The Landscape of Intersecting Brane Models

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Outline

1. Motivation – predictions from string theory

2. Intersecting brane models

3. Proof of finiteness, estimates for $\mathcal{N}_G$

4. Distribution of generations

5. Conclusions

1. Motivation – predictions from string theory

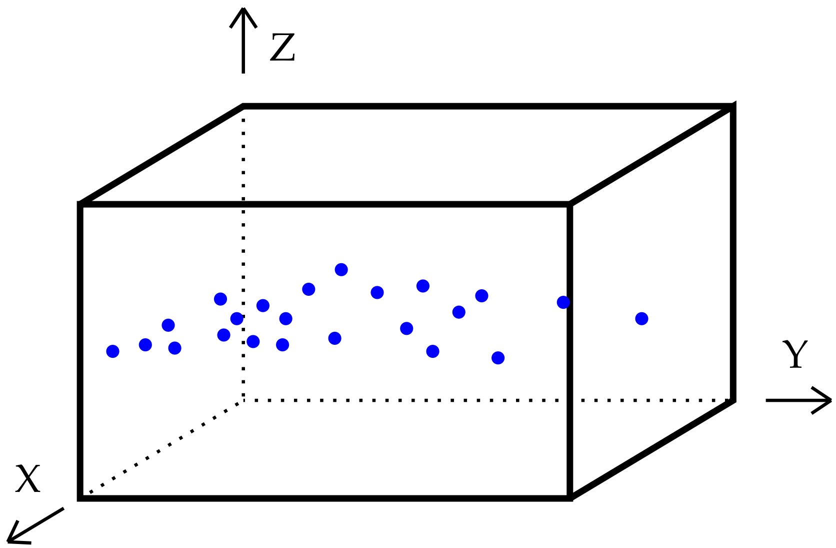
- No nonpert. definition of string theory in general background (R-R fluxes, de Sitter, background independence)

⇒ No dynamics, no hints for vacuum selection

- Extent of landscape of vacua undetermined
  - # Calabi-Yau, non-Kähler, nongeometric fluxes, ...
  - Swampland versus landscape: which 4D theories lift?

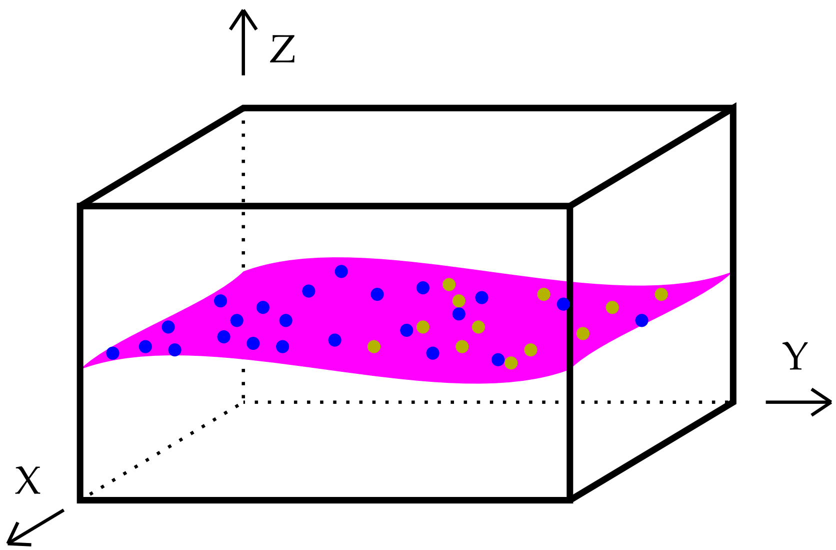
⇒ "Representative sample" currently out of reach

- Program: find “correlations in corners”



– Find family of vacua with computable EFT parameters X, Y, Z

- Program: find "correlations in corners"

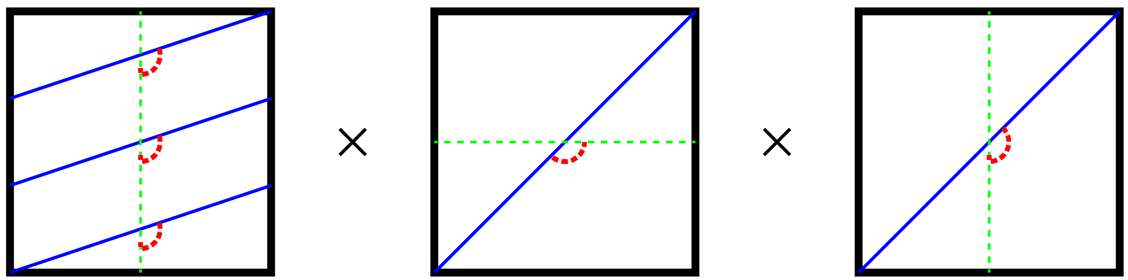


– Find family of vacua with computable EFT parameters X, Y, Z

– Find constraints/correlations

– Look under all lamp posts, compare

2. Intersecting brane models on a toroidal orientifold



- IIA: Factorizable D6-branes with windings (n_i, m_i)
- IIB: Magnetized D9-branes with fluxes
- U(N) on multiple branes
- Chiral fermions on intersecting branes

$$I = \prod_{i=1}^{3} (n_i \hat{m}_i - \hat{n}_i m_i)$$

Most studied orientifold: $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$

$\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold action

$$\begin{aligned}(z_1, z_2, z_3) &\rightarrow (-z_1, -z_2, z_3) \\ (z_1, z_2, z_3) &\rightarrow (z_1, -z_2, -z_3)\end{aligned}$$

Orientifold $z_i \rightarrow \bar{z}_i$

- Used to construct 3-generation models
  containing standard model gauge group
  [Cvetic/Shiu/Uranga, Cvetic/Li/Liu, Cremades/Ibanez/Marchesano]
- Moduli not stabilized
  but promising arena for real flux compactification
  [Marchesano/Shiu, . . .]

Questions to address:

- How many intersecting brane models on fixed orientifold?
  finite/infinite?

- Distribution of gauge group G, # generations, Yukawas, ...
  correlations/constraints?

Previous statistical analysis:

[Blumenhagen/Gmeiner/Honecker/Lust/Weigand]

- Based on one-year computer search
- Suggested gauge group, # generations are fairly independent
- Suggests $\sim 10^{-9}$ of models have $SU(3) \times SU(2) \times U(1)$
  and 3 generations of chiral fermions

Outline of basic results

Technical advances in `hep-th/0606109`:

- Analytic proof of finiteness

- Focus on models *containing* G
  w/ possible extra ("hidden sector") branes

Physics results

- Analytic estimates for $\mathcal{N}_G = \#$ models containing G

- G, # generations essentially independent
  (modulo some number theoretic features,
  + bound on G, large $G \Rightarrow$ bounds smaller # generations)

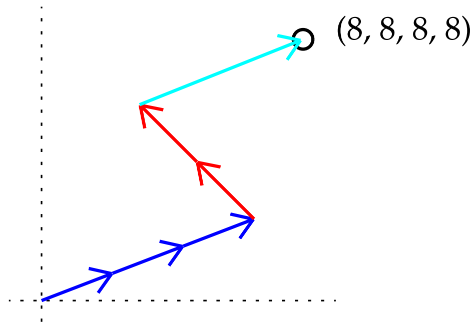
3. Proof of finiteness, estimates for $\mathcal{N}_G$

Finding SUSY IBM models $\sim$ partition problem

$$\sum_a (P_a, Q_a, R_a, S_a) \quad = \quad (T, T, T, T) = (8, 8, 8, 8)$$

$$P = n_1 n_2 n_3 \qquad\qquad Q = -n_1 m_2 m_3$$

$$R = -m_1 n_2 m_3 \qquad\qquad S = -m_1 m_2 n_3$$



SUSY conditions (when $P, Q, R, S > 0$):

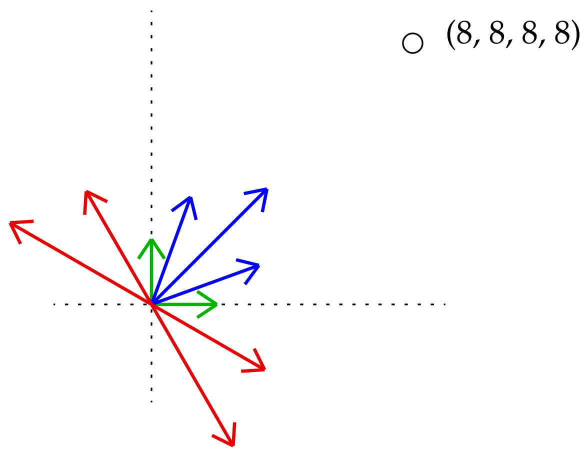
$$\frac{1}{P} + \frac{j}{Q} + \frac{k}{R} + \frac{l}{S} = 0, \qquad P + \frac{1}{j}Q + \frac{1}{k}R + \frac{1}{l}S > 0\,.$$

Why proof of finiteness is nontrivial

Can have negative tadpoles, *e.g.*

$$n = (3, 1, -1), \qquad m = (1, 1, -1)$$

$$\Rightarrow (P, Q, R, S) = (-3, 3, 1, 1)$$



3 kinds of branes (up to S_4 symmetry):

A: $- + + +$, **B**: $+ + 0\ 0$, **C**: $+ 0\ 0\ 0$

Proof of finiteness: example piece of proof

Take **A**-brane with $S_a < 0$,

$$\begin{aligned} P_a + \frac{1}{j} Q_a + \frac{1}{k} R_a + \frac{1}{l} S_a &= P_a + \frac{Q_a}{j} + \frac{R_a}{k} - \frac{1}{\frac{1}{P_a} + \frac{j}{Q_a} + \frac{k}{R_a}} \\ &\geq \frac{2}{3} \left(P_a + \frac{1}{j} Q_a + \frac{1}{k} R_a \right) \end{aligned}$$

using

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{3}{x+y+z}$$

So for a general configuration positive tadpoles give

$$\sum_{a+} P_a + \frac{1}{j} \sum_{a+} Q_a + \frac{1}{k} \sum_{a+} R_a + \frac{1}{l} \sum_{a+} S_a \leq \frac{3}{2} T (1 + \frac{1}{j} + \frac{1}{k} + \frac{1}{l})$$

implies, assuming wlog $1 \leq j \leq k \leq l$

$$\sum_{a+} P_a \leq 6T \,.$$

So negative P's at most sum to $\mathcal{O}(T)$; similar arg for Q.

General results on scaling

$$
\begin{aligned}
1\text{A} \quad &: \quad \sim (-T^3, T, T, T) \\
2\text{A} \quad &: \quad \sim \left.\begin{array}{c} (-T^5, T^3, T, T) \\ (T^5, -T^3, T, T) \end{array}\right\} \to (-T^3, T, T, T)
\end{aligned}
$$

Worst scaling $\sim (T^5, T^3, T, T)$

$\Rightarrow$ Finite number of SUSY configurations

Also: can estimate numbers of configurations with fixed gauge group

Counting configurations with gauge subgroup G

- Expect $>\sim \mathcal{O}(e^T)$ solutions to partition problem
- Look at configurations given G undersaturating tadpole
  $\Rightarrow$ polynomial number of solutions

e.g. $U(N)$ from N **A**-branes (allow "hidden sector" **B**, **C**'s)

$$\sim (-T^3/N^3, T/N, T/N, T/N)$$

$$\mathcal{N}_{NA} \sim \frac{\pi^6 T^3}{6^4(\zeta(3))^3 N^3}$$

e.g. $U(N) \times U(M)$ from $N\mathbf{A} + M\mathbf{B}$

$$A \sim (-T^3/N^3, T/N, T/N, T/N), \qquad B \sim (T^3/(N^2 M), T/M)$$

$$\mathcal{N}_{NA+MB} \sim \mathcal{O}(\frac{T^7}{N^5 M^2})$$

Generally, $U(N)$ factor suppressed by $1/N^\nu, \nu \geq 2$.

e.g., expect

$$\mathcal{N}_{aa} \sim (T/N)^6, \quad \mathcal{N}_{ab} \sim (T/N)^7, \quad \mathcal{N}_{bb} \sim (T/N)^4,$$

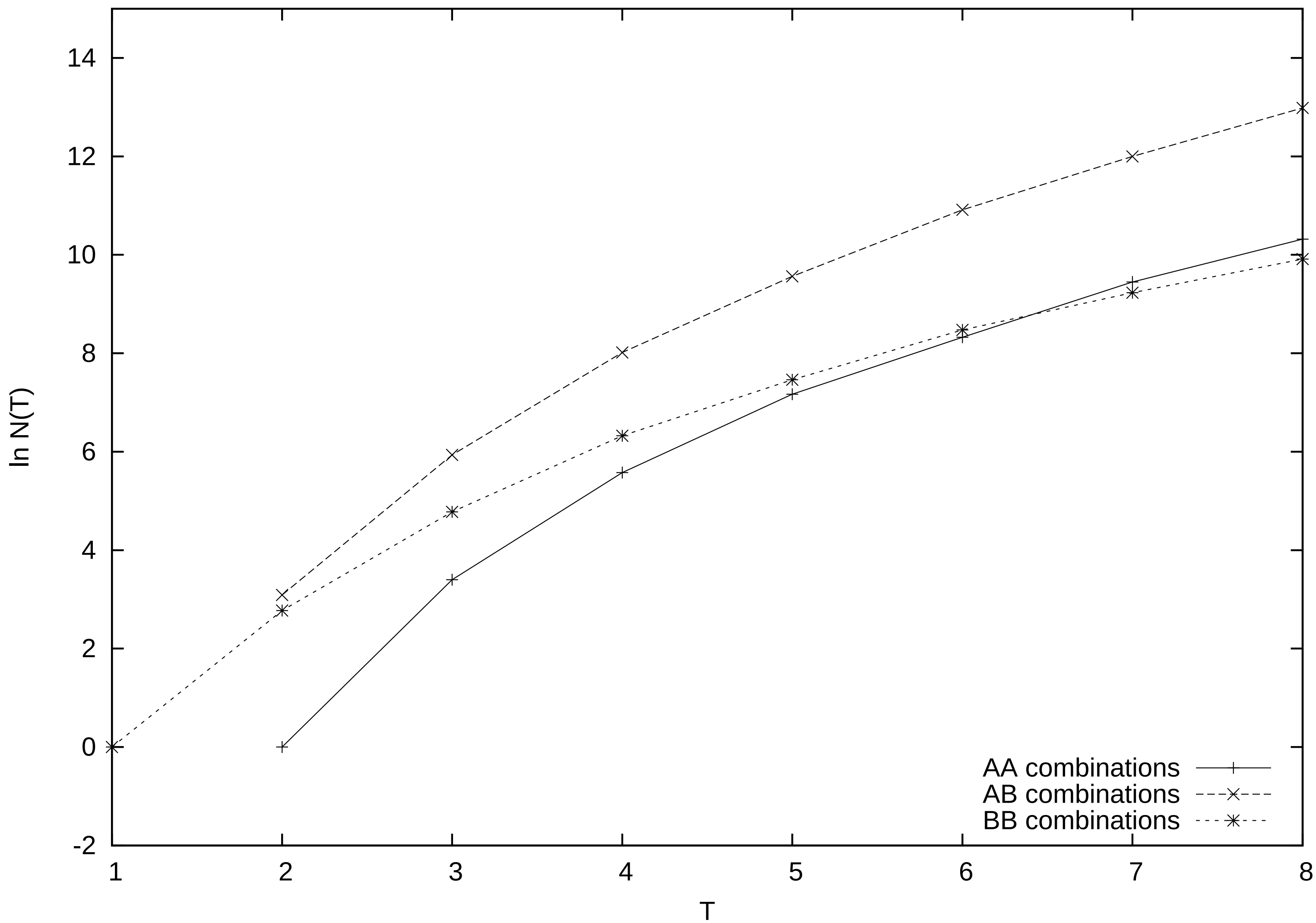
at $T = 8$ [$U(1) \times U(1)$]

$$\begin{aligned} \mathcal{N}_{aa}(8) &= 30,255 \\ \mathcal{N}_{ab}(8) &= 434,775 \\ \mathcal{N}_{bb}(8) &= 20,244 \end{aligned}$$

at $T = 4$ [$U(2) \times U(2)$ at $T = 8$]

$$\begin{aligned} \mathcal{N}_{aa}(4) &= 264 \\ \mathcal{N}_{ab}(4) &= 3,029 \\ \mathcal{N}_{bb}(4) &= 558 \end{aligned}$$

Growth as expected (contains extra logs)



(Log of) number of type **AA**, **AB**, **BB** branes for varying T

Estimates for $\mathcal{N}_G$

- Analytic estimates for # configurations with gauge subgroup G
- Efficient algorithms to scan all valid configurations
- Expect

$$\begin{aligned} \mathcal{N}_{SU(3)\times SU(2)\times U(1)} &\sim 10^{10} \\ \mathcal{N}_{SU(4)\times SU(2)\times SU(2)} &\sim 10^{7} \end{aligned}$$

- Each configuration admits $\sim e^{\Delta T}$ hidden sectors

4. Distribution of generation numbers

Generations of chiral fermions between $(n, m), (\hat{n}, \hat{m})$ branes from

$$I = \prod_{i=1}^{3} (n_i \hat{m}_i - \hat{n}_i m_i)$$

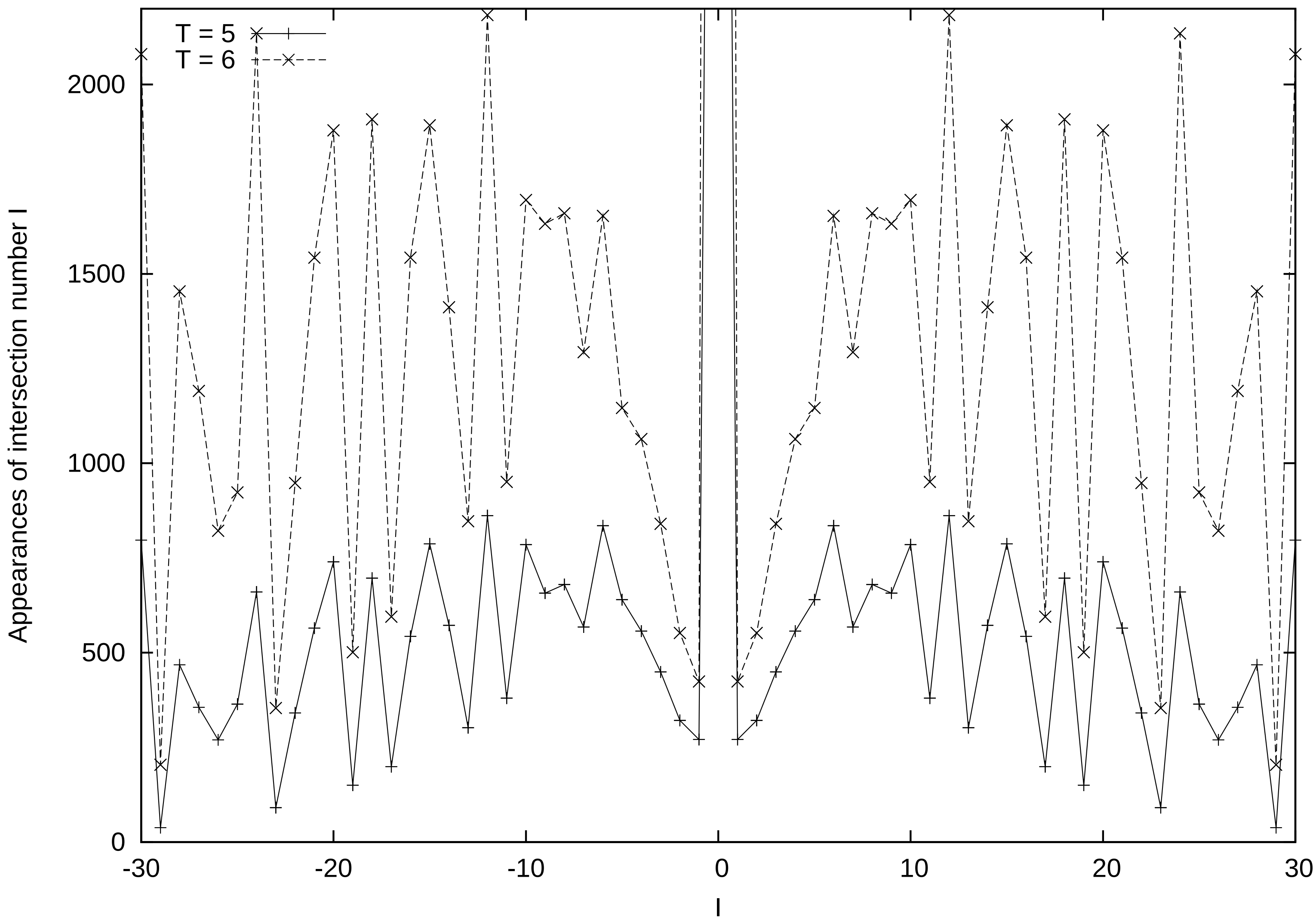
Look at distribution of I

- Intersection #'s for *e.g.* $U(N) \times U(N) \sim 10^3/N^5$ w/o extra $\mathbf{A}$'s
  generally expect $\sim (T/N)^7$

- Mild enhancement of composite I's

- Intersection numbers distributed quite independently

  e.g. for $\mathbf{AB\hat{B}}$, $T = 3$, $\quad H(I_{ab}) = H(I_{a\hat{b}}) = 4.553$
  mutual entropy $2H(I_{ab}) - H(I_{ab}, I_{a\hat{b}}) \sim 0.085$



Frequencies of small intersection numbers

Specific models

- For 3-generation $SU(3) \times SU(2) \times U(1), SU(4) \times SU(2) \times SU(2)$
  expect $\mathcal{O}(10-100)$ models

- Parity constraint from image branes $I_{xy} + I_{xy'} \equiv 0(\mathrm{mod}\ 2)$
  $\Rightarrow$ odd generations only w/ **C** branes, discrete B/skew tori

- $\mathcal{O}(10)$ models found in previous constructions
  [Cvetic/Shiu/Uranga, Cvetic/Li/Liu, Cremades/Ibanez/Marchesano]

- We identified 2 additional models:

$$4\mathbf{A} \quad : \quad n = (3, 1, -1), \qquad m = (1, 1, -1), \qquad (P, Q, R, S) = (-3, 3, 1, 1)$$

$$2\mathbf{C} \quad : \quad R = 1$$

$$2\mathbf{C} \quad : \quad S = 1$$

$+$ two distinct hidden sector $\mathbf{A} + \cdots$ combinations

5. Summary

- Proved finiteness for IBM on toroidal orientifold
- Estimates for numbers of models with fixed gauge group
- Analyzed generation numbers, no significant correlations
- No strong constraints on G, # generations
- Expect $\mathcal{O}(10 - 100)$ 3-generation $G_{\rm SM}$ models for this orientifold, found 2 new

Future directions

- Look at other orientifolds, CY
- Find larger families (10^9?) of 3-generation $G_{\rm SM}$ models
- Compute more detailed properties (Yukawas etc.), look for constraints
- Include fluxes, stabilize closed + open moduli
- Consider other corners (RCFT, heterotic, M-theory, etc. – SVP)