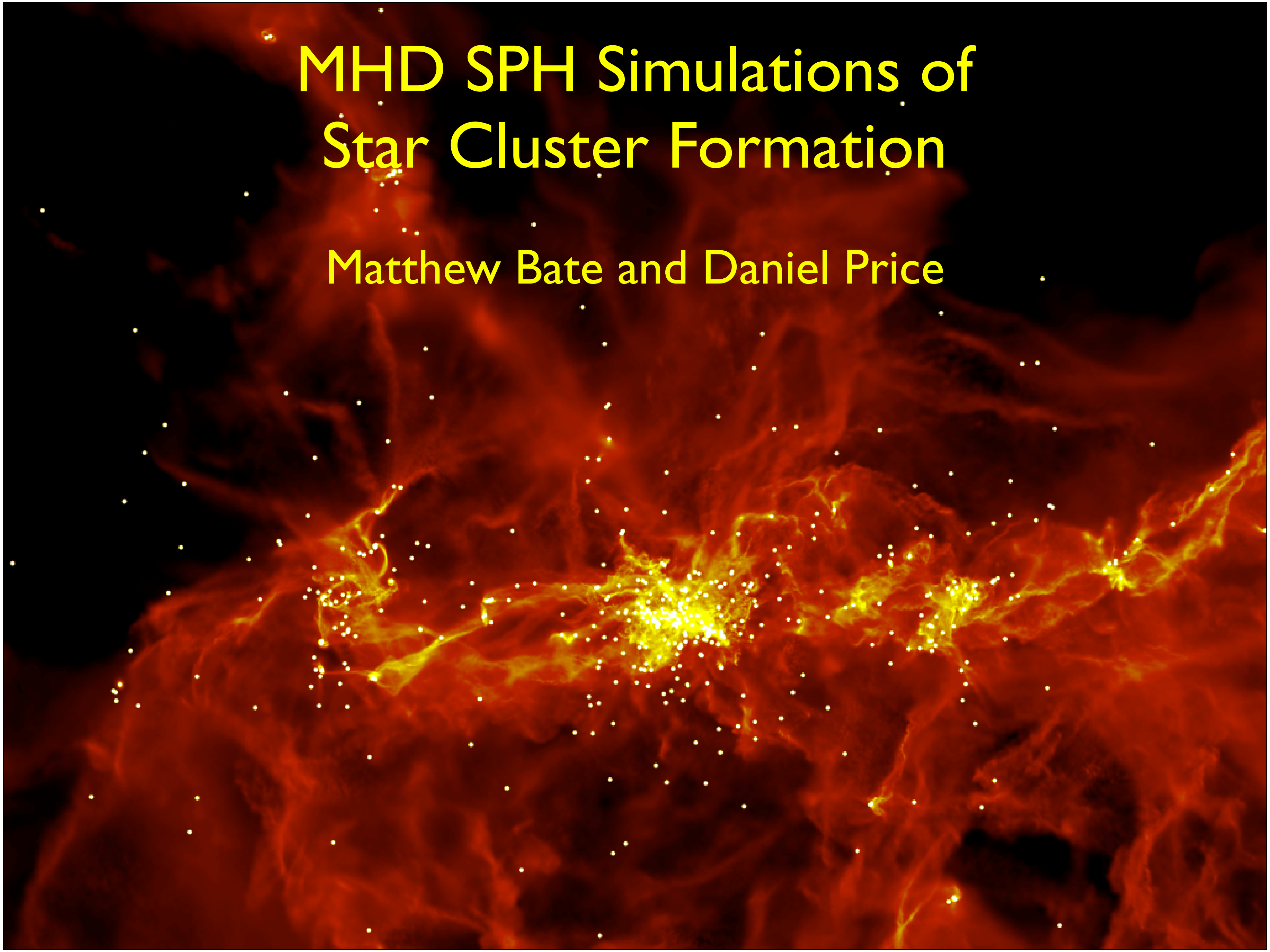
MHD SPH Simulations of Star Cluster Formation

Matthew Bate and Daniel Price



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http://www.astro.ex.ac.uk/people/mbate

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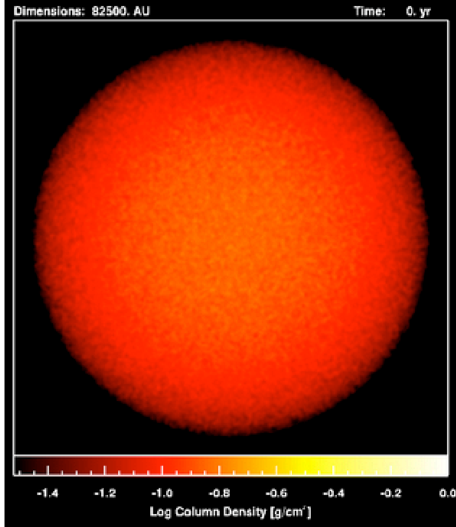
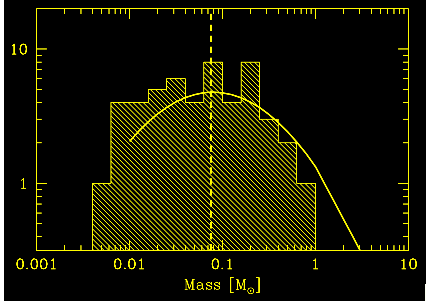
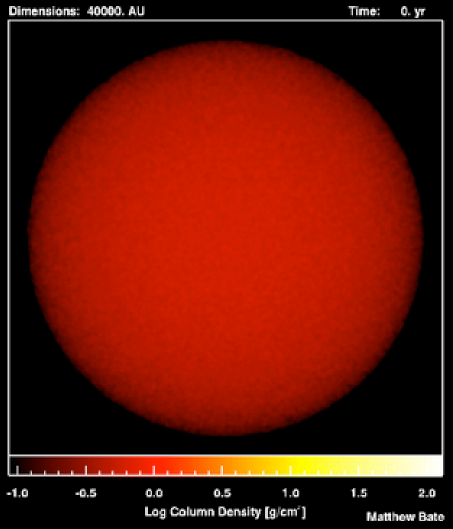
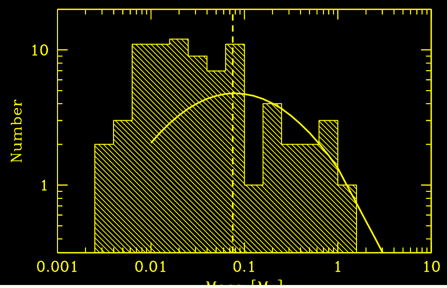
Typical molecular cloud
Jeans mass 1 M$_\odot$, Opacity limit 3 M$_J$, $P(k) \propto k^{-4}$



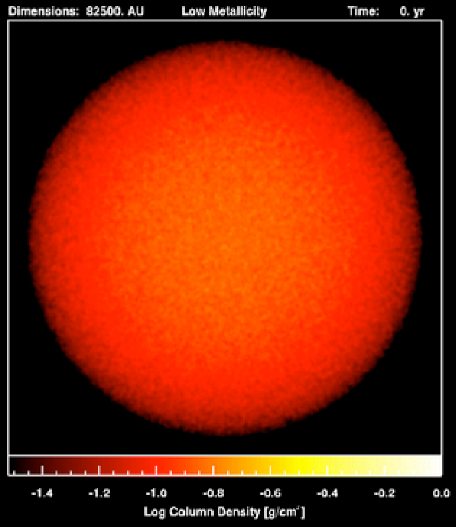


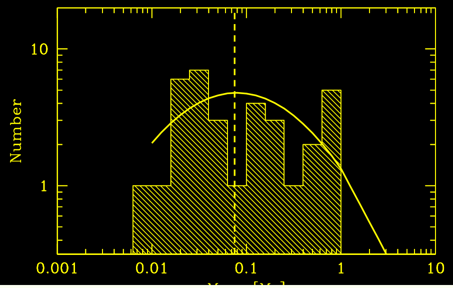
Denser cloud
Jeans mass 1/3 M$_\odot$



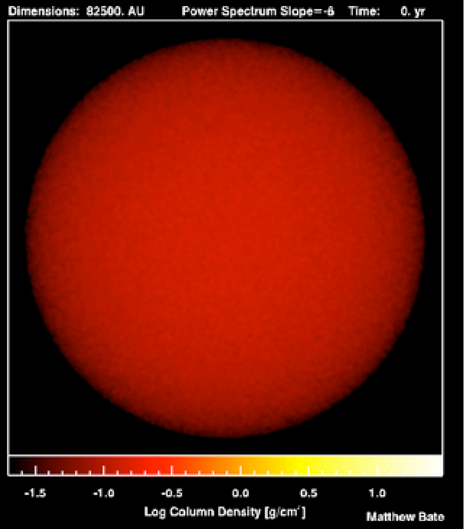


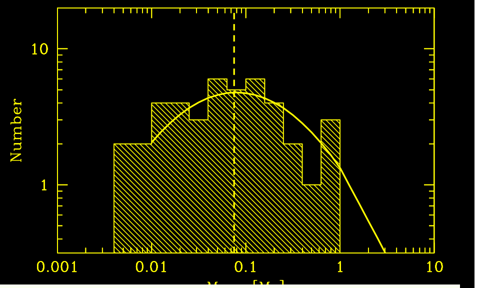
Lower metallicity cloud
Opacity limit 9 M$_J$

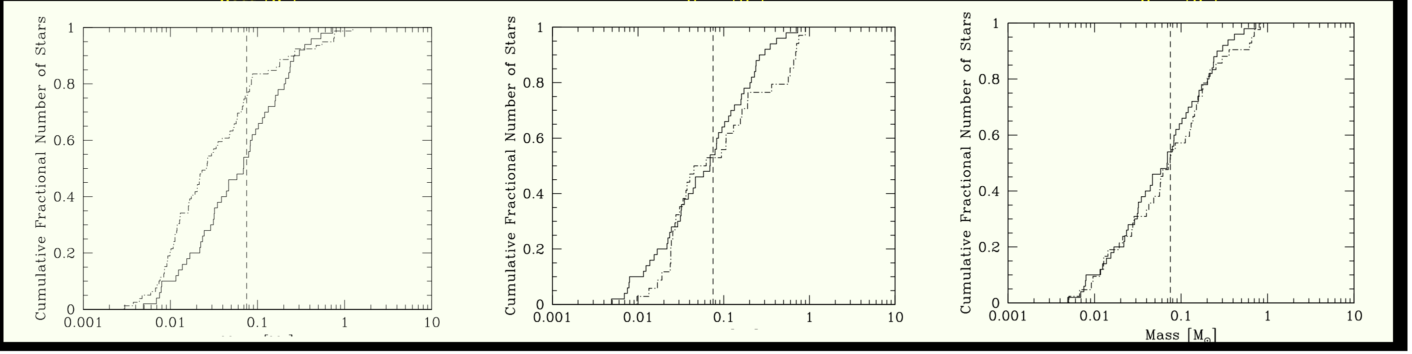




Large-scale `turbulence'
$P(k) \propto k^{-6}$







Real Star Cluster Formation

- Just finished a calculation 10 times more massive than Bate, Bonnell & Bromm 2003, Bate & Bonnell 2005, Bate 2005
  - 500 $M_\odot$ cloud, using 35,000,000 SPH particles
  - Resolves opacity limit for fragmentation
  - Follows:
    - Binaries to 1 AU and discs to ~10 AU radius
    - All binaries (0.02 AU) and discs to ~1 AU radius

- Statistics much improved over earlier calculations
  - 1254 objects at 1.50 t_{ff}
  - Binaries: 146 Triples: 40 Quadruples: 25 Quintuples: 20

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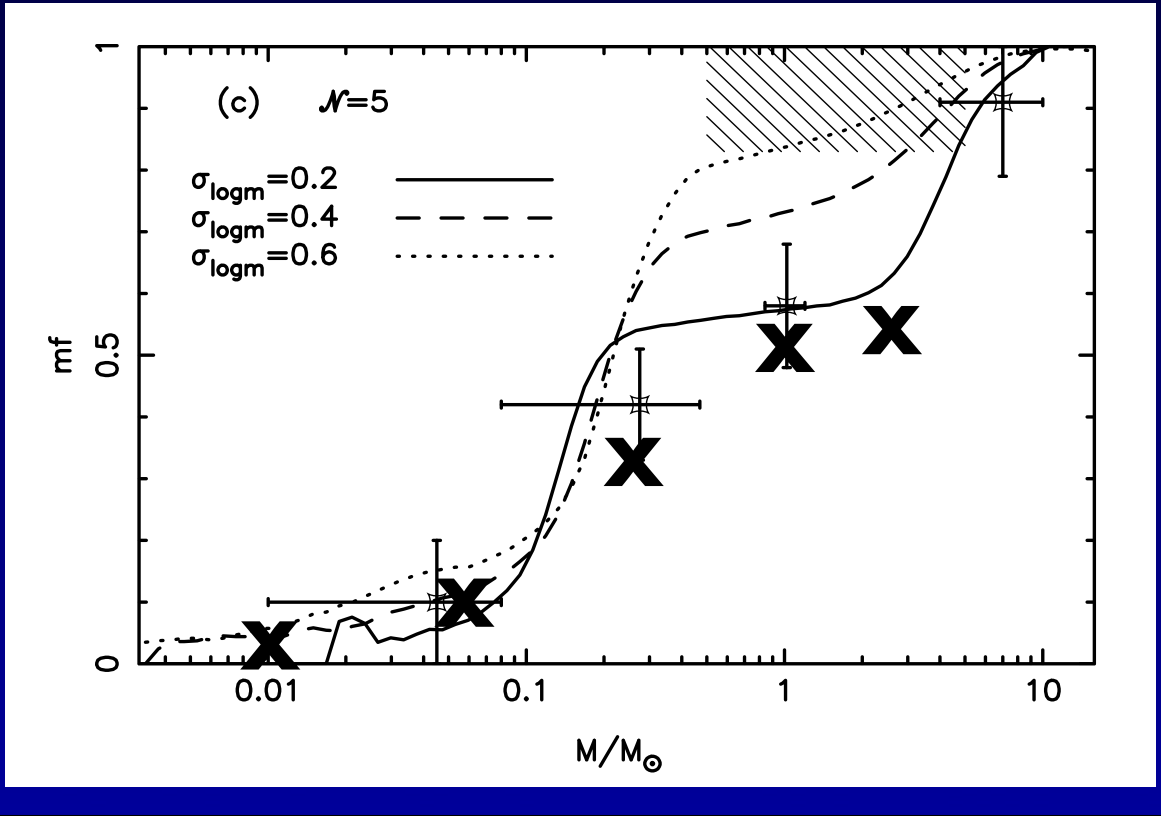
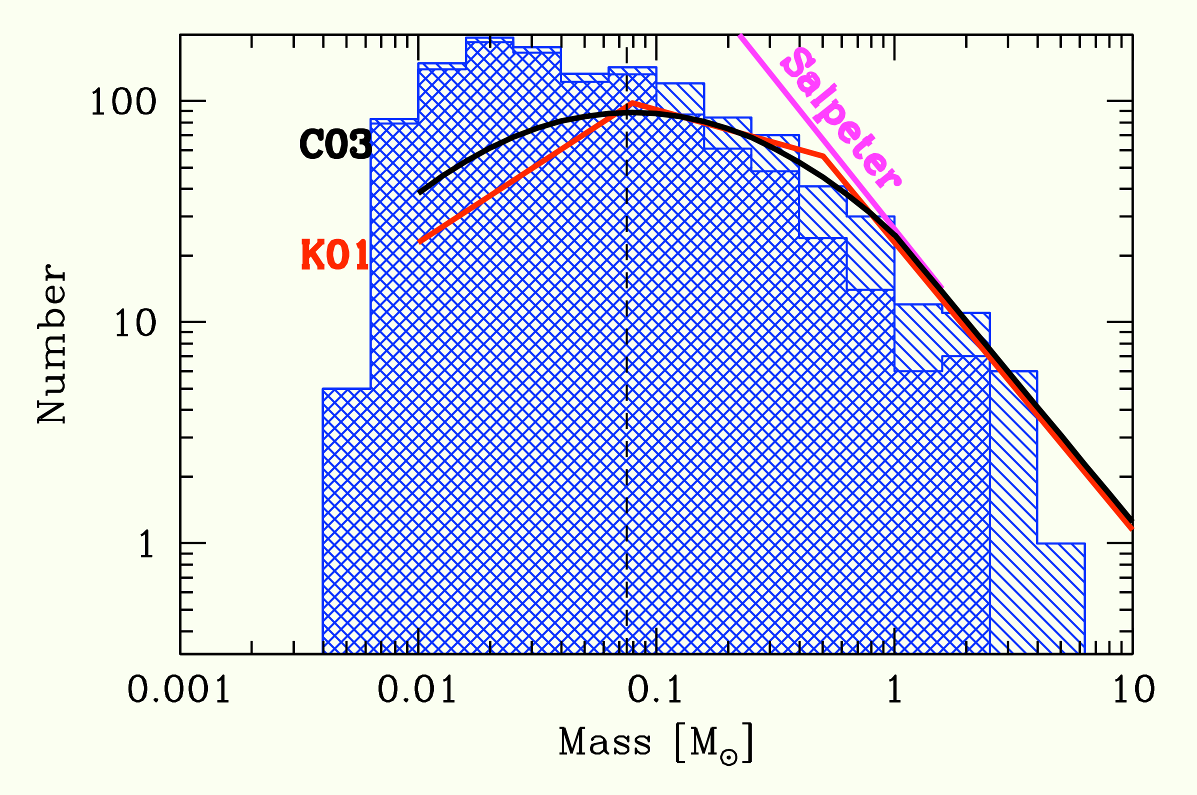
Binarity as a Function of Primary Mass

- Figure from Hubber & Whitworth (2005)
  - Observations: Martin et al. 2000; Fisher & Marcy 1992; Duquennoy & Mayor 1991; Shatsky & Tokovinin 2002
  - New large cluster calculation: **X**



Stellar Mass Distribution

- Competitive accretion/ejection gives
  - Salpeter-type slope at high-mass end
  - Low-mass turn over
- ~4 times as many brown dwarfs as a typical star-forming region
  - Not due to sink particle approximation - results almost identical for different sink parameters



Where to now?

- Statistics now good enough to know that pure hydrodynamics + sink particles cannot reproduce observations in detail
- Need to include additional physics
- Radiative transfer
  - Have developed a method for flux-limited diffusion within SPH (Whitehouse, Bate & Monaghan 2005; Whitehouse & Bate 2006)
  - Currently performing star cluster simulations with radiative transfer
- Magnetic fields
  - Price & Monaghan 2005; Price & Rosswog 2006; Rosswog & Price 2007
  - Star formation simulations: Price & Bate 2007

Magnetohydrodynamics (MHD)

- One-fluid approximation to plasma physics
- Ideal MHD implies "flux frozen-ness" (gas flows along field lines)

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla\cdot\left[\left(P+\frac{1}{2}\frac{B^2}{\mu_0}\right)\mathbf{I}-\frac{\mathbf{BB}}{\mu_0}\right]$$

$$\frac{du}{dt} = -\frac{P}{\rho}\nabla\cdot\mathbf{v}$$

$$\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right) = \left(\frac{\mathbf{B}}{\rho}\cdot\nabla\right)\mathbf{v}$$

$$\nabla\cdot\mathbf{B} = 0$$

SPH with MHD

- Some formulations of the momentum equation are unstable
  - We use the formulation of Morris (1996)
- Shocks
  - Use artificial dissipation terms for discontinuities in velocity, magnetic field, energy (Price & Monaghan 2004a)
- Maintaining $\nabla \cdot B = 0$: Use Euler potentials
  - Euler (1770), Stern (1976), Phillips & Monaghan (1985)
  - Use accurate SPH derivatives (Price 2004)

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

$$\frac{d\alpha}{dt} = 0, \frac{d\beta}{dt} = 0$$

‘advection of magnetic field lines’

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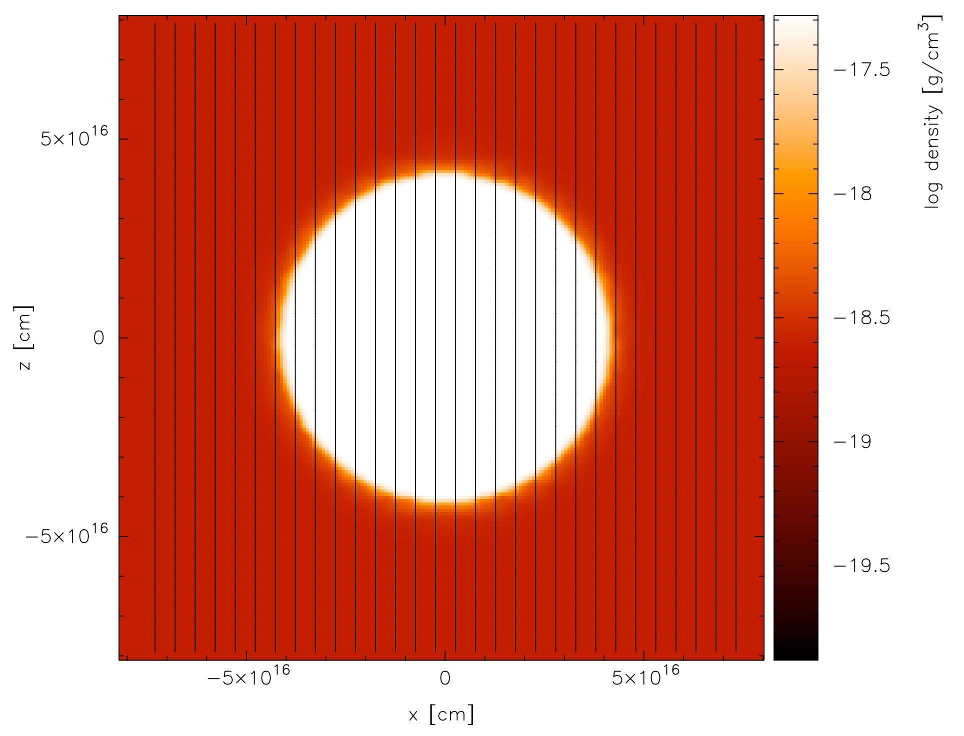
Single & Binary Star Formation
Price & Bate 2007



Resolution ~ 300,000 particles in core
(30,000 required to resolve Jeans mass,
ie. fragmentation)

- Dense core R=4x10^{16}cm=0.013pc =2674 AU
- Embedded in warm, low density medium
- M=1 $M_\odot$ in core
- Initial uniform B_z field
- T ~10K
- Solid body rotation
- Equation of state:

$$P = K\rho^{\gamma}$$

$$\begin{aligned} \gamma &= 1, & \rho &\leq 10^{-14}\mathrm{g\ cm}^{-3}, \\ \gamma &= 7/5, & \rho &> 10^{-14}\mathrm{g\ cm}^{-3}, \end{aligned}$$

Important Parameters

$$\alpha = \frac{\mathrm{E}_{\mathrm{therm}}}{\mathrm{E}_{\mathrm{grav}}}$$

Gravity vs pressure

$$\beta = \frac{\mathrm{E}_{\mathrm{rot}}}{\mathrm{E}_{\mathrm{grav}}}$$

Gravity vs rotation

$$\left(\frac{M}{\Phi}\right) / \left(\frac{M}{\Phi}\right)_{crit}$$

Gravity vs magnetic field

Field orientation: B_z or B_x

Important Parameters

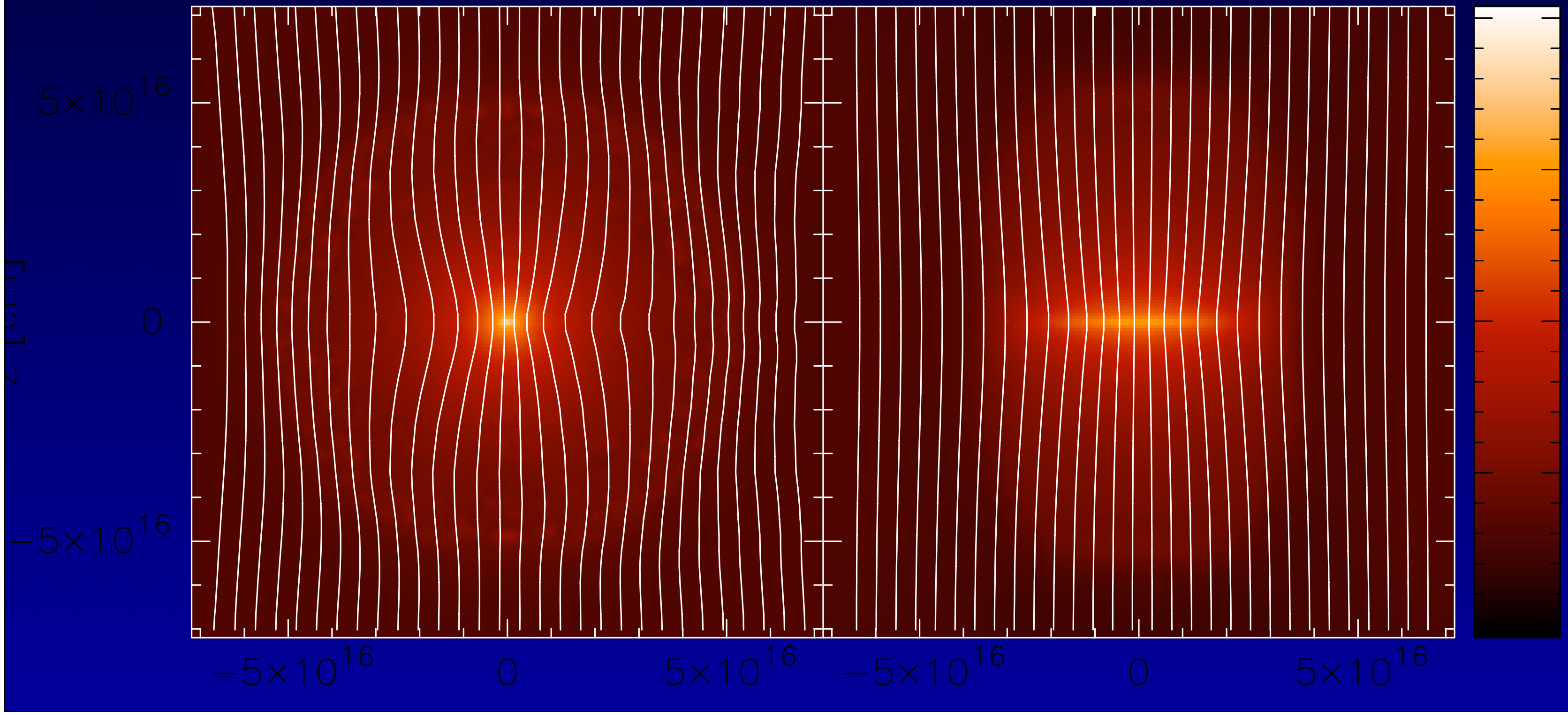
$\alpha = 0.35$ $\beta = 0.005$

($\Omega = 1.77 \times 10^{-13}$ rad/s)

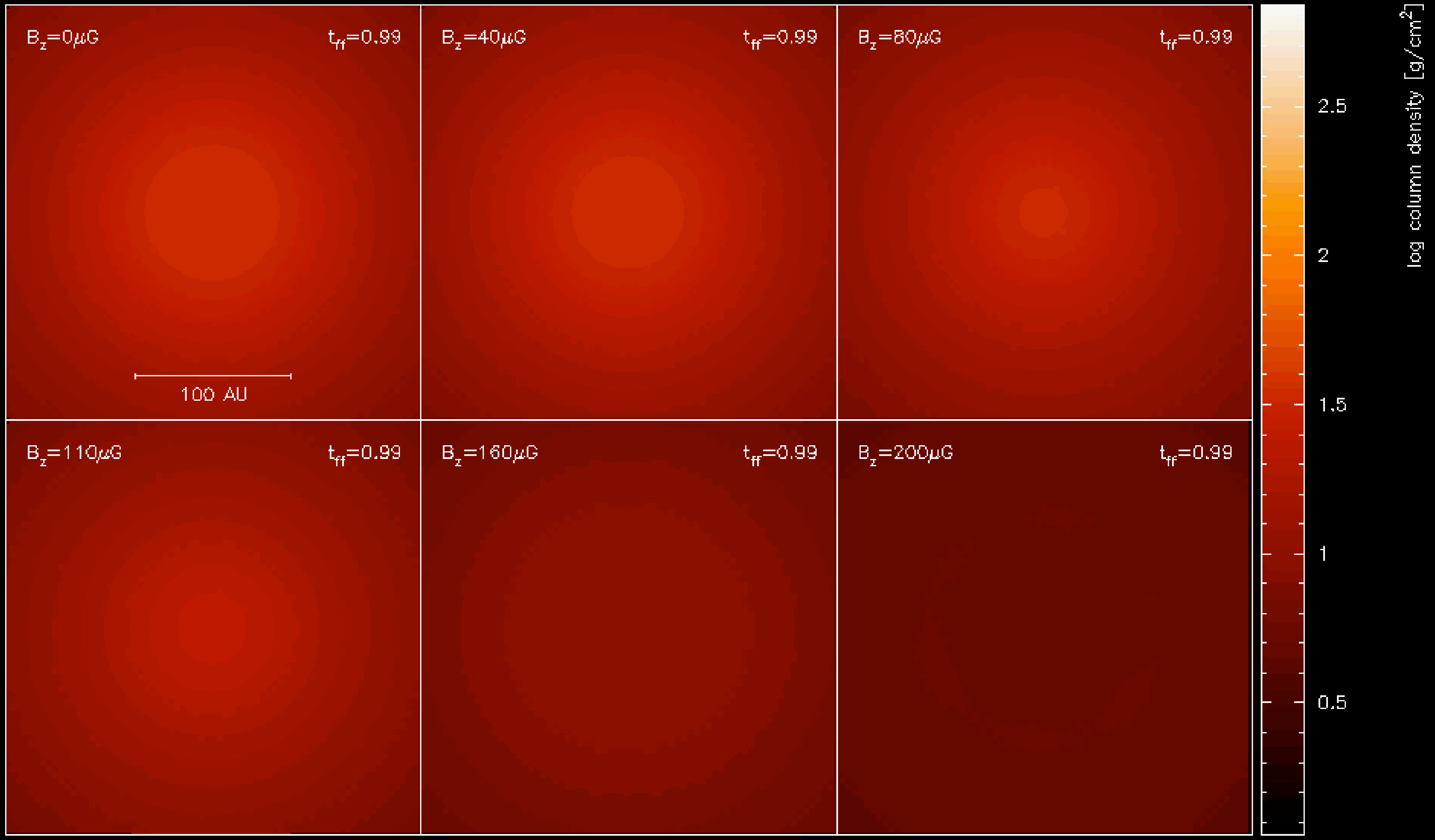
low B
field lines dragged in by
collapse

vary M/Φ

high B
collapse along field
lines



Effect of Magnetic Fields on Discs: B_z



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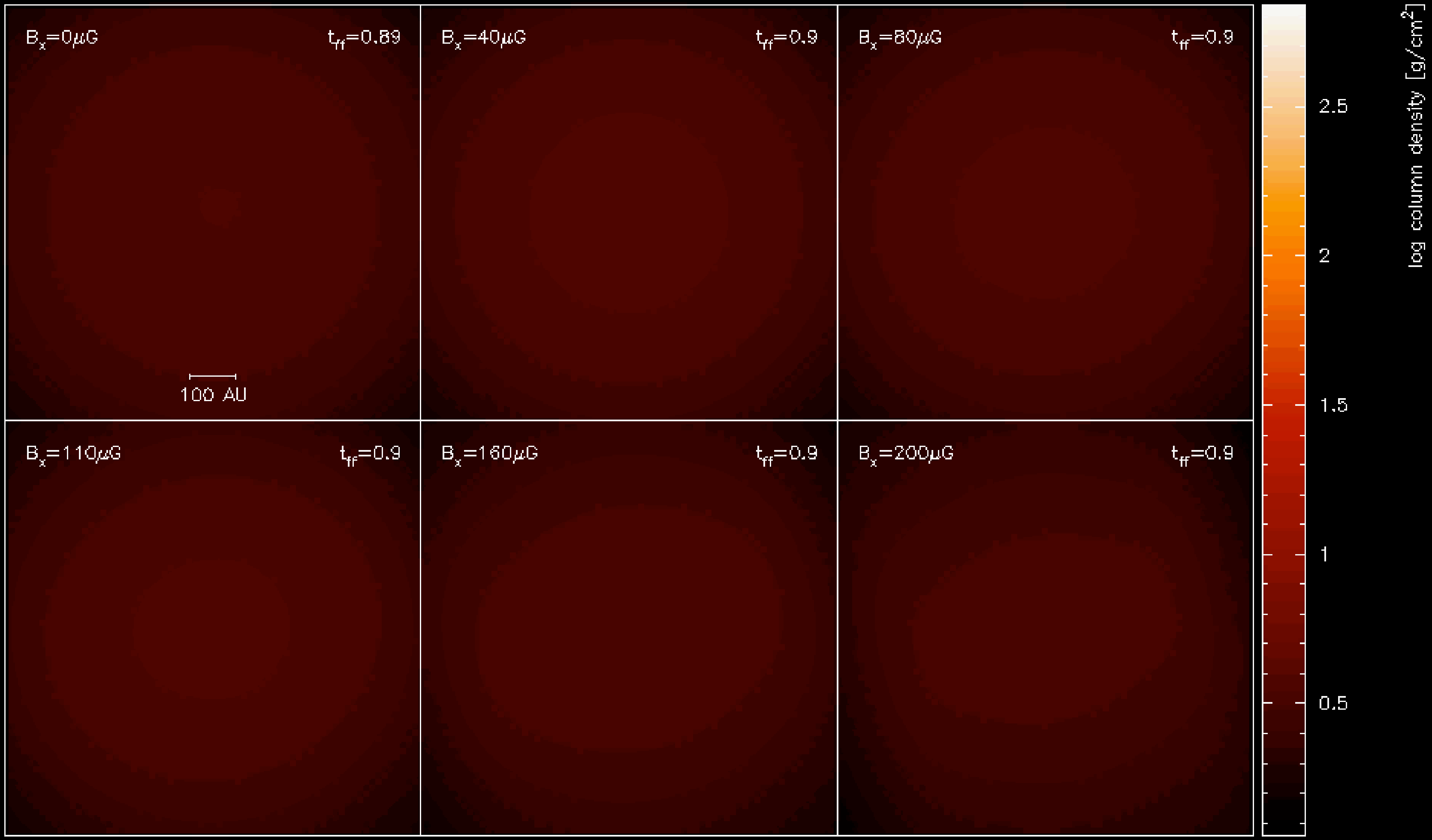
Effect of Magnetic Fields on Discs: B_x



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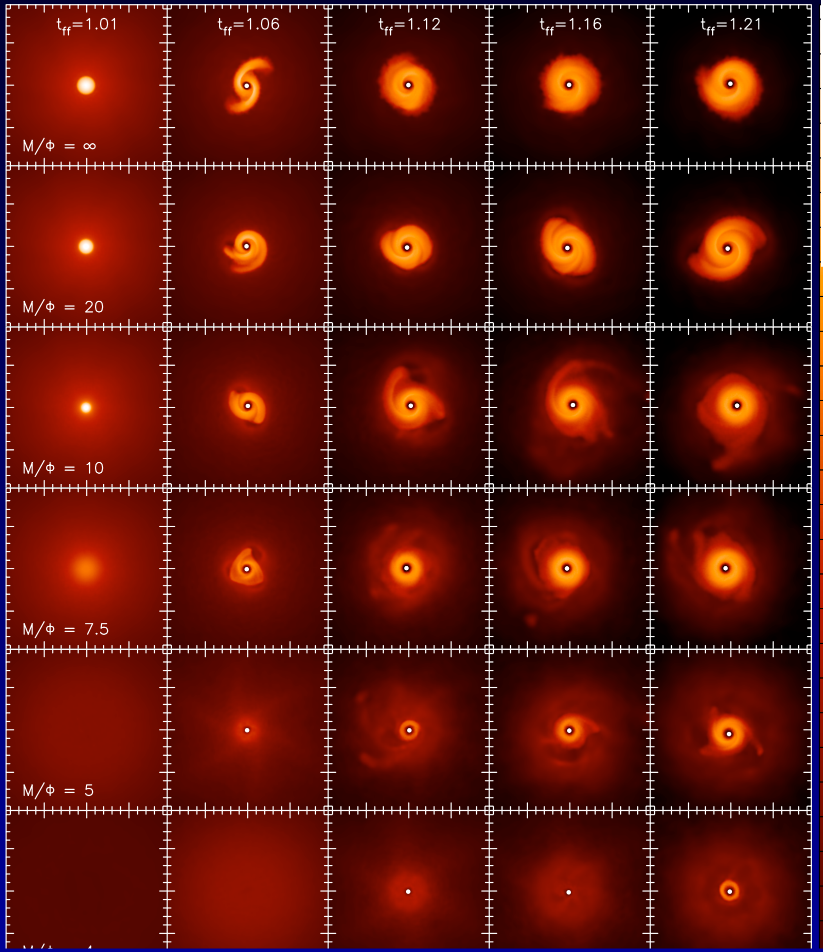
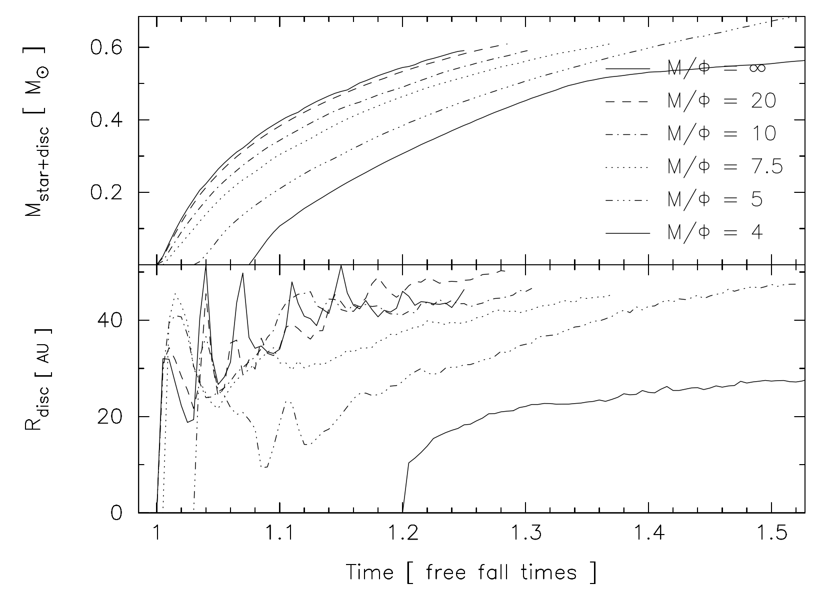
Effect of magnetic fields on circumstellar disc formation:





- Discs form later
- Less massive
- Smaller
- Slower accretion rates
- Less prone to gravitational instability

Binary Star Formation

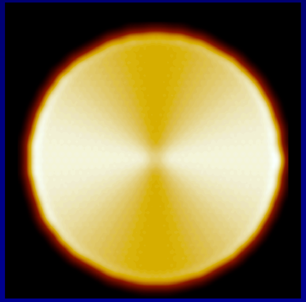
e.g. Boss & Bodenheimer (1979), Burkert & Bodenheimer (1993), Bate & Burkert (1997)

$\alpha = 0.26$

$\beta = 0.16$

$$\rho = \rho_0 (1 + 0.1 \cos{(m\phi)})$$

$m=2$



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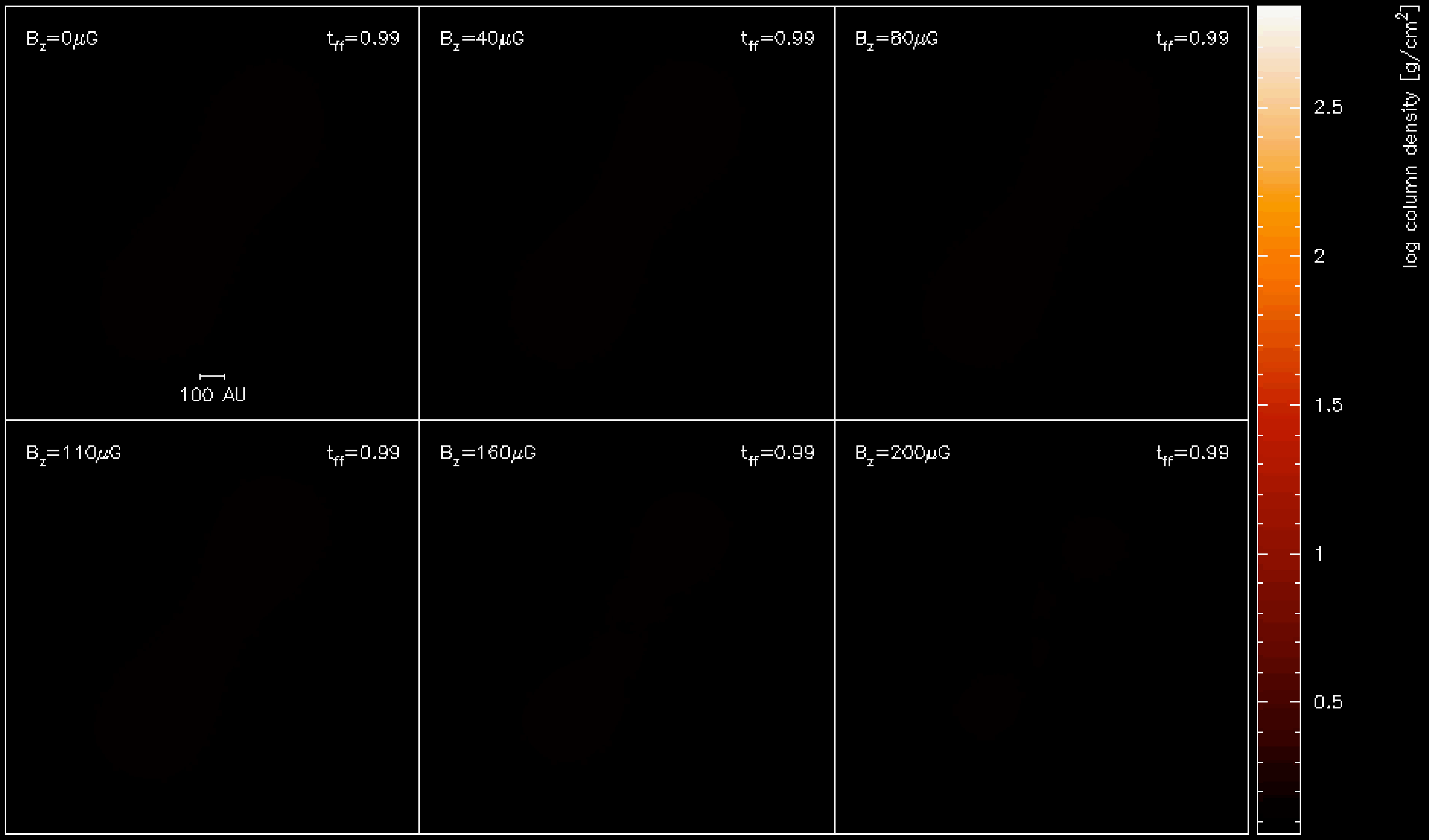
Magnetic Fields and Binary Formation: B_z



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Magnetic Fields and Binary Formation: B_x

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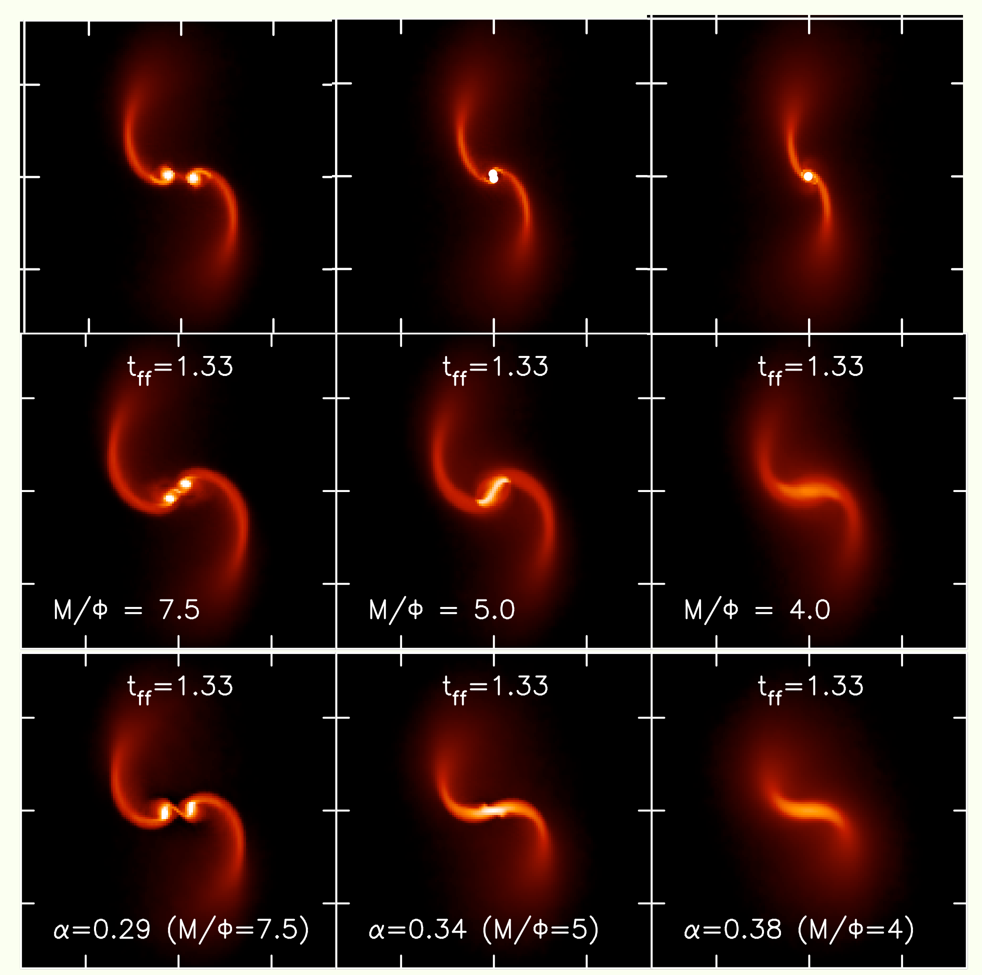
Impact of Magnetic Fields on Binary Formation

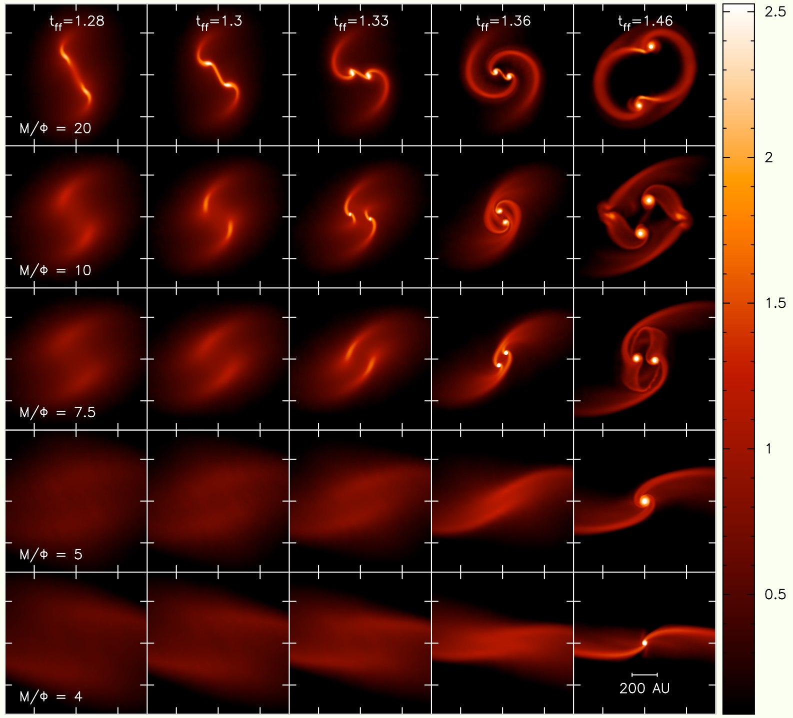
- Magnetic braking is usually thought of as primary effect of magnetic fields
  - e.g. Hosking & Whitworth 2004; Machida et al. 2004, 2005, 2006; Banerjee & Pudritz 2006

- We find magnetic pressure is the dominant cause of suppressed fragmentation
  - see also Boss 2005



Magnetic Tension Forces Can Aid Fragmentation

Full MHD (B_x field)



No magnetic tension forces

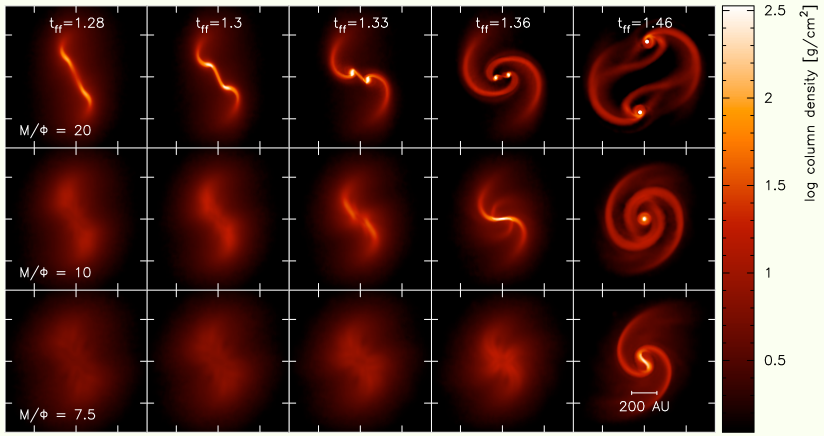
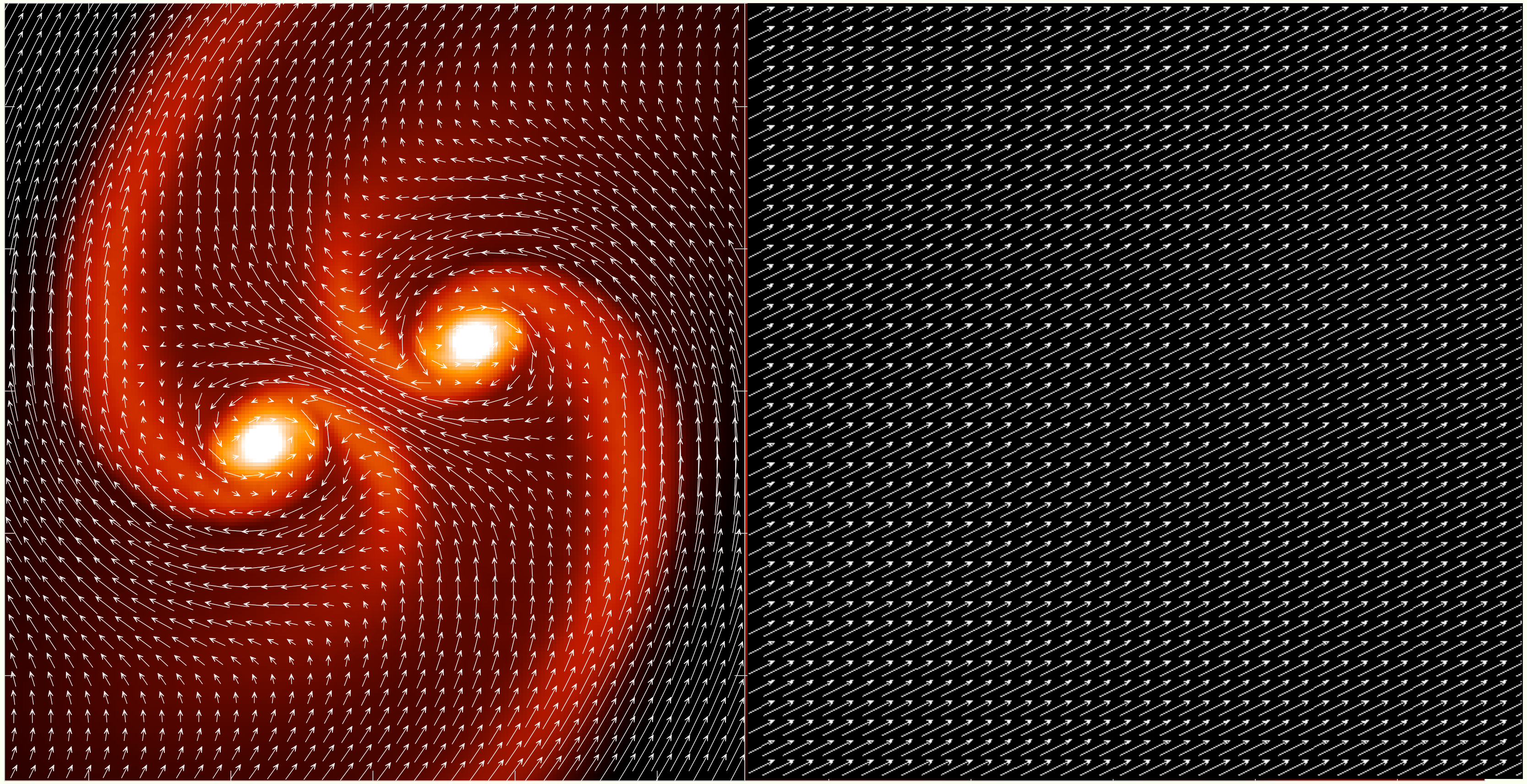


Figure 10. Results of binary star formation calculations using a magnetic field initially oriented perpendicular to the rotation axis (ie. initial field in the

- Effect of magnetic tension strongly dependent on field orientation
- Tension acts to increase fragmentation (c.f. Boss 2000,2002)

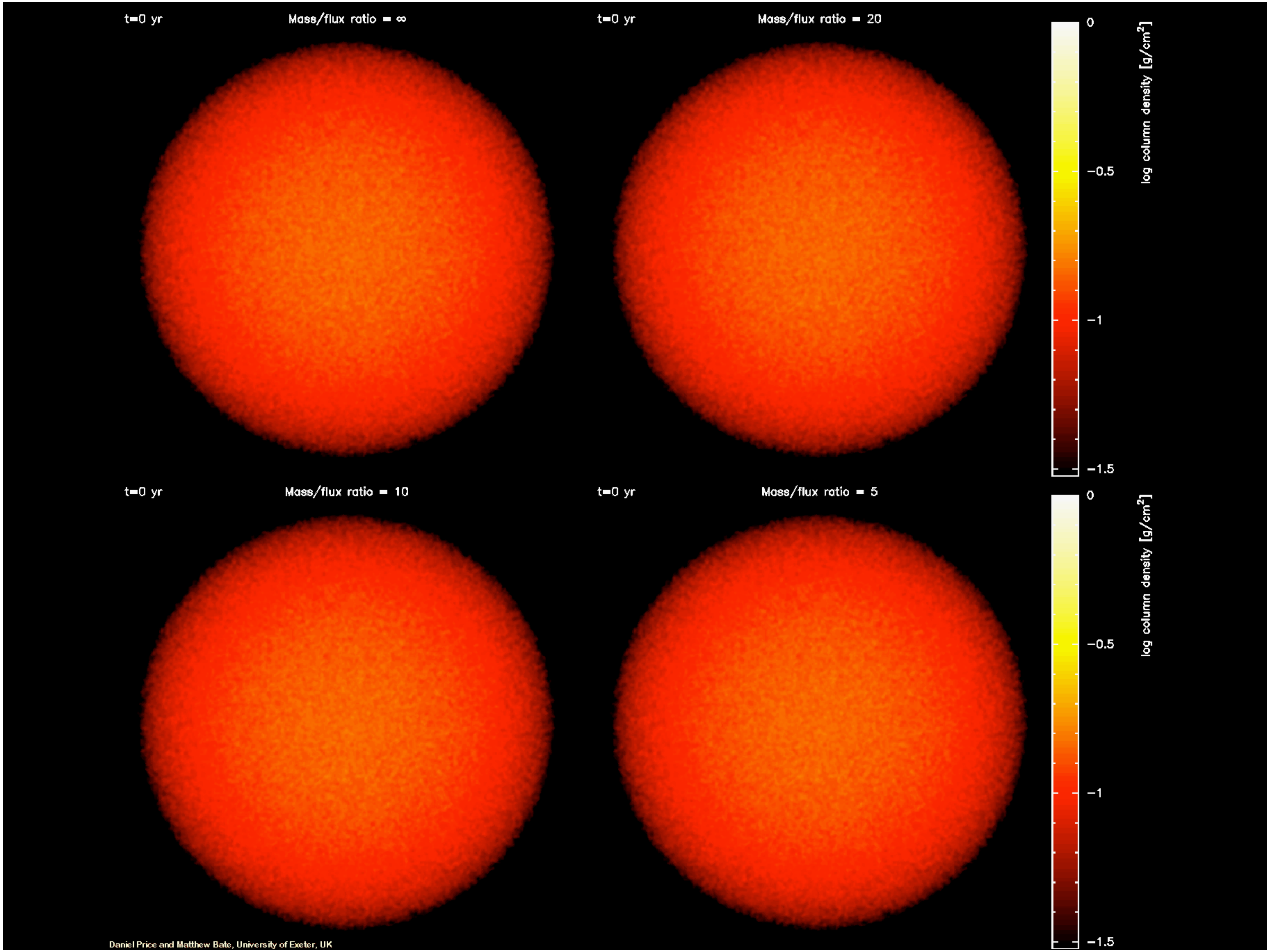
Magnetic Cushioning



First MHD Star Cluster Formation Calculations

- Repeat Bate, Bonnell & Bromm (2003)
  - 50 $M_\odot$ cloud, diameter 0.4 pc, mean thermal Jeans mass 1 $M_\odot$
  - Same resolution: 3,500,000 particles
    - Four times fewer particles per $M_\odot$ than Price & Bate (2007)
  - Magnetic field: 0, 10, 20, 40 microgauss
  - Mass-to-flux (M/Φ) ratio: Infinity, 20, 10, 5
  - Plasma beta (ratio of thermal to magnetic pressure) : Infinity, 7, 3.5, 2

- Follow to 1.6 initial cloud free-fall times (same as original calculation)
  - 300,000 yrs


t=0 yr
Mass/flux ratio = ∞
t=0 yr
Mass/flux ratio = 20
0
−0.5
−1
−1.5
log column density [g/cm^2]
t=0 yr
Mass/flux ratio = 10
t=0 yr
Mass/flux ratio = 5
0
−0.5
−1
−1.5
log column density [g/cm^2]
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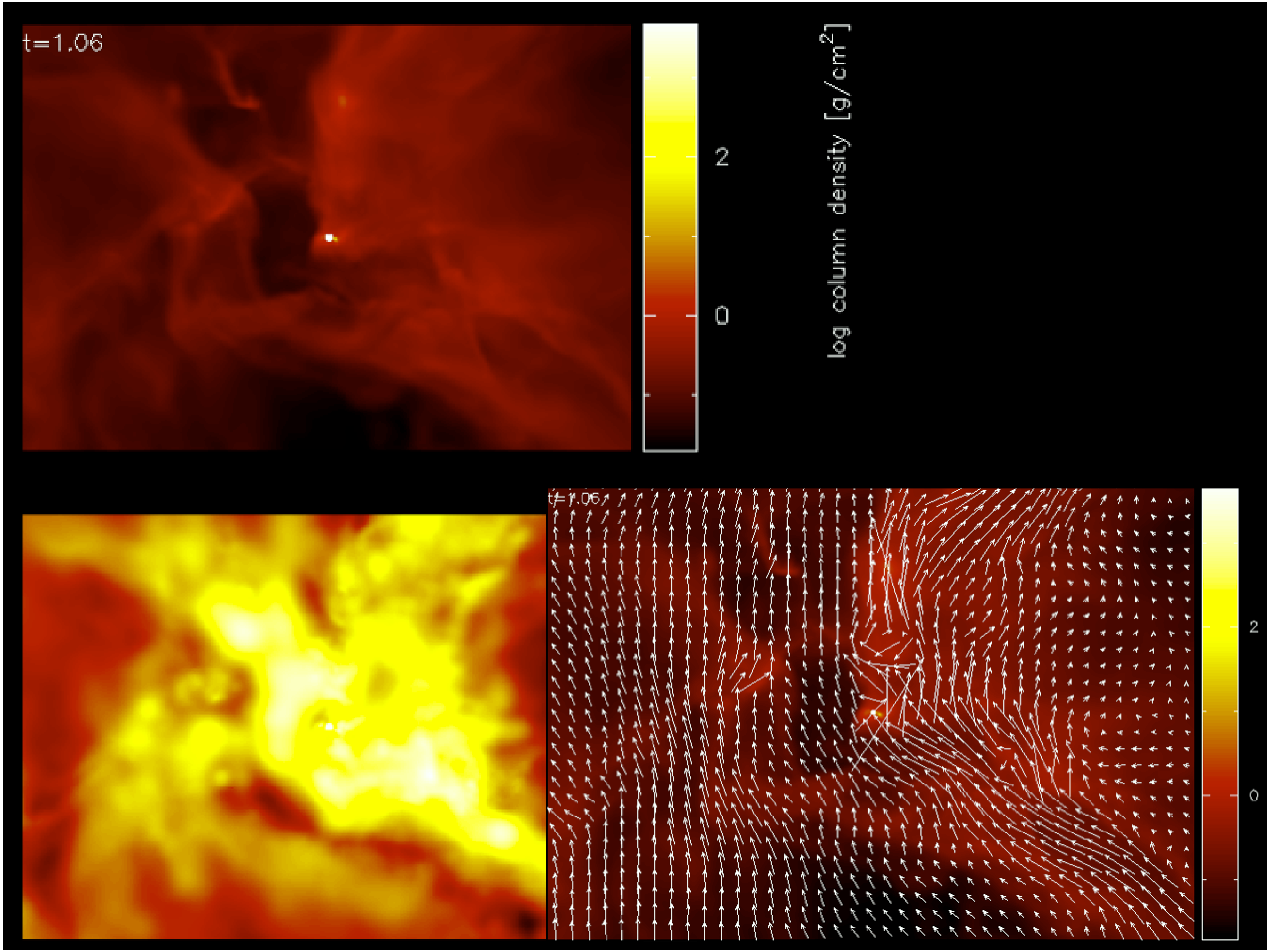
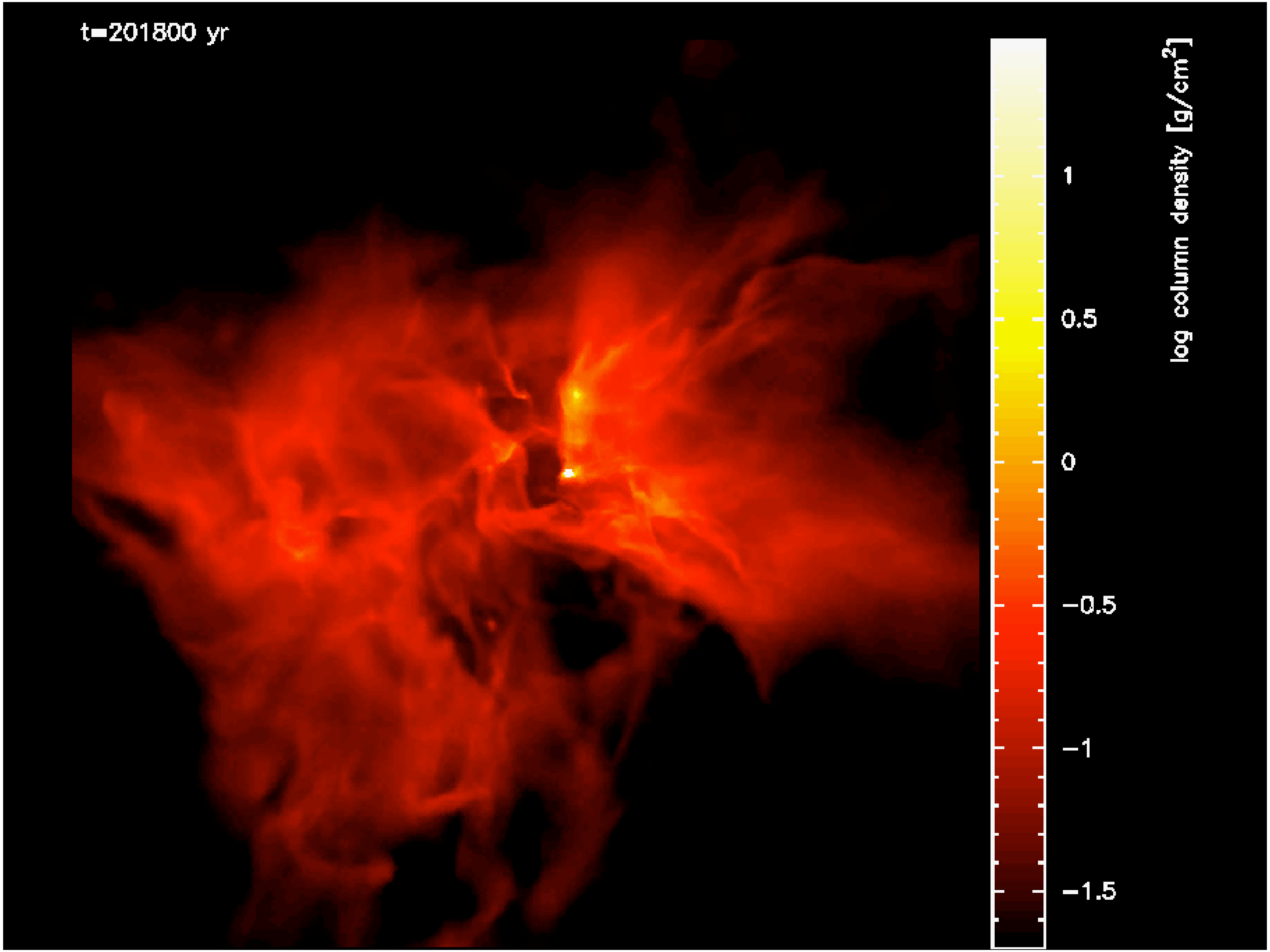
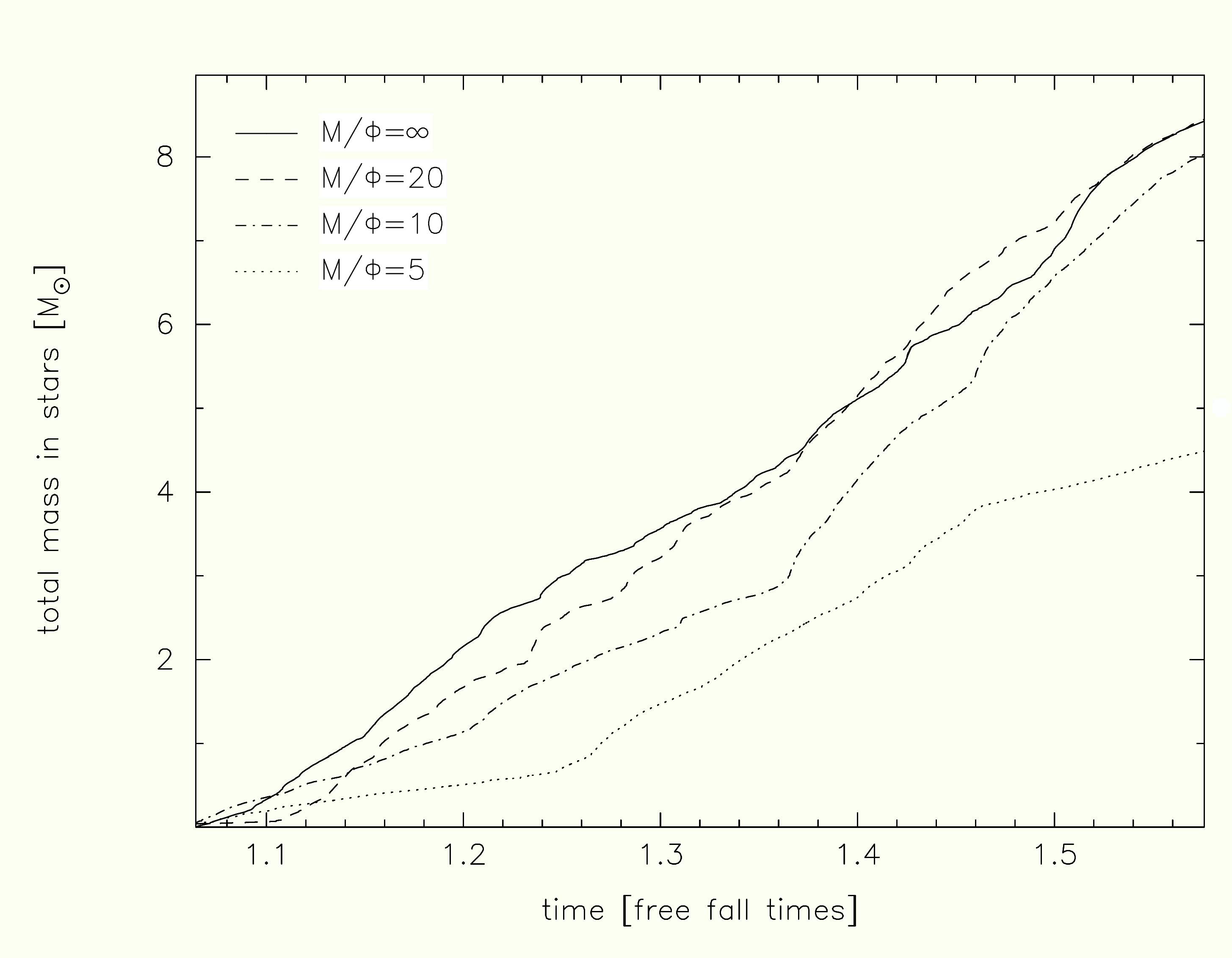

t=1.06
2
0
log column density [g/cm^2]
t=1.06
2
0


t=201800 yr
log column density [g/cm^2]
1
0.5
0
−0.5
−1
−1.5

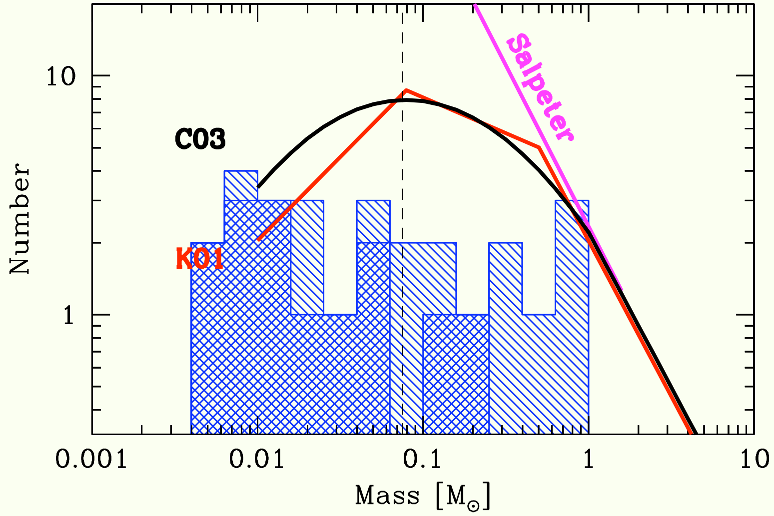
Star Formation Efficiency



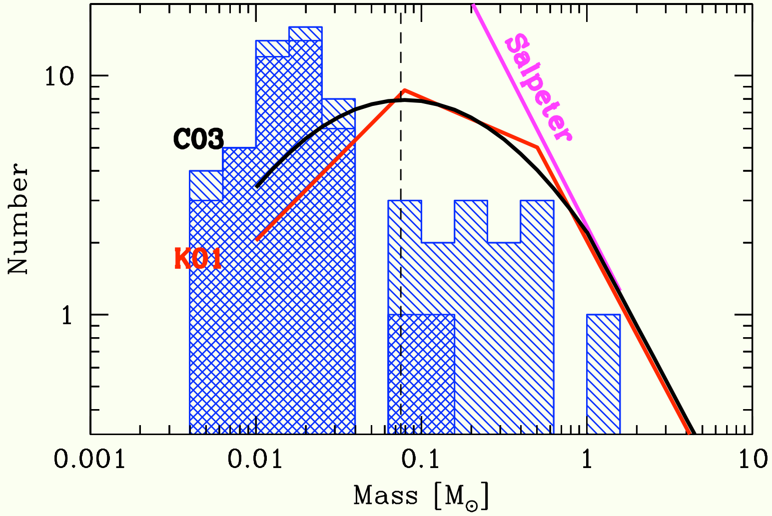
20% SFE_{ff}

Mass Functions

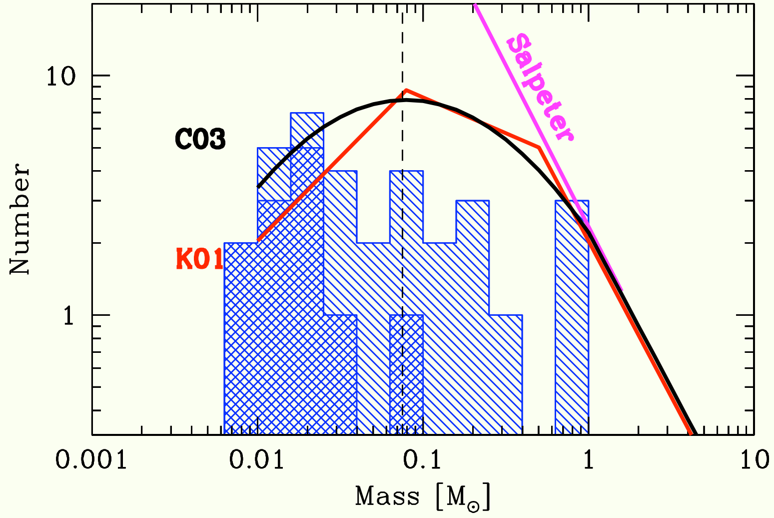
M/Φ=Infinity



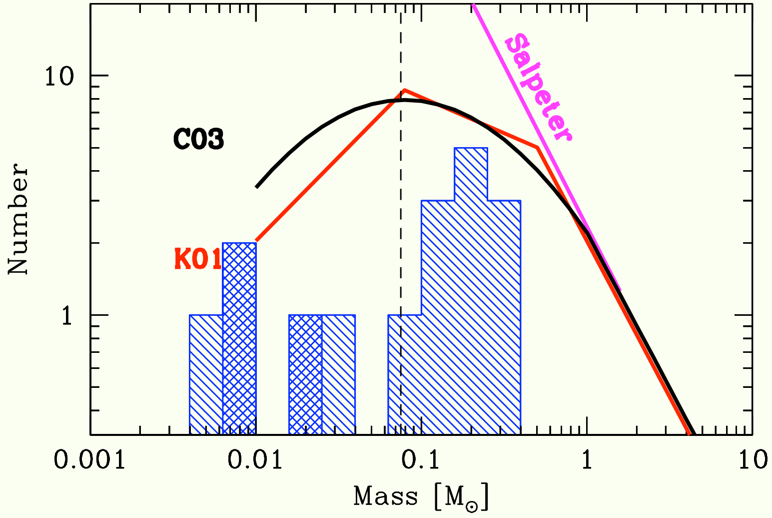
M/Φ=20



M/Φ=10



M/Φ=5



Conclusions

- MHD star formation calculations now possible with SPH
- The effects of magnetic fields are complicated (Price & Bate 2007)
  - Magnetic pressure can be more important than magnetic tension in inhibiting fragmentation
  - Although magnetic tension is responsible for magnetic braking, it can aid binary formation
  - Binary formation still possible even with strong fields ($M/\Phi \sim 3$) for perturbed clouds
- Star cluster formation
  - Strong magnetic fields ($M/\Phi \sim 5$)
    - Decrease the star formation efficiency
    - May decrease the ratio of low to high mass objects (i.e. fewer brown dwarfs)
    - Can produce large-scale voids and magnetic structures in the gas
  - Weak fields do not appear to drastically alter the hydrodynamic picture