
- The anomaly in ρ_{xx} near T_C can be explained by r_2 except for just on T_C .

Comparison to uniform non-linear susceptibilities

$$\mathbf{M}_{tot} = \mathbf{M}_{tot,0}(T) + \chi_0^{uni} \mathbf{H} + \chi_1^{uni} \mathbf{H}^2 + \chi_2^{uni} \mathbf{H}^3 + \dots,$$

$$\chi_1^{uni} = \beta^2 \left\langle \left(\mathbf{M}_{tot} - \langle \mathbf{M}_{tot} \rangle \right)^3 \right\rangle \text{ 1st order nonlinear susceptibility}$$

$$\delta \Delta R_{ISHE} \propto r_2 \propto \chi_2^{uni} \approx \beta^3 \left\langle \left(\mathbf{M}_{tot} - \langle \mathbf{M}_{tot} \rangle \right)^4 \right\rangle \text{ 2nd order nonlinear susceptibility}$$



Summary

- ❖ We have studied inverse spin Hall effects (ISHE) of weakly ferromagnetic $\text{Ni}_x\text{Pd}_{1-x}$ alloys ($x = 0.07 \sim 0.09$). **Anomalies in R_{ISHE} near T_C were clearly observed for each Ni concentration.** The shape of the anomaly is asymmetric with respect to T_C and does not depend on x .
- ❖ The experimentally observed anomaly can be well-explained with the generalized version of **the Kondo's theory**, which was **originally developed to explain the anomaly in anomalous Hall effect (AHE)** in pure Ni and Fe.
- ❖ **The higher-order spin fluctuations (i.e., r_1 and r_2 terms)** introduced in the generalized Kondo's theory give an **essential difference between SHE and AHE.**