# Coisotropic Model Building

#### Fernando Marchesano Ludwig Maximilians Universität <u>– München</u>



# Coisotropic Model Building

#### Fernando Marchesano Ludwig Maximilians Universität – München

In collaboration with: Anamaría Font and Luis Ibáñez hep-th/0607219

A classical question in string theory is if we can reproduce the SM as an effective theory

A classical question in string theory is if we can reproduce the SM as an effective theory

Four observable dimensions

3 Quarks & Leptons generations Spontaneous EWSB

 $SU(3) \times SU(2) \times U(1)$ 

Chirality

Gauge coupling constants

Yukawa couplings

and more...

Problem: we have not solved String/M-theory

- Problem: we have not solved String/M-theory
- Strategy: we explore a region of the theory which is under control 11-D Supergravity



- Problem: we have not solved String/M-theory
- Strategy: we explore a region of the theory which is under control
  11-D Supergravity



Classical scheme: high string scale,  $\Lambda_4 = 0$ , D=4 N=1 gravity & MSSM sector + SUSY source

- D=4 gravity
- U(N) theory
- Chiral fermions



U(N) theory

#### Chiral fermions









**D6-branes wrap 3-cycles**  $\Pi_3^a \subset CY_3$ 

- **D6-branes wrap 3-cycles**  $\Pi_3^a \subset CY_3$
- Each intersection gives rise to a D=4 chiral fermion



Berkooz et al. '96

 $q^2$ 

- **D6-branes wrap 3-cycles**  $\Pi_3^a \subset CY_3$
- Each intersection gives rise to a D=4 chiral fermion

Berkooz et al. '96

Chiral spectrum:  $I_{ab} = [\Pi_3^a] \circ [\Pi_3^b] \quad (N_a, \overline{N_b})$ 

q 1

Blumenhagen et al. '00 Aldazábal et al. '00

Matter replication

Mariño et al. '99

#### SUSY conditions

#### Mariño et al. '99

- F flatness  $\mathcal{F} + iJ|_{\Pi_3} = 0$  Lagrangian.
- D flatness  $Im \Omega|_{\Pi_3} = 0$  Special Lagrangian.



Mariño et al. '99

F - flatness  $\mathcal{F} + iJ|_{\Pi_3} = 0$  Lagrangian.

D - flatness  $Im \Omega|_{\Pi_3} = 0$  Special Lagrangian.

Gauge kinetic function

 $f_a = \int_{\Pi_3^a} e^{-\phi} \operatorname{Re} \Omega + iC_3$  Volume







# ...didn't we miss something??





- **D6-branes wrap 3-cycles**  $\Pi_3^a \subset CY_3$
- Each intersection gives rise to a D=4 chiral fermion

Berkooz et al. '96

Matter replication!!

Chiral spectrum:  $I_{ab} = [\Pi_3^a] \circ [\Pi_3^b] \quad (N_a, \overline{N_b})$ 

**q**<sup>1</sup>

 $q^2$ 

- Problem: we have not solved String/M-theory
- Strategy: we explore a region of the theory which is under control 11-D Supergravity



- Problem: we have not solved String/M-theory
- Strategy: we explore a region of the theory which is under control 11-D Supergravity



There are three kinds of space-filling D-branes in type IIA

- D4-branes
- D6-branes
- D8-branes

There are three kinds of space-filling D-branes in type IIA



There are three kinds of space-filling D-branes in type IIA



If we consider D4-branes, they should be wrapping a I-cycle  $\Pi_1$ . However, in generic CY's  $b_1 = 0$ , so the Chern-Simons action

$$\int_{M_4 \times \Pi_1} C_5$$

vanishes

There are three kinds of space-filling D-branes in type IIA



If we consider D4-branes, they should be wrapping a I-cycle  $\Pi_1$ . However, in generic CY's  $b_1 = 0$ , so the Chern-Simons action

$$\int_{M_4 \times \Pi_1} C_5$$

vanishes  $\implies$  No central charge

There are three kinds of space-filling D-branes in type IIA

- D4-branes X
  D6-branes
- D8-branes ?
- If we consider D4-branes, they should be wrapping a I-cycle  $\Pi_1$ . However, in generic CY's  $b_1 = 0$ , so the Chern-Simons action

$$\int_{M_4 \times \Pi_1} C_5$$

vanishes  $\Rightarrow$  No central charge  $\Rightarrow$  No BPS D4-brane
Naively, the same argument works for D8-branes:

$$b_1 = b_5 = 0 \quad \Rightarrow \quad \Pi_5 \text{ is homologically trivial}$$

$$\int_{M_4 \times \Pi_5} C_9 = 0$$

Naively, the same argument works for D8-branes:

$$b_1 = b_5 = 0 \quad \Rightarrow \quad \Pi_5 \text{ is homologically trivial}$$

$$\int_{M_4 \times \Pi_5} C_9 = 0$$

However, D8-branes can carry internal worldvolume fluxes  $\mathcal{F}_{\mu\nu} = B_{\mu\nu} + 2\pi \alpha' F_{\mu\nu}$ , and so the CS action reads  $\int_{M_4 \times \Pi_7} C_9 + \int_{M_4 \times \Pi_7} C_7 \wedge \mathcal{F}$ 

Naively, the same argument works for D8-branes:

$$b_1 = b_5 = 0 \quad \Rightarrow \quad \Pi_5 \text{ is homologically trivial}$$

$$\int_{M_4 \times \Pi_5} C_9 = 0$$

However, D8-branes can carry internal worldvolume fluxes  $\mathcal{F}_{\mu\nu} = B_{\mu\nu} + 2\pi \alpha' F_{\mu\nu}$ , and so the CS action reads  $\int_{M_{\star} \times \Pi_{\pi}} C_{9} + \int_{M_{\star} \times \Pi} C_{7} \wedge \mathcal{F}$ 

The second term will not vanish iff there is a dissolved D6-brane charge on our D8-brane Douglas '95

- Idea: we can have a D8-brane with
  - Trivial D8-brane charge
  - Non-trivial induced D6-brane charge



#### induced D6



So, in principle, we can also have BPS D8-branes !!!

In terms of topological strings, having BPS D8-branes implies that there are A-branes which are not Lagrangian

- In terms of topological strings, having BPS D8-branes implies that there are A-branes which are not Lagrangian
- Such possibility was pointed out by Kapustin and Orlov. In a  $\mathbf{CY}_n$  we can have
  - Typical A-branes: Lagrangian *n*-cycle,  $\mathcal{F} = 0$
  - Exotic A-branes: Coisotropic (n+2k)-cycle,  $\mathcal{F} \neq 0$

- In terms of topological strings, having BPS D8-branes implies that there are A-branes which are not Lagrangian
- Such possibility was pointed out by Kapustin and Orlov. In a  $\mathbf{CY}_n$  we can have
  - Typical A-branes: Lagrangian *n*-cycle,  $\mathcal{F} = 0$
  - Exotic A-branes: Coisotropic (n+2k)-cycle,  $\mathcal{F} \neq 0$
- We are interested in the case n=3, which means that coisotropic A-branes must wrap 5-cycles with  $\mathcal{F} \neq 0$
- Because those 5-cycles are trivial in homology, they are difficult to construct. No examples in the coisotropic literature

The BPS conditions for A-branes read

#### D6-branes

**F-flatness**  $\mathcal{F} + iJ|_{\Pi_3} = 0$ 

**D-flatness**  $\operatorname{Im} \Omega|_{\Pi_3} = 0$ 

The BPS conditions for A-branes read

	D6-branes	D8-branes	
F-flatness	$\mathcal{F} + iJ _{\Pi_3} = 0$	$(\mathcal{F}+iJ)^2 _{\Pi_5}=0$	Kapustin & Orlov'01
D-flatness	$\operatorname{Im}\Omega _{\Pi_3}=0$	$\mathcal{F}\wedge \mathrm{Im}\Omega _{\Pi_5}=0$	Kapustin & Li'03

The BPS conditions for A-branes read

D6-branesD8-branesF-flatness $\mathcal{F} + iJ|_{\Pi_3} = 0$  $(\mathcal{F} + iJ)^2|_{\Pi_5} = 0$ Kapustin & Orlov '01D-flatness $\operatorname{Im} \Omega|_{\Pi_3} = 0$  $\mathcal{F} \wedge \operatorname{Im} \Omega|_{\Pi_5} = 0$ Kapustin & Li'03

Let us consider the case where  $\mathcal{F} = 2\pi \alpha' F$ , and  $[\Pi_3^F]$  to be the Poincaré dual of  $[F/2\pi]$ . Then the BPS conditions suggest

$$J|_{\Pi_3^F} = 0 \quad \sim \quad \mathcal{F} \wedge J|_{\Pi_5} = 0$$
  
$$\operatorname{Im} \Omega|_{\Pi_3^F} = 0 \quad \sim \quad \mathcal{F} \wedge \operatorname{Im} \Omega|_{\Pi_5} = 0$$

So  $\Pi_3^F$  looks like an special Lagrangian 3-cycle in  $\mathbf{CY}_3$ 

## A toroidal example

#### Let us consider ${f T}^2 imes {f T}^2 imes {f T}^2$ and the D8-brane



$(T^2)_1$		$(T^2)_2$	$(T^2)_3$
$\Pi_5$	=	$(1,0)_1 \times (\mathbf{T}^2)_2$	$\times (\mathbf{T}^2)_3$
$F/2\pi$	=	$dx^2 \wedge dx^3 - dy^2$	$^{2} \wedge dy^{3}$

## A toroidal example

#### Let us consider ${f T}^2 imes {f T}^2 imes {f T}^2$ and the D8-brane



$(T^2)_1$		$(T^2)_2$	$(T^{2})_{3}$
$\Pi_5$	=	$(1,0)_1 \times (\mathbf{T}^2)_2$	$ imes (\mathbf{T}^2)_3$
$F/2\pi$	=	$dx^2 \wedge dx^3 - dy^2$	$^{2} \wedge dy^{3}$

The BPS conditions read

$$\begin{array}{c|c} & \text{D-term} & \xrightarrow{} & \text{Trivial} \\ & & \text{F-term} & \xrightarrow{} & \begin{cases} J_c \wedge F = 0 & \checkmark \\ J_c^2 + F^2 = 0 & \Longleftrightarrow & T_2T_3 = 1 \\ & & J_c = B + iJ & T_j = A_j + iB_j \end{array}$$

# A toroidal example

Let us consider  ${f T}^2 imes {f T}^2 imes {f T}^2$  and the D8-brane





Notice that

 $[\Pi_3^F] = [(1,0)(1,0)(1,0)] + [(1,0)(0,1)(0,-1)]$ 

and so the D6-brane charge is not of the form (I-cycle) x (I-cycle) x (I-cycle), like for CFT D6-branes



## **D8-branes on Z**<sub>2</sub> x Z<sub>2</sub>

Notice that  $b_1(\mathbf{T}^6) \neq 0$ , so finding BPS D8-branes is not that surprising.

## **D8-branes on Z**<sub>2</sub> x Z<sub>2</sub>

- Notice that  $b_1(\mathbf{T}^6) \neq 0$ , so finding BPS D8-branes is not that surprising.
- Let us set  $b_1 = 0$  by orbifolding our theory by  $\mathbb{Z}_2 \times \mathbb{Z}_2$ :



# Tadpoles

## **Tadpoles**

Because is the only surviving one, D8-branes contribute to RR tadpoles via its induced D6-brane charge

$$\sum_{a \in D6} N_a[\Pi_3^a] + \sum_{b \in D8} N_b[\Pi_3^{F_b}] = 4[\Pi_3^{O6}]$$

# **Tadpoles**

Because is the only surviving one, D8-branes contribute to RR tadpoles via its induced D6-brane charge

$$\sum_{a \in D6} N_a[\Pi_3^a] + \sum_{b \in D8} N_b[\Pi_3^{F_b}] = 4[\Pi_3^{O6}]$$

SUSY guarantees the cancellation of NSNS tadpoles, which are related to D-branes tensions. These tensions are also related to the gauge coupling constants:

$$\frac{1}{g_b^2} = \int_{\Pi_5^b} e^{-\phi} \frac{F_b}{2\pi} \wedge \operatorname{Re}\Omega = \int_{\Pi_3^{F_b}} e^{-\phi} \operatorname{Re}\Omega$$

In fact, the gauge kinetic function reads

$$f_b = \int_{\Pi_3^{F_b}} e^{-\phi} \operatorname{Re} \Omega + iC_3$$

# Chirality

# Chirality

Chirality arises from a mixture of intersection and magnetization mechanisms, as in type IIB

# Chirality

Chirality arises from a mixture of intersection and magnetization mechanisms, as in type IIB

However, the net number of chiral fermions is given by  $I_{ab} = [\Pi_3^{D8_a}] \circ [\Pi_3^{D8_b}] \quad \text{or} \quad I_{ab} = [\Pi_3^{D6_a}] \circ [\Pi_3^{D8_b}]$ 

### Yukawa couplings

## Yukawa couplings

For D6-branes Yukawas arise from worldsheet instantons  $T^2$   $T^2$   $T^2$   $T^2$ 



## Yukawa couplings



The supersymmetry conditions can be understood from the effective scalar potential

$$V_{NSNS} = \sum_{a} T_{D6_a} + \sum_{b} T_{D8_b} - 4T_{O6}$$

The supersymmetry conditions can be understood from the effective scalar potential

$$V_{NSNS} = \sum_{a} T_{D6_a} + \sum_{b} T_{D8_b} - 4T_{O6}$$

A D8-brane contribution is given by

$$2\text{Re}\,f_8\,V_{D8} = (\int_{\Pi_5} \mathcal{F} \wedge \text{Im}\,\Omega)^2 + e^{-2\phi}||l_i||^2 (\int_{\mathbf{T}^4} (\mathcal{F} + iJ)^2)^2$$

The supersymmetry conditions can be understood from the effective scalar potential

$$V_{NSNS} = \sum_{a} T_{D6_a} + \sum_{b} T_{D8_b} - 4T_{O6}$$

A D8-brane contribution is given by

$$2\operatorname{Re} f_8 V_{D8} = (\int_{\Pi_5} \mathcal{F} \wedge \operatorname{Im} \Omega)^2 + e^{-2\phi} ||l_i||^2 (\int_{\mathbf{T}^4} (\mathcal{F} + iJ)^2)^2$$

The supersymmetry conditions can be understood from the effective scalar potential

$$V_{NSNS} = \sum_{a} T_{D6_a} + \sum_{b} T_{D8_b} - 4T_{O6}$$

A D8-brane contribution is given by



 $X_i$ : open string field (location + Wilson line)  $n = n^{xy}n^{yx} - n^{xx}n^{yy}$ 

This kind of open-closed superpotentials are very attractive for moduli fixing.
# Superpotential

- This kind of open-closed superpotentials are very attractive for moduli fixing.
- However, one should be careful before concluding that Kähler moduli are stabilized because of the D8-brane. There could be extra open string moduli, coupled as

 $W_{open} = X_i X_j X_k$ 

just like happens for D6-branes.

Douglas'98

# Superpotential

- This kind of open-closed superpotentials are very attractive for moduli fixing.
- However, one should be careful before concluding that Kähler moduli are stabilized because of the D8-brane. There could be extra open string moduli, coupled as

 $W_{open} = X_i X_j X_k$ 

just like happens for D6-branes.

Douglas'98

In this case the F-flatness condition reads

$$\frac{\partial W}{\partial X_i} = (T_j T_k + X_j X_k - n) = 0$$

and only a combination of open and closed string moduli is fixed (like for D-term potentials).

#### $\mathbf{Z}_2 imes \mathbf{Z}_2$

		xx $xy$ $yx$ $yy$
$D8_a$	$N_a = 3 + 1$	$(1,0)_1 \times (1,3,-3,-10)_{23}$
$D6_b$	$N_b = 1$	$(0,1)_1(1,0)_2(0,-1)_3$
$D6_c$	$N_c = 1$	$(0,1)_1(0,-1)_2(1,0)_3$



Let us consider the  $Z_2 \times Z_2$  orientifold background and the following set of D8 and D6-branes

$D8_a$	$N_a = 3 + 1$	$(1,0)_1 \times (1,3,-3,-10)_{23}$
$D6_b$	$N_b = 1$	$(0,1)_1(1,0)_2(0,-1)_3$
$D6_c$	$N_c = 1$	$(0,1)_1(0,-1)_2(1,0)_3$

xx

xu

yx

UU



Let us consider the  $Z_2 \times Z_2$  orientifold background and the following set of D8 and D6-branes

$D8_a$	$N_a = 3 + 1$	$(1,0)_1 \times (1,3,-3,-10)_{23}$
$D6_b$	$N_b = 1$	$(0,1)_1(1,0)_2(0,-1)_3$
$D6_c$	$N_c = 1$	$(0,1)_1(0,-1)_2(1,0)_3$

xx

xu

yx

UU



Very similar to the previous D6-model...



Very similar to the previous D6-model...

Cremades et al. '03 7.M.& Shiu '04

 $\begin{array}{|c|c|c|c|c|c|c|c|}\hline N_{\alpha} & (n_{\alpha}^{1}, m_{\alpha}^{1}) & (n_{\alpha}^{2}, m_{\alpha}^{2}) & (n_{\alpha}^{3}, m_{\alpha}^{3}) \\ \hline N_{a} = 3 + 1 & (1, 0) & (3, 1) & (3, -1) \\ N_{b} = 1 & (0, 1) & (1, 0) & (0, -1) \\ N_{c} = 1 & (0, 1) & (0, -1) & (1, 0) \\ \hline \end{array}$ 



Let us consider the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold background and the following set of D8 and D6-branes

$D8_a$	$N_a = 3 + 1$	$(1,0)_1 \times (1,3,-3,-10)_{23}$
$D6_b$	$N_b = 1$	$(0,1)_1(1,0)_2(0,-1)_3$
$D6_c$	$N_c = 1$	$(0,1)_1(0,-1)_2(1,0)_3$



xxxuyxUU

SU(2) <sub>R</sub>

We can introduce additional D-branes to cancel tadpoles:

xx

 $xy \quad yx \quad yy$ 

$D8_Z$	$N_Z = 1$	$(0,1)_1(0,-1,-1,0)_{23}$
$D6_M$	$N_M = 2$	$(-2,1)_1(-3,1)_2(-3,1)_3$
$D6_F$	$N_F = 4$	$(1,0)_1(1,0)_2(1,0)_3$

We can introduce additional D-branes to cancel tadpoles:

xx

xy

yx yy

$D8_Z$	$N_Z = 1$	$(0,1)_1(0,-1,-1,0)_{23}$
$D6_M$	$N_M = 2$	$(-2,1)_1(-3,1)_2(-3,1)_3$
$D6_F$	$N_F = 4$	$(1,0)_1(1,0)_2(1,0)_3$

The D6-brane charge induced on the D8-branes is

 $D8_a : (1,0)_1 \times (1,3,-3,-10)_{23} = (1,0)_1 \times [(3,1)(3,-1) + (1,0)(1,0)]$  $D8_Z : (0,1)_1 \times (0,-1,-1,0)_{23} = (0,1)_1 \times [(1,0)(0,-1) + (0,-1)(1,0)]$ 

Many more variants may be built...

The chiral spectrum of this model is quite minimal

sector	Matter	Representation
ab	$Q_L + L$	$3\left(3+1,2,1 ight)$
ac	$Q_R + R$	$3(\overline{3}+1,1,2)$
bc	H	(1,2,2)
bM	L'	$6\left(1,2,1;2_{M} ight)$
сM	R'	$6\left(1,1,2;2_{M} ight)$

and one may get rid of the extra matter by performing a Higgsing of the form  $U(2)_M \to SO(2)_M$ 



#### Conclusions

- There is more than meets the eye: D6-branes need not be the only BPS objects of a Calabi-Yau compactification
- We have shown this by explicitly constructing BPS coisotropic D8-branes, in the sense of Kapustin and Orlov, in a  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold

### Conclusions

- There is more than meets the eye: D6-branes need not be the only BPS objects of a Calabi-Yau compactification
- We have shown this by explicitly constructing BPS coisotropic D8-branes, in the sense of Kapustin and Orlov, in a  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold
- These D8-branes show interesting model building features, like producing D=4 chiral fermions when intersecting other D8 or D6-branes
- We have analyzed the effective theory of these D8's. Many features are similar to D6-branes, but others are new, like a superpotential involving Kähler moduli





We have constructed new examples of MSSM-like vacua by means of coisotropic D8's and intersecting D6's

## Conclusions

- We have constructed new examples of MSSM-like vacua by means of coisotropic D8's and intersecting D6's
- Recently a statistical analysis of semi-realistic models in the same  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold has been performed.

Blumenhagen et al. '04 Douglas & Taylor'06

This analysis did not take into account the presence of coisotropic D8-branes, so the statistical results could in principle be modified.