Black hole singularities in Yang-Mills theories

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based on

Guido Festuccia, HL
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Understanding *spacelike* singularities is a major Challenge for theoretical physics

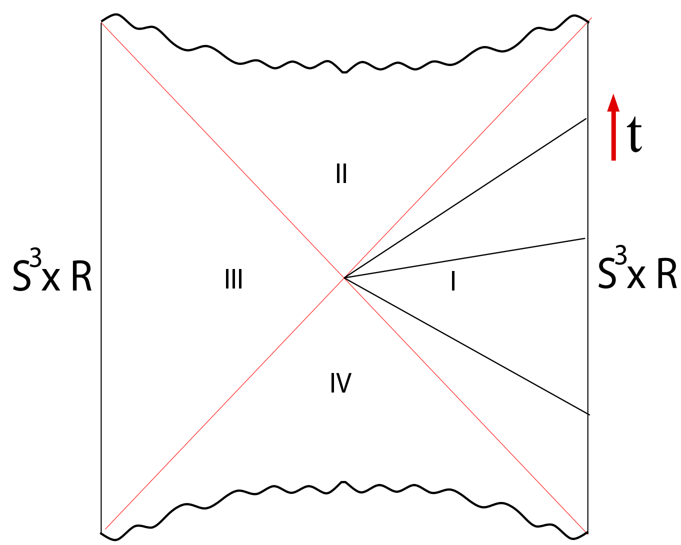
Big Bang/Big Crunch, Black holes

A non-perturbative formulation of string theory should provide a well-defined framework to address the problem.

Schwarzschild black holes in AdS

Quantum gravity in an $\mathrm{AdS}_5 \times S^5$ black hole background can be described by an $\mathbb{N} = 4$ $SU(N)$ Super Yang-Mills at finite temperature on $\underline{S^3}$.

Witten, Maldacena,...



AdS/CFT provides a full non-perturbative framework to address the problem.

Horowitz and Ross, Maldacena, ...

Understanding the black hole singularity from thermal Yang-Mills ?

The problem is hard !

1. Classical gravity corresponds to a *strongly* coupled YM theory

$$\frac{R}{l_p} \leftrightarrow N$$

$$\frac{R}{l_s} \leftrightarrow \lambda = g_{ym}^2 N$$

Supergravity limit: $N \to \infty, \ \lambda \to \infty$

2. Black hole singularities are nasty.

There is, however, one possible way out ......

It could be that the black hole singularity is so nasty that α' *effects are not enough to smooth it*.

Then the resolution of the singularity will boil down to understanding the *large N limit* of Yang-Mills theory and may *not* sensitively depend on the strongly coupled physics.

Information loss v.s. thermalization

An SYM theory on S^3 is a *bounded* many-body quantum mechanical system (with discrete energy levels)

$$\text{\textit{generic} initial states} \qquad \Rightarrow \qquad \text{thermal equilibrium}$$

In thermal equilibrium, details of initial states are essentially lost (although in principle recoverable).

The thermalization process can be interpreted in the bulk as the *gravitational collapse*.

This (in principle) solves the information loss problem for a black hole. Strong coupling is *not* essential to the resolution.

Singularities v.s. thermalization

The singularity problem is much more puzzling.

The existence of AdS/CFT does *not* automatically imply the resolution of the singularity, since the singularities lie beyond the horizons.

Given almost universal formation of singularities from gravitational collapse, it seems plausible that the black hole singularity simply reflects of some (possibly universal) dynamical aspect the underlying many-body system at equilibrium.

We would like to search for an underlying principle which could "in principle" solve the singularity problem.

Understanding the black hole singularity from thermal Yang-Mills ?

Find manifestations of the black hole singularity in the large N and large ‘t Hooft coupling limit of the YM theory;

L. Fidkowski, V. Hubeny, M. Kleban and S. Shenker

From these manifestations, understand whether (and how)

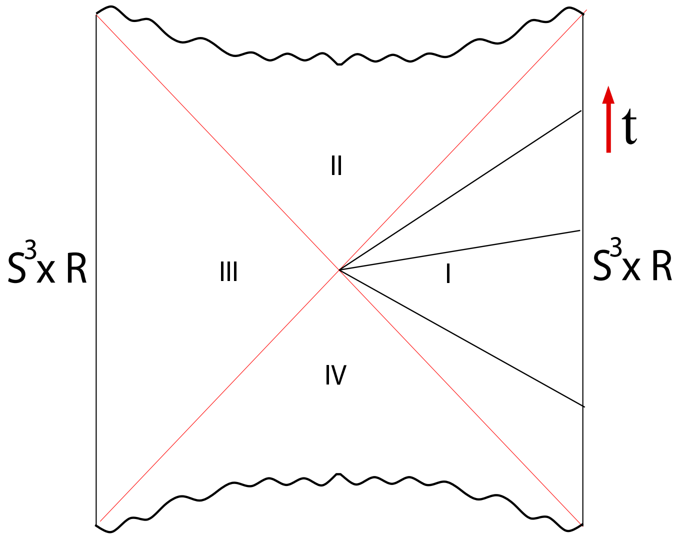
- finite N effects (g_s), or

- finite ‘t Hooft coupling effects (α')

resolve the singularity.

Main Challenge

Describe the physics beyond the horizon in AdS/CFT:



Balasubramanian, Ross; Louko, Marolf, Ross; Maldacena; Kraus, Ooguri, Shenker ...

L. Fidkowski, V. Hubeny, M. Kleban and S. Shenker

Outline

1. Establish a direct connection between boundary momentum space correlators and the bulk geometry.

2. Find signals of the singularity in momentum space correlators.

3. A curious property of strongly coupled SYM theory on S^3

4. Discuss the resolution of the singularity at finite N.

Thermal YM correlation functions

We are interested in finite temperature real-time correlation functions, e.g.

$$G_+(t,\vec{x}) = \mathrm{Tr}\left(e^{-\beta H} O(t,\vec{x}) O(0,\vec{0})\right)$$

β: inverse temperature

H: Hamiltonian of Yang-Mills theory

$\vec{x}$: a point on S^3.

O is a typical gauge invariant operator in Yang-Mills theory, dual to a bulk scalar field ϕ of mass m.

The conformal dimension Δ of O given by

$$\Delta = 2 + \nu, \qquad \nu = \sqrt{m^2 + 4}$$

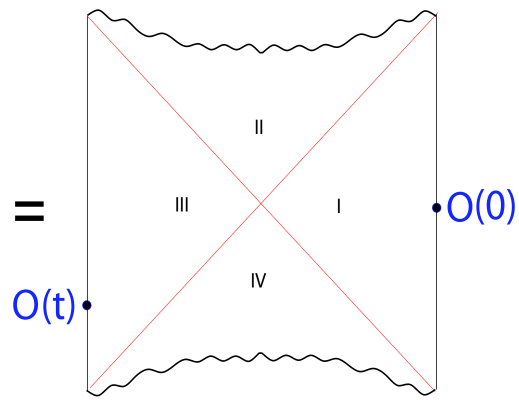
It is convenient to consider the Fourier transform $G_+(\omega, l)$.

ω: frequency,

l: angular momentum on S^3.

One can also consider (spatial coordinates suppressed)

$$G_{12}(t) = G_{+}(t - i\frac{\beta}{2}) =$$



Note

$$G_{12}(\omega, l) = e^{-\frac{\beta\omega}{2}} G_{+}(\omega, l)$$

Boundary correlators from gravity

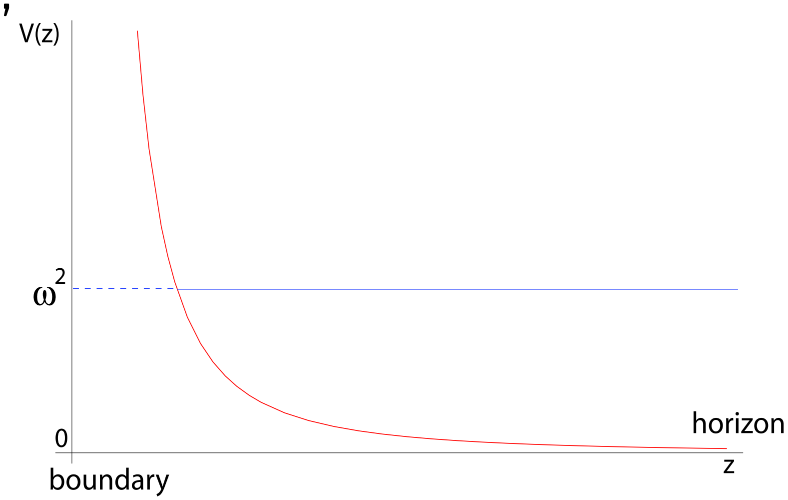
$G_+(\omega, l)$ can be obtained via AdS/CFT by solving the Laplace equation for the bulk scalar field ϕ, which in momentum space becomes the Schrodinger equation

$$\left(-\partial_z^2 + V_l(z)\right)\phi_{\omega l}(z) = \omega^2 \phi_{\omega l}(z)$$

z: tortoise coordinate.

$\phi_{\omega l}$ (z): Fourier component of ϕ.

For l not too large,



We consider normalizable modes $\phi_{\omega l}$

$$\phi_{\omega l}(z) \approx e^{-i\omega z - i\delta_\omega} + e^{i\omega z + i\delta_\omega}, \qquad z \to +\infty \ .$$

This determines

$$\phi_{\omega l}(z) \approx C(\omega, l) z^{\frac{1}{2}+\nu} + \cdots , \qquad z \to 0 \ .$$

Then

$$G_+(\omega, l) = \frac{(2\nu)^2}{2\omega} \frac{e^{\beta\omega}}{e^{\beta\omega} - 1} C^2(\omega, l)$$

For $d \geq 4$, the bulk Laplace equation cannot be solved exactly.

Various methods can be used to find $G_+(\omega, l)$ approximately.

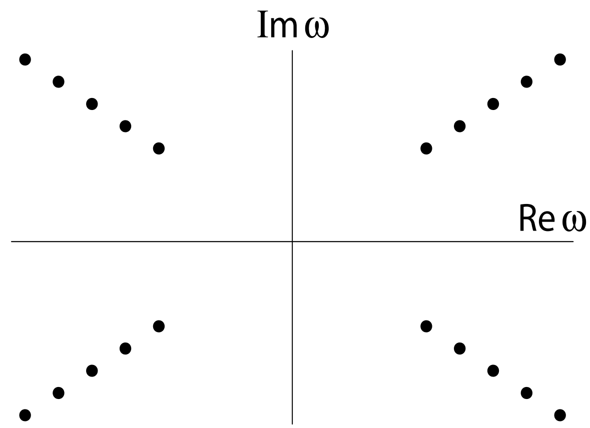
Nunez and Starinets; Cardoso, Natario and Schiappa; Siopsis

Liu and Festuccia

Of special interests is the analytic behavior of $G_+(\omega, l)$ in the complex ω-plane for fixed l.

Analytic properties of G_+

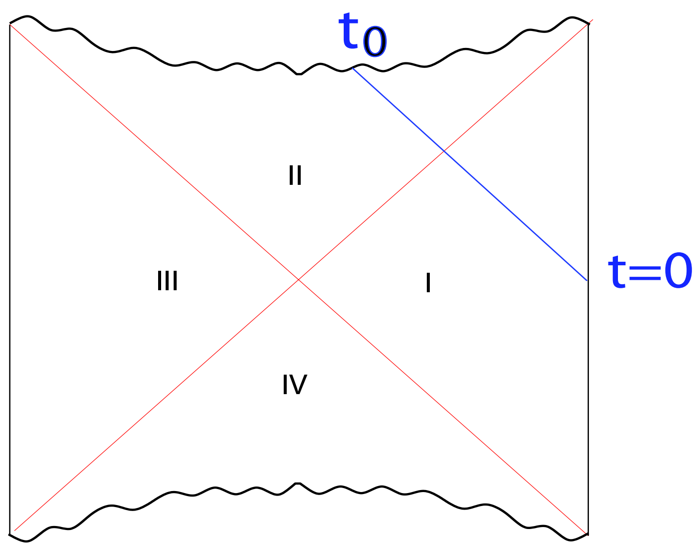
For l not too large,



pole spacing : $\delta \sim \frac{1}{\mathcal{B}}, \qquad \mathcal{B} = \tilde{\beta} + i\beta$

$$e^{-\frac{1}{2}\beta\omega} G_+(\omega, l) \sim e^{-\frac{1}{2}\beta|\omega|}, \qquad \omega \to \infty \text{ (near } \textit{real} \text{ axis)}$$

$$e^{-\frac{1}{2}\beta\omega} G_+(\omega, l) \sim e^{-\frac{1}{2}\tilde{\beta}|\omega|}, \qquad \omega \to \infty \text{ (near } \textit{imaginary} \text{ axis)}$$



$$4t_0 = \mathcal{B} = \tilde{\beta} + i\beta$$

Hard to extract information about the bulk geometry directly from $G_+(\omega, l)$, like a *quantum inverse scattering* problem.

We would like to know whether the presence of the bulk singularity is reflected in $G_+(\omega, l)$, and if yes, how.

We will be able to reduce the problem to a *classical inverse scattering* problem.

Large operator dimension limit

To make connection with the bulk geometry, consider the limit

$$\omega = \nu u, \qquad l = \nu k, \qquad \nu \gg 1, \qquad (\nu = \sqrt{m^2 + 4})$$

Then G_+ can be expanded in the large ν limit as

$$G_+(\nu u, \nu k) \approx 2\nu \, e^{\nu Z(u,k)} \, (1 + \cdots) + \cdots$$

$Z(u,k)$ and higher order terms of the expansion can be worked out explicitly from the Schrodinger equation.

In the large ν limit, the mass of the corresponding bulk particle is large and its propagation should approximately *follow geodesics*.

Relation with bulk geodesics I

Since the bulk geometry has Killing vectors along t and S^3 directions, a bulk geodesic can be characterized by integrals of motion (E, q).

We find that $Z(u, k)$ can be identified as the Legendre transform of the geodesic distance of a bulk spacelike geodesic with

$$u = iE, \qquad k = iq$$

Note: $\omega = \nu u$, $l = \nu k$.

For each complex pair (ω, l)

$$G_+(\omega, l) \quad \to \quad Z(u, k) \quad \to$$

$$\to \quad \text{complex bulk geodesic with } E = -iu, \;\; q = -ik$$

The geodesic starts and ends at the (complexified) boundary and has a turning point $r_c(E, q)$.

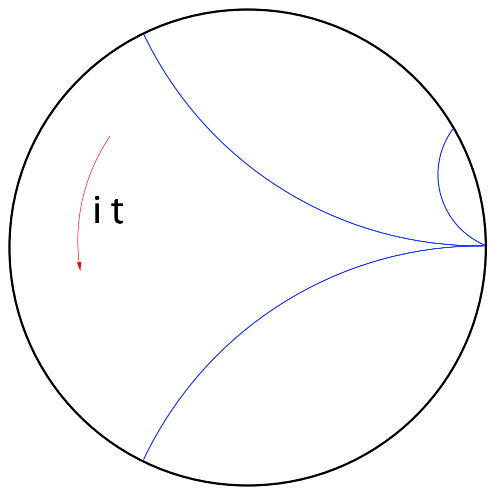
Thus we have a mapping between the complexified momentum space and complexified bulk geometry

$$(\omega, l) \to r_c(\omega, l)$$

We will now look at some examples with $k = 0$.

Relation with geodesics II

For real ω, the geodesic lies in the Euclidean section of the complexified spacetime

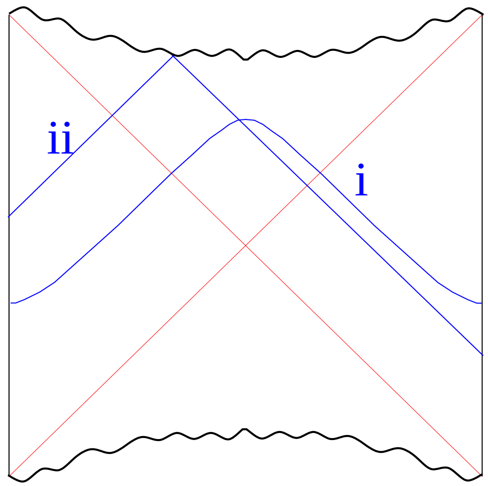


For real $\omega \to \pm\infty$, the geodesic approaches the boundary

$$r_c \propto |\omega|$$

UV/IR connection

For ω pure imaginary, the geodesic probes the region inside the horizon.



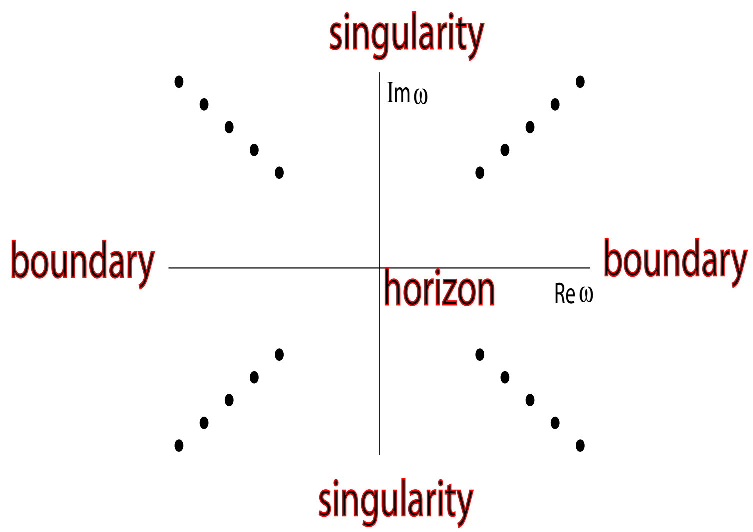
As $|\omega| \to \infty$, the geodesic *approaches the singularity:*

$$r_c \propto \frac{1}{|\omega|}$$

Time inside the horizon is dynamically generated.

Summary

We thus find



The lines of poles of $G_+(\omega, l)$ "create" new asymptotic regions in the complex-ω plane corresponding to the regions around the singularity.

Signals of the singularity

To probe the singularity, we consider $\omega \to \pm i\infty$

- $G_+(\omega)$ decays exponentially (for any ν)

$$e^{-\frac{1}{2}\beta\omega} G_+(\omega, l) \sim \omega^{2\nu} e^{-\frac{1}{2}\tilde{\beta}|\omega|}$$

- In the large ν limit ($\omega = i\nu E$, $G_+ \sim e^{\nu Z}$)

$$\lim_{E\to\infty} \left. \frac{d^{2n} Z(iE, k)}{dk^{2n}} \right|_{k=0} \sim E^{2n-2} \to \infty, \qquad n > 1$$

This leads to divergences when Fourier transformed (along certain contours) to coordinate space correlation functions.

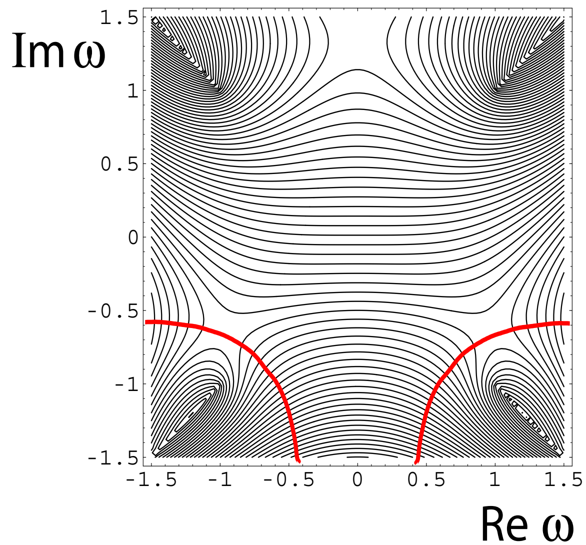
Coordinate space correlation functions

$$G_{12}(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, e^{-i\omega t} \, G_{12}(\omega)$$
$$\sim \int du \, e^{-i\nu u t - \frac{1}{2}\nu u \beta} \, e^{\nu Z(u,k)}$$

- The Fourier integral can be evaluated using the saddle point approximation in the large ν limit.

- Bulk geodesics with end point separation given by t appear as saddle points of the Fourier integral.

- Fourier integrals give a precise prescription for summing over geodesics.



Contour plot of the imaginary part of the exponent of the integrand.

The saddle on the imaginary axis corresponds to a geodesic passing inside the horizon.

The geodesic which goes inside the horizon does not contribute to the correlation function in the saddle point approximation.

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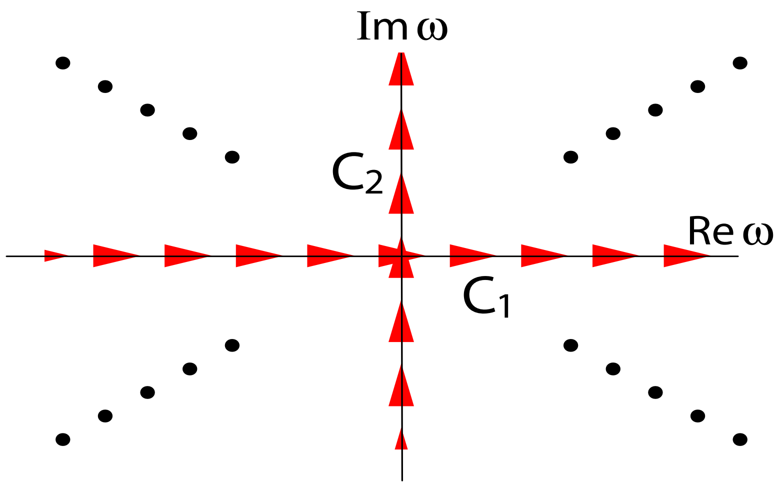
New gauge invariant observables and signals of the singularity (II)

New observables:

$$H_{12}(\tau) = \int_{C_2} \frac{d\omega}{2\pi} e^{-i\omega\tau} G_{12}(\omega)$$



The divergence of $H_{12}(\tau)$ for $\tau \to \pm\frac{\tilde{\beta}}{2}$ reflects the divergence of a spacelike geodesic approaching the singularity.

Signal amplification process

The geodesic which goes inside the horizon does *not* contribute to coordinate space correlation function $G_+(t)$ in the saddle point approximation.

However, through the process

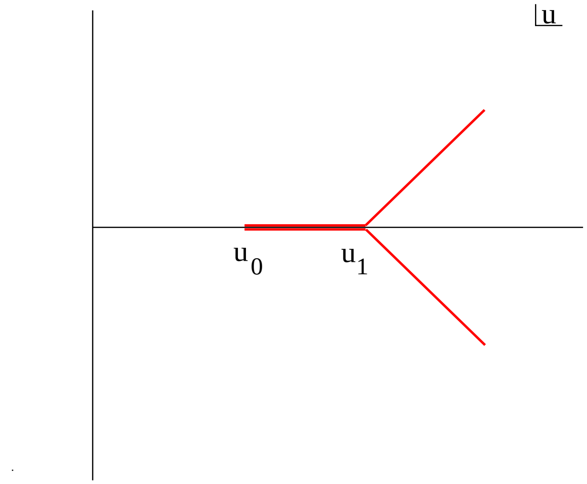
$$G_+(t) \quad \rightarrow \quad G_+(\omega) \quad \rightarrow \quad G_{12}(\omega) \quad \rightarrow \quad H_{12}(\tau)$$

the signal of the singularity is amplified.

Recall $G_{12}(\omega, l) = e^{-\frac{\beta\omega}{2}} G_+(\omega, l)$.

A curious property of YM theory on S^3

For sufficiently large $l > l_c$ ($l_c \sim T^4$ in large T limit),



the boundary theory contains very long-lived quasi-particles which almost never thermalize.

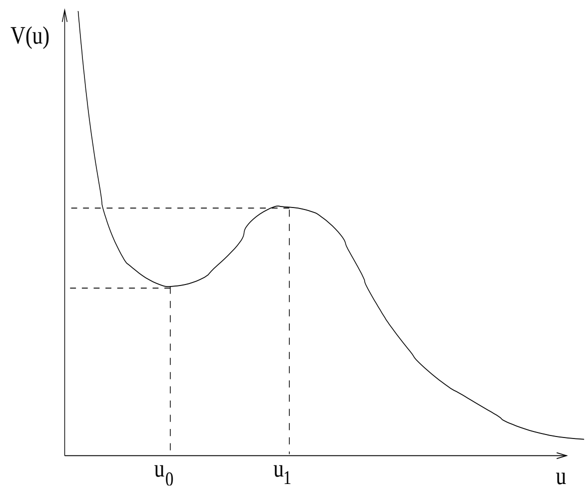
As $T \to \infty$, the angular momenta of these quasi-particles go to infinity much faster than T.

In the large T limit, the sphere decompactifies and the theory goes over to that of flat space.

Thus they are *not* present in the flat space limit of the boundary theory.

A simple explanation from classical gravity

When $l > l_c$, one finds that the potential becomes



When the angular momentum on S^3 is large enough, there exist stable classical time-like orbit which never falls into the black hole.

Yang-Mills theory at finite N ?

At finite N, the YM theory on S^3 has a *discrete* spectrum, i.e. *finite* number of states below any given energy.

This should be true even for coupling of order $O(1)$.

In particular

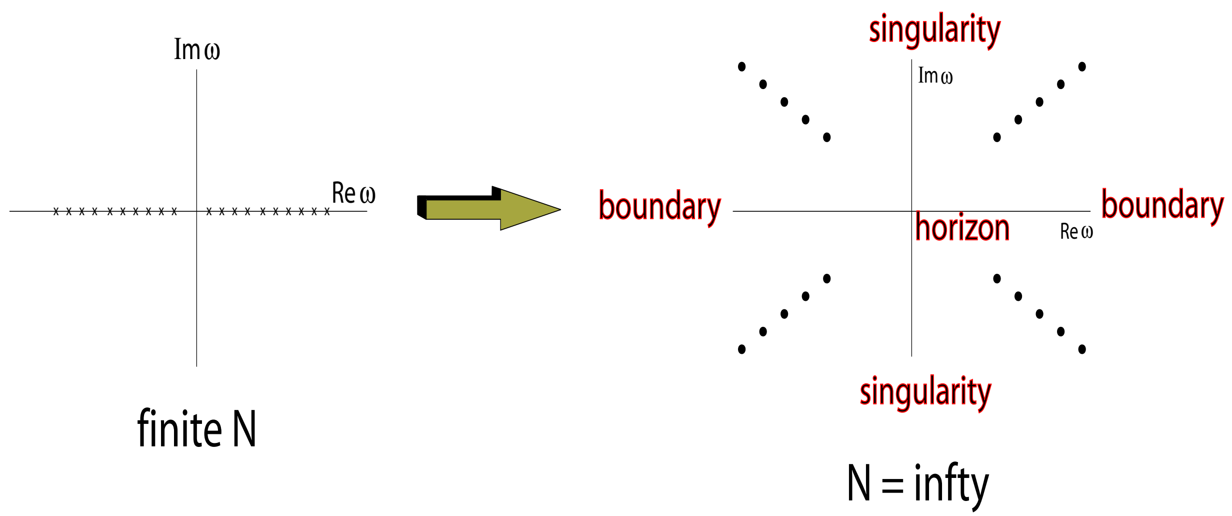
$$G_+(\omega) = 2\pi \sum_{m,n} e^{-\beta E_m} \rho_{mn} \delta(\omega - E_n + E_m)$$

m, n sum over the physical states of the theory.

The bulk analysis shows that in the *large* N limit the boundary YM theory has a continuous spectrum despite being on S^3.

The appearance of a continuous spectrum is tied to the presence of the horizon in the bulk.

Resolution of the singularity at finite N ?



The discrete spectrum at finite N appears to be *in conflict* with the presence of the horizon.

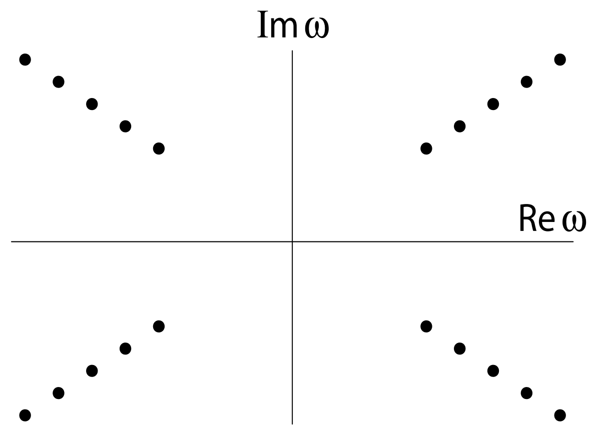
This suggest the horizon and the singularity are *approximate* concepts valid *only in large N expansion*.

Future directions (I)

Understand whether the analytic behaviors we observed following from supergravity analysis (i.e. corresponding to strong coupling in SYM) exist at weak coupling.

If yes, this will provide strong evidence that singularity may persist at finite α'.

It will also enable us to study the singularity at weak coupling by focusing on the large N effect.



$$e^{-\frac{1}{2}\beta\omega} G_+(\omega, l) \sim e^{-\frac{1}{2}\beta|\omega|}, \qquad \omega \to \infty \ \text{(near \textit{real} axis)}$$

$$e^{-\frac{1}{2}\beta\omega} G_+(\omega, l) \sim e^{-\frac{1}{2}\tilde{\beta}|\omega|}, \qquad \omega \to \infty \ \text{(near \textit{imaginary} axis)}$$

Future directions (II)

The fact that the geometry inside the horizon appears to be encoded in Yang-Mills theories along the imaginary axis suggests that there might exist an alternative description of AdS black holes in which the regions inside the horizon are manifestly represented.