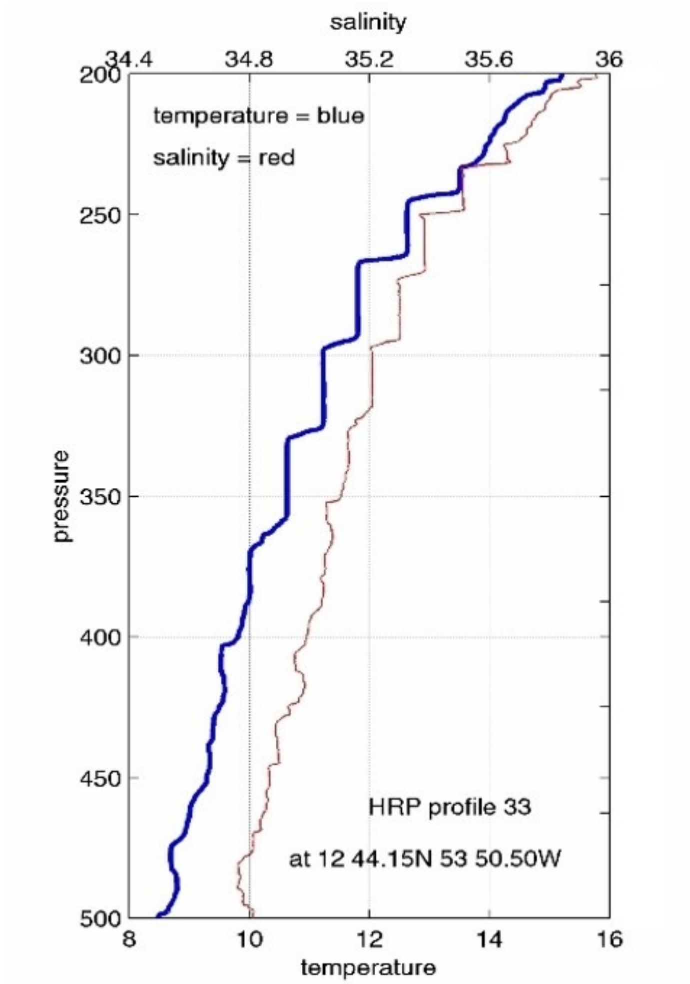
Thermohaline layering in tropical and mid-latitude oceans

Timour Radko (NPS)



Schmitt et al. (2005)

What is the origin of *permanent* staircases?

A “simple” answer:
Double-Diffusive Convection

A set processes driven by the difference in the molecular diffusivities of density components (e.g., temperature and salinity of sea-water)

$$\begin{cases} \dfrac{d\vec{\mathrm{v}}_{tot}}{dt} = -\dfrac{\nabla p}{\rho_0} + g(\alpha T - \beta S)\vec{k} + \nu\nabla^2\vec{\mathrm{v}} \\ \nabla\cdot\vec{\mathrm{v}} = 0 \\ \dfrac{dT}{dt} + w\bar{T}_z = k_T\nabla^2 T \\ \dfrac{dS}{dt} + w\bar{S}_z = k_S\nabla^2 S \end{cases}$$

Governing equations
(Navier-Stokes, incompressible)

$$k_T \approx 1.4\cdot 10^{-7}\, m^2/s$$

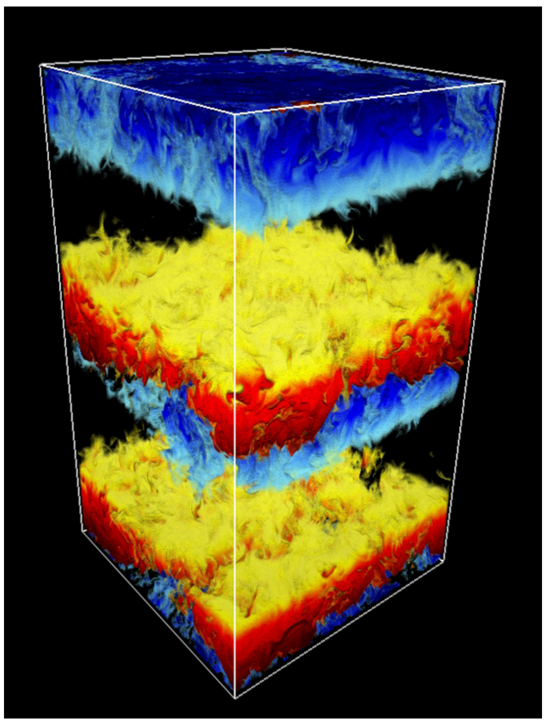
$$k_S \approx 1.1\cdot 10^{-9}\, m^2/s$$

$$k_S \ll k_T$$

Double-Diffusive Convection

salt fingering

warm, salty

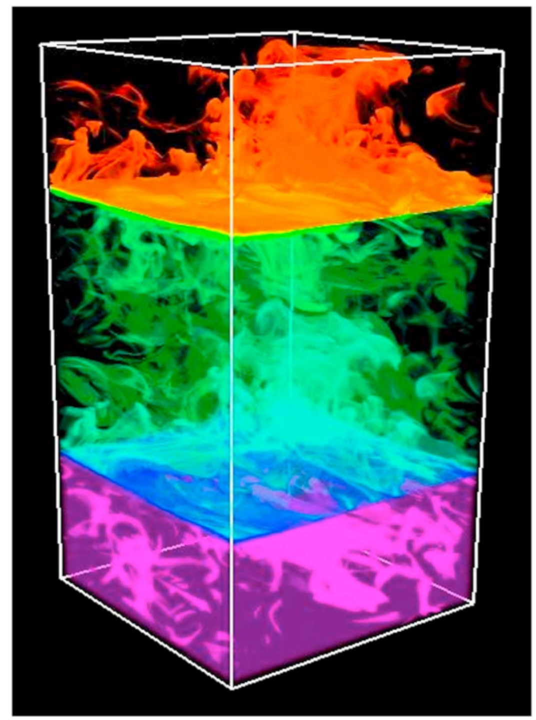


cold, fresh

$$T = T_{tot} - \overline{T}$$

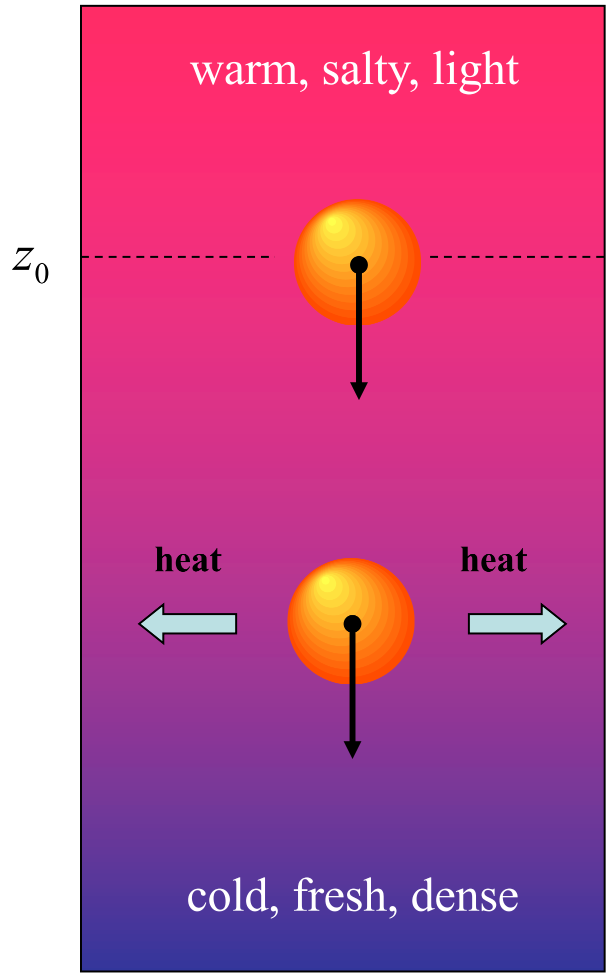
diffusive convection

cold, fresh



warm, salty

Salt Fingers (Physics)



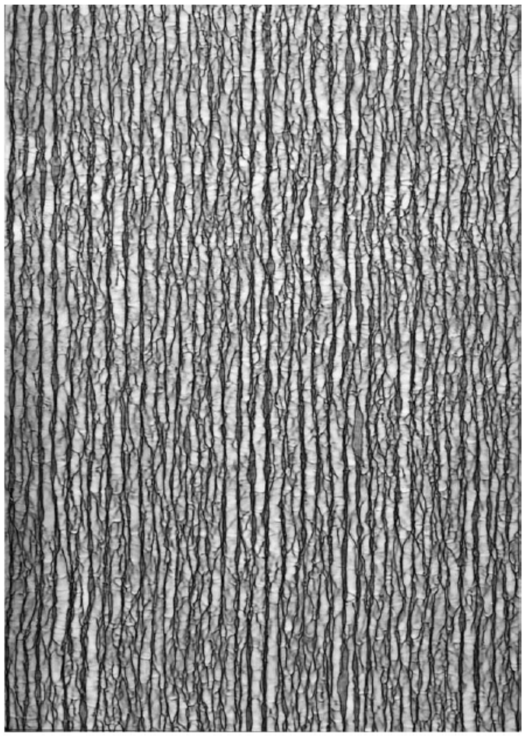
Stern (1960)

Instability of convectively stable (bottom-heavy) systems

Salt Fingers

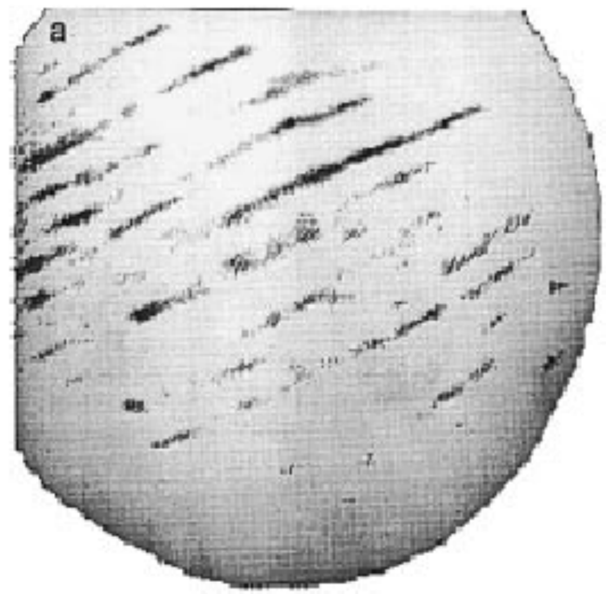
lab experiments

sugar / salt



Krishnamurti (2003)

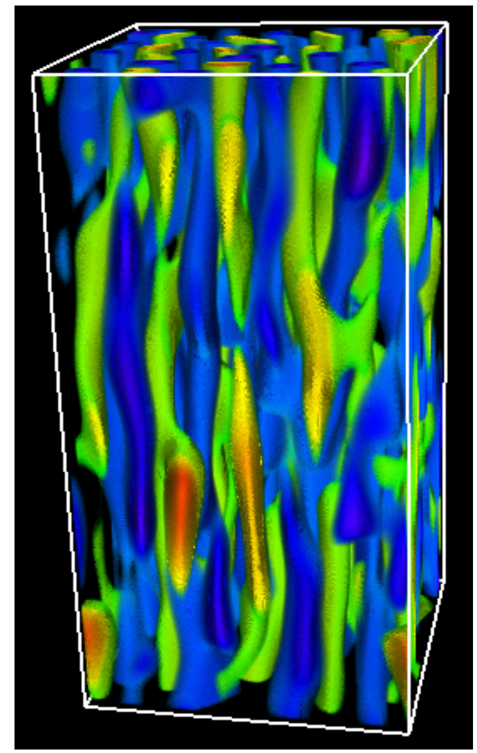
the ocean



10cm

St. Laurent and Schmitt (1999)

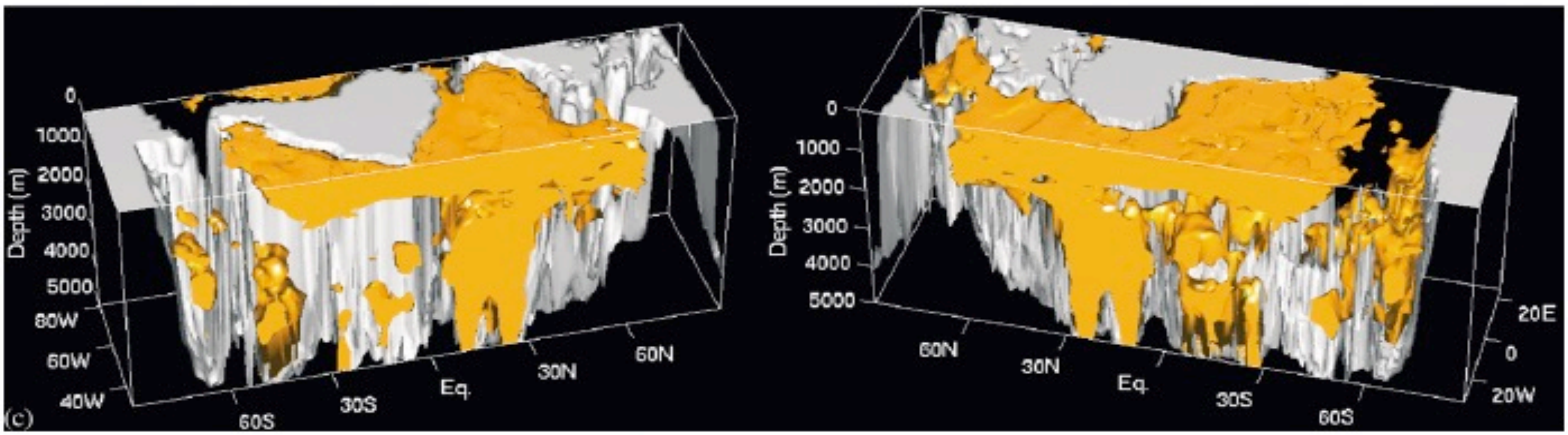
simulations



Radko and Smith (2012)

Favorable conditions for salt fingers

$$\frac{\partial T}{\partial z} > 0, \quad \frac{\partial S}{\partial z} > 0, \quad \frac{\partial \rho}{\partial z} < 0$$ ~30% of the world ocean



You (2002)

large-scale stratification

95% of the main thermocline in the Atlantic is unstable to salt-fingering

Some of us don't need a reason to love salt fingers.

For others:

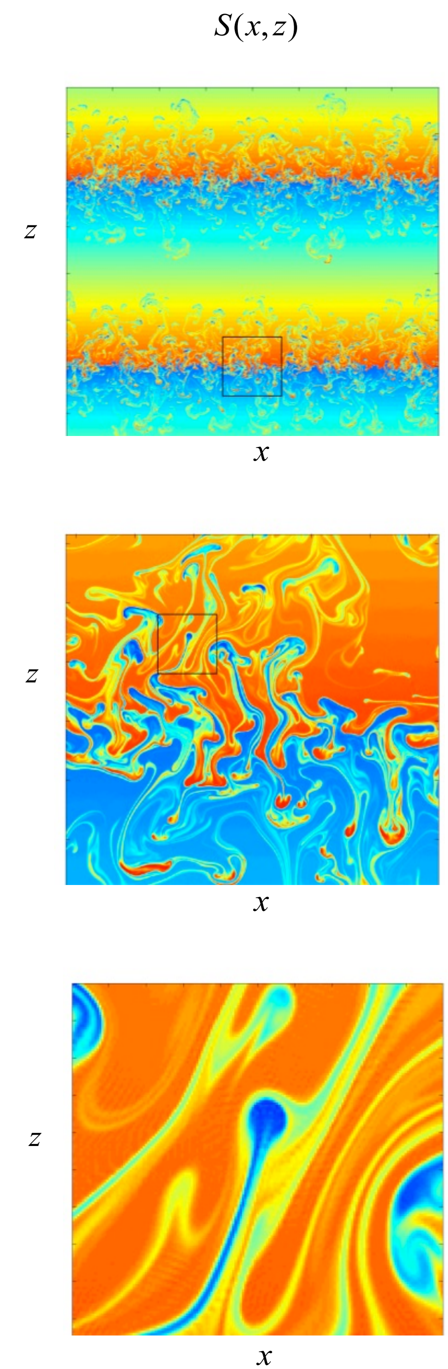
- SF frequently control vertical mixing
 (St Laurent and Schmitt, 1999; Schmitt et al., 2005)

- SF often determine the pattern of the *T-S* relationship
 (Schmitt, 1981)

- SF produce lateral intrusions (Stern, 1967)

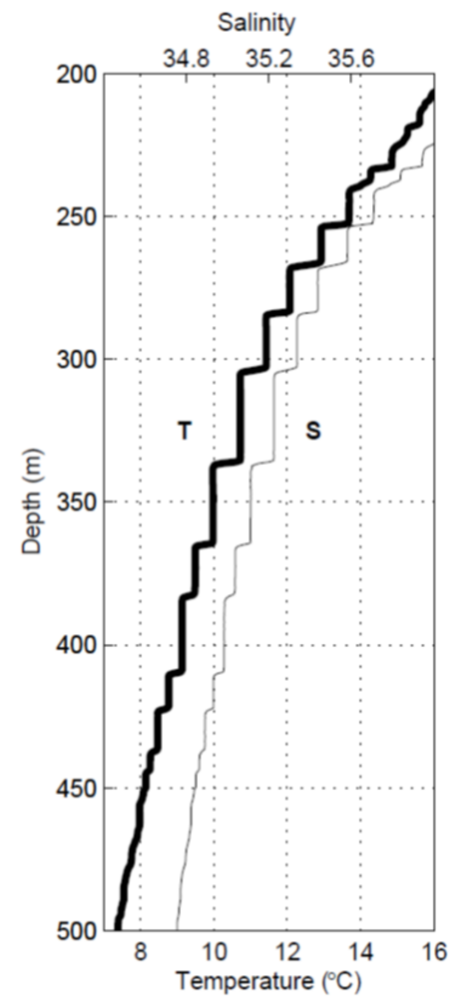
… and the most amazing phenomenon …

- SF produce **thermohaline staircases**



Thermohaline staircases:

- Fascinating effect
- Naval applications
- **Vertical mixing**



Schmitt et al. (2005)

Eddy diffusivities in the NA Central thermocline:

K_s=1 cm^2/s – salt fingers forming the staircase (Schmitt et al., 2005; Veronis, 2007);
K_s= 0.1 cm^2/s – salt fingers in a smooth gradient (St. Laurent and Schmitt, 1999; Stern et al., 2001);
K_{turb}= 0.05 cm^2/s – overturning gravity waves (Gregg, 1989; Polzin et al., 1995)

Thermohaline staircases: We should know what is going on

Salt flux due to salt fingers through the Caribbean staircase:

$$F_{SF} \approx A_{C-SALT} K_S \frac{\partial S}{\partial z} \sim 3.5 \cdot 10^6 \,\text{psu m}^3\,\text{s}^{-1}$$

Salt flux due to internal waves through the entire subtropical gyre:

$$F_{turb} \approx A_{GYRE} K_S \frac{\partial S}{\partial z} \sim 1.5 \cdot 10^6 \,\text{psu m}^3\,\text{s}^{-1}$$

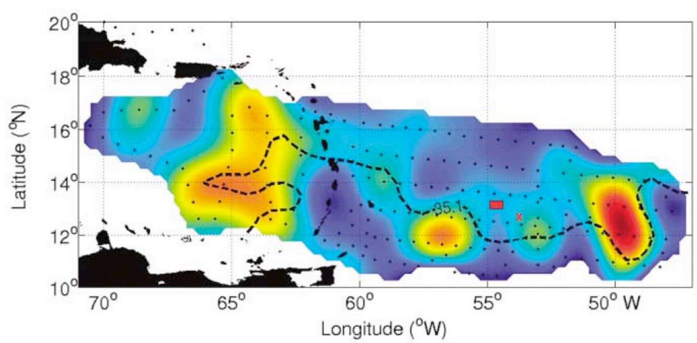
salt

$$F_{SF} > F_{turb}$$

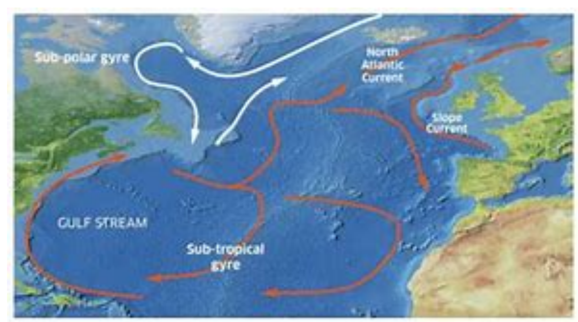
heat

$$H_{SF} \sim H_{turb}$$

Caribbean Staircase



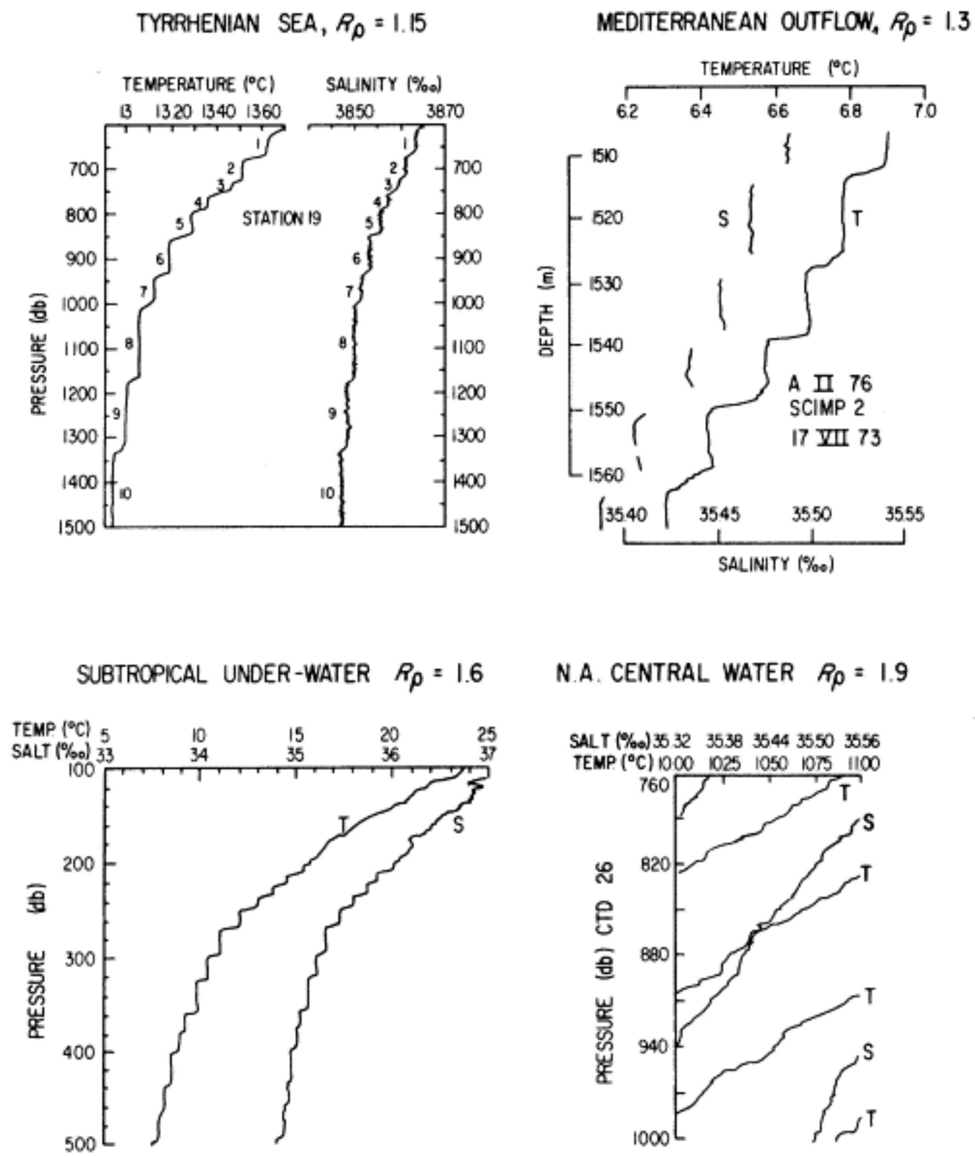
Schmitt et al. (2005)



$$A_{GYRE} \sim 15 \times A_{C-SALT}$$

Thermohaline staircases: What do we know?

$$R_\rho = \frac{\alpha T_z}{\beta S_z} \quad \text{– the density ratio}$$

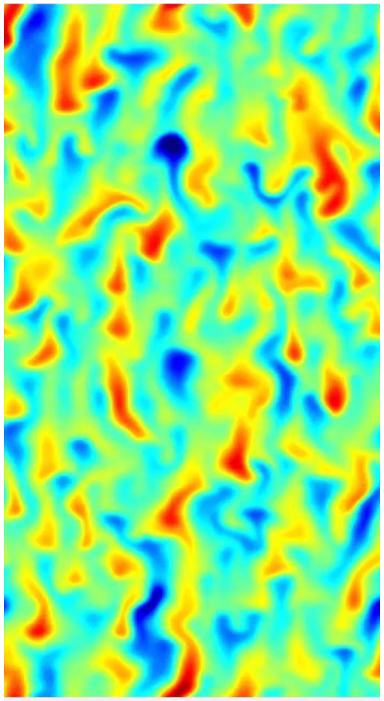


Conditions for thermohaline layering:

$$R_\rho = \frac{\alpha T_z}{\beta S_z} < 1.8$$

Schmitt (1981)

$$T = T_{tot} - \bar{T}, \quad R_\rho = 2$$



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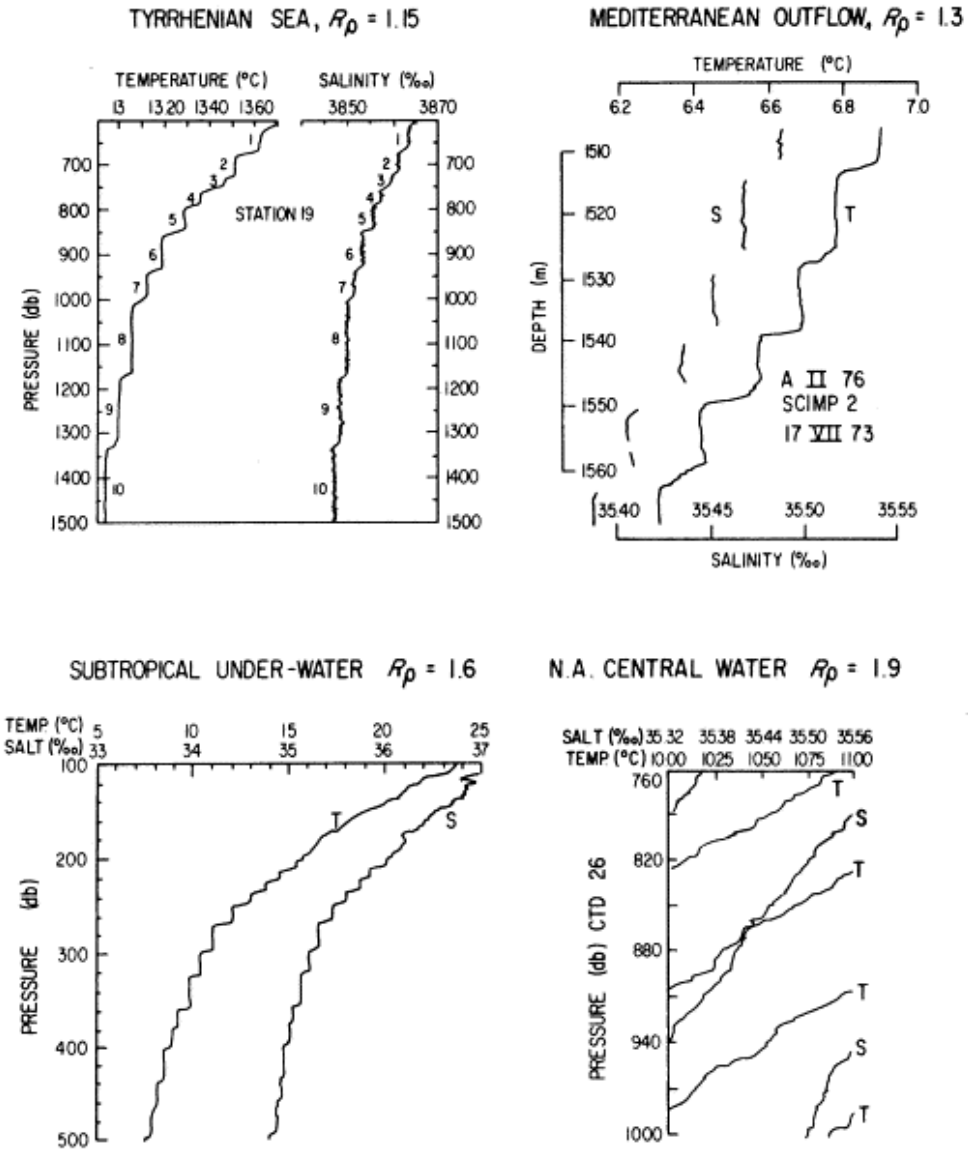
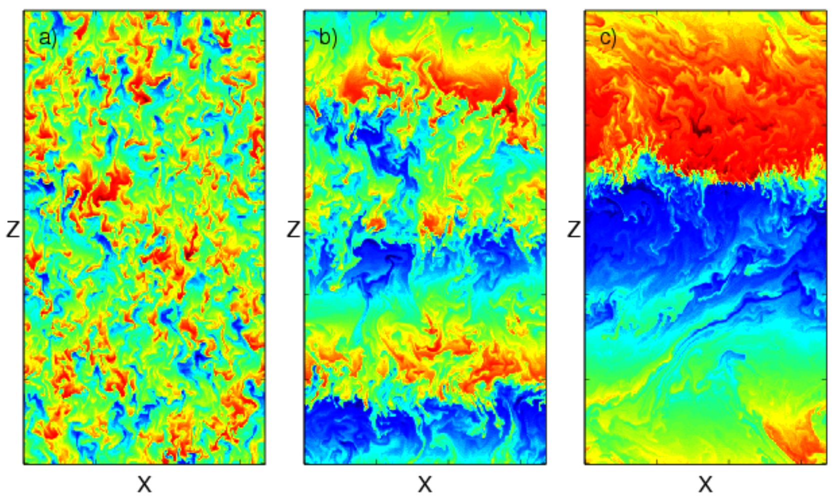


Schmitt (1981)

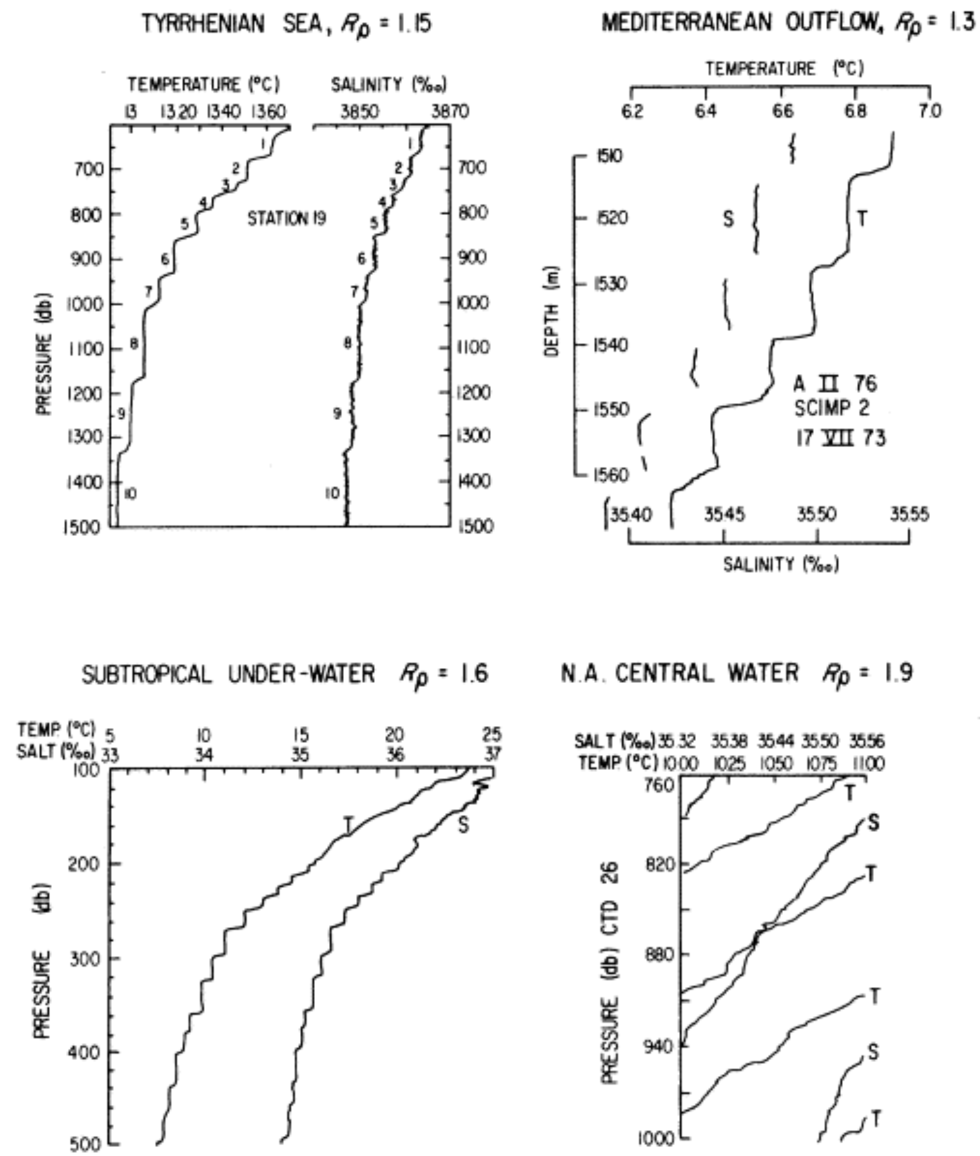
$$T = T_{tot} - \overline{T}, \quad R_{\rho} = 1.1$$



$t_{\text{dim}} \sim 12$ *hours* $\qquad t_{\text{dim}} \sim 4$ *days* $\qquad t_{\text{dim}} \sim 8$ *days*

Conditions for thermohaline layering:

$$R_{\rho} = \frac{\alpha T_z}{\beta S_z} < 1.8$$



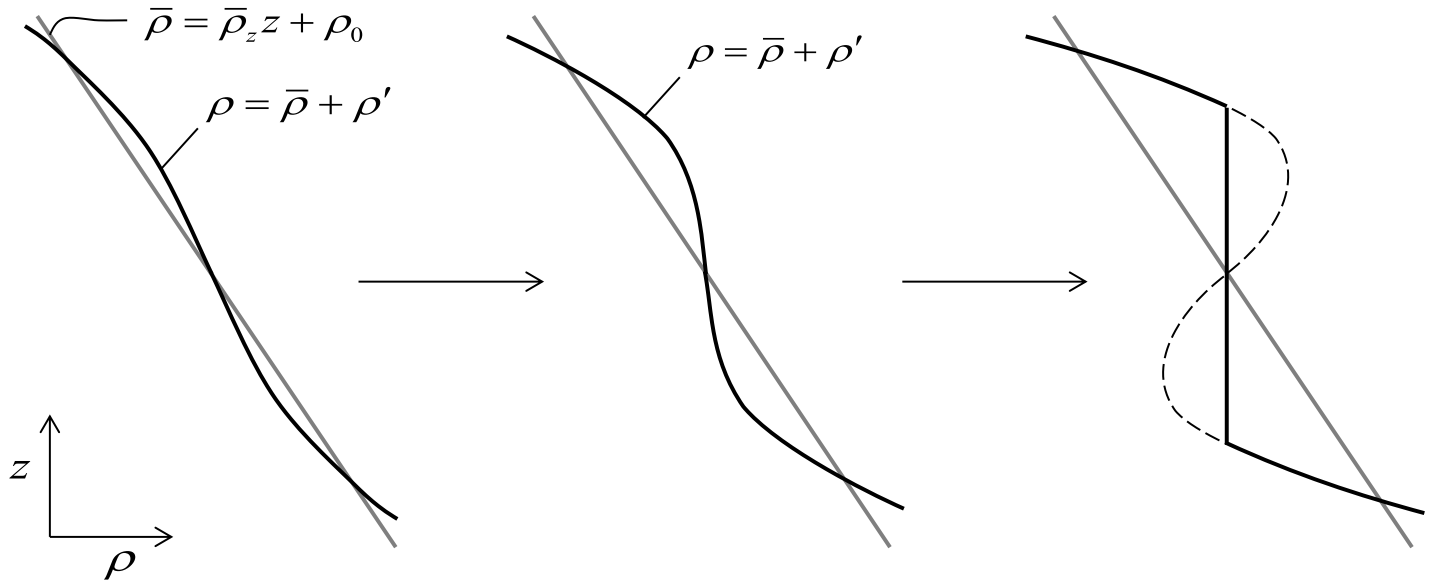
Schmitt (1981)

What is the mechanism of layering and why is it so sensitive to R_ρ?

Explanations of layering

- Collective instability (Stern, 1969)
- Metastable equilibria forced by external disturbances (Stern and Turner, 1969)
- Mechanisms unrelated to double-diffusion (Phillips,1972; Posmentier,1977)
- Negative density diffusion (Schmitt, 1994)
- Lateral intrusions which develop into a staircase (Merryfield, 2000)
- **Instability of the flux-gradient laws** (Radko, 2003, 2019)

$$F_T = F_T(T_z, S_z)$$
$$F_S = F_S(T_z, S_z)$$



Instability of a uniform gradient – 1D model

Large-scale conservation

$$\frac{\partial T}{\partial t} = -\frac{\partial F_T}{\partial z}$$
$$\frac{\partial S}{\partial t} = -\frac{\partial F_S}{\partial z}$$

Flux-gradient laws

$$F_T = F_T(T_z, S_z)$$
$$F_S = F_S(T_z, S_z)$$

Dimensional arguments

$$F_T = k_T T_z Nu(R_\rho)$$
$$\beta F_S = \frac{\alpha F_T}{\gamma(R_\rho)}$$
$$R_\rho = \frac{\alpha T_z}{\beta S_z}$$

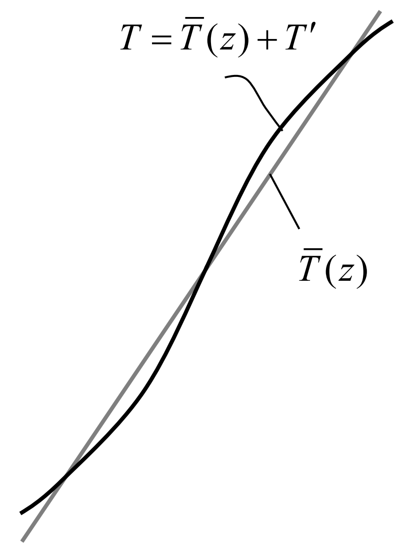
the flux ratio

Linearization:

$$T = \overline{T}(z) + T'$$
$$S = \overline{S}(z) + S'$$
$$R_\rho = R_0 + R'$$

$(\overline{T}, \overline{S}, R_0)$ – uniform background gradient

(T', S', R') – small perturbation



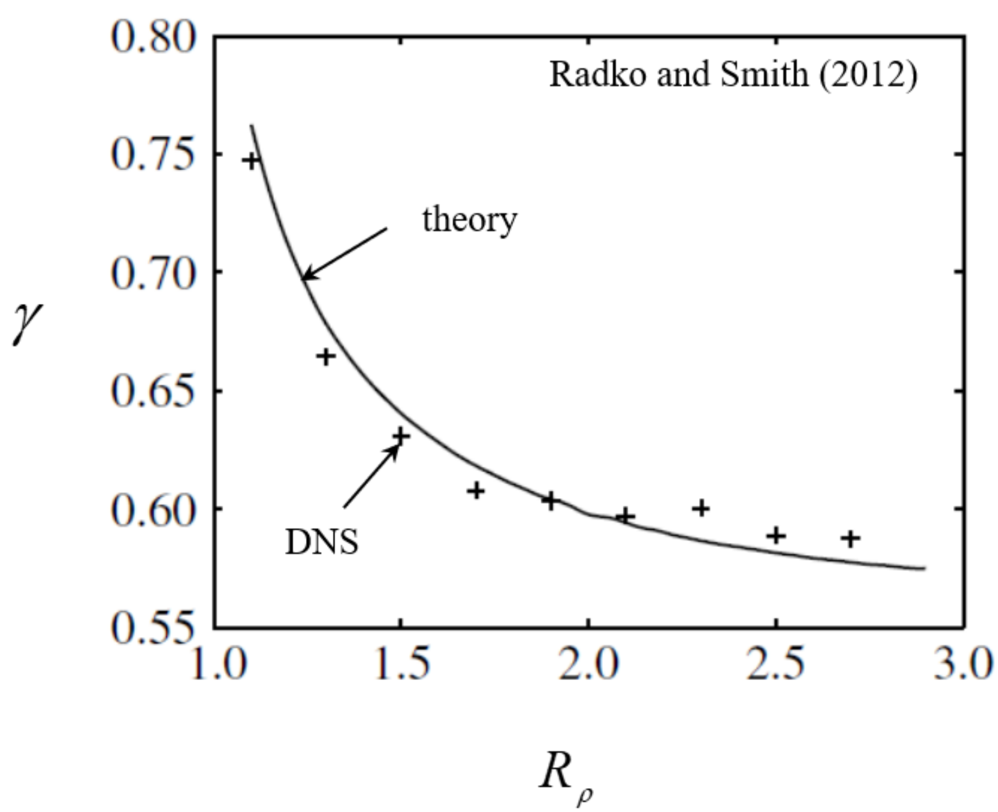
Normal mode analysis:

$$(T', S') = (\hat{T}, \hat{S}) \sin(mz) \exp(\lambda t)$$

Eigenvalue equation for the growth rate:

$$\lambda^2 + \lambda(\ldots)m^2 + \frac{\partial \gamma}{\partial R_\rho}\frac{Nu^2(R_0)m^4}{\gamma^2} = 0$$

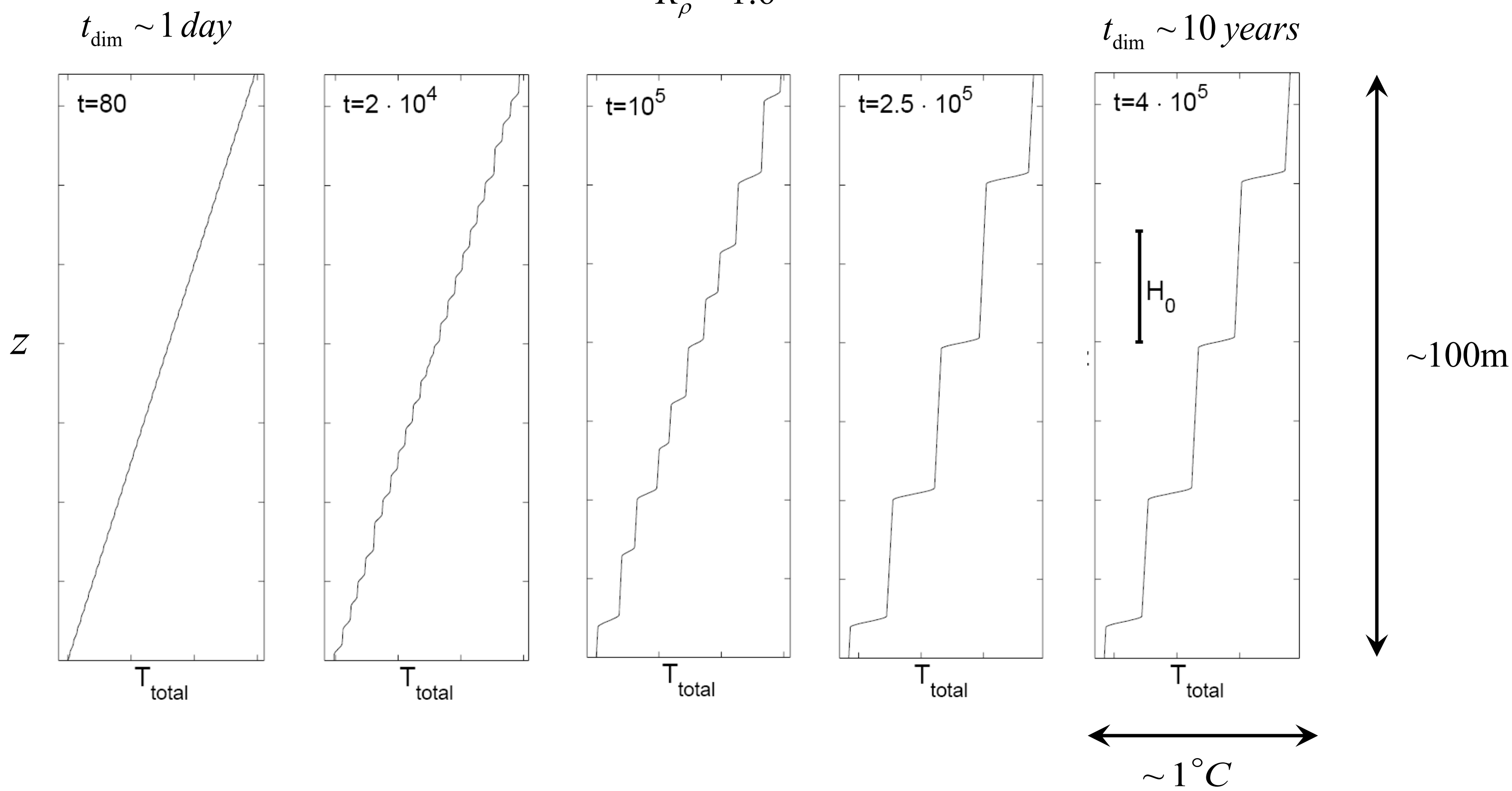
If the flux ratio **decreases** with R_ρ, the uniform gradient is **unstable**

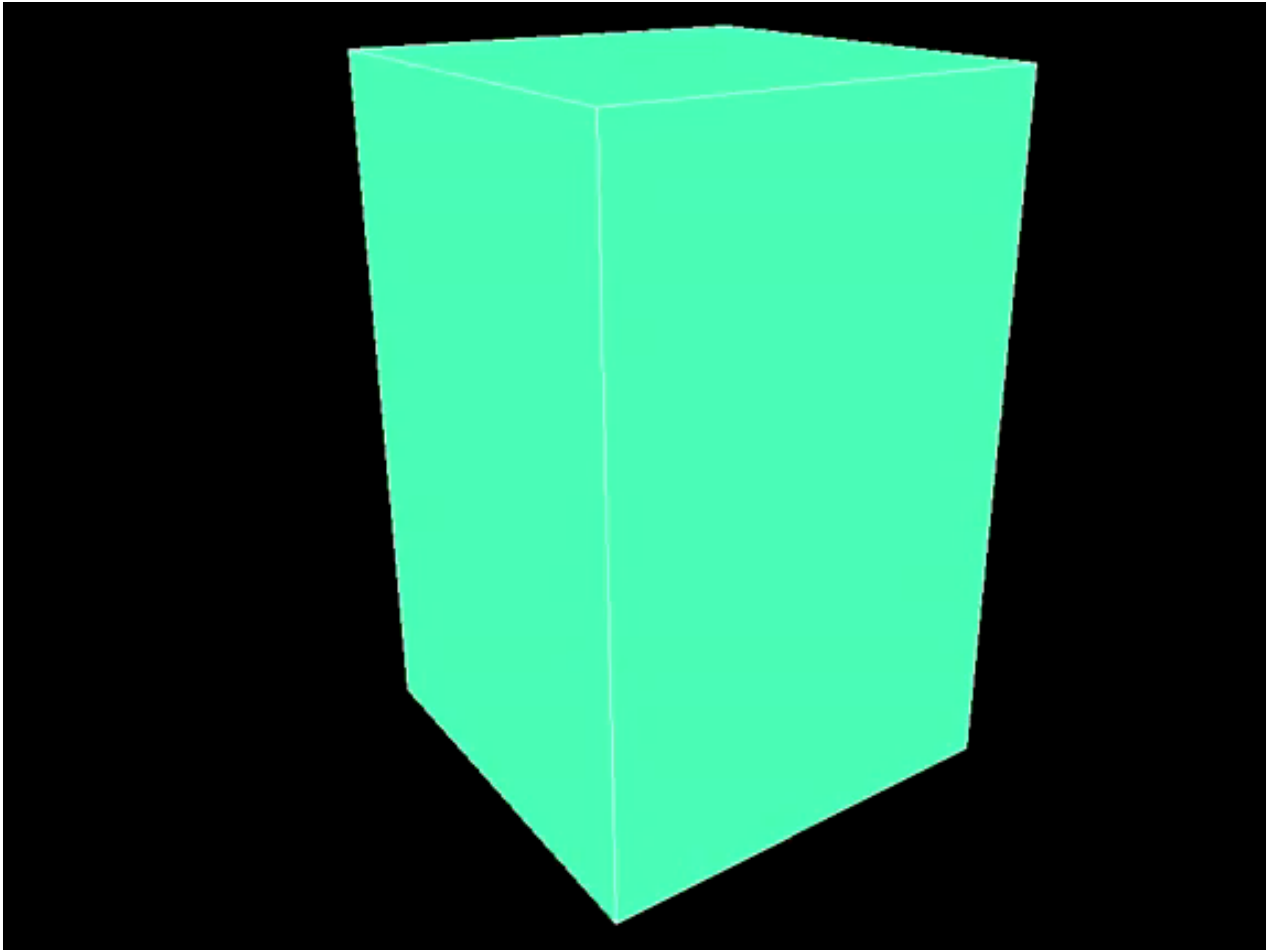


$$\frac{\partial T}{\partial t} = -\frac{\partial F_T}{\partial z}$$
$$\frac{\partial S}{\partial t} = -\frac{\partial F_S}{\partial z}$$

The layering instability
Parametric flux-gradient model (nonlinear version)

$R_\rho = 1.6$

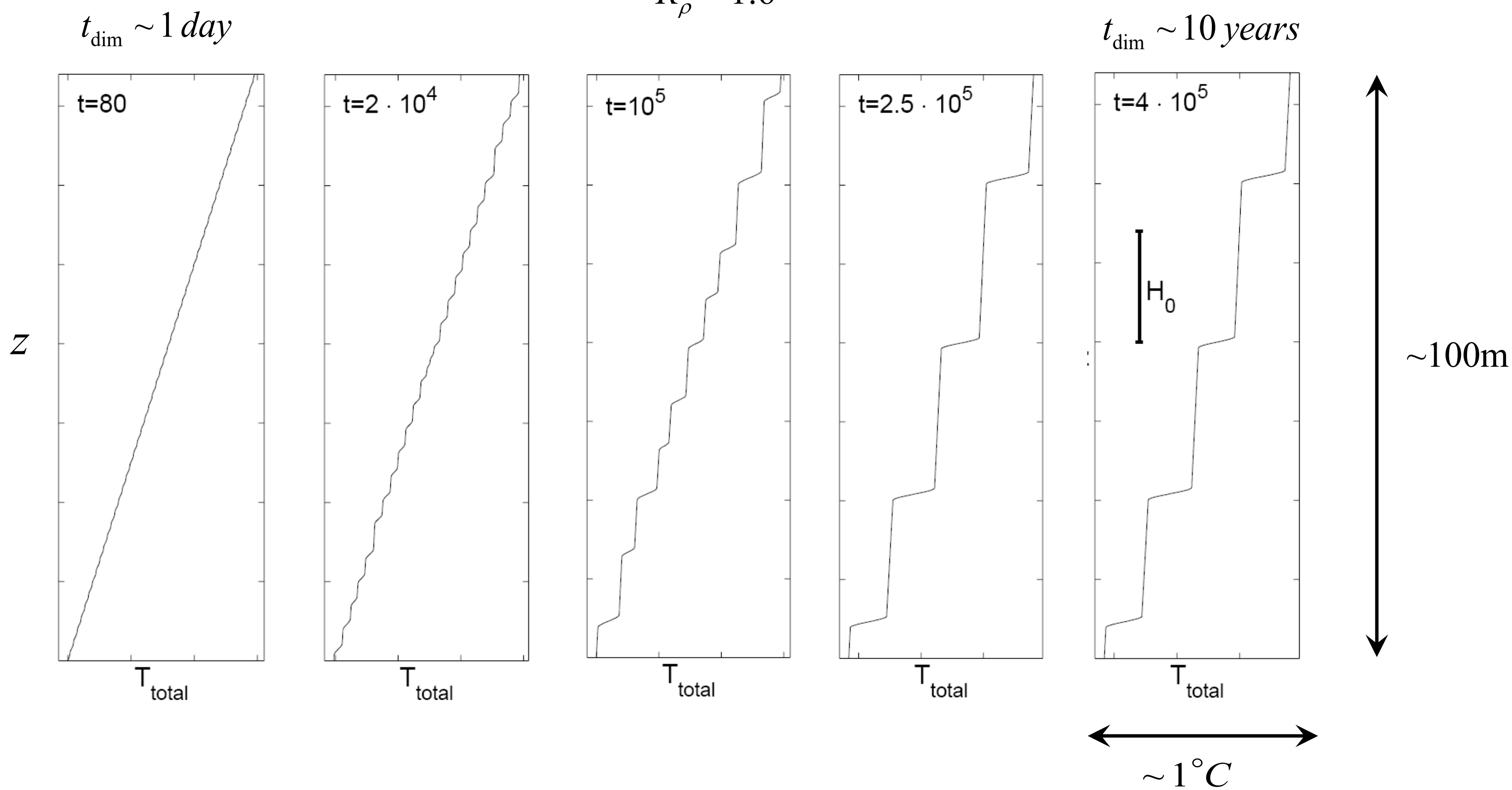




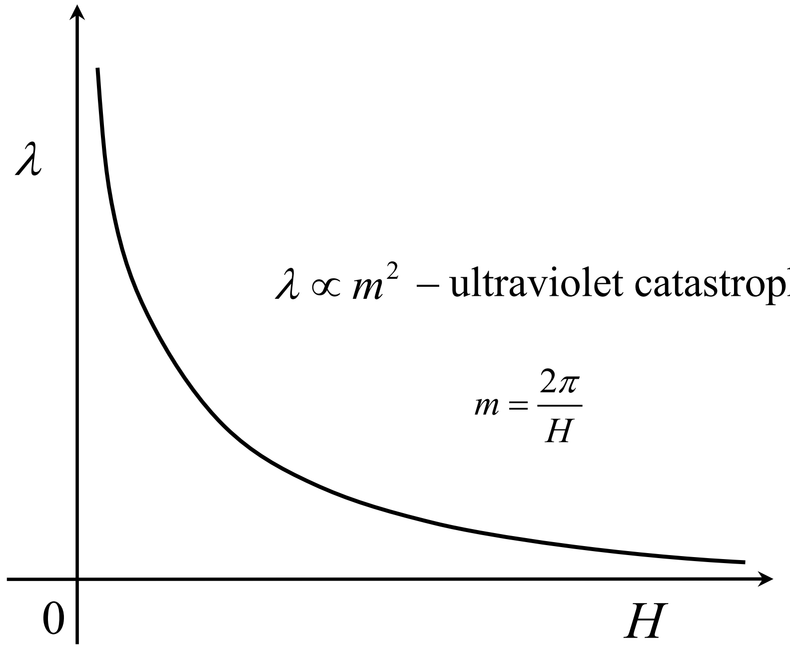
The layering instability
Parametric flux-gradient model (nonlinear version)

$$\frac{\partial T}{\partial t} = -\frac{\partial F_T}{\partial z}$$
$$\frac{\partial S}{\partial t} = -\frac{\partial F_S}{\partial z}$$

$R_\rho = 1.6$



The layering instability
Remaining questions/problems



$\lambda \propto m^2$ – ultraviolet catastrophe

$$m = \frac{2\pi}{H}$$

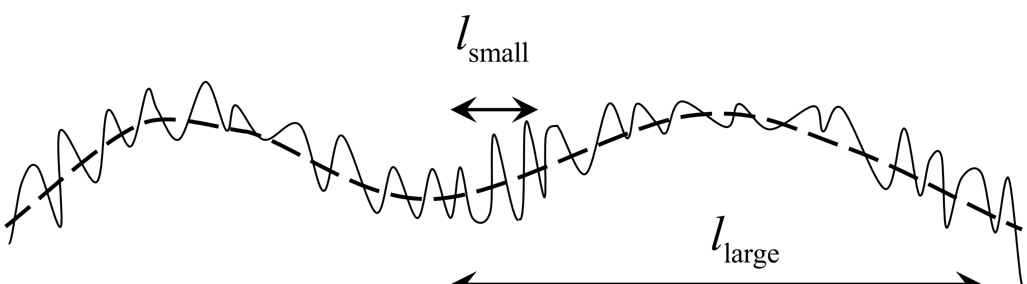
$$F_T = F_T(T_z, S_z)$$
$$F_S = F_S(T_z, S_z)$$

- What is the origin of ultraviolet catastrophe in the model?
- What determines the growth rate of layering instability?
- What determines the preferred wavelength?
- Why does the gamma-instability theory predict layering even for $R_\rho > 1.8$?
- Where is microstructure?

Thermohaline layering on the microscale

$$\cancel{\begin{aligned} F_T &= F_T(T_z, S_z) \\ F_S &= F_S(T_z, S_z) \end{aligned}}$$

Alternative approach: **multiscale mechanics**



homogenization method

modulational stability analysis

8D hydrodynamics

$$Z = \varepsilon z, \quad t_2 = \varepsilon^2 t$$

$$\varepsilon = \frac{l_{\text{small}}}{l_{\text{large}}}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \varepsilon^2 \frac{\partial}{\partial t_2}, \quad \frac{\partial}{\partial z} \to \frac{\partial}{\partial z} + \varepsilon \frac{\partial}{\partial Z} \ .$$

Governing equations (2D, non-dimensional)

$$\begin{cases} \dfrac{\partial T}{\partial t} + J(\psi, T) + \dfrac{\partial \psi}{\partial x} = \nabla^2 T \\ \dfrac{\partial S}{\partial t} + J(\psi, S) + \dfrac{1}{R_0}\dfrac{\partial \psi}{\partial x} = \tau \nabla^2 S \\ \dfrac{\partial}{\partial t}\nabla^2 \psi + J(\psi, \nabla^2 \psi) = \Pr\left[\dfrac{\partial}{\partial x}(T - S) + \nabla^4 \psi\right] \end{cases}$$

$$T = \tilde{T}_0(x,z,t) + \boldsymbol{T}_{ls}(Z,t_2) + \varepsilon \breve{\boldsymbol{T}}_{\boldsymbol{1}}(Z,t_2)\tilde{T}_1(x,z,t) + \ldots$$

Solve a sequence of balances at each order in ε

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \varepsilon^2 \frac{\partial}{\partial t_2}, \quad \frac{\partial}{\partial z} \to \frac{\partial}{\partial z} + \varepsilon \frac{\partial}{\partial Z} \ .$$

If the expansion is truncated at $O(\varepsilon^2)$

$$\begin{cases} \dfrac{\partial T_{ls}}{\partial t_2} = K_1 \dfrac{\partial^2 T_{ls}}{\partial Z^2} + K_2 \dfrac{\partial^2 S_{ls}}{\partial Z^2} \\ \dfrac{\partial S_{ls}}{\partial t_2} = K_3 \dfrac{\partial^2 T_{ls}}{\partial Z^2} + K_4 \dfrac{\partial^2 S_{ls}}{\partial Z^2}. \end{cases}$$

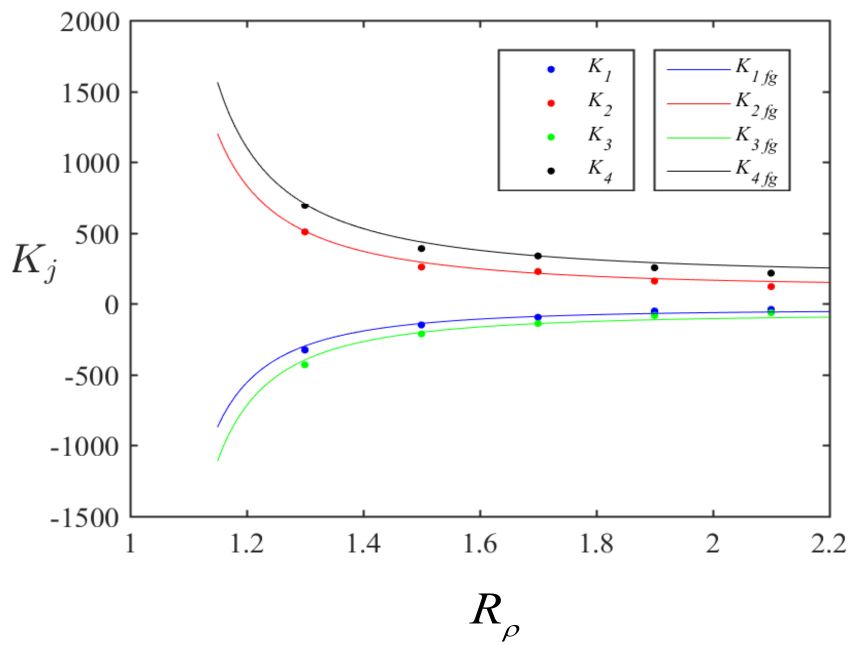
$$K_1 = 1 + \left[\left\langle \tilde{\psi}_{1T} \frac{\partial \tilde{T}_0}{\partial x} - \frac{\partial \tilde{\psi}_0}{\partial x} \tilde{T}_{1T} \right\rangle\right], \ \ldots$$

$$Z = \varepsilon z, \quad t_2 = \varepsilon^2 t$$

If the expansion is truncated at $O(\varepsilon^2)$:

$$\begin{cases} \dfrac{\partial T_{ls}}{\partial t} = K_1 \dfrac{\partial^2 T_{ls}}{\partial z^2} + K_2 \dfrac{\partial^2 S_{ls}}{\partial z^2} \\ \dfrac{\partial S_{ls}}{\partial t} = K_3 \dfrac{\partial^2 T_{ls}}{\partial z^2} + K_4 \dfrac{\partial^2 S_{ls}}{\partial z^2}. \end{cases}$$

$\lambda \propto m^2$ – ultraviolet catastrophe (still)



The multiscale expansion at $O(\varepsilon^2)$ simply reproduces the flux-gradient model

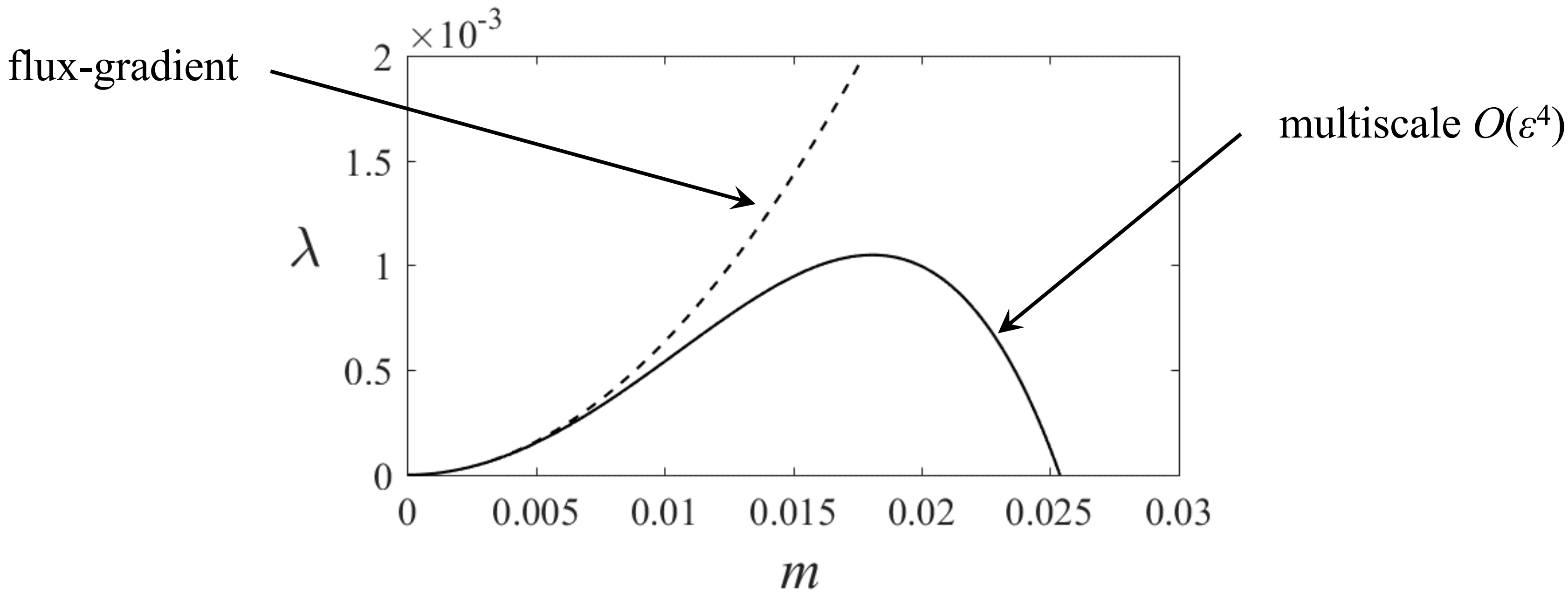
If the expansion is extended to $O(\varepsilon^4)$:

$$\begin{cases} \dfrac{\partial T_{ls}}{\partial t} = K_1 \dfrac{\partial^2 T_{ls}}{\partial z^2} + K_2 \dfrac{\partial^2 S_{ls}}{\partial z^2} + K_5 \dfrac{\partial^4 T_{ls}}{\partial z^4} + K_6 \dfrac{\partial^4 S_{ls}}{\partial z^4} \\ \dfrac{\partial S_{ls}}{\partial t} = K_3 \dfrac{\partial^2 T_{ls}}{\partial z^2} + K_4 \dfrac{\partial^2 S_{ls}}{\partial z^2} + K_7 \dfrac{\partial^4 T_{ls}}{\partial z^4} + K_8 \dfrac{\partial^4 S_{ls}}{\partial z^4} \end{cases}$$

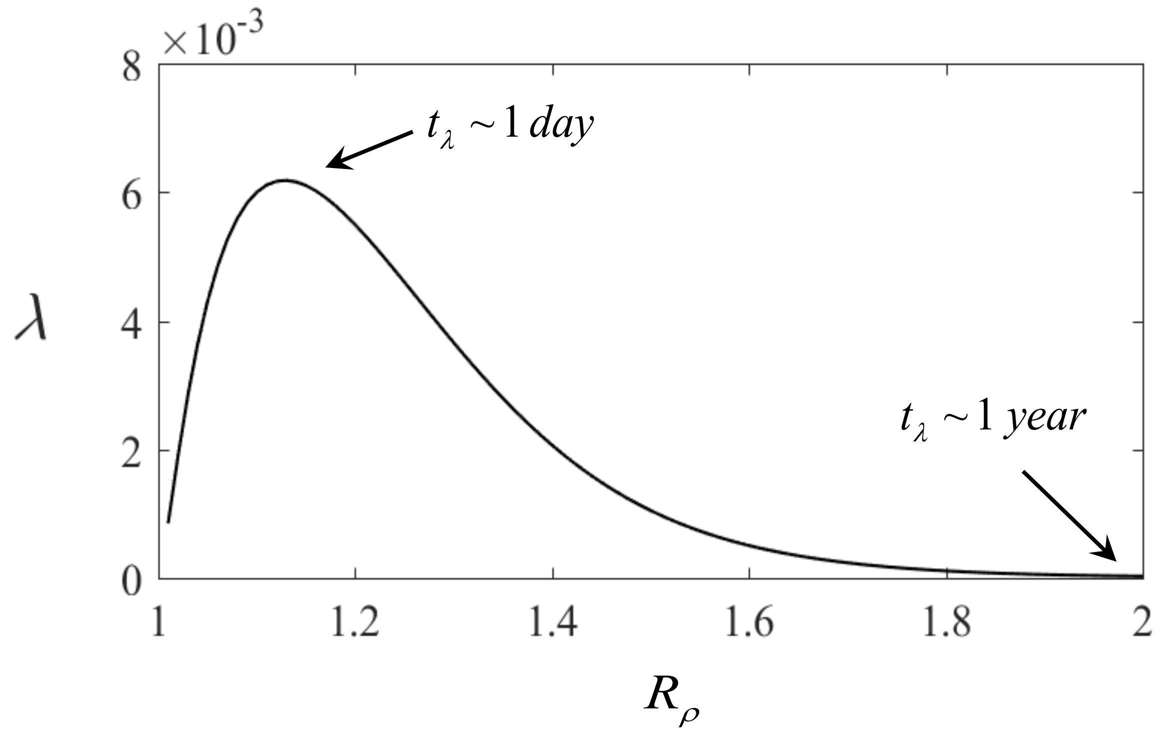
Normal mode analysis

$$(T_{ls}, S_{ls}) = (\hat{T}, \hat{S}) \sin(mz) \exp(\lambda t)$$

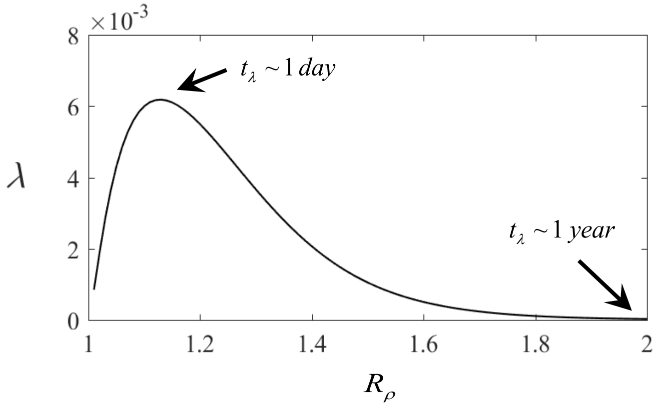
$$\lambda = \lambda(m)$$

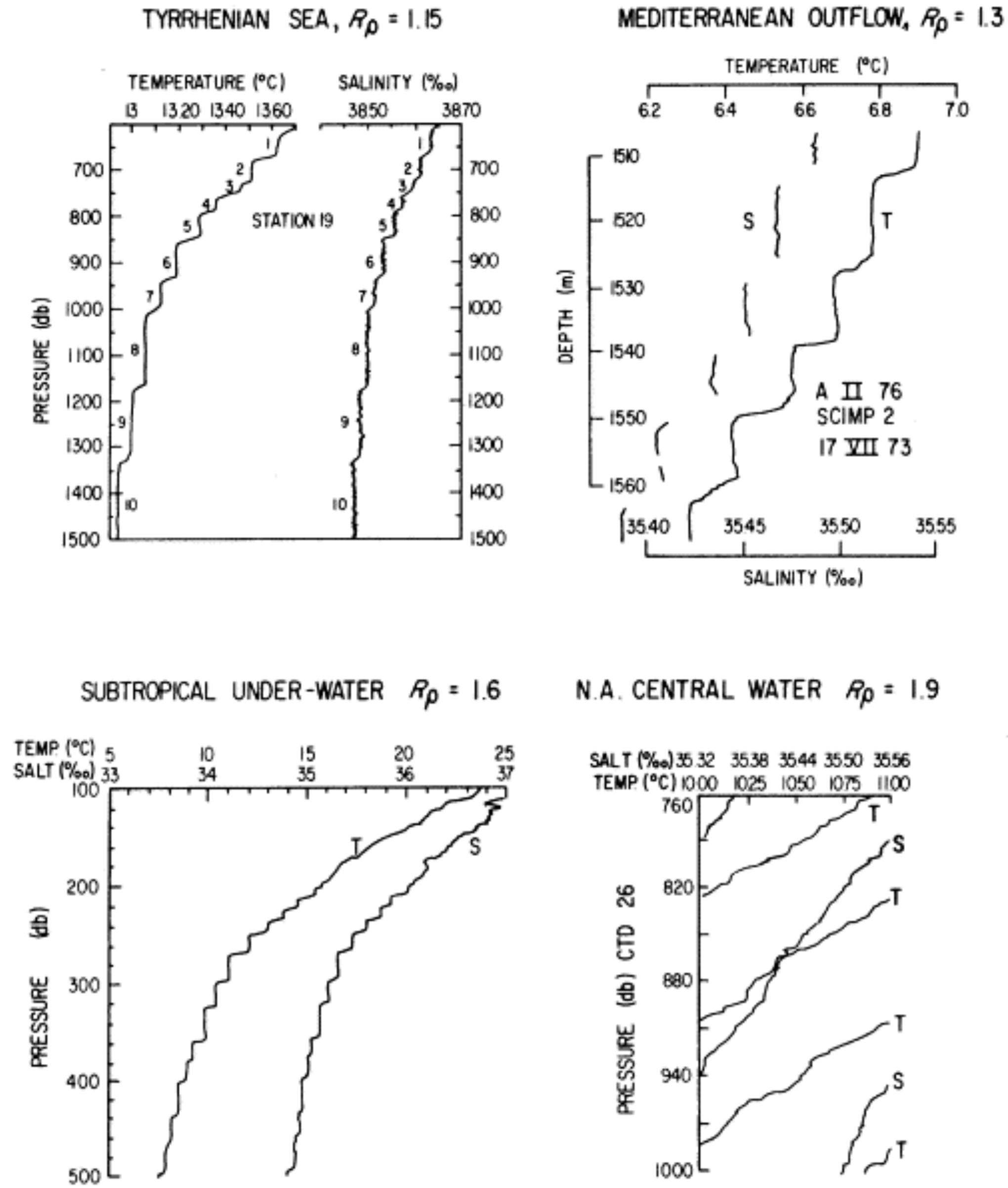


Growth rates of layering instability



$$R_\rho = \frac{\alpha T_z}{\beta S_z} \quad \text{– the density ratio}$$





Schmitt (1981)

Conditions for thermohaline layering:

$$R_\rho = \frac{\alpha T_z}{\beta S_z} < 1.8$$

Final comments:

- Let us keep the flame burning

  "… there remains a host of unresolved problems in double diffusion." (Schmitt, 2012)

- Some aspects of thermohaline layering can be explained using flux-gradient laws

- The complete theory of staircases demands the modification of Fick's law

$$\cancel{F_T = -K_T \frac{\partial T}{\partial z}} \quad \leftarrow \quad F_T = -K_T \frac{\partial T}{\partial z} + K_{T3} \frac{\partial^3 T}{\partial z^3} + \ldots$$

- Physical oceanographers should embrace 8D hydrodynamics