Entropy Bounds: Can we live without them?

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Based on work w/R. Sorkin
Also w/ S. Ross and D. Minic

Bekenstein Bound: $S < \alpha RE$

Holographic Bound: $S < A/4l_p^2$

A common story

Black Hole Thermodynamics

⇓

Entropy Bounds ⟺ $S \leq A/4l_p^2$, $S \leq \alpha RE.$

Loophole!

Outline:

1. Review of a lovely debate: Bekenstein, Unruh, and Wald: Entropy Bounds vs. Floating Boxes.
2. Loopholes: Preserving the 2^{nd} law without Bekenstein's bound ($S < \alpha RE$).
3. The observer dependence of entropy.
4. Save holographic bound ($S<A/4l_p^2$) for questions.

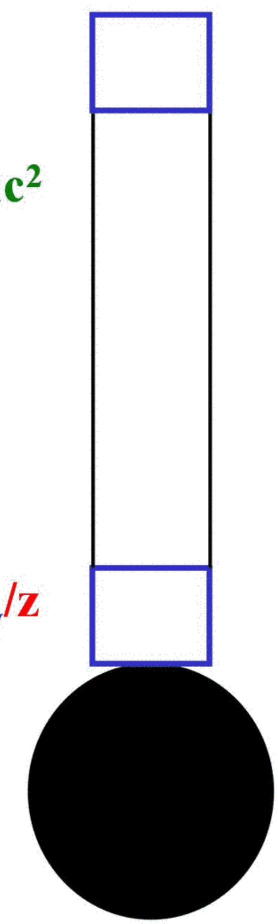
1.Bekenstein's Concern:

Adiabatic:

- No kinetic energy
- Energy redshifted to almost zero.

$S_{infinity} > 0$

$E_{infinity} = \mathbf{mc^2}$



$S_{box} = S_{infinity}$

$E_{box} = E_{infinity}/z \sim 0$

Bekenstein's Concern: Entropy

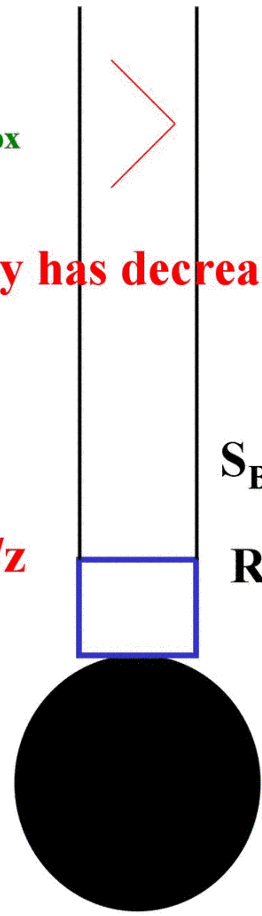
(1973)

Before

$S = S_{BH} + S_{box}$

After

$S = S_{BH}$



Entropy has decreased????

$$S_{BH} = \frac{4\pi R^2}{4 l_p^2}$$

$R = 2MG/c^2$

$E_{box} = E_{infinity}/z \sim 0$

$S_{box} = S_{infinity}$

M, R, and S_{BH} are same before and after!

The Bekenstein Bound:

$E_{box} > 0$

Black Hole grows!

Before

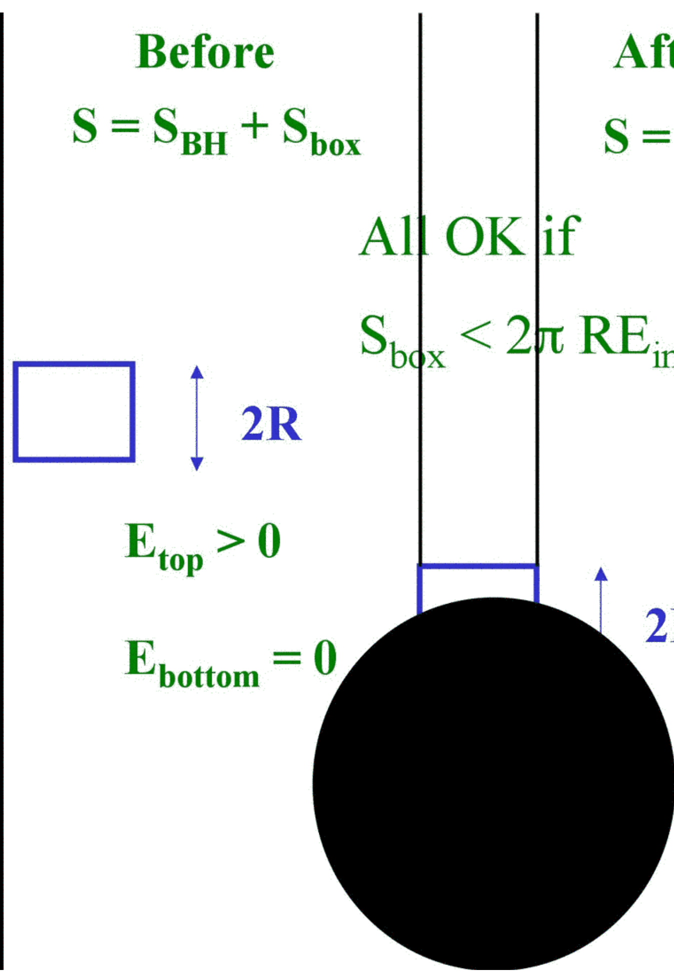
$S = S_{BH} + S_{box}$

After

$S = S_{bigger\ BH}$

All OK if

$S_{box} < 2\pi R E_{infinity}/c\hbar$



2R

$E_{top} > 0$

$E_{bottom} = 0$

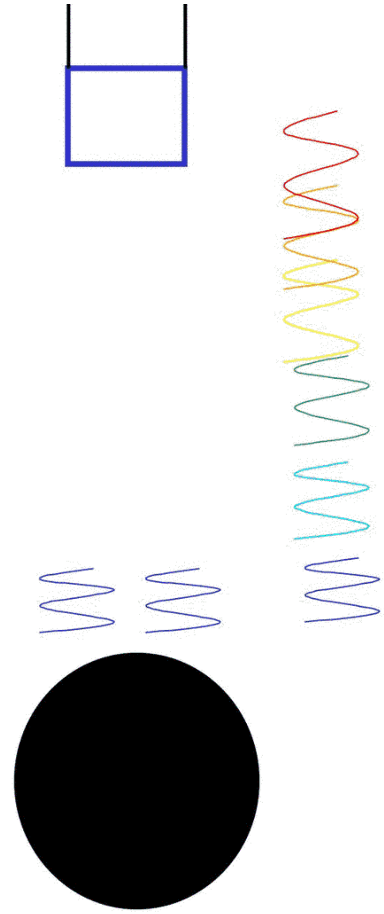
2R

1b. Unruh and Wald: An Alternate Response

(1981)

What about the Hawking radiation?

Weak or Strong?



Weak! Cold!

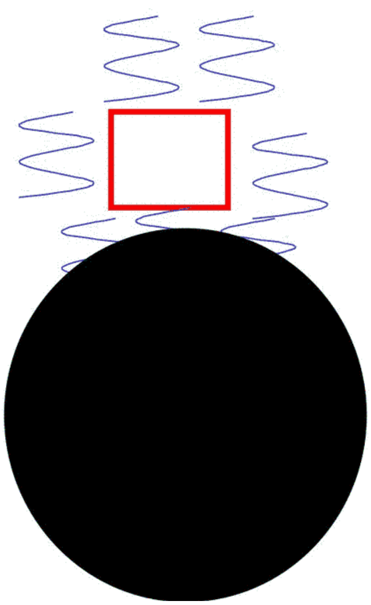
Hot! Strong close to black hole!

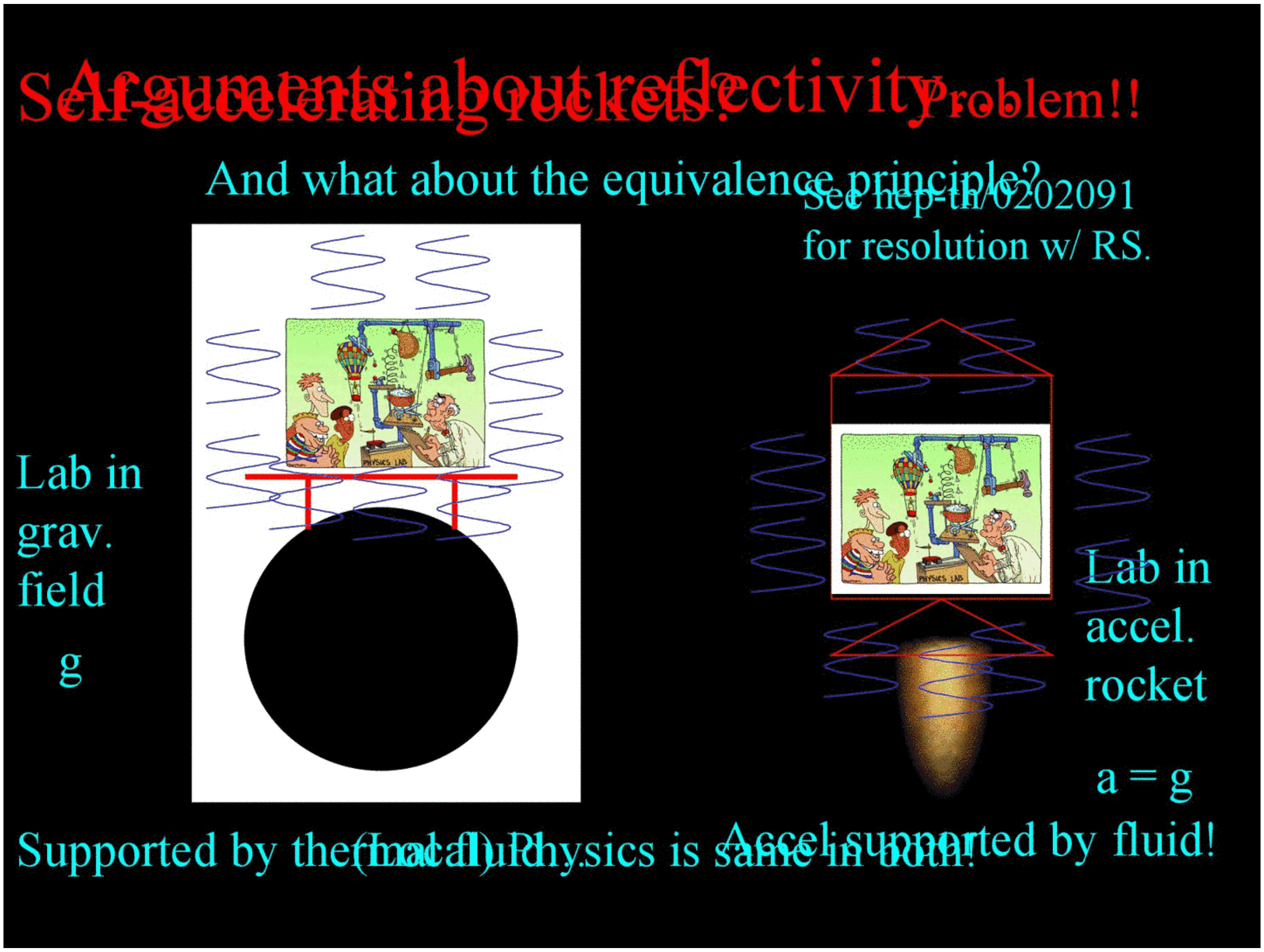
More accurately: a thermal fluid

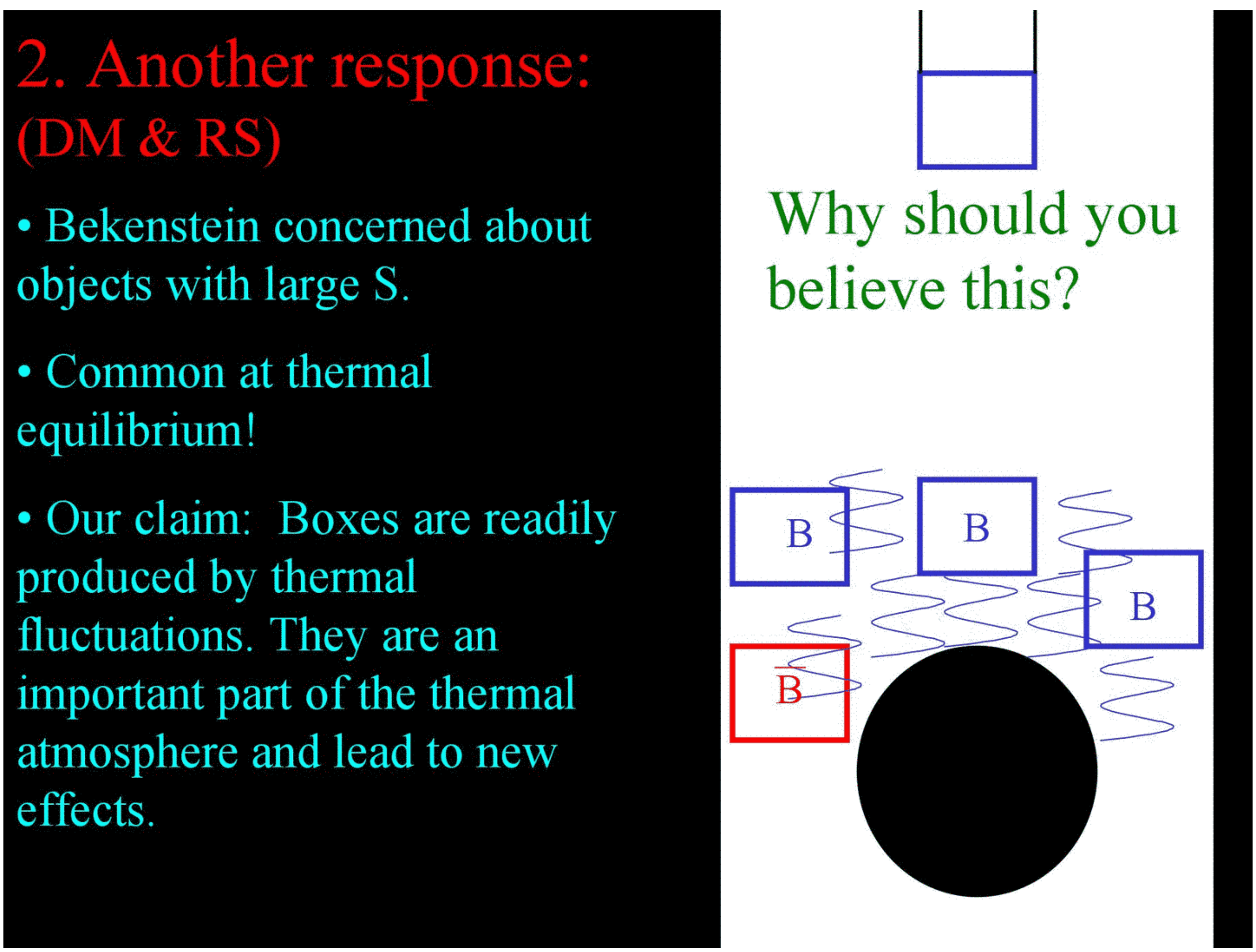
Box Floats! (Archimedes)

- Box does not reach horizon.
- Even for $R_{box} = 0$, E_{box} does not reach zero!
- Black Hole grows if box dropped in.
- BH Entropy increases!

Claim: Takes care of Bekenstein's concern w/o new bounds!




Arguments about reflectivity
Self-accelerating rockets?
Problem!!
And what about the equivalence principle?
See hep-th/0202091
for resolution w/ RS.
Lab in
grav.
field
g
Lab in
accel.
rocket
a = g
Supported by thermal fluid
(Local) Physics is same in both!
Accel supported by fluid!


2. Another response:
(DM & RS)
• Bekenstein concerned about objects with large S.
• Common at thermal equilibrium!
• Our claim: Boxes are readily produced by thermal fluctuations. They are an important part of the thermal atmosphere and lead to new effects.
Why should you believe this?
B
B
B
B

Bekenstein's Concern:

(80's and 90's)

- Let box fall freely from far away.
- Hawking radiation is a small effect. E_{box} is constant.
- S_{BH} grows. S_{box} gone!
- But ΔS_{BH} depends only on E_{box} – not on S_{box}!

$S_{infinity} > 0$

$E_{infinity} = mc^2$

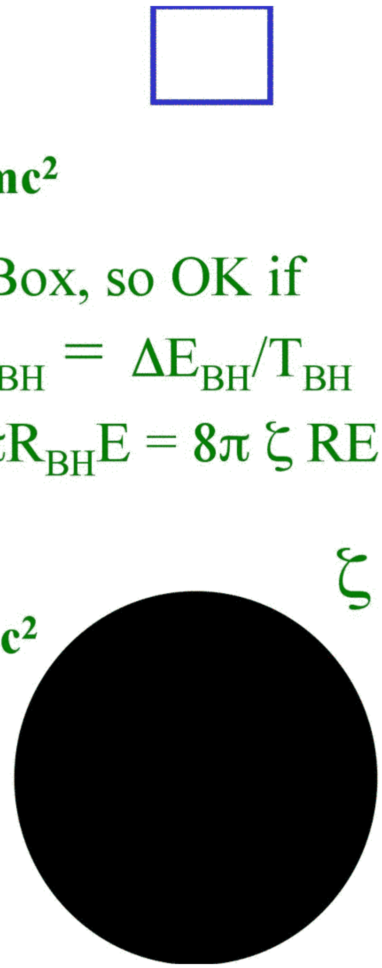
Small Box, so OK if

$S < \Delta S_{BH} = \Delta E_{BH}/T_{BH}$

$= 4\pi R_{BH} E = 8\pi \zeta RE$

$\zeta = \frac{R_{BH}}{2R}$

$E_{box} = mc^2$



But Wald...

Boxes and Thermal Fluctuations

(DM & R Sorkin 2002)

- These boxes have lots of S.
- What is their free energy?

F is negative!

Violation: $S > 8\pi \zeta RE$

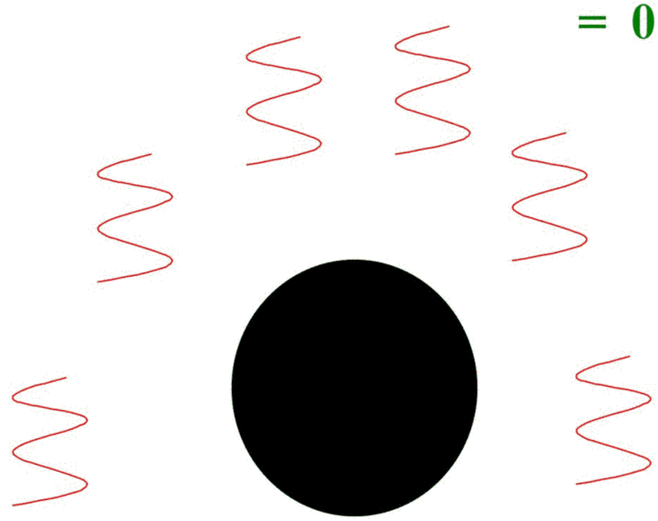
$T = 1/(4\pi R_{BH})$

$\zeta = \frac{R_{BH}}{2R}$

$F = E - TS$

$< E - \frac{8\pi \zeta RE}{4\pi R_{BH}} = E - E\zeta(2R/R_{BH})$

$= 0$



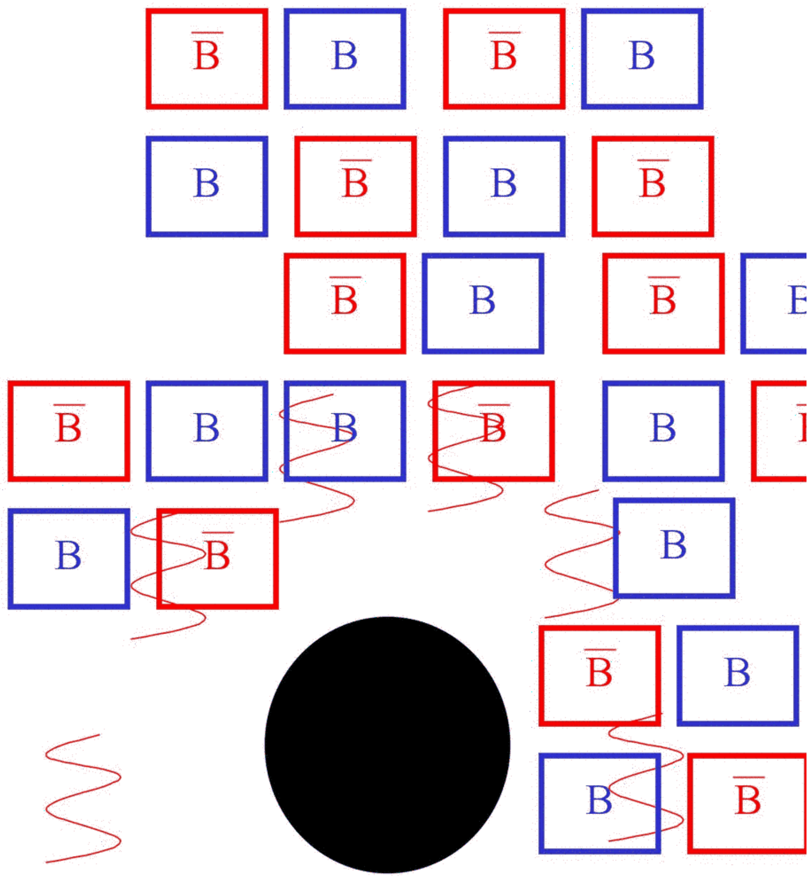
Negative Free Energy?

$$Z = \Sigma_{micro}\, e^{-E/T}$$

$$= \Sigma_{macro}\, e^{S-E/T}$$

$$= \Sigma_{macro}\, e^{-F/T}$$

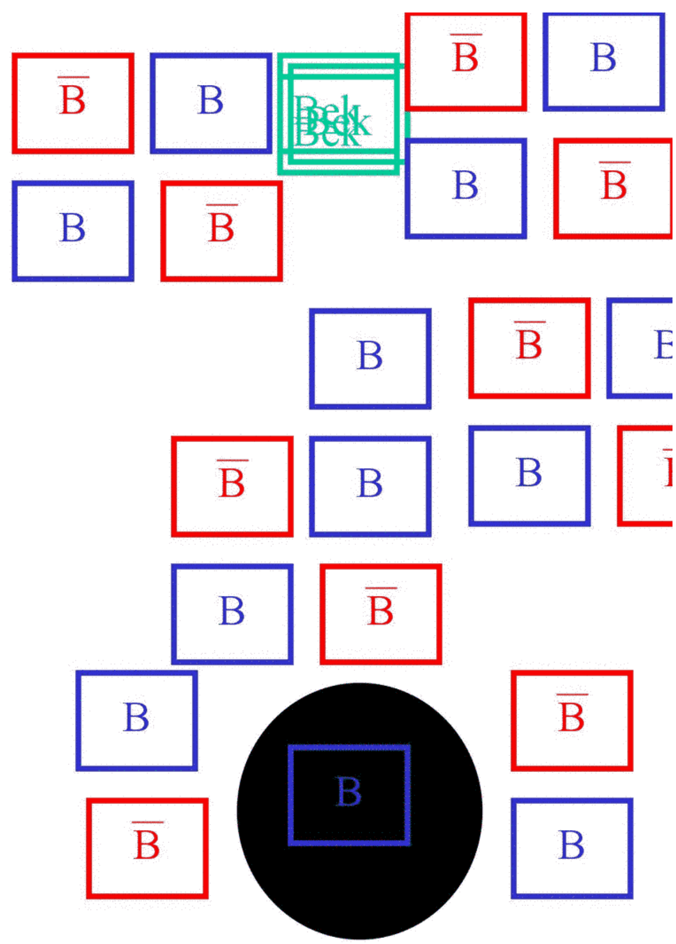
Any macrostate of box w/ this E more likely to be present than not!



What happens to Bekenstein's Box?

- Bounce off,
- Annihilate,
- Or, if transparent, another box comes out.

Bek's box is already present in *equilibrium*. So are *all* similar boxes.



Complete Generality

Drop E, S into *any* black hole.
(Assume small change)

$$\Delta S_{total} = \Delta S_{BH} - S$$
$$= E/T - S$$
$$= F/T$$

Sign of LHS = Sign of RHS

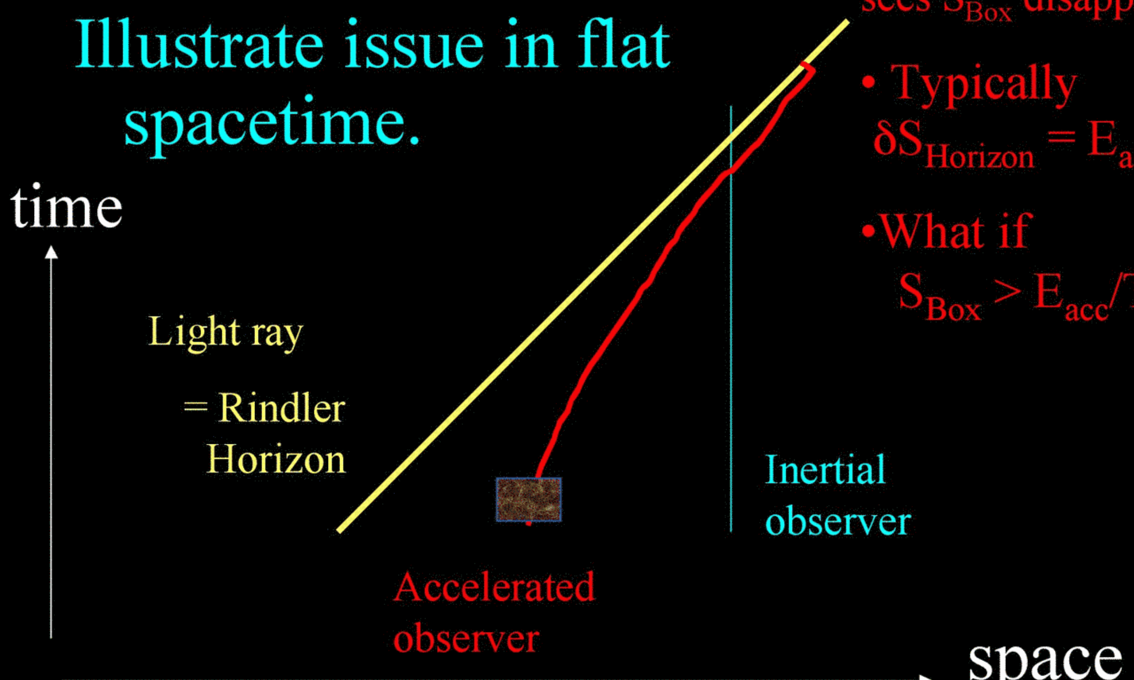
Boxes in thermal atmosphere
& New effects

Similar story for
"Holographic Bound." $S < A/4l_p^2$.

3. Equilibrium + 1?

Free fields (w/ S. Ross & D. Minic)

Illustrate issue in flat spacetime.



- Rindler observer sees S_{Box} disappear.
- Typically $\delta S_{Horizon} = E_{acc}/T$.
- What if $S_{Box} > E_{acc}/T$?

Let's calculate E &S:

	Inertial observer (for reference)	Accelerated observer
No object	$\|0>_M$	ρ_0 (thermal)
One object (microstate)	$\|1>_M$	ρ_1 (not thermal)
One object (macrostate)	$\rho = N^{-1} \Sigma \|1_i><1_i\|$	ρ_2 (not thermal)
Change in E	E	$E_{acc} = \mathrm{Tr}\,[H\, \delta\rho]$
Change in S	$S_{box} = \ln N$	$S_{acc} = -\,\delta\, \mathrm{Tr}[\rho \ln \rho]$
		$\delta\rho = \rho_2 - \rho_0$

Reminder

Take $\delta\rho$ small since $N > e^{E/T}$

$$S_{acc} = -\,\delta\, \mathrm{Tr}[\rho \ln \rho]$$
$$= -\,\mathrm{Tr}\,[\,\delta\rho\,(1 + \ln \rho)]$$
$$= -\,\mathrm{Tr}\,[\,\delta\rho\,(\not{1} - H/T)]$$
$$= E_{acc}/T$$

Explicit calculations for one-particle state w/ N free fields in limit of large N:

$$S_{box} = \ln N \quad \neq \quad S_{acc} = E_{acc}/T$$

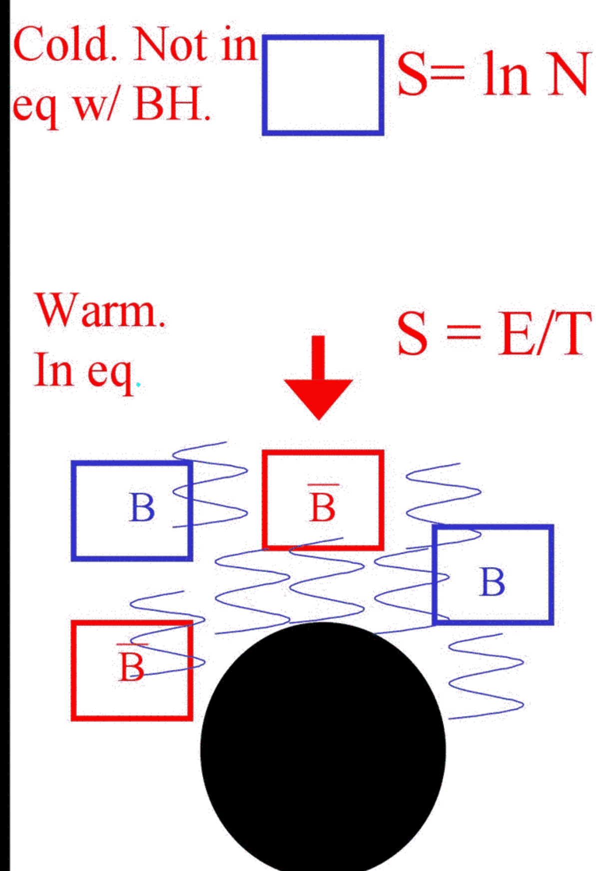
$E >> T$: $\qquad E_{acc} = E$

Why does our intuition fail?

- Can't rely on intuition from distinguishable particles for $S - E/T > 0$ because….
- QFT says to sum over # of particles at finite T.
- e^S states for one particle, $(e^S)^n$ states for n particles.
- $\Sigma_n e^{-nE/T} (e^S)^n = \Sigma_n e^{(S-E/T)n}$ -- diverges; i.e., infinite # of particles preferred.
- Conclude: Indistinguishability critical in this regime (though details of statistics irrelevant).

Consider again fall into BH (free case):

- 2nd *already* violated? (BEFORE object enters black hole!)
- Like sending (unpolarized) low E photon into hot cavity.
- Other radiation (objects) must leak out!



Review

1. Arguments for a fundamental ``Bekenstein Entropy Bound'' contain a loophole.

2. This loophole comes from the ***high*** probability that highly entropic objects will be created by thermal fluctuations. B $\bar{B}$

3. Important effect for free fields: a new *observer dependence* of S!

4. Similar loophole in arguments for the `holographic bound.' (Expands on Wald.)

My Viewpoint on Bekenstein bound ($S < \alpha RE$)

1. Not motivated by GSL.
2. Does not explain BH entropy or other properties. So, no real motivation at all.
3. Does not depend on G. No known definition that holds for all QFT's. In particular, violated w/ large # of species.
4. Hard to believe as a fundamental principle (and why bother?).
5. Nevertheless, not ruled out that some version holds in our Universe (at least at some scale). Experimental tests welcome.

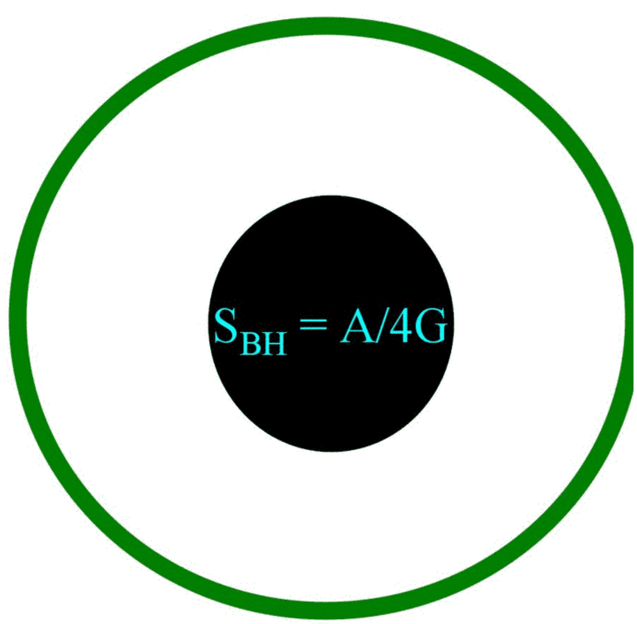
My Viewpoint on holographic bound ($S < A/4l_p^2$)

1. Not motivated by GSL.
2. But would help to explain BH entropy if true.
3. Supported by AdS/CFT, Susskind/Witten in particular.
4. Depends on G. Much more robust than Bek. bound. No clear violations of Bousso form known, even w/ large # of species.
5. Worth investigating as fundamental principle, though status unclear. Def of S? Jury still out.
6. Experimental tests would be wonderful!!

4. The Holographic Bound $S < A/4G$

(Susskind, ‘t Hooft)

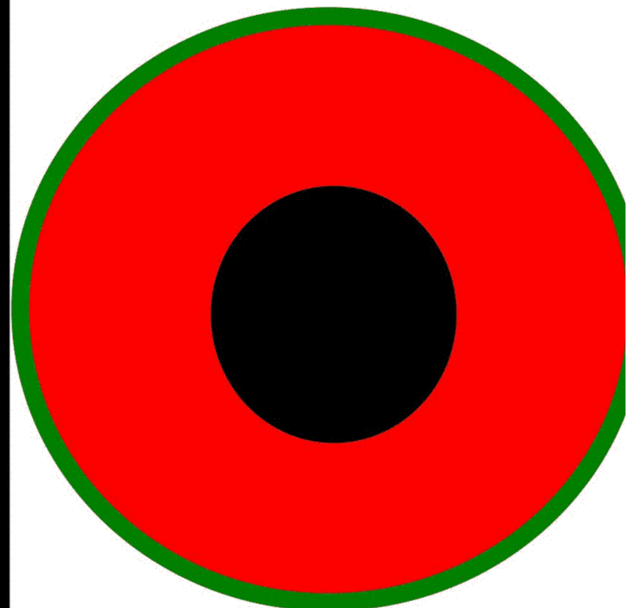
1. Suppose Q,J=0 object with $S > A/4G$.
2. Not a Black Hole, $E < E_{BH}$ at same A.
3. Add shell of mass; make into BH of same A.



So, original S is smaller.

Wald: Some subtlety?

1. Suppose large S due to large number N of scalar fields.
2. Hawking radiation is N times as great!
3. Semiclassical result: Black Hole evaporates in T ~ R. Note that this is time for shell to fall.
4. BH is a fluctuation, not equilibrium.

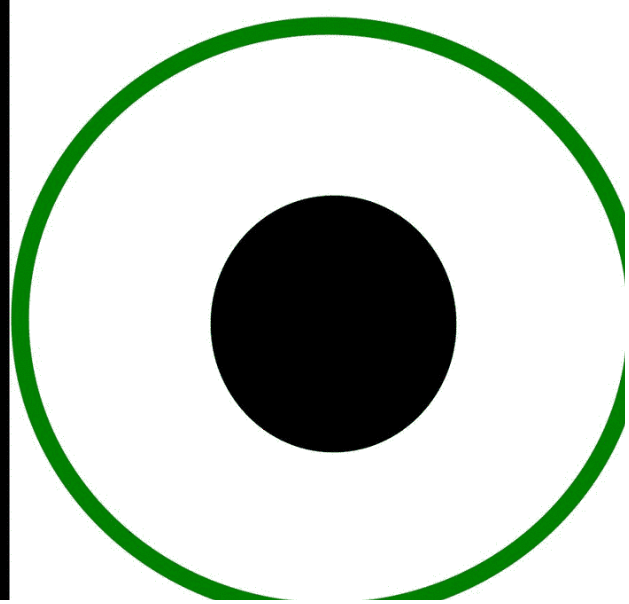


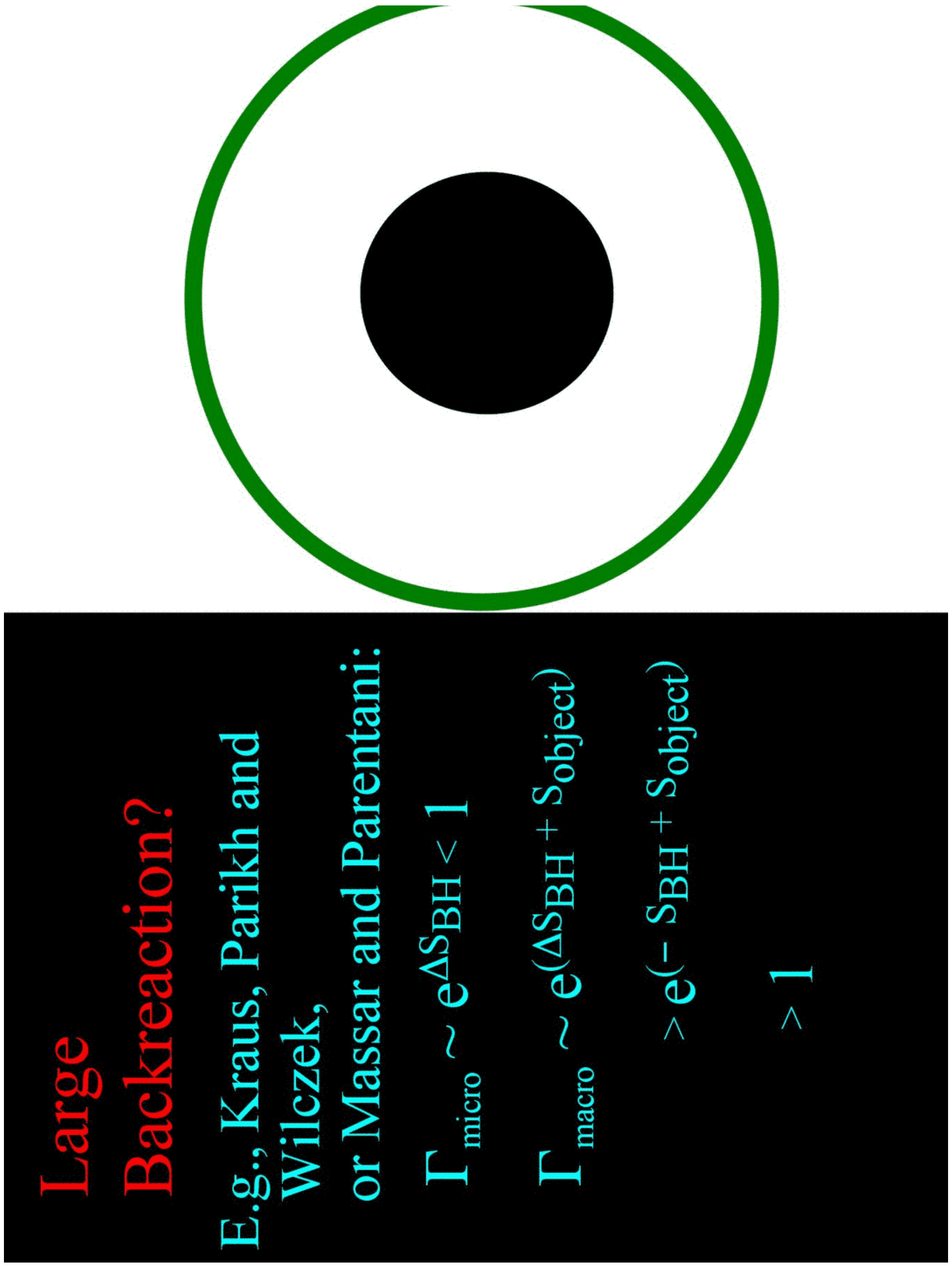
(Artist's Interpretation)

Our Calculation

1. Suppose *any* object with $S > A/4 = \pi R^2$.
2. Not Black Hole, so $E < M_{BH} = 2R$.
3. Suggestion: BH may quickly produce a copy of object through Hawaking-like process.
4. Can study this when $E << M_{BH}$ and back-reaction is small.

$$\begin{aligned} F &= E - TS \\ &< E - \frac{\pi R^2_{BH}}{4\pi R_{BH}} \\ &= E - M_{BH}/2 \quad < 0 \end{aligned}$$




Large
Backreaction?
E.g., Kraus, Parikh and
Wilczek,
or Massar and Parentani:
$\Gamma_{micro} \sim e^{\Delta S_{BH}} < 1$
$\Gamma_{macro} \sim e^{(\Delta S_{BH} + S_{object})}$
$> e^{(- S_{BH} + S_{object})}$
> 1