Freedom and constraints in the landscape of magnetized/intersecting brane models

KITP workshop: Fundamental Aspects of Superstring Theories

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Based on:

arXiv:0903.0386, w/ V. Kumar
hep-th/0606109, w/ M. R. Douglas
arXiv:090??.?, w/ V. Rosenhaus
Outline

1. Philosophy
2. Magnetized Brane Models
3. Intersecting Brane Models
Can string theory make quantitative predictions for 4D low-E physics?

- A priori, unlikely q. gravity at $10^{19}$ GeV affects physics at 1 TeV
- Long hoped: low-energy physics computable from string theory (e.g. $e^-$ mass)
- Landscape: threatens predictability below Planck scale
- Issues:
  - Can’t define theory completely
  - Probably understand only set of measure 0 of vacua ($G_2$, non-Kaehler, non-geometric, . . .)
  - Difficult to compute in known vacua (no simple world-sheet description w/R-R fields, etc.)
- Probabilities on landscape: difficult to interpret w/o understanding time-dependence & cosmology in ST

So how can we hope for quantitative predictions?
Searching for stringy constraints

Given a large set $\mathcal{V}$ of vacua
(e.g. II flux vacua on CY, IBM on toroidal orientifolds, . . .)

And observables $\mathcal{O}$
(e.g. gauge group, matter content, couplings, . . .)

How are observables distributed?

Extreme cases

A: “Anything goes” (perhaps B: “Bounded”), C: “Constrained”
If there are constraints on low-energy physics

These constraints should be visible in every corner of the landscape

Common constraint on $\mathcal{O}$ in disparate sets of vacua $\mathcal{V}_1, \mathcal{V}_2$

$\Rightarrow$ hints at global constraint

Motivates looking at classes of vacua, looking for:

- Mathematical characterization of classes of vacua
- Range of low-energy field theories available in classes

— Complementary to standard approach of looking for realistic model

— Secondary motivation: find algorithms for constructing classes of models with specific properties like $H \subset G$ for $H = SU(2) \times SU(3)$
Landscape/Swampland problem for predictive phenomenology:
What range of EFT’s arise in allowed (metastable) string vacua?

**Possibility A: “Anything goes”**

Perhaps virtually any variation on standard model below 100 TeV can be realized in string theory
(change masses, couplings, matter content, extensions, . . . )
— Predictivity difficult, maybe impossible in practice
— Progress requires real dynamics, measure etc.

**Possibility C: Low-Energy Constraints**

Perhaps not all low-E field theories allowed from ST
— May give constraints, (e.g. 3/19 of SM parameters)
Focus on theories w/SUSY —why?

– If SUSY at intermediate scale:

  - If no constraints from ST on SUSY theory, low-energy constraints derivable from SUSY FT and SUSY breaking mechanism
  - If constraints from ST on SUSY theory:
    ⇒ constraints on broken SUSY theory at sub TeV scales

– If SUSY breaking at Planck scale: ⇒ predictions very tough

So: look for constraints arising from ST on SUSY field theory + gravity in 4 and higher dimensions

For today, focus on gauge group $G$, matter content (representations, # generations)

Don’t worry about moduli, cosmology

Question: does ST make definite predictions anywhere in landscape?
2. Magnetized Brane Models

Magnetized brane models on K3  [V. Kumar/WT arXiv:0903.0386]

Result: Clean mathematical characterization of class of vacua:
6D $\mathcal{N} = 1$ SUSY models w/ rank 16 gauge group

Setup: Type I/heterotic SO(32) $(\text{Spin}(32)/\mathbb{Z}_2)$ on K3
16 D9’s (+ orientifold images) w/fluxes $\sim$ T-dual of IBM

"Stacks" of $N_a$ branes w/ flux $f_a$

\[
F = \begin{pmatrix}
\vdots \\
0 \\
\vdots \\
0 \\
f_1 \\
f_2 \\
\vdots \\
0
\end{pmatrix}
\]

$N_1$ $N_2$
Constraints on fluxes

- Fluxes $f_a$ integrally quantized

$$f_a \in H^2(K3, \mathbb{Z}) = \Gamma^{3,19} = U \oplus U \oplus U \oplus E_8(-1) \oplus E_8(-1) \quad [U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}]$$

- $f_a$ generate even integral lattice $\Lambda \subset \Gamma^{3,19}$

Tadpole constraint:

$$\frac{1}{8\pi^2} \int_s \text{Tr} \mathcal{F} \wedge \mathcal{F} = -\frac{1}{2} \int_S \rho_1(R) \Rightarrow \sum_a N_a f_a \cdot f_a = -24$$

SUSY constraints:

$$\int f^a \wedge \Omega = 0 \quad \int f^a \wedge \bar{\Omega} = 0 \quad \int f^a \wedge J = 0$$

$\Lambda$ negative definite $\Rightarrow \exists \Omega, J$ giving SUSY (pos. def. 3-plane in $\Gamma^{3,19}$)

Constraints for SUSY vacuum:

Define matrix $m_{ab} = f_a \cdot f_b$

$$\sum_a N_a m_{aa} = -24, \quad m_{ab} \text{ negative semidefinite}$$
Parameters \( N_a, m_{ab} \Rightarrow \text{gauge group, matter content in 6D} \)

Gauge group:
\[
G = U(N_1) \times U(N_2) \times \cdots \times U(N_K) \times SO(32 - 2 \sum_a N_a)
\]

[Technical points: 1) Some U(1)’s anomalous \( \rightarrow \) massive, 2) \( G \) may be enhanced when \( J \cdot f = 0, f^2 = -2 \)]

Matter content: depends only on \( N_a, m_{ab} = f_a \cdot f_b \)

<table>
<thead>
<tr>
<th>Rep. (+ c.c.)</th>
<th># hypermultiplets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjoint ( U(N_a) )</td>
<td>1</td>
</tr>
<tr>
<td>(( N_a, \bar{N}_b ))</td>
<td>((-2 - (f_a - f_b)^2))</td>
</tr>
<tr>
<td>(( N_a, N_b ))</td>
<td>((-2 - (f_a + f_b)^2))</td>
</tr>
<tr>
<td>Antisym. ( U(N_a) )</td>
<td>((-2 - 4f_a^2))</td>
</tr>
<tr>
<td>(( N_a, 2M ))</td>
<td>((-2 - f_a^2))</td>
</tr>
<tr>
<td>Neutral</td>
<td>20</td>
</tr>
</tbody>
</table>

Anomalies: \( F^4, R^4 \) cancel (e.g. \( n_H - n_V + 29n_T = 273 \))

[Green/Schwarz/West, Intriligator, Honecker]
So for this class of models

\[ N_a, m_{ab} \text{ parameterize 6D physics} \]

\[ \text{Tadpole + SUSY: } \sum_a N_a m_{aa} = -24, \quad m_{ab} \text{ neg. semidefinite} \]

**Question:** Up to these constraints, what \( N_a, m_{ab} \) possible?

**Answer:** A, “Anything goes”

**Furthermore,** realization of given \( N_a, m_{ab} \) unique up to duality
— Some discrete redundancy (generalize Banerjee/Sen dyon #’s)

**Key:** Nikulin results on lattice embeddings
Embedding theorems: vector (1D lattice) embeddings as example

Given lattice $\Lambda$ with $\langle x, y \rangle \in \mathbb{Z}$, desired norm $\nu$

Existence: $\exists x \in \Lambda : \langle x, x \rangle = \nu$?
Uniqueness: Is such an $x$ unique (up to $\Lambda$ automorphisms?)

Positive definite
e.g. Cartesian $\delta^a_b$

$5 = 2^2 + 1^2$

$\exists$ : not guaranteed
$(7 \neq a^2 + b^2)$

$!$ : not guaranteed
$(8^2 + 1^2 = 7^2 + 4^2)$

Even unimodular
e.g. $E_8$ root lattice

$E_8 = \{(x_1, \ldots, x_8) : \sum x_i \in 2\mathbb{Z}, x_i \in \mathbb{Z} \text{ or } \mathbb{Z} + 1/2 \ \forall i\}$

$\exists$ : Guaranteed!

$!$ : not guaranteed
(e.g. $\nu = 14 \Rightarrow 2x$'s w/ $\langle x, x \rangle = 14$)

Even unimodular
indefinite signature
rank $\geq 4 + |\text{sig.}|$
e.g. $\Gamma^{2,2} = U \oplus U$,

$\exists$ : Guaranteed
$((1, x, 0, 0)^2 = 2x)$

$!$ : Guaranteed unique
(Wall's theorem)
$!$ primitive ($x \neq nx'$)
$!$ to automorphism
Nikulin’s theorem

Let $\mathcal{M}$ be an even lattice of signature $(t_+, t_-)$ and let $\mathcal{L}$ be an even, unimodular lattice of signature $(l_+, l_-)$. There exists a unique primitive embedding $\eta : \mathcal{M} \hookrightarrow \mathcal{L}$, provided the following conditions hold:

1. $l_+ - t_+ > 0$ and $l_- - t_- > 0$.
2. $l_+ + l_- - t_+ - t_- \geq 2 + l(A(\mathcal{M})_p) \quad \forall \text{ primes } p \neq 2.$
3. $l_+ + l_- - t_+ - t_- \geq l(A(\mathcal{M})_2)$; if $=\text{ then } A(\mathcal{M}) \cong \mathbb{Z}_2^3 \oplus A'$.

Definitions:

$A(\mathcal{M}) = \mathcal{M}^*/\mathcal{M}$, \quad $A(\mathcal{M})_p = \text{ subgroup of elements of order } p^k$ (any $k$)

$l(A(\mathcal{M})_p) = \# \text{ generators of } A(\mathcal{M})_p$

primitive: $\eta(\mathcal{M}) = \text{Span}(\eta(\mathcal{M})) \cap \mathcal{L}$

Note: $l(A(\mathcal{M})_p) \leq t_+ + t_-$

so when $l_+, l_- = 3, 19, t_+ = 0, \Rightarrow$ always OK for $t_- \leq 10$
Some consequences of Nikulin’s theorem
(after some fiddling with special cases)

- For every $N_a, m_{ab}$ satisfying tadpole + SUSY, $m \Rightarrow \Lambda$ embeds into $\Gamma^{3,19}$, $m, \Lambda \rightarrow f \rightarrow \Omega, J$ (“Anything goes”)

- $\Lambda \hookrightarrow \Gamma^{3,19}$ can be primitive in all cases except $(-2)^{12}$.

- Redundancy from overlattices w/ discrete embeddings (generalizes Banerjee & Sen dyon invariants)

Result: separates

Constraints (tadpole, SUSY, $N, m \rightarrow 6D$ physics)

Freedom (any $N, m : \sum_a N_a m_{aa} = -24, \ m \leq 0$)
Example of lattice embeddings

Kummer lattice $K \subset \Gamma^{3,19}$ from orbifold

Basis $E_i, i = 1, \ldots, 16, \{\sum_{i \in H} \frac{1}{2} E_i : H \text{ affine hyperplane in } \mathbb{Z}_2^4\}$
inner product $\langle E_i, E_j \rangle = -2\delta_{ij}$

can embed $(-2)^{11}$

can embed $(-2)^{12}$ but not in primitive fashion!
Application: find all models w/ $SU(3) \times SU(2) \subset G$

- Straightforward (ignoring possible enhancement of $G$)
- $N_1 = 3, N_2 = 2$

$$m = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix} = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

$$A < 0, \ C < 0, \ 3A + 2B \geq -24, \ AC - B^2 > 0$$

71 models. # "quarks" in $(3, 2) + (3, \bar{2})$ is $(-4 - 2m_{11} - 2m_{22}) \geq 4$

Point: just plug in values for desired quantities.

No extra info needed about rest of model (unlike IBM toroidal orientifold models—next part)
Comments on K3 rank 16 models

- For \( e.g. \ SO(32 - N) \times SU(N) \), construction gives all anomaly-allowed #'s of hypers in allowed rep's.

- These are special points in moduli space. Turning on nonabelian fluxes \( \rightarrow \) reduces rank, more general.

- For complete analysis need to include enhancement from small instantons on shrinking cycles \[ \text{[Aspinwall/Morrison, Intriligator]} \]
3. Intersecting Brane Models

Intersecting Brane Models in IIA

Well known, simple models

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold well studied, $\Rightarrow \supset$ SM gauge group, 3 gens.

[Blumenhagen/Körß/Lüst(/Görlich/Ott), Ibáñez/Marchesano/Rabadan, Aldazabal/Franco/Ibáñez/Rabadan/Uranga, Cvetic/Shiu/Uranga, Cvetic/Li/Liu, Cremades/Ibanez/Marchesano, Kumar/Wells, March./Shiu]

Systematically studied:

Blumenhagen/Gmeiner/Honecker/Lüst/Weigand (Computer search)

Douglas/Taylor, Rosenhaus/Taylor (Complete analysis)
IBM on $T^6/Z_2 \times Z_2$

Winding numbers on each torus

Tadpoles: $P = n_1 n_2 n_3$, $Q = -n_1 m_2 m_3$, $R = -m_1 n_2 m_3$, $S = -m_1 m_2 n_3$

Cancellation: $\sum_a P_a = \sum_a Q_a = \sum_a R_a = \sum_a S_a = 8$

SUSY conditions (when $P, Q, R, S > 0$):

$$\frac{1}{P} + \frac{j}{Q} + \frac{k}{R} + \frac{l}{S} = 0, \quad P + \frac{1}{j} Q + \frac{1}{k} R + \frac{1}{l} S > 0.$$

3 kinds of branes (up to $S_4$ symmetry):

- $a$: $- + + +$
- $b$: $+ + 0 0$
- $c$: $+ 0 0 0$

moduli + $a$ branes (negative tadpoles) make problem tricky.
Blumenhagen et al. [hep-th/]

Fix moduli $\vec{U} = (\hat{h}, \hat{j}, \hat{k}, \hat{l}) \in \mathbb{Z}^4$ ($j = \hat{j}/\hat{h}$, $k = \hat{k}/\hat{h}$, $l = \hat{l}/\hat{h}$),

Scan over $U = |\vec{U}|$, fixed $|\vec{U}| \rightarrow$ p.d. condition $\rightarrow$ finite solutions

- Scanned to $U = 12$, found $\sim 10^8$ models, complexity exponential
- Seemed found most models, decreasing tail
- $G$ components, # generations $\sim$ random

Douglas/Taylor [hep-th/0606109]

- Proved finite # total solutions, analytic bounds on a-brane combos

Rosenhaus/Taylor (to appear)

- Completed construction of all a-brane combinations (99,479)
- Allows construction of models w/ desired features
- Confirmed $\sim$ random distribution
- “Diversity in tail”
Example: consider $H = SU(3) \times SU(2) \times U(1) \subset G$ (distinct $U(1)$)

— For each of $10^5$ a-brane combinations, add b’s, c’s
— Find all $3 + 2 + 1$ combos ($\sim 13.4$ M)
— Of these, for $\sim 12.9$M moduli fixed, typical $U \sim 1000$

Explanation:

- More diverse $3 + 2 + 1$ combos w/ large negative tadpole from a’s
- Large negative tadpole $\rightarrow$ large moduli
- Fewer ways to complete models at large moduli w/ “extra” sector

Generations: # “quarks” $\sim \mathcal{O}(10)$, no suppression of 3 generations.
K3 Magnetized Brane Models

- Nikulin theorem $\Rightarrow$ simple math. characterization of model space
- Only constraints SUSY, tadpole, $N_a, m_{ab} \rightarrow 6D$ physics
- $\sum N_a m_{aa} = -24, m \leq 0 \Rightarrow$ exists lattice embedding
- Models unique up to dualities, discrete redundancy from overlattices
- Direct construction of models w/ desired properties

IBM on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- Completed analysis of allowed negative tadpole (a-brane) combos
- Allows direct construction of models w/ desired properties
- “Diversity is in the tail” of this landscape slice
Further Directions

- Extend analysis of 6D $N = 1$ SUSY models (nonabelian bundles, gauge enhancement, discrete $B$, bundles w/o v.s.)

- Apply K3 results, lattice embedding theorems in other contexts (less SUSY, lower dimensions, . . .)

- Find more general theoretical structure $\Rightarrow$ finite # IBM SUSY models

- Consider IBM/MBM on more general Calabi-Yau manifolds (proof of finite # solutions, . . .)