Seismology of rapidly rotating stars

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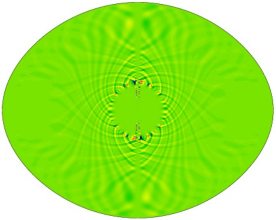
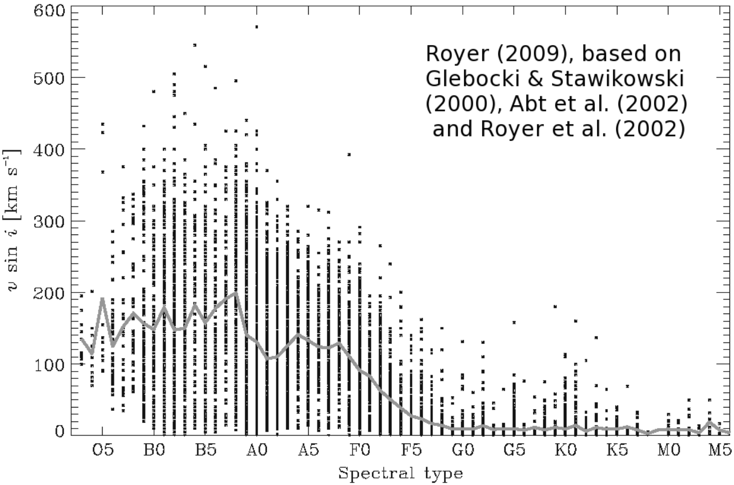
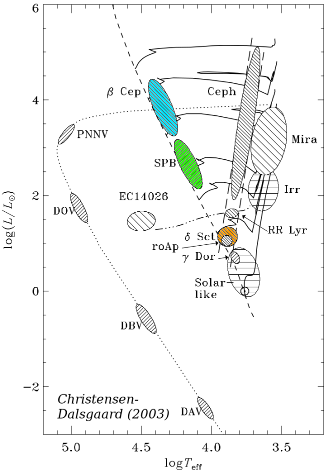


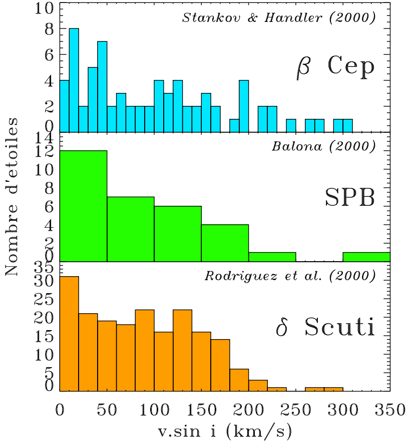
Introduction



- many massive and intermediate mass stars are rapid rotators

Introduction



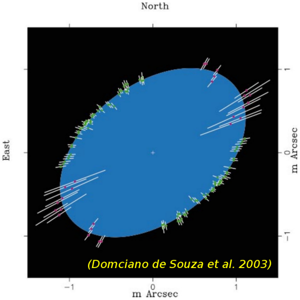


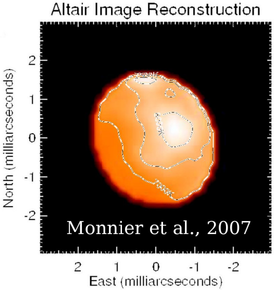
- the same applies to pulsating stars

The challenges of rapid rotation

Stars/stellar models

- centrifugal deformation
- gravity darkening
- baroclinic flows (differential rotation, meridional circulation)
- turbulence, mixing, modified evolution





The challenges of rapid rotation

Pulsations

- no longer described by single spherical harmonics
  - $\Rightarrow$ currently no automatic classification scheme
- lack of *simple* frequency patterns
  - p-modes: superposition of multiple independent patterns (Lignières & Georgeot 2008, 2009)
  - g-modes: varying period separation (e.g. Berthomieu et al. 1978) + numerous inertial modes
  - $\Rightarrow$ difficult to identify observed modes
- usually classical pulsators $\Rightarrow$ amplitudes are difficult to predict

1. Introduction
2. Theory
   - Pulsation calculations
   - Numerical implementation
3. Seismology
   - Search for frequency patterns
   - Observational mode identification methods
     - Non-adiabatic calculations
     - Amplitude ratios and phase shifts
     - LPVs
4. α Ophiuchi
5. Conclusion

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Pulsation calculations in rapidly rotating stars

Different approaches for including rotation

- perturbative approach
  - e.g. Saio (1981), Soufi et al. (1998)
- traditional approximation
  - e.g. Berthomieu et al. (1978), Lee & Saio (1987), Townsend (1997)
- 2D calculations
  - e.g. Reese et al. (2006), Lovekin et al. (2009), Ouazzani et al. (2015)
- ray dynamics, characteristics
  - e.g. Dintrans & Rieutord (2000), Lignières & Georgeot (2009)

Pulsation equations – adiabatic case

$$
\begin{aligned}
0 &= \frac{\delta\rho}{\rho_o} + \vec{\nabla}\cdot\vec{\xi} \\
0 &= \Delta\Psi - 4\pi G\left(\rho_o\frac{\delta\rho}{\rho_o} - \vec{\xi}\cdot\vec{\nabla}\rho_o\right) \\
0 &= [\omega + m\Omega]^2\,\vec{\xi} - 2i\vec{\Omega}\times[\omega+m\Omega]\,\vec{\xi} - \vec{\Omega}\times\left(\vec{\Omega}\times\vec{\xi}\right) \\
&- \vec{\xi}\cdot\vec{\nabla}\left(\varpi\Omega^2\vec{e}_\varpi\right) - \frac{P_o}{\rho_o}\vec{\nabla}\left(\frac{\delta P}{P_o}\right) + \frac{\vec{\nabla}P_o}{\rho_o}\left(\frac{\delta\rho}{\rho_o} - \frac{\delta P}{P_o}\right) - \vec{\nabla}\Psi \\
&+ \vec{\nabla}\left(\frac{\vec{\xi}\cdot\vec{\nabla}P_o}{\rho_o}\right) + \frac{\left(\vec{\xi}\cdot\vec{\nabla}P_o\right)\vec{\nabla}\rho_o - \left(\vec{\xi}\cdot\vec{\nabla}\rho_o\right)\vec{\nabla}P_o}{\rho_o^2} \\
\frac{\delta P}{P_o} &= \Gamma_1\frac{\delta\rho}{\rho_o}
\end{aligned}
$$

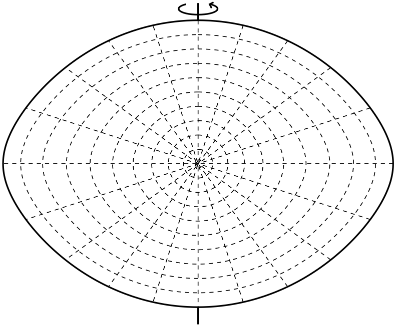
- neglects energy exchanges during oscillations

Pulsation equations – non-adiabatic case

$$
\begin{aligned}
0 &= \frac{\delta\rho}{\rho_o} + \vec{\nabla}\cdot\vec{\xi} \\
0 &= \Delta\Psi - 4\pi G\left(\rho_o\frac{\delta\rho}{\rho_o} - \vec{\xi}\cdot\vec{\nabla}\rho_o\right) \\
0 &= [\omega + m\Omega]^2\,\vec{\xi} - 2i\vec{\Omega}\times[\omega+m\Omega]\,\vec{\xi} - \vec{\Omega}\times\left(\vec{\Omega}\times\vec{\xi}\right) \\
&- \vec{\xi}\cdot\vec{\nabla}\left(\varpi\Omega^2\vec{e}_\varpi\right) - \frac{P_o}{\rho_o}\vec{\nabla}\left(\frac{\delta P}{P_o}\right) + \frac{\vec{\nabla}P_o}{\rho_o}\left(\frac{\delta\rho}{\rho_o} - \frac{\delta P}{P_o}\right) - \vec{\nabla}\Psi \\
&+ \vec{\nabla}\left(\frac{\vec{\xi}\cdot\vec{\nabla}P_o}{\rho_o}\right) + \frac{\left(\vec{\xi}\cdot\vec{\nabla}P_o\right)\vec{\nabla}\rho_o - \left(\vec{\xi}\cdot\vec{\nabla}\rho_o\right)\vec{\nabla}P_o}{\rho_o^2} \\
i\left[\omega+m\Omega\right]\rho_o T_o\delta S &= \epsilon_o\rho_o\left(\frac{\delta\epsilon}{\epsilon_o} + \frac{\delta\rho}{\rho_o}\right) - \vec{\nabla}\cdot\delta\vec{F} \\
&+ \vec{\xi}\cdot\vec{\nabla}\left(\vec{\nabla}\cdot\vec{F}_o\right) - \vec{\nabla}\cdot\left[\left(\vec{\xi}\cdot\vec{\nabla}\right)\vec{F}_o\right] \\
\delta\vec{F}^{\mathrm{R}} &= \left[\left(1+\chi_T\right)\frac{\delta T}{T_o} + \chi_\rho\frac{\delta\rho}{\rho_o}\right]\vec{F}_o^{\mathrm{R}} \\
&- \chi_o\left[T_o\vec{\nabla}\left(\frac{\delta T}{T_o}\right) + \vec{\xi}\cdot\vec{\nabla}\left(\vec{\nabla}T_o\right) - \vec{\nabla}\left(\vec{\xi}\cdot\vec{\nabla}T_o\right)\right]
\end{aligned}
$$

- and perturbed EOS, and opacities

Numerical implementation



- explicit expression in spheroidal coordinates
- projection onto spherical harmonics
- radial discretization using Chebyshev polynomials

Numerical implementation

N_r	N_h	Memory (in Gb)	Time (in min)	Num. proc.
400	10	3.5		
400	15	7.9		
400	20	13.4	5	4
400	29	28.0	10	8
400	40	52.7	22	8
400	50	82.3	26	16

- estimated accuracy based on variational expression and work integral:
  - frequencies: $\sim 10^{-4}$
  - excitation/damping rates: 10^{-2} to 10^{-1}

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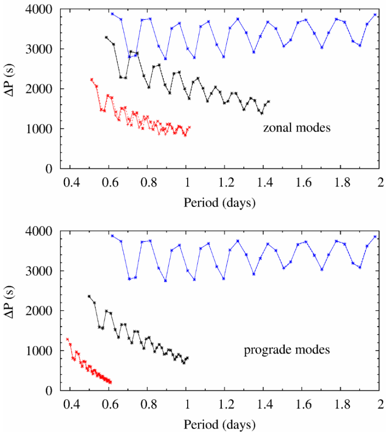
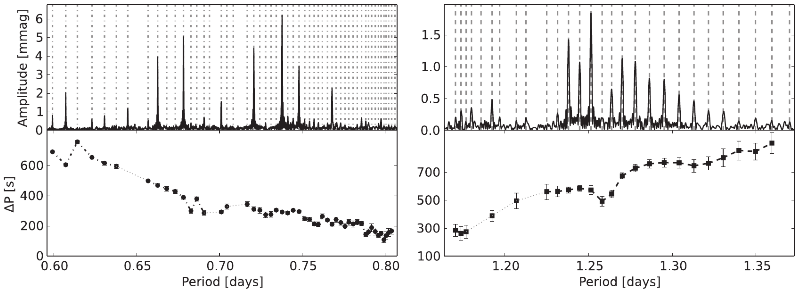
Search for frequency patterns – g-modes



(Ouazzani et al. 2016)

- see Ehsan Moravveji's talk
- linear trend between period spacing and period.
  - slope depends on azimuthal order and rotation rate
- based on traditional approximation (e.g. Berthomieu et al. 1978, Lee & Saio 1987, Townsend 2003)
- qualitatively confirmed with 2D calculations (e.g. Ballot et al. 2011, Ouazzani et al. 2016)

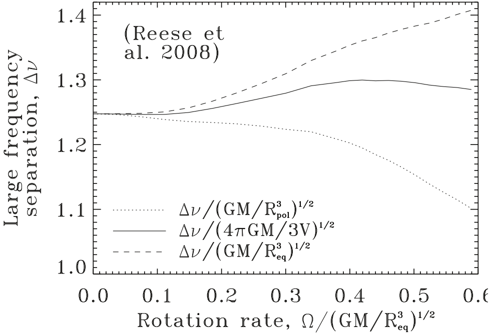
Search for frequency patterns – g-modes



(Van Reeth et al . 2015)

- observational confirmation (Bedding et al. 2015, Van Reeth et al. 2015)

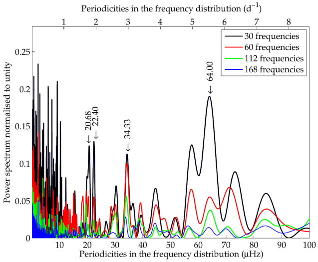
Search for frequency patterns – p-modes



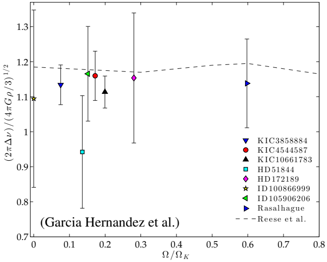
(Reese et al. 2008)

- large frequency separation, $\Delta\nu$, scales with the mean density, at arbitrary rotation rates (Reese et al. 2008)

Search for frequency patterns – p-modes

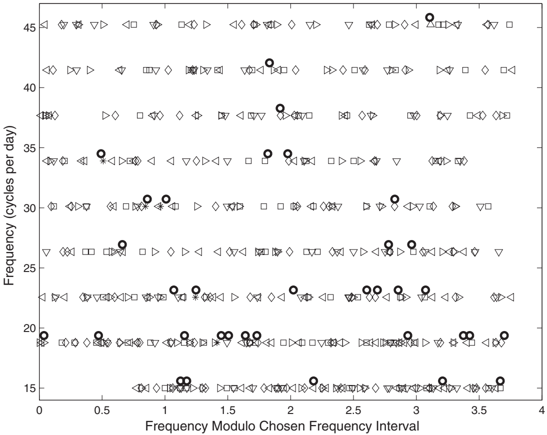


(García Hernández et al. 2013)



(García Hernández et al. 2015)

- observational confirmation based binary systems with independent estimates for the mean density (García Hernández et al. 2015)



(Deupree et al. 2012)

- however, identification of individual modes remains difficult

Observational mode identification methods

- use supplementary observations to constrain mode geometry
- **multicolor photometry:** amplitude ratios and phase differences
- **spectroscopy:** line profile variations
- need for consistent calculation of $\delta T_{\rm eff}/T_{\rm eff}$

- non-adiabatic calculations
  - provide $\delta T_{\rm eff}/T_{\rm eff}$
  - predict which modes are excited

Non-adiabatic calculations

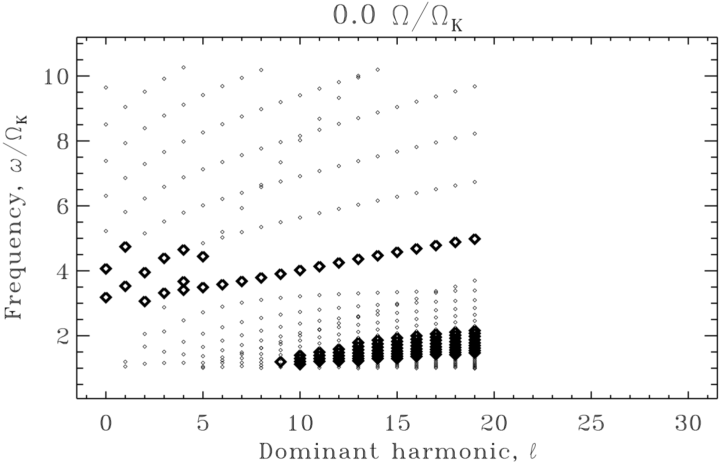
Model

- calculated with the ESTER code (Rieutord et al. 2016)
- 9 $M_\odot$ models
- $\Omega = 0.0$ to $0.8\,\Omega_K$
- $z = 0.025$
- OPAL opacities

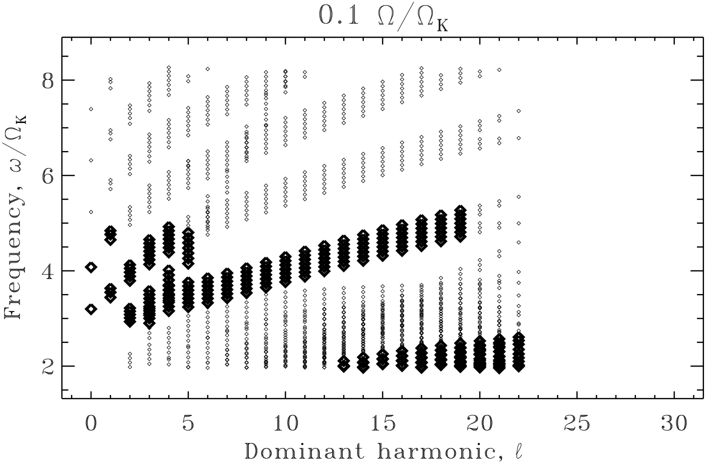
Modes

- calculated with the TOP code (Reese et al. 2006, 2009)
- β Cep type pulsations
- p and g modes
- excited by iron opacity bump at $\log(T) = 5.3$

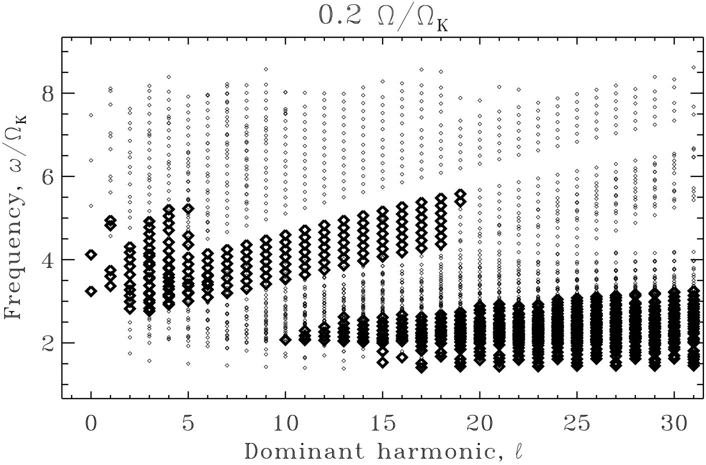
Frequencies



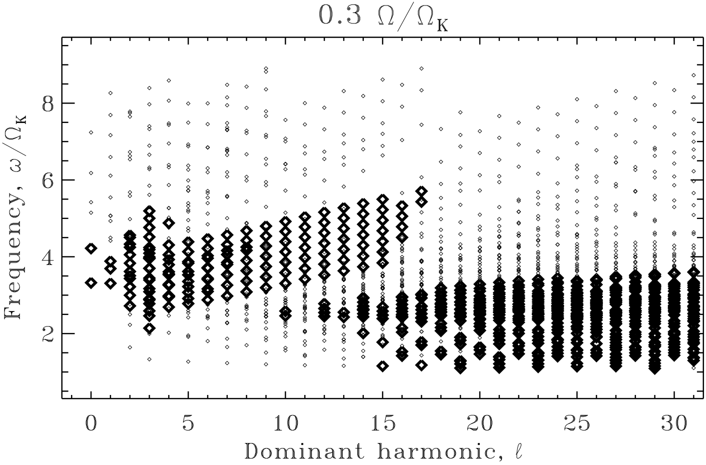
Frequencies



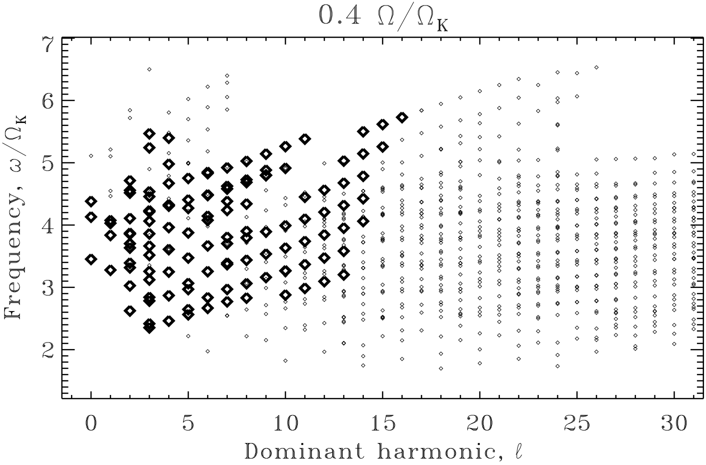
Frequencies



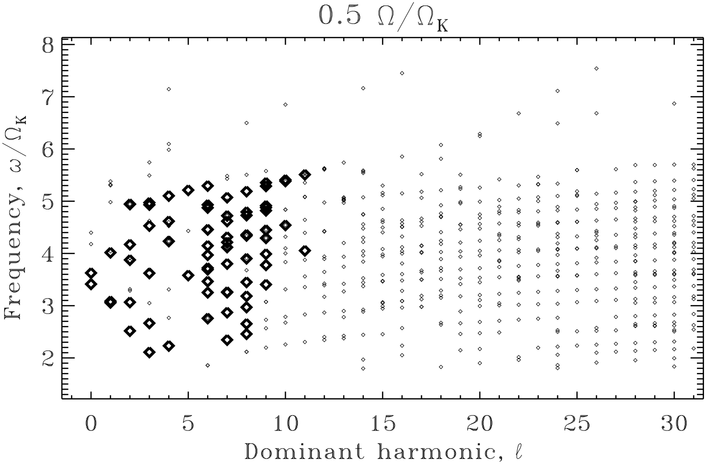
Frequencies



Frequencies



Frequencies



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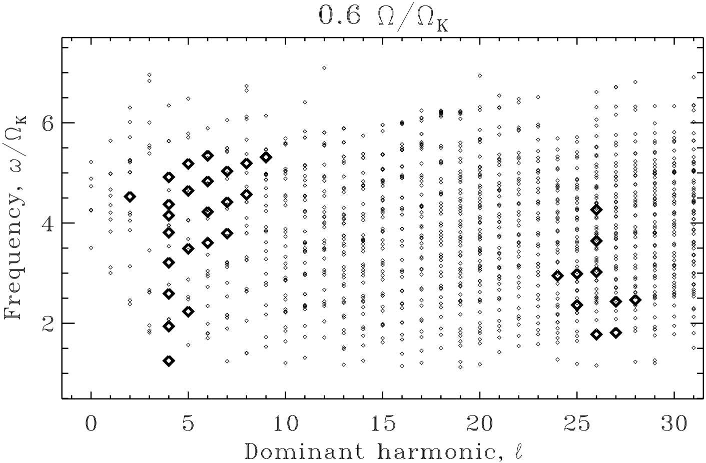
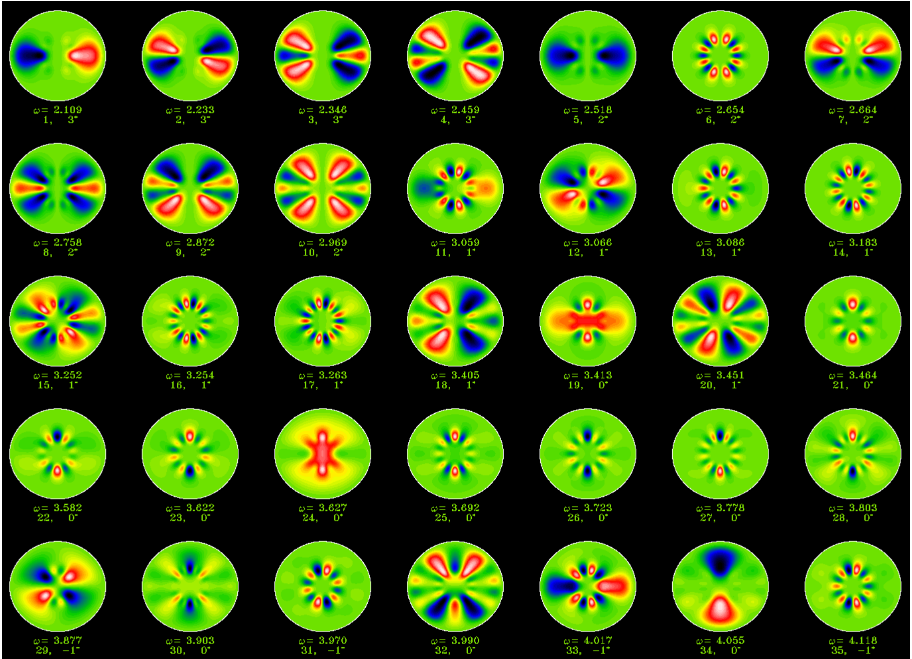
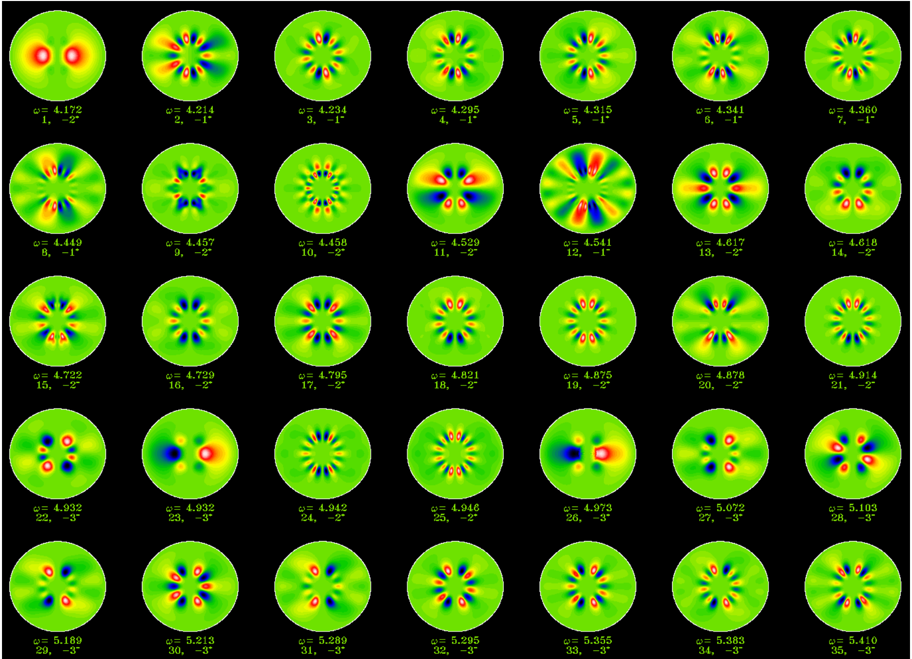


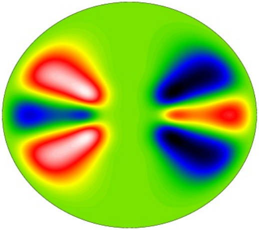


$\Omega = 0.5\ \Omega k$

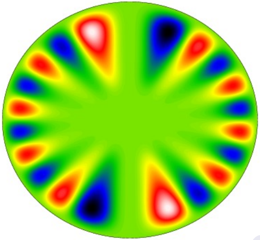


$\Omega = 0.5\ \Omega k$

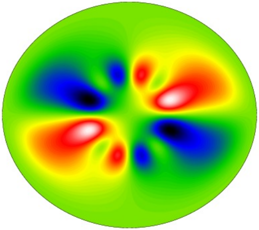
Excited modes



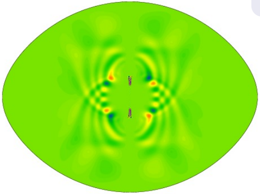
Island



Whisp. gallery



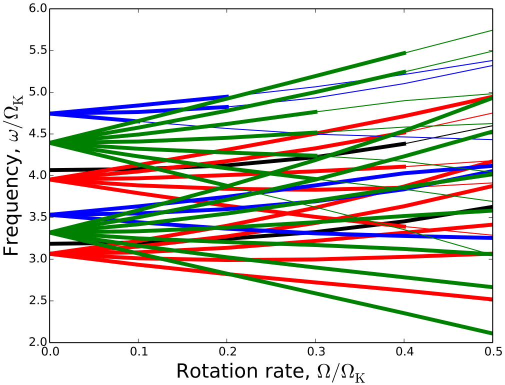
Mixed



Rosette

- modes from different classes are excited

Multiplets



- prograde modes remain unstable longer
- Lee (2008) also found a preference for prograde modes

Amplitude ratios and phase shifts

Basic principle

- measure pulsation modes in different photometric bands
- calculate ratio of mode amplitudes and phase differences between different bands
- these will depend on mode structure, thereby constraining mode ID

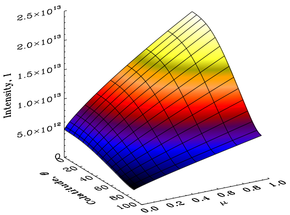
Advantages

- independent of intrinsic mode amplitudes
- independent of inclination and azimuthal order only in non-rotating case (e.g. Daszyńska-Daszkiewicz et al. 2002, Townsend 2003)

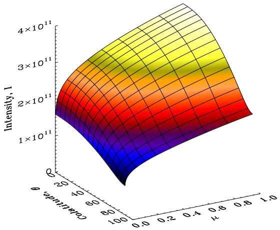
Intensities

$$I(T_{\rm eff}, g_{\rm eff}, \mu) = I_0(T_{\rm eff}, g_{\rm eff}) h(\mu, T_{\rm eff}, g_{\rm eff})$$

- $I_0(T_{\rm eff}, g_{\rm eff})$ from blackbody spectrum
- $h(\mu, T_{\rm eff}, g_{\rm eff})$ from Claret (2000)
- bolometric, Strömgren, and Johnson-Cousins photometric bands

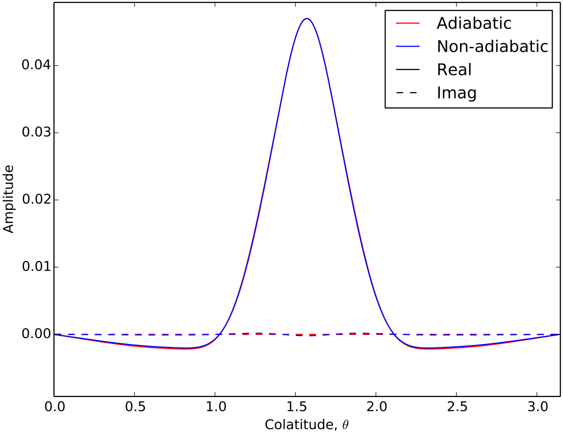


Bolometric



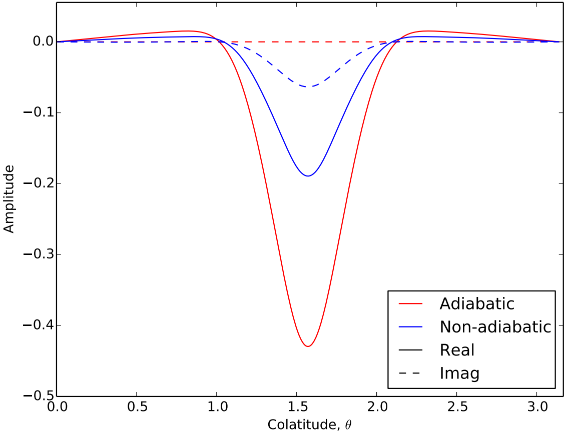
Strömgren, u

Adiabatic vs. non-adiabatic $\delta T_{\mathrm{eff}} / T_{\mathrm{eff}}$



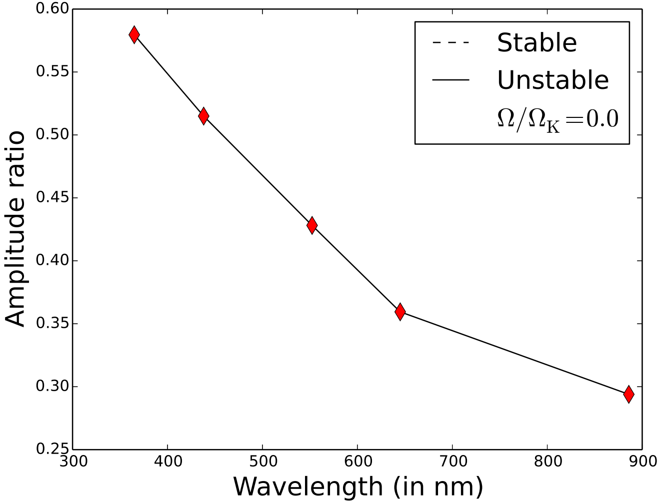
ξ_r

Adiabatic vs. non-adiabatic $\delta T_{\mathrm{eff}} / T_{\mathrm{eff}}$

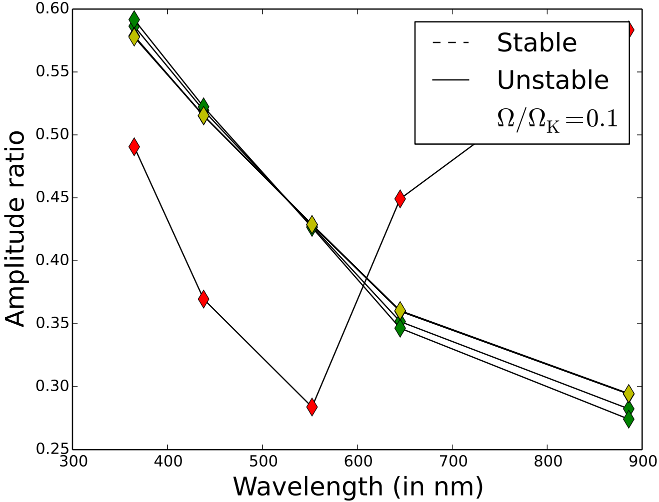


$\delta T_{\mathrm{eff}} / T_{\mathrm{eff}}$

Amplitude ratios for an $\ell = 3$ multiplet ($i = 30°$)



Amplitude ratios for an $\ell = 3$ multiplet ($i = 30°$)



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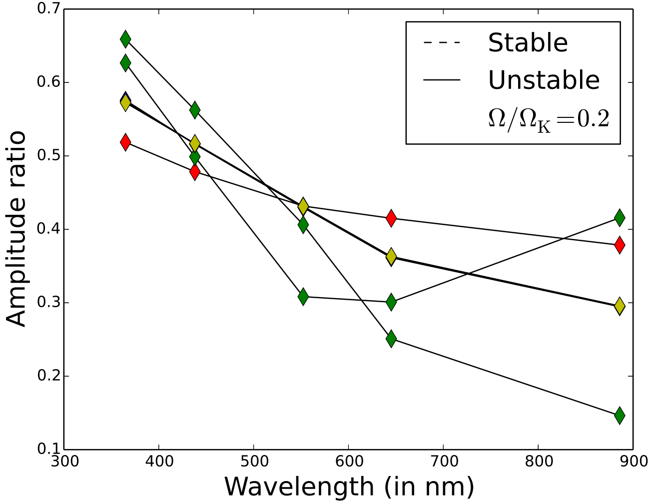

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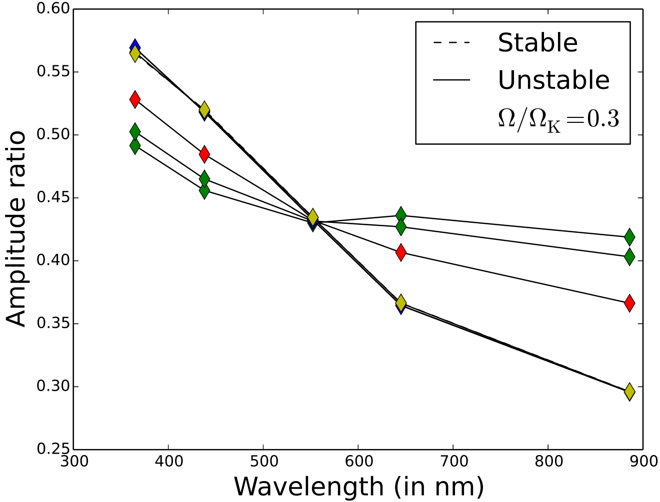

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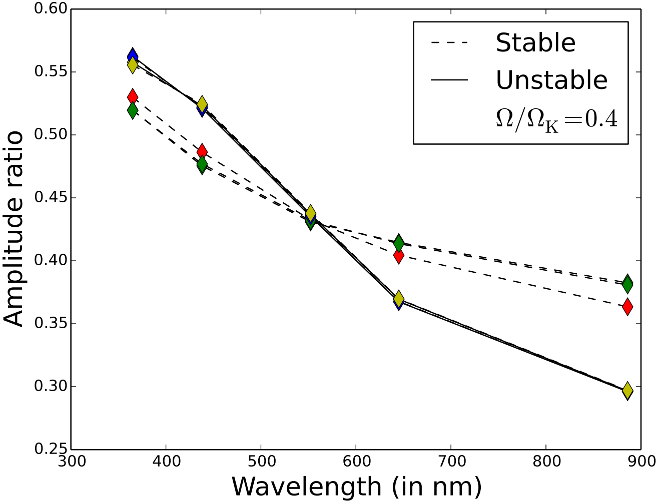
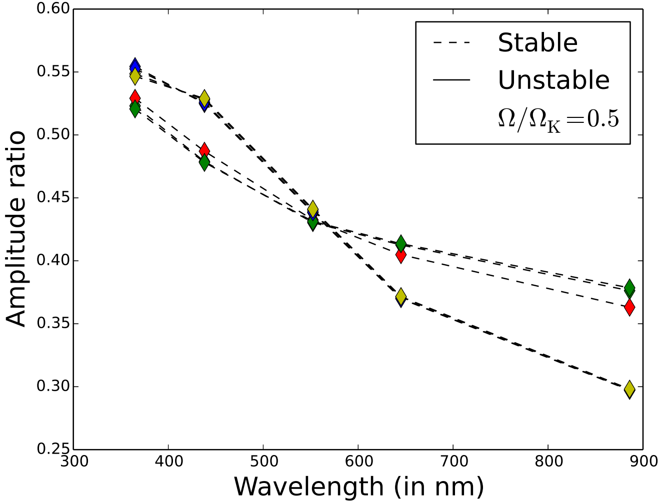
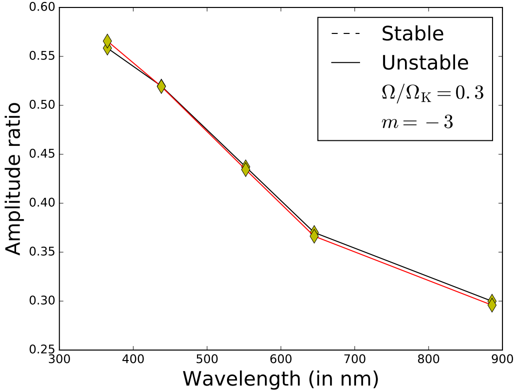


Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



Amplitude ratios for modes with the same (ℓ, m) values



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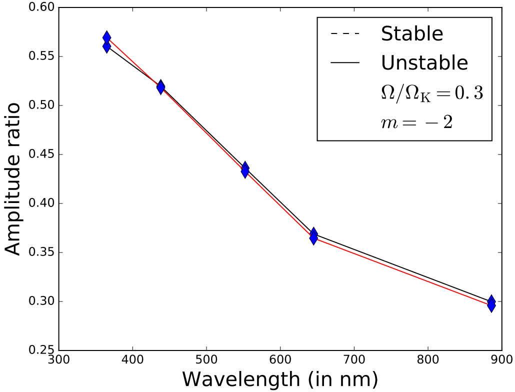

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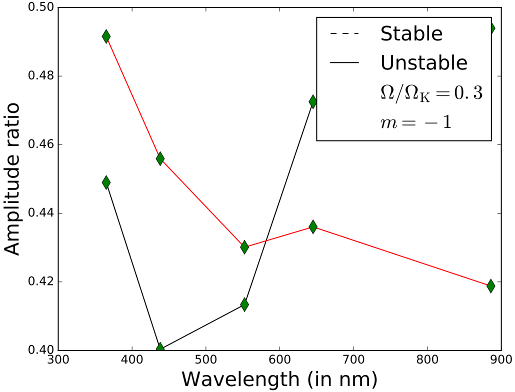

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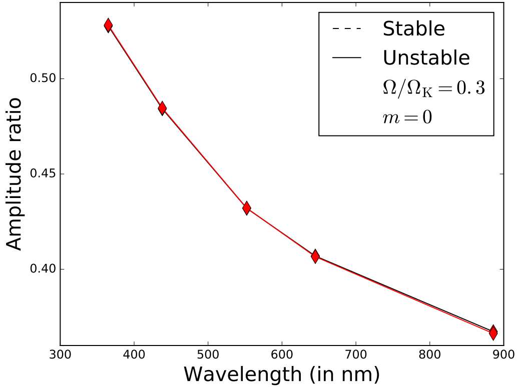

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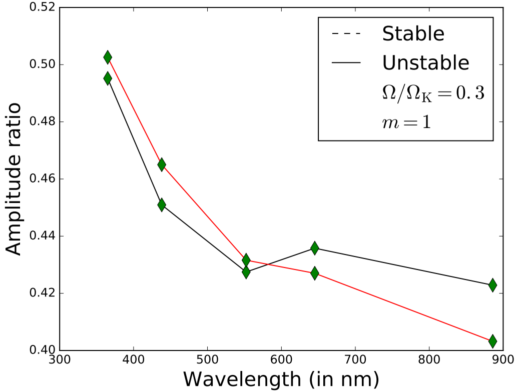

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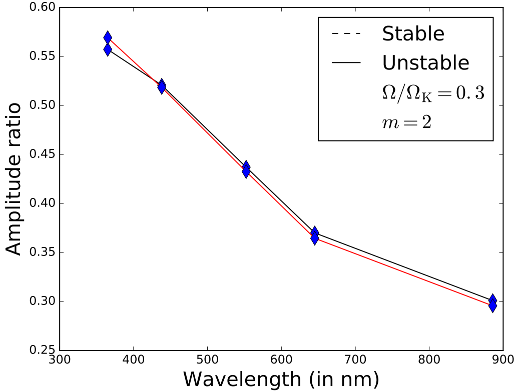

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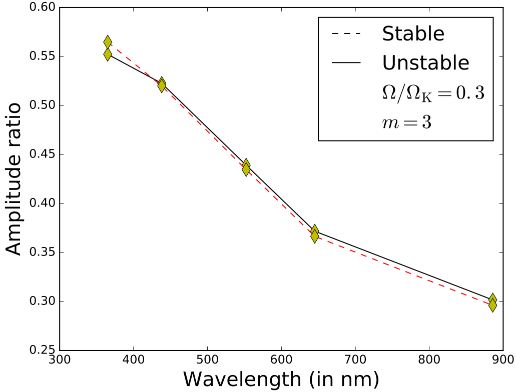

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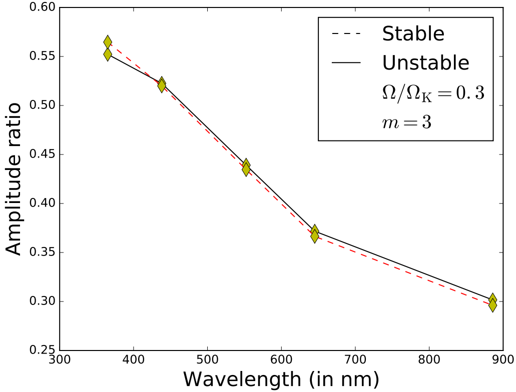
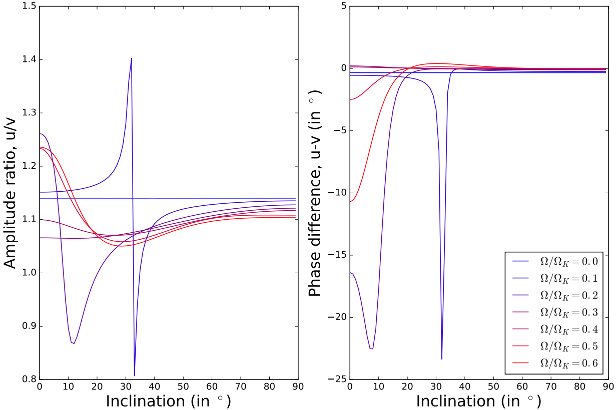


- similar amplitude ratios – may be used to identify similar modes (Reese et al. 2017)

Phase shifts



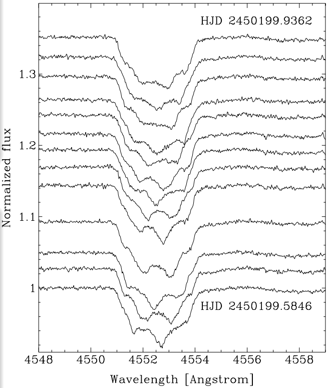
Line Profile Variations (LPVs)

Previous works

- Clement (1994): 2D calculations
- Townsend (1997): the traditional approximation, but realistic stellar spectra

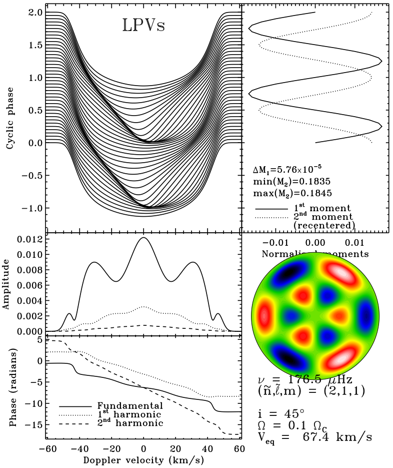
Description

- includes Doppler shifts and $\delta(d\vec{S})$
- δT_{eff} and δg_{eff} neglected
- use of blackbody spectrum (incl. gravity darkening)
- rudimentary description of limb darkening

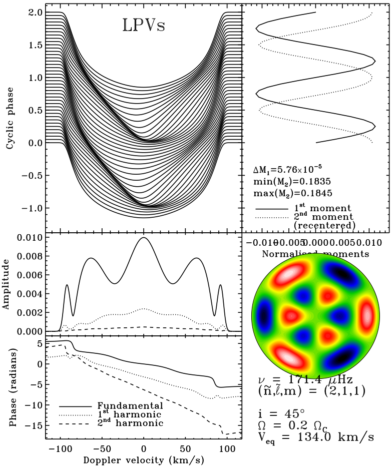


(Telting & Schrijvers, 1998)

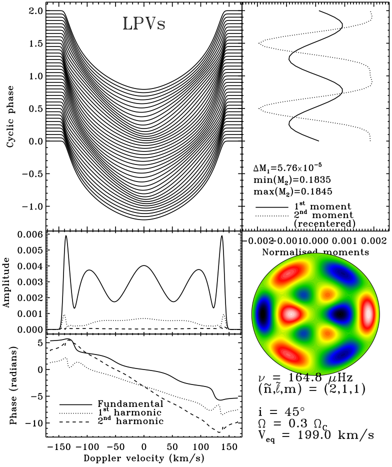
Increasing rotation rates



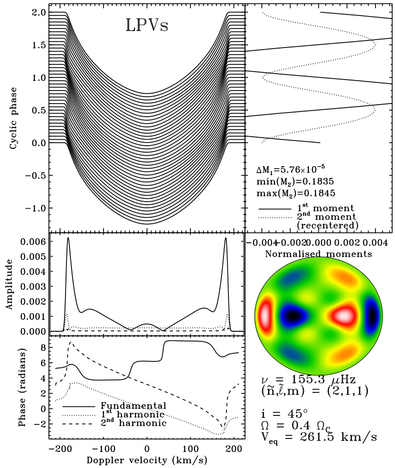
Increasing rotation rates



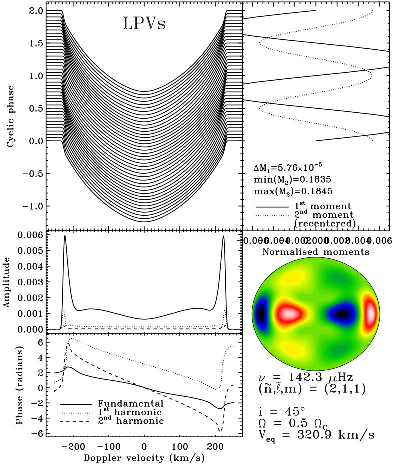
Increasing rotation rates



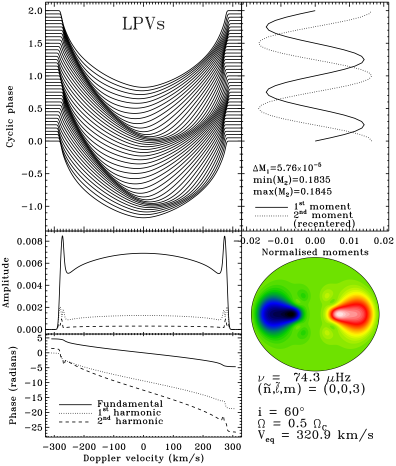
Increasing rotation rates



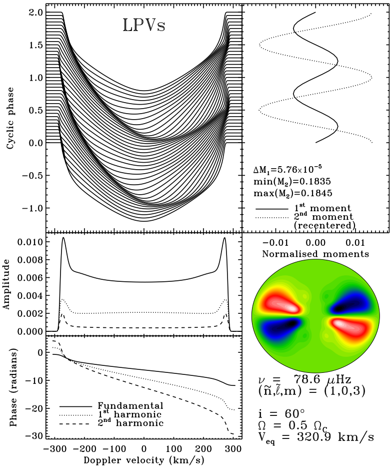
Increasing rotation rates



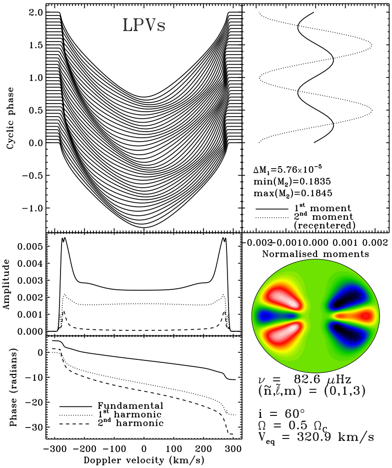
Increasing ℓ value



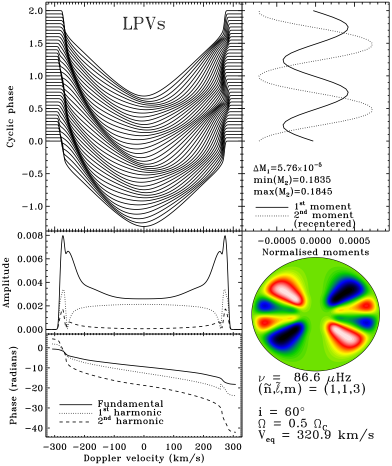
Increasing ℓ value



Increasing ℓ value

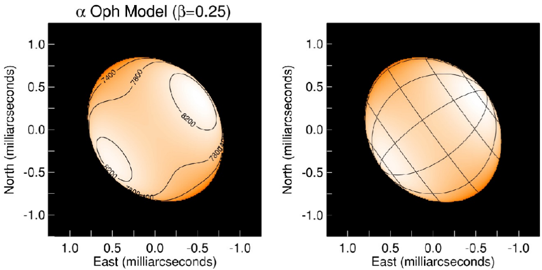


Increasing ℓ value



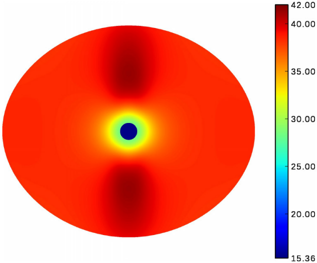
α Ophiuchi

- binary system: A5III + K6V (Cowley 1969 et al. + Hinkley et al. 2011)
- $v_{eq} = 240 \mathrm{km.s}^{-1}$
- polar and equatorial radii determined through interferometry (Zhao et al. 2009)
- 57 pulsation frequencies from photometry (Monnier et al. 2010)

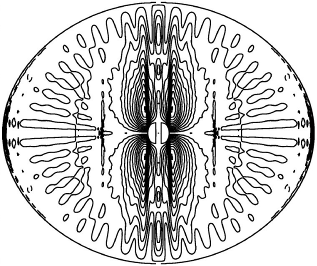


(Zhao et al. 2009)

Characteristics of the model



Rotation profile

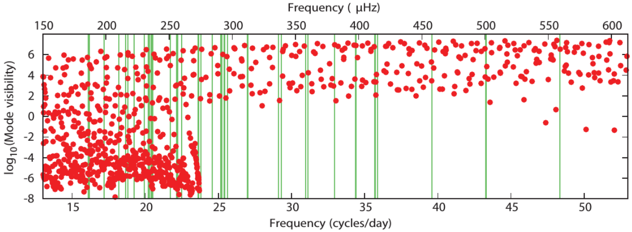


Meridional circulation

(Mirouh et al. 2013, Mirouh, PhD, 2016)

- calculated with ESTER
- mass: 2.22 $M_\odot$
- $Z = 0.02$, $X = 0.7$, $X_c = 0.26$

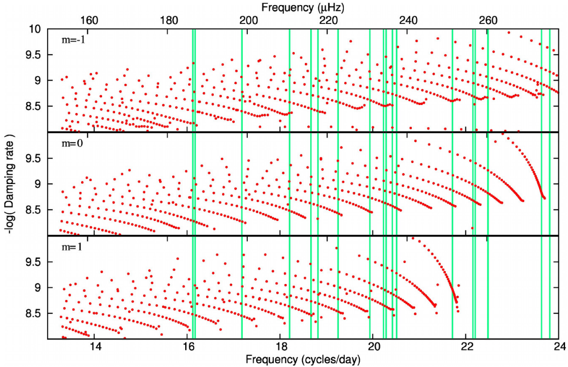
Initial results



(Mirouh et al. 2013, Mirouh, PhD, 2016)

- large selection of theoretical models around each observed frequencies

Initial results



(Mirouh et al. 2013, Mirouh, PhD, 2016)

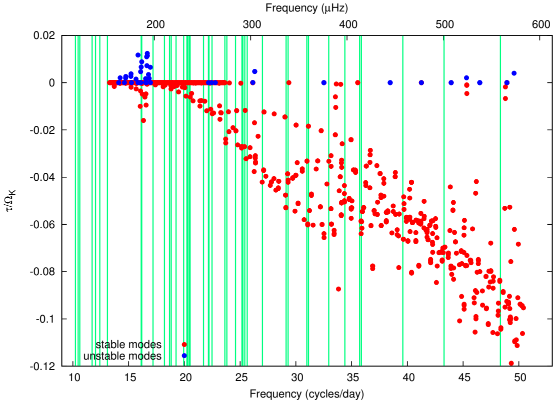
- find excitation rate through quasi-adiabatic approximation
  - no unstable modes!

New calculations

- new fully non-adiabatic calculations

New calculations

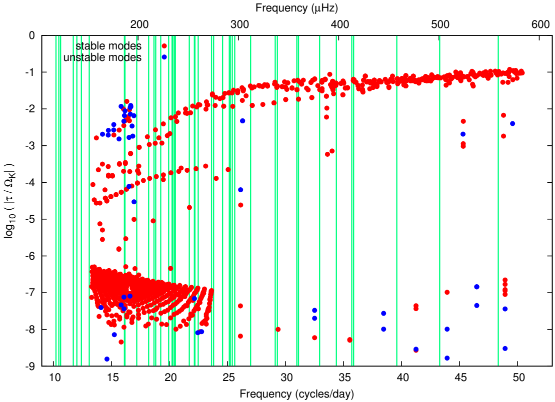
- new fully non-adiabatic calculations
- unstable modes appear



(Mirouh et al. in prep)

New calculations

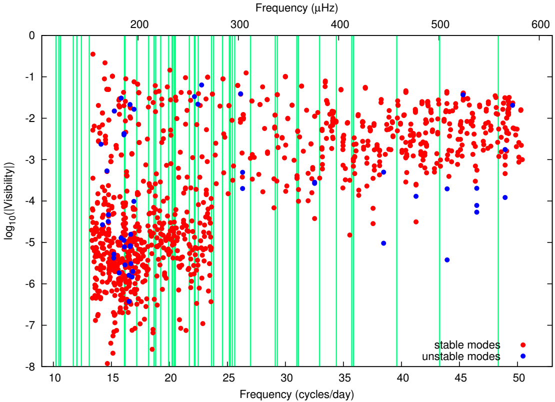
- new fully non-adiabatic calculations
- unstable modes appear



(Mirouh et al. in prep)

New calculations

- new fully non-adiabatic calculations
- unstable modes appear



(Mirouh et al. in prep)

Conclusion

- non-adiabatic calculations are an important step forward:
  - can now predict which modes are unstable
  - can calculate amplitude ratios, phase shifts, and LPVs

Prospects

- use realistic atmospheres in calculating visibilities and LPVs
- identify modes in observed stars
  - e.g. through multicolor photometry – see Gerald Handler's talk
- constrain internal structure of rapidly rotating stars

Supplementary material

Work integral

- it is possible to derive an integral expression for the complex frequencies:

$$A\omega^2 + 2B\omega + C = 0$$

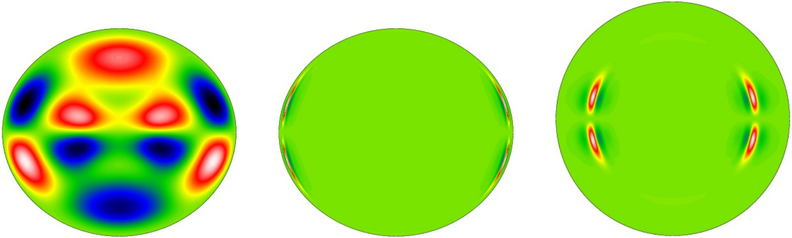
where

$$\begin{aligned} A &= \int_V \rho_0 \xi^2 \mathrm{dV}, \\ B &= \int_V \rho_0 \left[m\Omega\xi^2 - i\vec{\Omega} \cdot \left(\vec{\xi} \times \vec{\xi}^* \right) \right] \mathrm{dV}, \\ \Re(C) &= \text{a complicated expression} \\ \Im(C) &= -\int_V \Im \left\{ \frac{\delta P \delta \rho^*}{\rho_0} \right\} \mathrm{dV} \end{aligned}$$

- From this we deduce the excitation rate:

$$\Im(\omega) = -\frac{\Im(C)}{2\left(A\Re(\omega) + B\right)}$$

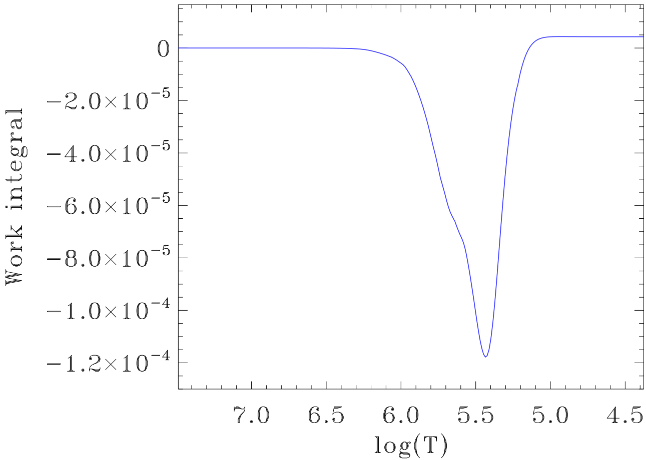
Work integral



Acoustic   Work   Work (vs. log T)

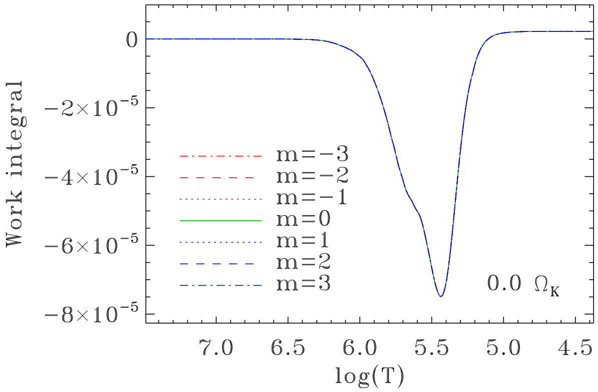
- red = driving regions
- blue = damping regions

Work integral



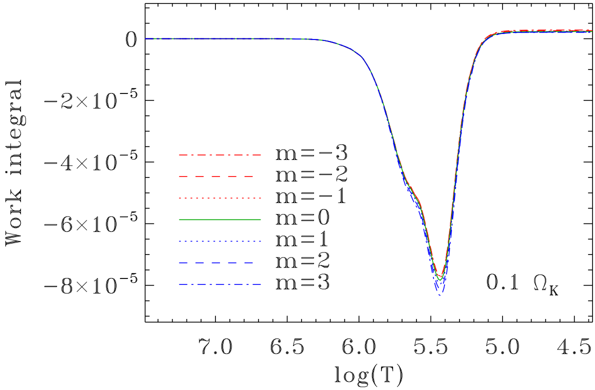
- obtained by integrating in horizontal direction + vertical anti-derivative

A multiplet



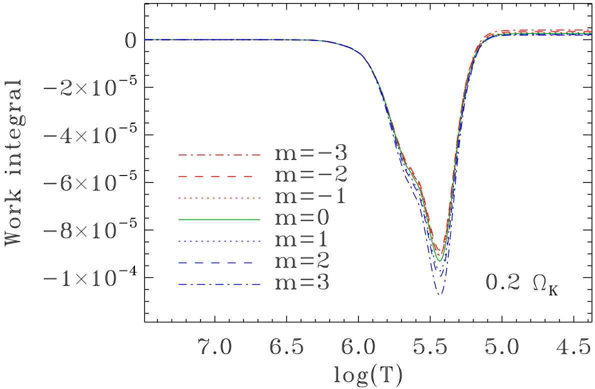
- rotation rate $= 0.0 \quad \Omega_K$, $\varepsilon = 0$

A multiplet



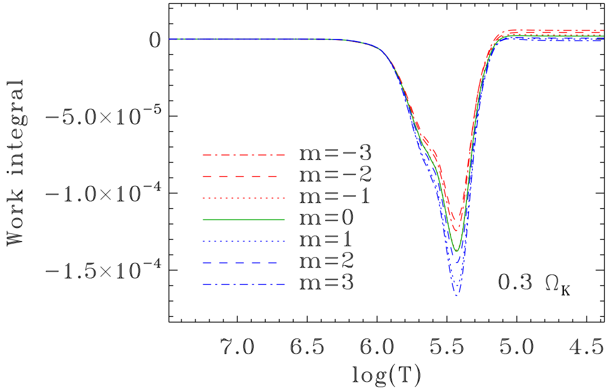
- rotation rate = 0.1 Ω_K, $\varepsilon = 4.9 \times 10^{-3}$

A multiplet



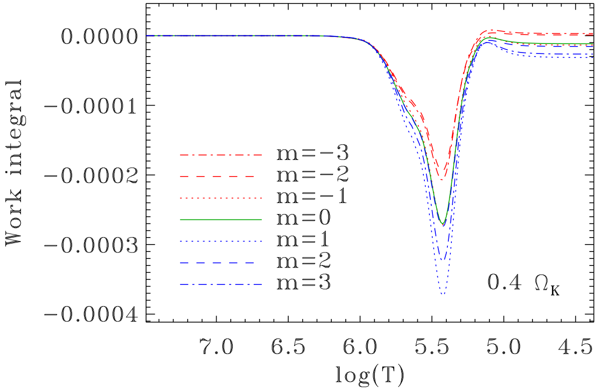
- rotation rate $= \quad 0.2 \quad \Omega_{\mathrm{K}}, \quad \varepsilon = 1.9 \times 10^{-2}$

A multiplet



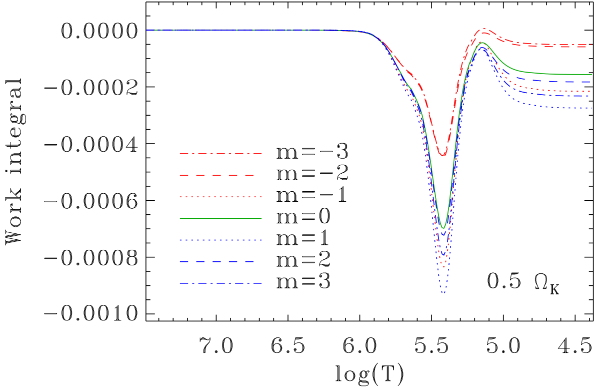
- rotation rate = 0.3 Ω_K, $\varepsilon = 4.3 \times 10^{-2}$

A multiplet



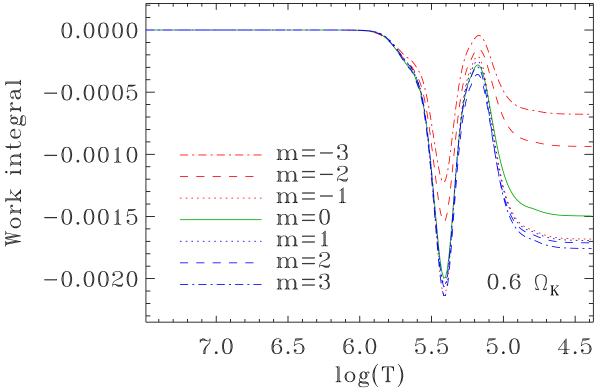
- rotation rate = 0.4 Ω_K, $\varepsilon = 7.4 \times 10^{-2}$

A multiplet



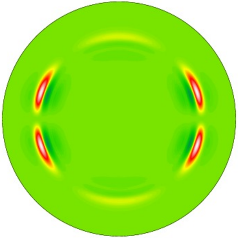
- rotation rate = $0.5 \quad \Omega_{\mathrm{K}}$, $\varepsilon = 11.2 \times 10^{-2}$

A multiplet

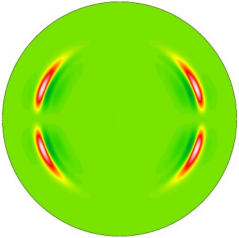


- rotation rate = $0.6\ \Omega_{\mathrm{K}}$, $\varepsilon = 15.5 \times 10^{-2}$

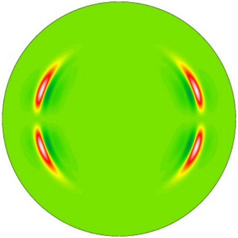
A multiplet



m = 0
Stable



m = −2
Excited



m = 2
Stable

- rotation rate $= 0.4\ \Omega_{\mathrm{K}}$, $\varepsilon = 7.4 \times 10^{-2}$