Complex/Symplectic Mirrors

Alessandro Tomasiello

KITP, Santa Barbara,
Oct. 12, 2005

Mainly based on
hep-th/0510042 [Chuang, Kachru, AT]
with review of results from
hep-th/0505212 [Graña, Minasian, Petrini, AT]
hep-th/0311122 [Fidanza, Minasian, AT]

Introduction

"Mirror symmetry exchanges complex and ~~Kahler~~ moduli"
symplectic

complex $(K = 0)$

$\exists\, \Omega \mid \Omega \wedge \bar{\Omega}$ nowhere zero; $d\Omega = 0$

(complex three-form)

almost complex $(c_1 = 0)$

$\exists\, \Omega \mid \Omega \wedge \bar{\Omega}$ nowhere zero

symplectic

$\exists\, J \mid J \wedge J \wedge J$ nowhere zero; $dJ = 0$

(real two-form)

almost symplectic

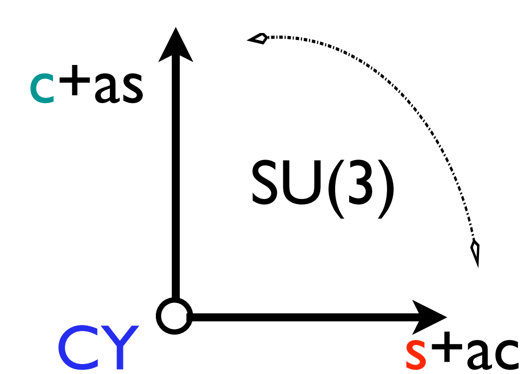
$\exists\, J \mid J \wedge J \wedge J$ nowhere zero

almost complex + almost symplectic : SU(3) structure

complex + symplectic : CY

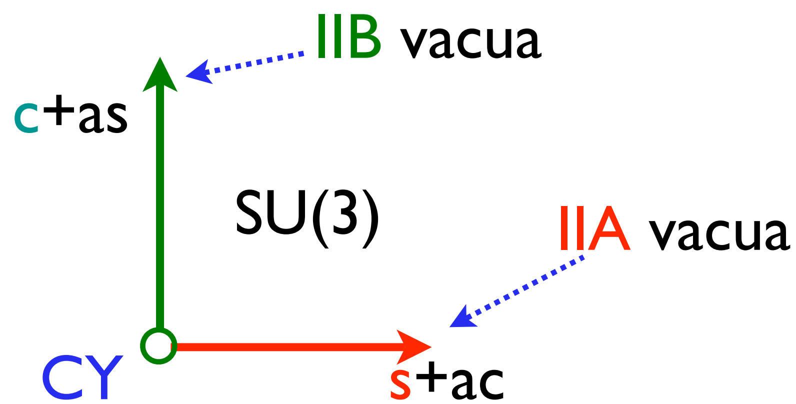
[with some compatibility]

Might it be almost symplectic + complex $\longleftrightarrow$ almost complex + symplectic ?



Evidence so far:

- $\mathcal{N} = 1$ RR vacua



IIA $\quad de^{iJ} = 0 \qquad d\Omega = F$

IIB $\quad d\Omega = 0 \qquad de^{iJ} = F$

$$e^{iJ} = 1 + iJ - \frac{1}{2} J \wedge J - \frac{i}{6} J \wedge J \wedge J$$

- Direct T-duality computations

e.g. "$e^{iJ} \longleftrightarrow \Omega$" $\qquad (\nabla J + H)_{ijk} \longleftrightarrow (\nabla J - H)_{i\bar{j}\bar{k}}$

Both results have direct interpretation in terms of

Generalized complex geometry

- In particular, both symplectic and complex particular cases of the same condition:

both Ω and e^{iJ} are pure spinors for $T \oplus T^*$

(tangent + cotangent)

We want to produce examples.

- Method: transitions

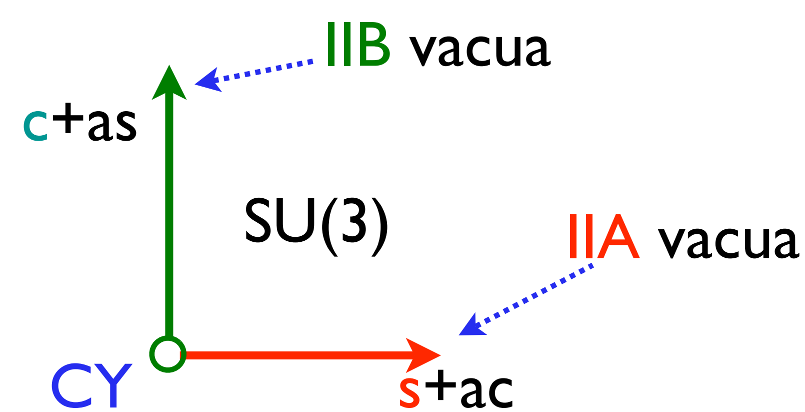
IIA → symplectic (+almost complex)

(CY:) complex + symplectic

IIB → complex (+almost symplectic)

what allows them to happen in string theory is RR flux on the CY

- Resulting vacua are not from 10d supergravity; but still



- They come by construction in mirror pairs

Plan

- Review evidence
  - New vacua
    - Their geometrical interpretation; consequences for the general picture

Review of previous evidence

- preserved $\mathcal{N}=1$ (RR $\neq 0$):

[Graña, Minasian, Petrini,AT]

IIA

$$(d+H\wedge)(e^{2A-\phi}\Phi_+)=0$$

$$(d+H\wedge)(e^{2A-\phi}\Phi_-)=dA\wedge\Phi_-^* +(a^2-b^2)e^{\phi}F-i(a^2+b^2)e^{\phi}*F$$

IIB

$$(d+H\wedge)(e^{2A-\phi}\Phi_+)=dA\wedge\Phi_+^* +(a^2-b^2)e^{\phi}F-i(a^2+b^2)e^{\phi}*F$$

$$(d+H\wedge)(e^{2A-\phi}\Phi_-)=0$$

For now $\Phi_+=e^{iJ}$, $\Phi_-=\Omega$

A warping
ϕ dilaton
a, b normalizations

$$\Phi_\pm^\dagger\wedge\Phi_\pm=ab\,\mathrm{vol}$$
$$a^2-b^2=c\,e^{-A}$$
$$a^2+b^2=c'\,e^{A}$$

The two equations are exchanged by

Mirror map:

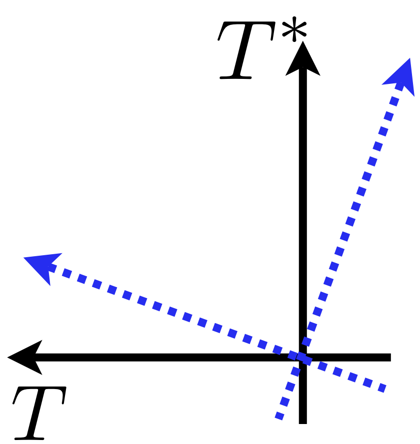
IIA		IIB
Φ_+	$\rightarrow$	$i\Phi_-$
Φ_-	$\rightarrow$	$-i\Phi_+$
F	$\rightarrow$	iF

Consequences: (IIA) $(d + H\wedge)e^{iJ} = 0 \;\Rightarrow\; dJ = 0$

(IIB) $(d + H\wedge)\Omega = 0 \;\Rightarrow\; d\Omega = 0$

SU(3) on T $\Longrightarrow$ SU(3) on T^* $\Longrightarrow$ SU(3)xSU(3) on $T \oplus T^*$

can happen more generally



Correspondingly: Φ_+ more general than e^{iJ}

Φ_- more general than Ω

(Example: $\Phi_+ = e^{ij} \wedge (v + iw)$ 4d +2d mix)

For general Φs: Generalized complex geometry

[Hitchin, Gualtieri, Witt...]

- same mathematical properties (pure spinor on $T \oplus T^*$)
- supersymmetry equations still valid!

- **T-duality:**

[Fidanza,Minasian,AT; Gurrieri, louis,Micu,Waldram]

  - assume SLag T^3 fibration $\longrightarrow J, \ \Omega$
  - dualize the torus $\longrightarrow \tilde{J}, \ \tilde{\Omega}$
  - $(dJ, d\Omega) \longleftrightarrow (d\tilde{J}, d\tilde{\Omega})$

(not obvious a priori: $J \overset{?}{\leftrightarrow} \Omega$)

(intrinsic torsions) $\begin{matrix} \Omega \llcorner dJ \\ (dJ)^{2,1}_{\rm prim} \\ \ldots \end{matrix} \longrightarrow ?$

Results are actually best summarized using e^{iJ} and Ω

"$e^{iJ} \longleftrightarrow \Omega$"

$T \oplus T^* \circlearrowleft$

Compare: $\Omega \leftrightarrow e^{B+iJ}\sqrt{Td}$ for branes on Calabi-Yau's

$$\begin{matrix} & & & \Phi_+ & & & \\ & & * & & * & & \\ & * & & * & & * & \\ \Phi_-^* & & * & & * & & \Phi_- \\ & * & & * & & * & \\ & & * & & * & & \\ & & & \Phi_+^* & & & \end{matrix}$$

expand $(d + H\wedge)\Phi_\pm$ in a "pure Hodge diamond":

Pairs (Φ_+, Φ_-) make mirror symmetry more manifest.

More covariant intrinsic torsion:

$$\langle \Phi_+, (d+H\wedge)\Phi_- \rangle$$
$$\langle (dx^m \wedge + J^{mn}\iota_n)\Phi_-, (d+H\wedge)\Phi_- \rangle$$
$$\ldots$$

Action of T-duality **rotates the diamond**

Intermezzo: CY transitions

[Candelas,Green,Hubsch...]

IIB: At a point $p \in \mathcal{M}_{\rm cpl}$

- N three-cycles B_a shrink
- they satisfy R relations in homology

$h^{2,1}$ vector multiplets

$h^{1,1}+1$ hypermultiplets

- N new massless hypers B_a
- charged under vectors $\int_{B_a} C_4$
- but the charge matrix has kernel of dim. R

"F-term"

$$\sum_a Q^a{}_b B_a^\dagger \sigma^\alpha B_a = 0$$

New vacua, $\mathcal{N}=2$, $B_a \neq 0$

What is their 10d interpretation?

On the new branch:

- Higgs mechanism: lose $N-R$ vectors
- gain N hypers; lose $N-R$

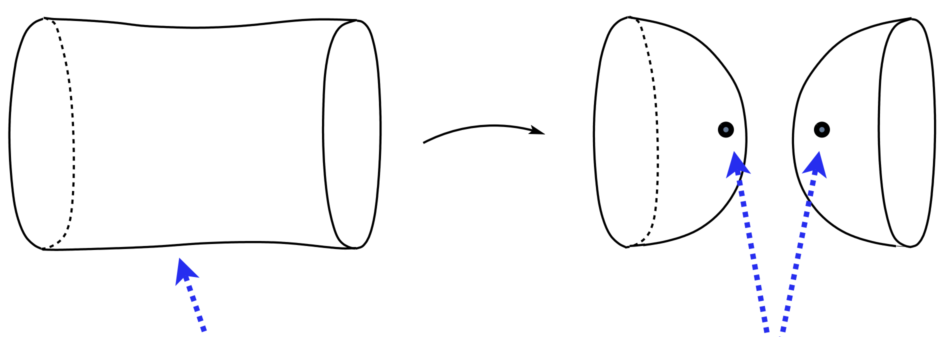
$h^{2,1}-N+R$ vector multiplets

$h^{1,1}+R$ hypermultiplets

Proposal: transitions (topologically: "surgery")

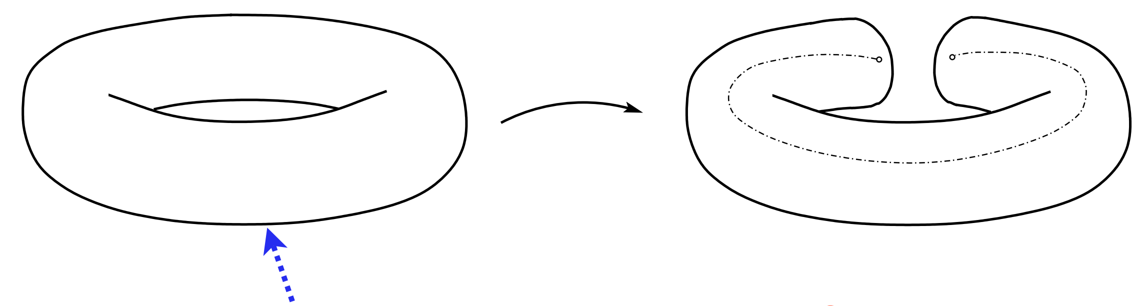
replace the three-cycles with two-cycles $M_6 \to \tilde{M}_6$

noncompact case:



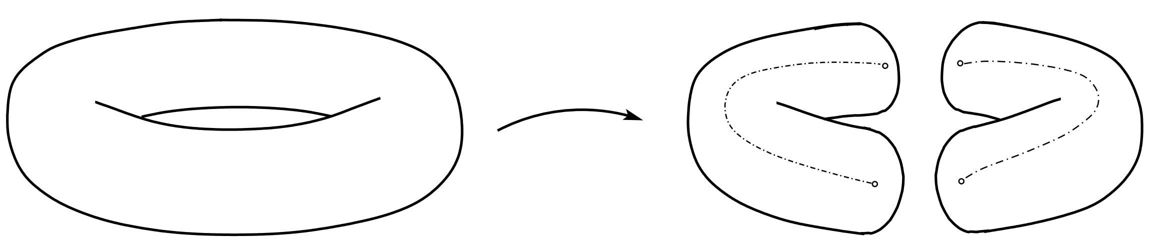
non-trivial S^3 non-trivial S^2

compact case (no relations):



non-trivial S^3 S^2 is trivial

compact case (with relations):



two dependent S^3 shrink

one new non-trivial S^2

In general:

$$\tilde{b}_2 = b_2 + R$$
$$\frac{\tilde{b}_3}{2} = \frac{b_3}{2} - N + R$$

- agrees with the multiplets

- Q: If M_6 is CY; is $\tilde{M}_6$ CY too?

  A: when $R \neq 0$ [Werner]

New branch:
IIB on $\tilde{M}_6$

Flux gives new vacua (IIB)

Let us now suppose no relations. To fix ideas:

$B^\dagger \sigma^\alpha B = 0$: no branch with $B \neq 0$

Only the cycle A_1 shrinks

But switch on $F_3 \mid \int_{B_1} F_3 = n^1$ ⟶ contribution to the potential; vacuum only when A_1 shrinks

At that point: new hyper B

this time $B^\dagger \sigma^\alpha B = e^{2\phi} n^1 \delta^\alpha{}_3$ ⟶ $B = \binom{\sqrt{e^{2\phi} n^1}}{0}$

New (Higgs) branch!

[Polchinski,Strominger]

what happens to the multiplets?

- vectors: one becomes massive (A_1 shrinks)
- hypers: gain B; it + universal ⟶ one massless, one massive

$h^{2,1}(\mathrm{CY}) - 1$	vector multiplets
$h^{1,1}(\mathrm{CY}) + 1$	hypermultiplets

+ one vector and one hyper have paired up and become massive

Interpretation of the vacua

[Chuang,Kachru,AT]

- Counting of massless states consistent with the topological counting

$$\tilde{b}_2 = b_2 + R$$
$$\frac{\tilde{b}_3}{2} = \frac{b_3}{2} - N + R$$

for $R = 0$

- reasonable: going to the new branch only affects CY close to shrinking A_1

This last point is not automatical for IIA

Example: with $F_6 \longrightarrow$ the whole (quantum) volume of M_6 shrinks

"Localized" cases: $\exists p \in \mathcal{M}_{spl}$ in which (e.g.) only one curve shrinks;

switch on F_4 and drive the CY to that point.

(example where p exists: elliptic fibration over $\mathbb{F}_1$)

This time we will have

$$\tilde{b}_2 = b_2 - N + R$$
$$\tilde{b}_3 = b_3 + R$$

ok for $R = 0$

So the new vacua should come from $\tilde{M}_6$

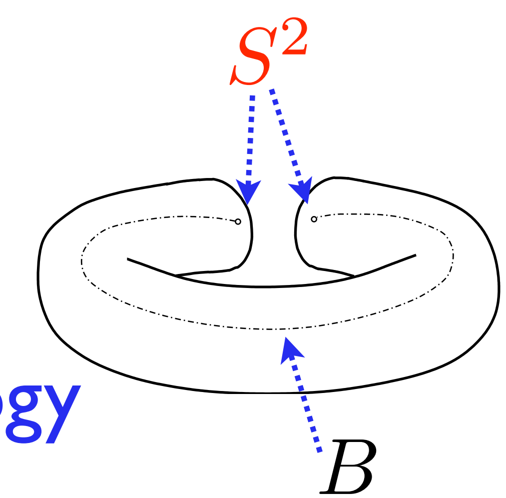
whose topology is given by surgery

Q: what about their differential-geometric properties?
they cannot be CY. What else?

A: More generally than for CY

	M_6 complex $\Rightarrow \tilde{M}_6$ complex	M_6 symplectic $\Rightarrow \tilde{M}_6$ symplectic
$S^3 \to S^2$	yes	if $R > 0$
$S^2 \to S^3$	if $R > 0$	yes

[Smith, Thomas, Yau]
[Freedman; Tian]
[Werner]



Why? first case: S^2 is holomorphic but trivial in homology

$$\int_B dJ = \int_{S^2} J = \mathrm{vol}(S^2) \neq 0$$

For us,
M_6 is CY
(complex+ symplectic) $\longrightarrow$

IIB: complex (almost symplectic)

IIA: symplectic (almost complex)

Can we check this picture?

Try: find the fields which got a mass by Higgs directly on $\tilde{M}_6$

- KK for 10d supergravity on $\tilde{M}_6$. Should it work?
  - Actually, these vacua cannot be coming from 10d sugra (it is impossible without negative sources)
  - computation with brane hyper B is valid when S^2 is small

Let us compare anyway. Idea:

- On both sides we have an $\mathcal{N}=2$ gauging, due to (IIB):
  - on M_6, to F_3
  - on $\tilde{M}_6$, to dJ

$\mathcal{N}=2$ 4d supergravity "moment map" $\mathcal{P}^\alpha$ → gaugings, potential

Rigid limit, $\mathcal{N}=1$ subalgebra → $\mathcal{P} \sim \mathcal{W}$ superpotential

On M_6

$$\mathcal{P}^3 = B^\dagger \sigma^3 B + \int_{M_6} F_3 \wedge \Omega$$

can expand using harmonic forms

On $\tilde{M}_6$

$$\mathcal{P}^3 = \int_{\tilde{M}_6} dJ \wedge \Omega$$

uh?

Prescription: [Gurrieri,Louis,Micu,Waldram; Grana,Louis,Waldram]

- (J,Ω) determine a metric $\quad (g_{i\bar{j}} = -iJ_{i\bar{j}})$
- use it to find eigenforms $\quad \Delta\omega = m^2\omega$
- $[d,\Delta]=0 \;\Rightarrow\; d$ acts on a given mass level as a finite matrix

In our case: there should be a mass level determined by S^2

$$d\omega_2 = m\omega_3$$

massive

Write $\begin{aligned} J &= t_{\rm mass}\,\omega_2 + J_{\rm harm} \\ \Omega &= X_{\rm mass}\,\omega_3 + \Omega_{\rm harm} \end{aligned}$ and use in $\int dJ\Omega$

Q: $\int dJ\Omega = -\int J d\Omega = 0$?? remember: complex ($K=0$) has $d\Omega = 0$

A: in fact, $d\Omega = 0$ only on the vacuum!

on M_6 $\quad X^1 \leftrightarrow X_{\rm mass}$ $\quad$ on $\tilde{M}_6$

$\int_{A_1} \Omega$ $\qquad\qquad$ $\Omega = X_{\rm mass}\,\omega_3 + \Omega_{\rm harm}$

vacuum at $X^1 = 0$:

A_1 cycle shrinks

vacuum at $X_{\rm mass} = 0$:

$d\Omega = 0$

massive hyper:

$J = t_{\rm mass}\omega_2 + J_{\rm harm}$ $\longleftrightarrow$ massive combination of g_s and B

brane hyper

- This method
  - required guessing properties of spectrum of Δ
  - obscures the expected integrality of the gaugings

Is there a more "cohomological" understanding of dJ and $\int dJ \wedge \Omega$?

- We propose one should think of dJ as $\in H^3(\tilde{M}_6, S^2)$ relative cohomology

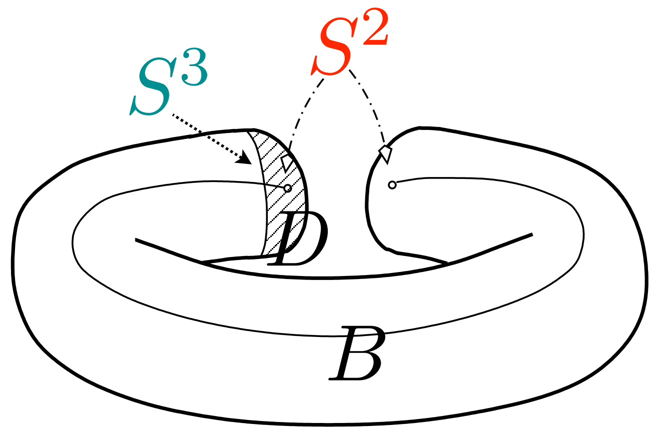
$\int_B dJ = \mathrm{vol}(S^2)$ $\qquad (B, S^2) \in H_3(\tilde{M}_6, S^2)$ relative homology

fixed by being a holomorphic curve

- also, $d\Omega \neq 0$ should be dual to a pair $(S^3, D) \in H_4(\tilde{M}_6, S^3)$

- a "linking number" between S^3 and S^2 would be the gauge charge

($\int dJ \wedge \Omega$ looks like a gen. Chern-Simons)



- Reid's fantasy:

- many 19-dim. moduli spaces of algebraic K3's; later recognized as $\subset$ 20-dim. moduli space of K3

something similar for three-folds?

- does string theory realize a version of this for complex and symplectic manifolds?

the massive fields we discussed lead us off-shell

Conclusions

- String theory has vacua on complex or symplectic manifolds
  - They follow patterns suggested by supergravity
    - Mirror symmetry still holds