Acceleration from D-cceleration

with D. Tong

hep-th/0310nnn

A few recent refs on related topics:

"k-flation", "k-essence"

"tachyon matter"

"bouncing branes"

Inflation in string theory KKLMMT cf Sandip's talk

Scalar field dynamics is important, for example in cosmology

- inflation, quintessence
- dynamics of (approximate) moduli
- tunneling effects

It is typically studied using the 2-derivative action

$$S_{2\text{-deriv}} = \int d^4x \sqrt{g} \left(G_{ij}(\phi) g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j - V(\phi) \right)$$

In highly SUSY cases, $G_{ij}(\phi)$ and $V(\phi)$ are highly constrained, and rigorous study of SUSY QFT has focused on determining G_{ij} and V (and BPS states)

For example, in the $\mathcal{N}=4$ SYM theory, $V(\Phi) = \frac{1}{g_{YM}^2} \mathrm{Tr}\,[\Phi^a, \Phi^b]^2$,

6 $N \times N$ adjoint matrices $a,b = 1 \ldots 6$

and $G_{ij}(\phi) g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j = \frac{1}{g_{YM}^2} \mathrm{Tr}\, \partial_\mu \Phi^a \partial_\nu \Phi^a g^{\mu\nu}$

are both exact statements.

So along the flat directions

$$\Phi = \begin{pmatrix} \phi_1 & & 0 \\ & \ddots & \\ 0 & & \phi_N \end{pmatrix} \Leftrightarrow V(\Phi) = 0,$$

the metric G_{ij} is δ_{ij}:

$$S_{2\text{-deriv}}(\phi_i) = \frac{1}{g_{YM}^2} \int d^4x \sqrt{g}\, \partial_\mu \phi_i \partial^\mu \phi_i$$

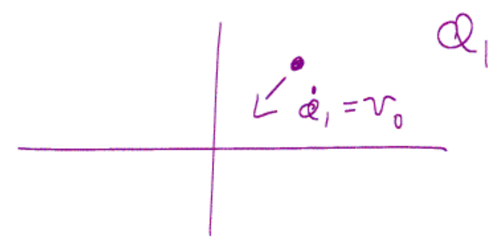
If the 2-derivative action sufficed, then the motion on the CFT moduli space would be very simple:

Let us consider $\phi_1 \neq 0$ $\phi_{i \neq 1} = 0$

i.e. $\Phi = \begin{pmatrix} \phi_1 & 0 \\ 0 & 0 \end{pmatrix}$

$U(N) \to U(1) \times U(N-1)$

$\delta S_{2\text{-deriv}} = 0 \Rightarrow \ddot{\phi}_1 = 0 \Rightarrow \dot{\phi} = v_0$ (constant)



However, the dynamics of Φ_1 is not well approximated by the equations of motion arising from $S_{2\text{-derivative}}$ as $\Phi_1 \to 0$

because as $\Phi_1 \to 0$ new light degrees of freedom emerge

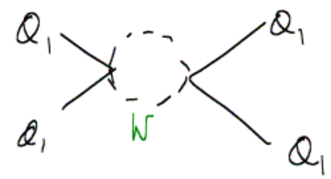
e.g. Φ_{1j} ($j \neq 1$), W^μ_{1j} ($j \neq 1$) + fermions } "W bosons" $m_W = \Phi_1 \equiv \Phi$

2 effects

① radiative corrections to $S[\Phi_1]$ (virtual W bosons)

② production of on-shell W bosons

①



$+ \ldots$

$$\sim N \int \frac{d^4 p}{(p^2 + \Phi^2)((p+q)^2 + \Phi^2)} + \ldots \qquad \Phi^2 \sim m_W^2$$

$$\to \mathcal{L} = \frac{\dot\Phi^2}{g_{YM}^2}\left(\frac{1}{2} + \frac{\lambda}{4}\frac{\dot\Phi^2}{\Phi^4} + \cdots\right)$$

where $\lambda = g_{YM}^2 N$ ('t Hooft coupling)

That is, we have corrections scaling like $\lambda \frac{\dot\Phi^2}{\Phi^4}$ from virtual W bosons.

$\Rightarrow$ cannot ignore these "higher derivative" effects as we move to origin $\Phi \to 0$

Note that these higher derivative effects are not suppressed by powers of m_s (string mass scale) or m_p (Planck mass scale), but only by powers of ϕ

Our main results will be:

- Strong coupling CFT: These $\frac{d\dot{\phi}}{\phi^4}$ effects sum up to produce dramatic "slow down" (relative to naive moduli space approx.) cf Kabat Lifshitz
- Combining this effect with gravity and other ingredients preserves the slow roll and leads to acceleration $\ddot{a}(t) > 0$: "k-inflation" occurs in strongly coupled CFT coupled to gravity.

② W pair production

Since $m_W = \phi$, and ϕ depends on time as we move to the origin, the "W" mass m_W depends on time $\Rightarrow$ pair production of "W"s is possible.

The strength of particle production for a t-dependent mass (or t-dependent harmonic oscillator frequency) is controlled by

$$\frac{\dot{m}_W}{m_W^2} = \frac{\dot{\phi}}{\phi^2}$$

So if we start moving toward the origin, governed by the 2-derivative action, $\ddot{\phi} = 0 \Rightarrow \dot{\phi} = v_0$,

The virtual W corrections ① become important at $\frac{\lambda \dot{\phi}^2}{\phi^4} = \frac{\lambda v_0^2}{\phi^4} \gtrsim 1$

i.e. at $\boxed{\phi^2 = \sqrt{\lambda}\, v_0}$

The W production becomes important at $\boxed{\phi^2 = v_0}$

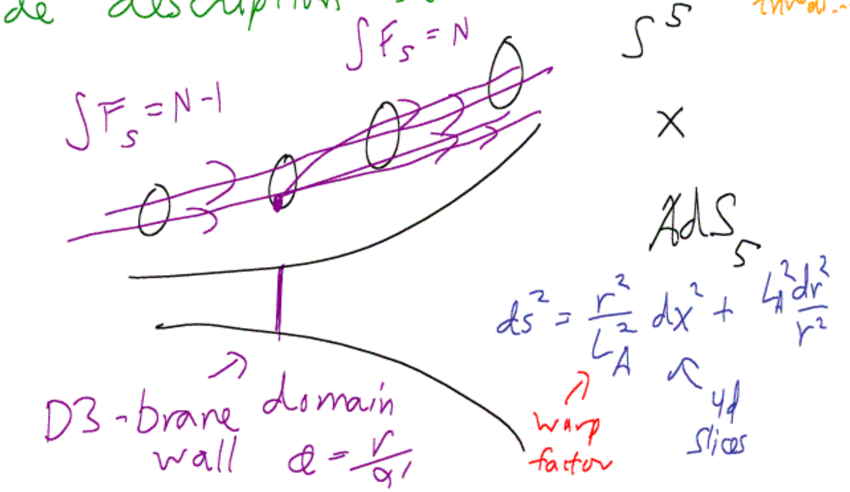
So at weak coupling, W production is encountered first. At strong coupling, we will see that indeed the virtual effect ① dominates

Strong CFT coupling $\lambda \gg 1$:

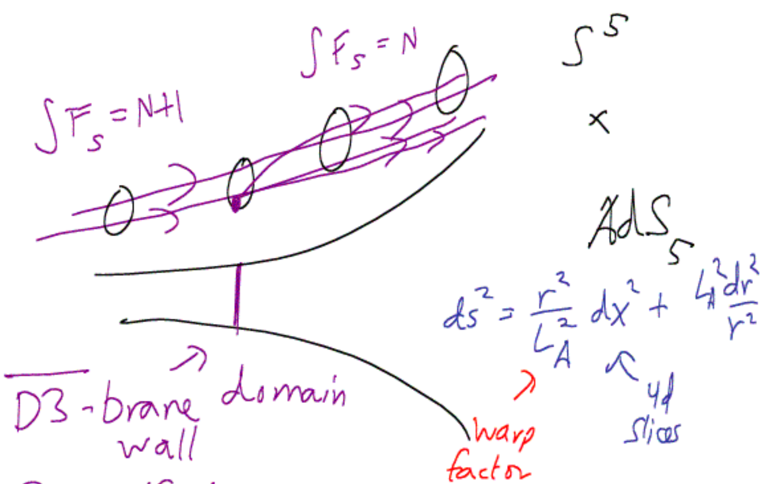
This regime is described by the gravity side of AdS/CFT in the large radius regime $\frac{L_{AdS}}{l_s} = \lambda^{\frac{1}{4}}$

The Coulomb branch in the gravity side description is:

Maldacena
Kraus Larsen
Trivedi ...



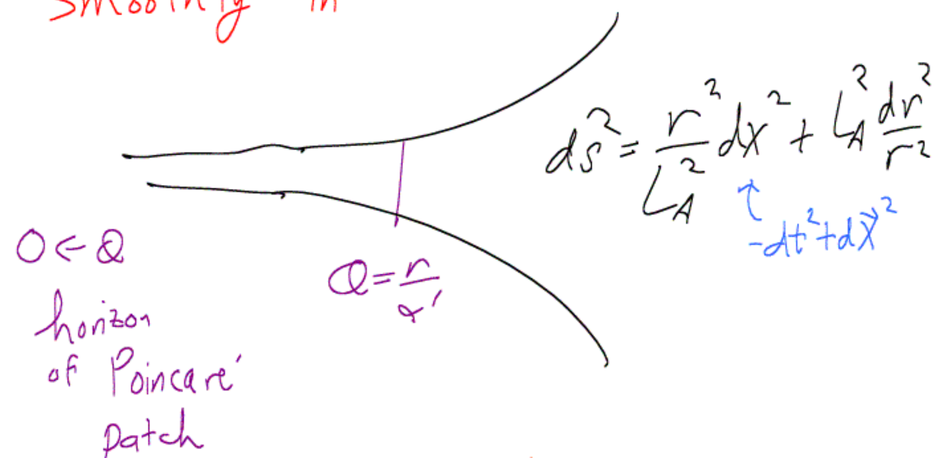
There is also an interesting generalization in which we replace the D3B with a $\overline{D3B}$ (anti 3-brane) cf Kachru-Verlinde



$\Phi = r/\alpha'$

This is in some sense a strong coupling limit of the brane-antibrane system that Sen et al discussed (we have N branes and 1 antibrane)

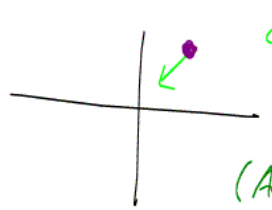
It is now immediately clear that going to the origin of the Coulomb branch ($\Phi \to 0$) cannot occur smoothly in finite time t:



- In probe approximation, takes forever to reach the origin
- speed limit $v_{proper}^2 = \frac{\lambda \dot{\Phi}^2}{\Phi^4} \leq 1$

So we find that the 2-derivative "moduli space approximation" is wildly wrong:

$S_{2\text{-deriv}}$

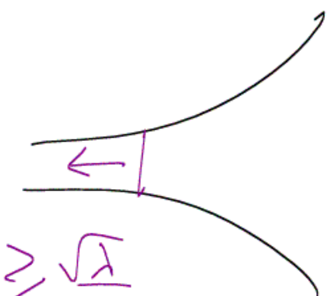


$\dot{\phi} = v_0$

unconstrained

(Autobahn)

S_{full}



$\phi \gtrsim \frac{\sqrt{\lambda}}{t}$

$(\dot{\phi}^2 \leq \lambda \phi^4)$

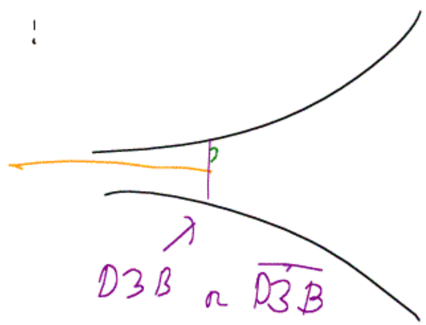
(Mumbai/L.A./Tehran in rush hour)

<u>Plan</u>: 1) Analyze <u>how</u> ϕ "slows down" (relative to naive moduli space approximation) in a controlled approximation scheme.

2) Couple to gravity (+ other sectors) and study cosmological evolution following from $\phi(t)$ matter sector (*including potential that may be generated)

→ since ϕ "slows down", this may provide a new mechanism for slow-roll in inflation (cf "k-essence", "k-flation")

Spectrum:



① string oscillator modes

warp factor $\frac{r}{L_A} \Rightarrow m_s^{eff} = \frac{1}{\sqrt{g'}} \frac{r}{L_A} \simeq \phi \frac{\sqrt{g'}}{L_A}$

$$\boxed{m_s^{eff} = \frac{\phi}{\lambda^{1/4}}}$$

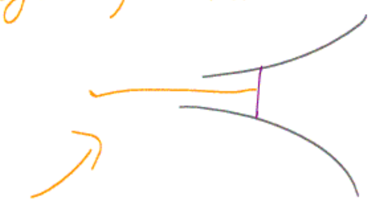
② stretched strings

D3B: W bosons

$$m_W = \underbrace{\int_0^r dr' \frac{L_A}{r'} \frac{1}{\alpha'}}_{dm} \underbrace{\frac{r'}{L_A}}_{\text{warp factor}} = \frac{r}{\alpha'} = \phi \quad \text{(BPS)}$$

$$\boxed{m_W = \phi}$$

$\overline{D3B}$: At weak coupling $\lambda \ll 1$, the lowest 3-$\bar{3}$ string is a tachyon.
In our strong coupling (= large L_A) regime, this mode is <u>not</u> tachyonic:



(up to order 1 coefficients)

$$m_T^2 \simeq -m_{s,eff}^2(\phi) + m_W^2(\phi)$$
$$= -\frac{\phi^2}{\sqrt{\lambda}} + \phi^2 \underset{\lambda \gg 1}{\sim} \phi^2$$

$$\boxed{m_T^2 \sim \phi^2 > 0}$$

③ massless brane modes $A_\mu, \delta\phi, \ldots$

④ bulk KK modes & bulk closed strings

The effective action for our system is the Dirac-Born-Infeld Lagrangian, coupled to gravity, plus corrections

$$S_T = \int d^4x \left[\frac{1}{2}\sqrt{-g}\,(M_p^2 + \phi^2)\,R + \mathcal{L}_0 + \dots \right]$$

where

$$\mathcal{L}_0 = -\frac{1}{g_{YM}^2}\left(f(\phi)^{-1}\sqrt{1+f(\phi)\, g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi} + V(\phi) \mp f(\phi)^{-1}\right)$$

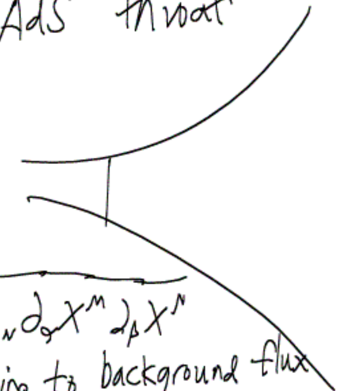
$$\mp \; \begin{cases} D3B \\ \overline{D3B} \end{cases}$$

where

$$f(\phi) = \frac{\lambda}{\phi^4} \quad \text{for } AdS_5$$

I.

pure CFT $\leftrightarrow$ ∞ AdS throat



pure AdS geometry

$$\mathcal{L} = \mathcal{L}^{CFT}_{DBI} = \sqrt{\det G_{MN}\,\partial_\alpha X^M \partial_\beta X^N}$$

+ coupling to background flux

$$\to S[\phi] = -\frac{N}{\lambda^2}\int d^4x\,\phi^4\left(\underbrace{\sqrt{1-\frac{\lambda\dot\phi^2}{\phi^4}}}_{\sqrt{1-v_{proper}^2}} \mp 1\right)$$

quartic potential $\leftrightarrow$ conformal invariance.

Includes planar loops of open strings (coupling of probe to classical geometry) but does not include

- closed or open string production
  small acceleration
- back reaction of the probe on the geometry

Will check these corrections are small in our solutions pure CFT:

Let us start by analyzing the evolution for the pure CFT ($M_p \to \infty$, $g_{\mu\nu} \to \eta_{\mu\nu}$, $V(\Phi) \to 0$)

$$S \to -\frac{N}{\lambda^2}\int d^4x\, \Phi^4 \left(\underbrace{\sqrt{1-\frac{\lambda\dot\Phi^2}{\Phi^4}}}_{\sqrt{1-v^2_{proper}}} \mp 1\right)$$

$H = p_\Phi \dot\Phi - \mathcal{L} \to$

$$E = \frac{N}{\lambda^2}\Phi^4\left(\frac{1}{\sqrt{1-\frac{\lambda\dot\Phi^2}{\Phi^4}}} \mp 1\right)$$

$$= \frac{1}{g_{YM}^2}\left(\frac{1}{2}\dot\Phi^2 + \frac{1}{8}\frac{\lambda\dot\Phi^4}{\Phi^4} + \cdots\right) + \frac{1}{\lambda g_{YM}^2}\Phi^4(1\mp1)$$

Solve for $\dot\Phi \to$

$$\dot\Phi = \frac{\Phi^2\sqrt{\lambda E(\lambda^2 E + 2N\Phi^4)}}{(\lambda^2 E + N\Phi^4)}$$

consider $\Phi \to 0$ (heading to origin).

For $\Phi^4 \ll \frac{\lambda^2 E}{N}$,

$$\dot\Phi \simeq \frac{\Phi^2}{\sqrt\lambda} \Rightarrow \Phi(t) \to \frac{\sqrt\lambda}{t}$$

at late times

- We approach the speed limit (speed of light on gravity side)
- From the CFT description, Φ slows down dramatically in its motion on moduli space

Background check:

a) acceleration

$$a_p \sim \frac{\lambda^{\frac{1}{4}}}{\phi} \frac{d}{dt}\left(\frac{\sqrt{\lambda}\,\dot\phi}{\phi^2}\right) << \frac{1}{\sqrt{\alpha'}} \quad \checkmark$$

b) probe approximation:

Our fast-moving brane carries a lot of energy and has its field lines squashed into a transversely spreading pancake.

To maintain probe approx on AdS:

$$\frac{1}{R} \gg E_p\, l_5^3 \Rightarrow \boxed{E < \frac{\phi^4}{g_s^2}}$$

$$E_p \sim E\left(\frac{R}{r}\right)^4$$

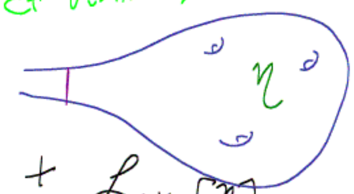
still have $v_p \sim 1$ phase in the window

$$\boxed{E < \frac{\phi^4}{g_s^2} < \frac{E\lambda^2}{N g_s^2}} \quad \checkmark$$

II. For Cosmology:

CFT coupled to 4d gravity + other sectors $\leftrightarrow$ throat attached to compactification

cf Verlinde, GKP



$$\mathcal{L} = \mathcal{L}^{CFT}_{DBI} + \mathcal{L}_{Einstein}[\mathcal{R}] + \mathcal{L}_{\substack{\text{other}\\ \text{sectors}}}[\eta]$$

$$- \phi^2 \mathcal{R} + \cdots \quad (\text{curvature corrections } \mathcal{R}/\phi^2\,)$$

$$- \frac{\phi^2(\partial\eta)^2}{M_*^2} + \cdots$$

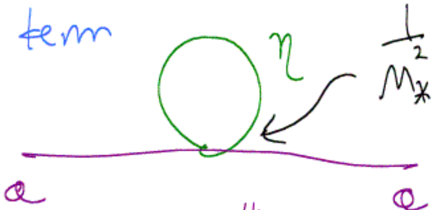
higher dimension operators coupling CFT to other sectors

Let us treat the combination in effective field theory, using the gravity side of AdS/CFT for the QFT $\leftrightarrow$ throat.

the higher dimension operators coupling the CFT to other sectors generate corrections to the probe action $S[\phi]$:

a) Potential $V(\phi)$:

e.g. mass term $\frac{1}{M_*^2}\phi^2(\partial\eta)^2$

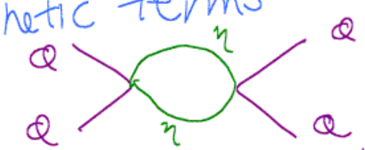


$$\rightarrow \quad m_\phi^2 = \frac{\Lambda_{uv}^4}{M_*^2}$$

*Leading effect (CFT alone had no such mass term)

b) DBI kinetic terms

e.g. $\dot\phi^4$ term



$$\rightarrow \frac{\dot\phi^4}{M_*^4}\log\left(\frac{\Lambda_{uv}}{M_*}\right) \ll \frac{\lambda\dot\phi^4}{g_{YM}^2\phi^4}$$

small ϕ

* subleading effect

So the general setup produces corrections (such as $m^2\phi^2$) to the potential while leaving intact the $\mathcal{L}_{DBI}$ higher derivative effects to good approximation.

ultimately this deforms AdS geometry, but the effect is subleading in the probe action.

Another way to say it:

deform $\mathcal{L} \rightarrow \mathcal{L} - m^2\phi^2$ by hand.

The $\mathcal{L}_{DBI}$ kinetic terms come from integrating out W's with $m_W^2 = \phi^2$.

The deformation will shift the W mass by less than $m_W^2 \rightarrow \phi^2 + m^2$

So get same results as before for $\phi > m$.

So the effective action for our system is the Dirac-Born-Infeld Lagrangian, coupled to gravity, plus corrections

$$S_T = \int d^4x \left[\frac{1}{2}\sqrt{-g}\,(M_p^2 + \phi^2)\,R + \mathcal{L}_0 + \cdots\right]$$

where

$$\mathcal{L}_0 = -\frac{1}{g_{YM}^2}\left(f(\phi)^{-1}\sqrt{1 + f(\phi)\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi} + V(\phi) \mp \underbrace{f(\phi)^{-1}}_{\{D3B,\ \overline{D3B}\}} \right)$$

where

$$f(\phi) = \frac{\lambda}{\phi^4} \text{ for } AdS_5$$

The fine print:

This action includes loops of open strings on the brane but no handles (bulk closed string loops); it is valid for arbitrary velocity ($v_{proper} \leq 1$) but small proper acceleration $\ll m_s^2$

The $(\cdots)$ includes corrections to the existing terms which scale like powers of $\mathcal{R}/\phi^2$.

As long as particle & string production is suppressed, perturbations don't grow, and the $(\cdots)$ are small in a given solution, this action S_T governs the dynamics. We will study the evolution using S_T and check for self-consistency

Now let us consider the solutions of the theory coupled to gravity

→ cosmological applications

Some results:

1) "slow" roll of $\Phi \to 0$ permits inflation away from the usual $\Phi \gg M_p$, $\left(\frac{V'}{V}\right)^2 M_p^2 \ll 0$, $M_p^2 \frac{V''}{V} \ll 0$ regime

In particular, the kinetic mechanism for slow roll, natural (and as $\Phi \to 0$ unavoidable!) for 3Bs and $\overline{3B}$s in warped throats, seems to ameliorate the problems highlighted in KKLMMT (Trivedi talk)

2) Motion on moduli space gets stuck near origin if we start heading that way

3) We find scale factor $a(t) \propto t^{\frac{2}{3}}$ (like for matter domination) for $V = V_4 \Phi^4 > 0$. (cf tachyon matter.)

In the above solutions, particle production & density perturbations don't cause large back reaction.

More details:

plug in metric ansatz

$$ds^2 = -dt^2 + a(t)^2 dx^2$$

and again $\phi = \phi(t)$

Let $\gamma^{-1} = \sqrt{1 - \frac{\lambda \dot\phi^2}{\phi^4}}$

Then $\delta S \Rightarrow$ stress-energy tensor $\rightarrow$

$$g_{YM}^2 \rho = \frac{\phi^4}{\lambda(\gamma^{-1})} + \left(V \mp \frac{\phi^4}{\lambda}\right) = \frac{1}{2}\dot\phi^2 + V + \cdots$$

$$g_{YM}^2 p = -\gamma^{-1}\frac{\phi^4}{\lambda} - \left(V \mp \frac{\phi^4}{\lambda}\right) = \frac{1}{2}\dot\phi^2 - V + \cdots$$

We will find solutions with $\phi \to \frac{\sqrt{\lambda}}{t} \Rightarrow \gamma^{-1} \to 0$
$\Rightarrow$ far from $\rho = \frac{1}{2}\dot\phi^2 + V$

$$3H^2 = \frac{1}{M_p^2}\rho$$

$$2\frac{\ddot a}{a} + H^2 = -\frac{1}{M_p^2} p$$

$$\ddot\phi + \frac{3f'}{2f}\dot\phi^2 - \frac{f'}{f^2} + 3H\gamma^{-2}\dot\phi + \left(V' + \frac{f'}{f^2}\right)\gamma^{-3} = 0$$

where $f = \frac{\lambda}{\phi^4}$

Note since $\gamma^{-1} \sim \sqrt{1 - v_p^2} \to 0$
the potential terms (and conformal coupling $R\phi^2$) do not affect ϕ evolution much.

A useful way to solve these eqns is the Hamilton Jacobi method

$$\frac{d}{dt}\left(3H^2 = \frac{1}{M_p^2}\rho\right)$$

$$\Rightarrow \quad 6HH'\dot{\Phi} = -\frac{1}{M_p^2 g_s}\frac{3H\dot{\Phi}^2}{\gamma}$$

$$\dot{\Phi} = -2M_p^2 g_s \gamma H' \qquad \sqrt{1-\frac{\lambda\dot{\Phi}^2}{\Phi^4}}$$

$$\Downarrow$$

$$\dot{\Phi} = \frac{-2H'}{\sqrt{\frac{1}{M_p^4 g_s^2}+\frac{4\lambda}{\Phi^4}H'^2}} \longrightarrow \text{plug into } 3H^2 = \frac{1}{M_p^2}\rho$$

$$\Rightarrow \quad V = 3M_p^2 g_s H^2 - M_p^2 g_s\frac{\Phi^4}{\lambda}\sqrt{\frac{1}{M_p^2 g_s}+4\frac{\lambda}{\Phi^4}H'^2} \pm \frac{\Phi^4}{\lambda}$$

Given V, solve for $H[\Phi]$, then for $\Phi(t)$

$H[\Phi(t)] = \frac{\dot{a}}{a} \rightarrow$ solve for $a(t)$

$\longrightarrow$ Analyze the eqns of motion for different regimes of V_0, V_2, V_4

$V_0 = 0$ (i.e. subdominant):

$$H = h_1\Phi + \dots$$

(We are interested in $\Phi \to 0$ regime)

Hamilton-Jacobi equation becomes

$$V_2\Phi^2 = \left(3h_1^2 - \frac{2h_1}{\sqrt{\lambda}}\right)M_p^2 g_s \Phi^2 + \mathcal{O}(\Phi^4)$$

$$\Rightarrow \quad h_1 = \frac{1}{3\sqrt{\lambda}}\left(1+\sqrt{1+3\frac{V_2\lambda}{M_p^2 g_s}}\right)$$

$$\dot{\Phi} = \frac{-2H'}{\sqrt{\frac{1}{M_p^2 g_s}+4\frac{\lambda H'^2}{\Phi^4}}} = \frac{-2h_1\Phi^2}{\sqrt{\frac{\Phi^4}{M_p^4 g_s^2}+4\lambda h_1^2}}$$

For $\mathcal{Q}^4 < 4\lambda h_1^2 g_s^2 M_p^4$

$$\dot{\mathcal{Q}} \to -\frac{1}{\sqrt{\lambda}}\mathcal{Q}^2 \Rightarrow \mathcal{Q} \to \frac{\sqrt{\lambda}}{t}$$

(speed limit)

Then $H = \frac{\dot{a}}{a} = h_1 \mathcal{Q} = h_1 \frac{\sqrt{\lambda}}{t}$

$$\Rightarrow a(t) \to a_0 t^{h_1\sqrt{\lambda}}$$

$$h_1 = \frac{1}{3\sqrt{\lambda}}\left(1+\sqrt{1+3\frac{V_2\lambda}{M_p^2 g_s}}\right)$$

① $V_2 = 0$ $\quad h_1 = \frac{2}{3\sqrt{\lambda}}$ $\quad h_1\sqrt{\lambda} = \frac{2}{3}$

$$a(t) = a_0 t^{2/3}$$

It is as if we have dust $\left(w = \frac{p}{\rho} = 0\right)$ even though we have no massive matter

cf tachyon matter

② $V_2 \neq 0$ Look for inflation:

$\ddot{a} > 0 \Leftrightarrow h_1\sqrt{\lambda} > 1$

From our above solution for h_1

$$\Rightarrow \boxed{\frac{V_2\lambda}{g_s M_p^2} > 1 \text{ for acceleration}} \;(*)$$

- No need for flat potential for slow roll
- $\mathcal{Q} \ll M_p$ consistent with inflation

Because of the back reaction at the far IR end of the throat, we do not have (controlled) eternal acceleration.

$$N_e = \log \frac{a(t_f)}{a(t_0)} = h_1\sqrt{\lambda}\,\log\left(\frac{t_f}{t_0}\right)$$

$$N_e \gg 1 \iff h_1\sqrt{\lambda} \gg 1$$
$$\iff V_2 \gg \frac{g_s M_p^3}{\lambda}$$

Another accelerating phase:

$$V_2 = 0$$

$$V_4 = -\frac{1}{\lambda}\left(\sqrt{1+36\lambda h_3^2 (g_s^2 M_p^4)} + 1\right)$$

$$V_6 = 3h_3^2 g_s M_p^2$$

$$a \propto t \quad \text{at late times}$$

$$a = a_0\, e^{-\frac{1}{t^2}\frac{\left(\frac{1}{g_s M_p^2} + 36\lambda h_3^2\right)^{3/2}}{2^4 3^3 h_3^2}}$$

acceleration $\rightarrow$ steady state

Self-consistency:

1) $R\phi^2$ coupling is subdominant if $\phi \ll M_p$:

gravity eqn of motion:

$$\mathcal{L} = (\phi^2 + M_p^2)\, R + \cdots$$

scalar eqn of motion:

$$\ddot{\phi} + \frac{3f'}{2f}\dot{\phi}^2 - \frac{f'}{f^2} + 3H\gamma^{-2}\dot{\phi} + \left(V' + \frac{f'}{f^2}\right)\gamma^{-3} = 0$$

where $f = \frac{\lambda}{\phi^4}$

$R\phi$ appears here multiplied by $\gamma^{-3} \to 0$

(An $R\phi^2$ coupling kills ordinary inflation)

R/ϕ^2 corrections:

- In inflationary phase, these scale like $\frac{H^2}{\phi^2} = \frac{h_1^2 \lambda / t^2}{\lambda / t^2} = h_1^2$

Recall

$$h_1 = \frac{1}{3\sqrt{\lambda}}\left(1 + \sqrt{1 + 3\frac{V_2 \lambda}{M_p^2 g_s}}\right) \sim \sqrt{\frac{V_2}{M_p^2 g_s}}$$

So we can arrange $h_1 \ll 1$ by taking $V_2 \ll M_p^2 g_s$ ✓

- In noninflationary phases, $H \sim \frac{\dot{a}}{a} \leq \mathcal{O}\left(\frac{1}{t}\right) \Rightarrow \frac{H^2}{\phi^2} \ll 1$ ✓

2) Perturbations

- Acceleration small so little closed string radiation off the brane
- particle production? suppressed: t-dependent masses

$$\frac{\dot m_W}{m_W^2} = \frac{\dot\varphi}{\varphi^2} \sim \frac{1}{\sqrt{\lambda}} \ll 1$$

for $\lambda \gg 1$

$$\frac{\dot m_s}{m_s^2} \sim \frac{\dot\varphi/\lambda^{1/4}}{\varphi^2/\sqrt{\lambda}} \sim \frac{1}{\lambda^{1/4}} \ll 1$$

for $\lambda \gg 1$

(string oscillators on or near the brane

$\omega \sim \gamma\sqrt{N_{osc} m_s(\varphi)^2 + \vec{p}^{\,2}}$

$\dot\omega/\omega^2 \ll 1$

perturbations of φ:

$\varphi \to \varphi + \delta\varphi$ e.g. in inflationary phase:

$$\delta\ddot\varphi + \left(\frac{6}{t} + 3H\right)\delta\dot\varphi + \left(\frac{6}{t} + c\lambda\frac{k^2}{M_p^2 a^2 V_2 t^4} + \frac{H}{t}\right)\delta\varphi = 0$$

For $\frac{k^2}{a^2} \leq M_p^2$

$\to \leq \frac{1}{V_2 t^4}$

$\exists$ modes and t periods for which this is subdominant to

$$m_{\delta\varphi}^2(t) \sim \frac{H}{t} = \frac{h_1\sqrt{\lambda}}{t^2}$$

$$\frac{\dot{m}_{\delta\varphi}}{m_{\delta\varphi}^2} \sim \frac{\sqrt{\frac{h_1\sqrt{\lambda}}{t^2}}}{\frac{h_1\sqrt{\lambda}}{\cancel{t^2}}} \sim \frac{1}{\sqrt{h_1\sqrt{\lambda}}} \ll 1$$

so $\delta\varphi$ particle production is suppressed

Solution when $\frac{H}{t}$ dominates (late enough t):

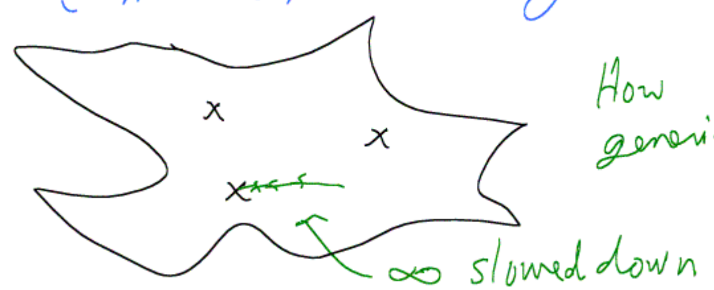
$$\delta\varphi_1 \propto t^{-2}, \quad \delta\varphi_2 \propto t^{-3}$$

so perturbations do not grow (and continue to shrink after horizon exit)

Many open questions:

1) Complete model of inflation?
   - generation of $V_2\varphi^2$ in explicit string compactification?
   - exit, reheating in cut off throat?
   - Spectrum of density perturbations

2) motion of scalar fields ("moduli") in string cosmology



How generic?

3) Other finite distance singularities in moduli space

- $\mathcal{N}=2$ SYM, conifold

- weak coupling case of $\mathcal{N}=4$ SYM

Sum up the $\alpha' \to \alpha'_{singularity}$ corrections?

In general, when is there a speed limit?

$\Leftrightarrow$ geometric interpretation of internal scalar field variable