Spontaneous and forced symmetry breaking in binary fluid convection

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Binary fluid convection: Dimensionless equations

$$
\begin{aligned}
\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla P + PrR[T + SC]\hat{\mathbf{z}} + Pr\nabla^2\mathbf{u}, \\
T_t + (\mathbf{u}\cdot\nabla)T &= \nabla^2 T, \\
C_t + (\mathbf{u}\cdot\nabla)C &= \tau\nabla^2 C - \nabla^2 T,
\end{aligned}
$$

where $\mathbf{u} = (u, w)$ in (x, z) coordinates. The Prandtl number Pr, Lewis number τ, Rayleigh number R and the separation ratio S are defined by

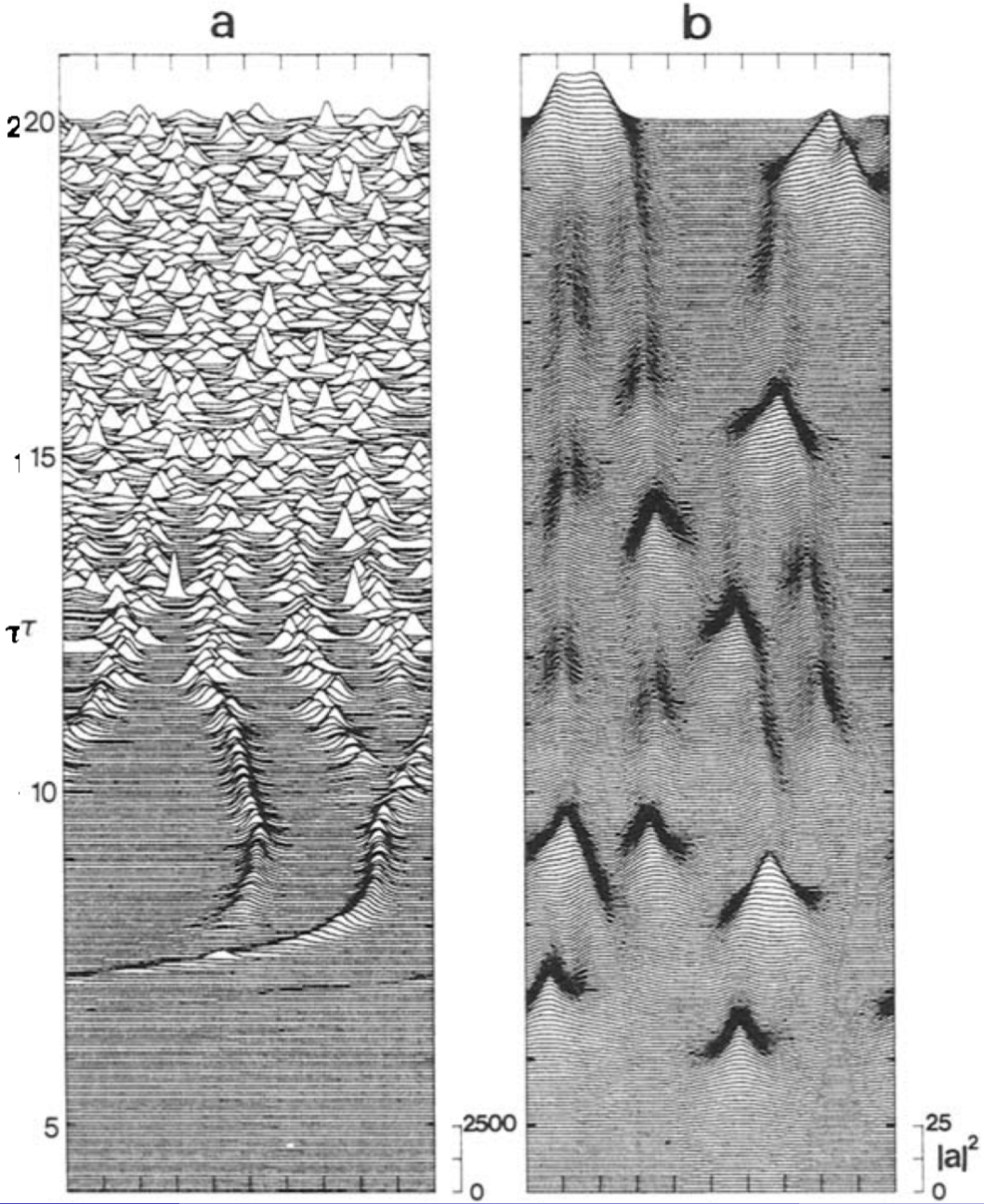
$$
Pr = \frac{\nu}{\kappa_T}, \qquad \tau = \frac{\kappa_C}{\kappa_T}, \qquad R = \frac{g\alpha_T \Delta T \ell^3}{\nu\kappa_T}, \qquad S = \frac{\alpha_C}{\alpha_T} S_{Soret}
$$

The boundary conditions are

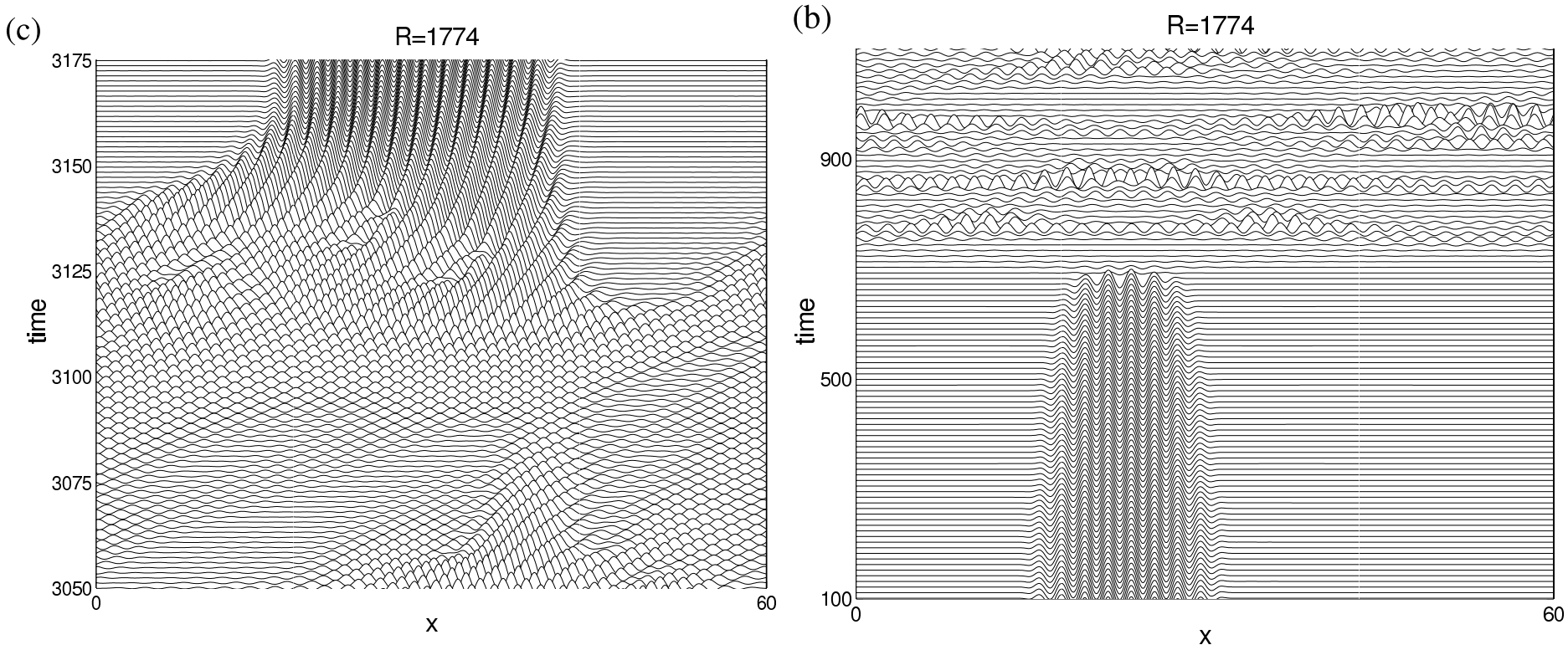
$$
\begin{aligned}
\text{at} \quad z = 1: \quad & u = w = T = \eta_z = 0, \\
\text{at} \quad z = 0: \quad & u = w = T - 1 = \eta_z = 0,
\end{aligned}
$$

with periodic boundary conditions (PBC) with period Γ in x. Here $\eta \equiv C - T$ whose gradient is proportional to the mass flux. We are interested in the regime $\tau < 1$, $S < 0$ (double diffusive convection).

Dispersive chaos: Bretherton and Spiegel (1983)

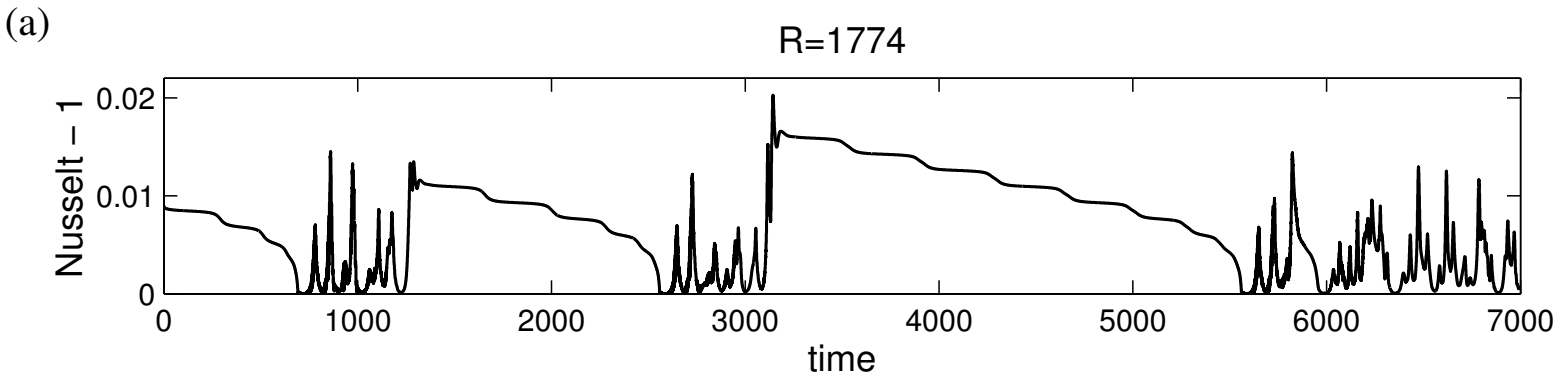


Formation of a convecton



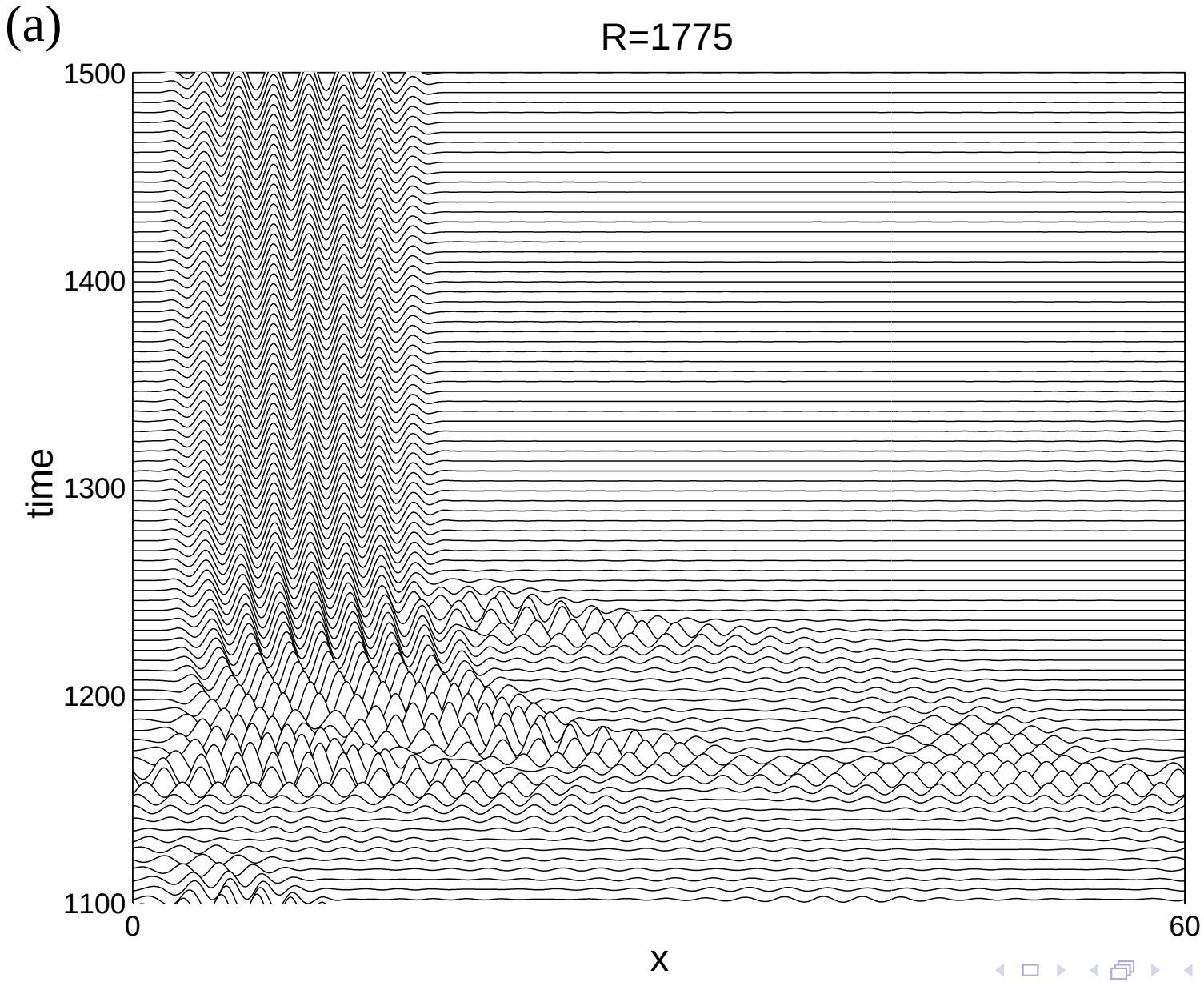
Batiste et al., J. Fluid Mech. **560**, 149–158 (2006)

Relaxation oscillations at $R = 1774$

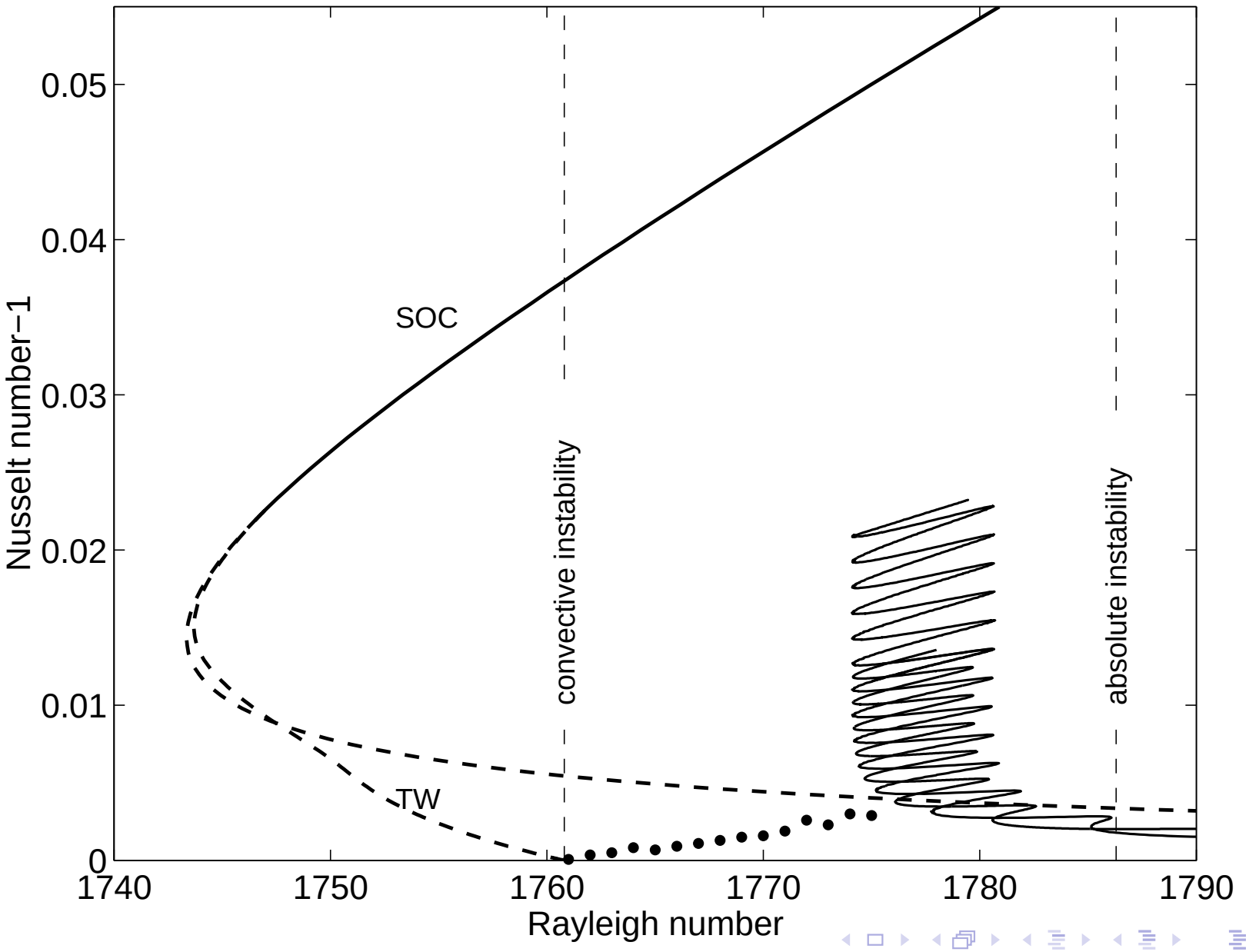


Batiste et al., J. Fluid Mech. **560**, 149–158 (2006)

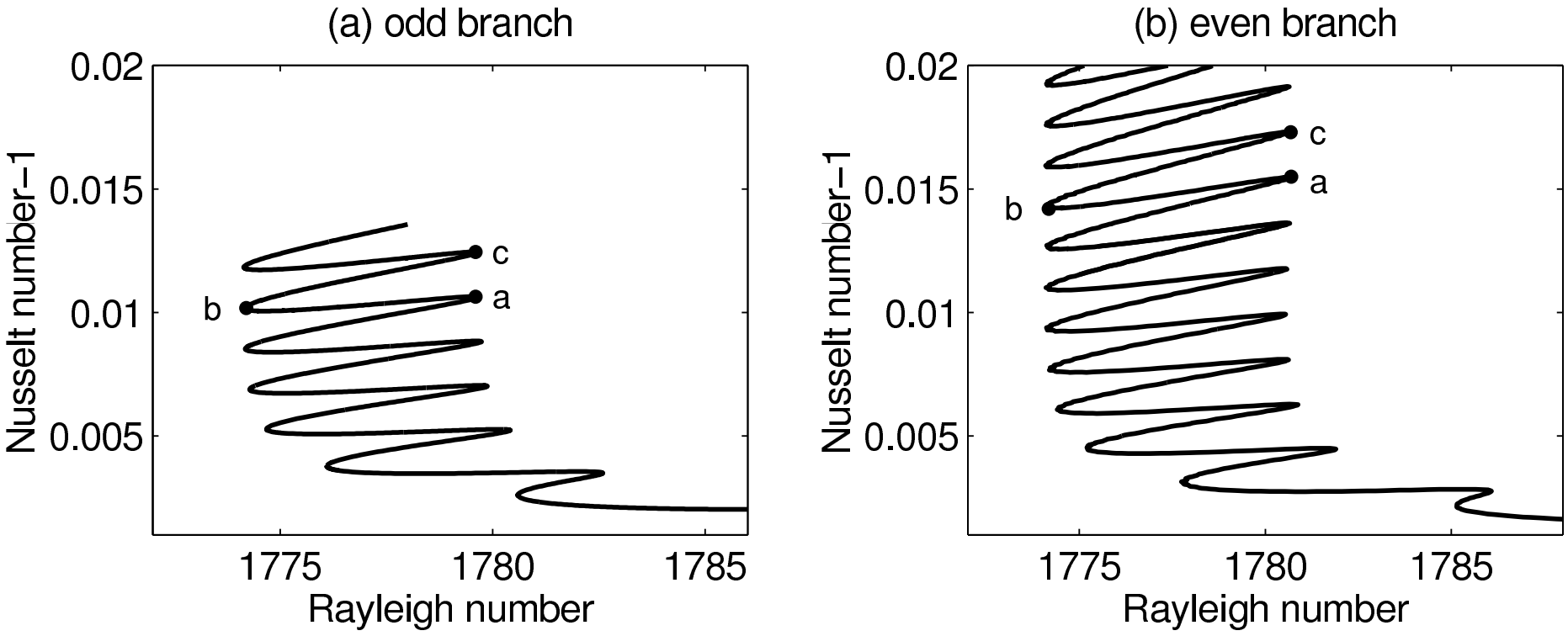
Formation of a convecton



Bifurcation diagram

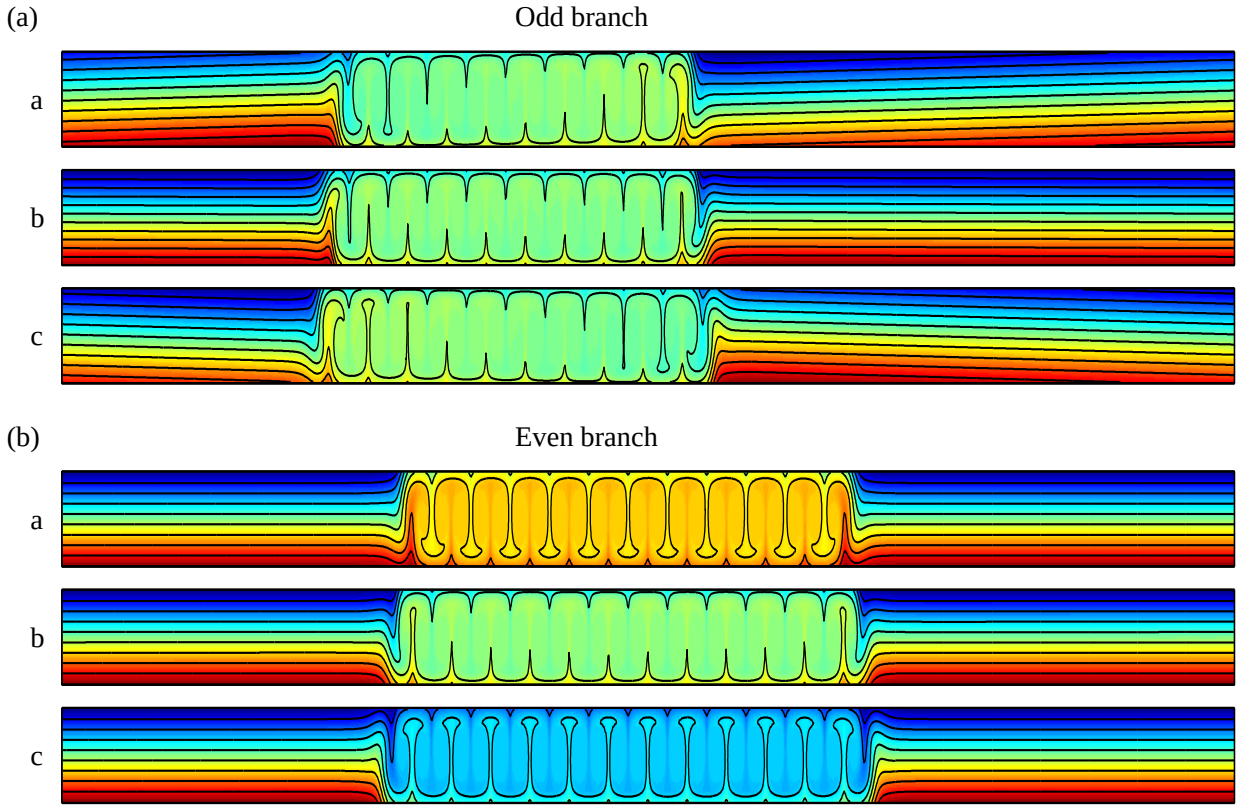


Odd and even convectons



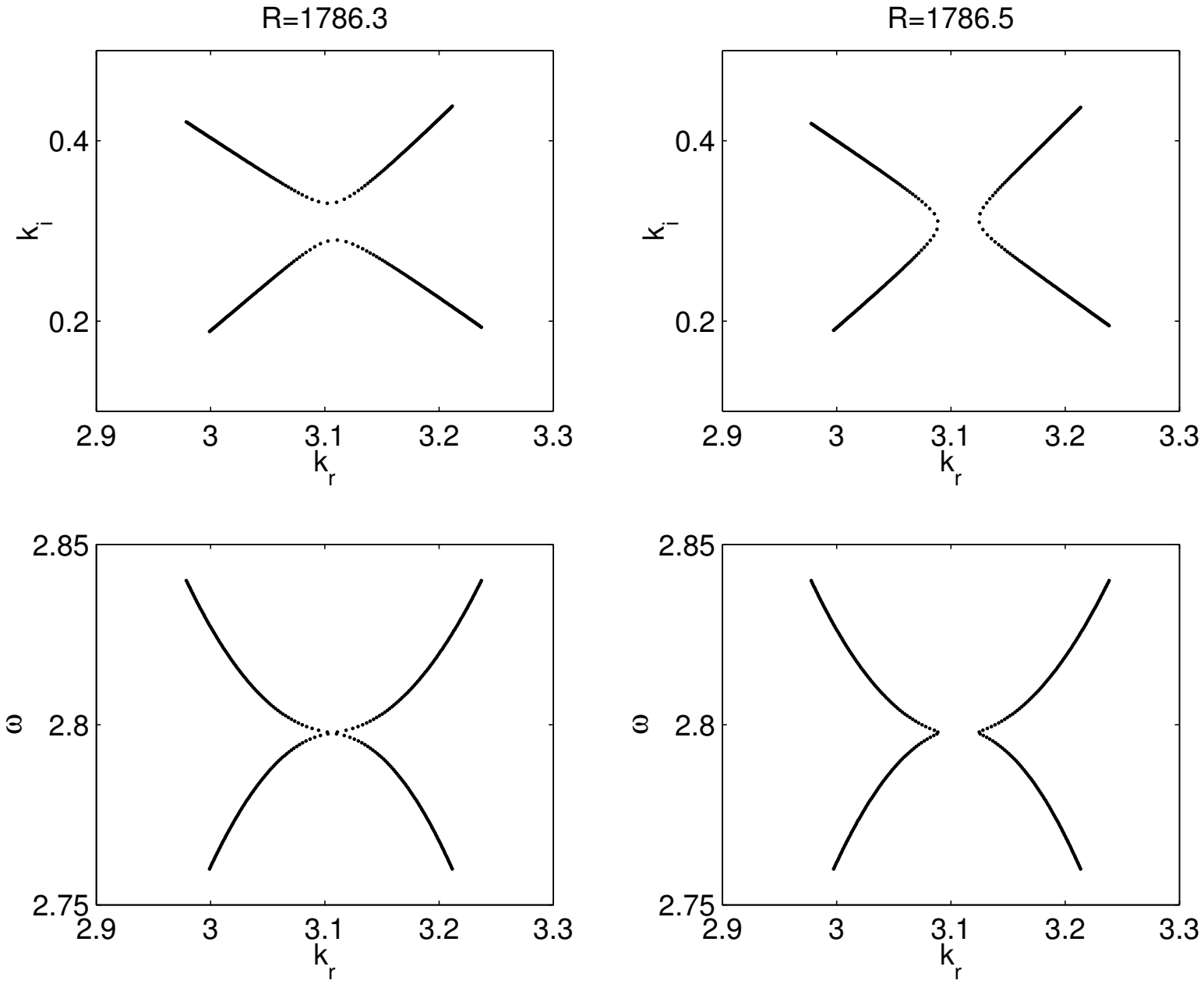
Batiste et al., J. Fluid Mech. **560**, 149–158 (2006)

Odd and even convectons

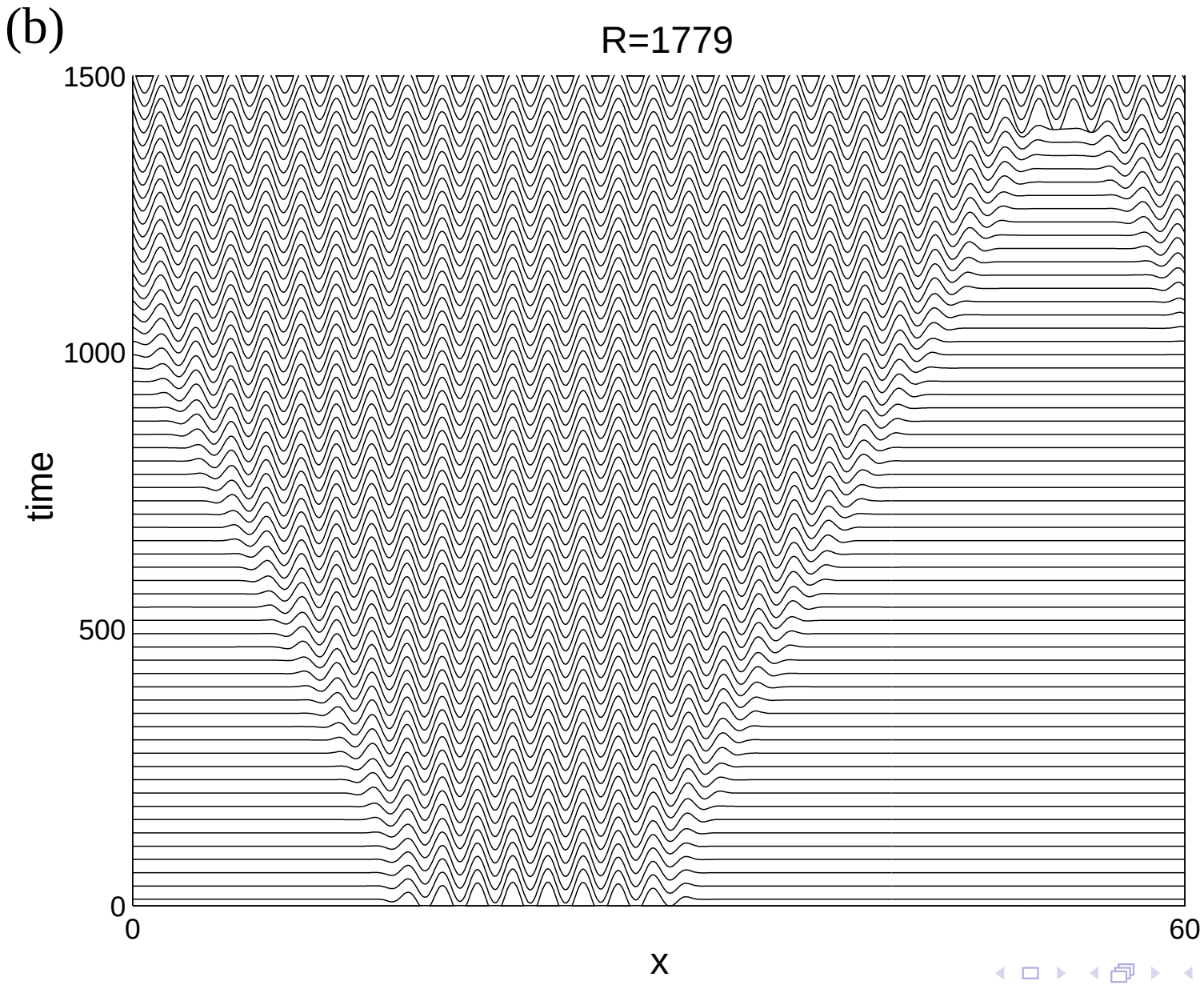


Both are stationary structures because of reflection symmetry:
$R_2 \circ R_1$ (odd convectons) and R_1 (even convectons)

Stability of the convectons



Depinning: $\Gamma = 60$



Swift-Hohenberg equation in one spatial dimension

The Swift-Hohenberg equation

$$u_t = ru - \left(q_c^2 + \partial_x^2\right)^2 u + f(u)$$

is very simple but has very remarkable properties. These are a consequence of the following:

- Fourth order in space
- Intrinsic length scale $2\pi/q_c$
- Bistability due to competing nonlinear terms: $f(u) = b_3 u^3 - b_5 u^5$
- Symmetries: $R_1 : x \to -x, u \to u$; $R_2 : x \to x, u \to -u$
- Variational dynamics

$$u_t = -\frac{\delta F}{\delta u},$$

where

$$F = \int_{-\infty}^{\infty} dx \left\{ -\frac{1}{2} r u^2 + \frac{1}{2}\left[(q_c^2 + \partial_x^2) u\right]^2 - \int_0^u f(v)\, dv \right\}$$

In the following we think of $F[u]$ as the (free) energy of the system

Snakes-and-ladders structure of the pinning region: SH35

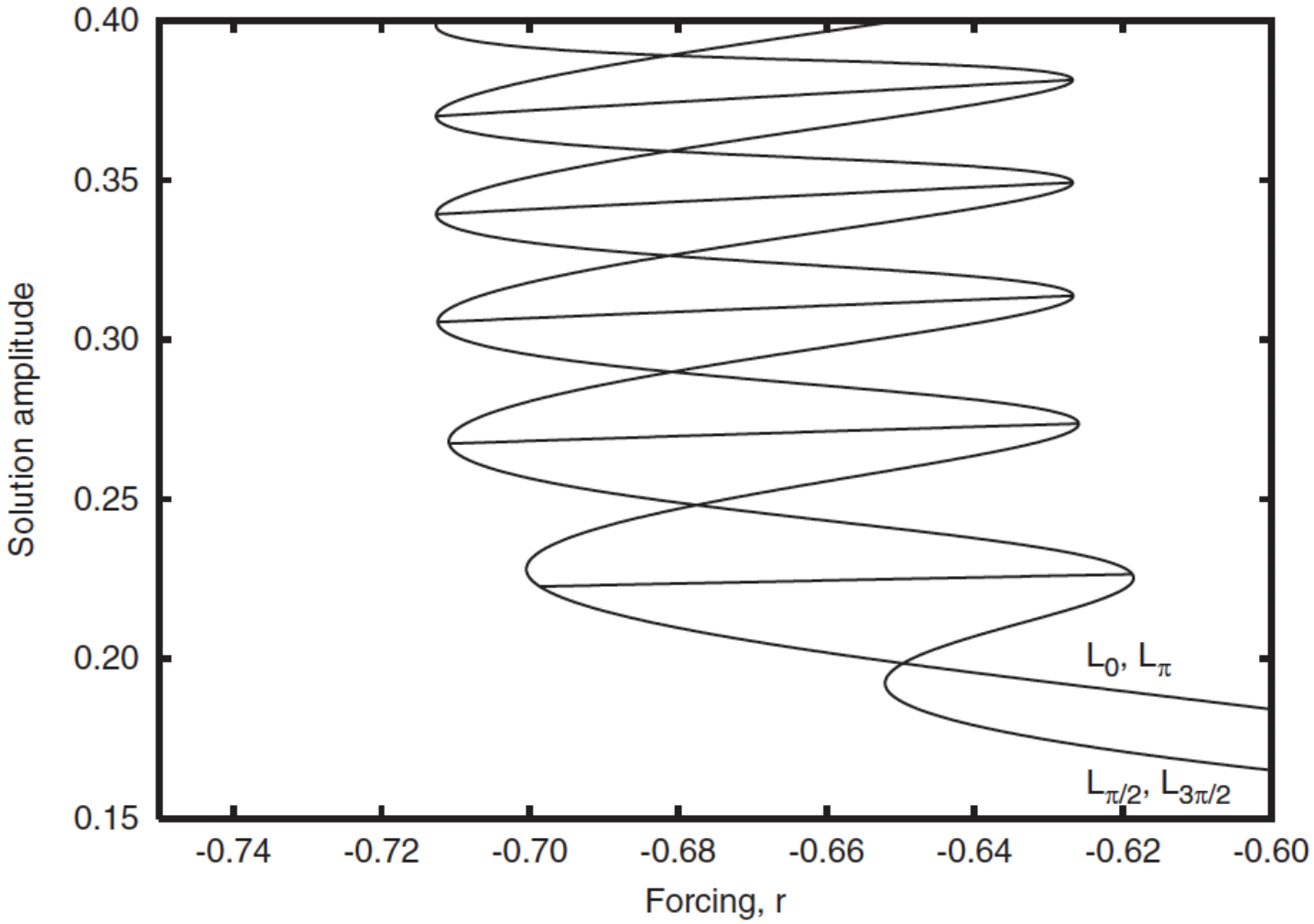
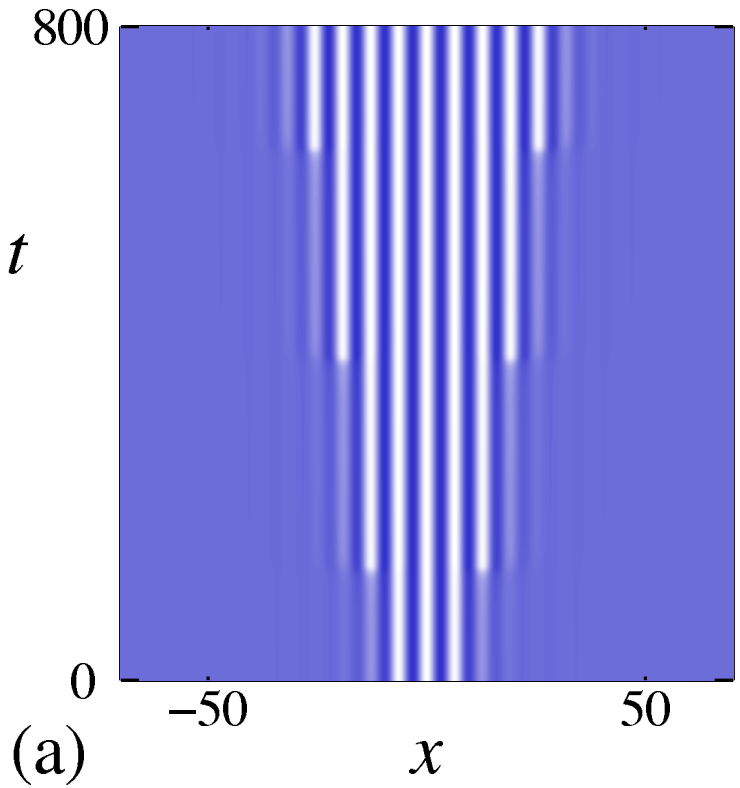
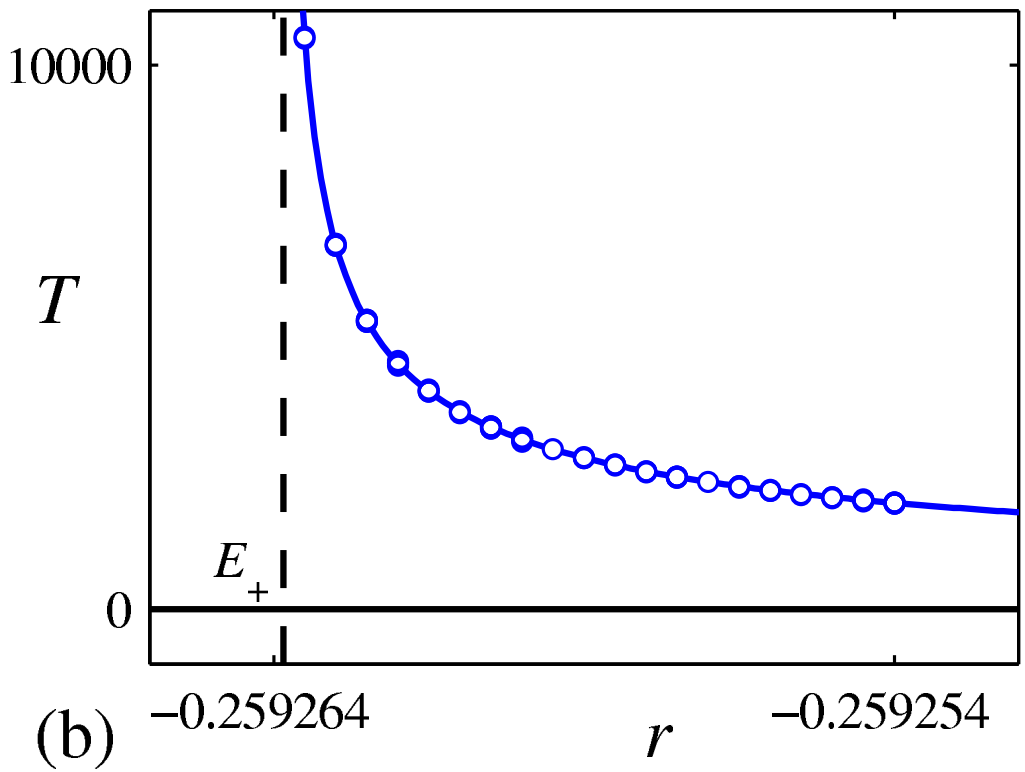


FIG. 1. The snakes-and-ladders structure of the pinning region $-0.713 < r < -0.626$ in SH35, Eq. (1), with $b_3 = 2$, $\epsilon = 0$.

Depinning: SH23

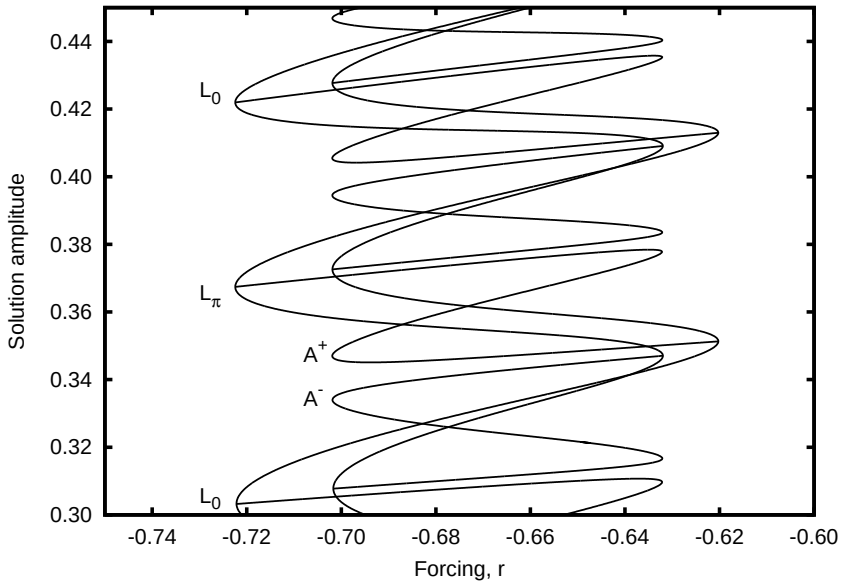




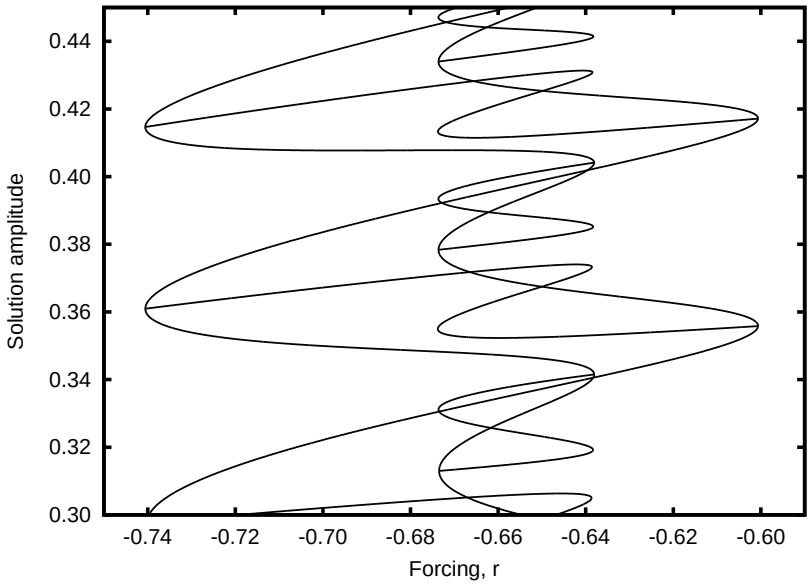
New cells are nucleated symmetrically on either side of the structure

Collisions of LS: SH35 with broken R_2 symmetry

$$u_t = ru - \left(1 + \partial_x^2\right)^2 u + 2u^3 - u^5 + \epsilon(\partial_x u)^2$$



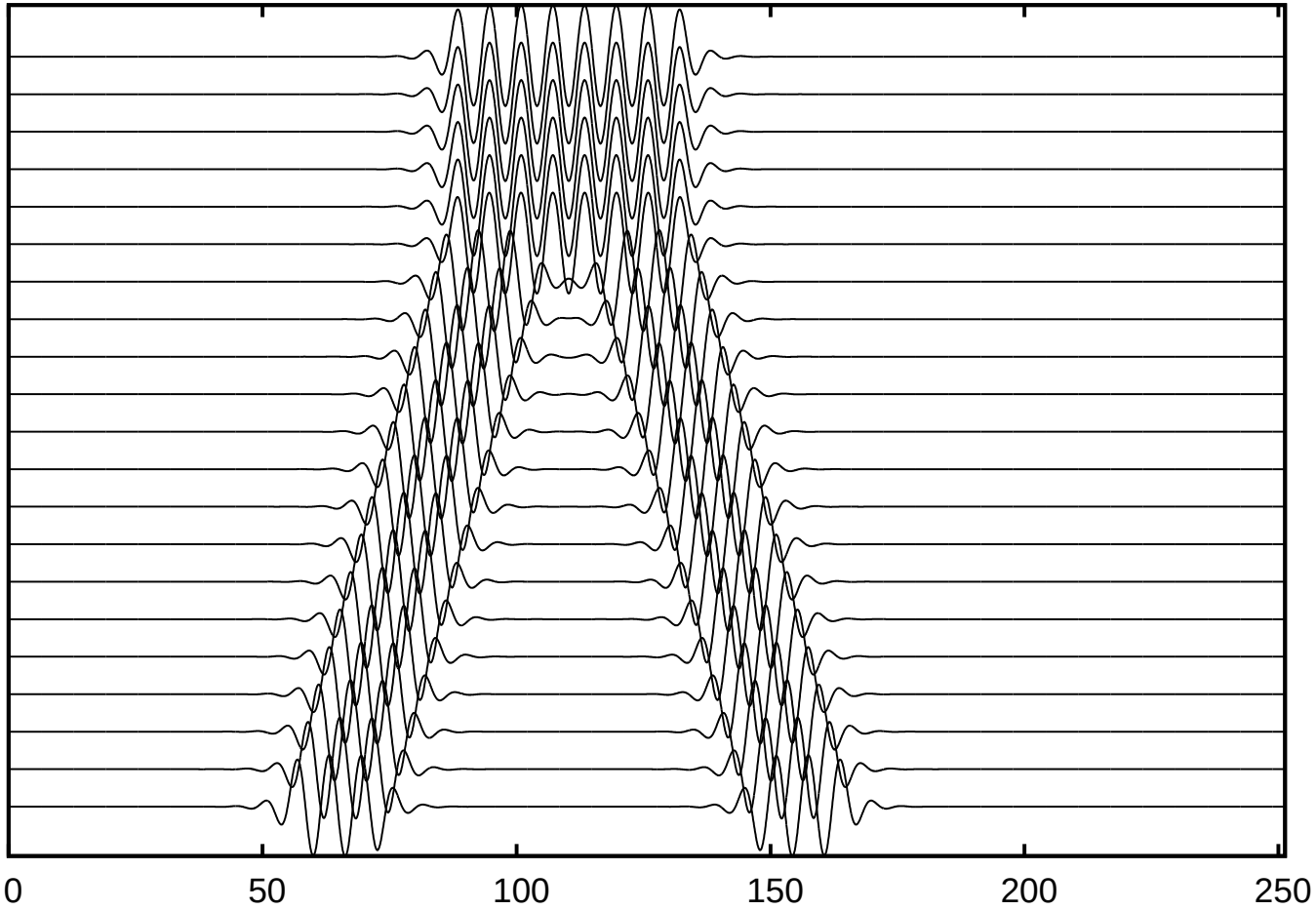
$\epsilon = 0.03$



$\epsilon = 0.1$

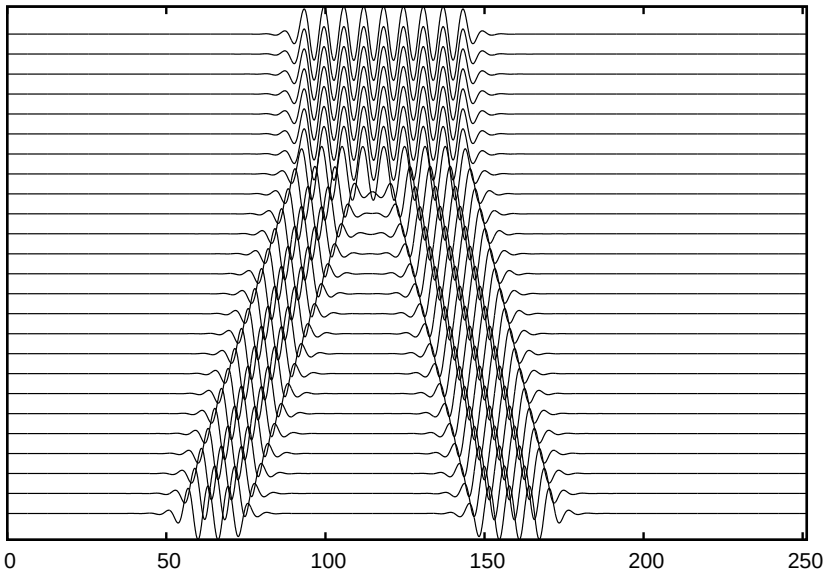
Asymmetric states are no longer stationary

Collision of like pulses for $r = -0.65$, $\epsilon = 0.1$

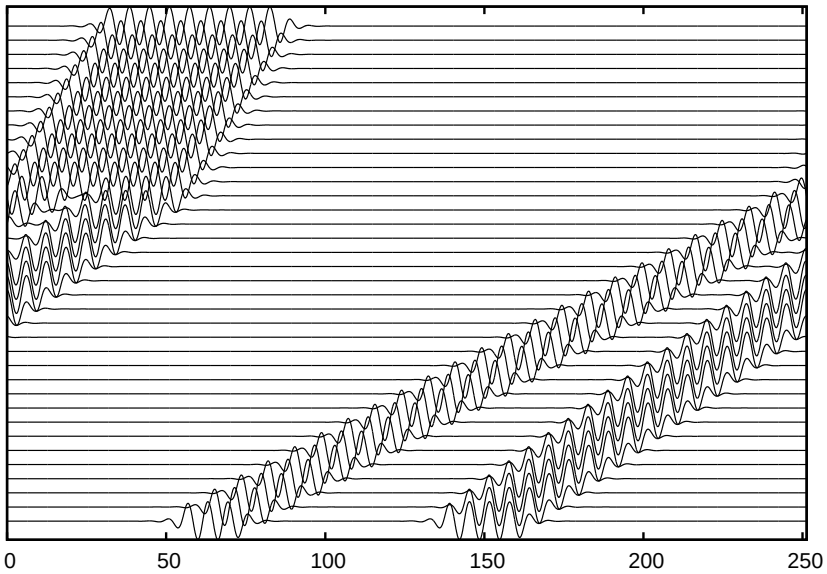


Houghton and Knobloch, PRE **84**, 016204 (2011)

Collision of unlike pulses: $\epsilon = 0.1$, $r = -0.65$



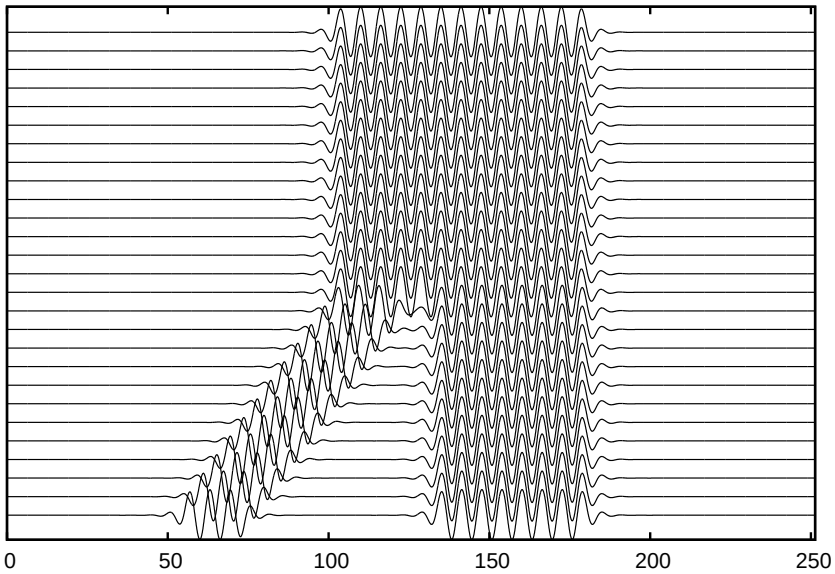
repulsion



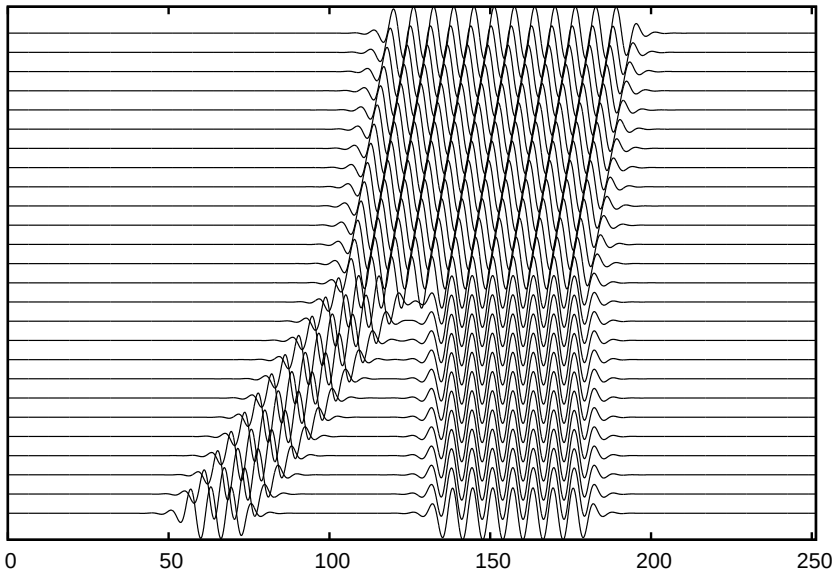
attraction

Like fronts repel, unlike fronts attract

Collision of unlike pulses: $\epsilon = 0.1$, $r = -0.65$



attraction



repulsion

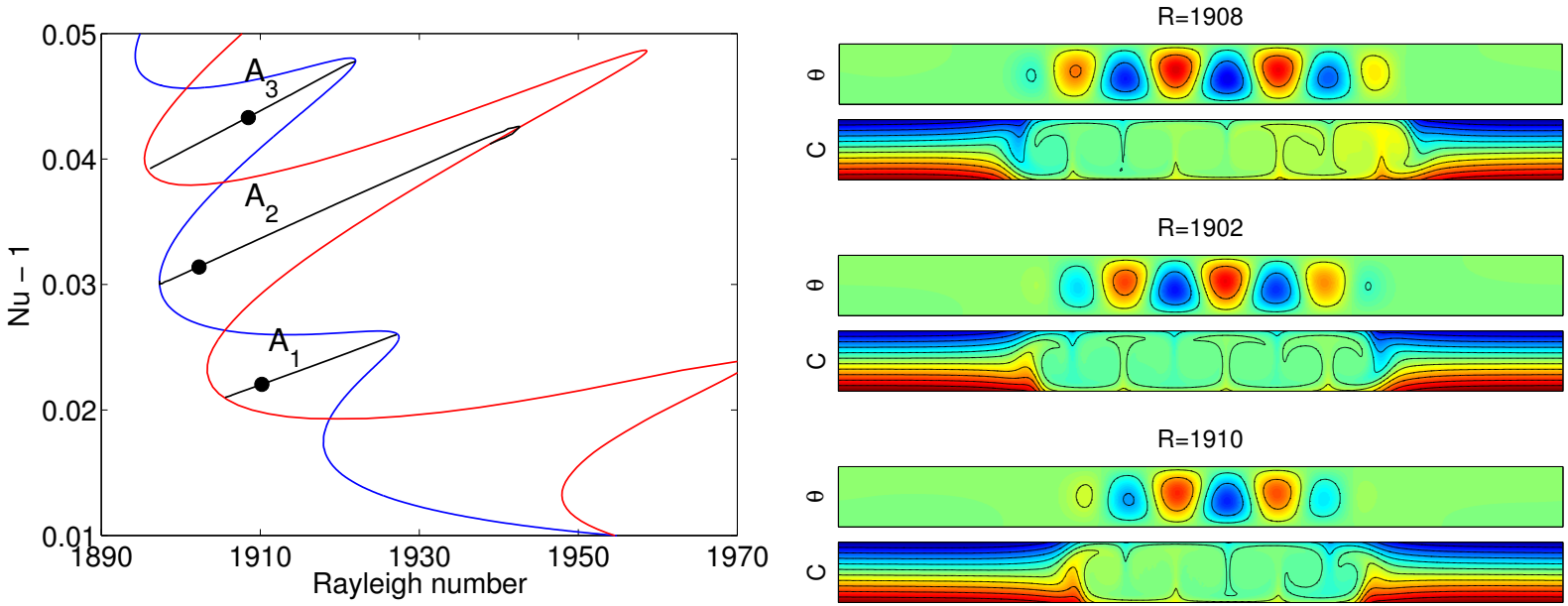
Like fronts repel, unlike fronts attract

Binary fluid convection with $S = -0.1$, $\tau = 0.01$, $\sigma = 7$

Newton's law of cooling:

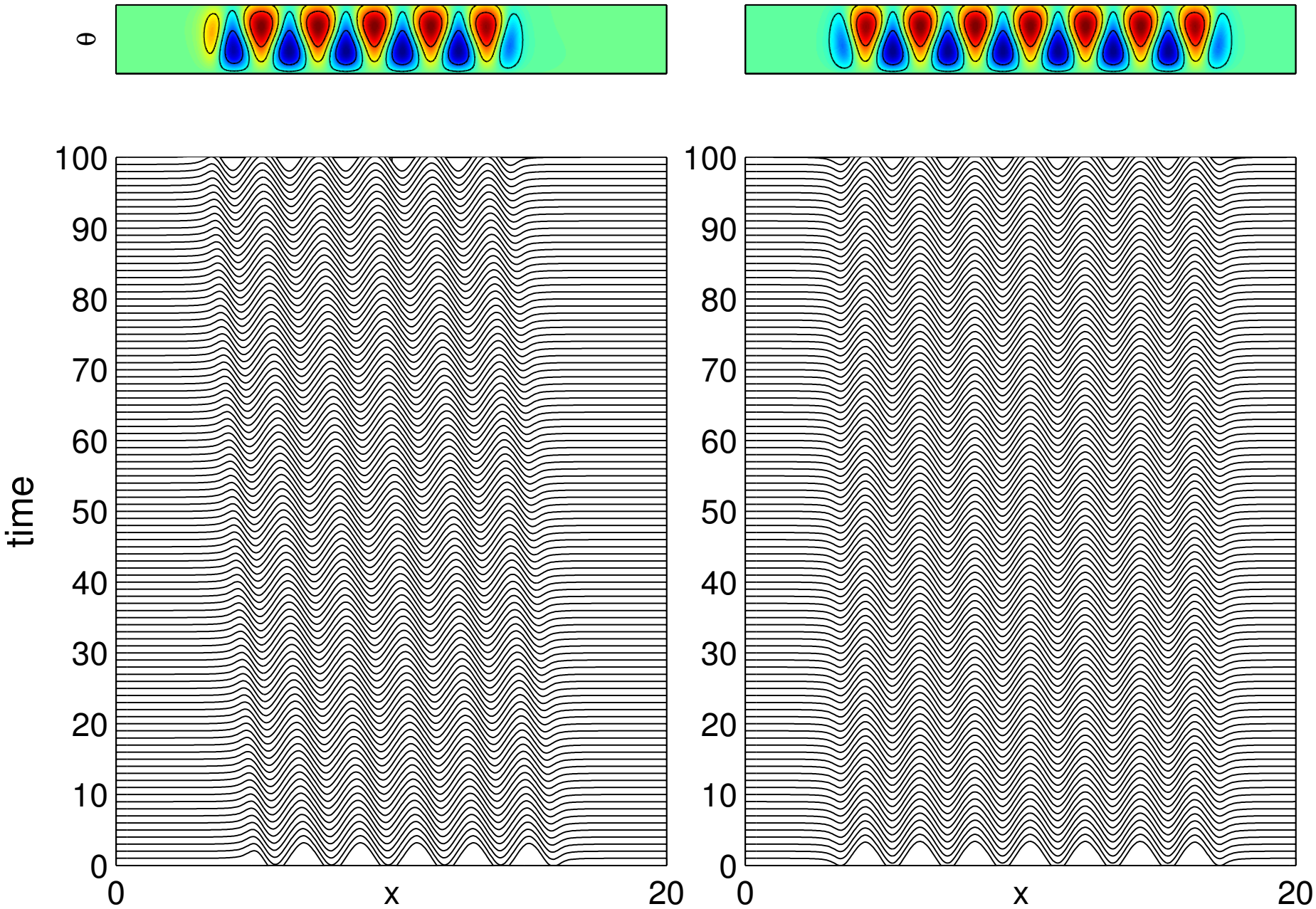
$$(1-\beta)\theta_z + \beta\theta = 0 \quad \text{on} \quad z = 1, \qquad \theta = 0 \quad \text{on} \quad z = 0.$$

Here $\beta = 1$:

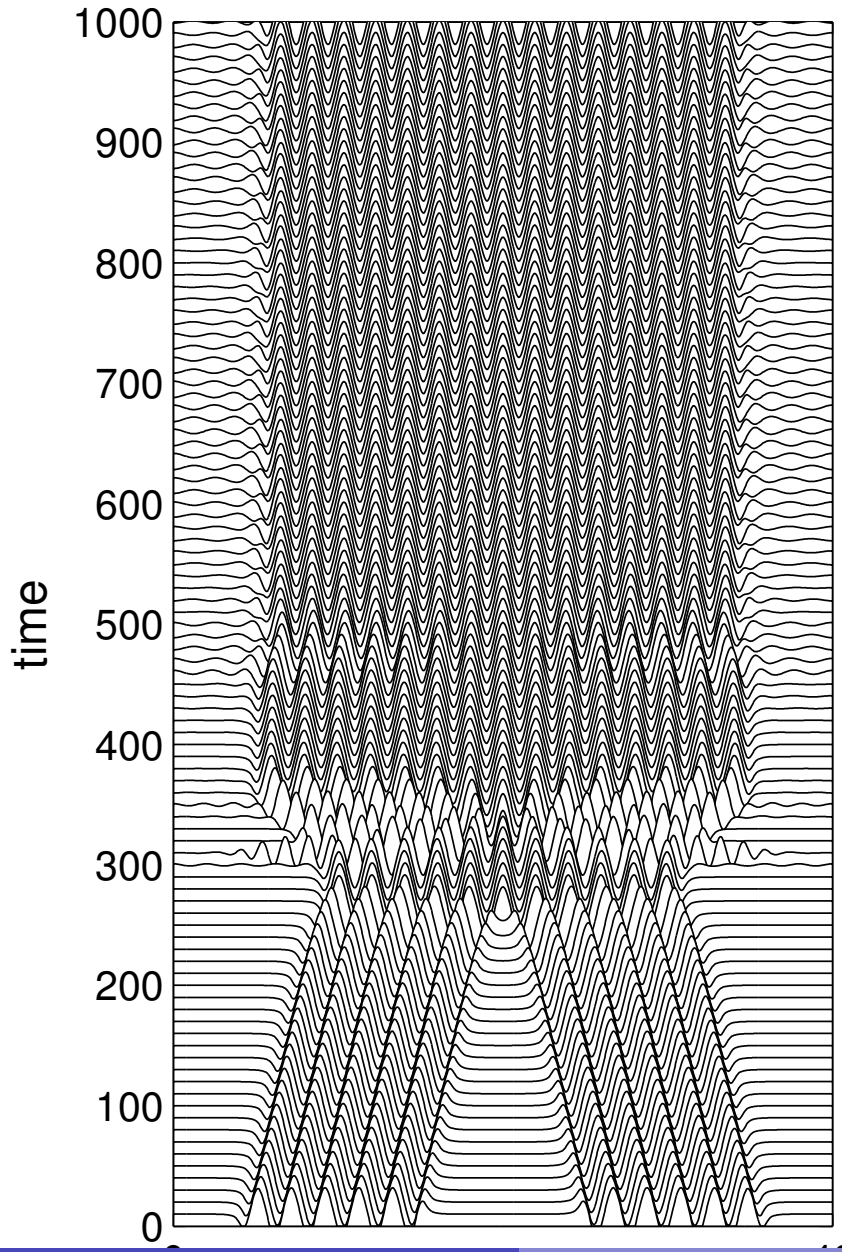


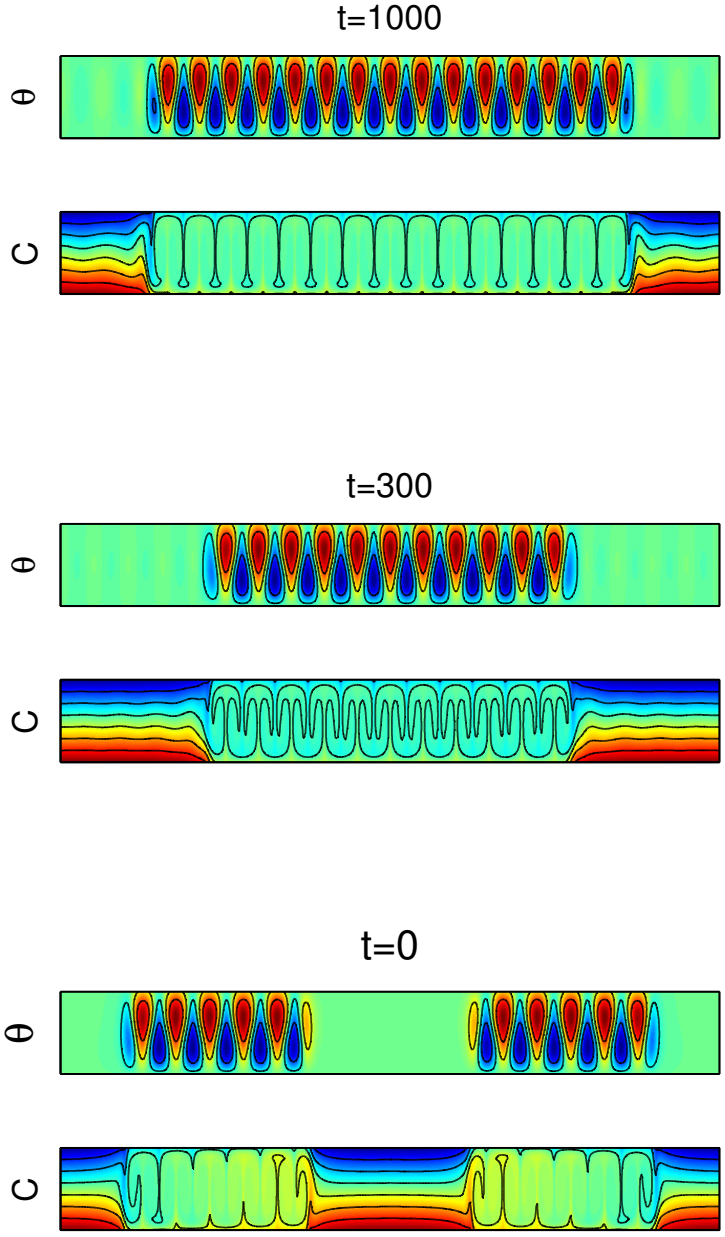
Mercader et al., JFM **722**, 240 (2013)

$\Gamma = 20,\ S = -0.5,\ \sigma = 0.6,\ \tau = 0.03,\ R = 2750,\ \beta = 0.9$



Binary fluid convection with heat loss: $\beta = 0.9$





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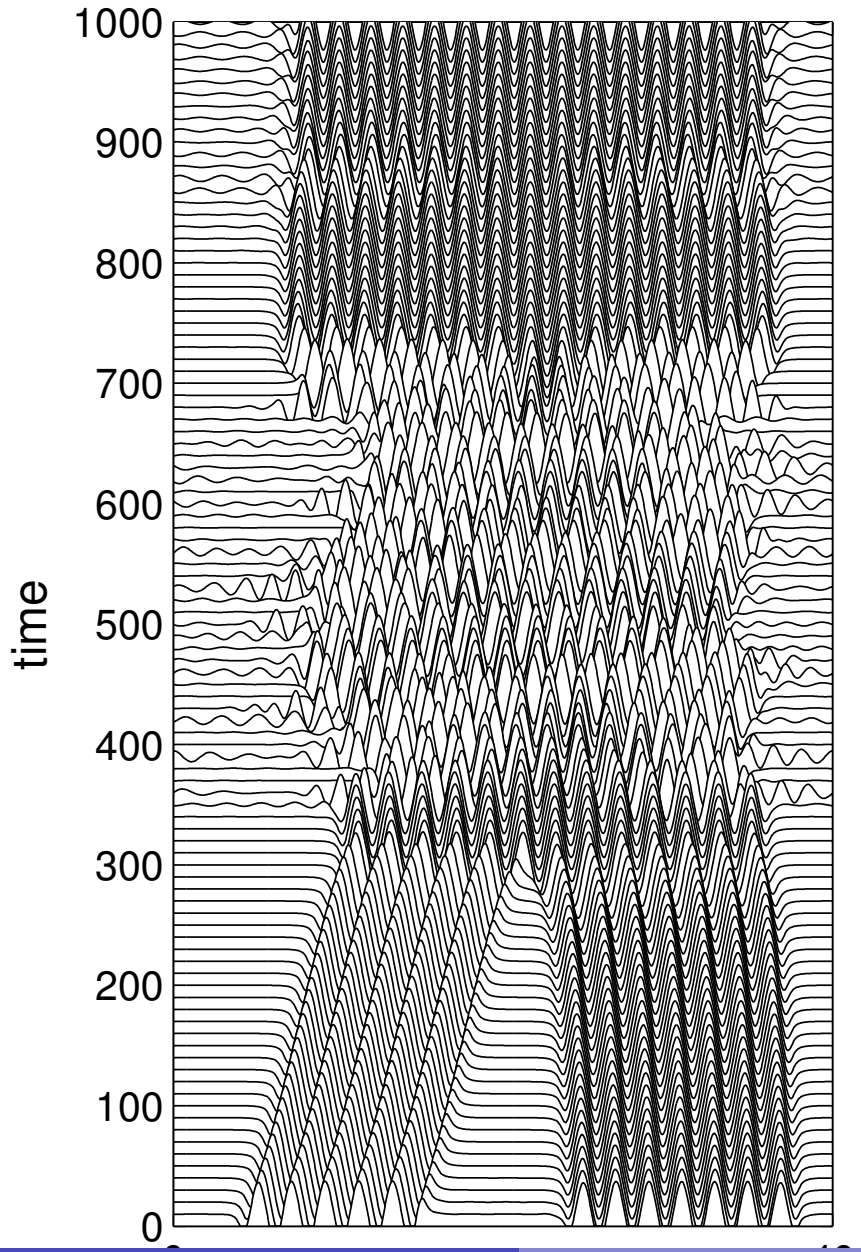

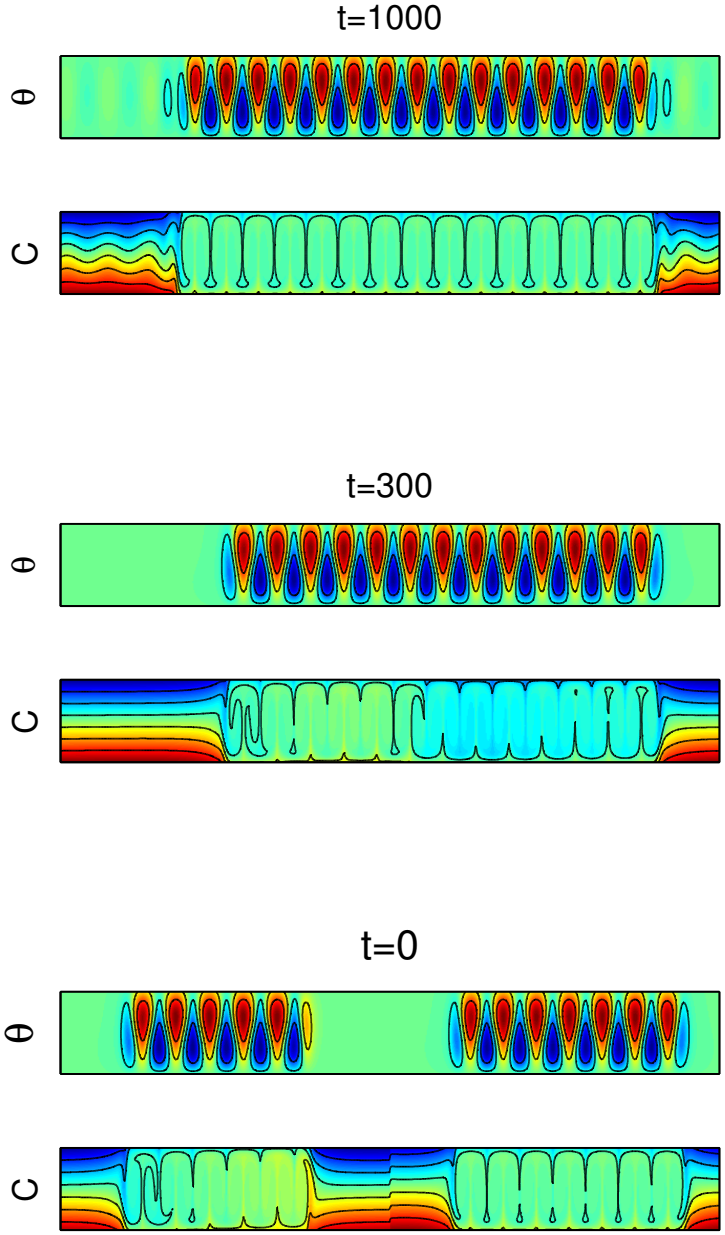

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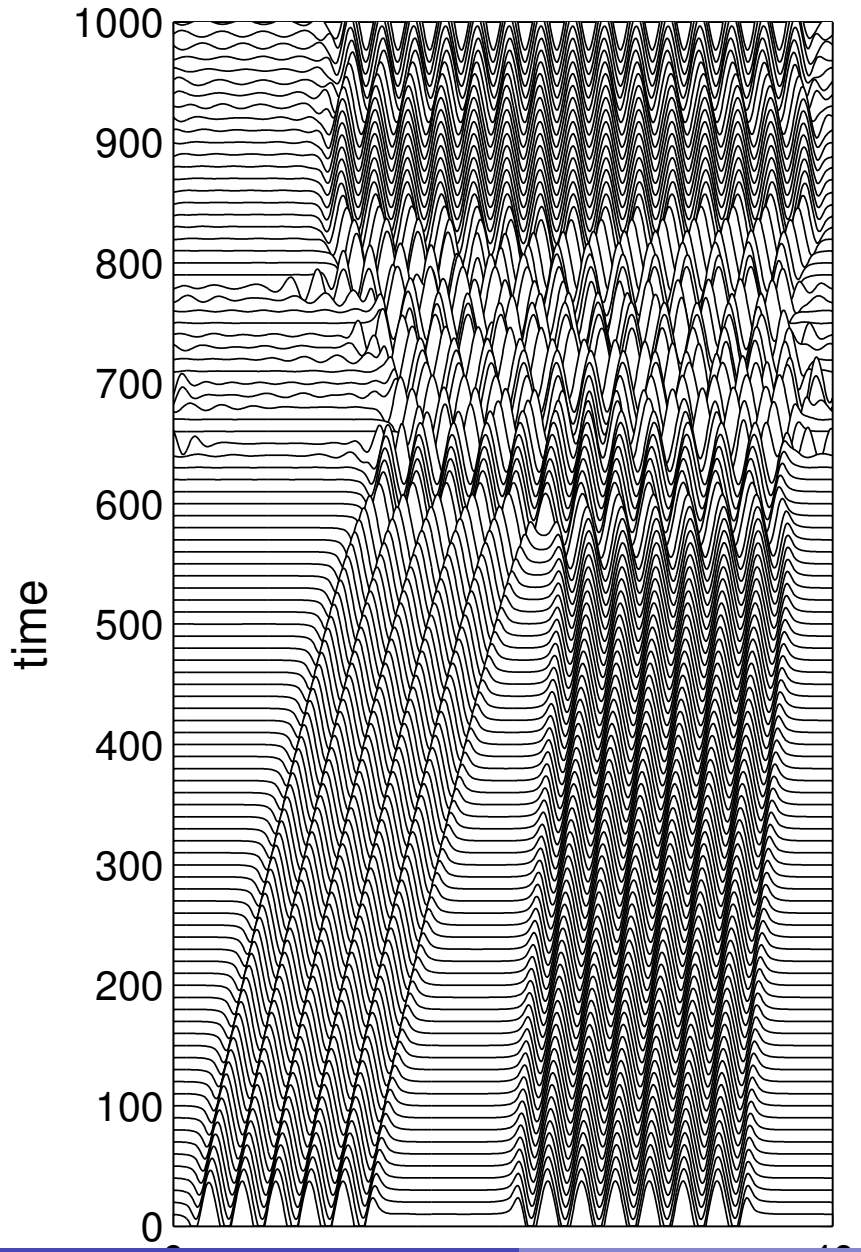

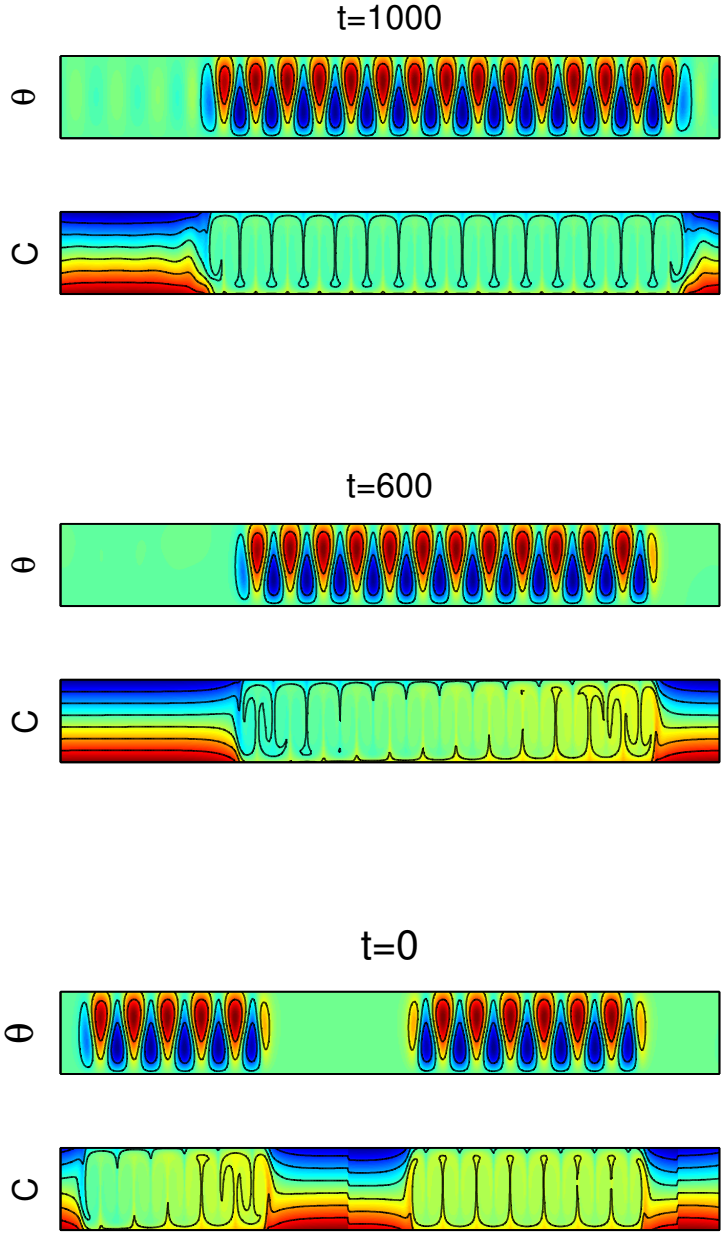

Conclusions

The fidelity of the SHE as a model of double diffusive convection is quite astonishing. This is despite

- its simplicity and even its variational structure
- no double diffusive effects are explicitly included
- the model cannot in fact be 'derived' by any systematic procedure

It is a 'minimal' model that contains an intrinsic scale and bistability, no more, no less.

Is this a case (in the immortal words of P Diamond) of

- all models are wrong, but some models are useful, or
- some models are too good to be true, others are too true to be good?