Aspects of brane cosmology

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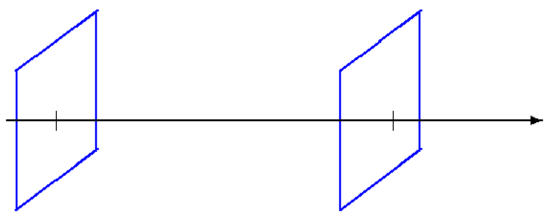
Introduction

Two generic situations are usually considered:

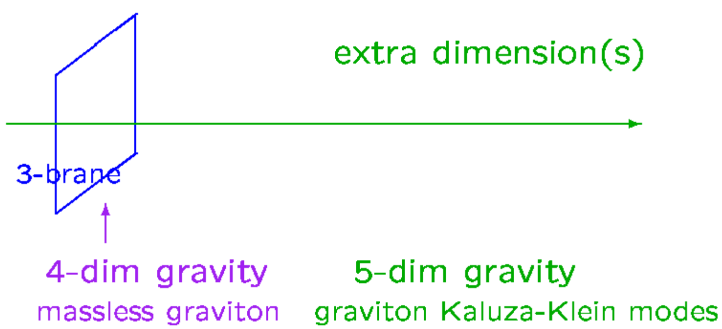
• the dimensions orthogonal to the brane are compact in order to recover 4-dimensional gravity in the effective low energy theory.



e.g. RS-I, Randall-Lykken, ...

Often the branes are at the end of space
e.g. Hořava, Witten

• 4-dimensional gravity is itself localized on the brane
Rubakov, Shaposhnikov; Randall, Sundrum;..



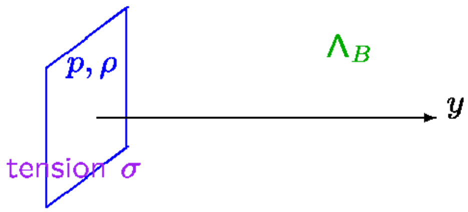
Because of the attractiveness of the brane scenario, many have been tempted to push it beyond the strict string framework where it was defined, and to generalize it to topological defects on which matter is localized.

Since brane universes are new set ups, they may provide new twists to old problems (hierarchy problem, baryogenesis, dark matter, late acceleration of the universe or cosmological constant).

ADD...

Toy models

Take the simplest 5-dimensional model (codimension 1)



5-dimensional Einstein equation + Israel junction conditions on the brane

→ generalized Friedmann equation on the brane:

$$H^2 = \frac{1}{6M_5^3}\Lambda_B + \frac{1}{36M_5^6}\sigma^2 + \frac{1}{18M_5^6}\sigma\rho + \frac{1}{36M_5^6}\rho^2 + \frac{\mathcal{C}}{a_0^4} - \frac{k}{a_0^2}$$

P.B., Deffayet, Ellwanger, Langlois; Csaki, Graesser, Kolda, Terning; Cline, Grojean, Servant ;...

with $8\pi G_5 \equiv M_5^{-3}$, a_0 cosmic scale factor on the brane,

to be compared with the standard Friedmann equation

$$H^2 = \frac{\lambda}{3} + \frac{1}{3M_P^2}\,\rho - \frac{k}{a^2}$$

- 4-d cosmological constant λ receives contributions both from the bulk vacuum energy Λ_B and from the string tension σ^2

- Comparing the linear terms in ρ allows a cosmological determination of the 4-dimensional Planck scale M_P

$$M_P^2 = 6\frac{M_5^6}{\sigma} \quad .$$

- new contribution in ρ^2. This shows that the cosmology of a 4-dimensional brane universe is generically different from the cosmology of a 4-dimensional universe

↔ notion of extrinsic curvature i.e. the way the brane is curved within the higher-dimensional spacetime: the presence of localized energy induces extrinsic curvature.

importance of the non-conventional ρ^2 term increases as one goes back in time → possible role in the early universe (inflation models)

- the dark radiation term $\mathcal{C}/a_0^4$ represents the effect of the gas of bulk gravitons on the cosmological evolution of the brane. The brane does not represent a closed system.

In general relativity, Birkhoff's theorem: a spherically symmetric gravitational field in empty space must be static, with a metric given by the Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)\, dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}\, dr^2 + r^2 d\Omega^2 \quad .$$

Generalization of Birkhoff's theorem valid for this set up: the general solution for the bulk metric is static with metric given by the the AdS-Schwarzschild solution:

$$\begin{aligned} ds^2 &= -h(y)\, dt^2 + \frac{1}{h(y)} dy^2 + y^2\, d\Sigma_k^2 \quad , \\ h(y) &= k - \frac{\mathcal{C}}{y^2} + \frac{y^2}{\ell^2} \quad , \end{aligned}$$

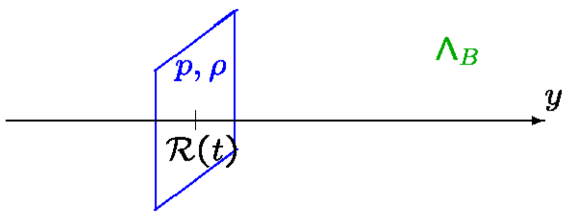
where

$$\ell \equiv \sqrt{\frac{6M_5^3}{|\Lambda_B|}}$$

is the AdS_5 curvature radius. Thus $\mathcal{C}$ may be interpreted as the mass of the 5-dimensional black hole that is formed in the bulk because of graviton radiation from the brane.

In this set up, time dependence i.e. cosmological evolution only comes from the motion of the brane in the bulk.

Kraus



$\mathcal{R}(t)$ is the cosmic scale factor on the brane:

$$\begin{aligned} ds^2\big|_{\rm ind} &= -\left[h(\mathcal{R}) - \frac{1}{h(\mathcal{R})}\left(\frac{d\mathcal{R}}{dt}\right)^2\right] dt^2 + \mathcal{R}^2 d\Sigma_k^2 \\ &\equiv -d\tau^2 + \mathcal{R}^2(\tau) d\Sigma_k^2 \end{aligned}$$

When should one recover the 4-dimensional picture?

When the physics on the brane is 4-dimensional, i.e.

- if the extra dimension is compact and its radius is stabilized

e.g. Hořava-Witten theory

Lukas, Ovrut, Waldram

- if the extra dimension is noncompact but the 4-dim. graviton is localized

Rubakov, Shaposhnikov; Akama; Gogberashvili; Randall,Sundrum

e.g. in Randall-Sundrum (RS-II) case, warped geometry in the extra dimension i.e. the cosmic scale factor has a y dependence:

$$a(y) = e^{-|y|/\ell}, \quad \ell^2 = \frac{6M_5^3}{|\Lambda_B|} \quad AdS_5 \text{ curvature radius}$$

Minkowski (M_4) constraint: $\frac{1}{3}\lambda = \frac{1}{6M_5^3}\Lambda_B + \frac{1}{36M_5^6}\sigma^2 = 0$

Planck scale : $M_P^2 = M_5^3 \int_{-\infty}^{+\infty} e^{-2|y|/\ell} dy = M_5^3 \ell$

Agrees with the cosmological evaluation:

$$M_P^2 = \frac{6M_5^6}{\sigma} = M_5^3 \left(\frac{6M_5^3}{|\Lambda_B|}\right)^{1/2} = M_5^3 \ell$$

Models with codimension > 1

Gherghetta, Shaposhnikov; Cline, Descheneau, Giovannini, Vinet; Schwindt, Wetterich;...

Situation is very different because of

- different ultraviolet behavior
- more interdependence between brane and bulk

e.g.

$$ds^2 = a(\rho) g_{\mu\nu} dx^\mu dx^\nu + \gamma(\rho) d\theta^2 + d\rho^2$$

As $\rho \to 0$, $\gamma(\rho) \to (1 - \lambda/2\pi)^2 \rho^2$

deficit angle λ

A bulk particle identifies the singularity by circling it (circumference at radius ρ : $(2\pi - \lambda)\rho$).

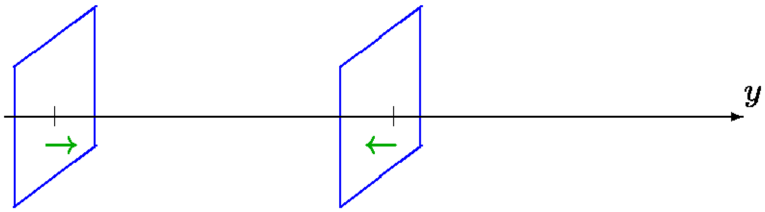
Searching for unstable configurations

Many of the possible brane configurations are unstable.

⊕ good for a cosmological scenario

e.g. brane inflation

Dvali, Tye



⊕ important to identify the few stable configurations

Ex.1 graviton instability for negative tension branes at an orbifold fixed point

C. Charmousis, J.F. Dufaux; P.B.

Modify the RS-I set up (2 branes of opposite tensions localized at Z_2 orbifold fixed points $y = 0$ and $y = y_c$) by adding a Gauss-Bonnet term

$\mathcal{L}_{GB} = R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd}$

$$S_{\rm bulk} = \int d^5x\sqrt{-g}\left\{\frac{M_5^3}{2}R - \Lambda_B + \frac{M_5^3}{2}\alpha\mathcal{L}_{GB}\right\}$$

$$S_{\rm brane} = -\int_{y=0} d^4x\sqrt{-g_4}\,\sigma_1 - \int_{y=y_c} d^4x\sqrt{-g_4}\,\sigma_2$$

RS-type solution :

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\ ,\ k = \sqrt{\frac{1}{4\alpha}\left(1 - \sqrt{1 + \frac{4}{3}\alpha\frac{\Lambda_B}{M_5^3}}\right)}.$$

with the condition : $\sigma_1 = -\sigma_2 = 2kM_5^3(3 - 4\alpha k^2)$.

This set up has a graviton tachyonic mode:

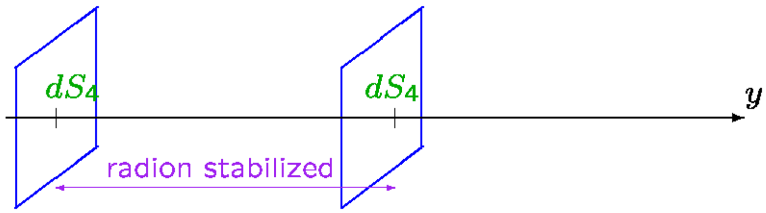
$$ds^2 = e^{-2ky}\left(\eta_{\mu\nu} + h_{\mu\nu}^{(m)}(x)\psi_m(y)\right)dx^\mu dx^\nu + dy^2$$

with $h_{\mu\nu}^{(m)}$ transverse and traceless and $\Box h_{\mu\nu}^{(m)}(x) = m^2 h_{\mu\nu}^{(m)}(x$

There exists a solution $\psi_m(y)$ with $m^2 = -\mu^2 < 0$.

Ex.2 radion instability with de Sitter branes

Martin, Felder, Frolov, Peloso, **Kofman**



Strong tachyonic instability in the radion mode :

$$ds^2 = a(y)^2 \left[(1+2\Phi)\ dy^2 + (1+2\Psi)\left(-dt^2 + e^{2Ht}\mathbf{dx}^2\right)\right]$$

Einstein eqns $\Rightarrow \quad \Psi = -\Phi/2$

$$\Phi(x^\mu, y) = \Phi^{(m)}(x^\mu)\ \varphi_m(y)$$

with $\Lambda\Phi^{(m)}(x) = m^2\Phi^{(m)}(x)$

Solution $\varphi_m(y)$ with $m^2 < 0$ $(m^2 = -4H^2 + ...)$

Meeting observation : predictions for CMB

- Perturbations: computing the CMB spectrum

Kodama, Ishibashi, Seto; Maartens; Langlois; van de Bruck, Dorca, Brandenberger, Lukas; Koyama, Soda; Deruelle, Dolezel, Katz; Langlois, Maartens, Sasaki, Wands; Deffayet; Riazuelo, Vernizzi, Steer, Durrer...

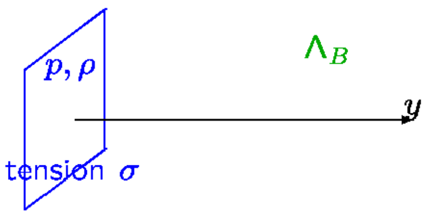
To this day no complete evaluation from the higher-dimensional point of view of the C_l distribution for the cosmic microwave background.

problems already arise in the simplest case of the Randall-Sundrum model because the Kaluza-Klein modes do not decouple, even to linear order: this is a reflection of the fact that the brane is not a closed system.

In the two-brane case, need to define precisely which brane fluctuation modes are allowed (dynamical brane, orbifold fixed point, orientifold...) and, if needed, to properly stabilize the radion field.

Does the brane picture shed new light on the cosmological constant problem?

Braneworld set up



Cosmological constant on the brane:

$$\lambda = \frac{1}{2M_5^3}\,\Lambda_B + \frac{1}{12M_5^6}\,\sigma^2$$

M_5 5-dimensional Planck scale

⊖ requires fine-tuning between string tension and bulk vacuum energy (Randall-Sundrum condition)

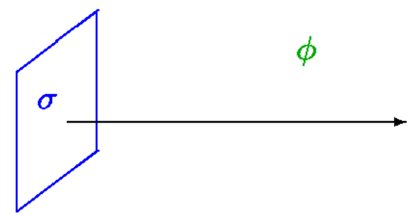
⊕ decouples the cosmological constant from the brane vacuum energy i.e. the tension σ

Is there an adjustment mechanism, i.e. a way of evading Weiberg's no-go theorem?

Brane universes and self-tuning

Arkani-Hamed, Dimopoulos, Kaloper, Sundrum; Kachru, Schultz, Silverstein

SELF TUNING: introduce a bulk scalar field ϕ which couples conformally to matter on the brane



$$\mathcal{S} = \mathcal{S}_{bk} + \mathcal{S}_{br} = M_5^3 \int d^5x \sqrt{|g_5|}\left[\frac{1}{2}R^{(5)} - \frac{3}{2}\partial^m\phi\,\partial_m\phi - 3\mathcal{V}(\right.$$

$$+ \int_{\text{brane}} d^4x \sqrt{|g_4|}\, f^2(\phi)\,(-\sigma),$$

One finds static spatially flat solutions to the classical equations of motion for any value of the brane tension

e.g. $\quad \mathcal{V}(\Phi) = 0 \quad , \quad f^2(\phi) = Ce^{\mp\phi}$

Warped background $ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$

$$A(y) = \frac{1}{2}\ln\left(1 - \frac{|y|}{y_c}\right)$$

$$\phi(y) = \phi_0 \pm \ln\left(1 - \frac{|y|}{y_c}\right)$$

Naked singularity at $|y| = y_c > 0$

• Fine tuning associated with the presence of the singularity: one may cure the singularity by adding a second brane but the content of the second brane is then fine-tuned.

Forste, Lalak, Lavignac, Nilles

• Include one-loop corrections to gravity (Gauss-Bonnet) in the bulk

P.B., C. Charmousis, S. Davis, J.-F. Dufaux

There exist solutions with no naked singularity at finite distance from the brane

$$A(y) = A_0 + x \ln\left(1 + \frac{|y|}{y_c}\right)$$
$$\phi(y) = \phi_0 - \frac{2}{\zeta} \ln\left(1 + \frac{|y|}{y_c}\right)$$

Planck mass finite for $x < -1/2$

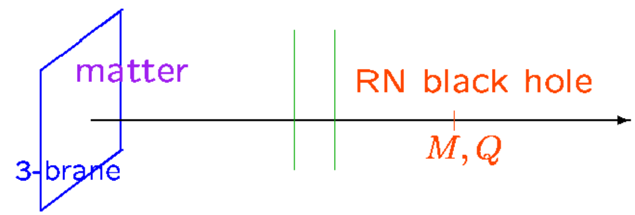
but fine tuning reappears.

• Replace the bulk scalar field by a black hole configuration in the bulk

→ Flat branes in black hole backgrounds

Csaki, Erlich, Grojean

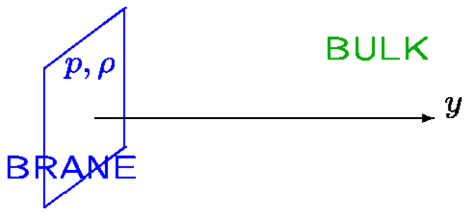
Possible to find flat brane solutions without naked singularities, with enough parameters to avoid fine tuning of the theory



In the case of AdS-Reissner-Nordstrom black hole, enough parameters (M, Q) so that they can be tuned to take into account variations of the brane vacuum energy

Brane cosmology and an alternative to dark energy : modification of gravity at cosmological distances

Take the simplest 5-dimensional model (single extra dimension)



5-dimensional Einstein equation + Israel junction conditions on the brane → generalized Friedmann equation on the brane:

$$H^2 = \frac{1}{36 M_f^6}\, \rho^2$$

with M_f higher-dimensional Planck scale,

to be compared with the standard Friedmann equation

$$H^2 = \frac{1}{3 m_{Pl}^2}\, \rho$$

If the two are present, expect an equation of the form

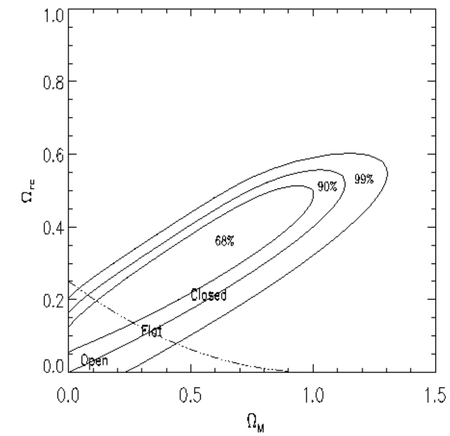
$$\alpha H^2 + \beta H \sim \rho$$

For example cosmology of induced gravity model

Deffayet; Deffayet, Dvali, Gabadadze

$$H^2 = \left(\sqrt{\frac{\rho}{3 m_{Pl}^2} + \frac{1}{4 r_c^2}} + \frac{1}{2 r_c} \right)^2 - \frac{k}{a^2}$$

Hence acceleration at late time, without a need for a cosmological constant!



$\Omega_{r_c} \equiv 1/4 r_c^2 H_0^2$

Ω_M

Modification of gravity at large distance

- Precursor model:GRS model

Gregory, Rubakov, Sibiriakov

where a collection of massive graviton modes contribute to form a 4-dimensional unstable massless graviton bound-state on the brane.

→ 5-d gravity (r^{-3}) recovered at large distance.

- multi-gravity models

Kogan, Mouslopoulos, Papazoglou, Ross

e.g. bi-gravity: set-up corresponds to two branes, each localizing gravity: for a finite distance between the branes, the corresponding graviton zero modes mix, giving a massless graviton plus a very light graviton whose presence modifies gravity at large distance.

- induced gravity models

Dvali, Gabadadze, Porrati

gravity includes 5d gravity plus induced 4d gravity on the brane:

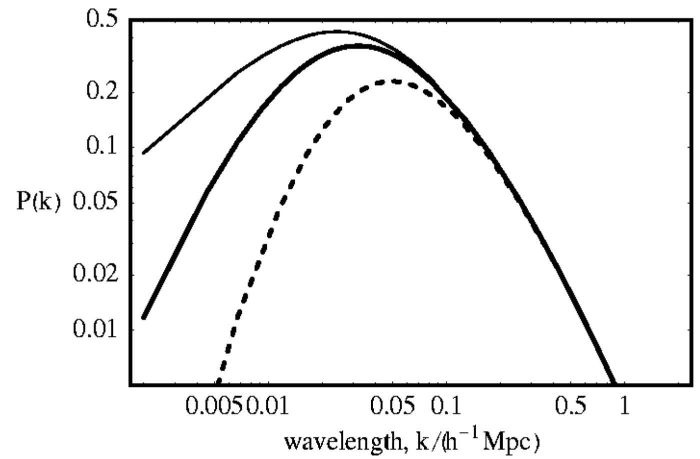
$$\mathcal{S} = \int d^5x \, \sqrt{-g} M_f^3 \, \frac{1}{2} R^{(5)} + \int_{\text{brane}} d^4x \, \sqrt{-h} M_{Pl}^2 \, \frac{1}{2} R^{(4)}$$

For distances r larger than a critical distance r_c given by

$$r_c = \frac{M_{Pl}^2}{2M_f^3}$$

gravity "leakage" into the extra dimension: gravitational force in r^{-3}.

For all these models, the critical distance must be cosmological: CMB P.B., J. Silk, weak lensing Bernardeau,Uzan



All known models either have ghosts or become strong at distances of a few kilometers.

e.g. DGP model

Rubakov; Luty, Porrati, Rattazzi

There are scalar and vector modes with dim. 5 interaction terms that scale as $M_P^2/M_5^{9/2}$: the theory becomes strong at $E = \left(M^9/M_P^4\right)^{1/5}$.

String models with codimension larger than 1

Because of their different ultraviolet properties, theories with codimension $n > 1$ behave very differently:

- Newton's law is always 4-dimensional on the (infinitely thin) brane:

$$G(\mathbf{p}, \mathbf{y}=\mathbf{0}) = \frac{D_n(\mathbf{p})}{M_f^{n+2}\left[1 + r_c^n \mathbf{p}^2 D_n(\mathbf{p})\right]}, \quad D_n(\mathbf{p}) = \int d^n q \frac{1}{\mathbf{p}^2 +}$$

where $r_c^n = M_P^2/M_f^{2+n}$ $\quad D_n(\mathbf{p})$ divergent for $n \geq 2$

- for a brane of width w, there is a critical scale above which gravity is screened

Kiritsis, Tetradis, Tomaras

$$R_c = w\left(\frac{r_c}{w}\right)^{n/2}$$

Explicit model: type II superstring on $M_4 \times X_6$, with X_6 non-compact Ricci flat 6-dimensional manifold

Antoniadis, Minasian, Vanhove

$$\mathcal{S} = \frac{M_S^8}{(2\pi)^7}\int_{M_{10}} e^{-2\phi}\mathcal{R}_{10} - \frac{M_S^2}{3(4\pi)^7}\int_{M_{10}}\left(2\zeta(3)e^{-2\phi} \mp \frac{2\pi^2}{3}\right) R^4,$$

yields

$$\mathcal{S} = \frac{M_S^8}{(2\pi)^7}\int_{M_{10}} e^{-2\phi}\mathcal{R}_{10} + \frac{\chi M_S^2}{3(4\pi)^7}\int_{M_4}\left(-2\zeta(3)e^{-2\phi} \pm 4\zeta(2)\right) \mathcal{R}$$

requires Euler characteristics $\chi < 0$.

Conclusions

- Braneworld set-up is not unique. String theory consistency should constrain the number of possibilities.

- Most braneworld backgrounds do not lead to satisfactory late time cosmology (Brans Dicke fields...).

- If a satisfactory braneworld background is found, then it is in principle possible to compute perturbations: the fact that the brane is not a closed system imposes to perform a computation in the higher-dimensional theory (if one wants to keep intact the brany characteristics)

- Braneworld cosmology may lead to unexpected effects: violations of Lorentz invariance, modification of gravity at cosmological distances, late acceleration...

- The braneworld idea has given a new set up to discuss old problems but has not (yet?) provided striking new solutions.