Strings 03, Santa Barbara
Oct. 2003

Cosmological brane-bulk energy exchange and new sources of acceleration

with: G. Kofinas
N. Tetradis
T. Tomaras
V. Zarikas

hep-th/0207060

and "to appear"

Plan

- Introduction.
- Cosmological equations
- Study of inflow-outflow
  - exact solution,
  - fixed points
  - stability
- New mechanisms for inflation-acceleration.

①

Introduction

The brane-world idea is an abstraction of

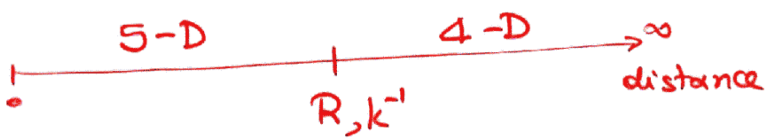
- Heterotic M-theory
- type-I (orientifolds) strings

with alternative realisation

- RS brane worlds

Features:

- Gravity is not allways 4D.
  eg (compactification or RS)



Associated Cosmology:

- 4D at late times
- " – " at early times

②

Important effect: emission of KK graviton states → energy loss on the brane.

Inverse: acretion of bulk energy (if bulk is not empty)

Typically:

- If evolution period is 4D emission is supressed.
- if evolution period is "5D" emission is important.

So:

- New cosmological eras with "tracking" of the realisation of gravity
- New effect: brane bulk energy exchange.

(3)

Other factors affect both the realisation of 4D gravity and associated energy exchange.

Example: brane-induced gravity

Dvali, Gabadadze, Porrati.

All theories (higher D) have a non-trivial 4D gravity realisation

$\Rightarrow$ non-trivial periods of energy exchange.

- Provide important constraints on the physics
- Give new (and useful) effects like inflation and late-time acceleration.

(4)

The context: RS-cosmology

$$S = M^3\int d^5x\sqrt{g}\,(R_{(5)} - \Lambda) + \int d^4x\sqrt{g}\,(-V)$$

$M \to$ 5D Planck scale

$V \to$ 4D cosmological constant

$\gg$

$\Lambda \to$ 5D

define: $k = \dfrac{V}{M^3}$

$$M_P^2 = \frac{M^6}{V} = \frac{M^3}{k}$$

$$\lambda = \Lambda + \frac{V^2}{M^3} \to \text{effective 4D cosmological constant}$$

0 —— 4D —— k —— 5D —— ∞ $\to$ Energy

$$kr \gg 1 \quad V(r) = \frac{1}{4\pi M_P^2}\cdot\frac{1}{r}\left\{1 + \frac{1}{2k^2r^2} + \cdots\right\}$$

Randall Sundrum

$$kr \ll 1 \quad V(r) = \frac{1}{4\pi^2 M^3}\,\frac{1}{r^2}\left\{1 + kr + O(r^2)\right\}$$

E.K. Tetradis Tomaras

5

RS cosmology

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{\rho^2}{M^6} + \frac{\rho}{M_P^2} - \frac{K}{a^2} + \frac{\lambda}{6M^3}$$

Binetruy, Deffayet, Ellwanger, Langlois

Define dimensionless $\tilde\rho = \frac{\rho}{V}$

$$H^2 = k^2\left(\tilde\rho^2 + \tilde\rho\right) - \frac{K}{a^2} + \frac{\lambda}{6M^3} \quad *$$

$\tilde\rho \ll 1 \Rightarrow$ 4D evolution

$\tilde\rho \gg 1 \Rightarrow$ "5D" evolution

measuring time in k-units:

$$H^2 = \tilde\rho^2 + \tilde\rho - \frac{\tilde K}{a^2} + \frac{1}{6}\left(1 + \frac{\Lambda}{M^3 k^2}\right)$$

5a

Brane-bulk energy exchange: the RS case

We must estimate the KK emission.

Example:

$$\left.\frac{d\rho}{dt}\right|_{lost} = \langle n_a n_b \sigma_{a+b\to c+KK}\cdot v\, E_{KK}\rangle$$

thermal average

$$(\sigma_{a+b\to c+KK}\cdot v) \sim \frac{1}{M^3}\int_0^T dm\, |\psi_m(0)|^2$$

$$\psi_m(0) = \begin{cases} \sqrt{\frac{m}{k}} & m \ll k \\ 1 & m \gg k \end{cases}$$

$$\Rightarrow \sigma_{a+b\to c+KK}\cdot v \sim \frac{1}{M_P^2}\left(\frac{T}{k}\right)^2 \qquad T \ll k$$

$$\sim \frac{1}{\ }\left(T\right) \qquad T \gg k$$

(5b)

assume that the particles a, b belong to the "driving" density

with $\rho = w\, P$

then: $n_{a,b} \sim T^{3(1+w)-1}$, $T \sim \frac{1}{a}$

$E_{kk} \sim T$

since $\tilde{\rho} \sim T^{3(1+w)}$

we can reexpress everything in terms of $\tilde{\rho}$

$$\frac{d\rho}{dt}\Big|_{lost} \sim T^{6(1+w)+1} \sim \tilde{\rho}^{\,2+\frac{1}{3(1+w)}} \quad \text{for } \tilde{\rho} \ll 1$$

$$\sim T^{6(1+w)} \sim \tilde{\rho}^{\,2} \quad \text{for } \tilde{\rho} \gg 1$$

$$\frac{\frac{d\rho}{dt}\Big|_{lost}}{\frac{d\rho}{dt}\Big|_{dilution}} \sim \; \longrightarrow \tilde{\rho}^{\,\frac{1}{2}+\frac{1}{3(1+w)}} \ll 1$$

$$\longrightarrow O(1)$$

(6)

General approach: arbitrary bulk and brane matter.

$$G_{AB} = \frac{1}{2M^3}\, T_{AB}$$

$$T = T^{bulk}_{vac} + T^{bulk}_{matter} + T^{brane}_{vac} + T^{brane}_{matter}$$

$$T^{brane}_{vac} = \delta(z)\,(+V, V, V, V, 0)$$

$$T^{brane}_{matter} = \delta(z)\,(-\rho, P, P, P, 0)$$

$$T^{bulk}_{vac} = (+\Lambda, -\Lambda, -\Lambda, -\Lambda, -\Lambda)$$

$\Rightarrow$ Equations (FRW) on the brane

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = -T^{0}{}_{5}$$

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \lambda - \frac{1}{M^6}\Big(V(3P-\rho) + \rho(3P+\rho)\Big) - \frac{T^{5}{}_{5}}{M^3}$$

to proceed further (1) must have a concrete solution to bulk equations or (2) make an approximation

- Neglect $T_5^5|_{brane} \longrightarrow \frac{T_{matter}}{V} \gg \frac{T_5^5}{\Lambda}$ (+)
- Simplification
- Generically expected to hold except at special points

Rewrite the system as:

$T \equiv T_5^0$

$$\dot\rho + 3(1+w)\frac{\dot a}{a}\rho = -T$$

$$H^2 = \frac{\rho^2}{M^6} + \frac{\rho}{M_P^2} - \frac{k}{a^2} + \underline{\underline{\chi}} + \lambda$$

$$\dot\chi + 4\frac{\dot a}{a}\chi = \left(\frac{\rho}{M^6} + \frac{1}{M_P^2}\right)T$$

This is the definition of χ.

Ansatz: $T \equiv T(\rho)$

typically $T(\rho) \sim A\,\rho^\nu$

(from conformal invariance)

Study special solutions in small density (4D) regime.

(8)

$$\dot\rho + 3(1+w)H\rho = -T \qquad \lambda \neq 0$$

$$H^2 = \frac{\rho}{M_P^2} + \chi - \frac{k}{a^2} + \lambda \qquad \dot\chi + 4H\chi = \frac{T}{M_P^2}$$

- $\boxed{w = \frac{1}{3}}$ define $\hat\rho = \rho + M_P^2\chi$

$$\Rightarrow \quad \dot{\hat\rho} + 4H\hat\rho = 0$$

$$H^2 = \frac{\hat\rho}{M_P^2} - \frac{k}{a^2} + \lambda$$

$\hat\rho$ is standard radiation density

$\chi \longrightarrow$ mirage (bulk) radiation

$T \neq 0$ transforms $\rho \leftrightarrow \chi$ but $\hat\rho$ constant

E.g: $T \sim A\cdot\rho$

$$\rho = \rho_0\left(\frac{a_0}{a}\right)^4 e^{-At}$$

$$\chi = \frac{1}{M_P^2}A\left(\int_0^t \frac{a(u)}{a_0}e^{-Au}du\right)\rho_0\left(\frac{a_0}{a}\right)^4$$

brane radiation decays into "mirage" radiation that still affects evolution

This solution is similar for $w \neq \frac{1}{3}$, but $T \sim A\cdot\rho$

Accelerating solutions

9

fixed points
$H \to H_*$
$\rho \to \rho_*$ constant
$\chi \to \chi_*$
$T \to T_*$

$$3 H_* (1+w) \rho_* = - T(\rho_*)$$
$$H_*^2 = \frac{\rho_*}{M_P^2} + \chi_*$$
$$2 H_* \chi_* = \frac{T(\rho_*)}{M_P^2}$$

polynomial equations with $\propto$ finite # of solutions

possible iff $T<0$ (inflow)

and $-1 < w < 1/3$

Stability of acceleration fixed points

$$v = \left.\frac{d \log T}{d \log \rho}\right|_* \qquad T \sim \rho^v$$

$$\frac{d}{dt}\begin{pmatrix}\delta\rho\\ \delta\chi\end{pmatrix} = \frac{T_*}{\rho_*} \mathcal{M} \begin{pmatrix}\delta\rho\\ \delta\chi\end{pmatrix}$$

$$M_{1,2} = \frac{7+3w-3v(1+w) \pm \sqrt{24(2v-3)(1+w) + (7+3w-3v(1+w))^2}}{6(1+w)}$$

stable for: $-1 < w < \frac{1}{3}$
$0 < v < 3/2$

Non-flat solutions

10

$v=1$, $k=1$, $w>-\frac{1}{3}$ (outflow)

$\Longrightarrow$ eternal deceleration

For $k=-1$, $w=0$, $v=1$

first deceleration then acceleration

(without outflow $\to$ no acceleration in the end)

(12)

Some general properties:

- For $q>0$ we must necessarily have $\underline{\chi<0}$

However χ contributes a negative term in Friedman equation

$$H^2 \sim e+\chi$$

$\to$ matter must compensate for χ

$\Rightarrow$ $\Omega_m>2$, $\Omega_\chi<-1$ $k=0$

$\Rightarrow$ we need more dark energy

But at $k=-1$ we can have $\Omega_k=0.96$, $\Omega_\chi=-0.06$, $\Omega_m\simeq 0.1$

- For $T>0$ (outflow) e decreases for all expanding solutions

- $q(e)$ always lies below the parabola : $(1-3w)\gamma\rho-(1+3w)\delta\rho^2$

(11a)

acceleration versus e equation.

$$\left[3(1+w)\rho + \epsilon T(e) / \sqrt{(1-3w)\gamma e-(1+3w)\delta e^2-\frac{k}{a^2}-q(e)}\right]\frac{dq}{de}$$

$$+\,\epsilon T(e)\left(2(1+3w)\delta\rho-(1-3w)\gamma\right)/\sqrt{}$$

$$-\,4q(e) + (1+3w)\left(2(2+3w)\delta\rho-(1-3w)\gamma\right)\rho=0$$

THE FULL NON-LINEAR SYSTEM 11

Equations:

$\beta \sim \frac{1}{M^6}$

$\gamma \sim \frac{1}{M_p^2}$

$$a\frac{d\rho}{da} = -3(1+\omega)\rho - \frac{\epsilon T(\rho)}{\sqrt{\beta\rho^2 + 2\gamma\rho - \frac{k}{a^2} + \chi}}$$

$$a\frac{d\chi}{da} = -4\chi + \frac{2\epsilon(\beta\rho+\gamma)T(\rho)}{\sqrt{\beta\rho^2 + 2\gamma\rho - \frac{k}{a^2} + \chi}}$$

and

$$\left(3(1+\omega)\rho\sqrt{} + \epsilon T(\rho)\right)\frac{d\chi}{d\rho} = 4\chi\sqrt{} - 2\epsilon(\beta\rho+\gamma)T(\rho)$$

$$q \equiv \frac{\ddot{a}}{a} = -(2+3\omega)\beta\rho^2 - (1+3\omega)\gamma\rho - \chi(+2)$$

Using this we can also derive a first-order equation of $q(\rho)$

* $\epsilon = 1 \Rightarrow$ expansion

$\epsilon = -1 \Rightarrow$ contraction

13

- For $k=0$ and 1, $\omega > -\frac{1}{3}$ there is always deceleration at large enough ρ
- For $k=0$ and 1, $\omega \geq \frac{1}{3}$ There is deceleration at all times
- For $k=0$ the system becomes autonomous and for $T \propto A\rho^\nu$ it can be solved in the "4D" regime

$$|H|^{\frac{5+w-3\nu(1+\omega)}{2}} = C\left|\rho^{1-\nu}|H| - \frac{A(\nu-1)}{5+3w-3\nu(1+\omega)}\right|$$

(14)

Fixed point of full system and stability analysis.

All fixed points are accelerating

stability:

$$\frac{d}{da}\begin{pmatrix}\delta\rho\\ \delta q\end{pmatrix} = \frac{9(1+w)^2\rho_*^3}{2T_*^2}\begin{pmatrix}m_1 & m_2\\ m_3 & m_4\end{pmatrix}\begin{pmatrix}\delta\rho\\ \delta q\end{pmatrix}$$

$$m_1 = 3(1+w)\Big((3-\nu)(1+3w)b\rho_* + (\nu-2)(1-3w)\gamma\Big)$$

$$m_2 = 3(1+w)$$

$$m_3 = \Big((1-3w)\gamma - 2(1+3w)b\rho_*\Big)\times$$
$$\times\Big((1+3w)(7+9w-3\nu(1+w))b\rho_* - (1-3w)(4+6w-3\nu(1+w))\gamma\Big)$$

$$m_4 = -\Big(2(1+3w)^2 b\rho_* + (1-3w)^2\gamma\Big)$$

For $w=0$, $\nu=1$, if $|A| \le \frac{3\gamma}{\sqrt{8b}}$ there are two positive real roots:

$$\rho_*^{\pm} = \left(1 \pm \sqrt{1-\frac{8bA^2}{9\gamma^2}}\right)\frac{\gamma}{2b}$$

ρ_*^+ is always a saddle point

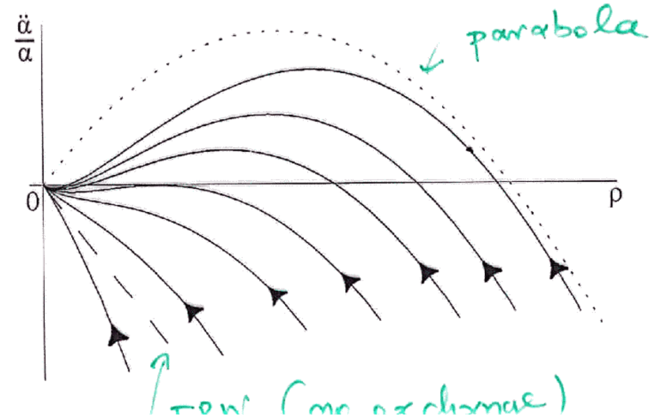
ρ_*^- is stable

(15)

$w=0$, $k=0$ $\nu=1$ (outflow)

two families:

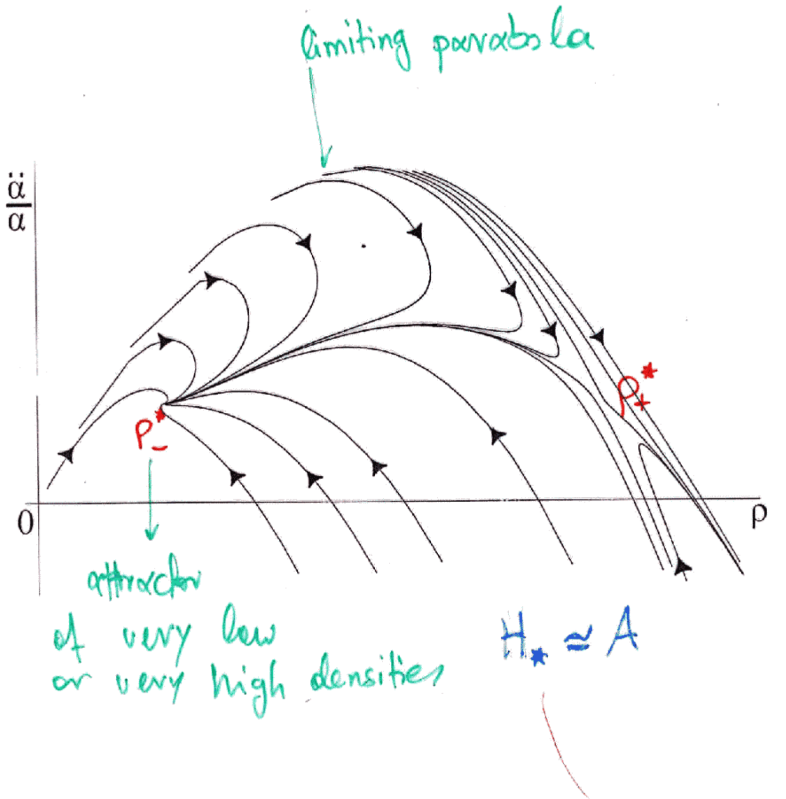
- Deceleration always
- Intermediate acceleration



(16)

Influx, $k=0$, $w=0$, $\nu=1$

3 classes of behavior



(17)

"Locking" of density (ρ) and dark energy (χ).

$\underline{\nu=\frac{3}{2}}$: $T \sim A\rho^{3/2}$: $A=\mu-\frac{1}{\mu}$

$\underline{k=0}$

Solution (4D regime)

$$\left(\frac{|H|}{\sqrt{\rho}}-\mu\right)^{-\frac{2}{\mu}+8\mu}\left(\frac{|H|}{\sqrt{\rho}}+\frac{1}{\mu}\right)^{-2\mu+\frac{8}{\mu}}=C\,e^{\mu+\frac{1}{\mu}}$$

and

$$\left(\frac{|H|}{\sqrt{\rho}}-\mu\right)^{2\mu^2}\left(\frac{|H|}{\sqrt{\rho}}+\frac{1}{\mu}\right)^{2}=C'a^{-(\mu^2+1)}$$

For $A>-\frac{3}{2}$ and late times

$H^2 \sim \mu^2\rho + \dots$ could potentially

$\rho \sim a^{-4+\frac{1}{\mu^2}} + \dots$ "simulate" dark

$\chi \sim (-1+\mu^2)\rho + \dots$ non-relativistic matter.

$T \sim A\rho^{3/2}$ from ... $\chi \sim a^{-3}$

SUMMARY

(18)

- Cosmological energy exchange is a generic (unavoidable?) feature of all high-D realisations of gravity.
- Our "phenomenological" analysis indicates:
  - → conversion of radiation to dark (mirage) radiation
  - → generic stable de Sitter points (inflow)

outflow → eternal deceleration
→ intermediate acceleration

Inflow → generic fixed points → de Sitter
→ passage acceleration ↔ deceleration

speculation: start with a brane "hotter" than bulk

→ deceleration + radiation → acceleration (inflation)
$\left(H_* = \frac{M^3}{M_P^2}\right)$ → deceleration (exit)

→ super cooling → inflow → attractor late time acceleration $(H_* = A)$

Open problems

(19)

- Investigate the validity of the approximation

$$\frac{T_{55}}{V_5} \ll \frac{T_{00}}{V_4}$$

Exact solutions will help

- Investigate further the physically interesting solution as well as the microscopic cross-sections for in/out-flow.
- Generalize to compactified gravity.