GUTs in Type IIB Orientifold Compactifications

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based on

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Introduction and Motivation

Particle phenomenology in Type II string theories

Realistic gauge theories and matter interactions from Type IIB intersecting 7-branes?

example: $SU(5)$ Georgi-Glashow GUT and its susy and higher-dim. extensions

- Standard model particles fit nicely into $SU(5)$ representations:

$$\mathbf{24} \rightarrow (\mathbf{8},\mathbf{1})_{0_Y} + (\mathbf{1},\mathbf{3})_{0_Y} + (\mathbf{1},\mathbf{1})_{0_Y} + (\mathbf{3},\mathbf{2})_{5_Y} + (\overline{\mathbf{3}},\mathbf{2})_{-5_Y}$$

$\Rightarrow$ gauge fields

$$\mathbf{10} \rightarrow (\mathbf{3},\mathbf{2})_{1_Y} + (\overline{\mathbf{3}},\mathbf{1})_{-4_Y} + (\mathbf{1},\mathbf{1})_{6_Y}$$

$$\overline{\mathbf{5}} \rightarrow (\overline{\mathbf{3}},\mathbf{1})_{2_Y} + (\mathbf{1},\mathbf{2})_{-3_Y}$$

$$\mathbf{1}_N \rightarrow (1,1)_{0_Y}$$

$\Rightarrow$ quarks, leptons and neutrinos

$$\mathbf{5}_H \rightarrow (\mathbf{3},\mathbf{1})_{-2_Y} + (\mathbf{1},\mathbf{2})_{3_Y} \qquad \overline{\mathbf{5}}_H \rightarrow (\overline{\mathbf{3}},\mathbf{1})_{2_Y} + (\mathbf{1},\mathbf{2})_{-3_Y}$$

$\Rightarrow$ Higgs doublet

- gauge coupling unification is natural at the GUT scale

GUT models and Compactification

- Recently, there has been much progress in realizing GUT models in local F-theory constructions on intersect. 7-branes Beasley,Heckman,Vafa; Donagi,Wijnholt Heckman,Marsano,Saulina,Schäfer-Nameki,Vafa; Marsano,Saulina,Schäfer-Nameki

  ⇒ F-theory treats Type IIB string backgrounds with a varying dilaton

  ⇒ strong coupling enhancements of gauge groups to exceptional groups

  ⇒ various new insights for models for which gravity can be decoupled (for example new mechanism to break GUT group)

  However:

  New GUT breaking in local F-theory models requires knowledge about global geometry.

  Cannot address global constraints. Restrictive?

  Cannot address moduli stabilization in local set-ups. Value of couplings?

- Construction of compact scenarios with all the desired properties is more challenging:

  ⇒ general F-theory background: construction of viable compact Calabi-Yau fourfolds

  ⇒ new consistency conditions (such as tadpole cancellation)

In this talk:

- work in the weak coupling regime but compact set-ups

GUTs in Type IIB Calabi-Yau orientifolds with intersecting D7 branes

1. Building Models in Type IIB orientifolds
   - D7 branes with gauge flux
   - Consistency conditions

2. $SU(5)$ GUTs and their breaking
   - Georgi-Glashow $SU(5)$ GUT
   - Hypercharge flux

3. Concrete compact GUT models
   - GUTs on del Pezzo transitions of the Quintic $\mathbb{P}_{1,1,1,1,1}[5]$
   - GUTs on del Pezzo transitions of $\mathbb{P}_{1,1,1,6,9}[18]$

Building Models in Type IIB orientifolds

➪ Calabi-Yau Orientifolds: Calabi-Yau space Y + orientifold involution

- orientifolds with $O3$ / $O7$ planes: $\Omega_p\,(-1)^{F_L}\,\sigma$
  (Ω_p world-sheet parity, σ is holomorphic isometry)

- orientifold involution σ: splits $H^p(Y) = H^p_+(Y) \oplus H^p_-(Y)$

  $\Rightarrow$ bulk spectrum (Kähler deformation sector)

$$e^{-B} \wedge (e^{-\phi}\text{Re}(e^{iJ}) + i(C_0 + C_2 + C_4)) = \tau\, 1_+ + G^i \omega_i^- + T_I \tilde{\omega}_+^I$$

R-R axions $\quad H^0_+ \quad H^2_- \quad H^4_+$

  $\Rightarrow$ bulk Kähler potential (large volume): Giddings,Kachru,Polchinski; TG,Louis

$$K(\tau + \bar{\tau}, G + \bar{G}, T + \bar{T}) = -2 \log \left[e^{-2\phi} \int_Y J \wedge J \wedge J \right]$$

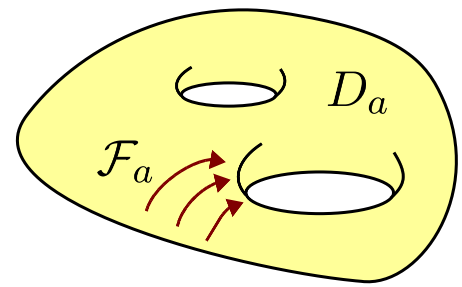
  $\Rightarrow$ K is independent of R-R axions

- axions might become gauged in the presence of D7-branes:
  D-term: $D = X^{T_I} \partial_{T_I} K + X^{G^i} \partial_{G^i} K$

➪ D7 branes with gauge bundles: Calabi-Yau manifold Y + D-branes

- stack of N_a space-time filling D7 branes wrapped on susy four-cycle $\iota : D_a \hookrightarrow Y$
  $\Rightarrow U(N_a)$ gauge group, preserve $\mathcal{N} = 1$ susy on world-volume

- D7-branes can carry a gauge flux bundle $\mathcal{F}_a$
  $\Rightarrow$ restrict to $\mathcal{F}_a$ of rank one: line bundles



$$\mathcal{F}_a = \mathbf{1}_{N_a}(F_a^{(0)} + \iota^* B) + \textstyle\sum_i \mathbf{T}_i F_a^{(i)} \qquad (\mathrm{tr}(\mathbf{T}_i) = 0)$$

line bundles are uniquely determined by their first Chern class:

$$c_1(L_a^{(0)}) = \frac{1}{2\pi}(F_a^{(0)} + \iota^* B) \in H^2(D_a) \qquad c_1(L_a^{(i)}) = \frac{1}{2\pi} F_a^{(i)} \in H^2(D_a)$$

– $L_a^{(0)}$ induces split $U(N_a) \to SU(N_a) \times U(1)_a$

– $L_a^{(i)}$ can break $SU(N_a)$ further: split of $U(1)$ factors

➪ D- and F-terms form gauge bundles on D7 branes:

- gauge-flux $\mathcal{F}_a$ might induce a gauging of bulk scalars G^i and T_I: Jockers,Louis

$$\text{D-term} \quad \propto \quad \int_{D_a} \iota^* J \wedge (F_a^{(0)} + \iota^* B) \qquad\qquad (J \text{ is Kähler form on } Y)$$

However: $H^2(D_a)$ can have elements which are non-trivial or trivial in $H^2(Y)$

$\Rightarrow$ non-trivial parts of L_a:

$\Rightarrow$ massive $U(1)$ via Green-Schwarz mechanism

$\Rightarrow$ trivial parts of L_a: do not couple to bulk scalars (at large volume)

$\Rightarrow$ massless $U(1)$

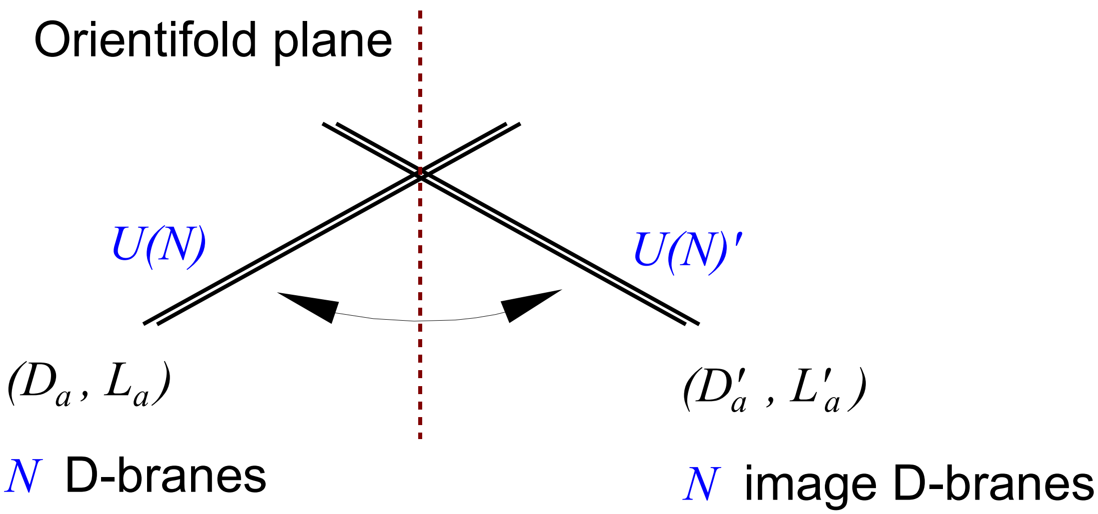
- D7-brane superpotential Witten

$$W = \int_{\mathcal{C}_5} F_a^{(0)} \wedge \Omega \qquad\qquad D_a \subset \partial \mathcal{C}_5$$

- obtained e.g. from Witten's holomorphic Chern-Simons action
- dimensional reduction keeping non-dynamical three-forms TG,Ha,Klemm,Klevers

➪ Orientifold planes and D-branes:

- orientifold involution σ maps D-brane to image D-brane:
  line bundles $\mathcal{F}'_a = -\sigma^* \mathcal{F}_a$



➪ Tadpole cancellation: vanishing of all induced tadpoles in the compact Y/σ

- D7-tadpole:

$$\sum_a N_a\left([D_a]+[D'_a]\right)=8\,[D_{\rm O7}]$$

- D5-tadpole: induced D5-charge due to non-trivial line-bundle on D7-brane

$$\forall\omega\in H^2_-(Y):\qquad \sum_a N_a\int_Y \omega\wedge\Big(\,[D_a]\wedge\mathrm{tr}(\mathcal{F}_a)+[D'_a]\wedge\mathrm{tr}(\mathcal{F}'_a)\,\Big)=0$$

- D3-tadpole:

$$\frac{\chi(CY_4)}{12}=(N_{\rm D3}+N_{\rm D3'})+N_{\rm flux}-\sum_a\frac{N_a}{4\pi^2}\Big(\int_{D_a}\mathrm{tr}(\mathcal{F}_a^2)+\int_{D'_a}\mathrm{tr}(\mathcal{F}'^2_a)\Big)$$

$\chi(CY_4)$: - O3-charge

- gravitational D3-charges $\propto\chi(D)$ of D7 and O7

Remarks on Tadpole cancelation:

- also $c_1(L_a)$ on trivial cycles in Y will contribute to D3 tadpole
- discrete B-field flux in $H^2_+(Y)$ contributes tadpole: $c_1(L) \to c_1(L) + B_+$

Additional constraints:

- Freed-Witten anomaly: quantization condition on $F_a = \mathcal{F}_a - \mathbf{1}_{N_a} \cdot \iota^* B$

$$\frac{1}{2\pi}[F_a]_{ij} + \delta_{ij}\frac{1}{2}c_1(K_{D_a}) \;\in\; H_2(D_a, \mathbb{Z}) \quad \Rightarrow \quad L_a^{(i)} \text{ can be fract. quantized}$$

- K-theory constraints (generalized charge quantization)

Supersymmetry constraints:

Becker[2],Strominger; Marino,Minasian,Moore,Strominger

D-term constraints restricts values of Kähler form and B-field

$$\text{D-terms} \quad \propto \quad \int_{D_a} \iota^* J \wedge (F^{(0)} + \iota^* B_2) = 0$$

$\Rightarrow$ needs to be satisfied inside the Kähler cone

➪ Spectrum from intersecting D7-brane

- adjoint matter from D7 branes:

  $h^{2,0}(D_a)$ deformations of the brane

  $h^{1,0}(D_a)$ Wilson line moduli

  $\Rightarrow$ both absent for special four-cycles such as del Pezzo surfaces

- chiral matter from intersections (full matter content using Ext-groups)

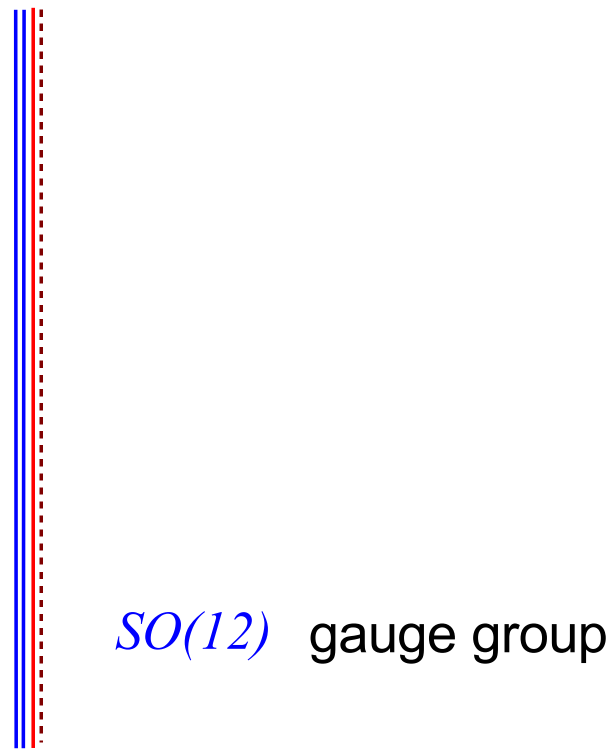
Representation	Multiplicity	
$(\overline{N}_a, N_b)$	I_{ab}	bifundamental reps.
(N_a, N_b)	$I_{a'b}$	
A_a	$\frac{1}{2}(I_{a'a} + 2I_{\mathrm{O7a}})$	anti-symmetric reps.
S_a	$\frac{1}{2}(I_{a'a} - 2I_{\mathrm{O7a}})$	symmetric reps.

example: $$I_{ab} = -\int_X [D_a] \wedge [D_b] \wedge (\, c_1(L_a) - c_1(L_b)\,)$$

$SU(5)$ GUTs and their breaking

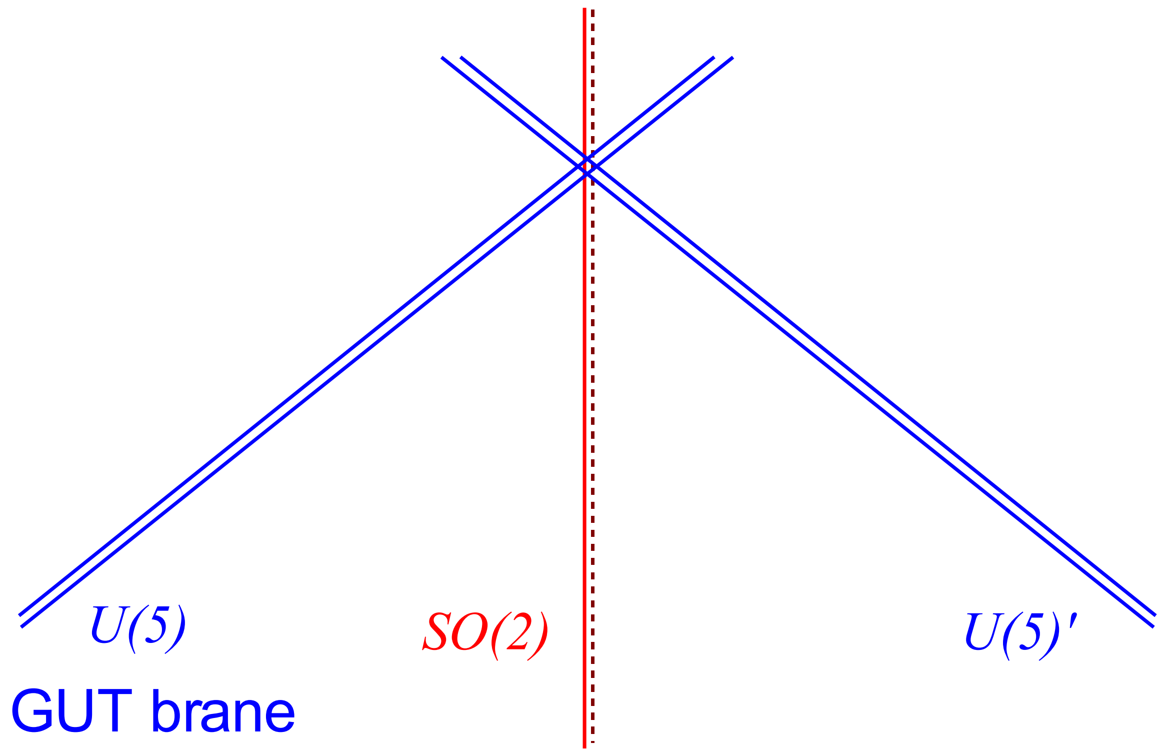
➪ Schematics of a GUT model from D7 branes (1):

- Start with 12 D7 branes on top of O7 plane



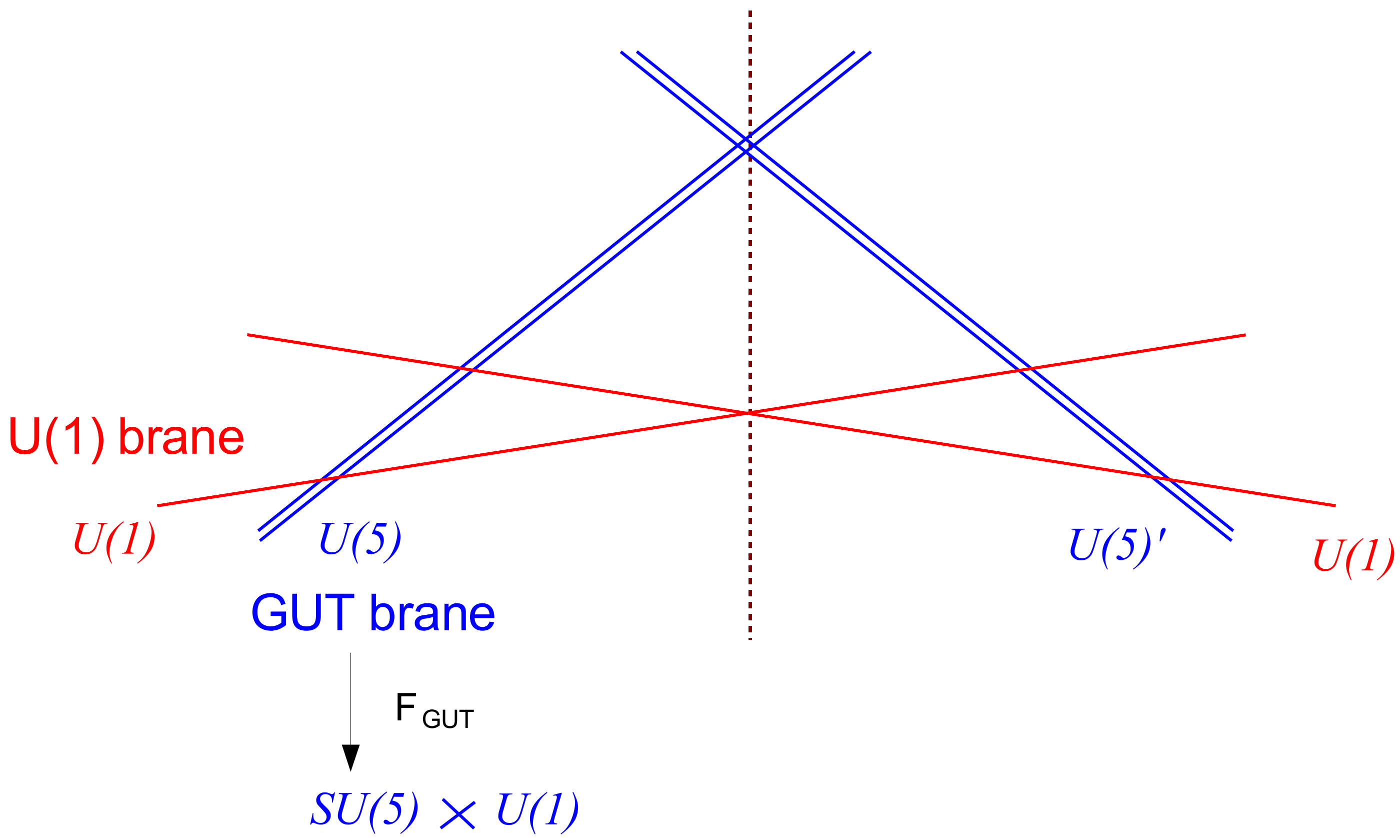
➪ Schematics of a GUT model from D7 branes (2):

- Move 5 D7 branes and their images off the O7 plane



➪ Schematics of a GUT model from D7 branes (3):

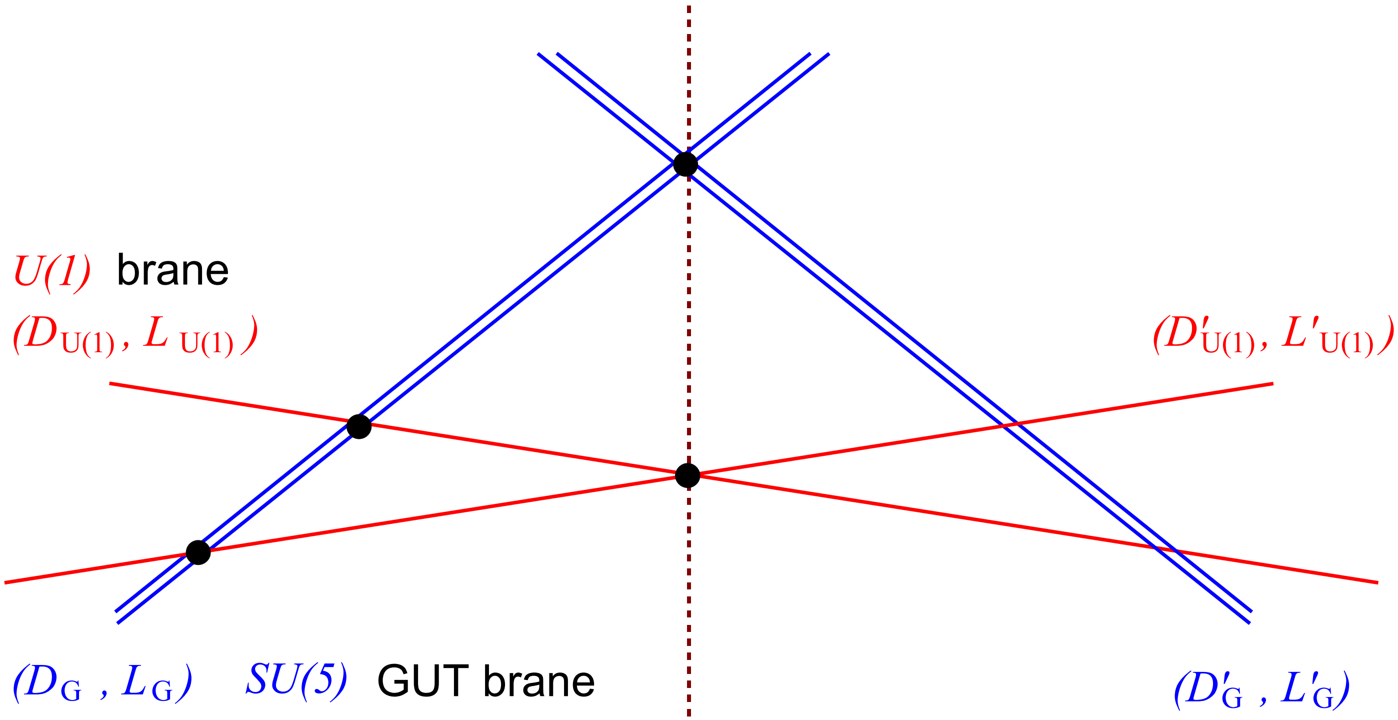
- Move 1 D7 branes and its image off the O7 plane



- adjoint of $SO(12)$ splits under $SU(5) \times U(1)_a \times U(1)_b$ and yields the needed GUT representations:

$$\mathbf{66} = \mathbf{24}^{(0,0)} + \mathbf{1}^{(0,0)} + \mathbf{10}^{(2,0)} + \overline{\mathbf{10}}^{(-2,0)} + \mathbf{5_H}^{(1,-1)} + \overline{\mathbf{5}}_{\mathbf{H}}{}^{(-1,1)} + \mathbf{5}^{(1,1)} + \overline{\mathbf{5}}^{(-1,-1)}$$

➪ Schematics of a GUT model from D7 branes (4):



$\mathbf{10}$	3	$D_G \cap D'_G$
$\overline{\mathbf{5}}$	3	$D_G \cap D'_{U(1)}$
$\mathbf{1}_N$	3	$D_{U(1)} \cap D'_{U(1)}$
$\mathbf{5}_H + \overline{\mathbf{5}}_H$	$1+1$	$D_G \cap D_{U(1)}$

➪ Hypercharge and GUT breaking:

$$U(5) \;\rightarrow\; SU(5)\times U(1) \;\rightarrow\; SU(3)\times SU(2)\times U(1)_Y \times U(1)$$

- F-theory with GUT 7-brane: Beasley,Heckman,Vafa; Donagi,Wijnholt

  use L_Y with $c_1(L_Y)\in H^2(D_{\rm G})$ trivial in $H^2(Y)$ to break GUT to MSSM

  $\Rightarrow$ no scalar to render the $U(1)_Y$ massive (no direct het. analog ?)

  hypercharge flux for MSSM: Buican,Malyshev,Morrison,Verlinde,Wijnholt

- Freed-Witten anomaly: L_Y and L have mixed embedding for non-spin divisors

  (relevant for branes on del Pezzo surfaces)

$$F_{\rm GUT} = \mathbf{1}_{N_a}\left(F^{4D} + c_1(L) + \frac{2}{5}c_1(L_Y) + \frac{1}{2}c_1(K_{D_a})\right) + T_Y\left(F_Y^{4D} + \frac{1}{5}c_1(L_Y)\right)$$

with $T_Y = \text{diag}(-2,-2,-2,3,3)$ $\qquad c_1(L)\in\frac{1}{2}\mathbb{Z}$ $\qquad c_1(L_Y)\in\mathbb{Z}$

$\Rightarrow$ factor $\frac{1}{5}$ ensures that $H^*(D_a, L_Y)$ can be zero (no vector-like exotics)

Blumenhagen,Braun,TG,Weigand

- Compact models: TG,Klemm; Blumenhagen,Braun,TG,Weigand
  - need to check that not all elements of $H^2(D_{\rm G})$ are non-trivial in Y (hypercharge), but that additional $U(1)$'s become massive
  - check by computing BPS numbers (GV invariants) for the curve classes
  - simple compact examples are obtained by generic four-cycle transitions

➪ non-perturbative generation of missing Yukawa couplings $\mathbf{10}^{(2,0)}\ \mathbf{10}^{(2,0)}\ \mathbf{5_H}^{(1,-1)}$

Blumenhagen,Cvetic,Lüst,Richter,Weigand

– main obstacle to get fully realistic models at weak coupling

– in compact models one can check for the presence of 4-cycles which can support the appropriate D3-brane instanton ($I_{\rm GUT,inst} = 1,\ I_{\rm U(1),inst} = -1$) in the Kähler cone

$\Rightarrow$ non-pert. $\mathbf{10\ 10\ 5_H}$ scale $\propto \exp(-{\rm Vol}_{\rm inst}/g_s)$

Concrete compact GUT models

➪ D7 branes on del Pezzo surfaces

- Why del Pezzo surfaces?
  - local F-theory constructions: Beasley, Heckman, Vafa; Donagi, Wijnholt

    del Pezzos are shrinkable $\Rightarrow$ decoupling of gravity

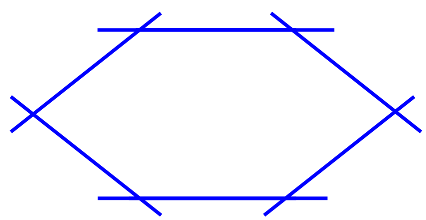
    leaves 10 choices: $\mathbb{P}^1 \times \mathbb{P}^1$, $\mathbb{P}^2$ blown up at $n = 0, \ldots, 8$ points
  - compact models: del Pezzo $\rightarrow$ $h^{2,0} = h^{1,0} = 0$ $\rightarrow$ no adjoint matter

    also allow dP_9 or even more blow-ups (not shrinkable)

- geometry of del Pezzo surfaces is intimately linked to representations of groups
  - $h^{1,1}(dP_n) = 1 + n$ anti-canonical class $-K$

    n simple roots of $A_2 \oplus A_1, D_4, D_5, E_6, E_7, E_8$
  - geometry of lines on del Pezzos (genus 0, degree 1 curves)

dP_3 example:

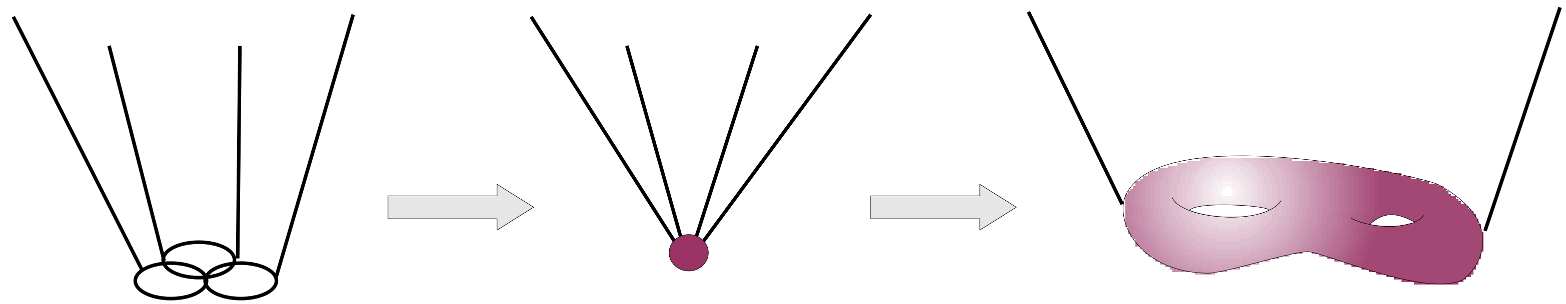


6 intersecting lines

dP_6 example: 27 intersecting lines

dP_7 example: 56 intersecting lines

➪ Del Pezzo surfaces in a compact Calabi-Yau: del Pezzo transitions



Simple Examples:

- E_8 del Pezzo transition starting with $\mathbb{P}_{1,1,1,6,9}[18]$ Morrison, Vafa

  $h^{(1,1)} = 2,\ h^{(2,1)} = 272 \quad \longrightarrow \quad h^{(1,1)} = 3,\ h^{(2,1)} = 243$

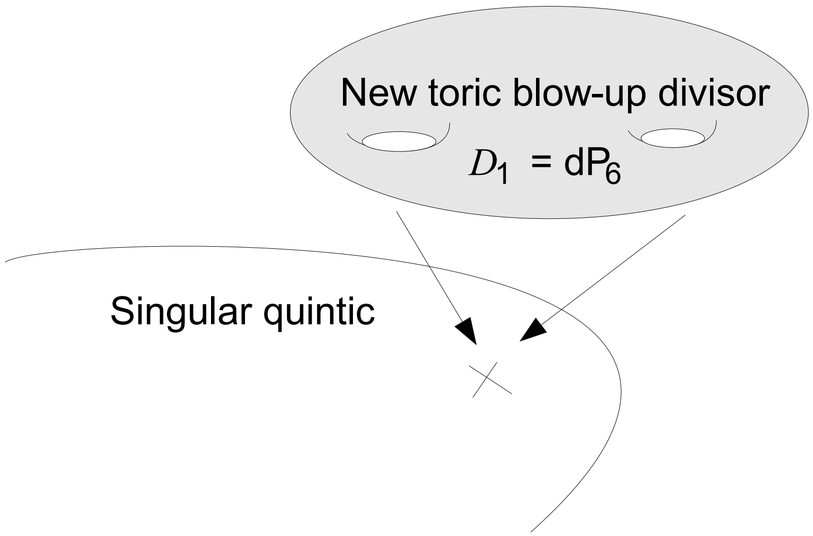
- E_6 del Pezzo transition starting with quintic hypersurface

  $h^{(1,1)} = 1,\ h^{(2,1)} = 101 \quad \longrightarrow \quad h^{(1,1)} = 2,\ h^{(2,1)} = 90$

$\Rightarrow$ there exist whole chains of del Pezzo transitions realized in toric geometry

➪ 1 Transitions of the Quintic hypersurface

- begin with quintic in $\mathbb{P}^4$, i.e. $\mathbb{P}_{1,1,1,1,1}[5]$, perform del Pezzo transitions torically:



add point $\nu_1 = (0,0,0,1)$ to toric data of $\mathbb{P}^4$

$$\chi(D_1) = \int_{D_1} c_2(D_1) = 9$$

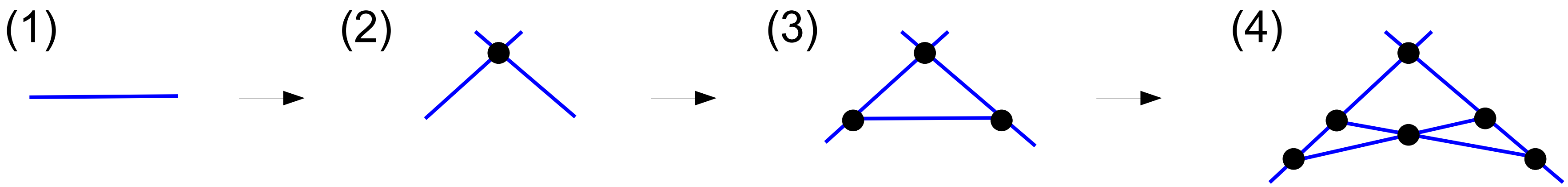
$$K^2 = \int_{D_1} c_1^2(D_1) = 3$$

How many classes of dP_6 are non-trivial in new CY?

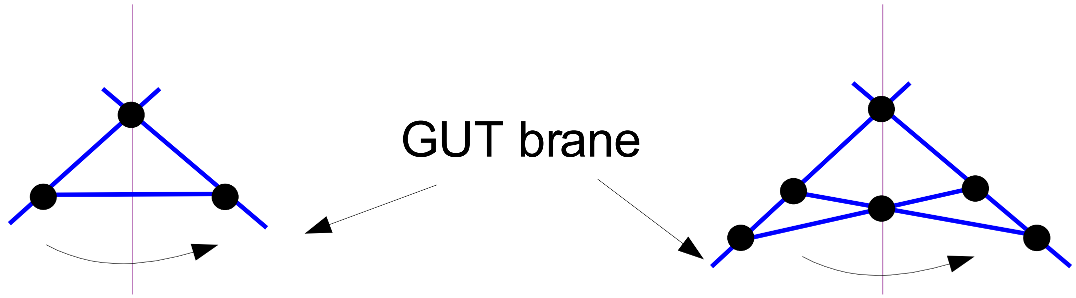
- check that $\Delta h^{1,1}_{CY} = 1$ (only divisor class)
- check that $\Delta \chi_{CY} = 24 = 2 \times C_{E_6}$
- count curves in homology class of globally non-trivial cycle (computation of BPS/GV invariants)
  $\Rightarrow$ $n = 27$ - number of degree one curves in dP_6
  $\Rightarrow$ whole E_6 lattice is trivial in new CY

➪ continue to perform del Pezzo transitions torically (add further divisors)

Transition (1):	generate one generic dP_6
Transition (2):	generate two intersecting dP_7 (intersecting in $\mathbb{P}^1$)
Transition (3):	generate three intersecting dP_8
Transition (4):	generate four intersecting dP_9

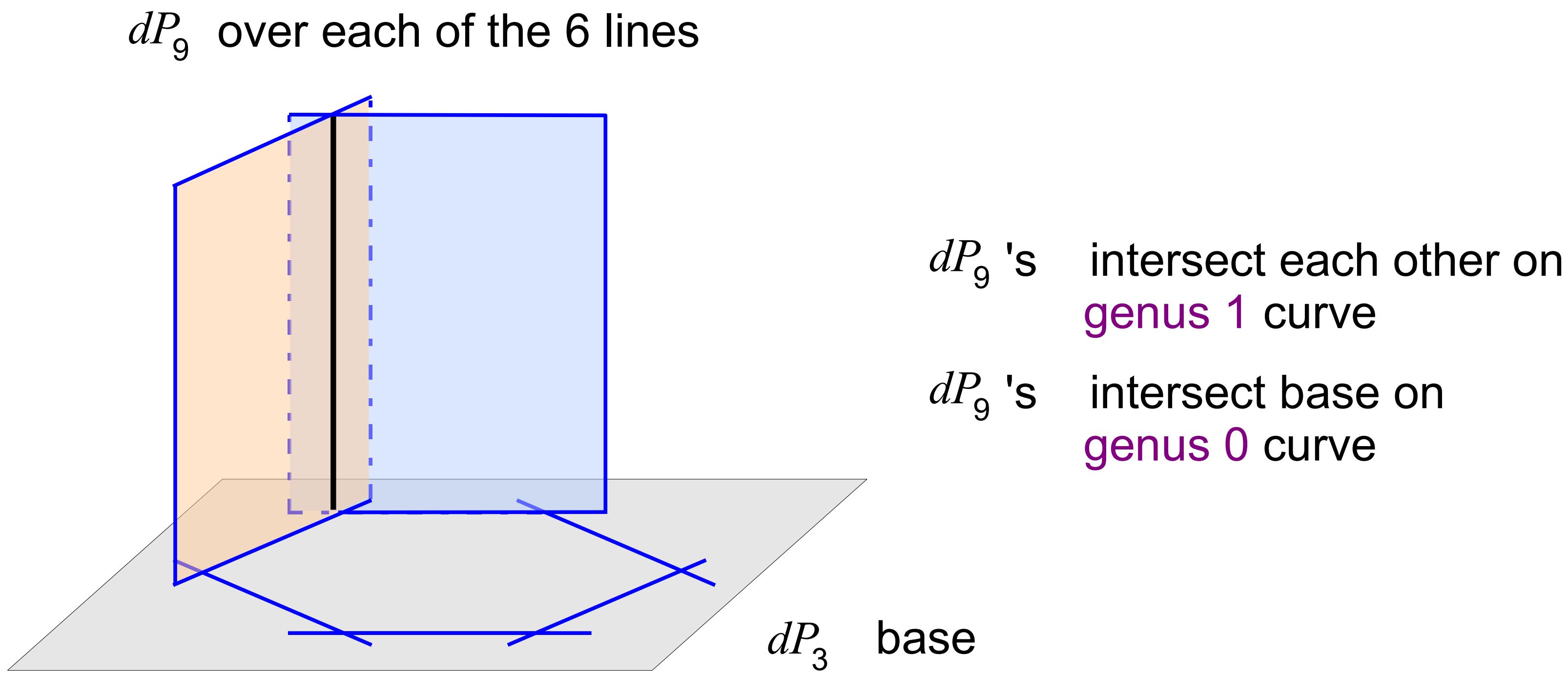


- note that always $\Delta\chi = 24 = 2 \times C_{E_6}$, indeed one checks that there are always E_6 lattices trivial in the CY threefolds $\Rightarrow$ can support hypercharge flux
- simple quintic involutions extend to transitioned spaces



➪ 2 Elliptic fibration over dP_r - Transitions of $\mathbb{P}_{1,1,1,6,9}[18]$

Example: toric base dP_3 $\longrightarrow$ CY as hypersurface in toric space



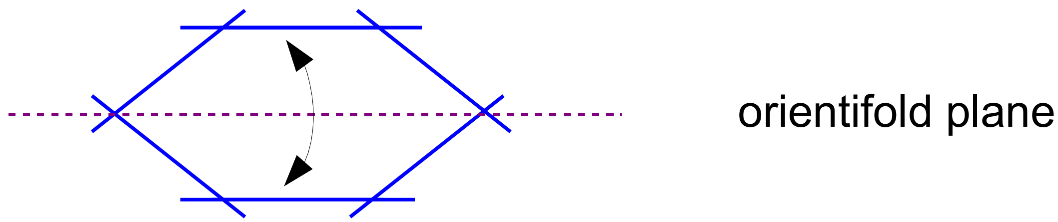
$\Rightarrow$ there are actually 18 topological phases connected via flop transitions

$\Rightarrow$ one phase corresponds to $3 \times dP_8$ - transition of $\mathbb{P}_{1,1,1,6,9}[18]$

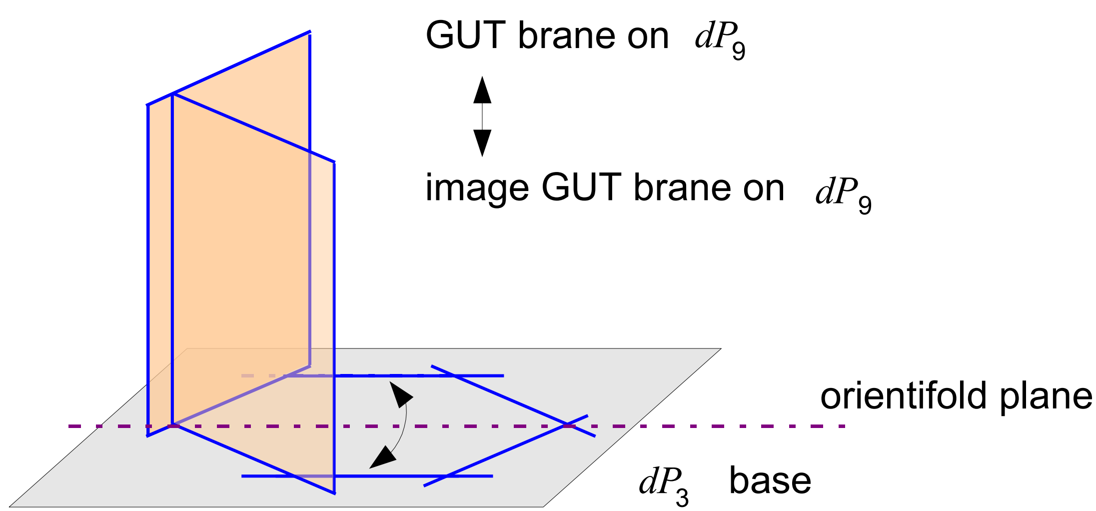
➪ Involutions on del Pezzo base   classified all involutions on all del Pezzo surface

Blumenhagen, Braun, TG, Weigand

example: dP_3 - base



➪ base involution extends to elliptically fibered threefold: exchange of dP_9 fibers



property	mechanism	status
globally consistent	tadpoles + K-theory	✓
D-term susy	vanishing FI-terms inside Kähler cone	✓
gauge group $SU(5)$	$U(5) \times U(1)$ stacks	✓
3 chiral generations	choice of line bundles $L_{\mathrm{GUT}}, L_{\mathrm{U(1)}}$	✓
no vector-like matter	localisation on $\mathbb{P}^1$ curves	✓
1 vector-like of Higgs	choice of line bundles	✓
no adjoints	rigid 4-cycles $\leftarrow$ del Pezzo	✓
GUT breaking	$U(1)_Y$ flux L_Y on trivial 2-cycles	✓
3-2 splitting	Wilson lines on $g = 1$ curve	✓
3-2 split + no dim=5 p^+-decay	local. of H_u, H_d on disjoint comp.	✓
$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ Yukawa	presence of appropriate D3-instanton	✓
Majorana neutrino masses	presence of appropriate D3-instanton	✓

$\Rightarrow$ found models which satisfy <u>all</u> criteria,

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Majorana neutrino masses	presence of appropriate D3-instanton	✓

$\Rightarrow$ found models which satisfy all criteria, but not yet simultaneously

$\Rightarrow$ generalization: models with more complicated del Pezzo configurations and orientifold involutions

Conclusions

- Discussed constructions of GUTs in Type IIB orientifold compactifications
  - many of the local F-theory mechanisms can be realized (e.g. GUT breaking)
  - new consistency conditions arise (e.g. tadpole cancellation, Kähler cone conditions)
  - non-perturbative generations of the missing couplings needed
- Construction of promising class of compact CY orientifolds
  - intersecting del Pezzo and other rigid surfaces
  - involutions and O-planes can be determined explicitly
    $\Rightarrow$ globally consistent D-brane configurations
  - gauge bundles on D7 branes $\Rightarrow$ spectrum, GUT breaking etc.
  - appealing phenomenological feature (MSSM chiral spectrum, no exotics, etc.)
- Open problem: - lift to compact GUT models in F-theory
  - combination with moduli stabilization, susy breaking