Axions in String Theory

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To the QCD action, it is possible to add a CP-violating interaction

$$I_\theta = \frac{\theta}{8\pi^2}\int \mathrm{tr} F \wedge F.$$

Since, the instanton number $\frac{1}{8\pi^2}\mathrm{tr} F \wedge F$ is an integer, θ is a periodic variable with period 2π. Experimentally, from limits on EDM of neutron and mercury,

$$|\theta| \lesssim 10^{-10}.$$

The standard model already violates CP in the weak sector, hence the extreme smallness of the theta angle poses a naturalness problem: *the strong CP problem.*

In the literature, there have been three proposals for solving the strong CP problem:

- vanishing of the up quark mass $m_u = 0$.
- axion, a light scalar particle makes θ dynamical and allows it to relax to $\theta = 0$.
- spontaneous breaking of underlying CP symmetry while preserving $\theta = 0$.

If $m_u = 0$, then a chiral rotation of the up quark $\psi_u \to e^{-i\gamma_5 \theta}\psi_u$ sets $\theta = 0$ hence, it θ does not have physical effects. This seems to be at odds with the current algebra result

$$\frac{m_u}{m_d} \sim \frac{1}{1.8}.$$

However, m_u has the same quantum numbers as $\frac{m_d m_s}{\Lambda_{QCD}}$ so m_u could be generated by strong QCD dynamics.

Recent lattice gauge theory computations disfavor $m_u = 0$ (MILC).

The second possibility is that the underlying theory is CP invariant, so that the bare θ_{bare} vanishes. The CP symmetry is then broken spontaneously. The theta angle receives contribution from the determinant of the quark mass matrix

$$\theta = \theta_{bare} + \det(M),$$

so one has to break CP in such way that M is complex but its determinant is real and positive. This can be done in a technically natural way (Nelson, Barr).

In this talk we will concentrate on the third solution to strong CP problem: axions.

One starts with an global $U(1)$ symmetry, under which some quarks are charged. This symmetry is assumed to have a $SU(3)_c \times SU(3)_c \times U(1)_{PQ}$ anomaly. If the Peccei-Quinn symmetry is unbroken, some of the quarks must be massless so it leads to the $m_u = 0$ solution of the strong CP problem.

If it is broken, the pseudo-Goldstone boson is the PQ axion.

Axion is a light scalar with a Peccei-Quinn symmetry

$$a \to a + \delta,$$

that is broken by QCD instantons. It couples to QCD instanton density $\mathrm{tr}F \wedge F$ because the gauge anomaly. Its action is

$$\mathcal{S} = \int -\frac{F_a^2}{2} da \wedge \star da + \frac{ra}{8\pi^2} \mathrm{tr}F \wedge F,$$

where we normalized a to have period 2π and r is an integer. F_a is the axion decay constant or the axion coupling parameter. Setting $\tilde{a} = aF_a$ to get canonical kinetic term

$$\mathcal{S} = \int -\frac{1}{2} d\tilde{a} \wedge \star d\tilde{a} + \frac{r\tilde{a}}{8\pi^2 F_a} \mathrm{tr}F \wedge F,$$

shows that axion couples with strength $1/F_a$.

With the axion coupling to QCD instanton number, the QCD theta angle becomes dynamical

$$\bar{\theta} = \theta + ra.$$

The axion gets a nontrivial potential of height $\sim m_\pi^2 F_\pi^2$ from QCD instantons. This potential has minimum at $\theta = 0$. (Vafa-Witten)

The axion rolls down to the minimum of the potential at $\theta = 0$, solving the strong CP problem.

The axion's couplings to matter are proportional to $1/F_a$. Hence, if F_a is too small, axion couples too strongly and too many axions are produced in various astrophysical processes, which leads to i.e. fast cooling of red giants...

$$F_a \geq 10^9 \mathrm{GeV}.$$

Cosmological considerations put upper bound on F_a. The minimum of the axion potential depends on low energy data, i.e. the phases of the fermion masses, that are unimportant in the early universe, so one would expect that the axion starts at a random value different from $\theta = 0$.

Once the Hubble scale is comparable to axion's mass, the axion starts oscillating and its density decreases as $1/a(t)^3$. For

$$F_a > 10^{12}\text{GeV},$$

the axion mass $m_a \sim m_\pi F_\pi / F_a$ is too small, and the axion would not have enough time to dilute its density.

In the early days of string theory, it has been noted, that the weakly coupled heterotic string theory leads to axions with $F_a \approx 10^{16-18}\mathrm{GeV}$, which is above the standard cosmological bound.

In a forthcoming paper with E. Witten, we revisit the question to what extent string theory compactifications can lead to value of F_a that is compatible with cosmological bounds. We compute F_a in a wide range of models, in order to get an overall picture.

We find that in many models, including most GUT-like models, F_a is in the range from $10^{16}\mathrm{GeV}$ to $10^{18}\mathrm{GeV}$. We also point out some modes, in which axions satisfy the cosmological bound.

Let's review the situation in weakly coupled $E_8 \times E_8$ heterotic string theory.

The B-field polarized along the four noncompact dimensions is dual to the model independent axion

$$dB = \star da.$$

The SUGRA terms in the action

$$\int (dB + \omega(A))^2,$$

where $\omega(A) = \mathrm{tr} A dA + A^3$ gives rise to the $\int a \; \mathrm{tr} F \wedge F$ coupling. The axion has axion decay constant

$$F_a = 1.1 \times 10^{16} \mathrm{GeV}.$$

The model dependent axions come from the B-field polarized within the compactification manifold

$$B = \sum_i a_i \, \omega_i \qquad i = 1, \ldots, b_2(X),$$

where ω_i are harmonic representatives of $H^2(X)$. The kinetic term for the axion comes the kinetic term of the three form $H = dB$

$$S_{kin} = -\frac{1}{2} \int H \wedge \star H = - \int_X \omega \wedge * \omega \int d^4x \, \frac{1}{2} \partial_\mu a \partial^\mu a.$$

The axion coupling comes from the Green-Schwarz anomaly cancelation term

$$\int B \wedge (\alpha \mathrm{tr} F^4 + \beta \mathrm{tr} F^2 \wedge \mathrm{tr} F^2 + \ldots).$$

These axions have F_a around $10^{16-18}\mathrm{GeV}$.

In summary, the weakly coupled heterotic string theory has $b_2(X) + 1$ axions, all of which violate the cosmological bound $F_a < 10^{12}\text{GeV}$.

The Peccei-Quinn symmetry is valid perturbatively: the coupling of B field to strings

$$\int_\Sigma B$$

vanishes, since in perturbation theory $\Sigma = \partial\Omega$.

Nonperturbatively, the PQ symmetry of model dependent axions is violated by worldsheet instantons with strength

$$\exp(-S) = \exp(-A_\Sigma).$$

The model independent axion couples other strongly coupled gauge groups that violate the PQ symmetry besides QCD.

Supressing these additional violations of PQ symmetry leads to significant constraints on the models.

Taking heterotic string theory to strong coupling leads to M-theory compactified on a CY times an interval, $X \times I$, with matter fields living on an E_8 boundary.

Here, one has a model independent axion, coming from dualizing $C_{\mu\nu 10}$ and $b_2(X)$ model dependent axions from $C = a\ \omega_Q \wedge dx^{10}$.

One lowers F_a by taking the length of the interval I large. However, for large I one runs into a strong coupling singularity on the hidden E_8 boundary…hence, this model does *not* lower F_a significantly.

If one abandons GUT unification, one can achieve lower F_a by compactifying string theory on manifolds with QCD fields supported on a 'vanishing cycle'.

In this geometry, F_a scales differently from M_p. Taking the compactification manifold large, lowers F_a relative to M_p.

Example: take the heterotic M-theory model with matter fields living on the 'smaller' of the two E_8 boundaries.

Another example: M-theory on a G_2 manifold, with QCD fields living on an ADE singularity along a small 3-cycle Q in X.

The axion comes from $C = a\omega_Q$ a harmonic 3-form dual to Q. At an ADE singularity, there is a coupling

$$2\pi \int_{R^{3,1}\times Q} C \wedge \frac{\mathrm{tr} F \wedge F}{8\pi^2}$$

that leads to the standard axionic coupling.

The size of Q is determined by the strength of QCD coupling to be $\sim few\ \ell_{11}$. To get low F_a one takes a large manifold X.

Another example: Intersecting D-brane models with QCD living on $D(3+q)$ branes wrapping a q-cycle Q, with $q > 0$. The axion comes from zero mode of the RR q-form gauge field $C_q = a\omega_Q$.

One can achieve small F_a in GUT-like unified models with anomalous $U(1)$ gauge symmetries. These $U(1)$ symmetries have QCD anomaly canceled by the Green-Schwarz mechanism.

If the $U(1)$ gauge symmetry gets Higgsed by eating a 'would be axion', a global $U(1)_{PQ}$ symmetry survives below string scale. If this symmetry is spontaneously broken at a lower scale, one gets an axion with a lower F_a.

Example: compactifications of $E_8 \times E_8$ heterotic string theory with a poly-stable gauge bundle: the low energy gauge group often contains anomalous $U(1)$'s. The axion comes from a phase of a E_8 gauge field whose VEV breaks the $U(1)$.

For this model to work, the explicit breaking of $U(1)$ by world-sheet instantons must be subdominant.

As discussed above, the PQ symmetries of string axions are violated, besides QCD by

- gauge instantons
- gravitational instantons
- worldsheet instantons
- D-brane instantons

The strong CP problem gets solved if QCD effects are dominant for at least one of the axions.

Instantons with action S generate potential

$$V(a) \sim M^4 e^{-S}(1 - \cos(a + \phi)),$$

where ϕ is an $\mathcal{O}(1)$ phase mismatch between the instantons and QCD instantons. One needs $S \geq 200$ to suppress $V(a)$, which puts a strong constraint on models.

In models with low scale SUSY breaking, the instantons generate a superpotential $W \sim M^3 \exp(-S)$ which leads to a potential $V = |DW|^2$ with scale

$$V(a) \sim m_{SUSY}^2 M^2 \exp(-S)$$

from interference of the one-instanton term with a SUSY-breaking term $DW/D\Phi \sim m_{SUSY}^2$. This relaxes the constraint on S. Extra fermion zero modes on the instanton worldvolume can further suppress the instanton contribution.

There have been several proposals made for evading the cosmological bound $F_a < 10^{12}\text{GeV}$

- anthropic explanations
- late entropy production due to particle decay
- arranging for QCD to be more strongly coupled in the early universe
- low scale inflation

The axion can have additional couplings to matter. The PQ symmetry $a \to a + constant$ allows it to have arbitrary derivative couplings to quarks and leptons.

In addition to the axion coupling to QCD instanton density, the axion may have similar coupling to two photons

$$a \ \mathrm{tr} F_{em} \tilde{F}_{em} \propto a \ E \cdot B$$

which is the basis searches for axions.

This coupling is not implied by the similar coupling of axion to QCD instanton density, however it is present in many models. In particular in $SU(5)$ GUT models, there is a precise prediction for its value (Srednicki, Kaplan).

In summary, a large class of string theory compactifications predict value of F_a above the range allowed by cosmological arguments.

Whether or not axion satisfies the cosmological bound, finding it in the range, $10^9\mathrm{GeV} < F_a < 10^{18}\mathrm{GeV}$, which includes the range $10^{16}\mathrm{GeV} < F_a < 10^{18}\mathrm{GeV}$ predicted by many string theory models would clarify the situation.